Kepler's Laws and N-Body Simulations

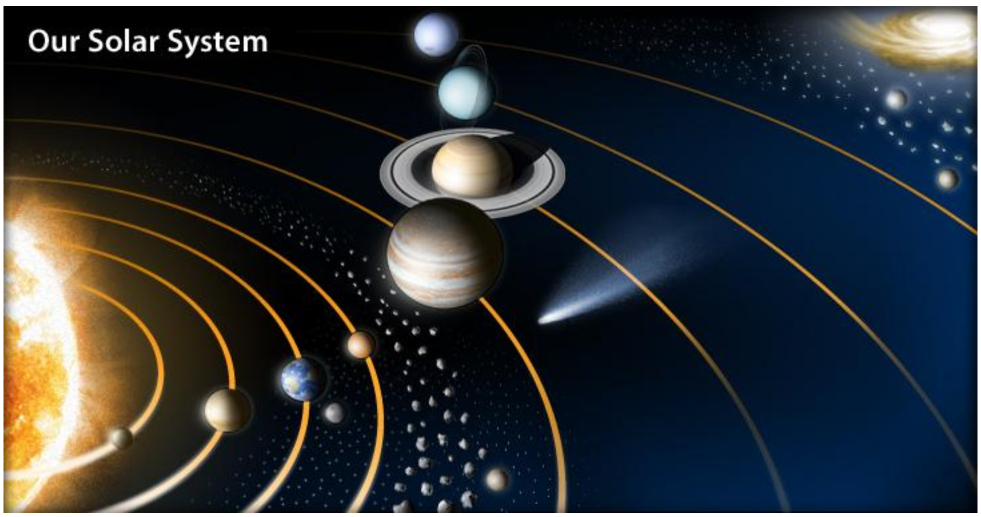


Image: http://solarsystem.nasa.gov/planets/images/splash-planets.jpg

Student Name: ______________________________

Student Number: ______________________________

Date: ______________________________

Lab Session (Day/Time): ______________________________

Lab Demonstrator: ______________________________

2017 version by Michele Trenti and Daniela Carrasco; based on text by Nicole Darman and Jack Line.

Contents

1 Introduction

1.1 Circular Motion

Uniform circular motion can be described as the motion of an object in a circle at a constant speed. As an object moves in a circle, it is constantly changing direction. At all instances, the object is moving tangent to the circle. Since the direction of the velocity vector is the same as the direction of the object's motion, the velocity vector is directed tangent to the circle as well. The diagram to the right depicts this by means of a vector arrow. An object moving in a circle is accelerating. Accelerating objects are objects which are changing their velocity - either the speed (i.e., magnitude of the velocity vector) or the direction. An object undergoing uniform circular motion is moving with a constant speed. Nonetheless, it is accelerating due to its change in direction. The direction of the acceleration is inwards.

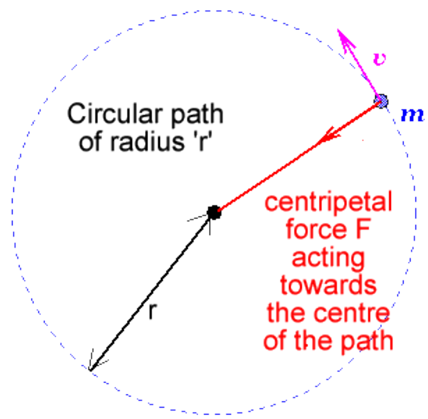


Figure 1: Circular motion example

The final characteristic motion for an object undergoing uniform circular motion is the net force. The net force acting upon such an object is directed towards the centre of the circle. The net force is said to be an inward or centripetal force. Without such an inward force, an object would continue in a straight line, never deviating from its direction. Yet, with the inward net force directed perpendicular to the velocity vector, the object is always changing its direction and undergoing an inward acceleration.

The basic reason why the planets orbit around the Sun is that the gravity of the Sun keeps them bound. Just as the Moon orbits the Earth because of the pull of Earth's gravity, the Earth orbits the Sun because of the pull of the Sun's gravity.

