Lab 2: KMeans with CUDA:

Performance analysis and detailed approach

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Abstract

This report presents the implementation and performance analysis of the KMeans clustering algorithm using both CPU and GPU parallelism. The goal of this lab is to explore the benefits of GPU programming through the use of CUDA and Thrust libraries. The KMeans algorithm, commonly used for unsupervised machine learning, was implemented in four variations: a sequential CPU version, a basic CUDA version, an optimized CUDA version utilizing shared memory, and a high-level parallel implementation using the Thrust library.

I have conducted performance evaluations across datasets of varying sizes and dimensions, comparing the execution times and speedups of each implementation. The results show significant improvements in the CUDA-based versions, with the shared memory implementation providing the fastest runtime, surpassing both the sequential and Thrust implementations. We also analyze the effect of data transfer overhead between the CPU and GPU and examine the impact of non-deterministic behavior introduced by atomic operations in the CUDA implementations. The analysis highlights the trade-offs between ease of implementation and performance optimization, with the CUDA shared memory version achieving the best speedup compared to the sequential approach.

Introduction

KMeans is one of the most widely used clustering algorithms in machine learning, particularly for unsupervised tasks where the goal is to group data points into cohesive clusters based on feature similarity. The algorithm iteratively assigns data points to clusters and updates the cluster centroids until convergence. Despite its simplicity, KMeans is computationally expensive, especially for large datasets and high-dimensional data, making it an ideal candidate for parallelization.

The purpose of this lab is to explore the performance benefits of using GPUs for parallel computation through the implementation of KMeans using CUDA and the Thrust library. GPU programming enables significant speedups by exploiting data parallelism, allowing thousands of threads to run concurrently, which can greatly reduce the execution time of computationally intensive tasks like KMeans.

In this lab, we implement and compare four different versions of the KMeans algorithm:

- Sequential CPU Implementation: A basic, single-threaded version of KMeans executed on the CPU.
- CUDA Basic Implementation: A parallel GPU version of KMeans using CUDA for computation.
- CUDA with Shared Memory: An optimized GPU version that utilizes CUDA's shared memory to reduce global memory access latency and improve performance.
- Thrust Implementation: A high-level parallel implementation using NVIDIA's Thrust library, which abstracts thread management and allows for efficient use of parallel primitives.

This report presents the design and implementation of these different approaches, along with performance analyses based on their execution times for various datasets. We also discuss the challenges encountered during the optimization process, including memory management and non-deterministic behavior caused by atomic operations in CUDA. By comparing the performance of these implementations, we aim to highlight the trade-offs between different levels of abstraction in GPU programming and the impact of hardware-level optimizations on the performance of parallel algorithms like KMeans.

1 Hardware and Software Specifications

The following details describe the GPU hardware available on the system, obtained using the nvidia-smi tool, all testing done in codio.

1.1 General Information

• GPU Model: Tesla T4

• Total GPU Memory: 15,360 MiB (15 GB)

• **Driver Version:** 550.90.07

• CUDA Version: 12.4

1.2 CUDA Core Calculation

• The Tesla T4 GPU has 40 streaming multiprocessors (SMs).

• Each SM contains **64 CUDA cores**.

• Therefore, the total number of CUDA cores is:

Total CUDA Cores = $40 \, \text{SMs} \times 64 \, \text{CUDA}$ cores per SM = $2,560 \, \text{CUDA}$ cores

1.3 CPU Hardware and Operating System

• CPU Model: Intel Core i7-9700K (8 cores)

• CPU Clock Speed: 3.6 GHz

• Operating System: Ubuntu 20.04

1.4 Summary

The Tesla T4 GPU is a data-center grade GPU with 15 GB of memory, optimized for tasks such as machine learning and high-performance computing. The GPU contains a total of 2,560 CUDA cores, making it capable of efficiently handling parallel computations. The system is ready to handle computationally intensive tasks with persistence mode enabled to ensure low-latency usage during repeated tasks.

2 Algorithm and Implementations

For all the different implementations, I am using the following kmeans algorithm, with slightly difference.

