

Project1

TWW

2024-05-04

Exponential Distribution and Central Limit Theorem

Overview

In this project, we will investigate the exponential distribution in R and compare it with the Central Limit Theorem. We will simulate the distribution of averages of 40 exponentials and examine its properties.

Simulations

We will use the `rexp()` function in R to simulate the exponential distribution. The mean and standard deviation of the exponential distribution are both equal to $1/\lambda$, where λ is the rate parameter. We set $\lambda = 0.2$ for all simulations.

To simulate the distribution of averages of 40 exponentials, we will perform 1000 simulations:

```
set.seed(42)
lambda <- 0.2
n <- 40
num_sims <- 1000

sim_means <- replicate(num_sims, mean(rexp(n, lambda)))

#Sample Mean versus Theoretical Mean
#The theoretical mean of the exponential distribution is 1/lambda. Let's compare the sample mean from o

theoretical_mean <- 1/lambda
sample_mean <- mean(sim_means)

paste("Theoretical Mean:", theoretical_mean)

## [1] "Theoretical Mean: 5"

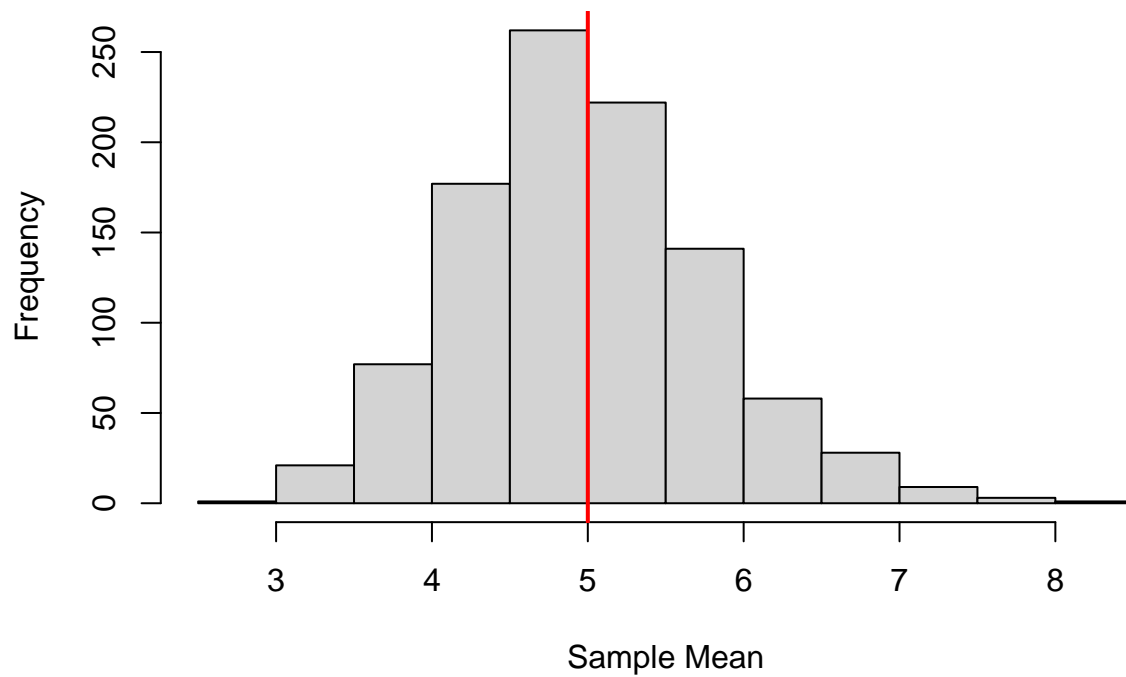
paste("Sample Mean:", sample_mean)

## [1] "Sample Mean: 4.98650831745453"

#We can visualize the distribution of sample means using a histogram:

hist(sim_means, main = "Distribution of Sample Means", xlab = "Sample Mean")
abline(v = theoretical_mean, col = "red", lwd = 2)
```

Distribution of Sample Means



#The red vertical line represents the theoretical mean.