Why, then, does the Earth travel in an elliptical orbit around the Sun, rather than just getting pulled in all the way? This happens because the Earth has a velocity in the direction perpendicular to the force of the Sun's pull. If the Sun weren't there, the Earth would travel in a straight line. But the force of gravity due to the Sun alters its course, causing it to travel around the Sun, in a shape very near to a circle..

1.2 Kepler's Laws

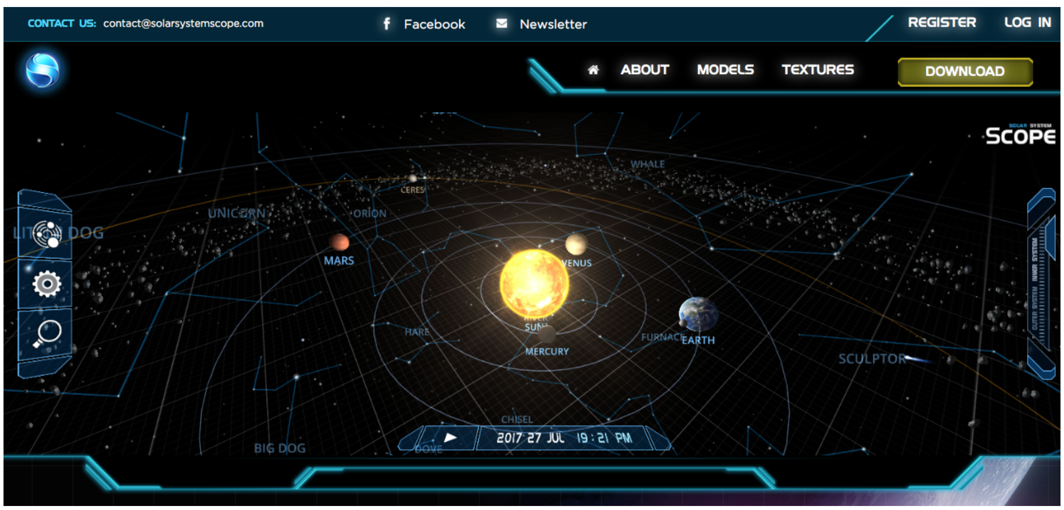
In the early 1600s, Johannes Kepler proposed three laws of planetary motion. Kepler was able to summarise the carefully collected data of his mentor - Tycho Brahe - with three statements that described the motion of planets in a Sun-centred solar system. Kepler's efforts to explain the underlying reasons for such motions are no longer accepted; nonetheless, the actual laws themselves are still considered an accurate description of the motion of any planet or satellite.

1.3 N-Body Simulations

Gravitational N-body simulations are numerical solutions of the equations of motions for N particles interacting gravitationally. These are widely used tools in astrophysics, with applications from few body or solar system like systems, all the way up to galactic and cosmological scales. These simulations have allowed significant advances in our knowledge of the universe and continue to be used actively in research projects today.
⋆ **Refer to question 1.**

> It is now time to look at some simulations. To do this, you must use the computers in the labs. Download the notes (this same file) into the computer. Open it and click on the links (in blue) given throughout the labs.

> **OPEN THE SOLAR SYSTEM LINK NOW**
> www.solarsystemscope.com



Once the initial slideshow has finished playing, take some time to explore the solar system using the menu on the left of the screen.

