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# Solution to the motion of a delta manipulator with three degrees of freedom

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#### **ABSTRACT**

Based on the mechanical structure of a Delta manipulator with three degrees of freedom, the kinematic characteristics of the parallel manipulator were analyzed, and the vector relation between the static and moving platforms was given. The inverse kinematic function of the three-degree-of-freedom (3-DOF)parallel manipulator was deduced by the vector equation. The simulated forward kinematic solution in the moving platform was carried out by Pro/E modeling software and virtual prototype simulation software ADAMS, which is proved simple. These research contents provide a valuable reference for the adjustment and operation of 3-DOF Delta manipulator prototypes with respect to its dynamic performance, motion track planning, mechanical structure, system control, etc.

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#### **KEYWORDS**

Three degrees of freedom; parallel manipulator; kinematic simulation; Pro/E; ADAMS

#### 1. Introduction

High-speed Delta manipulator is a mechanism that achieves high-speed 3-DOF translation. It features flexible motion, compact structure and high rigidity with the driving device installed in the fixed bracket and the slave arm made of lightweight rod [1-3]. These advantages allow it to be widely used in high-speed sorting and packing in fields like pharmaceuticals, food and electronics [4]. As the Delta manipulator operates in a moving platform, the motion trajectory of the platform determines the rotation angle functions of the three driving joints in the static platform. If the 3-DOF parallel manipulator has to complete different jobs, the motion trails of the platform should be changed accordingly, and thus the three rotation angle functions should be effort fully re-calculated. In this paper, based on the mechanical structure of a Delta manipulator with three degrees of freedom, the kinematic characteristics of the parallel manipulator are analyzed, and the vector relation between the static and moving platforms is given. The kinematic inverse function of the three-degree-offreedom (3-DOF) parallel manipulator is deduced by the vector equation. The simulated forward kinematic solution in the moving platform is carried out by Pro/E modeling software and virtual prototype simulation software ADAMS, which is proved simple. These research contents provide a valuable reference for the adjustment and operation of 3-DOF Delta

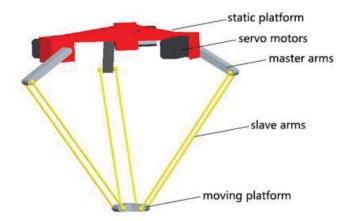


Figure 1. The delta manipulator with three degrees of freedom.

manipulator prototypes including its dynamic performance, motion track planning, mechanical structure and system control.

# 2. Delta manipulator analysis

# 2.1. Structural analysis

Delta manipulator is mainly composed of a static platform, a moving platform, 3 master arms, 3 slave arms, 3 servo motors and 3 reducers. The connectivity between the static and moving platforms is realized by the corresponding connection of the three axisymmetric evenly-distributed kinematic chains. The servo motor and reducer are installed in the static platform. Each chain is composed by the master arm and the slave arm. The slave arm is a closed-loop

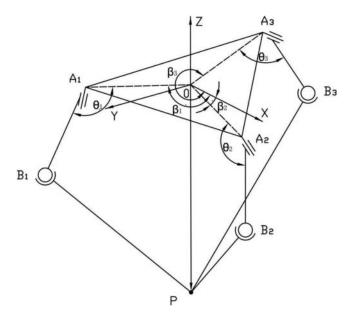


Figure 2. Schematic diagram of the parallel manipulator.

mechanism with four spherical pairs and rods, as shown in Fig. 1. These components enable the slave arm to fix to the master arm in series. With the starting angles and rotation angles of the three master arms as the input parameters, the moving platform moves under control in different trajectories. The moving platform is the output of the manipulator which can link to a variety of apparatuses as required, so as to fulfill corresponding functions.

# 2.2. Motion solution analysis

In the center of the static platform is a reference coordinate system O-xyz established, with the moving platform simplified to a point P to ease the mechanism calculation [4], as shown in Fig. 2. To seek the solutions to the motion, it is necessary to provide an input drive to determine the output component's motion parameters such as displacement, velocity and acceleration. The key is to determine the position of the output component because it is the basis for the calculation of speed and acceleration [5]. The position solution is divided into forward solution and backward solution. The former one is to solve the position and attitude of the moving platform with given position parameters of the three driving joint, and the latter one is the inverse [6].