#Sample Variance versus Theoretical Variance

#The theoretical variance of the exponential distribution is $(1/\lambda)^2$. Let's compare the sample var

```
theoretical_var <- (1/lambda)^2
```

```
sample_var <- var(sim_means)
```

```
paste("Theoretical Variance:", theoretical_var)
```

```
## [1] "Theoretical Variance: 25"
```

```
paste("Sample Variance:", sample_var)
```

```
## [1] "Sample Variance: 0.634440520668948"
```

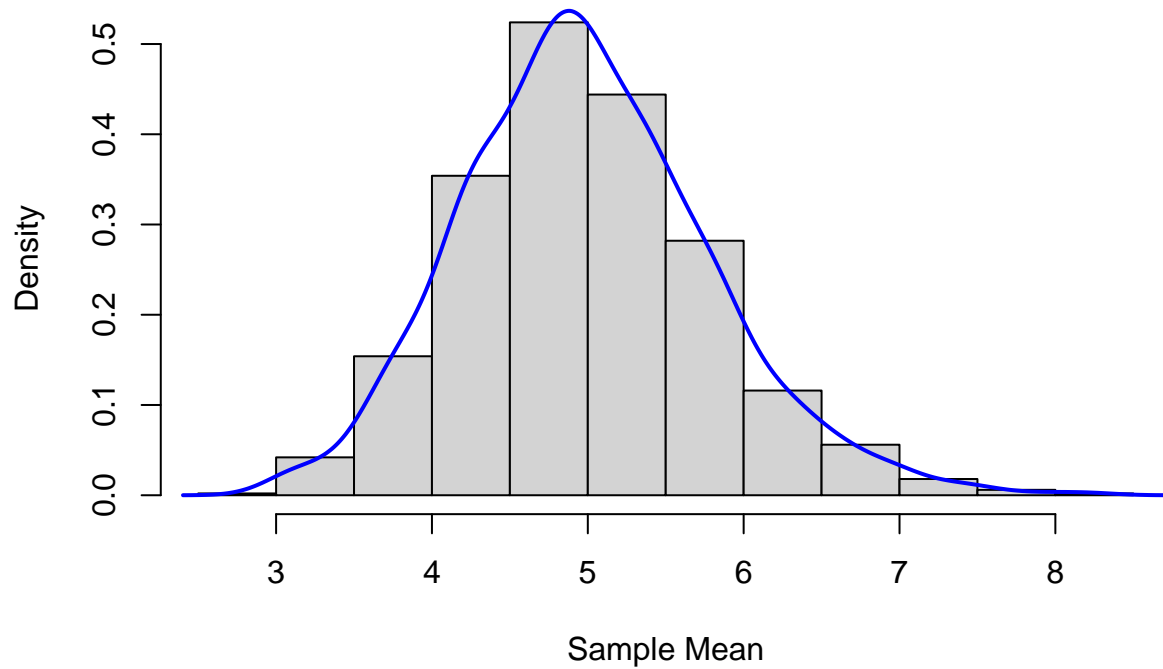
#Distribution

#To check if the distribution of sample means is approximately normal, we can create a histogram and a

```
hist(sim_means, main = "Distribution of Sample Means", xlab = "Sample Mean", prob = TRUE)
```

