In solving the questions in this assignment, I worked together with my classmate Addison Weatherhead 1005691128. I confirm that I have written the solutions/code/report in my own words.

Lisa Yu
1005786866
Yulisa5

Part I

(Q1)

1.1) points on L : $\vec{P} = \vec{P_0} + t\vec{d} = \underbrace{\begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix}}_{\vec{P_0}} + t \underbrace{\begin{bmatrix} d_x \\ d_y \\ d_z \end{bmatrix}}_{\vec{d}}$ for all $t \in \mathbb{R}$

pixel coordinate:

$$\vec{p} = \begin{pmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X_0 + td_x \\ Y_0 + td_y \\ Z_0 + td_z \end{pmatrix}$$

$$= \begin{pmatrix} f(X_0 + td_x) + p_x(Z_0 + td_z) \\ f(Y_0 + td_y) + p_y(Z_0 + td_z) \\ Z_0 + td_z \end{pmatrix}$$

$$= \begin{pmatrix} fX_0 + ftd_x + p_x Z_0 + p_x td_z \\ fY_0 + ftd_y + p_y Z_0 + p_y td_z \\ Z_0 + td_z \end{pmatrix} = \begin{pmatrix} \frac{fX_0 + ftd_x + p_x Z_0 + p_x td_z}{Z_0 + td_z} \\ \frac{fY_0 + ftd_y + p_y Z_0 + p_y td_z}{Z_0 + td_z} \\ 1 \end{pmatrix}$$

to get pixel coordinates of vanishing point, take limit as $t \to \infty$.

$$x = \lim_{t\to\infty} \frac{fX_0 + ftd_x + p_x Z_0 + p_x td_z}{Z_0 + td_z} = \lim_{t\to\infty} \frac{\frac{fX_0}{t} + fd_x + \frac{p_x Z_0}{t} + p_x d_z}{\frac{Z_0}{t} + d_z} = \frac{fd_x + p_x d_z}{d_z}$$

$$y = \lim_{t\to\infty} \frac{fY_0 + ftd_y + p_y Z_0 + p_y td_z}{Z_0 + td_z} = \lim_{t\to\infty} \frac{\frac{fY_0}{t} + fd_y + \frac{p_y Z_0}{t} + p_y d_z}{\frac{Z_0}{t} + d_z} = \frac{fd_y + p_y d_z}{d_z}$$

$\Rightarrow$ the pixel coordinates of the vanishing point is $\begin{pmatrix} \frac{fd_x + p_x d_z}{d_z} \\ \frac{fd_y + p_y d_z}{d_z} \\ 1 \end{pmatrix}$

1.2)

plane: $\vec{n} \cdot \vec{d} = 0$, for a fixed $\vec{n} = \begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix}$

suppose we fix d_y, d_z then $d_x = \frac{-n_y d_y - n_z d_z}{n_x}$

$\Rightarrow$ points that lie on the plane is

$$\vec{P} = \vec{P_0} + t \begin{bmatrix} \frac{-n_y d_y - n_z d_z}{n_x} \\ d_y \\ d_z \end{bmatrix} \quad \text{for all } t, d_y, d_z \in \mathbb{R}$$

Similarly as 1.1), we get pixel coordinate $\vec{p} = \begin{pmatrix} \frac{fX_0 + ftd_x + p_x Z_0 + p_x td_z}{Z_0 + td_z} \\ \frac{fY_0 + ftd_y + p_y Z_0 + p_y td_z}{Z_0 + td_z} \\ 1 \end{pmatrix}$

where $d_x = \frac{-n_y d_y - n_z d_z}{n_x}$ for any d_y, d_z.

Then we can take limit as $t \to \infty$ as previous and get the pixel coordinate of the vanishing point:

$$\vec{p} = \begin{pmatrix} \frac{fd_x + P_x d_z}{d_z} \\ \frac{fd_y + P_y d_z}{d_z} \\ 1 \end{pmatrix} \quad \text{where } d_x = \frac{-n_y d_y - n_z d_z}{n_x} \quad \text{for any } d_y, d_z$$

$$\Rightarrow \vec{p} = \begin{pmatrix} \frac{f\left(\frac{-n_y d_y - n_z d_z}{n_x}\right) + p_x d_z}{d_z} \\ \frac{fd_y + p_y d_z}{d_z} \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