### 3. Manipulator movement solution

#### 3.1. Backward kinematic solution analysis

The backward position solution is to give the position coordinate of the point P to determine the rotation angles  $\theta_1$ ,  $\theta_2$ , and  $\theta_3$  of the master arms 1, 2, and 3, respectively. The position vector  $\mathbf{r} = (\mathbf{x} \ \mathbf{y} \ \mathbf{z})^T$  of the end P can be expressed as:

$$\begin{cases}
\overrightarrow{OA}_1 + \overrightarrow{A}_1B_1 + \overrightarrow{B}_1P = \overrightarrow{OP} \\
\overrightarrow{OA}_2 + \overrightarrow{A}_2B_2 + \overrightarrow{B}_2P = \overrightarrow{OP} \\
\overrightarrow{OA}_3 + \overrightarrow{A}_3B_3 + \overrightarrow{B}_3P = \overrightarrow{OP}
\end{cases}$$
[1]

We set the radius of the static platform R, according to the geometrical relationship; we can get  $A_i$  point in the static platform coordinate system o-xyz position vector

$$\begin{cases} \overrightarrow{OA}_1 = (R\cos\beta_1 & R\sin\beta_1 & 0)^T \\ \overrightarrow{OA}_2 = (R\cos\beta_2 & R\sin\beta_2 & 0)^T \\ \overrightarrow{OA}_3 = (R\cos\beta_3 & R\sin\beta_3 & 0)^T \end{cases}$$
 [2]

The position vector of  $\overrightarrow{OB}_i$  can be acquired by the position relation of the active arm  $A_iB_i$  and Vector  $\overrightarrow{OA}_i$  in Fig. 2, and its position vector is

$$\begin{cases}
\overrightarrow{OB}_1 = \left[ (R + A_1 B_1 \sin \theta_1) \cos \beta_1 & (R + A_1 B_1 \sin \theta_1) \sin \beta_1 & A_1 B_1 \cos \theta_1 \right]^T \\
\overrightarrow{OB}_2 = \left[ (R + A_2 B_2 \sin \theta_2) \cos \beta_2 & (R + A_2 B_2 \sin \theta_2) \sin \beta_2 & A_2 B_2 \cos \theta_2 \right]^T \\
\overrightarrow{OB}_3 = \left[ (R + A_3 B_3 \sin \theta_3) \cos \beta_3 & (R + A_3 B_3 \sin \theta_3) \sin \beta_3 & A_3 B_3 \cos \theta_3 \right]^T
\end{cases}$$
[3]

We calculate the equation (1), and obtain

$$\begin{cases}
\overrightarrow{B_1P} = \overrightarrow{OP} - \overrightarrow{OB_1} \\
\overrightarrow{B_2P} = \overrightarrow{OP} - \overrightarrow{OB_2} \\
\overrightarrow{B_3P} = \overrightarrow{OP} - \overrightarrow{OB_3}
\end{cases} [4]$$

We substitute the equation (3) and the equation (4), and obtain

$$\begin{cases}
\overrightarrow{B_1P} = [x - (R + A_1B_1\sin\theta_1)\cos\beta_1 & y - (R + A_1B_1\sin\theta_1)\sin\beta_1 & z - A_1B_1\cos\theta_1]^T \\
\overrightarrow{B_2P} = [x - (R + A_2B_2\sin\theta_2)\cos\beta_2 & y - (R + A_2B_2\sin\theta_2)\sin\beta_2 & z - A_2B_2\cos\theta_2]^T \\
\overrightarrow{B_3P} = [x - (R + A_3B_3\sin\theta_3)\cos\beta_3 & y - (R + A_3B_3\sin\theta_3)\sin\beta_3 & z - A_3B_3\cos\theta_3]^T
\end{cases} [5]$$

Because the length of the vector  $\overrightarrow{B_iP}$ ,  $\overrightarrow{A_iB_i}$  is known, and the length is equal,  $\overrightarrow{B_iP} \mid = m, \mid \overrightarrow{A_iB_i} \mid = n$ , so

$$m^{2} = [x - (R + n\sin\theta_{i})\cos\beta_{i}]^{2} + [y - (R + n\sin\theta_{i})\sin\beta_{i}]^{2} + [z - n\cos\theta_{i}]^{2}$$
 [6]