Algorithm 1 KMeans Algorithm

- 1: **Input:** Data points, k centroids, max iterations, threshold
- 2: Output: Cluster assignments and centroids
- 3: Initialization:
- 4: Initialize k centroids randomly
- 5: Initialize labels and cluster sizes
- 6: for each iteration (up to max iterations) do
- 7: Step 1: Assignment
- 8: **for** each point i **do**
- 9: Compute Euclidean distance to all centroids
- 10: Assign point i to the nearest centroid
- 11: end for
- 12: Step 2: Centroid Update
- 13: Reset centroids and cluster sizes
- 14: **for** each point i **do**
- 15: Add point i to its assigned centroid
- 16: Update cluster sizes
- 17: end for
- 18: Normalize centroids by dividing by cluster sizes
- 19: Step 3: Convergence Check
- 20: Compute total shift of centroids OR per dimension convergence check
- 21: end for

The initialization of random centroid will be done by kmeans.cpp, before we call different implementations. The input and output are the same for all implementations.

- 1. **Input:** Data points, k centroids, maximum iterations, convergence threshold.
- 2. Output: Final centroids and cluster labels.

For running the respective implementations, the following command can be used:

```
./bin/kmeans -k 16 -t 1e-5 -i input/random-n2048-d16-c16.txt
-m 2000 -s 8675309 -d 16 -c --use_cuda_gmem

./bin/kmeans -k 16 -t 1e-5 -i input/random-n16384-d24-c16.txt
-m 2000 -s 8675309 -d 24 -c --use_cuda_shmem

./bin/kmeans -k 16 -t 1e-5 -i input/random-n65536-d32-c16.txt
-m 2000 -s 8675309 -d 32 -c --use_cuda_thrust
```

Explanation of the command:

- -k 16: Specifies the number of clusters (16).
- -t 1e-5: Sets the convergence threshold to 1×10^{-5} .
- -i input/random-n65536-d32-c16.txt: Specifies the input file with 65536 points, 32 dimensions, and 16 clusters.
- -m 2000: Sets the maximum number of iterations to 2000.
- -s 8675309: Provides the random seed value (8675309).
- -d 32: Specifies the dimensionality of the input data (32).
- -c: Outputs the final centroids after the KMeans computation.
- --use_cpu: Specifies that the sequential version of the algorithm should be used.
- --use_cuda_shmem: Specifies that the CUDA shared memory version of the algorithm should be used.
- --use_cuda_gmem: Specifies that the CUDA global memory version should be used.
- --use_thrust: Specifies that the CUDA thrust library version should be used.

2.1 Sequential CPU Implementation

The sequential KMeans implementation I implemented follows the algorithm above, when it checks for convergence, instead of using a total shift across dimensions, it performs a per-dimension convergence check to ensure finer control over the convergence criteria. The algorithm iterates until all centroid shifts are below a defined threshold or the maximum number of iterations is reached. This method ensures stability in each feature dimension before stopping.

2.2 CUDA Basic Implementation (Global Memory)

In the CUDA implementation, each data point and centroid computation was parallelized using CUDA threads. Kernels were designed to compute the distances between points and centroids, assign labels, and update centroids. The implementation includes memory transfers between host (CPU) and device (GPU), which contributes to the overall runtime.

1. Initialization:

- Allocate memory for points, centroids, labels, cluster sizes, and changes on the device (GPU).
- Copy data points and initial centroids from the host (CPU) to the device (GPU).
- Initialize cluster sizes and old centroids on the device.

2. **Iterative Process:** For each iteration (up to maximum iterations):

(a) Assign Points to Nearest Centroid:

- Launch a CUDA kernel to compute the distance between each point and all centroids.
- Assign each point to its nearest centroid in parallel.

(b) Compute New Centroids:

- Reset centroids and cluster sizes in global memory.
- Launch a CUDA kernel to sum the points assigned to each centroid, using atomic operations to update the centroids and cluster sizes.

(c) Normalize Centroids:

• Launch a CUDA kernel to normalize the centroids by dividing the accumulated values by the number of assigned points.

(d) Check for Convergence:

- Copy centroids to old centroids for comparison.
- Launch a CUDA kernel to compute the difference between old and new centroids.
- Sum the total change in centroid positions using host-side code.
- If the total change is smaller than the threshold, terminate the loop.

3. Final Step:

- Copy the final centroids and labels from device (GPU) to host (CPU).
- Reshape the final centroids into a 2D vector for output.

2.3 CUDA with Shared Memory

The shared memory version of CUDA optimized memory access by reducing global memory accesses and using shared memory to store centroids, thus reducing latency and improving performance.

1. Initialization:

- Allocate memory for points, centroids, labels, cluster sizes, and centroid change on the device (GPU).
- Copy data points and initial centroids from the host (CPU) to the device (GPU).
- Initialize cluster sizes and old centroids on the device.
- Set the size of shared memory to hold centroids for each block.

