

# Reinforcement Learning & Lab3

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# Outline

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- Reinforcement Learning for Gomoku
- POMDP for Spoken Dialog Systems
- Lab3

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# **Reinforcement Learning for Gomoku**

# Reinforcement Learning for Gomoku

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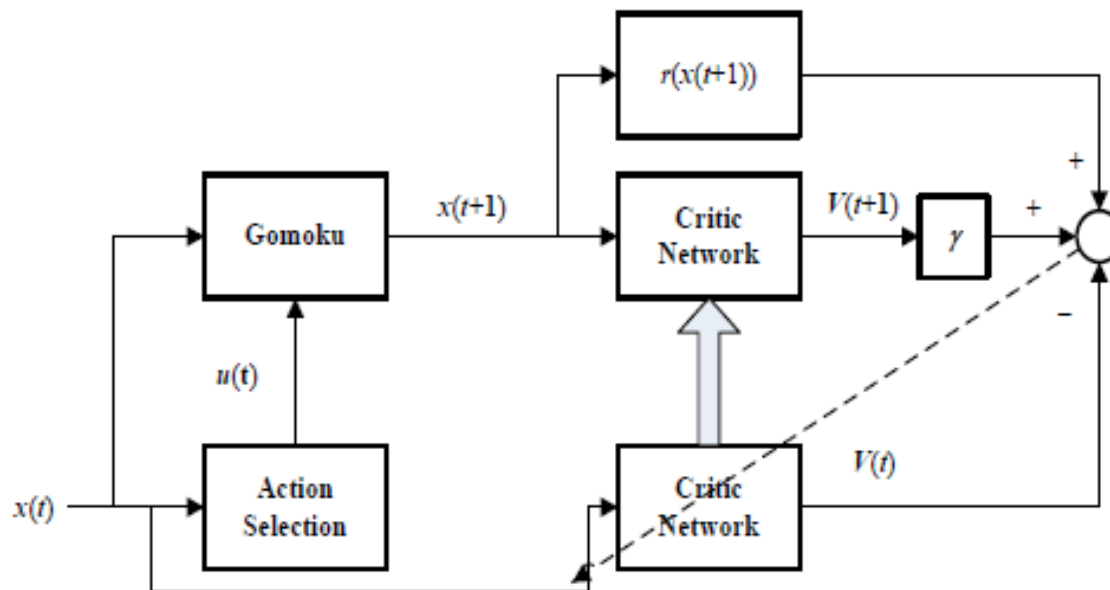
- Adaptive Dynamic Programming for Gomoku
- ADP with MCTS for Gomoku

# ADP for Gomoku

- Key idea of ADP

In TD learning, the action decision or value function can be described in continuous form, approximated by nonlinear function such as neural network

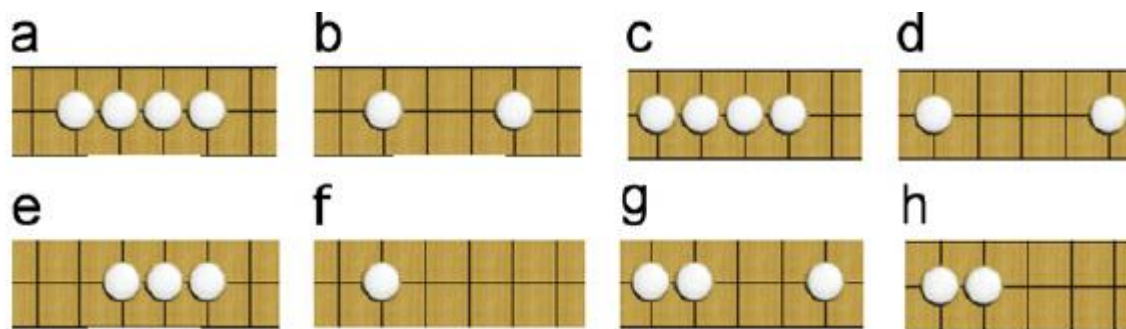
- The ADP structure



# ADP for Gomoku

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- Input state to describe a board situation
  - 20 patterns for each of two players, totally 40 patterns



- Whose turn to move
- In the offensive/defensive(Who is first to move)

# ADP for Gomoku

- Input state to describe a board situation
  - Five input nodes indicate the number of every pattern except for five-in-a-row ( $n$  denotes the number of a pattern)

Value of $n$	Input 1	Input 2	Input 3	Input 4	Input 5
0	0	0	0	0	0
1	1	0	0	0	0
2	1	1	0	0	0
3	1	1	1	0	0
4	1	1	1	1	0
$> 4$	1	1	1	1	$(n-4)/2$

- The number of the special pattern five-in-a-row, is represented by 1 input node. If this pattern shows up, then its input is 1, otherwise 0

# ADP for Gomoku

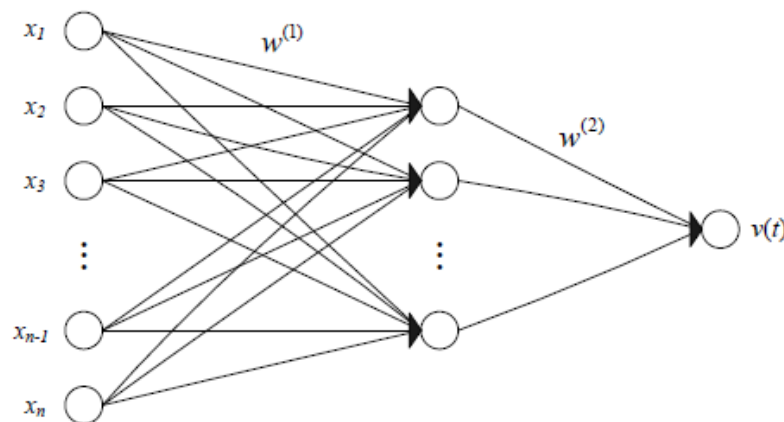
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- Input state to describe a board situation
  - For each pattern we assign two input nodes to represent the turn
  - Use two input nodes to indicate which player is the first to move
  - Totally  $19*5*2+1*1*2+40*2+2 = 274$  input nodes



# ADP for Gomoku

- Critic Network in the ADP
  - Used to evaluate board situations(winning probability of player1)
  - A feed forward three-layer fully connected neural network



- Unnecessary to be neural network, you can try other functions

# ADP for Gomoku

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- Critic Network in the ADP
  - Train the neural network
    - Define the prediction error

$$e(t) = \alpha[r(t+1) + \gamma V(t+1) - V(t)]$$

- To minimize the objective error

$$E(t) = \frac{1}{2}e^2(t)$$

# ADP for Gomoku

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- Action
  - Player 1 chooses the move that leads to the state with the maximal output value obtained from the neural network.
  - Player 2 selects the move that leads to the state with the minimal output value obtained by the same neural network.

