

## Environment setup

Neuron fires spontaneously, this introduces noise in processing the signals. In this coursework, I will model neural connections in full connected forward model and Poisson model, and see how the firing behaviour is influenced by noise or stimuli strength in different layers of neurons. This will help me understand how the models work, and why we prefer Poisson models. First I initialize 1000 neurons. For simplicity reason, all neurons are modeled with integration and fire model. The parameters are set to reasonable default value as required.

I simulate the neurons for 400 ms. In the first 200 ms, The neurons receive a 2 nA current, and a independent Gaussian noise ( $M=0, sd=150$ ) and a common noise ( $M=0, sd=50$ ). The parameter is chose according to the lab materials. The relatively large 0 centered value ensures the neuron to receive noise (from membranes and of the stimuli) in a large range and equal likely to be hyper-polarize and depolarized by the noise. These also ensure behaviors to be various between different neurons. And the stimuli from 200 ms to 400 ms is set to be 40 nA. No common noise for all neuron is added to full fill the requirement of the task. As a result, the average firing rate for the whole 400ms is 37.57, the average firing rate of the first half period is 2.025, and 73.12 for the second period. The distribution of spikes in the 1000 neuron, stimulus and peristimulus time histogram is shown in the figure below.

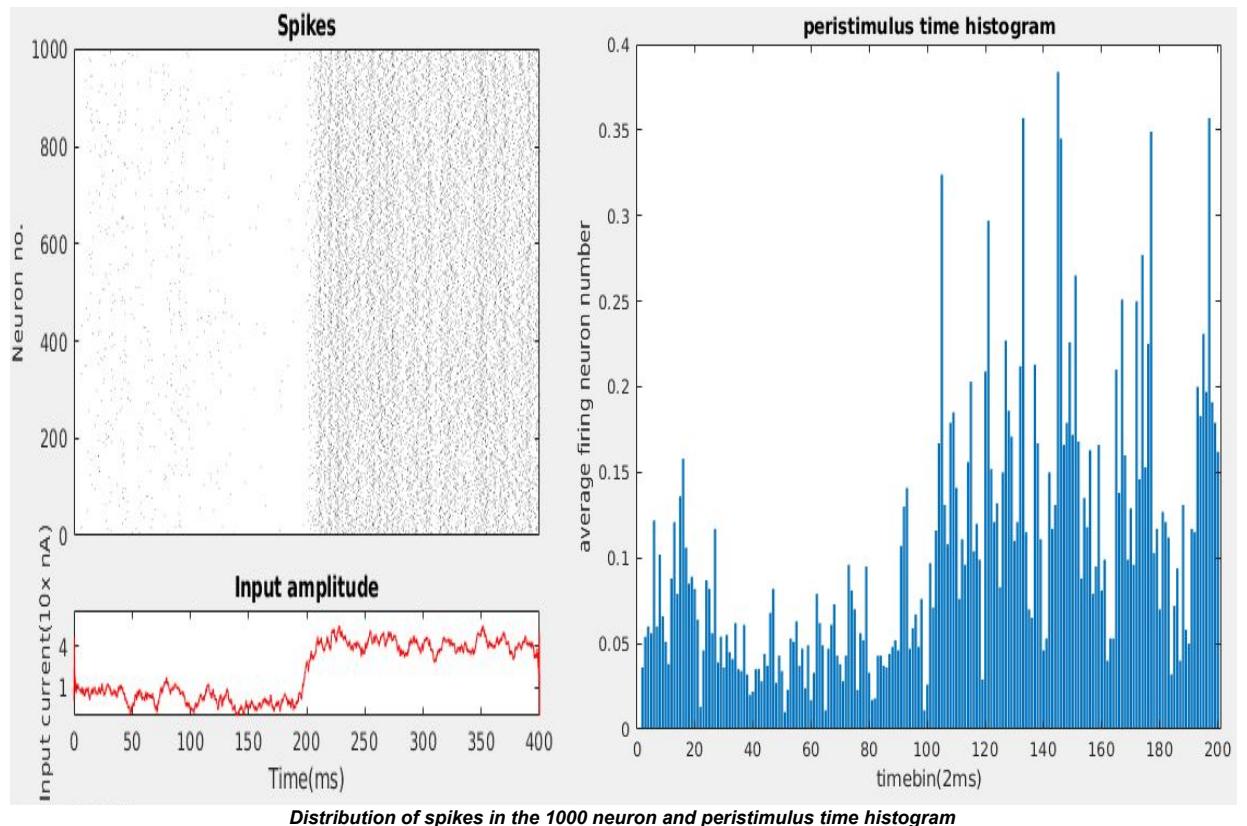


Figure 1: The distribution of spikes: stimulus and peristimulus time histogram

**Question 1** (20 marks) Examine how 1) the background stimulus strength, and 2) the standard deviation of the noise influence the response of the population when the stronge stimulus is presented. Explain the findings. (No fully extensive parameter sweeps needed).

### Answer 1

#### 1) Stimulus strength

The firing rate should increase nearly linearly with the increase of strength of stimulus. The integration model does not introduce any control of saturation. And after reset the membranes potential, the neuron can fire again in not time. The firing behavior is totally depend on the membranes potential. So the firing rate increase nearly linearly. This is also reflect in deduction of equations:

$$F = -1/\tau * \log(1 - (V_{threshold} - V_{rest})/I_{stim} * R_m)$$

(equation adapted from Lecture 5)

Firing rate is approximately in proportion to  $I_{stim}$ .

