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  - $R \circ S = \{xy \in \Sigma^* : x \in R \wedge y \in S\}$.
  - $\bar{R} = \{x \in \Sigma^* : x \notin R\}$.
  - $R' = \{x \in \Sigma^* : x' \in R\}$.
  - $R^i = \begin{cases} \{\varepsilon\}, & i = 0 \\ L^{i-1}L, & i > 0 \end{cases}$.
  - $R^* = \bigcup_{i=0}^{\infty} R^i$.
  - $R^+ = \bigcup_{i=1}^{\infty} R^i$.

The Pumping Lemma for Regular Sets

Let L be a regular set. Then there is a constant n such that if $z \in L$ and $|z| \geq n$, we may write $z = uvw$ in such a way that $|uv| \leq n$, $|v| \geq 1$, and for all $i \geq 0$, $uv^iw \in L$. Furthermore, n is no greater than the number of states of the smallest FA accepting L.

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  - $L_2 = \{0^i 1^i : i \geq 1\}$.
  - $L_3 = \{s : s = s^R\}$.
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