# ADP for Gomoku

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- Action
  - Reduce the action space
    - Only considering the empty positions near the ones occupied
    - When there are several alternative actions which have equally high evaluation, we simply choose the one that is last found

# ADP for Gomoku

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- Action
  - Cope with the exploration and exploitation dilemma
    - Let player 2 randomly select his first move, meanwhile player 1 place his piece on the center of the board if he is in the offensive and select his first move randomly if he is in the defensive
    - Let both players select moves following  $\epsilon$ -greedy policy

$$a(t) = \begin{cases} \arg \max_a V(t+1) & \text{with probability } 1-\epsilon \\ \text{random action} & \text{with probability } \epsilon \end{cases}$$

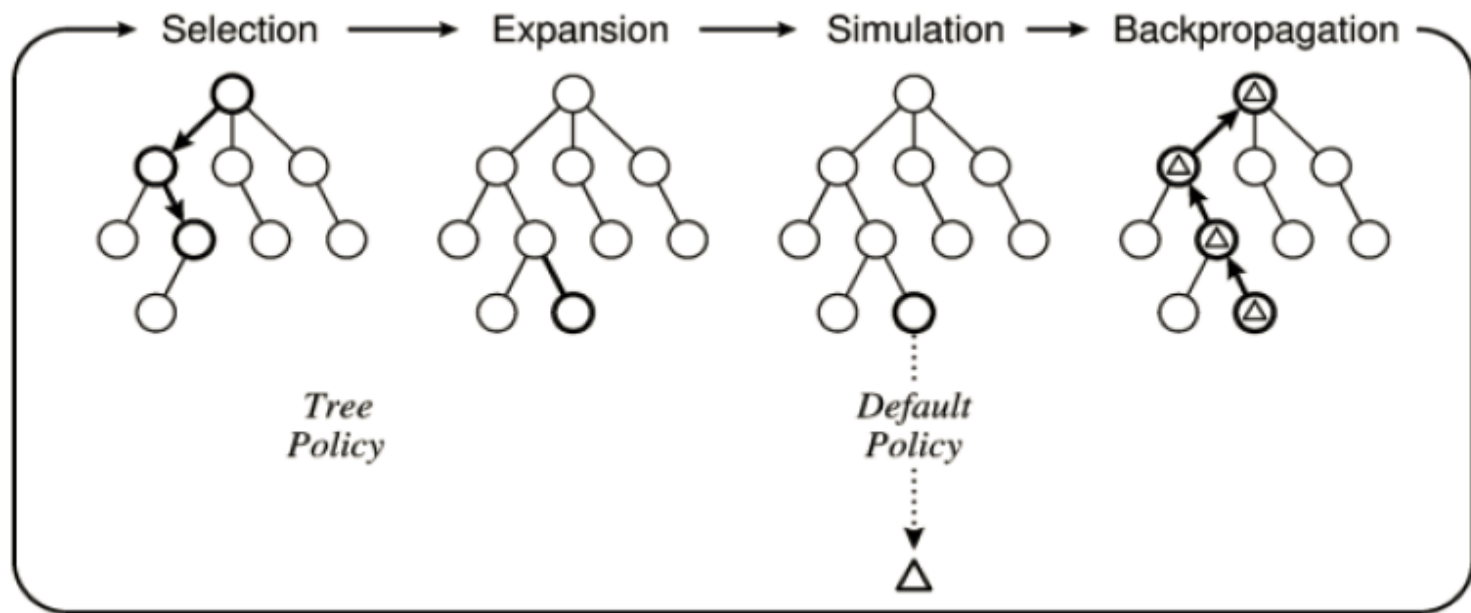
# ADP for Gomoku

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- Reward
  - The reward is set to 0 during the game.
  - After a game, if player 1 wins, the final reward is 1, if he loses, the reward is 0

# ADP with MCTS for Gomoku

- Monte Carlo Tree Search(MCTS)



# ADP with MCTS for Gomoku

- Heuristic Monte Carlo Tree Search(HMCTS)
- Save more time in simulation than random sampling and get converge earlier

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## Algorithm 1: HMCTS for Gomoku

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```
input original state  $s_0$ ;  
output action  $a$  corresponding to the highest value of MCTS;  
add Heuristic Knowledge;  
obtain possible action moves  $M$  from state  $s_0$ ;  
for each move  $m$  in moves  $M$  do  
    reward  $r_{total} \leftarrow 0$ ;  
    while simulation times < assigned times do  
        reward  $r \leftarrow \text{Simulation}(s(m))$ ;  
         $r_{total} \leftarrow r_{total} + r$ ;  
        simulation times add one;  
    end while  
    add  $(m, r_{total})$  into  $data$ ;  
end for each  
return action Best( $data$ )
```

```
Simulation(state  $s_t$ )  
    if ( $s_t$  is win and  $s_t$  is terminal) then return 1.0;  
    else return 0.0;  
end if  
if ( $s_t$  satisfied with Heuristic Knowledge)  
    then obtain forced action  $a_f$ ;  
        new state  $s_{t+1} \leftarrow f(s_t, a_f)$ ;  
    else choose random action  $a_r \in$  untried actions;  
        new state  $s_{t+1} \leftarrow f(s_t, a_r)$ ;  
end if  
return Simulation( $s_{t+1}$ )
```