#### 2) Standard deviation of noise

Noise will lead the neuron to fire more erratically (as we found out in coursework 1), it induce difference between neurons, average firing behavior should not be affected to a large extend. However, with larger standard deviation, we get a flatter distribution and the noise distribution, which has a higher possibility to get a high value. In that case. When we extend the noise standard deviation, it is expected that we get firing rate a little bit higher with high standard deviation noise.

The result is illustrated in the picture with some possible values of strength or standard deviation of noise .

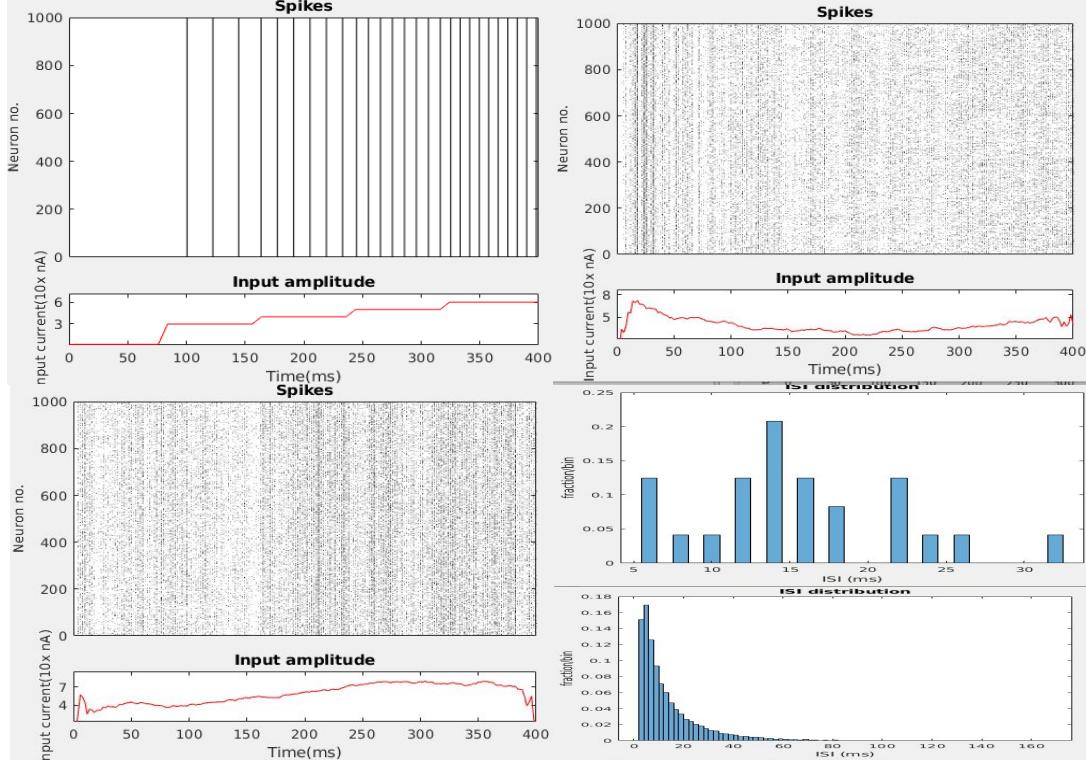


Figure 2: Example changes of firing behavior with some possible values of strength(left bottom) or standard deviation of noise(right top). The picture in left top shows the distribution with out noise and the picture in the right bottom shows the distribution of the inter spike interval of 0 standard deviation and 150 standard deviation.

As can be seen from the picture, the neuron fires nearly linearly more times when the strength increases, and the inter spike interval get more diverse given independent noise.

3) the noise of the current The noise of current is added as common noise, this induce small changes of input current and induce small changes that is same for all neurons. It is plotted under the individual neural behavior along with input strength. This noise makes neuron behaves differently to the input at different time. It should be averaged out though different runs. This noise does not is not included in the discussion.

**Question 2** (40 marks) What would happen if multiple populations from the previous question were chained in successive layers? Try to illustrate this using a simulation, you can take 5 layers each with  $N = 20$ , the first layer should receive input from a stimulus (as in Q1) and the rest only receive input through uniform connections from previous layers. You will need to use a sufficiently high gain  $W$  between layers.

### Answer 2

There are two very different ways to build connections between layers. The mean difference is whether the stimulus in the up layer depend on the firing of the layer below it. So I build two models to realize them independently.

#### model 1

I simulate the 5 layers of neurons with integrate and fire model. All the neurons are set with default parameters as question 1. The synapse transmission between layers is mimic with 'synaptic input' of integration and firing model: A neuron in the higher layer connects to all other neurons in the lower layer. And the weight of connection is 1. At a certain time interval  $itime$ , the input received by neurons in the higher layer is  $Istim$  if there is neuron firing in the previous layer, 0 if no neuron is firing in the previous layer. The membrane potential of one spike decayed given  $tausyn$ .

$$Isyn = I0 * (itime)$$

And the membranes potential after receiving synaptic input slowly decay to membrane potential.

$$dV = istim * Rm * (tausyn / (taum - tausyn)) * (\exp(-dt/taum) - \exp(-dt/tausyn))$$

$\tau_{syn}$  is set to be 2, and the equitation and value are adapted from lecture 5. The firing behavior of different layers of neurons are shown in Figure 3.