The angle  $\beta_i$  of the vector  $\overrightarrow{OA}_i$  and the x-axis is also known to be  $\beta_1 = \frac{5}{6}\pi$ ,  $\beta_2 = \frac{1}{6}\pi$ ,  $\beta_3 = \frac{3}{2}\pi$ , We substitute the equation (6), and obtain

$$\begin{cases} m^{2} = \left[ x + (R + n\sin\theta_{1}) \frac{\sqrt{3}}{2} \right]^{2} + \left[ y - (R + n\sin\theta_{1}) \frac{1}{2} \right]^{2} + \left[ z - n\cos\theta_{1} \right]^{2} \\ m^{2} = \left[ x - (R + n\sin\theta_{2}) \frac{\sqrt{3}}{2} \right]^{2} + \left[ y - (R + n\sin\theta_{2}) \frac{1}{2} \right]^{2} + \left[ z - n\cos\theta_{2} \right]^{2} \\ m^{2} = x^{2} + \left[ y + (R + n\sin\theta_{3}) \right]^{2} + \left[ z - n\cos\theta_{3} \right]^{2} \end{cases}$$
 [7]

We simplify each of the equations in formula (7), and get the general form:

$$U_i e_i^2 + V_i e_i + W_i = 0 \quad (i = 1, 2, 3)$$
 [8]

In the equation (8),  $e_i = tan(\frac{1}{2}\theta_i)$ ,  $U_i$ ,  $V_i$ ,  $W_i$  can be represented by known quantities x, y, z, r, m, n, so the equation (8) can be solved

$$e_i = \frac{-V_i \pm \sqrt{V_i^2 - 4U_i W_i}}{2U_i}$$
 [9]

Therefore, the active arm corner  $\theta_i = 2 \arctan(e_i)$  can be acquired directly. The angular velocity of the active arm can be acquired by  $\theta_i$  the first derivative, and the angular acceleration of the active arm can be acquired by  $\theta_i$  the second derivative.

Table 1. The size of the major manipulator parameters (unit: mm).

Parameter	Static platform R	Master arm n	slave arm $m$	Moving platform r
Size	300	150	600	75

#### 3.2. Forward position solution

For a parallel manipulator, the forward kinematic solution is to give the rotation angles of three mater arms to determine the coordinate of the P-point in the reference frame of the static platform. Among the available approaches like polynomial interpolation, numerical solutions to nonlinear equations, the least square method, and ADAMS software, ADAMS is of less calculation difficulty than the others. The ADAMS-based solution is as follows [7].

Build a 3D manipulator model in Pro / e and import it into the ADAMS; edit and define the constraints of the manipulator; select the geometric center of the platform as the key point to add a 3D point drive; through simulation, the ADAMS/view object measurement function allows us to measure the angular velocity and angular acceleration of three master arms. The measured curves are converted to spline curves by using editing tools, through which we obtain the discrete data points. By applying the processed curve to the AKISPL function in ADAMS, we can add the angle-time driving function to the three joints and accordingly obtain the motion trajectory of the manipulators. The motion trajectory is edited to generate a spline function, which is the forward position solution of the moving platform [8].

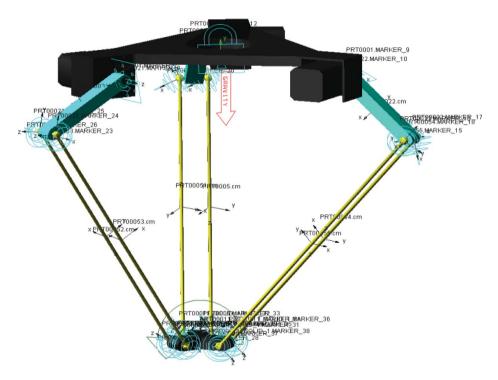


Figure 3. ADAMS manipulator model.

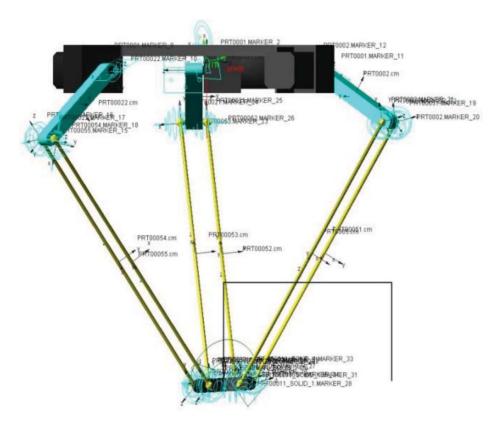


Figure 4. Kinematic simulation.

#### 3.2.1. Manipulator modeling

The 3D model of the static platform, master arms, slave arms and moving platforms is established and assembled in Pro/E [9]. The size of the parameters is shown in Table 1.

