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- Similarly, if $(D, \sqsubseteq, \vee)$ is a finite join semi-lattice, fixpoint theorem #2 applies even if some of the functions are $\vee$.

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- Do these ideas and techniques carry over to infinite sets?