Computing The Least Solution

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- This strategy corresponds exactly to the Jacobi method in iterative solvers for linear systems.
- The general approach is called round-robin scheduling of equations.

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- Worklist-based algorithm:
  - Initialize worklist with all equations.
  - Initialize solution vector v to the vector of all $\perp$s.
  - While the worklist is not empty:
    - Pick an equation from the worklist.
    - Evaluate RHS of this equation with the current value of the solution vector and update the entry corresponding to LHS variable in the solution vector v.
    - Add all equations that use this variable in their RHS to the worklist.

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- You can show that this algorithm will compute the least solution to the system of equations.