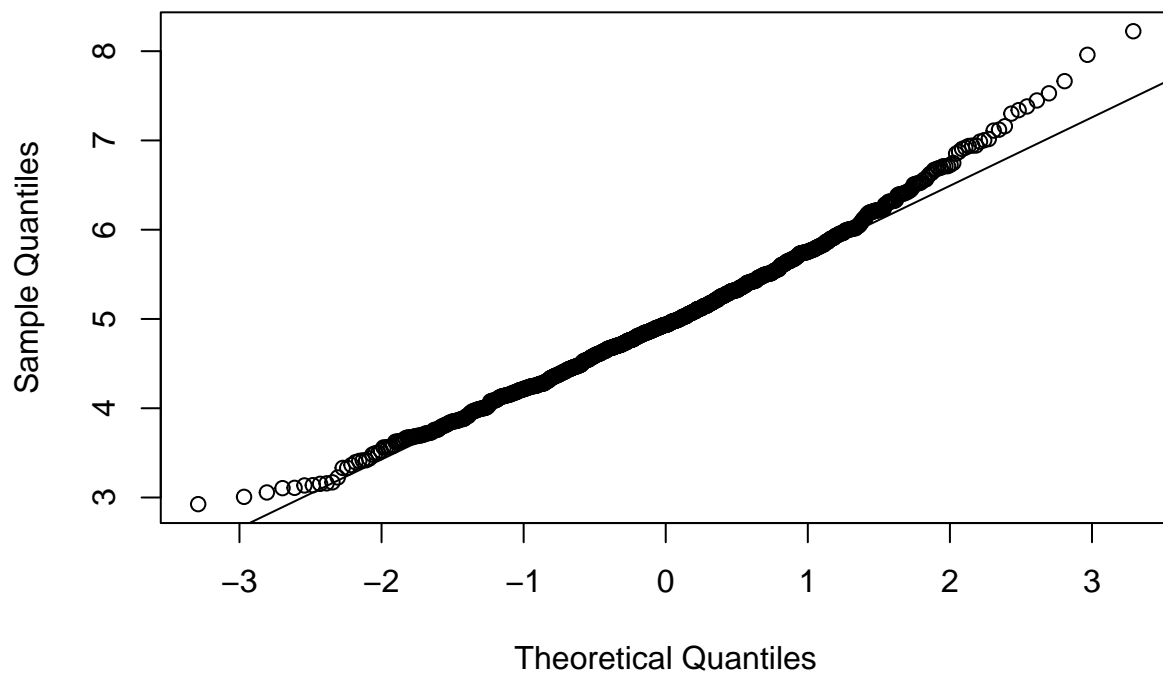
```
lines(density(sim_means), col = "blue", lwd = 2)
```

Distribution of Sample Means



```
qqnorm(sim_means)
qqline(sim_means)
```

Normal Q-Q Plot



#If the distribution is approximately normal, the histogram should resemble a bell curve, and the Q-Q plot should show points following a straight line.

#Tooth Growth Analysis

#Data Summary

#Let's load the ToothGrowth data and perform some basic exploratory data analysis:

```
data(ToothGrowth)
summary(ToothGrowth)
```

```
##      len      supp      dose
##  Min.   : 4.20   OJ:30   Min.    :0.500
## 1st Qu.:13.07   VC:30   1st Qu.:0.500
##  Median :19.25           Median :1.000
##   Mean  :18.81           Mean   :1.167
## 3rd Qu.:25.27           3rd Qu.:2.000
##   Max.  :33.90           Max.    :2.000
```

#Confidence Intervals and Hypothesis Tests

#We can use confidence intervals and hypothesis tests to compare tooth growth by supplement type (supp)

Compare tooth growth by supplement type

```
t.test(len ~ supp, data = ToothGrowth)
```

```
##
##  Welch Two Sample t-test
##
## data:  len by supp
## t = 1.9153, df = 55.309, p-value = 0.06063
## alternative hypothesis: true difference in means between group OJ and group VC is not equal to 0
## 95 percent confidence interval:
##  -0.1710156  7.5710156
## sample estimates:
## mean in group OJ mean in group VC
##      20.66333      16.96333
```

Compare tooth growth by dose

```
anova(lm(len ~ dose, data = ToothGrowth))
```

```
## Analysis of Variance Table
```

```
##
```

```
## Response: len
```

```
##      Df Sum Sq Mean Sq F value    Pr(>F)
## dose    1 2224.3  2224.30  105.06 1.233e-14 ***
## Residuals 58 1227.9    21.17
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

#Conclusions and Assumptions

#Based on the results of the confidence intervals and hypothesis tests, we can draw conclusions about t