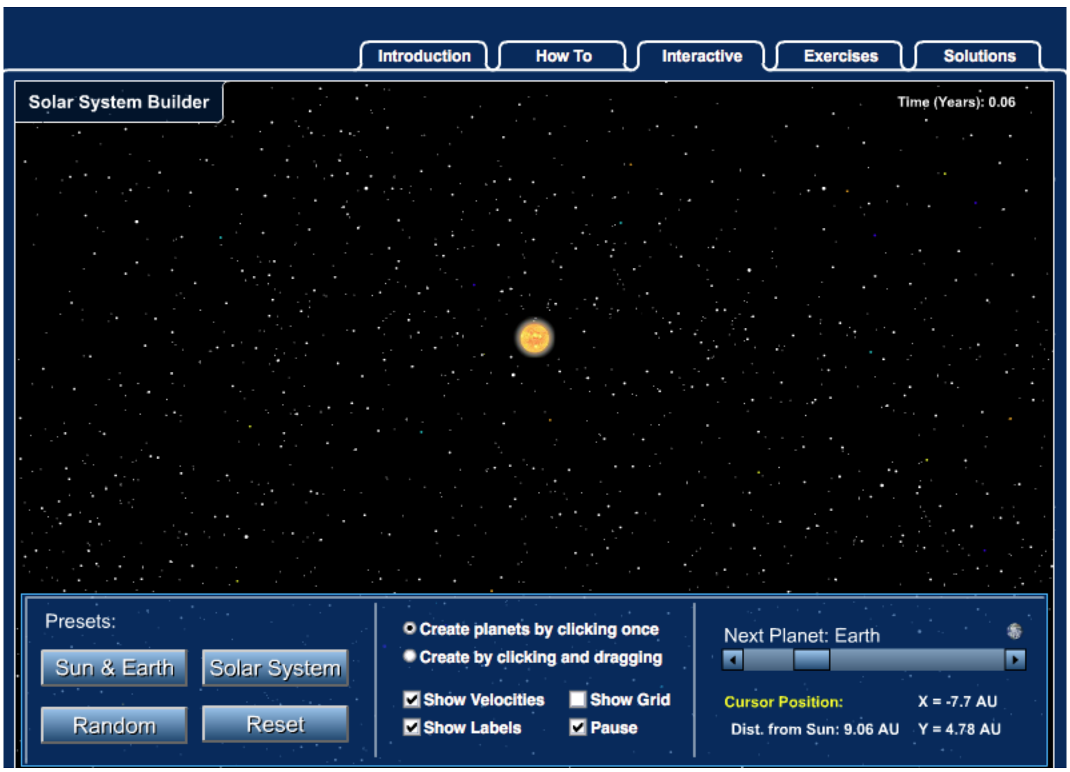
→ Click on the settings button on the left of the website, which is pictured here. Click on the squares to select the objects shown. Deselect everything except for 'Planets' and 'Dwarf Planets'. For them, all options should be marked. Hit 'Go'.
You can use the mouse to click and drag to change your point of view of the solar system, and the slide on the right of the screen can be used to zoom in and out.

⋆ **Refer to question 2-3.**

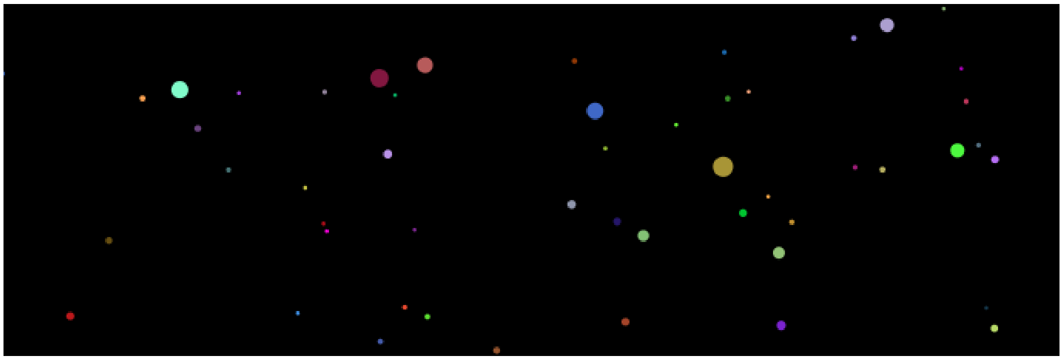
OPEN THE EARTH AND MARS LINK NOW

Go to
http://highered.mheducation.com/sites/0072482621/student_view0/interactives.html
Click on 'Solar System Builder', and a new window will pop up (See Figure below). At the bottom left, find the 'Sun and Earth' preset and click on it. You should see the Earth orbiting around the Sun. The elapsed time is shown on the top right corner. To have better measurements it's recommended that you select 'Show Grids'.
In order to study Mars now, go to the bottom right, click on the arrows until you find 'Mars', and place it at a distance of 1.5 AU from the Sun. Now the simulation includes the Earth, Mars, and the Sun. Using the grids estimate the time Mars takes to travel around the Sun.