As can be seen from the pictures, the higher level the neuron is in, the more likely that the neurons fire at the same time. The firing rate of the five layers have a decreasing trend. And they are, 75.38, 25.75, 11.5, 6.75, 8.63.

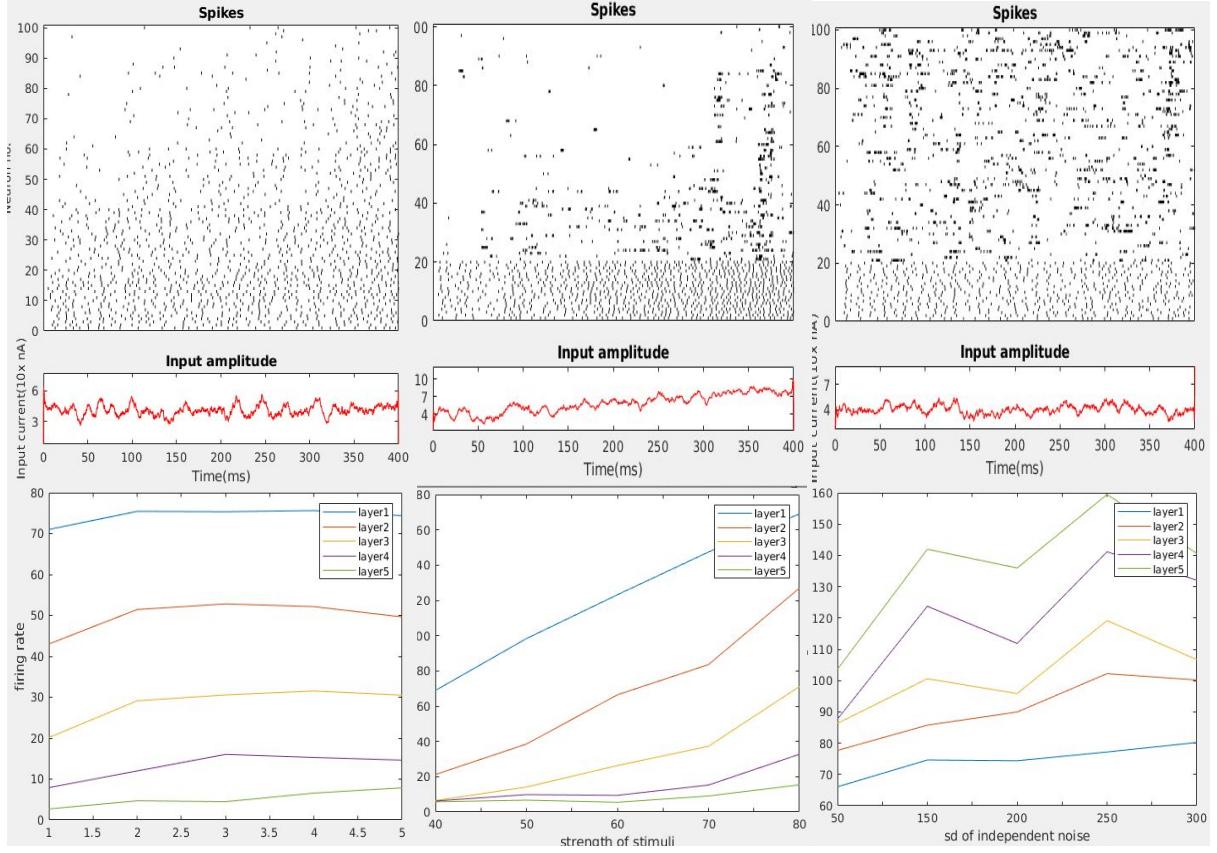


Figure 3: This figure contains The firing behaviors of all neurons (from bottom to top: 1 st layer to 5th layer) in the baseline condition(left). Changes of firing behavior with some possible values of strength(middle) or standard deviation of noise(right). The firing rate in the bottom picture is the average result of 20 runs.

1) Strength of Stimulus I changed the Strength of stimulus every 80 steps (8 ms). The time period is used to establish a relatively stable average firing behavior of neurons, thus the difference is more meaningful between different levels (than changing the stimulus per time bin). 5 levels of strength are used (40,50,60,70,80 nA), they are chose to be strong and different enough to yield different result than baseline. The firing rates of different layers increase with the strength of stimulus. The lower layer changes faster given the change. The first layer shows a near linear change in firing rate as the strength of stimuli increase (just as question 1). The increase of firing rate in 3-5 layers is much slower. This effect of stimulus strength might be restricted from the firing rate of the previous layers and the 'perceived increase' of stimuli strength is lower for the higher layers.

2) The standard deviation of noise The Standard deviation of noise is changed in 5 levels (50,150,200,250,300) The values are chose in similar ways with strength. The firing rate of different layers shows a trend of increasing with the standard deviation of the noise. The changes of different layers reversed comparing to the change with strength. The higher number of layers the neuron lays on, the more its firing rate increases with the increase of standard deviation of noise. At the same time, the firing behavior is more noisy. This might because: The increase membrane potential of higher levels of neuron is accumulated when the neuron in the lower level fires with decay. Some times they might have high membrane potential, but it decays before reaching the potential. A high standard deviation of noise at times gives high

values, provide a larger possibility for those trend to cross the threshold and makes the neuron more prepare for the spikes of lower layers. This also explains why the neuron tend to fire successively after the first spike (which is not realistic, and should be changed).