```
Best( $data$ )  
    return action  $a$  //the maximum  $r_{total}$  of  $m$  from  $data$ 
```

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# ADP with MCTS for Gomoku

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- Heuristic rules
  - If four-in-a-row is occurred in my side, the player will be forced to move its piece to the position where it can emerge five-in-a-row in my side.
  - If four-in-a-row is occurred in opposite side, the player will be forced to move its piece to the position where it can block five-in-a-row in opposite side.
  - If three-in-a-row is occurred in my side, the player will be forced to move its piece to the position where it can emerge four-in-a-row in my side.
  - If three-in-a-row is occurred in opposite side, the player will be forced to move its piece to the position where it can block four-in-a-row in opposite side.

# ADP with MCTS for Gomoku

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- Upper Confidence bounds for Tree(UCT)

- Based on Upper Confidence Bounds(UCB)

$$\frac{Q(v')}{N(v')} + c\sqrt{\frac{2 \ln N(v)}{N(v')}}}$$

- $\frac{Q(v')}{N(v')}$  is the average reward of node  $v'$ ,  $N(v')$  and  $N(v)$  is the visited count of node  $v'$  and  $v$
    - Balance the conflict between exploration and exploitation and find out the final result earlier

# ADP with MCTS for Gomoku

- Upper Confidence bounds for Tree (UCT)

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## Algorithm 2: UCT for Gomoku

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```
input create root node  $v_0$  with state  $s_0$ ;  
output action  $a$  corresponding to the highest value of UCT;  
while within computational budget do  
     $v_l \leftarrow \text{Tree Policy}(v_0)$ ;  
    Policy  $\leftarrow$  Heuristic Knowledge;  
    reward  $r \leftarrow \text{Policy}(s(v_l))$ ;  
    Back Update( $v_l, r$ );  
end while  
return action  $a(\text{Best Child}(v_0))$ 
```

```
Tree Policy(node  $v$ )  
    while  $v$  is not in terminal state do  
        if  $v$  not fully expanded then return Expand( $v$ );  
        else  $v \leftarrow \text{Best Child}(v, 1/\sqrt{2})$ ;  
    end if  
end while  
return  $v$  //this is the best child node
```

```
Expand(node  $v$ )  
    choose random action  $a \in$  untried actions from  $A(s(v))$ ;  
    add a new child  $v'$  to  $v$   
    with  $s(v') \leftarrow f(s(v), a)$  and  $a(v') \leftarrow a$ ;  
    return  $v'$  //this is the expand node
```

```
Best Child(node  $v$ , parameter  $c$ )  
    return arg max $v' \in \text{child}$  ( $(Q(v') / N(v')) + c\sqrt{2 \ln N(v) / N(v')}$ )
```

```
Policy(state  $s$ )  
    while  $s$  is not terminal do  
        if  $s$  satisfied with heuristic knowledge then  
            obtain forced action  $a$ ;  
        else choose random action  $a \in A(s)$  uniformly;  
        end if  
         $s \leftarrow f(s, a)$ ;  
    end while  
    return reward for state  $s$ 
```

```
Back Update(node  $v$ , reward  $r$ )  
    while  $v$  is not null do  
         $N(v) \leftarrow N(v) + 1$ ;  
         $Q(v) \leftarrow Q(v) + r$ ;  
         $v \leftarrow \text{parent of } v$ ;  
    end while
```

# ADP with MCTS for Gomoku

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- ADP with MCTS
  - Use ADP to train critic network, get 5 candidate moves and their ADP winning probabilities
  - Take candidate moves as the root node of MCTS and simulate, get their MCTS winning probabilities
  - Calculate the weighted sum of two winning probabilities:

$$w_p = \lambda w_1 + (1 - \lambda) w_2$$

# ADP with MCTS for Gomoku

- ADP with MCTS

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**Algorithm 3: ADP with MCTS**

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```
input original state  $s_0$ ;  
output action  $a$  correspond to ADP with MCTS;  
 $M_{ADP}, W_{ADP} \leftarrow \text{ADP Stage}(s_0)$ ;  
 $W_{MCTS} \leftarrow \text{MCTS Stage}(M_{ADP})$ ;  
for each  $w_1, w_2$  in pairs( $W_{ADP}, W_{MCTS}$ ) do  
     $w_p \leftarrow \lambda w_1 + (1-\lambda)w_2$ ;  
    add  $p$  into  $P$ ;  
end for each  
return action  $a$  correspond to max  $p$  in  $P$ 
```

ADP Stage(state  $s$ )

```
    obtain top 5 winning probability  $W_{ADP}$  from ADP( $s$ ) ;  
    obtain their moves  $M_{ADP}$  correspond to  $W_{ADP}$ ;  
    return  $M_{ADP}, W_{ADP}$ 
```

MCTS Stage(moves  $M_{ADP}$ )

