Lecture 3
The Stern-Gerlach Experiment II

Raffi Budakian

University of Waterloo

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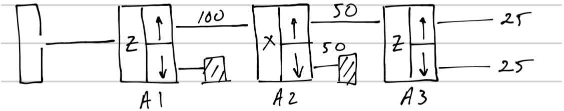
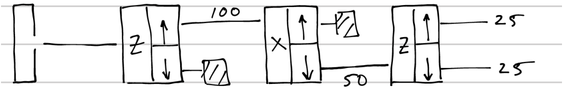
S-G Experiment 3

(A) $P[|+\rangle_{(1)}] :=$ Probability to exit A1



$$P[|+\rangle_{(1)}]\, P[|+\rangle_{x(2)}]\, P[|+\rangle_{(3)}] = 25\%$$
$$P[|+\rangle_{(1)}]\, P[|+\rangle_{x(2)}]\, P[|-\rangle_{(3)}] = 25\%$$

(B)



$$P[|+\rangle_{(1)}]\, P[|-\rangle_{x(2)}]\, P[|+\rangle_{(3)}] = 25\%$$
$$P[|+\rangle_{(1)}]\, P[|-\rangle_{x(2)}]\, P[|-\rangle_{(3)}] = 25\%$$

(C) Suppose we combine the two paths from A2 and send it to A3 without counting how many spins exit each of the two ports of A2.

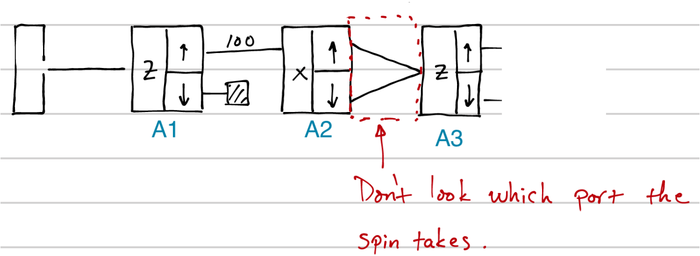
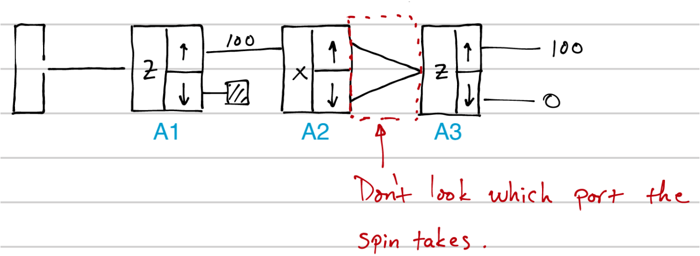


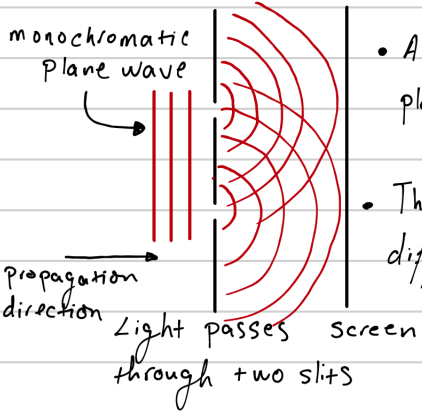
Figure: S-G Experiment 4

Classically, based on the results from Experiments 3A and 3B, we would expect the probability to exit the lower port of A3 to be $25 + 25 = 50$, and similarly $25 + 25 = 50$ to exit the top port.

S-G Experiment 4



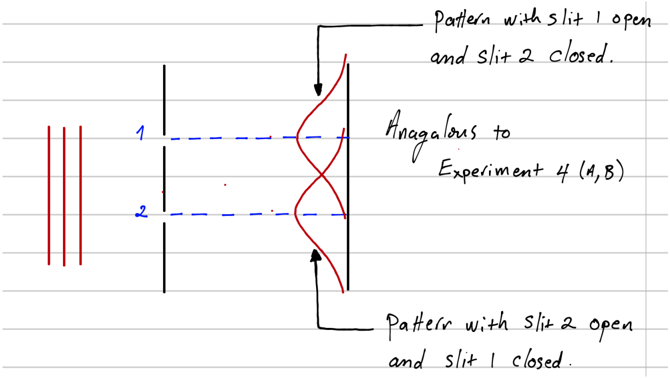
- By **not** observing which path the spin takes through the $x-$analyzer (A2), it's as if A2 was not there at all!
- This behaviour has no classical analog. However, this behaviour is consistent with the interference phenomenon observed for waves. In particular, the fact that no particles are observed in the lower port ($|-\rangle$) of A3 in S-G Exp.4 reminds us of two waves of equal amplitude that interfere destructively to produce zero amplitude.



- A monochromatic (single frequency) plane wave impinges onto two slits.
- The light through each slit is diffracted and emerges as spherical wave fronts.

If we cover one slit at a time, and allow the light from only one of the slits to fall on the screen, the resulting intensity pattern will look the one one shown below.

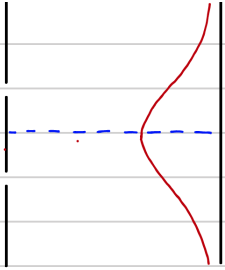
Young's Double Slit Interference Experiment



- Each pattern by itself resembles what we would expect for a classical distribution of particles arriving from the individual slits.
- If both slits are opened, then for classical particles, we would expect a distribution like the one shown below.

Young's Double Slit Interference Experiment

- For classical particles, the two single slit patterns would add.