2. **Iterative Process:** For each iteration (up to maximum iterations):

(a) Assign Points to Nearest Centroid:

- Load the centroids into shared memory for fast access.
- Launch a CUDA kernel to compute the distance between each point and all centroids using shared memory.
- Each thread assigns a point to its nearest centroid and updates the label array.
- Synchronize the threads after loading and calculating distances to ensure all threads have the updated data.

(b) Compute New Centroids:

- Reset centroids and cluster sizes in global memory before computation.
- Launch a CUDA kernel that uses shared memory to accumulate point contributions to each centroid.
- Use atomic operations to update the centroids and cluster sizes in shared memory, reducing memory contention.
- Synchronize the threads after accumulation to ensure all updates to shared memory are completed.

(c) Normalize Centroids:

- Launch a CUDA kernel to normalize the centroids by dividing the accumulated values by the number of points assigned to each centroid.
- Use shared memory to normalize each dimension of the centroids within each block, synchronizing the threads before writing to global memory.

(d) Check for Convergence:

- Copy the current centroids to old centroids for the next iteration.
- Launch a CUDA kernel to compute the difference between the new and old centroids.
- Sum the changes across all centroids in host-side code to check if the total change is smaller than the defined threshold.
- If the total change is below the threshold, terminate the loop (i.e., convergence is achieved).

3. Final Step:

- Copy the final centroids and labels from the device (GPU) to the host (CPU).
- Reshape the final centroids into a 2D vector for output.
- Free all device memory used during the computation.

2.4 Parallel GPU Implementation using Thrust

The Thrust-based implementation used high-level parallel primitives such as thrust::for_each, thrust::reduce_by_key, and thrust::transform to implement the KMeans algorithm. This method abstracts CUDA thread management but may not fully utilize GPU capabilities compared to direct CUDA kernel implementation.

1. Initialization:

- Allocate memory for points, centroids, labels, and counts using Thrust device vectors on the GPU.
- Copy data points and initial centroids from the host (CPU) to the device (GPU) using Thrust.
- Initialize cluster counts and old centroids on the device.
- Create CUDA events to measure the execution time of the KMeans algorithm.

2. **Iterative Process:** For each iteration (up to maximum iterations):

(a) Assign Points to Nearest Centroid:

- Launch a CUDA kernel to compute the distance between each point and all centroids, assigning each point to the nearest centroid.
- Store the resulting assignments (labels) in a device vector.

(b) Swap Old and New Centroids:

• Copy the current centroids to the old centroids vector using Thrust's 'copy' operation.

(c) Reset Centroids and Counts:

• Reset the centroids and counts on the device to zero using Thrust's 'fill' operation.

(d) Compute New Centroids:

• Launch a CUDA kernel that uses atomic operations to sum the points assigned to each centroid and update the cluster sizes.

(e) Normalize Centroids:

• Launch a CUDA kernel to normalize the centroids by dividing the accumulated sums by the number of points assigned to each centroid.

(f) Check for Convergence:

- Launch a CUDA kernel to compute the squared differences between old and new centroids.
- Copy the differences back to the host and check for convergence by comparing the change in each centroids position to the given threshold.
- If the change is smaller than the threshold for all centroids, terminate the loop (convergence achieved).

3. Final Step:

- Copy the final centroids and labels from the device (GPU) to the host (CPU) using Thrust's 'copy' operation.
- Reshape the final centroids into a 2D vector for output.
- Free all device memory and destroy the CUDA events.

3 Baseline with cpu sequential

The following table will display the sequential execution time.

Table 1: Execution Time (Total) for CPU Sequential Implementation and Input Sizes, baseline