$$x = \frac{f\left(\frac{-n_y d_y - n_z d_z}{n_x}\right) + p_x d_z}{d_z}, \quad y = \frac{fd_y + p_y d_z}{d_z} = \frac{f d_y}{d_z} + p_y \Rightarrow \frac{f d_y}{d_z} = y - p_y$$

$$\Rightarrow x = \frac{-n_y f d_y - n_z f d_z}{n_x d_z} + p_x n_x$$

$$= -\frac{n_y}{n_x}\left(\frac{f d_y}{d_z}\right) - \frac{n_z f d_z}{n_x d_z} + p_x n_x$$

$$= -\frac{n_y}{n_x}(y - p_y) - \frac{n_z f}{n_x} + p_x n_x$$

$$= -\frac{n_y}{n_x} y + \frac{n_y p_y}{n_x} - \frac{n_z f}{n_x} + p_x n_x$$

$$\Rightarrow y = \frac{x - \frac{n_y p_y}{n_x} + \frac{n_z f}{n_x} - p_x n_x}{-\frac{n_y}{n_x}}$$

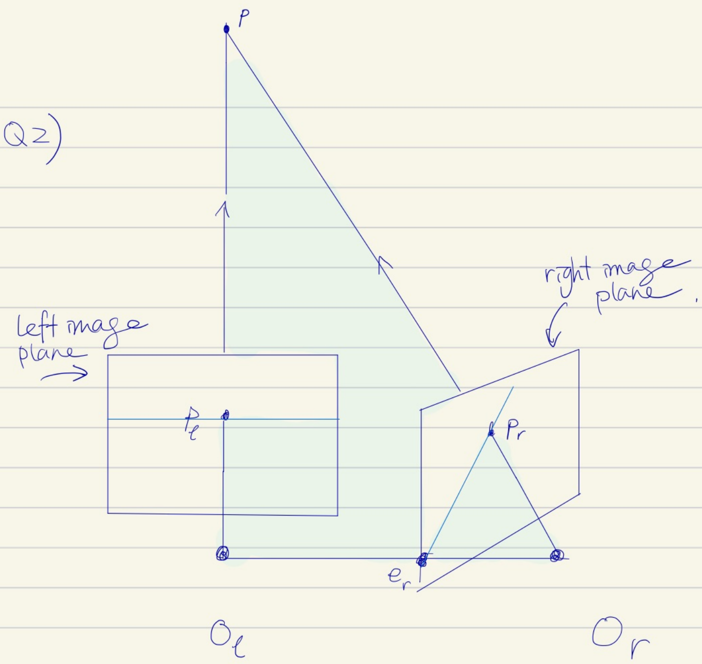
$$= \frac{n_x x - n_y p_y + n_z f - p_x n_x^2}{-n_y}$$

$$= \underbrace{\frac{-n_x}{n_y}}_{m} x + \underbrace{\frac{n_y p_y - n_z f + p_x n_x^2}{n_y}}_{b}$$

Since n_x, n_y, n_z, p_y, p_x, and y are all fixed constants, y can be writen as $y = mx + b$ for some constant m and b.

So the x-coordinate and y-coordinate has a linear relationship, hence proving that the vanishing points form a linear horizon for different d_y, d_z (i.e. for different points on the plane).

Q2)



The epipole on the right image plane is e_r as depicted in the drawing above. This is because this is at the intersection between the line connecting O_ℓ to O_r and the right image plane. The epipolar line on the right image is as drawn above since it passes through the right epipole and point P_r. If point P_r is higher than O_r, as drawn in the image above, then this epipolar line would slant upwards. (This line is where the right image plane intersects the triangular plane defined by points O_ℓ, O_r, and p.)

The epipole on the left image does not exist on the left image plane (i.e. it is at infinity) since the line connecting O_ℓ and O_r does not intersect the left image plane. Moreover, since the left image plane is not rotated, the left image plane intersects the triangular plane defined by points O_ℓ, O_r, and p at a straight horizontal line through point P_ℓ since O_ℓ is level with O_r. So the left epipolar line is horizontal through the point P_ℓ.