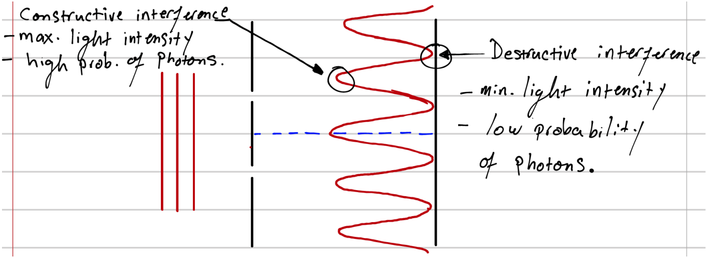


Classical probability distribution

For classical particles, the two single-slit intensity patterns would add.

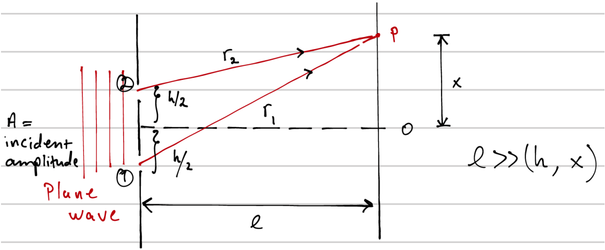
Young's Double Slit Interference Experiment

However, we observe that the waves from the two slits interfere.



- The key difference between classical particles and waves is the interference phenomenon observed for waves.
- Physically, we interpret the intensity as being proportional to the probability of observing a photon. The interference intensity is calculate by squaring the sum of the field amplitudes from the two slits.
- The S-G Experiment 4 exhibits an identical interference phenomenon as observed in Young's double slit experiment. This implies that the spins that take the upper and lower paths through A2 ($x-$analyzer) interfere constructively to produce all $|+\rangle$ spins through A3 ($z-$analyzer).

2-Slit Pattern Calculation



- $1/2$ of the incident wave amplitude exits from slits (1) and (2).
- The waves exiting each slit have a traveling wave solution given by

$$a_{(1,2)}(r_{(1,2)}, t) = \left(\frac{A}{2}\right) e^{i(kr_{(1,2)} - \omega t)} \tag{1}$$

where the wavenumber $k = \frac{2\pi}{\lambda}$, $\frac{\omega}{k} = c$, and

$$r_1 = \sqrt{l^2 + \left(\frac{h}{2} + x\right)^2}, \quad r_2 = \sqrt{l^2 + \left(\frac{h}{2} - x\right)^2}$$

2-Slit Pattern Calculation

- The wave amplitude at point P is:

$$a(x) = \frac{A}{2}\left(e^{ikr_1} + e^{ikr_2}\right)e^{-i\omega t} \tag{2}$$

- The wave intensity at point P is proportional to the magnitude square of the total wave amplitude at P.

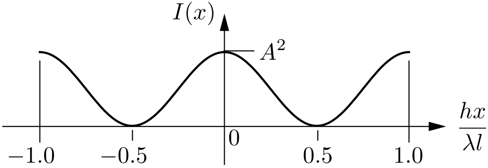
$$I(x) = \left(\frac{A^2}{4}\right)\left|e^{ikr_1} + e^{ikr_2}\right|^2 \tag{3}$$

$$= \left(\frac{A^2}{2}\right)\left[1 + \cos\left(k[r_1 - r_2]\right)\right] \tag{4}$$

2-Slit Pattern Calculation

$$r_1 - r_2 \approx \frac{hx}{l}, \qquad l \gg (h, x)$$

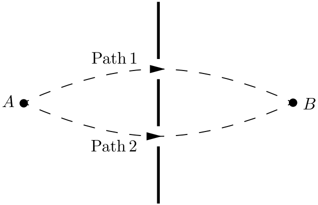
$$I(x) = \left(\frac{A^2}{2}\right)\left[1 + \cos\left(\frac{2\pi h}{\lambda l}\,x\right)\right]$$



- We can turn down the light level low enough so that there is only one photon going through the double slit device at any given time.

Single-Particle Interference

- According to our classical notion of reality, we think of a photon taking either one of two the paths to reach the screen. However, according to the laws of quantum mechanics, each particle takes **both** paths.



- If a photon (or any quantum particle) can take multiple paths to go between points A and B, the probability to go from A to B is calculated by summing the probability amplitudes for each of the possible paths, and calculating the square of the sum of the amplitudes. This is contrary to the classical way of calculating probability, where we add the probability of each event separately.

Single-Particle Interference

- Classical way of adding probabilities:

$$P_{\text{classical}} = \left|a_1 e^{i\phi_1}\right|^2 + \left|a_2 e^{i\phi_2}\right|^2$$

$$= a_1^2 + a_2^2$$

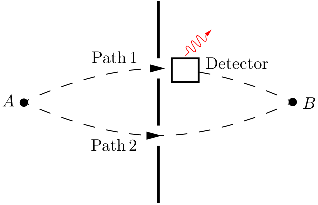
- Quantum mechanical way of adding probabilities:

$$P_{\text{quantum}} = \left|a_1 e^{i\phi_1} + a_2 e^{i\phi_2}\right|^2$$

$$= (a_1 e^{-i\phi_1} + a_2 e^{-i\phi_2})(a_1 e^{i\phi_1} + a_2 e^{i\phi_2})$$

$$= a_1^2 + a_2^2 + \underbrace{2a_1 a_2 \cos(\phi_1 - \phi_2)}_{\text{interference term}}$$

Single-Particle Interference



- If we place a device between A and B that can tell us which path the particle takes, then we don't observe interference, and instead recover the classical probability distribution. The act of observation localizes the particle to the path where the particle was observed to be. The particle can no longer exist in both paths, and therefore no interference takes place. This behaviour is sometimes referred to as the "collapse of the wavefunction." The notion of the wavefunction will be developed later in the course.