PHYS234: Quantum Physics 1 (Winter 2026)
Assignment 2

1. Calculations using quantum states:

$$|\psi_1\rangle = 3\,|+\rangle - i\,|-\rangle$$
$$|\psi_2\rangle = e^{i\pi/3}\,|+\rangle + |-\rangle$$
$$|\psi_3\rangle = 7i\,|+\rangle - 2\,|-\rangle$$

   (a) For each of the states $|\psi_j\rangle$ above ($j = 1\cdots 3$), find the corresponding normalized state $|\psi_j\rangle_N$.

   (b) Using the bra-ket notation, calculate all 9 inner products ${}_N\langle\psi_i|\psi_j\rangle_N$ for $i = 1\cdots 3$ and $j = 1\cdots 3$ using the normalized states.

   (c) For each state $|\psi_i\rangle$, find the state $|\phi_i\rangle$ with unit norm, $\langle\phi_i|\phi_i\rangle = 1$ that is orthogonal to it.
   Recall the orthogonality conditions for the basis states:
   $\langle +|+\rangle = \langle -|-\rangle = 1$ and $\langle +|-\rangle = \langle -|+\rangle = 0$.

   (d) Postulate 4 of quantum mechanics tells us that the complex square of the inner product $|\langle a|b\rangle|^2$ is the probability of measuring a particular quantum state. For each of the normalized states $|\psi\rangle_N$, calculate the probability of measuring each of the 6 states indicated below.

$$|1\rangle = |+\rangle$$
$$|2\rangle = |-\rangle$$
$$|3\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$$
$$|4\rangle = \frac{1}{\sqrt{2}}(|+\rangle - |-\rangle)$$
$$|5\rangle = \frac{1}{\sqrt{2}}(|+\rangle + i\,|-\rangle)$$
$$|6\rangle = \frac{1}{\sqrt{2}}(|+\rangle - i\,|-\rangle)$$

2. The state of a spin-$\frac{1}{2}$ particle that is spin up along the axis whose direction is specified by the unit vector

$$\mathbf{n} = \sin\theta\cos\phi\,\hat{\mathbf{x}} + \sin\theta\sin\phi\,\hat{\mathbf{y}} + \cos\theta\,\hat{\mathbf{z}}$$

where θ and ϕ are the polar and azimuthal angles, respectively, is given by

$$|+\rangle_n = \cos\left(\frac{\theta}{2}\right)|+\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|-\rangle$$

   (a) Verify that the state $|+\rangle_n$ reduces to the states $|+\rangle_x$ and $|+\rangle_y$ for the appropriate choice of the angles θ and ϕ.

   (b) Suppose that a measurement of $\hat{S}_z$ is carried out on a particle in the state $|+\rangle_n$. What is the probability that the measurement yields $(i)\,\hbar/2$ and $(ii) -\hbar/2$?

3. Suppose in a two-dimensional basis the operators $\hat{A}$ and $\hat{B}$ are represented by the 2×2 matrices

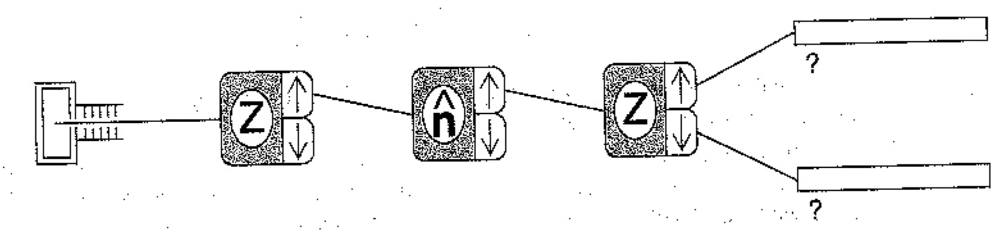
$$\hat{A} \to \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \quad \hat{B} \to \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$$

Show that $(\hat{A}\hat{B})^\dagger = \hat{B}^\dagger\hat{A}^\dagger$

4. Show that if the states $|a_n\rangle$ form an orthonormal basis, so do the states $\hat{U}\,|a_n\rangle$, provided $\hat{U}$ is unitary.

5. Show that the eigenvalues of a unitary operator can be written as $e^{i\theta}$.

6. A beam of spin-1/2 particles is sent through a series of three S-G analyzers, as shown in the figure. The second S-G analyzer is aligned along the $\hat{\mathbf{n}}-$direction.



   (a) Find the probability that particles transmitted through the first S-G analyzer are measured to have spin down at the third S-G analyzer.

   (b) How must the angle θ of the second S-G analyzer be oriented so as to maximize the probability that particles are measured to have spin down at the third S-G analyzer? What is this maximum fraction?

7. It is known that there is a 90% probability of obtaining $S_z = \frac{\hbar}{2}$ if a measurement of S_z is carried out on a spin-1/2 particle. In addition, it is known that there is a 20% probability of obtaining $S_y = \frac{\hbar}{2}$ if a measurement of S_y is carried out. Determine the spin state of a particle as completely as possible from this information. What is the probability of obtaining $S_x = -\frac{\hbar}{2}$ if a measurement of S_x is carried out?