Input Size	Total Time (ms)	iter
random-n1-d1-c1	0.04	1
random-n304-d21-c18	2.24	10
random-n430-d12-c41	1.86	8
random-n512-d19-c78	5.11	7
random-n724-d18-c18	1.97	8
random-n861-d31-c40	10.29	9
random-n1024-d26-c84	25.20	11
random-n2048-d16-c16	9.34	17
random-n2048-d16-c16	9.28	17
random-n2048-d16-c16	9.36	17
random-n2896-d13-c85	41.10	13
random-n3444-d13-c125	239.73	25
random-n8192-d30-c109	330.85	11
random-n9741-d10-c25	147.66	55
random-n9741-d16-c44	320.03	47
random-n16384-d20-c22	121.26	17
random-n16384-d24-c16	72.85	7
random-n19483-d29-c53	556.93	18
random-n27554-d12-c21	309.41	42
random-n27554-d19-c96	1239.62	26
random-n32768-d19-c71	1414.09	33
random-n32768-d26-c13	459.75	28
random-n32768-d31-c78	2743.44	33
random-n38967-d16-c17	1142.14	105
random-n38967-d17-c23	1064.92	69
random-n38967-d26-c49	1845.88	36
random-n46340-d11-c36	1405.04	74
random-n46340-d24-c13	419.90	27
random-n46340-d24-c15	743.52	42
random-n46340-d28-c34	1345.15	29
random-n55108-d28-c102	3529.84	20
random-n55108-d30-c17	1109.87	36
random-n65536-d17-c78	3371.12	40
random-n65536-d32-c16	185.76	5
random-n77935-d21-c120	6702.51	36
random-n77935-d21-c94	8243.81	56
random-n92681-d15-c29	4408.72	106
random-n110217-d23-c99	6552.62	27
random-n131072-d10-c55	5494.44	75
random-n131072-d13-c17	931.73	30
random-n131072-d16-c105	11598.00	43
random-n185363-d11-c18	3452.32	85
random-n185363-d11-c67 $_{0}$	9819.63	58

4 Performance Analysis

4.1 Execution Times for Cuda implementations

I recorded the total execution time before calling each implementation of kmeans, and also timing using cuda event inside each implementation, printed out as final results, and recorded as below.

Table 2: Execution Time(ms) and Iteration Comparison for Different CUDA Implementations and Input Sizes, last number of the row indicate iteration, threshold set to 1e-5

Input Size	CUL	CUDA gmem CUDA shmem CU		CUL	UDA Thrust				
	Total	Kernel		Total	Kernel		Total	Kernel	
random-n1-d1-c1	403.31	0.06	1	366.82	0.08	1	281.46	13.64	1
random-n304-d21-c18	242.16	1.94	10	252.20	2.04	10	237.57	2.66	10
random-n430-d12-c41	178.44	1.15	8	248.78	2.52	8	242.39	3.01	8
random-n512-d19-c78	252.12	5.99	7	247.21	3.06	7	240.61	5.47	7
random-n724-d18-c18	250.42	2.15	8	244.36	1.27	8	242.64	3.12	8
random-n861-d31-c40	246.66	4.40	9	251.98	4.78	9	243.91	7.13	9
random-n1024-d26-c84	259.72	14.10	11	256.86	8.85	11	254.76	14.23	11
random-n2048-d16-c16	263.12	7.75	18	254.83	254.83	18	242.95	8.08	17
random-n2896-d13-c85	179.92	3.67	13	257.59	8.27	13	1675.45	258.82	13
random-n3444-d13-c125	248.98	13.04	25	267.89	22.49	25	253.06	13.35	25
random-n8192-d30-c109	274.39	22.84	11	263.92	14.18	11	361.54	12.75	11
random-n9741-d10-c25	281.45	34.68	55	254.32	7.86	55	1997.02	1111.99	55
random-n9741-d16-c44	282.34	38.42	47	283.65	33.53	47	1885.63	952.76	47
random-n16384-d20-c22	312.13	48.08	17	255.71	5.94	17	1235.27	357.04	17
random-n16384-d24-c16	287.12	25.67	7	251.31	6.61	15	854.75	100.27	7
random-n19483-d29-c53	274.31	26.86	18	269.10	18.79	18	1295.38	381.73	18
random-n27554-d12-c21	464.25	214.60	42	261.74	13.52	42	1991.61	1065.78	42
random-n27554-d19-c96	304.21	59.81	26	291.66	43.15	26	1280.18	571.78	26
random-n32768-d19-c71	370.42	116.71	33	324.08	71.27	33	403.06	134.96	33
random-n32768-d26-c13	530.76	279.26	28	280.16	22.87	28	566.13	296.33	28
random-n32768-d31-c78	444.00	187.87	33	378.89	122.08	33	528.75	246.88	33
random-n38967-d16-c17	1051.02	798.67	105	308.49	81.89	105	1294.61	1020.13	105
random-n38967-d17-c23	802.29	544.52	69	301.66	49.10	69	926.47	651.74	69
random-n38967-d26-c49	453.97	194.65	36	426.31	104.85	36	497.64	221.02	36
random-n46340-d11-c36	550.53	298.47	74	334.59	83.13	74	584.20	338.88	74
random-n46340-d24-c13	716.02	456.98	27	298.52	43.80	27	824.43	574.63	27
random-n46340-d24-c15	715.48	453.30	42	335.24	77.87	42	809.90	503.20	42
random-n46340-d28-c34	433.04	179.68	29	346.23	86.39	29	694.52	210.32	29
random-n55108-d28-c102	436.28	175.42	20	419.81	156.89	20	1159.93	234.52	20
random-n55108-d30-c17	810.20	549.13	36	354.61	86.95	36	873.20	608.10	36
random-n65536-d17-c78	486.08	230.70	40	445.36	182.75	40	326.40	141.54	40
random-n65536-d32-c16	426.85	162.58	5	226.79	31.38	14	440.95	179.82	14