```
    for each move  $m$  in  $M_{ADP}$  do  
        create  $m$  as root node with correspond state  $s$   
        obtain  $w_2$  from  $MTCS(m, s)$   
        add  $w_2$  into  $W_{MCTS}$   
    end for each  
    return  $W_{MCTS}$ 
```

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# ADP with MCTS for Gomoku

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- ADP with MCTS
  - Compared to ADP :
    - Eliminate the neural network evaluation function's "short sight" defect, ensure the accuracy of the search
  - Compared to MCTS :
    - Save a large amount of time to find out the suitable action for Gomoku

# Reinforcement Learning for Gomoku

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- References:
  - Self-teaching adaptive dynamic programming for Gomoku
  - ADP with MCTS Algorithm for Gomoku

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# **POMDP for Spoken Dialog Systems**



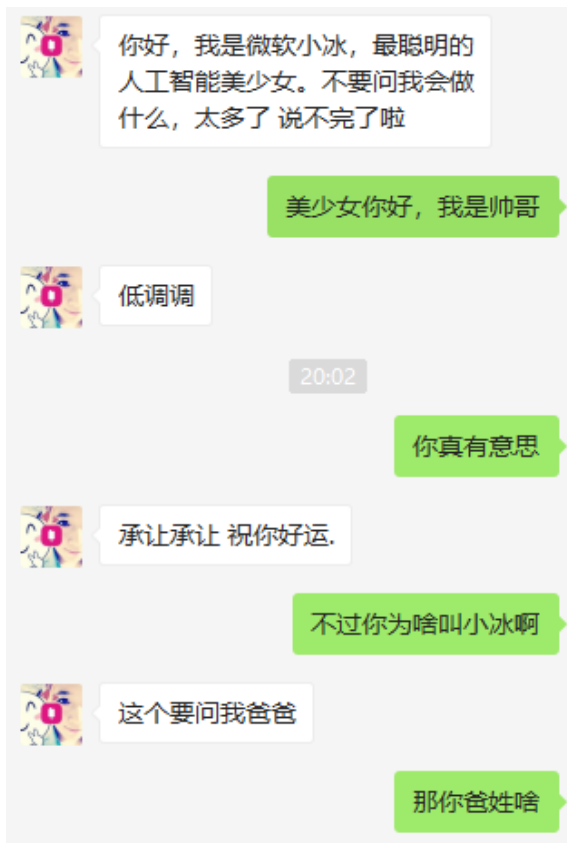
# POMDP for Spoken Dialog Systems

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- Spoken Dialog System(SDS)
- Partially Observable Markov Decision Process(POMDP)
- POMDP-Based SDSs

# Spoken Dialog System(SDS)

## ● Chatbot vs SDS



你好，我是微软小冰，最聪明的人工智能美少女。不要问我会做什么，太多了说不完了啦

美少女你好，我是帅哥

低调调

20:02

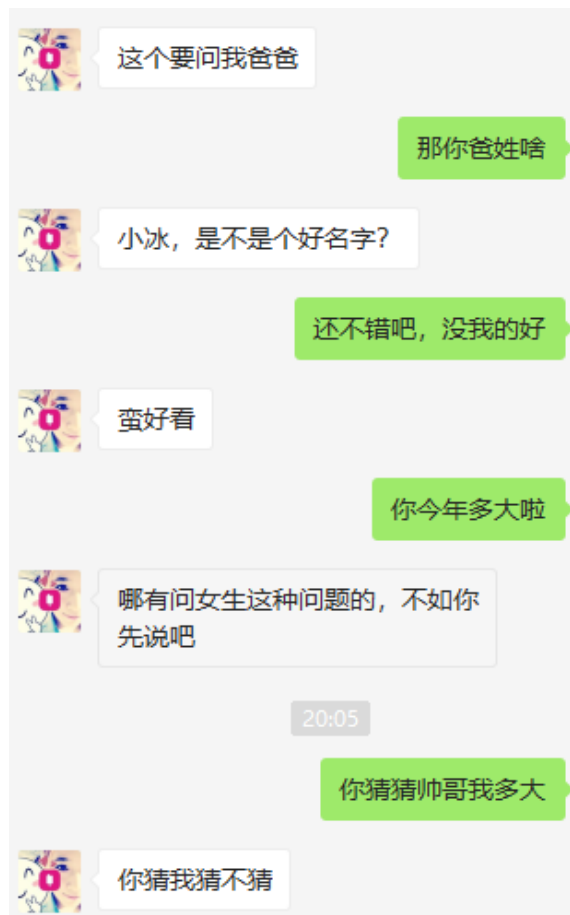
你真有意思

承让承让 祝你好运.

不过你为啥叫小冰啊

这个要问我爸爸

那你爸姓啥



这个要问我爸爸

那你爸姓啥

小冰，是不是个好名字？

还不错吧，没我的好

蛮好看

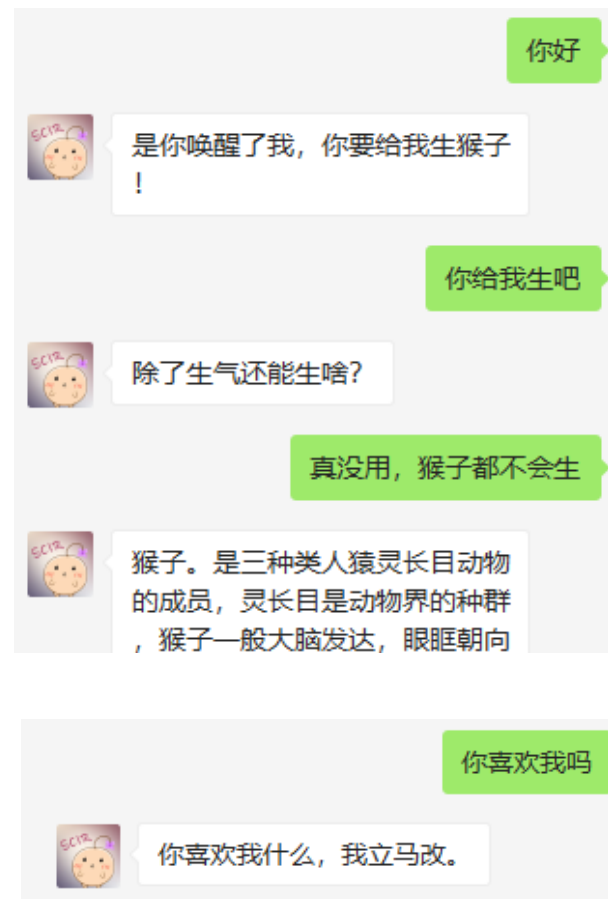
你今年多大啦

哪有问女生这种问题的，不如你先说吧

20:05

你猜猜帅哥我多大

你猜我猜不猜



你好

是你唤醒了我，你要给我生猴子！

你给我生吧

除了生气还能生啥？

真没用，猴子都不会生

猴子。是三类人猿灵长目动物的成员，灵长目是动物界的种群，猴子一般大脑发达，眼眶朝向

你喜欢我吗

你喜欢我什么，我立马改。

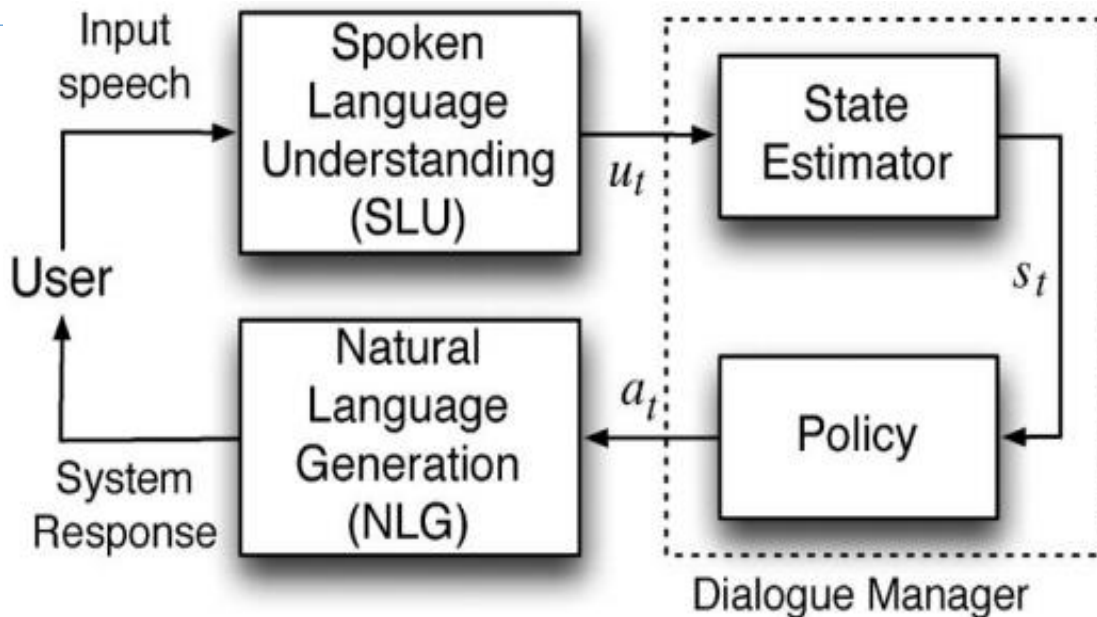
# Spoken Dialog System(SDS)

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- Chatbot vs SDS
  - Compared to chatbot, SDS is task-oriented and user has an intent



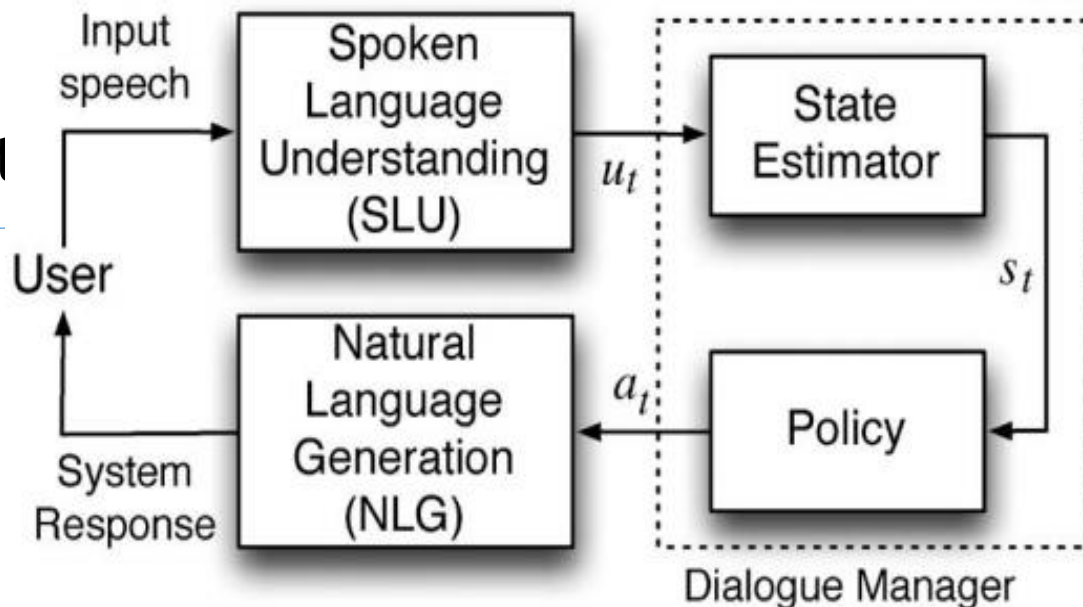
# Spoken Dialog System(SDS)



- **Structure**

- **SLU**: Map user's input speech into some abstract semantic representation  $u_t$
- **NLG**: Map the action  $a_t$  (semantic representation of DM's decisions) into response speech
- **DM**: Make decision according to  $u_t$ , which is our focus

# Spoken Dialog System



- Working process
- One approach is to fill the slot-value pairs until all slots are filled
  - At each turn  $t$ , SLU component converts each spoken input into an abstract semantic representation called a user dialog act  $u_t$
- Example: travel booking system
  - DM: system updates its internal state and a deterministic decision rule called a policy maps the state into an action  $a_t$
  - Input speech: "I want to go from Paris to London"
  - The system act  $a_t$  is then converted back into speech via NLG
  - component
  - $a_t$ : {time: \_}
  - System response: "When do you want to go? "

# Spoken Dialog System(SDSs)

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- Problems

- Large cost of laboriously handcrafting complex dialog manager
- Speech recognition error rate is still in the range 15%~30%,  
which make it fragile in operation

- Solution

- Take a good statistical approach such as MDP find a optimal policy through Reinforcement learning
- However, MDP assume that the entire state is observable

# POMDP

---

- Definition