⋆ **Refer to question 4-7.**



OPEN THE N-BODY INTERACTIONS LINK NOW
http://justfound.co/gravity/



⋆ **Refer to question 8.**

2 Kepler's Laws

2.1 Ellipticity

Ellipses can be described by the length of their two axes. A circle has the same diameter whether you measure it across or up and down, but an ellipse has diameters of different lengths. The longest one is called the major axis (a), and the shortest one is the minor axis (b). The ratio of these two lengths determines the ellipticity; the equation is $e = 1 - b/a$. Circles have e = 0, and very stretched-out ellipses have an ellipticity nearly equal to 1. Planets move in elliptical orbits, although in practice they are nearly circular. Comets are a good example of objects in our Solar System that may have very elliptical orbits. Compare the eccentricities and orbits of the objects in the diagram.

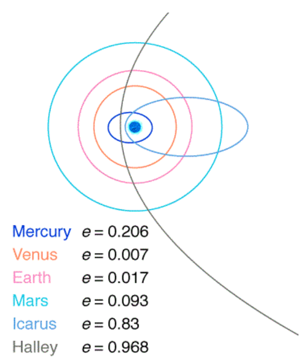


Figure 2: Elliptical orbits and their eccentricities.

2.2 Kepler's Three Laws

Kepler's three laws of planetary motion can be described as follows:

1. The path of the planets about the sun is elliptical in shape, with the center of the sun being located at one focus. (The Law of Ellipses)
2. An imaginary line drawn from the center of the sun to the center of the planet will sweep out equal areas in equal intervals of time. (The Law of Equal Areas)
3. The ratio of the squares of the periods of any two planets is equal to the ratio of the cubes of their average distances from the sun. (The Law of Harmonies)

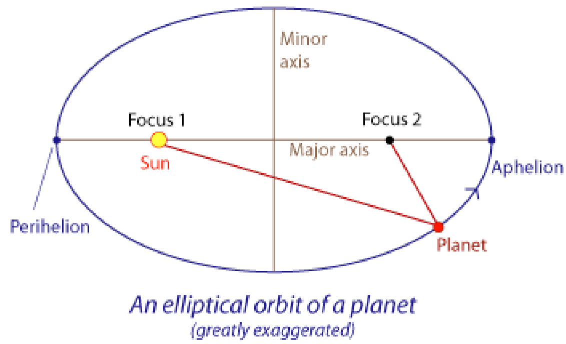


Figure 3: Foci points on an ellipse.

2.3 Kepler's First Law

Sometimes referred to as the law of ellipses - explains that planets are orbiting the sun in a path described as an ellipse. An ellipse can easily be constructed using a pencil, two tacks, a string, a sheet of paper and a piece of cardboard. Tack the sheet of paper to the cardboard using the two tacks. Then tie the string into a loop and wrap the loop around the two tacks. Take your pencil and pull the string until the pencil and two tacks make a triangle (see diagram at the right). Then begin to trace out a path with the pencil, keeping the string wrapped tightly around the tacks. The resulting shape will be an ellipse.

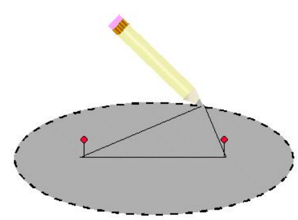


Figure 4: Kepler's first law. How to draw an ellipse using two tacks, string and a pencil.

An ellipse is a special curve in which the sum of the distances from every point on the curve to two other points is a constant. The two other points (represented here by the tack locations) are known as the foci of the ellipse. The closer together these points are, the more closely the ellipse resembles the shape of a circle. In fact, a circle is the special case of an ellipse in which the two foci are at the same location. Kepler's first law is rather simple - all planets orbit the sun in a path that resembles an ellipse, with the sun being located at one of the foci of that ellipse.