3) Generalize ability of the result The result showed in the picture is the average of 20 trials, the trend of the lines is different in different runs, but the order of the layers is stable.

## model 2

I then reduce the connection of neurons into simple forward firing rate model (like the one presented in network of Neurons in the second part of the lectures) with weight 0.4 between layers. All other settings include the sweeping of strength and standard deviation of noise are same to the previous model. (See Figure 4)

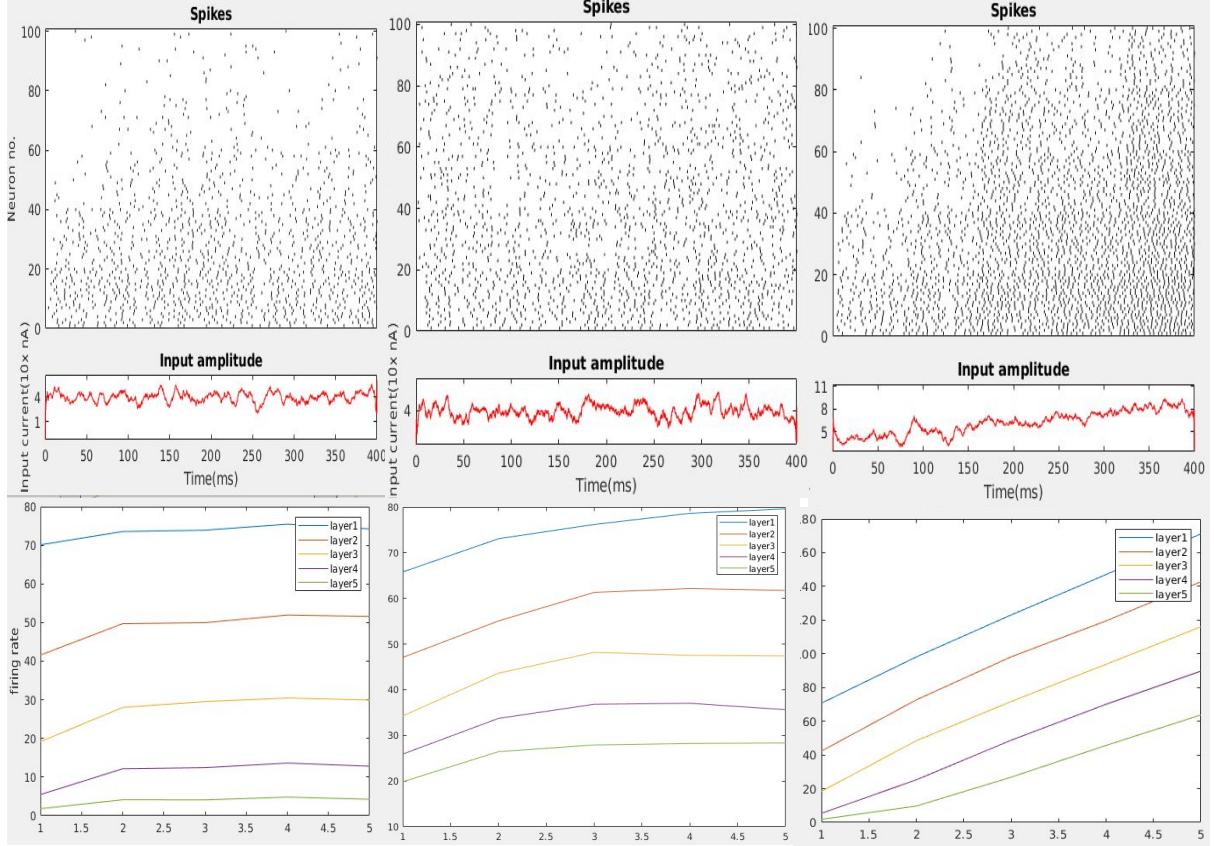


Figure 4: This figure contains The firing behaviors of all neurons (from bottom to top: 1 st layer to 5th layer) in the baseline condition(left). Changes of firing behavior with some possible values of standard diviation of noise(middle) or strength(right). The firing rate in the bottom picture is the average result of 20 runs.

1) Strength of Stimulus The result is similar to model 1, The firing rates of different layers increase with the strength of stimulus. All the layers changes near linearly with the change of stimuli strength. This might because the behavior of different layers is inherently similar, they are like neurons receiving different initial stimuli strength and increase at the same rate.

2) Sd of noise The result is very different compares to model 1. The The firing rates of dif-

ferent layers increase with the standard deviation of noise, the effect seems to reach saturation at the standard deviation of 200, and the degree they are influenced is similar to each other. Since the connection between the layers are mimic using firing rate multiplied by weight, it is expected that they behave in similar ways. Just as question 1, higher standard deviation do tend to provide, at times higher possibility of high membrane potential and lead to higher firing rate. Since the membrane potential is likely to be higher in the first layer, thus it also has the highest firing rate while less influenced by the saturation. It seems that the benefit relays on the original membrane potential.

3)Generalize ability of the result The result showed in the picture is the average of 20 trials, the trend of the lines is different in different runs, but the order of the layers is stable.