(Q3)

3.1).

let $l = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ such that $\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \cdot l = 0$ be the vector representation of line l

let $l' = \begin{bmatrix} a' \\ b' \\ c' \end{bmatrix}$ such that $\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \cdot l' = 0$ be the vector representation of line l'

then $l \times l' = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \times \begin{bmatrix} a' \\ b' \\ c' \end{bmatrix} = \begin{bmatrix} bc' - cb' \\ ca' - ac' \\ ab' - ba' \end{bmatrix} \overset{\text{normalize}}{=} \begin{bmatrix} \frac{bc'-cb'}{ab'-ba'} \\ \frac{ca'-ac'}{ab'-ba'} \\ 1 \end{bmatrix} = p.$

Show that p is on l:

$$\begin{bmatrix} \frac{bc'-cb'}{ab'-ba'} \\ \frac{ca'-ac'}{ab'-ba'} \\ 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = a\left(\frac{bc'-cb'}{ab'-ba'}\right) + b\left(\frac{ca'-ac'}{ab'-ba'}\right) + c$$

$$= \frac{abc' - acb' + bca' - bac' + cab' - cba'}{ab' - ba'}$$

* assuming l and l' are not parallel lines, then $ab' \neq ba'$

$$= \frac{0}{ab'-ba'} = 0 \Rightarrow \text{so } p \text{ is on the line } l.$$

Show that p is on l':

$$\begin{bmatrix} \frac{bc'-cb'}{ab'-ba'} \\ \frac{ca'-ac'}{ab'-ba'} \\ 1 \end{bmatrix} \cdot \begin{bmatrix} a' \\ b' \\ c' \end{bmatrix} = a'\left(\frac{bc'-cb'}{ab'-ba'}\right) + b'\left(\frac{ca'-ac'}{ab'-ba'}\right) + c'$$

$$= \frac{a'bc' - a'cb' + b'ca' - b'ac' + c'ab' - c'ba'}{ab' - ba'}$$

* assuming l and l' are not parallel lines, then $ab' \neq ba'$

$$= \frac{0}{ab'-ba'} = 0 \Rightarrow \text{so } p \text{ is on the line } l'$$

since l and l' are lines (not parallel or identical), then they must intersect at exactly one point (i.e. point $p = l \times l'$). //.

* when l and l' are parallel, then there is no intersection.

3.2)

Let $p = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$

let $p' = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$

then $p \times p' = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \times \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} y(1) - (1)y' \\ (1)x' - x(1) \\ xy' - x'y \end{bmatrix} = \begin{bmatrix} y - y' \\ x' - x \\ xy' - x'y \end{bmatrix} = l$.

<u>show p lies on l:</u>

$p \cdot l = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \cdot \begin{bmatrix} y - y' \\ x' - x \\ xy' - x'y \end{bmatrix} = x(y-y') + y(x'-x) + (xy' - x'y)$

$= xy - xy' + yx' - yx + xy' - x'y$

$= 0. \Rightarrow$ so point p lies on line l.

show p' lies on l:

$p' \cdot l = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} \cdot \begin{bmatrix} y - y' \\ x' - x \\ xy' - x'y \end{bmatrix} = x'(y-y') + y'(x'-x) + (xy' - x'y)$

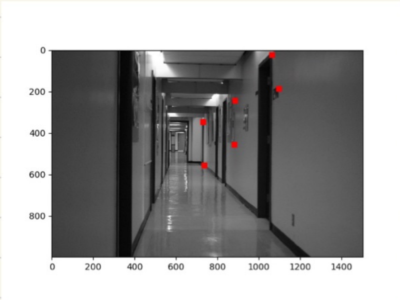
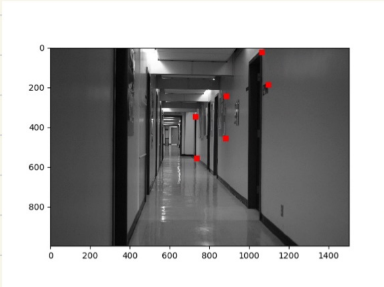
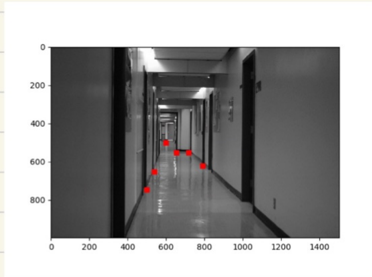
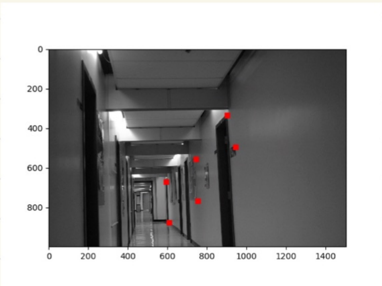
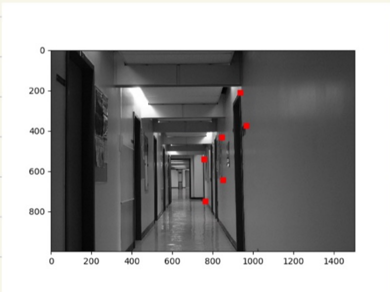
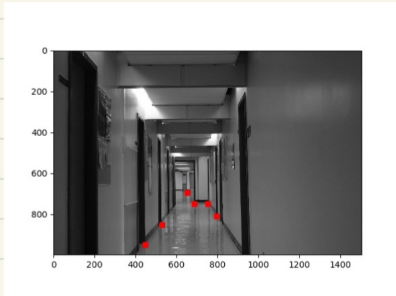
$= x'y - x'y' + y'x' - y'x + xy' - x'y$