⋆ **Refer to question 9.**

2.4 Kepler's Second Law

Sometimes referred to as the law of equal areas - describes the speed at which any given planet will move while orbiting the sun. The speed at which any planet moves through space is constantly changing. A planet moves fastest when it is closest to the sun and slowest when it is furthest from the sun. Yet, if an imaginary line were drawn from the centre of the planet to the centre of the sun, that line would sweep out the same area in equal periods of time. For instance, if an imaginary line were drawn from the earth to the sun, then the area swept out by the line in every 31-day month would be the same. This is depicted in the diagram below. As can be observed in the diagram, the areas formed when the earth is closest to the sun can be approximated as a wide but short triangle; whereas the areas formed when the earth is farthest from the sun can be approximated as a narrow but long triangle. These areas are the same size.

⋆ **Refer to question 10.**

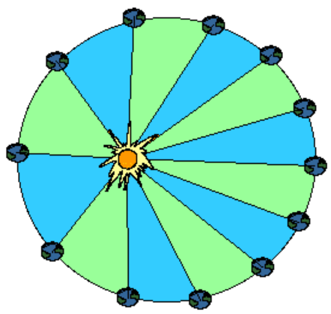


Figure 5: **The Law of Equal Areas.** An imaginary line drawn from the Sun to any planet sweeps out equal areas in equal amounts of time.

2.5 Kepler's Third Law

This law compares the orbital period and semi-major axis of the orbit of a planet. Unlike Kepler's first and second laws that describe the motion of a single planet, the third law makes a comparison between the different planets. In the Solar System, if we use the Earth's orbit as the standard orbit, then by measuring the period in years and the semi-major axis in AUs, then $P^2 = A^3$. **NOTE: $P^2 = A^3$ only applies to the planets in our solar system. For all other systems, like Jupiter's moons, P^2 is proportional to A^3.**

Kepler's third law describes the relationship between the period and semi-major axis for a planet's orbit about the Sun. Stated another way, the ratio P^2/A^3 of the planets' orbits about the sun is constant. This statement is also true for a group of satellites (whether moons or a man-made satellites) orbiting a specific planet. **Note that the value of the ratio changes, depending on the object at the centre of the orbits.** The more massive the central object is, the smaller the value of this P^2/A^3. ratio.