### Difference between models

The first model include the assumption that there are time related inter-connections between the layers in firing behaviors, the sum of firing behavior in the previous layer lead to the firing in the up layer. This casualty and time dependency is more similar to reality comparing to simplify neurons as firing rate.

This two models of connected neurons response similar to increasing stimuli strength, the firing rate increases in all layers. But they response differently to the standard deviation of the independent noise. This might be explained by the 'decay period' after receiving synaptic inputs. Unlike the firing rate model, the inputs received by the post synapse neuron is not evenly distributed in time. With in the decaying period, the neuron does not receive any input. While the membrane potential is higher than  $v_{rest}$ , it can not fire again unless being influenced by unexpected noise. This makes it more likely to be influenced by the noise than firing rate models.

**Question 3** (40 marks) Next, replace the background and stimulus current with synaptic input currents driven by a Poisson process. To do this you will need to set background and stimulus event rates  $rbg$ ,  $rstim$  and define a current amplitude  $ISyn0$ . Make sure this value is set sufficiently high such that spikes are generated ( $ISyn0 = 100$ ,  $rbg = 0.01$ ,  $rstim = 0.1$  should be a good starting point to explore). The synaptic events should decay exponentially with a time constant  $syn = 5ms$ . State how you implemented the Poisson input and verify that it has the desired properties. How can you get the high and low noise regimes from Question 1?

### Answer 3

The Poisson process is simulated in the following way (**A single Poisson process**):

Pre-synapse Neurotransmitter releasing is controlled by a Poisson process, The possibility of synaptic firing is set to be dependent on the tisi (the interval between spikes)

$$ps = (rstim + rbg) * \exp(-tisi * (rstim + rbg))$$

$rstim$  and  $rbg$  is the firing behavior of stimulus and background noise (per time bin (0.1)), they are set to default values (0.1 and 0.01). The  $rstim$  is updated for each layer in respect to the firing behavior of the neurons in this layer. After each spike, there are neuron transmitters (in this case 5 of them) to be released into the synapse, lead the membrane potential of the post synapse neuron increase.

$$v_{mem}(ie, itime) = v_{mem}(ie, itime) + isyn * dt * 5$$

This membrane potential lead the post synapse neuron to fire, at a certain rate (200-250), and release neuron transmitter according to the new firing rate. The releasing behavior is again modeled by the Poisson process.

In this case, with the time interval since the last spike getting larger, the possibility of releasing increases.

**(Neurons and Poisson Process)** A post synapse neuron receives input from a Poisson process calculated by the sum of its pre-synapse neurons' behavior (`rstim`), a common background noise in a certain rate (`rbg`). One neuron only receives input from one simulated Poisson process. The only difference between Poisson process the post synapse neurons receive is the random variable used to decide whether a release is placed or not in a single trial. The membrane potential induced by the neural transmitter decay with time constant `tausyn`.

The firing rate (`rstim`) used in generating the pre-synapse Poisson spikes are updated in each layer.