$= 0. \Rightarrow$ so point p' lies on line l

Hence we have shown that the line $l = p \times p'$ goes through the points p and p' as wanted. //

Part II

Q4) 4.1) selected points

	case A	case B	case C
I			
I2			

4.2)

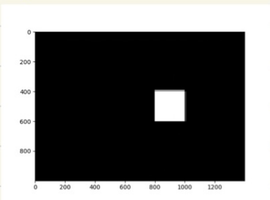
apply H to a square to see the effects of H.

case A

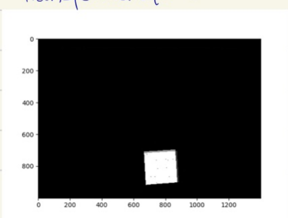
H=

```
[[-2.82730306e-03 -1.33239483e-04  5.51901156e-01]
 [-2.09441173e-05 -2.61585610e-03 -8.33897041e-01]
 [-2.57432643e-07  2.16623292e-08 -2.44640553e-03]]
```

original square:



transformed square:

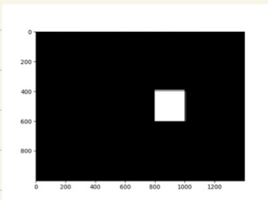


case B

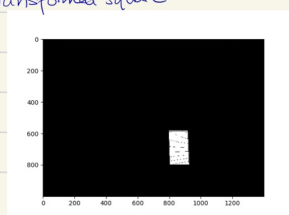
H=

```
[[ 1.16404235e-03 -6.48369258e-05  8.49659811e-01]
 [-9.93200442e-05  2.12481883e-03  5.27320275e-01]
 [-1.41121801e-07 -1.67955432e-07  2.37694487e-03]]
```

original square:



transformed square:

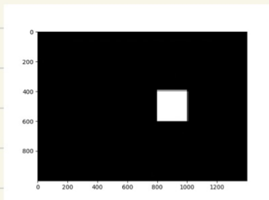


case C

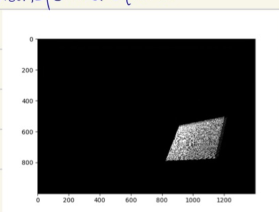
H=

```
[[-3.44909752e-04  6.78273190e-04 -7.14934396e-01]
 [ 5.72936537e-04 -5.28457729e-04 -6.99189075e-01]
 [ 6.77957370e-07  4.21137276e-07 -1.50316273e-03]]
```

original square:

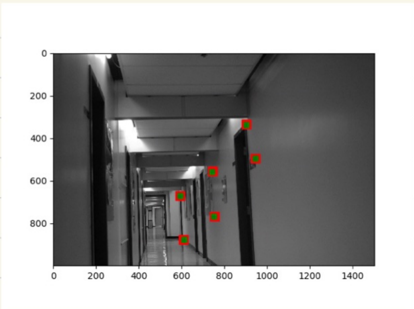
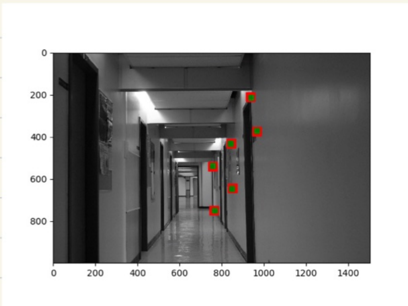
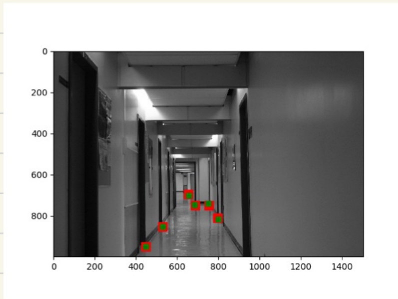


transformed square:

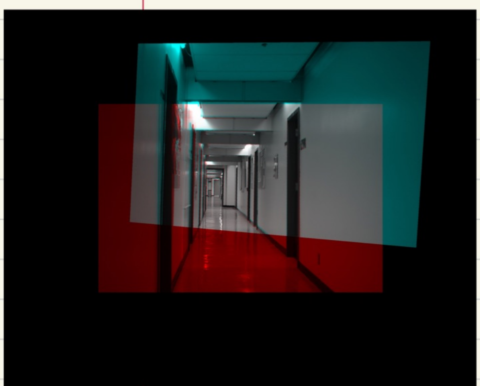
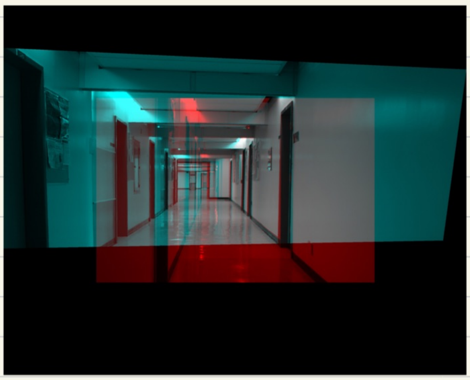
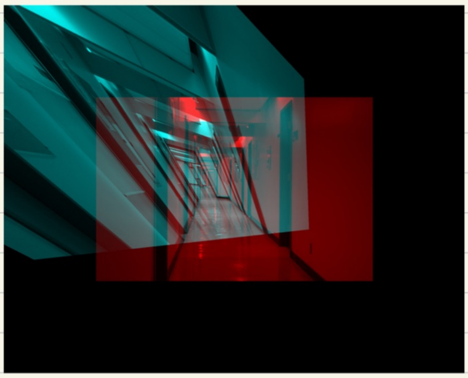


looking at the transformed square image, it appears that H applied a translation to the bottom left and a rotation counterclockwise	looking at the transformed square image, it appears that H applied a translation downwards and a horizontal scale (smaller)	looking at the transformed square image, it appears that H applied a translation downwards and a shear (and a little bit of scale to become larger)

4.3) ▨ = selected $(\tilde{x},\tilde{y})$ ▨ = estimated $(\tilde{x},\tilde{y})$

case A	case B	case C
		

4.4)

case A	case B	case C
		
The camera that took $\tilde{I}$ is higher and more to the right than the camera that took I. The camera for $\tilde{I}$ is also rotated a little clockwise.	The camera that took $\tilde{I}$ is closer (further down the hall) than the camera that took I. The camera for $\tilde{I}$ is also a little more to the right	The camera that took $\tilde{I}$ is closer (further down the hall) than the camera that took I. The camera for $\tilde{I}$ is also seems to be pointing more to the left.

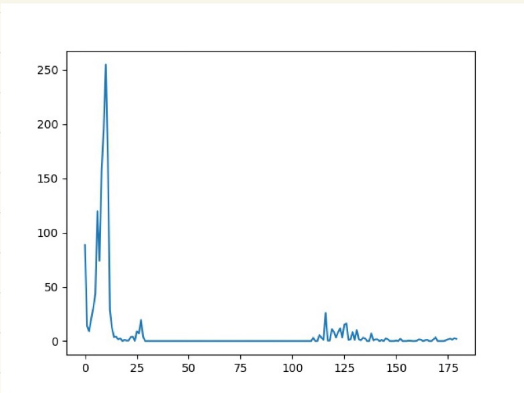
* The right wall appears to be more Lambertian since regardless of the angles, the pixels for the walls remain very similar hence overlapping to grey (in case A and B).