  - A MDP is defined as a tuple  $(S, A, T, R, \gamma)$
  - A POMDP is defined as a tuple  $(S, A, T, R, O, Z, \gamma, b_0)$ 
    - $S$  is a set of states with  $s \in S$ ;  $A$  is a set of actions with  $a \in A$
    - $T$  defines a transition probability of environment  $P(s_t | s_{t-1}, a_{t-1})$
    - $R$  defines the reward  $r(s_t, a_t) \in \mathbb{R}$
    - $O$  is a set of observations with  $o \in O$
    - $Z$  defines an observation probability  $P(o_t | s_t, a_{t-1})$
    - $b_0$  is an initial belief state

# POMDP

---

- Definition
  - The interpretation state  $s_t$  is not directly observable
  - A belief state  $b_t$  is maintained where  $b_t(s_t)$  indicates the probability of being in a particular state  $s_t$ , reflecting the uncertainty of the true environment state



# POMDP

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- Definition

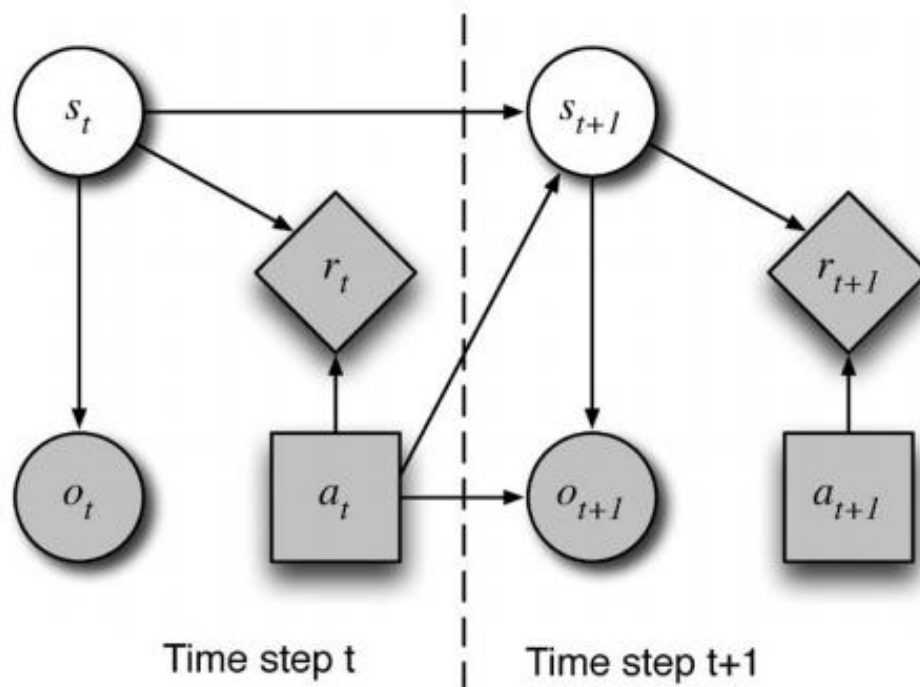
- The goal is to take actions at each time step that maximize its expected future discounted reward:

$$E \left[ \sum_{t=0}^{\infty} \gamma^t r_t \right]$$

- Because of the uncertainty of the true environment state, the agent must make decisions based on belief state

# POMDP

- Influence diagram



# POMDP

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- Belief Monitoring
  - Through interacting with the environment and receiving observations, the agent may update its belief in the true state (belief distribution of true state)

# POMDP

---

$$b_{t+1}(s_{t+1}) = p(s_{t+1}|o_{t+1}, a_t, \mathbf{b}) = \frac{p(o_{t+1}|s_{t+1}, a_t, \mathbf{b})p(s_{t+1}|a_t, \mathbf{b})}{p(o_{t+1}|a_t, \mathbf{b})}$$

$$= \frac{p(o_{t+1}|s_{t+1}, a_t) \sum_{s_t} p(s_{t+1}|a_t, \mathbf{b}, s_t)p(s_t|a_t, \mathbf{b})}{p(o_{t+1}|a_t, \mathbf{b})}$$

$$= k \cdot p(o_{t+1}|s_{t+1}, a_t) \sum_{s_t} p(s_{t+1}|a_t, s_t)b_t(s_t)$$

$k = 1/p(o_{t+1}|a_t, \mathbf{b})$  is a normalizing constant with

$$p(o_{t+1}|a_t, \mathbf{b}) = \sum_{s_{t+1}} p(o_{t+1}|s_{t+1}, a_t) \sum_{s_t} p(s_{t+1}|a_t, s_t)b_t(s_t)$$

# POMDP

---

- Belief MDP
  - A Markovian belief state allows a POMDP to be formulated as a MDP where every belief state is a state
  - Belief MDP is defined as a tuple  $(B, A, \tau, r, \gamma)$ 
    - $B$  is the set of belief states over the POMDP states
    - $A$  is the same finite set of action as for the original POMDP
    - $\tau$  is the belief state transition function
    - $r$  is reward function on belief states,  $r(b_t, a_t)$

# POMDP

---

- Belief MDP

- $\tau$  and  $r$  need to be derived from original POMDP

$$\tau(b_{t+1}|a_t, b_t) = \sum_{o_{t+1}} p(b_{t+1}|b_t, a_t, o_{t+1})p(o_{t+1}|a_t, b_t)$$

$$r(b_t, a_t) = \sum_{s_t} R(s_t, a_t)b_t(s_t)$$

- The belief MDP is not partially observable anymore, since at any given time the agent knows its belief state

# POMDP

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- Optimizing POMDP

- Value function

$$V^\pi(b_t) = r(b_t, \pi(b_t)) + \gamma \sum_{o_{t+1}} P(o_{t+1}|b_t, \pi(b_t)) V^\pi(b_{t+1})$$

- Bellman optimality equation

$$V^*(b_t) = \max_{a_t} \left[ r(b_t, a_t) + \gamma \sum_{o_{t+1}} P(o_{t+1}|b_t, a_t) V^*(b_{t+1}) \right]$$

- Optimal Policy

$$\pi^*(b_t) = \operatorname{argmax}_{a_t} \left[ r(b_t, a_t) + \gamma \sum_{o_{t+1}} P(o_{t+1}|b_t, a_t) V^*(b_{t+1}) \right]$$