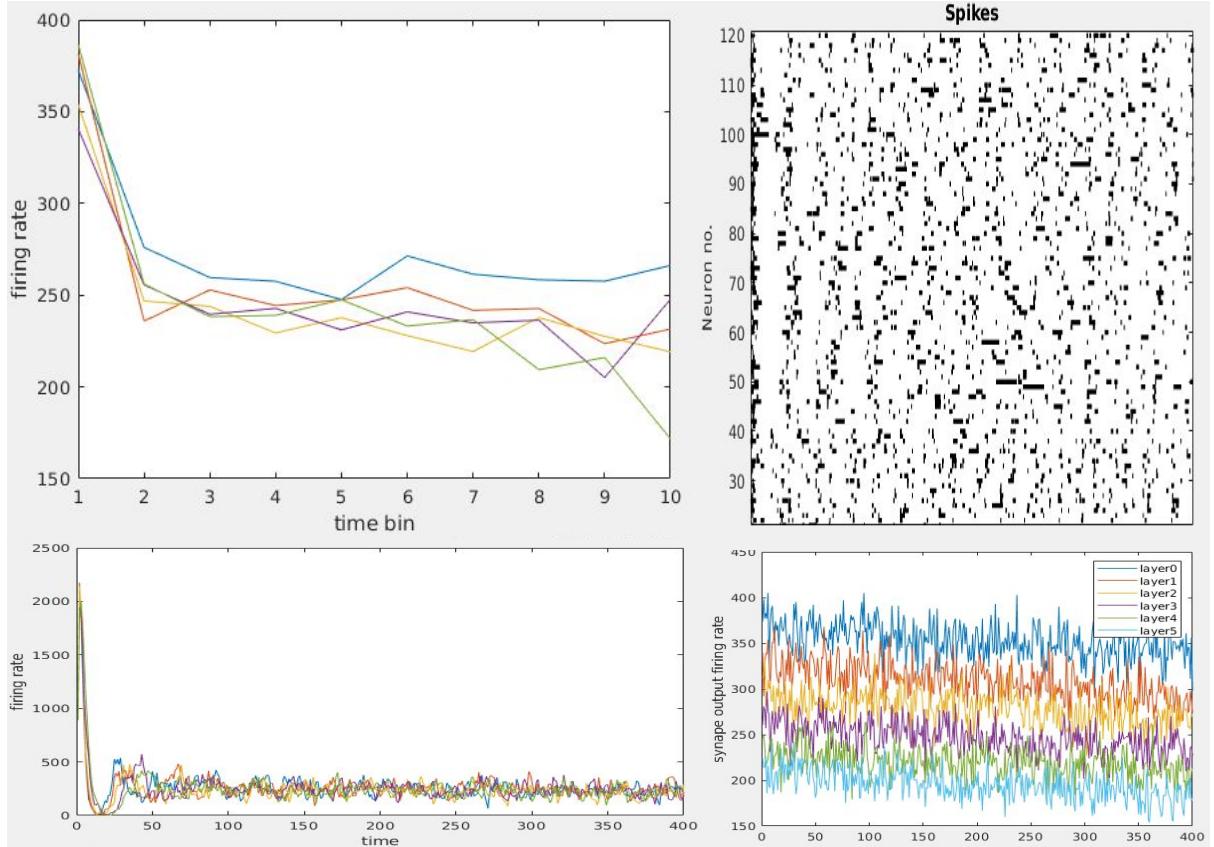


Figure 5: This figure contains The firing behaviors of all neurons (from bottom to top: 1 st layer to 5th layer) including firing rate (average, 10 time bins, left top, per time bin left bottom), and firing behaviour of single neuron (right top) neural transmitter release behavior(right bottom). all results are averaged in 50 runs.

**advantage of Poisson process** The model is simple and accurate. Poisson process simulation still treat neurons as firing rates, this is easy, fast to initialize and easy to control. But at the same time, it can mimic, very precise the dynamic of a synapse. The control is accomplished by the 'spiking interval'. It is not a common noise, which is same for all neurons, and it is not realized by adding large range of noise, which is not realistic and induce unpredictable changes of neuron potential. Instead, we mimic the noise using a very stable, predictable process of  $I_{syn}$ . Far less assumptions about how the noise is induced are made, we no longer have to

worry about the influence of too high or to low standard deviation of noise.

As a result the neurons produced by the Poisson models are very stable, the firing rate is stable after around 150 ms, it seems to has an intrinsic threshold of  $I_{syn} \geq 30$  mv, below which the neuron fires at a low rate (below 10). Achieving this standard, no matter how high the  $I_{syn}$  goes, the firing rate decays to 200-250 for all layers. This is very similar with the circumstance in real life situations. And allow the neurons to fire in a way that not influenced too much by the parameter settings.

This model is also more realistic than raw firing rate model in a way that it mimics the dynamic of synapse. With pre-synapse releasing controlled by Possion process and post synapse firing controled by membrane potential.

