PHYS 234: Quantum Physics 1 (Winter 2026)
Tutorial 2

1. Consider a spin-1/2 particle that is spin down along the direction specified by a unit vector

$$\hat{\mathbf{n}} = \cos\varphi \sin\theta \; \hat{\mathbf{x}} + \sin\varphi \sin\theta \; \hat{\mathbf{y}} + \cos\theta \; \hat{\mathbf{z}}, \tag{1}$$

where we are using spherical coordinates φ, θ (Figure 1). This means that the particle's state vector is an eigenstate of $\hat{\mathbf{n}} \cdot \mathbf{S}$ with eigenvalue $-\hbar/2$, so that the probability of a measurement of $\hat{\mathbf{n}} \cdot \mathbf{S}$ yielding $-\hbar/2$ is 1.

Note: the expression $\hat{\mathbf{n}} \cdot \mathbf{S}$ is shorthand for $\hat{\mathbf{n}} \cdot \mathbf{S} = n_x S_x + n_y S_y + n_z S_z$.

(a) Show that the state vector of the particle is

$$|-\rangle_{\hat{\mathbf{n}}} = \sin\left(\frac{\theta}{2}\right)|+\rangle - e^{i\varphi}\cos\left(\frac{\theta}{2}\right)|-\rangle. \tag{2}$$

Hint: Recall that, in the S_z eigenbasis, we have the matrix representations

$$S_x \doteq \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad S_y \doteq \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad S_z \doteq \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

(b) What are the possible values for measuring S_z? With what probabilities do each of them occur? Plot the probabilities as a function of θ.

(c) Give a qualitative explanation for why the probabilities in (b) are independent of φ.

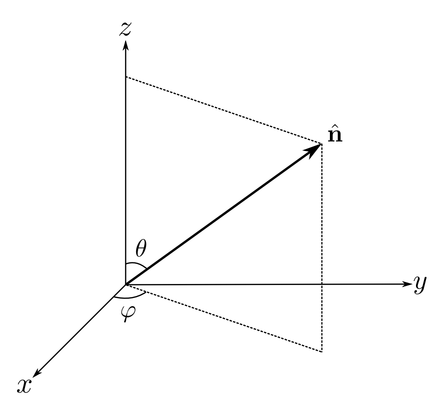

Figure 1: Direction of the spin specified in spherical coordinates.

Something to think about: Suppose a spin-1/2 particle is in state $|-\rangle_{\hat{\mathbf{n}}}$ (Eq. 2). Let $\hat{\mathbf{n}}_\perp$ be any unit vector orthogonal to $\hat{\mathbf{n}}$. First, without calculating, what do you expect for the probability of measuring $\hat{\mathbf{n}}_\perp \cdot \mathbf{S} = \pm\hbar/2$? Now confirm your expectation with a calculation.