Diffusion-based Time Series Imputation on Financial Data

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1 Introduction

Diffusion-based generative model is the most state-of-art technique proposed by [4], noising and denoising in the forward and backward process. Comparing with other deep generative models, Generative Adversarial Network (GAN) [1] is unstable in training where mode collapse might happen suddenly, likelihood-based methods like Variational AutoEncoder(VAE) [2] and flow-based generative model have elegant theoretical properties but their performance is a little weaker than GAN whose limitation might come from the prior distribution, score-based methods extend the original score to that with respect to data, and gradually tend to the underlying distribution with additive score and a small noise $x_{i+1} \leftarrow x_i + \epsilon \nabla_x \log p(x) + \sqrt{2\epsilon} z_i, z_i \in \mathcal{N}(0,1), \epsilon \to 0, i = 1, ...T, T \to \infty$. In each step, noise is added for the sake of converging to a distribution rather than a single point. Such naive sore-based method already utilize the idea as noising by additive term z_i and denoising by the score from data $\nabla_x \log p(x)$ at the same time.

Major pitfalls of naive score-based model are the hypothesis of data concentrating on a low dimensional manifold, inaccurate score matching especially in the region lacking of samples, and difficulty in recovery modes [3]. Diffusion model arises to handle these well, and the most important contribution from the perspective of statistical inference is that it provide a successful example to bypass the prior. More concretely, diffusion model consists of forward noising process $q(x_t|x_{t-1}) = \mathcal{N}(x_t; \sqrt{1-\beta_t} x_{t-1}, \beta_t I)$ and backward denoising process $p_\theta(x_{t-1}|x_t)$ to be trained, where $t = 1, ..., T, x_T \sim \mathcal{N}(0, I)$.

References

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[3] Calvin Luo. "Understanding diffusion models: A unified perspective". In: *arXiv preprint arXiv:2208.11970* (2022).

[4] Jascha Sohl-Dickstein et al. "Deep unsupervised learning using nonequilibrium thermodynamics". In: *International Conference on Machine Learning*. PMLR. 2015, pp. 2256–2265.