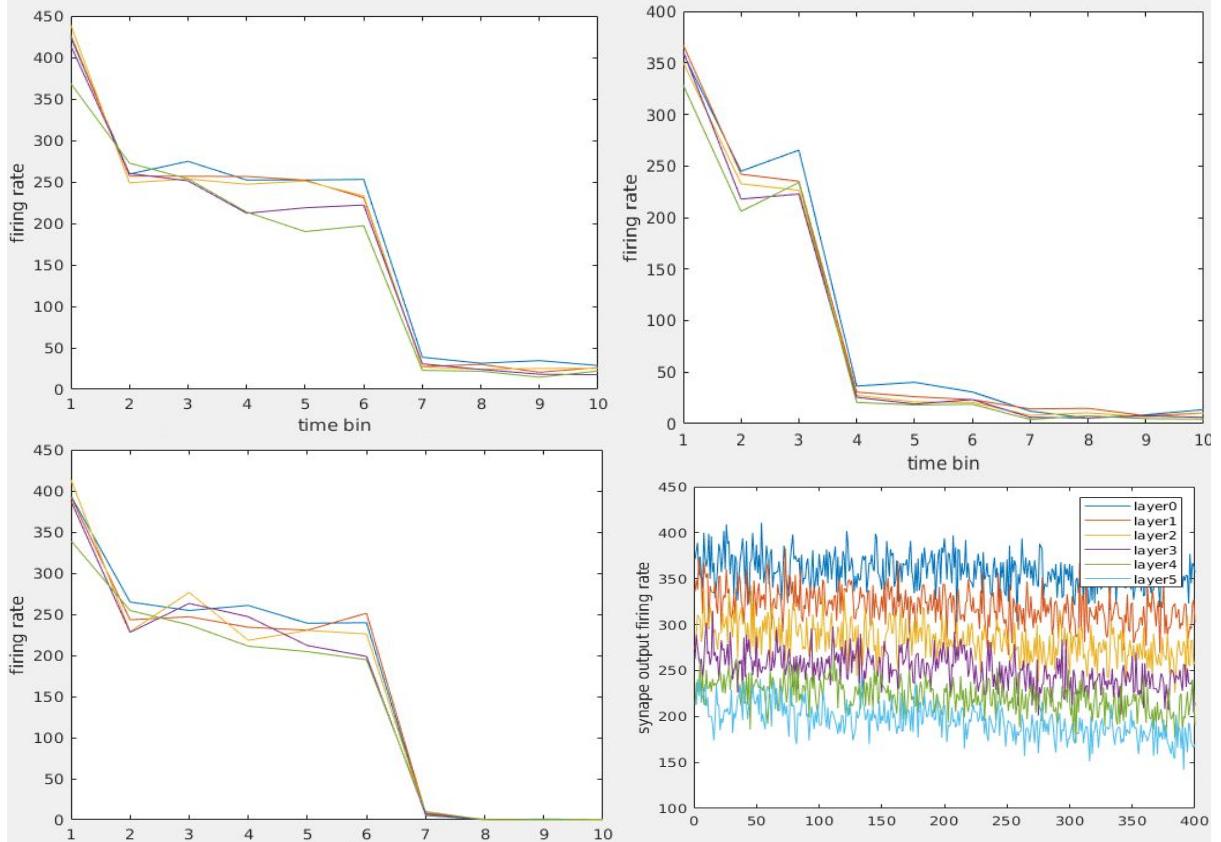


Figure 6: This figure contains The firing behaviors of all neurons (from bottom to top: 1st layer to 5th layer) in different  $I_{syn}$ . The three pictures of firing rate started as different values (100, 50, 150), the strength of  $I_{syn}$  is changed once at 120 ms (the 3/10 time bin) and changed again at 240 ms(the 3/10 time bin) to a lower value. The trend appears the same if the strength of  $I_{syn}$  does not go below 30mv. The bottom right picture shows the release of neuron transmitter of the 6 layers (0 layer is the pre-synapse layer for the first layer.)

Comparing between layers, all the layers are behaving similarly firing in rate 200-250, the high level neuron tend to fire less and send less neuron transmitters. The difference between layers is distinguishable.

### How the noise is added

In the Poisson process, noise is added as a background rate which indicate average random firing of the pre-synapse neuron., while the independent difference between neurons is inherently realized by the Poisson process. Compares to the connected model, this model does not

encourage the neuron (synapse) to fire in very close time bins. This makes it more realistic.

For independent differences between neurons, since the Poisson process is closely related to firing rate, the standard deviation of independent noise is same as the mean and thus is not changeable. But the independent variance should be inherently larger given higher firing rate. So, it would be like comparing different layers of the model, which, higher layers receive lower firing rate, and tend to fire less and release less.

The back ground noise is simulated as rbg, which include average random firing of the pre-synapse neuron. can be changed in the model. The following figure include a simulation of how the simulation changes with different background rate (0.01, 0.03 ,0.05) and (0, 0.05, 0.1)

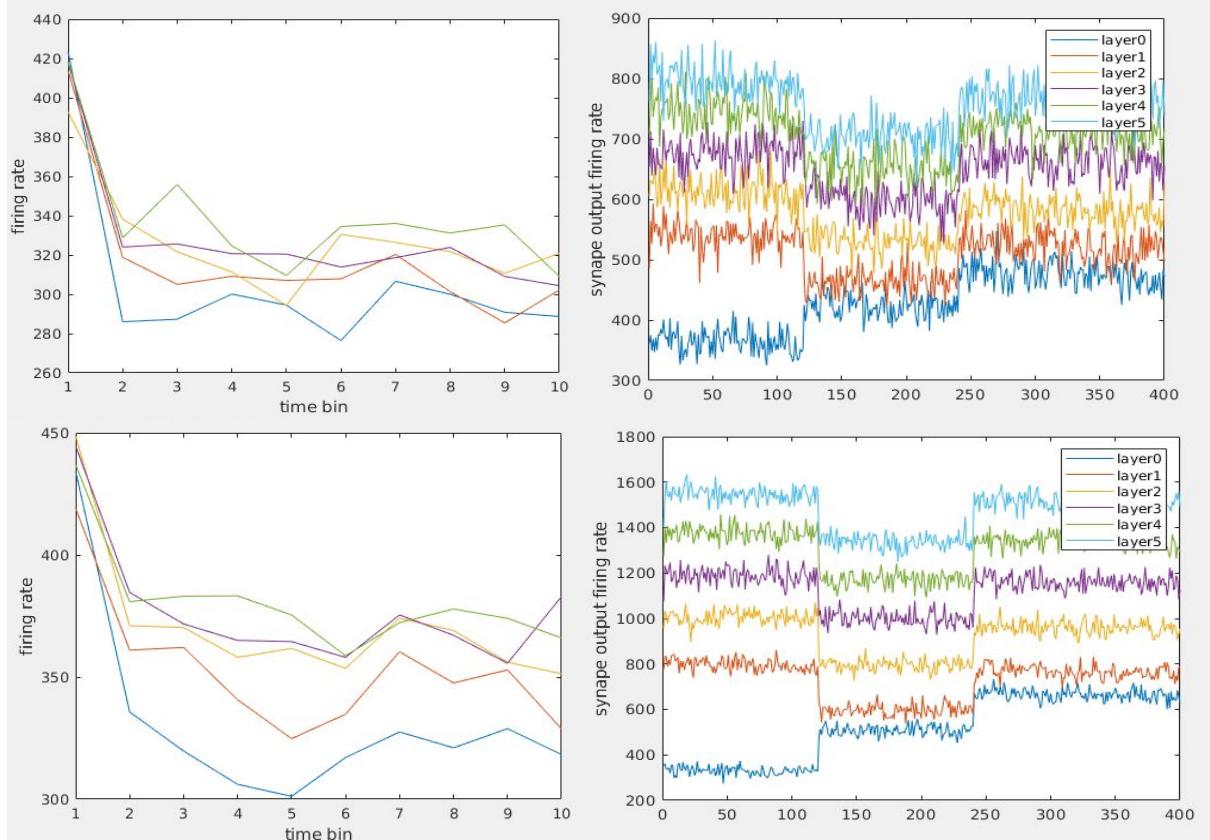


Figure 7: The rbg is changed in different scales in these simulations. It is changed once at 120 ms (the 3/10 time bin) and changed again at 240 ms. The top row is simulated with  $\text{rbg} = [0.01, 0.03, 0.05]$ , the bottom row with  $[0, 0.05, 0.1]$ , all result are averaged in 50 trials.