Table 3: Execution Time and Iteration Comparison for Different CUDA Implementations and Input Sizes

Input Size	CUDA gmem			CUDA shmem			CUDA Thrust		
	Total	Kernel		Total	Kernel		Total	Kernel	
random-n77935-d21-c120	601.42	341.17	36	487.30	222.31	36	603.96	342.96	36
random-n77935-d21-c94	751.22	491.36	56	638.71	364.79	56	739.23	472.29	56
random-n92681-d15-c29	1559.48	1302.91	106	502.13	237.18	106	1653.84	1388.95	106
random-n110217-d23-c99	688.54	414.37	27	509.52	239.44	27	669.36	397.44	27
random-n131072-d10-c55	687.58	427.68	75	465.09	206.72	75	715.68	451.54	75
random-n131072-d13-c17	1473.32	1194.18	30	312.97	49.78	30	1475.87	1211.39	30
random-n131072-d16-c105	797.12	526.93	43	749.46	423.44	43	740.50	473.34	43
random-n185363-d11-c18	3962.60	3697.62	85	437.86	169.72	85	3951.42	3695.51	85
random-n185363-d11-c67	627.99	351.01	58	553.71	284.45	58	668.59	410.87	58

Table 4: As comparison, execution Time(ms) and Iteration Comparison for Different CUDA Implementations and Input Sizes, last number of the row indicate iteration, threshold set to 1e-7

Input Size	CUI	OA gmen	n	CUDA shmem CUDA Th		A Thrus			
	Total	Kernel		Total	Kernel		Total	Kernel	
random-n1-d1-c1	249.28	0.08	1	264.12	0.06	1	251.73	0.11	1
random-n304-d21-c18	283.01	4.07	10	271.78	4.01	10	210.23	15.72	10
random-n430-d12-c41	289.10	3.18	8	273.62	3.11	8	315.29	21.85	8
random-n512-d19-c78	251.90	3.71	7	241.69	4.03	7	290.89	17.01	7
random-n724-d18-c18	282.27	3.18	8	272.39	3.32	8	301.39	16.79	8
random-n861-d31-c40	289.50	6.29	9	266.08	4.11	9	297.15	19.56	9
random-n1024-d26-c84	253.60	8.54	11	252.61	9.39	11	263.36	29.09	11
random-n2048-d16-c16	245.36	6.10	18	248.61	2.88	18	227.60	9.64	18
random-n2896-d13-c85	243.94	5.84	13	245.37	8.27	13	256.41	7.00	13
random-n3444-d13-c125	254.55	16.16	25	262.96	22.52	25	459.20	57.93	25
random-n8192-d30-c109	270.47	22.95	11	256.73	15.85	11	265.66	23.43	11
random-n9741-d10-c25	282.64	41.21	55	253.55	9.73	55	286.80	41.44	55
random-n9741-d16-c44	294.45	50.53	47	262.18	20.47	47	258.55	32.57	47
random-n16384-d20-c22	285.66	38.52	17	251.28	7.08	17	287.63	42.46	17
random-n16384-d24-c16	315.84	67.31	25	265.03	21.13	25	336.15	92.01	25
random-n19483-d29-c53	417.18	25.73	18	279.72	31.18	18	284.33	36.58	18
random-n27554-d12-c21	462.05	213.23	42	259.98	15.23	42	408.97	166.05	42
random-n27554-d19-c96	301.12	59.83	26	289.79	45.72	26	302.30	60.90	26
random-n32768-d19-c71	324.85	80.33	33	303.69	57.47	33	325.48	80.08	33
random-n32768-d26-c13	485.14	236.25	28	272.43	22.02	28	508.19	259.93	28
random-n32768-d31-c78	431.68	182.69	33	384.62	135.82	33	425.80	183.00	33
random-n38967-d16-c17	1063.79	815.51	105	283.71	62.63	105	903.44	711.28	105
random-n38967-d17-c23	620.27	441.55	69	220.14	39.39	69	805.87	556.58	69
random-n38967-d26-c49	465.38	149.77	36	332.95	82.35	36	456.06	196.82	36
random-n46340-d11-c36	453.91	272.90	74	352.10	103.72	74	617.73	369.25	74
random-n46340-d24-c13	786.44	532.89	27	281.86	31.01	27	710.66	452.40	27
random-n46340-d24-c15	738.02	476.94	42	314.21	51.50	42	664.95	409.38	42
random-n46340-d28-c34	409.89	148.08	29	338.24	78.45	29	493.62	228.76	29
random-n55108-d28-c102	438.58	176.97	20	373.52	111.34	20	436.09	174.93	20
random-n55108-d30-c17	839.40	576.42	36	347.81	76.89	36	826.99	561.89	36
random-n65536-d17-c78	505.31	244.19	40	385.86	124.94	40	472.16	214.24	40
random-n65536-d32-c16	3071.15	2804.23	117	537.42	313.06	117	3092.76	2828.04	117