#### 3.2.2. ADAMS simulation solution

We import the Pro/E assembly model into the ADAMS software and set the attributes of the properties of the static platform, master arms, slave arms and moving platform, including name, location, material and color. Also, the constraints between these components are redefined [10]. The edited and redefined model is shown in Fig. 3.

A three-dimensional point drive is added to the center of the moving platform. The relation equation between the position and time of the point drive is:

$$\mathbf{x} = \begin{cases} 0 & 0 \le t < 2 \\ 50(t-2) & 2 \le t \le 4 \\ 100 & 4 < t \le 5 \end{cases}$$

$$\mathbf{z} = \begin{cases} 40t & 0 \le t < 2 \\ 80 & 2 \le t \le 4 \\ 400 - 80t & 4 < t \le 5 \end{cases}$$

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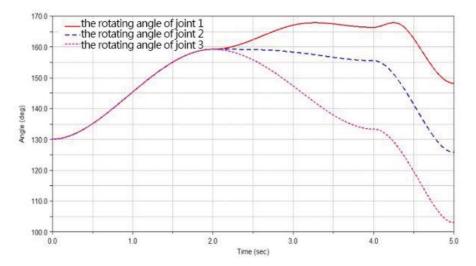
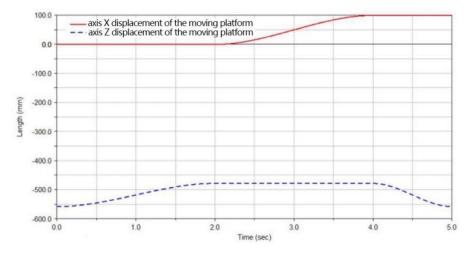


Figure 5. Simulation curves of the driving joint's rotation angle and displacement.

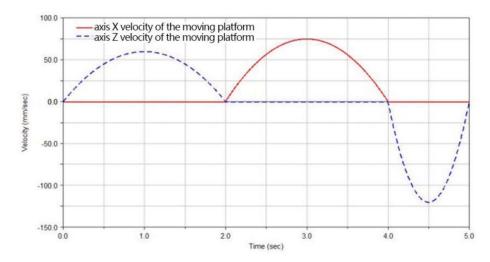
We simulate the manipulator for 5s and in 1000 steps. The center point P moves in desired trajectories, as shown in Fig. 4. The ADAMS / view object measurement function is used to measure the rotation angle of the three master arms' joints [11-12], and the corresponding curves are shown in Fig. 5.

After nullifying the drive and add in new drive to the three joints, we re-simulate the manipulator for 5s and in 1000 steps to obtain a new motion trajectory of the center point of the moving platform [13-16]. By measuring the position of the moving platform centers, we obtain the motion trajectory curve of the center point of the moving platform along x axis and z axis, with time as the x coordinate and displacement as the y coordinate. This curve is then converted to a spline curve, which is the forward position solution of the moving platform [17–20].

The re-simulation result shows that the motion trajectory of the point P coincides with Fig. 4. Therefore, our method is correct and feasible in finding the forward solution to the manipulator in motion.



**Figure 6.** The coordinate curve of the center position of the moving platform.



**Figure 7.** The speed curve of the center point of the moving platform.

#### 3.3. Solution to the speed of the moving platform

By taking the first-order derivative of the positions of the moving center points (Fig. 6), we obtain the moving velocity curve and show it in Fig. 7. This curve is edited into a spline function, which is the forward speed solution of the moving platform.

#### 4. Conclusions

Based on the analysis of the Delta manipulator motion model, the backward kinematic equation of the moving platform is obtained. After 3D Pro/E modeling and importing the model into the ADAMS software, we define the manipulator's attributes and constraints before conducting motion simulation. After giving the motion trajectory drive of the moving platform, we acquire the curve map of the three master arm joints' rotation angle. After editing and processing the curve into the driving function, we redrive the three master arms' joints, through which the moving platform's forward motion trajectory and velocity curves are obtained in a simple way. The coincidence between the two moving trajectories proves the feasibility and reliability of the ADAMS-based simulation of motion solution-seeking, and reproduces the kinematic performance of the manipulator in the direct form of curve maps. Our method provides a methodological reference for adjusting and operating the prototype and optimizing its motion trajectories. From the perspective of strategic control, our approach acts as an important basis for the optimization of system dynamics not only in a time-saving way but also reducing the research and development cost of Delta manipulator.

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