HW 2

Zongdi

2.

$$L(w,b,\xi,\alpha,\beta) = \frac{1}{2}\|w\|_2^2 + C\sum_{i=1}^{m}\xi_i - \sum_{i=1}^{m}\alpha_i(y_i(w^Tx_i+b)-1+\xi_i)$$

$$-\sum_{i=1}^{m}\beta_i\xi_i$$

$$\frac{\partial L}{\partial w} = w - \sum_{i=1}^{m}\alpha_i y_i x_i \overset{set}{=} 0 \Rightarrow w = \sum_{i=1}^{m}\alpha_i y_i x_i$$

$$\frac{\partial L}{\partial b} = \sum_{i=1}^{m}\alpha_i \cdot y_i \overset{set}{=} 0 \Rightarrow b = \sum_{i=1}^{m}\alpha_i y_i$$

$$\frac{\partial L}{\partial \xi_i} = C - \alpha_i - \beta_i \overset{set}{=} 0 \Rightarrow C = \alpha_i + \beta_i$$

$$\frac{\partial L}{\partial \alpha_i} = -y_i(w^Tx_i+b)-1+\xi_i \overset{set}{=} 0 \Rightarrow \alpha_i = 0 \text{ or } y_i(w^Tx_i+b) = 1-\xi_i$$

$$\frac{\partial L}{\partial \beta_i} = -\xi_i \overset{set}{=} 0 \Rightarrow \xi_i = 0 \text{ or } \beta_i = 0$$

Which are the KKT conditions

3.

$$K(x,x') = (\langle x, x'\rangle + c)^3$$

$$= (x_1x_1' + x_2x_2' + c)^3$$

$$= (x_1^2x_1^{2'} + x_2^2x_2^{2'} + 2x_1x_1'x_2x_2' + 2cx_1x_1' + 2cx_2x_2' + c^2)(x_1x_1' + x_2x_2' + c)$$

$$= x_1^3x_1^{3'} + x_1^2x_1^{2'}x_2x_2' + cx_1^2x_1^{2'} + x_2^2x_2^{2'}x_1x_1' + x_2^3x_2^{3'} + cx_2^2x_2^{2'}$$
$$+ 2x_1^2x_1^{2'}x_2x_2' + 2x_1x_1'x_2^2x_2^{2'} + 2cx_1x_1'x_2x_2' + 2cx_1^2x_1^{2'} + 2cx_1x_1'x_2x_2' + 2c^2x_1x_1'$$
$$+ 2cx_1x_1'x_2x_2' + 2cx_2^2x_2^{2'} + 2c^2x_2x_2' + c^2x_1x_1' + c^2x_2x_2' + c^3$$

$$= x_1^3x_1^{3'} + x_2^3x_2^{3'} + 3x_1^2x_1^{2'}x_2x_2' + 3x_1x_1'x_2^2x_2^{2'} + 6cx_1x_1'x_2x_2' + 3cx_1^2x_1^{2'}$$
$$+ 3cx_2^2x_2^{2'} + 3c^2x_1x_1' + 3c^2x_2x_2' + c^3$$

$$= \left[x_1^3,\ x_2^3,\ \sqrt{3}x_1^2 x_2,\ \sqrt{3}x_1 x_2^2,\ \sqrt{6c}\,x_1 x_2,\ \sqrt{3c}\,x_1^2,\ \sqrt{3c}\,x_2^2,\ \sqrt{3}c\,x_1,\ \sqrt{3}c\,x_2,\ \sqrt{c^3} \right] \begin{bmatrix} x_1'^3 \\ x_2'^3 \\ \sqrt{3}x_1'^2 x_2' \\ \sqrt{3}x_1' x_2'^2 \\ \sqrt{6c}\,x_1' x_2' \\ \sqrt{3c}\,x_1'^2 \\ \sqrt{3c}\,x_2'^2 \\ \sqrt{3}c\,x_1' \\ \sqrt{3}c\,x_2' \\ \sqrt{c^3} \end{bmatrix}$$

$\Phi(x)$

$\Phi(x') \rightarrow$