PHYS 234: Quantum Physics 1 (Winter 2026)
Assignment 1

1. Matrix Operations. Let

$$A = \begin{pmatrix} 1 & i \\ -i & 2 \end{pmatrix}, \qquad B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

(a) Compute AB and BA. Are they equal?

(b) Compute $A^\dagger$ (Hermitian conjugate).

(c) Is A Hermitian? Justify your answer.

(d) Compute $\det(A)$ and $\mathrm{Tr}(A)$.

2. Inverse and Identity. Let

$$M = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}.$$

(a) Find M^{-1}.

(b) Verify explicitly that $MM^{-1} = I$.

(c) For what values of a does

$$\begin{pmatrix} a & 1 \\ 1 & a \end{pmatrix}$$

fail to be invertible?

3. Eigenvalue Problem. Consider

$$H = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}.$$

(a) Find the eigenvalues of H.

(b) Find the corresponding normalized eigenvectors.

(c) Are the eigenvectors orthogonal?

(d) Is H diagonalizable?

4. Complex Arithmetic. Let $z = 1 + i$.

(a) Compute $|z|$ and $\arg(z)$.

(b) Write z in polar (exponential) form.

(c) Compute z^2 and e^{iz}.

(d) Show that $zz^* = |z|^2$.

5. Euler's Formula.

(a) Use Euler's formula to show that

$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}.$$

(b) Express $\sin\theta$ in terms of exponentials.

(c) Evaluate $\cos(\pi/3)$ using complex exponentials.

6. Inner Products. Let

$$|\psi\rangle = \begin{pmatrix} 1 \\ i \end{pmatrix}, \qquad |\phi\rangle = \begin{pmatrix} i \\ 1 \end{pmatrix}.$$

(a) Write $\langle\psi|$.

(b) Compute $\langle\psi|\phi\rangle$.

(c) Are the states orthogonal?

(d) Normalize $|\psi\rangle$.

7. Hermitian Conjugation. Let

$$A = \begin{pmatrix} 2 & 1+i \\ 1-i & 3 \end{pmatrix}.$$

(a) Compute $A^\dagger$.

(b) Verify whether $A^\dagger = A$.

(c) Compute $\mathrm{Tr}(A)$ and $\det(A)$.

(d) Show that $\mathrm{Tr}(A)$ is real.

8. Non-Commuting Matrices. Let

$$A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

(a) Compute AB and BA.

(b) Compute the commutator $[A, B]$ ($[A, B] := AB - BA$).

(c) Is either matrix Hermitian?

(d) Is the commutator Hermitian or anti-Hermitian?

9. Explicit Inversion. Let

$$M = \begin{pmatrix} 1+i & 1 \\ 1 & 1-i \end{pmatrix}.$$

(a) Compute $\det(M)$.

(b) Find M^{-1}.

(c) Verify that $MM^{-1} = I$.

(d) Is M^{-1} Hermitian?

10. Parameter Dependence. Let

$$A(\lambda) = \begin{pmatrix} \lambda & i \\ -i & \lambda \end{pmatrix}.$$

(a) Find $\det(A)$ as a function of λ.

(b) For which values of λ does A fail to be invertible?

(c) For invertible cases, compute A^{-1}.

11. Complex Matrix Eigenvalues. Let

$$H = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}.$$

(a) Find the eigenvalues of H.

(b) Find normalized eigenvectors.

(c) Are the eigenvalues real?

(d) Is H Hermitian?

12. Trace and Determinant Check. Let

$$A = \begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}.$$

(a) Compute the eigenvalues.

(b) Verify that the sum of eigenvalues equals $\mathrm{Tr}(A)$.

(c) Verify that the product of eigenvalues equals $\det(A)$.

(d) Are the eigenvectors orthogonal?

13. Polar Representation. Let

$$z = -1 + i\sqrt{3}.$$

(a) Compute $|z|$.

(b) Find $\arg(z)$ (principal value).

(c) Write z in exponential form.

(d) Compute z^3 using polar form.

14. Complex Conjugation. Let $z = a + ib$.

(a) Show that $z + z^* = 2a$.

(b) Show that $z - z^* = 2ib$.

(c) Show that $\frac{z}{z^*}$ lies on the unit circle (when $z \neq 0$).

(d) Evaluate $\frac{1+i}{1-i}$ and write it as $e^{i\theta}$.

15. Trigonometric Identities from Exponentials.

(a) Use exponentials to derive a formula for $\sin^2 \theta$.

(b) Show that $\cosh(ix) = \cos x$.

(c) Express $\sin(2\theta)$ using exponentials.

(d) Evaluate $\sin(\pi/4)$ using complex exponentials.

16. Oscillatory Functions.

(a) Write $e^{i(\omega t+\phi)}$ in terms of sine and cosine.

(b) Identify the real and imaginary parts.

(c) Show that $\left|e^{i\theta}\right| = 1$.

(d) Briefly explain why complex exponentials are useful in quantum mechanics.

17. Normalization. Let

$$|\psi\rangle = \begin{pmatrix} 2 \\ -i \end{pmatrix}.$$

(a) Compute $\langle\psi|\psi\rangle$.

(b) Normalize $|\psi\rangle$.

(c) Verify that the normalized state has unit norm.

(d) State the physical meaning of normalization (one sentence).

18. Orthogonality. Let

$$|\phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \qquad |\phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

(a) Compute $\langle\phi_1|\phi_2\rangle$.

(b) Are the states orthogonal?

(c) Are they normalized?

(d) Do they form a basis of $\mathbb{C}^2$?