Lecture 1
A Classical Description of Orbital Magnetic Moment

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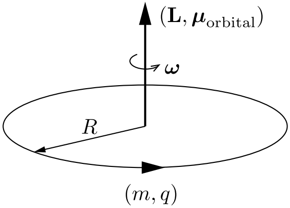
A Classical Description of Orbital Magnetic Moment

- In our development of quantum theory, we will focus on a set of experiments performed by Otto Stern and Walther Gerlach in 1922, which revealed the electron possesses an intrinsic magnetic moment that is quantized. The series of experiments that have come to be known as the Stern-Gerlach experiments were some of the first measurements that demonstrated the foundational concepts of quantum theory.
- The S-G experiments were done to test the Bohr model of the atom, which predicted that the negatively charged electrons moved in discrete orbits around a positively charged nucleus.
- Classically, a charged object moving in a circular motion will generate a magnetic moment.
- This magnetic moment will interact with an external magnetic field that varies as a function of space to produce a force.
- The deflection of atoms in a magnetic field gradient would test of the Bohr model.
- Before discussing the S-G experiments, let's look at the interaction of a magnetic moment with an external magnetic field.

A Classical Description of Orbital Magnetic Moment



- Consider the motion of a charged particle with mass m and charge q moving in a circular orbit or radius R.
- From the laws of classical mechanics, we know that the particle has an angular momentum given by

$$\mathbf{L} = I\boldsymbol{\omega} \tag{1}$$

where $I = mR^2$ is the moment of inertia of the particle, and $\boldsymbol{\omega}$ is the angular velocity.

A Classical Description of Orbital Magnetic Moment

- We also know from the laws of electricity and magnetism that a circulating loop of current produces a magnetic moment

$$\boldsymbol{\mu}_{\text{orbital}} = i\,\mathbf{A} \tag{2}$$

where i is the electrical current and $\mathbf{A}$ is the area vector, whose magnitude corresponds to the area enclosed by the current, and whose direction is normal to the plane of the orbit.

- If we express eqns.(1,2) in terms of the circular orbit, then we can find a simple expression for the magnetic moment in terms of the angular momentum.

$$\mathbf{L} = mR^2\boldsymbol{\omega} \tag{3}$$

$$\begin{aligned}\boldsymbol{\mu}_{\text{orbital}} &= \left(\frac{\Delta q}{\Delta t}\right)\pi R^2\hat{\mathbf{z}} \\ &= \frac{\omega q}{2\pi}\,\pi R^2\hat{\mathbf{z}}\end{aligned} \tag{4}$$

where the direction $\hat{\mathbf{z}}$ is taken to be normal to the plane of the orbit.

Interaction of a Magnetic Moment with Magnetic Fields

$$\begin{aligned} \boldsymbol{\mu}_{\text{orbital}} &= \frac{\omega q R^2}{2}\hat{\mathbf{z}} \\ &= \left(\frac{q}{2m}\right)\mathbf{L} \end{aligned}$$

$$\boxed{\boldsymbol{\mu}_{\text{orbital}} = \left(\frac{q}{2m}\right)\mathbf{L}} \tag{5}$$

Intrinsic Magnetic Moment (Spin)

- We see that from a purely classical description, a charged particle with angular momentum also has magnetic moment.
- Interestingly, the ratio of the angular momentum and the orbital magnetic moment is proportional only the charge-to-mass ratio, and does not involve quantities, such as the radius, that are particular to the orbit of the particle.
- The connection between angular momentum and magnetic moment, in fact is very deep. It turns out that elementary particles, such as the proton, electron, neutron, muon, etc. have an intrinsic angular momentum and a magnetic moment.
- The name given to the intrinsic magnetic moment is "spin"— the name motivated by the classical notion of a rotating charged object, having both angular momentum and magnetic moment.
- Though the term spin implies a rotating body, the particle is not physically spinning. Spin is a fundamental property of a particle, like its mass or charge.

The Spin Magnetic Moment

- The intrinsic or spin magnetic moment is expressed as

$$\boldsymbol{\mu}_{\text{spin}} = g\left(\frac{q}{2m}\right)\mathbf{S} \tag{6}$$

- The factor g, known as the "$g-$factor" is a multiplicative factor, that modifies the classical result we found earlier in eq.(5). For classical orbital motion $g = 1$.
- In 1928, Paul Dirac showed that for certain class of particles, such as electrons, $g = 2$, if quantum theory is made consistent with special relativity.
- The relativistic quantum theory also predicted antimatter.
- In 1933, Dirac and Schrödinger were awarded the Nobel Prize in Physics for "For the discovery of new productive forms of atomic theory."
- In 1965, Feynman, Schwinger, and Tomonaga were awarded the Nobel Prize in Physics for their theory of Quantum Electrodynamics (QED), which unified electricity and magnetism and quantum mechanics.

The Spin Magnetic Moment

- Using QED, it possible to calculate corrections to $g = 2$. For an electron the $g-$factor has been calculated to be

$$g \approx 2.00231930436256(35)$$

- This is the most precise constant of nature known.
- The calculated and measured values of the electron $g-$factor agree to 35 parts in 10^{14}. This is the equivalent of measuring the diameter of the earth to the precision of the width of a hair. This remarkable agreement is a triumph of quantum theory.
- You may have heard of the $g - 2$ experiment in Fermi Lab, whose goal is to measure the $g-$factor of the muons – a heavier cousin of the electron.
- The aim of these experiments is to probe the extent to which experiment and theory agree in predicting the muon $g-$ factor. Discrepancies could point to theories that lie outside of the Standard Model – our best theory describing elementary particles and forces.

Interaction of a Magnetic Moment with Magnetic Fields

- For the rest of this section, I will refer to $\boldsymbol{\mu}_{\text{orbital}}$ as $\boldsymbol{\mu}$.
- In the presence of a uniform magnetic field $\mathbf{B}$, a magnetic moment $\boldsymbol{\mu}$ experiences a torque that tries to make $\mathbf{B} \parallel \boldsymbol{\mu}$.
- The torque acts in a direction that tries to lower the potential energy of $\boldsymbol{\mu}$. Thus, we can express a conservative potential energy $U(\theta)$.

$$\begin{aligned} U(\theta) &= -\boldsymbol{\mu} \cdot \mathbf{B} \\ &= -|\boldsymbol{\mu}||\mathbf{B}| \cos\theta \end{aligned} \tag{7}$$

  with

$$\frac{dU}{d\theta} = |\boldsymbol{\tau}| = |\boldsymbol{\mu}||\mathbf{B}| \sin\theta \tag{8}$$

- In vector form:

$$\begin{aligned} \boldsymbol{\tau} &= \boldsymbol{\mu} \times \mathbf{B} \\ |\boldsymbol{\tau}| &= |\boldsymbol{\mu}||\mathbf{B}|\ \sin\theta \end{aligned} \tag{9}$$

  where θ is the interior angle made between $\boldsymbol{\mu}$ and $\mathbf{B}$.

Interaction of a Magnetic Moment with Magnetic Fields

- Because U is a conservative potential, its spatial derivatives are equal to the force on $\boldsymbol{\mu}$.

$$\mathbf{F} = -\boldsymbol{\nabla} U = \boldsymbol{\nabla}(\boldsymbol{\mu} \cdot \mathbf{B}) \tag{10}$$

- A magnetic moment experiences a torque, and no net force when placed in a uniform magnetic field. If however, the field varies in space, then it will experience a force in the direction of increasing magnetic field.