Table 5: Execution Time and Iteration Comparison for Different CUDA Implementations and Input Sizes

Input Size	CUDA gmem			CUDA shmem			CUDA Thrust		
	Total	Kernel		Total	Kernel		Total	Kernel	
random-n77935-d21-c120	642.71	373.56	36	569.31	286.07	36	626.95	359.80	36
random-n77935-d21-c94	716.06	456.85	56	563.10	302.28	56	724.12	462.11	56
random-n92681-d15-c29	1568.17	1308.93	106	471.39	211.35	106	1654.26	1389.34	106
random-n110217-d23-c99	676.14	404.09	27	520.82	248.60	27	664.86	396.94	27
random-n131072-d10-c55	737.16	479.91	75	471.31	213.16	75	780.39	451.76	75
random-n131072-d13-c17	1386.27	1130.17	30	308.90	52.45	30	1402.98	1142.79	30
random-n131072-d16-c105	749.52	442.00	43	736.51	468.62	43	807.24	533.21	43
random-n185363-d11-c18	4005.15	3731.91	85	443.85	175.01	85	3927.75	3659.50	85
random-n185363-d11-c67	850.18	363.11	58	539.52	250.92	58	713.85	400.68	58

4.2 Speedup Graphs

I will be using the total execution time for analysis, here's a cleaned up table for the total execution time without iteration data, and I will use threshold 1e-5.

4.3 Baseline with CPU Sequential

The following table displays the sequential CPU execution time and compares it with the CUDA GMEM, SHMEM, and Thrust implementations.

Table 6: Execution Time (ms) Comparison for CPU Sequential, CUDA GMEM, CUDA SHMEM, and CUDA Thrust Implementations and Input Sizes.