# POMDP-Based SDSs

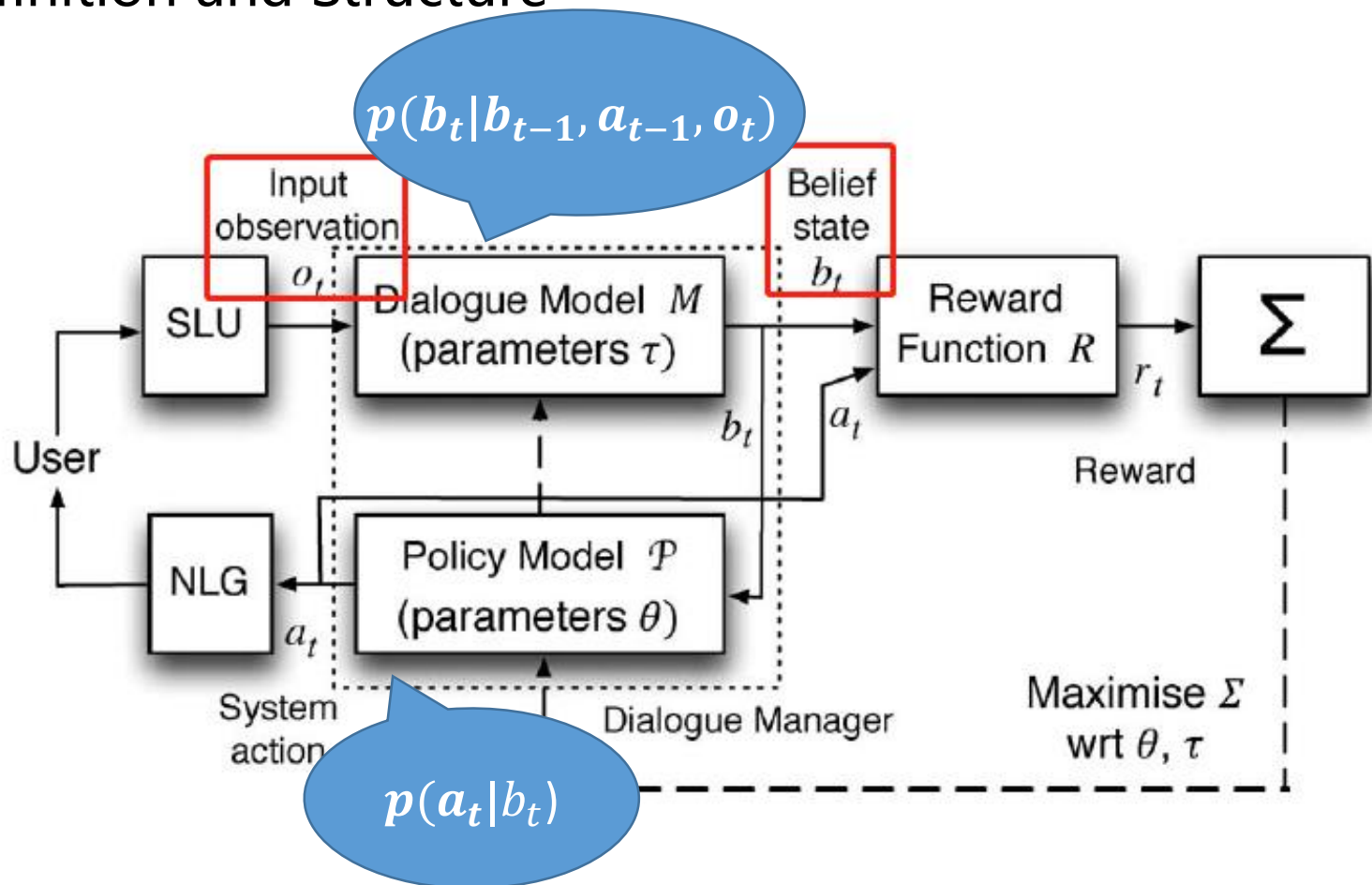
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- Why POMDP?
  - Modelling the inherent uncertainty of user utterances  
(provide robustness against the speech recognition errors)
  - Reduce the cost of laboriously handcrafting complex dialog managers



# POMDP-Based SDSs

- Definition and Structure



# POMDP-Based SDSs

---

- User Simulators

- Why need?

Algorithm requires around  $10^5$  dialogs to optimize a policy,

However the availability of large diverse corpora and/or large numbers of real users willing to interact with a partially trained dialog system is normally limited

# POMDP-Based SDSs

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- User Simulators
  - Operations
    - Given a sequence of user acts and system responses, the aim is to model the distribution of plausible user responses from which an actual user response can be sampled

$$p(u_t | a_t, u_{t-1}, a_{t-1}, u_{t-2}, \dots)$$

# POMDP-Based SDSs

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- User Simulators
  - N-gram based on Information State
  - Treat a dialogue as a sequence of pairs of speech acts and tasks

# POMDP-Based SDSs

---

- User Simulators
  - It takes as input the  $n-1$  most recent <speech act, task> pairs in the dialogue history, and uses the statistics of  $n$ -grams in the training set to decide on the next user action
  - If no  $n$ -grams match the current history, the model can back-off to smaller  $n$ -grams

# POMDP-Based SDSs

---

- User Simulators

- Example: flight reservation

- History:

ACT: opening, instruction, request\_info,  
[provide \_info], implicit\_confirm, request\_info,  
[provide \_info], implicit\_confirm, request\_info

TASK: meta\_greeting\_goobye, meta\_instruct, orig\_city,  
[orig\_city], orig\_city, dest\_city,  
[dest\_city], orig\_dest\_city, depart\_date

# POMDP-Based SDSs

---

- User Simulators

- An example 3-gram that would lead to the user action shown before

{

<system, implicit\_confirm, orig\_dest\_city > ,

< system, request\_info, depart\_date > ,

< user, [(provide\_info, depart\_date), (provide\_info, depart\_time)] >

}

# POMDP-Based SDSs

---

- User Simulators
  - N-gram need a large amount of handcrafting
  - Dynamic Bayesian network
  - Hidden Markov Model
  - Train a POMDP-based dialog system to behave like a user



# POMDP-Based SDSs

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- Problems

Standard POMDP do not scale to the complexity needed to represent a real-world SDSs, because the number of states, actions, and observations can each easily be extremely large

- Solution

- Approximation

# POMDP-Based SDSs

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- Dialog Model Representation(Belief state)

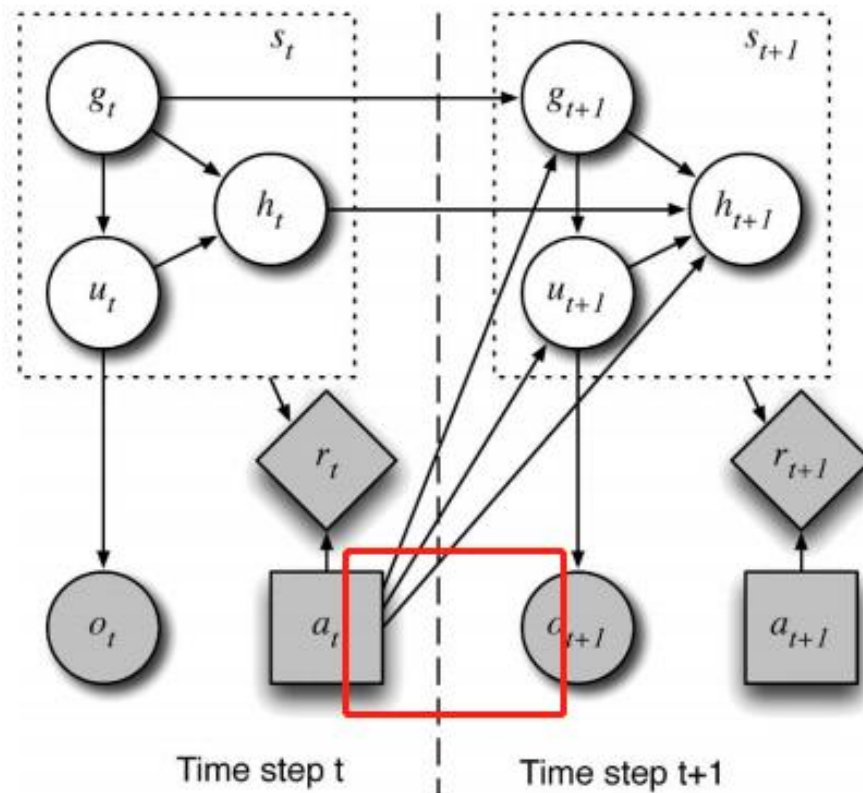
- Factorization

$$s_t = (g_t, u_t, h_t)$$

- The user's goal  $g_t$ , the intent of the most recent user utterance  $u_t$ , and the dialog history  $h_t$

# POMDP-Based SDSs

- Influence Diagram



## POMDP-Based SDSs

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- Dialog Model Representation
  - Plugging the factorization into the belief update