⋆ **Refer to question 11.**

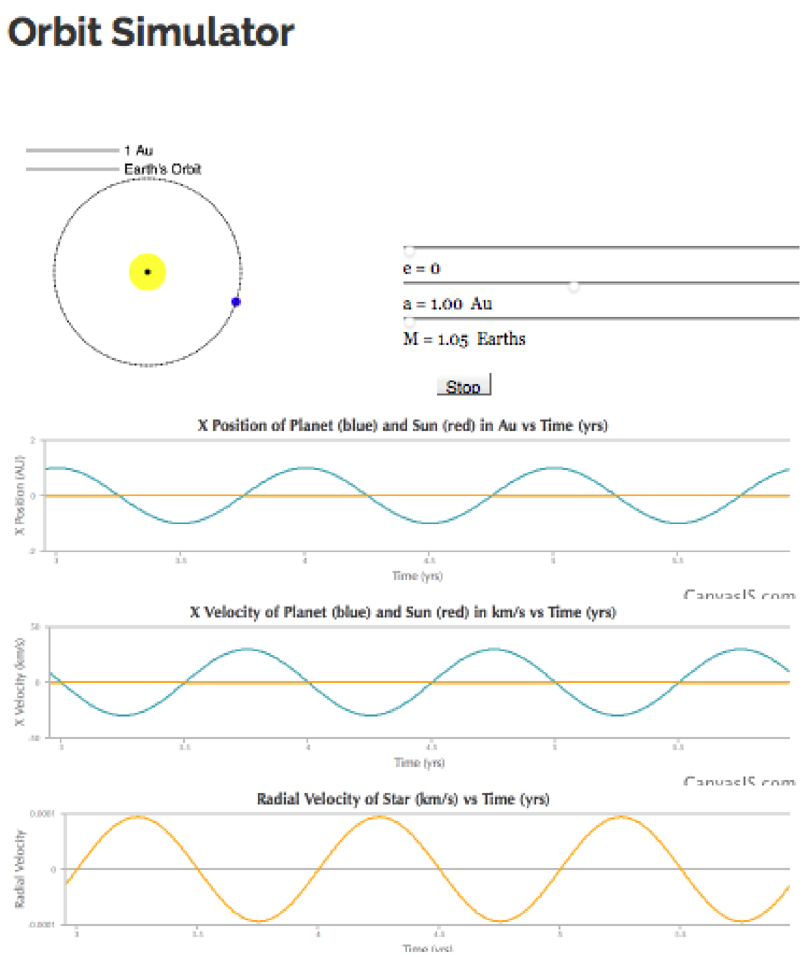
3 Two – Body Simulations

In classical mechanics, the two-body problem is to determine the motion of two point particles that interact only with each other. Common examples include a satellite orbiting a planet, a planet orbiting a star and two stars orbiting each other (a binary star system).

In this section, you will be varying characteristics of two bodies in order to see how they interact and orbit one another. Feel free to have a play with the software before answering the questions.

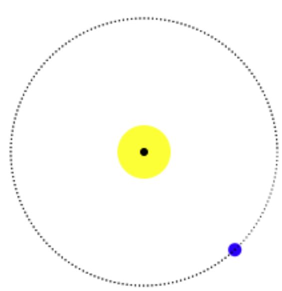
OPEN THE 2 BODY LINK NOW

http://adamdempsey90.github.io/html5/orbits/orbitsim.html

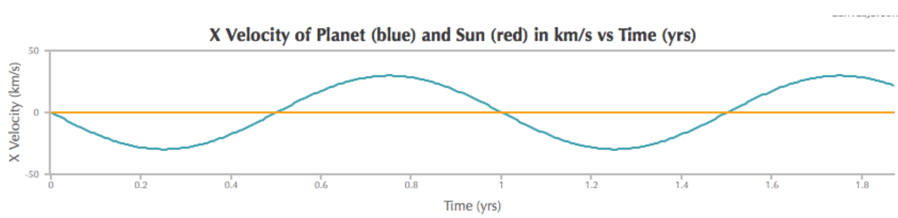


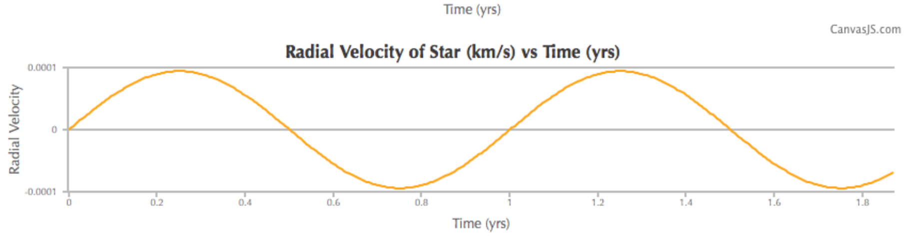
3.1 Understanding the plots

Privileged view: A bird's eye view (directly overhead) of the bodies in motion.



Radial velocity: The velocity of each body along the line of sight to the observer. To see the velocity you can place the arrow on top of the graphs.



