PHYS234: Quantum Physics 1 (Winter 2026)
Assignment 3

1. Verify for the operators $\hat{A}$, $\hat{B}$, and $\hat{C}$ that

   (a) $\left[\hat{A}, \hat{B}+\hat{C}\right] = \left[\hat{A}, \hat{B}\right] + \left[\hat{A}, \hat{C}\right]$

   (b) $\left[\hat{A}, \hat{B}\hat{C}\right] = \hat{B}\left[\hat{A}, \hat{C}\right] + \left[\hat{A}, \hat{B}\right]\hat{C}$

2. Show that the operator $\hat{C}$ defined through $\left[\hat{A}, \hat{B}\right] = i\hat{C}$ is Hermitian, provided the operators $\hat{A}$ and $\hat{B}$ are Hermitian.

3. Calculate ΔS_x and ΔS_y for an eigenstate of S_z for a spin-$\frac{1}{2}$ particle. Check to see if the uncertainty relation $\Delta S_x \Delta S_y \geq \hbar |\langle S_z \rangle|/2$. Repeat your calculation for an eigenstate of $\hat{S}_x$.

You can find the information needed to answer questions 4 and 5 in section 2.7 of McIntyre.

4. A spin-1 particle is in the state

$$|\psi\rangle \xrightarrow[S_z\,\text{basis}]{} \frac{1}{\sqrt{14}}\begin{pmatrix} 1 \\ 2 \\ 3i \end{pmatrix}$$

   (a) What are the probabilities that a measurement of S_z will yield the values $\hbar, 0$, or $-\hbar$ for this state? What is $\langle S_z \rangle$?

   (b) What is $\langle S_x \rangle$ for this state? *Suggestion:* Use matrix mechanics to evaluate the expectation value.

   (c) What is the probability that a measurement of S_x will yield the value $\hbar$ for this state?

5. A spin-1 particle is prepared in the state

$$|\psi_i\rangle = \sqrt{\frac{1}{6}}\,|1\rangle - \sqrt{\frac{2}{6}}\,|0\rangle + i\sqrt{\frac{3}{6}}\,|-1\rangle$$

   Find the probability that the system is measured to be in in the final state

$$|\psi_f\rangle = \frac{1+i}{\sqrt{7}}\,|1\rangle_y + \frac{2}{\sqrt{7}}\,|0\rangle_y - \frac{i}{\sqrt{7}}\,|-1\rangle_y$$

6. Determine the matrix representation of the spin-$\frac{1}{2}$ angular momentum operators S_x, S_y, and S_z using the eigenstates of S_y as a basis.

7. Show that the $\mathbf{S}^2$ operator commutes with each of the spin component operators S_x, S_y, and S_z. Do this once with matrix notation for a spin-$\frac{1}{2}$ system and a second time using only the component commutation relations

$$[S_x, S_y] = i\hbar S_z$$
$$[S_y, S_z] = i\hbar S_x$$
$$[S_z, S_x] = i\hbar S_y$$

and the definition $\mathbf{S}^2 = S_x^2 + S_y^2 + S_z^2$.