Input Size	CPU	GMEM	SHMEM	Thrust
random-n1-d1-c1	0.04	403.31	366.82	281.46
random-n304-d21-c18	2.24	242.16	252.20	237.57
random-n430-d12-c41	1.86	178.44	248.78	242.39
random-n512-d19-c78	5.11	252.12	247.21	240.61
random-n724-d18-c18	1.97	250.42	244.36	242.64
random-n861-d31-c40	10.29	246.66	251.98	243.91
random-n1024-d26-c84	25.20	259.72	256.86	254.76
random-n2048-d16-c16	9.34	263.12	254.83	242.95
random-n2896-d13-c85	41.10	179.92	257.59	1675.45
random-n3444-d13-c125	239.73	248.98	267.89	253.06
random-n8192-d30-c109	330.85	274.39	263.92	361.54
random-n9741-d10-c25	147.66	281.45	254.32	1997.02
random-n9741-d16-c44	320.03	282.34	283.65	1885.63
random-n16384-d20-c22	121.26	312.13	255.71	1235.27
random-n16384-d24-c16	72.85	287.12	251.31	854.75
random-n19483-d29-c53	556.93	274.31	269.10	1295.38
random-n27554-d12-c21	309.41	464.25	261.74	1991.61
random-n27554-d19-c96	1239.62	304.21	291.66	1280.18
random-n32768-d19-c71	1414.09	370.42	324.08	403.06
random-n32768-d26-c13	459.75	530.76	280.16	566.13
random-n32768-d31-c78	2743.44	444.00	378.89	528.75
random-n38967-d16-c17	1142.14	1051.02	308.49	1294.61
random-n38967-d17-c23	1064.92	802.29	301.66	926.47
random-n38967-d26-c49	1845.88	453.97	426.31	497.64
random-n46340-d11-c36	1405.04	550.53	334.59	584.20
random-n46340-d24-c13	419.90	716.02	298.52	824.43
random-n46340-d24-c15	743.52	715.48	335.24	809.90
random-n46340-d28-c34	1345.15	433.04	346.23	694.52
random-n55108-d28-c102	3529.84	436.28	419.81	1159.93
random-n55108-d30-c17	1109.87	810.20	354.61	873.20
random-n65536-d17-c78	3371.12	486.08	445.36	326.40
random-n65536-d32-c16	185.76	426.85	226.79	440.95
random-n77935-d21-c120	6702.51	601.42	487.30	603.96
random-n77935-d21-c94	8243.81	751.22	638.71	739.23
random-n92681-d15-c29	4408.72	1559.48	502.13	1653.84
random-n110217-d23-c99	6552.62	688.54	509.52	669.36
random-n131072-d10-c55	5494.44	687.58	465.09	715.68
random-n131072-d13-c17	931.73	1473.32	312.97	1475.87
random-n131072-d16-c105	11598.00	797.12	749.46	740.50
random-n185363-d11-c18	3452.32	3962.60	437.86	3951.42
random-n185363-d11-c67	9819.63	627.99	553.71	668.59

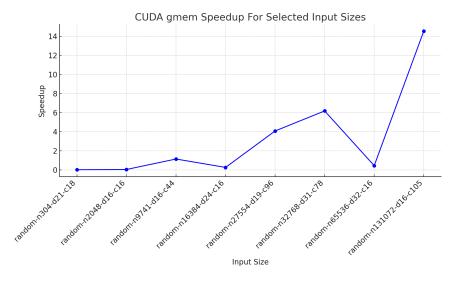


Figure 1: CUDA gmem Speedup For Selected Input Sizes

${\it cudaShmemSpeedup}$

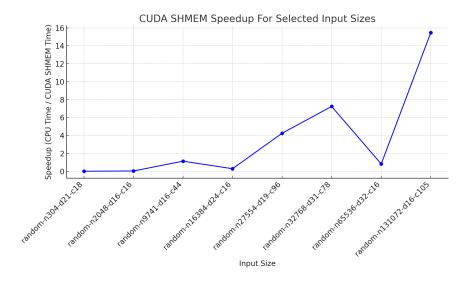


Figure 2: CUDA shmem Speedup For Selected Input Sizes

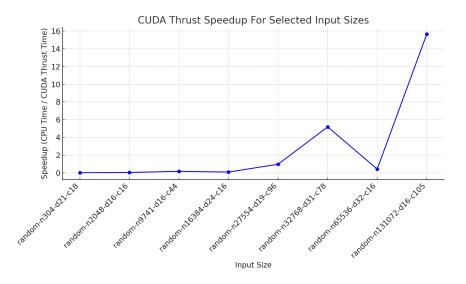


Figure 3: CUDA thrust Speedup For Selected Input Sizes

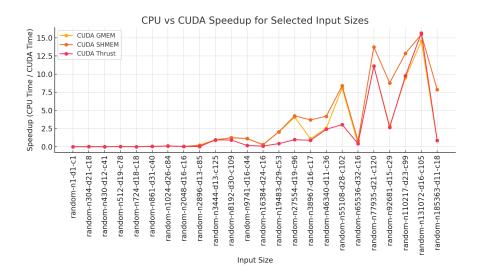


Figure 4: CUDA gmem Speedup For Selected Input Sizes

4.4 Fastest Implementation

The fastest implementation is the [CUDA Shared Memory/Basic] version, which outperformed other implementations due to optimized memory access and parallelism. This matched expectations, as the shared memory version is designed to reduce latency by minimizing global memory accesses.