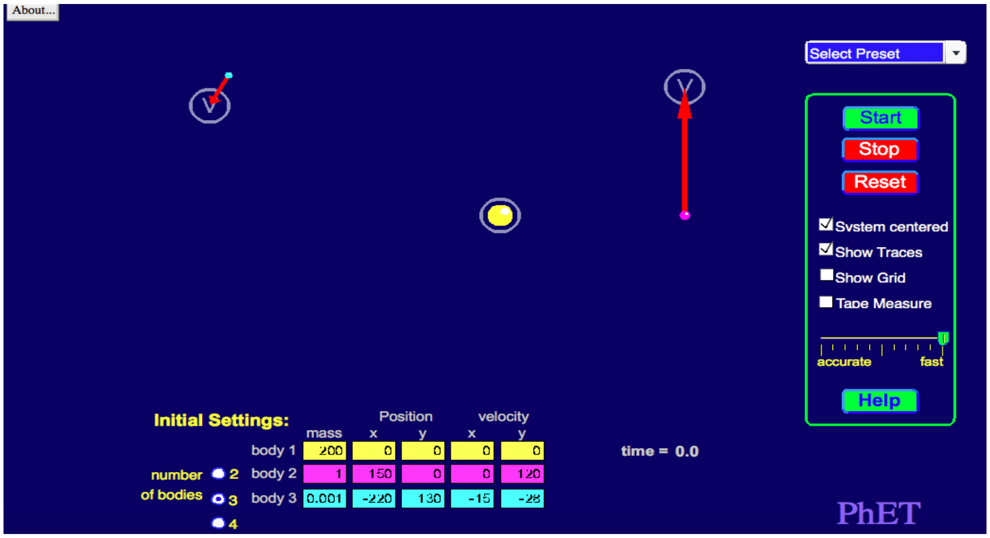


⋆ **Refer to questions 12-14.**

4 N-Body Simulations

The N-Body problem is the problem of predicting the motion of a group of celestial objects that interact with each other gravitationally. Solving this problem has been motivated by the need to understand the motion of the Sun, planets, stars and galaxies. Gravitational interactions are responsible for these motions, and these can be expressed in terms of differential equations – these are equations that contain the rate at which parameters change.

OPEN THE N BODY LINK NOW
https://phet.colorado.edu/sims/my-solar-system/my-solar-system_en.html



4.1 Stability

We can define a gravitational system as stable if the orbits of the objects do not change with time. In these simulations, it may be a bit tricky to establish whether a system is stable or unstable. A few things to think about while answering this section include:

- Do all the bodies appear to keep the same orbital shape during the simulation? If so, then it is considered stable.
- Do the bodies appear to start off in a predictable manner then become chaotic? If so then it is considered unstable.
- Do the bodies just interact and fly away after a short period of time? If so, then it is considered unstable.
- Do some of the bodies appear to coalesce to a point where they don't appear to be moving? If so, then the bodies have collided, and we consider such system unstable.

Consider different cases. Click on the configuration you want to see at the top right of the simulation and hit Start. Read below to see how to change the speed of your simulation to help you answer question 15.

Notes on software:

- You can choose standard configurations (Select Preset), or make your own by changing the Initial Settings table.
- Build your own system by dragging bodies and the V symbol or by typing into the Initial Settings table.
- The table on the right allows you to stop the simulation, manage the velocity, and reset to try a new configuration.

⋆ **Refer to questions 15.**

5 Links and Resources

5.1 Links

Intro
Solar System - www.solarsystemscope.com
Earth and Mars - http://highered.mheducation.com/sites/0072482621/student_view0/interactives.html
N-Body interactions - http://justfound.co/gravity/

Two Body Simulations
2 Body – http://adamdempsey90.github.io/html5/orbits/orbitsim.html

N-Body Simulations
N Body – https://phet.colorado.edu/sims/my-solar-system/my-solar-system_en.html

5.2 Resources

http://www.scholarpedia.org/article/N-body_simulations_(gravitational)
http://www.physicsclassroom.com/class/circles/u6l4a.cfm
http://www.physicsclassroom.com/mmedia/circmot/ucm.cfm
http://www.windows2universe.org/the_universe/uts/kepler1.html

Questions

Kepler's Laws and N-Body Simulations

Student Name: ____________________

Student Number: ____________________

Date: ____________________

Lab Session (Day/Time): ____________________

Lab Demonstrator: ____________________

2017 version by Michele Trenti and Daniela Carrasco; based on text by Nicole Darman and Jack Line.