$$\begin{aligned} b_{t+1}(g_{t+1}, u_{t+1}, h_{t+1}) = & \eta P(o_{t+1} | u_{t+1}) \\ & \cdot P(u_{t+1} | g_{t+1}, a_t) \\ & \cdot \sum_{g_t} P(g_{t+1} | g_t, a_t) \\ & \cdot \sum_{h_t} P(h_{t+1} | g_{t+1}, u_{t+1}, h_t, a_t) \\ & \cdot b_t(g_t, h_t). \end{aligned}$$

# POMDP-Based SDSs

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- Dialog Model Representation
  - Benefits of Factorization
    - Reduce the dimensions of the state transition matrix
    - Reduce the number of conditional dependencies

# POMDP-Based SDSs

---

- Dialog Model Representation
  - Further Approximation
    - N-best approach with pruning and recombination
    - The factored Bayesian network approach

# POMDP-Based SDSs

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- Policy Model Representation
  - Policy
    - A mapping between belief state  $b$  and appropriate system action  $a$
  - Objective
    - To find an optimal policy  $\pi^*$  that maximizes the expected sum of discounted rewards at the end of the dialog

# POMDP-Based SDSs

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- Policy Model Representation
  - Summary space
    - A compressed feature space in which both states and actions are simplified
    - A subspace of the full master space
    - Belief monitoring is performed in master space
    - Decision taking and policy optimization take place in summary space



# POMDP-Based SDSs

---

- Policy Model Representation
  - Operations of summary-space POMDP
    - After belief updating, the belief state  $b$  in master space is mapped to a vector of features  $\hat{b}$  and a corresponding set of candidate actions  $\{\hat{a}\}$
    - The policy is then used to select the best action to take  $\hat{b} \rightarrow \hat{a}$  from the set of candidate actions and a second heuristic is used to map  $\hat{a}$  back into a full action  $a$  in master space

# POMDP-Based SDSs

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- Policy Optimization
  - The Policy is now a function of the summary belief state and actions  $\hat{b} \rightarrow \hat{a}$
  - To find a optimal Q-function(for a deterministic policy)

$$\pi^*(\hat{b}) = \arg \max_{\hat{a}} \{Q^*(\hat{b}, \hat{a})\}.$$

# POMDP-Based SDSs

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- Policy Optimization
  - Planning Under Uncertainty
  - Value Iteration
  - Monte Carlo Optimization
  - Least Squares Policy Iteration
  - Natural Actor-Critic Optimization

# POMDP for Spoken Dialog Systems

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- References:
  - USING POMDPS FOR DIALOG MANAGEMENT
  - POMDP-Based Statistical Spoken Dialog Systems: A Review
  - Partially Observable Markov Decision Processes for Spoken Dialog Systems
  - User Simulation for Spoken Dialogue Systems: Learning and Evaluation
  - Learning User Simulations for Information State Update Dialogue Systems
  - Error Handling in Spoken Dialogue Systems

# Lab3

# Lab3

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- Problem:
  - Solve the Grid World Problem based on MDP
- Requirement:
  - Print optimal value of all states using value iteration and policy iteration
- Address: <http://10.88.3.60/problem.php?id=1002>

# Lab3

- Value Iteration