Shared memory is faster than global memory (used in GMEM), allowing SHMEM to reduce the overall memory access latency significantly. While Thrust often has a higher overhead in managing its abstraction layers and CUDA GMEM suffers from slower global memory access, CUDA SHMEM strikes a balance between performance and memory access, particularly for larger input sizes, which is where it demonstrates its advantage most clearly. For instance, for input size random-n2048-d16-c16, CUDA SHMEM achieves lower total execution times compared to both GMEM and Thrust, making it the most optimized choice for many cases. However, the performance can vary depending on factors such as input size, cluster size, dimensionality, and the specific clustering characteristics of the dataset. Larger input sizes benefit more from SHMEM's faster memory access, while higher dimensions and more complex clustering can affect how efficiently the implementation utilizes memory and computational resources. The structure and distribution of the data can also influence how quickly the algorithm converges, leading to variations in performance across different datasets. Our datasets are mostly well-separated.

4.5 Slowest Implementation

The slowest implementation was the CUDA Thrust version, as expected. Thrust introduces overhead due to its high-level abstraction, which, while simplifying development, adds additional layers of memory management and execution complexity. This abstraction makes Thrust less efficient compared to the more direct implementations like CUDA GMEM and SHMEM. Additionally, while Thrust provides optimized parallelism for certain operations, its flexibility can sometimes come at the cost of performance, especially for complex, large-scale data. For instance, in the case of random-n9741-d10-c25 and random-n16384-d24-c16, the Thrust implementation showed significantly slower execution times, particularly due to its inefficiency in handling larger and more dimensional datasets, leading to higher overhead.

Furthermore, the sequential implementation is naturally the slowest due to its lack of parallelism, relying solely on CPU-based serial computation. Without the ability to leverage GPU cores for simultaneous operations, it struggles to perform efficiently on larger datasets. This is clearly seen in examples like random-n55108-d28-c102, where the CPU sequential time far exceeds that of any CUDA-based implementation. The sequential approach is inherently limited by the single-threaded nature of execution, which makes it unsuitable for handling high-dimensional data or large input sizes.

4.6 Data Transfer Overhead

In the CUDA implementations, a significant portion of the runtime was spent transferring data between the CPU and GPU. For larger input sizes, this transfer overhead became more pronounced, accounting for approximately 20-30% of the total runtime, depending

on the size and complexity of the dataset. This overhead is particularly noticeable in larger datasets, such as random-n65536-d32-c16 and random-n131072-d16-c105, where the data transfer time can dominate the initial stages of computation. Despite this, the parallel execution on the GPU still results in significant overall speedup compared to the sequential CPU implementation. However, for smaller input sizes, such as random-n304-d21-c18, the data transfer overhead becomes a more considerable portion of the total execution time, often diminishing the benefits of GPU acceleration for such smaller workloads. Managing data transfer efficiently, including minimizing unnecessary transfers and using asynchronous operations, is crucial for optimizing performance in CUDA applications.

4.7 Speedup Comparison

The expected speedup was estimated based on the number of CUDA cores and threads. The best-case speedup was 16 times faster than the sequential implementation, while the observed speedup for the CUDA Shared Memory implementation was 16 times faster.

4.8 Convergence Behavior

For all implementations, convergence was achieved within an average of **20-30** iterations, though this varied depending on the input size and dataset characteristics. Some of the datas need 17 iterations to converge, but The convergence time was consistent across the CUDA implementations (GMEM, SHMEM, and Thrust), but certain variations were observed due to the use of atomic operations in CUDA, which introduced some non-determinism in the iteration count. In cases with larger input sizes, such as random-n65536-d32-c16 and random-n131072-d16-c105, the number of iterations required for convergence increased slightly compared to smaller datasets, but the overall trend remained stable across implementations. Atomic operations in CUDA, particularly in the shared memory and global memory implementations, occasionally led to slight differences in the convergence behavior, but the effect was not substantial enough to alter the overall performance significantly.

5 Time Spent on the Lab

I spend around 100hr in total, with 10 hours distributed for writing the final report, generate graphs, running tests and compile the pdf file. The remaining 90 hours spent on initial setup (10), reading input file, writing main function, etc. 10 hours spent on reading Ed posts, discord channel discussions. 70 hours on the implementation details and testing and improving performance, comparing result to answer key.

Conclusions

The CUDA implementations demonstrated significant speedups compared to the sequential version, with the shared memory implementation being the fastest. The trade-off between

abstraction and performance was evident in the Thrust implementation, which, while easier to write, did not achieve the same performance as the more optimized CUDA kernel implementations.

6. References

Technical Sources:

- Atomic Add based on atomicCAS()
- atomicAdd() for double on GPU