Finite Automaton: Formal Definition

- A finite automaton (FA) is defined as the 5-tuple

$$M = (Q, \Sigma, \delta, q_0, F), \text{ where}$$

- Q is a finite set of states;
- $q_0 \in Q$ is the start state;
- $F \subseteq Q$ is the set of accepting states;
- Σ is the input alphabet; and
- δ is the *transition function* describing the behavior of the FA.

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- δ is the *transition function* describing the behavior of the FA.
  - A FA with $\delta: Q \times (\Sigma \cup \{\varepsilon\}) \to 2^Q$ is called a nondeterministic finite automaton with ϵ-moves (ϵ-NFA).
  - A FA with $\delta: Q \times \Sigma \to 2^Q$ is called a nondeterministic finite automaton (NFA).
  - A FA with $\delta: Q \times \Sigma \to Q$ is called a deterministic finite automaton (DFA).

The Finite Automaton Viewed As A Graph

- Given the FA $M = (Q, \Sigma, \delta, q_0, F)$, define the labeled directed graph $G(M) = \big(V(M), E(M)\big)$, where
  - $V(M) = Q$; and
  - For each transition $\delta(p, \sigma) = q$, $E(M)$ contains the directed edge (p, q) with label σ.

FA Recognize Regular Languages

- The language recognized by $M = (Q, \Sigma, \delta, q_0, F)$ (denoted $L(M)$) is the set of input strings $s \in \Sigma^*$ for which M, starting from q_0 and scanning the symbols of s in sequence order, transitions to some state $q' \in F$.
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- In the graph view, a computation of M on input string s corresponds to a directed path $q_0 \rightsquigarrow \cdots \rightsquigarrow q'$ in $G(M)$ such that the concatenation of the edge labels along the path is equal to s.
  - $s \in L(M) \equiv q' \in F$.

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- *Theorem*: Every regular language can be recognized by a finite automaton, and the language recognized by every finite automaton is regular.

Examples of Finite Automata

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