```
function VALUE-ITERATION(mdp,  $\epsilon$ ) returns a utility function
  inputs: mdp, an MDP with states  $S$ , actions  $A(s)$ , transition model  $P(s' | s, a)$ ,
           rewards  $R(s)$ , discount  $\gamma$ 
            $\epsilon$ , the maximum error allowed in the utility of any state
  local variables:  $U$ ,  $U'$ , vectors of utilities for states in  $S$ , initially zero
                     $\delta$ , the maximum change in the utility of any state in an iteration

  repeat
     $U \leftarrow U'$ ;  $\delta \leftarrow 0$ 
    for each state  $s$  in  $S$  do
       $U'[s] \leftarrow R(s) + \gamma \max_{a \in A(s)} \sum_{s'} P(s' | s, a) U[s']$ 
      if  $|U'[s] - U[s]| > \delta$  then  $\delta \leftarrow |U'[s] - U[s]|$ 
  until  $\delta < \epsilon(1 - \gamma)/\gamma$ 
  return  $U$ 
```

**Figure 17.4** The value iteration algorithm for calculating utilities of states. The termination condition is from Equation (17.8).

# Lab3

- Policy Iteration

```
function POLICY-ITERATION(mdp) returns a policy
  inputs: mdp, an MDP with states  $S$ , actions  $A(s)$ , transition model  $P(s' | s, a)$ 
  local variables:  $U$ , a vector of utilities for states in  $S$ , initially zero
                   $\pi$ , a policy vector indexed by state, initially random

  repeat
     $U \leftarrow \text{POLICY-EVALUATION}(\pi, U, \text{mdp})$ 
    unchanged?  $\leftarrow$  true
    for each state  $s$  in  $S$  do
      if  $\max_{a \in A(s)} \sum_{s'} P(s' | s, a) U[s'] > \sum_{s'} P(s' | s, \pi[s]) U[s']$  then do
         $\pi[s] \leftarrow \operatorname{argmax}_{a \in A(s)} \sum_{s'} P(s' | s, a) U[s']$ 
        unchanged?  $\leftarrow$  false
  until unchanged?
  return  $\pi$ 
```

**Figure 17.7** The policy iteration algorithm for calculating an optimal policy.