As can be seen in the picture, the noise influence the different layers in a different way, the higher layers (5th etc) are more likely to be influenced by the background noise, their firing behavior changes a lot with higher noise, they tend to fire more, and release more neural transmitters in a whole picture. It is unclear why the releasing pattern shows a clear decay in the middle period. It might be due to the different increase speeds between layers.

A pure simulation for 400 ms is provided below. The background noise is set to 0, 0.05 and 0.1 and run for 400ms each.

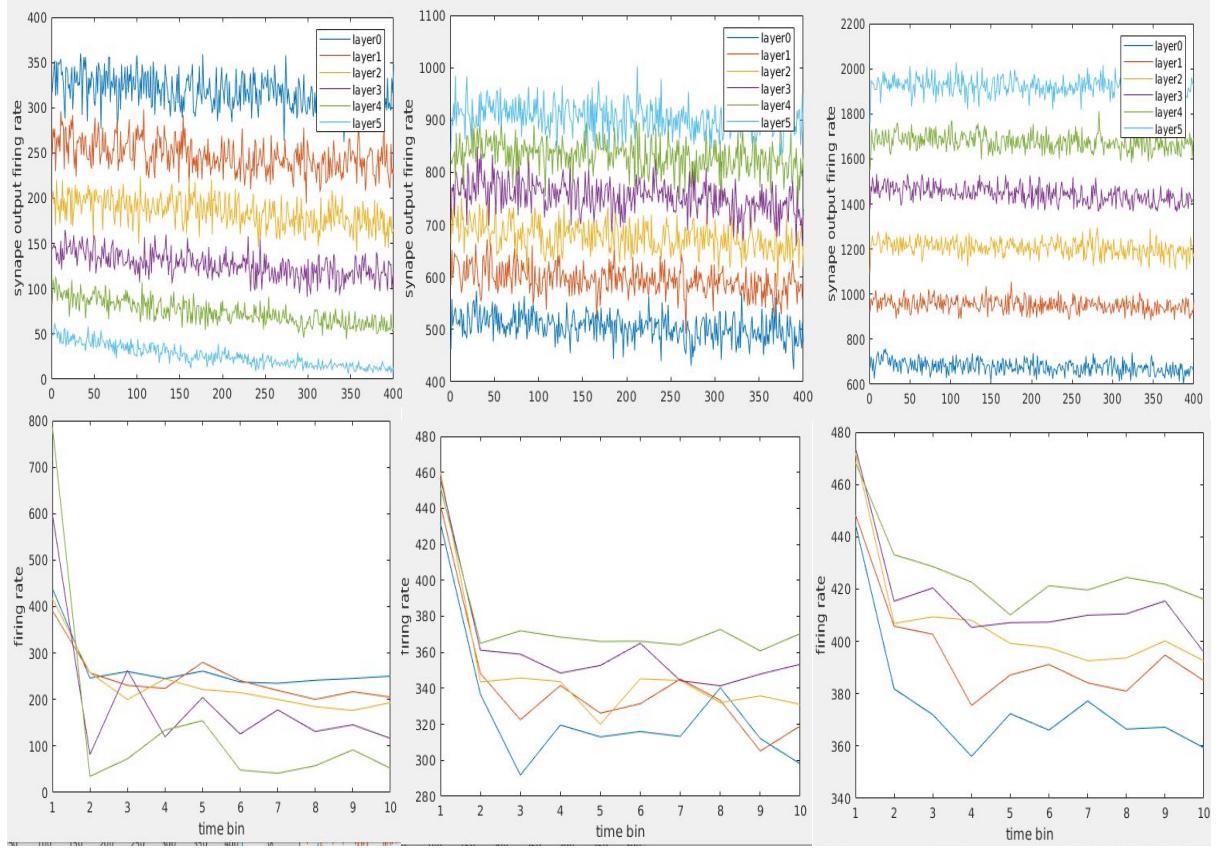


Figure 8: The rbg is changed in different scales in these simulations. left and side rbg=0, middle rbg=0.05, right rbg=0.1.

This simulation is more clear how the firing rate changes with the background noise.

The result is similar with the connected model we build in question 2. Higher level of neuron tend to be influenced more with the background noise rate get higher. The principle behind the process might be similar too. Since I am trying to mimic the time dynamic of neuron in the first model, this similarity might indicates that the Poisson process inherently include some properties of the connected simulated connections.

## Conclusion

In conclusion, in this coursework, I simulated the neural network in different ways: Fully connected forward model, and Poisson model. I adjust the parameters of strength and standard deviation of noise in the forward model, the background rate in the Poisson model. This provide better understanding of how the models work. All models are sensitive to the strength of inputs, that result in higher firing rate of all layers. The model assumed connections between firing neurons and The Poisson model both have the character that neuron fires together and higher levels of layers are more sensitive to noise.

## appendix

The code are included in the package code.