1. **In your own words, explain why planets orbit around stars (more massive bodies). See section 1.1 for help.**

2. **Describe the main differences between the orbits of dwarf planets and the ones of regular planets.**

3. **When observing directly from above the solar system. What shape do most planets appear to have for their orbits? Explain your answer.**

4. **From the Earth and Mars simulation, what is the orbital period of Mars?**

5. **We know that Earth is 1AU (astronomical unit) away from the Sun. How far away is Mars according to this simulation?**

6. **We know that increasing the orbit size will affect the orbital period if its velocity stays the same. Start a new simulation but now with Earth 4AU away from the Sun. What is its orbital period now?**

7. **Considering what you found for Question 5:**
   (a) **If Earth is now 4AU away, how far away must Mars be for the ratio between the two distances to be the same? (Ask your demonstrator if you need help with ratios).**

   (b) **What orbital period does this correspond to? (Hint: Drag Mars in the simulation out to the distance you found in part 7a.**

   (c) **In order to remain on a near-circular orbit, the planet orbital velocity depends on the distance from the Sun. If a system undergoes a change such as the one above, what is happening to its orbital velocity? Is it decreasing or increasing? Do you have an idea of why this is happening?**

8. **Fill the screen with objects (press 'K') and see what happens. Explore different cases by placing new objects in the simulation. Explain what you see in it, i.e. what happens when:**
   (a) **Two objects collide.**

(b) A high-mass object interacts with a low-mass object.

(c) Two objects get close but don't collide.

9. After reading about Kepler's first law, does your answer to question 3 change? Explain why. Draw a picture of where you think the two foci points are and include the Sun and a planet of your choice for reference. See figure 2.1 for help.

10. According to Kepler's second law, which triangle in figure 2.4 (A_1 or A_2) has a greater area if the time taken between points x_1 and x_2 is the same as the time taken between x_3 and x_4? Explain your answer.

11. Kepler's third law discusses the proportional relationship between the orbital period and orbital radius of satellites.

(a) **To test this, calculate the following for each planet:**

Planet	Orbital Period P (Julian Years)	Orbital Radius A (AU)	P^2/A^3
Mercury	0.24	0.38	
Venus	0.62	0.72	
Earth	1.00	1.00	
Mars	1.88	1.52	
Saturn	11.86	5.20	
Jupiter	29.47	9.54	
Neptune	84.07	19.18	
Pluto	164.81	30.06	

(b) **Using this table, you can now calculate the orbital parameters for any object orbiting the Sun. Considering an asteroid in our solar system, calculate its semi-major axis if it has a period of 5 years (i.e. $P = 5$).**

12. Make sure that both masses are set to 1 solar mass, $e = 0.5$, and $a = 100AU$.

(a) **Sketch the privileged view of the two bodies orbiting one another and their corresponding radial velocity plot (second plot).**

(b) When do the bodies have the fastest radial speed? Provide a worded answer below and label it on the diagram on part 13a. (Remember: Speed is the absolute value of the velocity. So if velocity $= -10ms^{-1}$, then speed $= 10ms^{-1}$).

(c) Do the bodies reach their peak speeds at the same time? Explain.

13. Repeat 12 now make mass two $M = 0.20 Suns$.

(a) Sketch the privileged view of the two bodies orbiting one another and their corresponding radial velocity plot.

(b) When do the bodies have the fastest radial speed? Provide a worded answer below and label it on the diagram on part 13a.

(c) Do the bodies reach their peak speeds at the same time? Explain.

(d) **What differences did you notice between the orbital paths and velocity between question 12 and question 13.**

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14. Using the table below, perform a thorough investigation into 2 body simulations by varying the second mass, separation and eccentricity. Draw a picture of the orbit.

Mass 1	Mass 2	Separation (A)	Eccentricity (E)	Max. Radial Speed	Orbit Diagram

15. Try six different configurations and fill in the following table. At least 3 of them have to be defined by you with the Initial Settings. Remember to run the simulations for an adequate amount of time to consider its stability. Consider also cases that are relevant and different from each other (ask your demonstrator

if you are not sure about this). When assessing the stability, briefly justify your answer.

Initial Diagram	**Initial Conditions**	**Stability**

Conclusion

Write a short, one paragraph conclusion on what you have learned in this lab. Be sure to mention Kepler's laws and N-body simulations.

Marks

CATEGORY	4	3	2	1	0
Participation					
Calculations					
Worded Questions					
Scientific Concepts					
Conclusion					
Total					

Total:_____/20 **Total:_____/10**