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 - $R \cap S = \{x \in \Sigma^* : x \in R \land x \in S\}.$
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 - $R \circ S = \{xy \in \Sigma^* : x \in R \land y \in S\}.$
 - $\overline{R} = \{x \in \Sigma^* : x \notin R\}.$
 - $R' = \{x \in \Sigma^* : x' \in R\}.$
 - $R^{i} = \begin{cases} \{\varepsilon\}, & i = 0 \\ L^{i-1}L, i > 0 \end{cases}$
 - $R^* = \bigcup_{i=0}^{\infty} R^i$.
 - $R^+ = \bigcup_{i=1}^{\infty} R^i$.

The Pumping Lemma for Regular Sets

Let L be a regular set. Then there is a constant n such that if $z \in L$ and $|z| \ge n$, we may write z = uvw in such a way that $|uv| \le n$, $|v| \ge 1$, and for all $i \ge 0$, $uv^iw \in L$. Furthermore, n is no greater than the number of states of the smallest FA accepting L.

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 - $L_2 = \{0^i 1^i : i \ge 1\}.$
 - $L_3 = \{s: s = s^R\}.$
 - $L_4 = \{0^i 1^i 2^i : i \ge 1\}.$
 - $L_5 = \{a^i b^j c^i d^j : i \ge 1, j \ge 1\}.$

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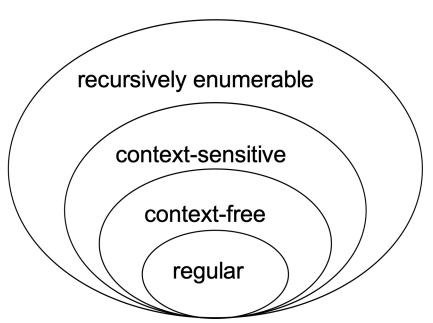
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