✻ The floor appears to be less Lambertian since in case C, although we have matched the floors after the transformation, the overlapped floor area still appears red/blue in lots of places because the reflection of light from different angles resulted in different pixel values (lighter/darker), so the overlap is not grey.

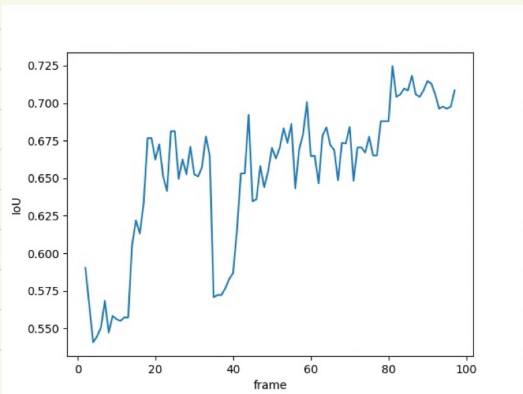
Q5) 5.1) using hue histogram:

Viola-Jones face detection on first frame:

hue histogram:



IoU on frame 2+:



(if more than one face detected, take the one most similar to previous frame's detected one) max IoU: (IoU = 72.5%, frame 81) min IoU: (IoU = 54.1%, frame 4)

□ = mean shift tracking

□ = Viola-Jones detection

□ = mean shift tracking

□ = Viola-Jones detection

```
5.1 max IoU = 0.7247381926624592, frame = 81
5.1 min IoU = 0.5405960342602371, frame = 4
5.1 percentage IoU higher than 70%: 0.14583333333333334
5.1 percentage IoU lower than 60%: 0.1875
```

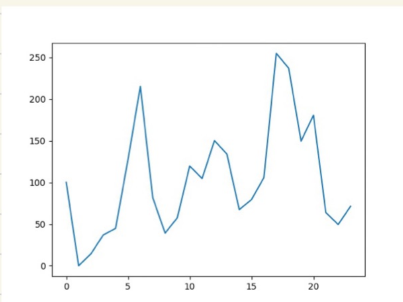
14.6% of IoU's are higher than 70%,
18.8% of IoU's are lower than 60%.

for small IoU (<60%), Viola-Jones detection seems to be correct more often than mean shift tracking. This could be because the initial hue histogram did not capture a good representation of face pixel distribution (i.e. the first frame's detection is bad).

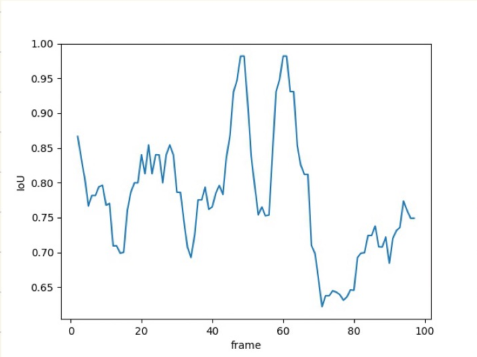
5.2) using gradient histogram:

* same face detection on first frame as 5.1).

gradient histogram:



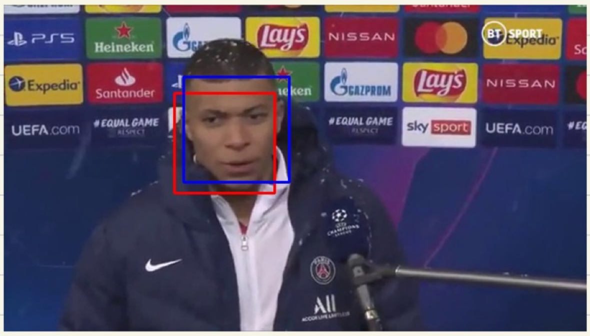
IoU on frame 2t:



(if more than one face detected, take the one most similar to previous frame's detected one)

max IoU: (IoU = 98.2 %, frame 49)

min IoU: (IoU = 62.2 %, frame = 71)



□ = mean shift tracking
□ = Viola-Jones detection

□ = mean shift tracking
□ = Viola-Jones detection

```
5.2 max IoU = 0.9820631442253064, frame = 49
5.2 min IoU = 0.621985275450622, frame = 71
5.2 percentage IoU higher than 80%: 0.3229166666666667
5.2 percentage IoU lower than 70%: 0.1875
```

32.3% of IoU's are higher than 80%
18.8% of IoU's are lower than 70%.