PHYS 234: Quantum Physics 1 (Winter 2026)
Tutorial 3

The goal of this tutorial session is to build familiarity with the matrix representation of operators and states in different bases.

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1. Consider a spin-1/2 particle in the state

$$|\psi\rangle = \frac{3}{5}|+\rangle + i\frac{4}{5}|-\rangle. \tag{1}$$

   (a) Write the matrix representation of $|\psi\rangle$ in
      i. the S_z eigenbasis
      ii. the S_y eigenbasis
      iii. the basis $\{\frac{3}{5}|+\rangle + i\frac{4}{5}|-\rangle, -\frac{4}{5}|+\rangle + i\frac{3}{5}|-\rangle\}$

   (b) For each matrix representation derived above, calculate the probability of a measurement of S_y yielding $\hbar/2$. Show that all three give the same result.

2. Consider the operator A, which has a matrix representation

$$A \doteq \begin{pmatrix} 1 & 2i \\ -2i & 1 \end{pmatrix}, \tag{2}$$

   in the S_z eigenbasis.

   (a) Does A satisfy the condition to be a valid observable?
   (b) If yes, what are the possible measurement outcomes?
   (c) Find the matrix representation of A in the S_y eigenbasis.
   (d) Suppose that a spin-1/2 particle is in the $|+\rangle_y$ state. Calculate the probabilities for the possible measurement outcomes of A using both the S_z and S_y representations.

Something to think about: Suppose a spin-1/2 particle is in the $\hat{\mathbf{n}} \cdot \mathbf{S}$ eigenstate $|+\rangle_{\hat{\mathbf{n}}}$. Show that the probability of a measurement of $\hat{\mathbf{n}}'.\mathbf{S}$ yielding $\hbar/2$ is $\cos^2(\alpha/2)$, where α is the angle between the two vectors $\hat{\mathbf{n}}$ and $\hat{\mathbf{n}}'$.

Hint: A clever choice of basis will make the algebra a lot less tedious.