PHYS 234: Quantum Physics 1 (Winter 2026)
Quiz 2 Solutions

We are told that $|+\rangle$ and $|-\rangle$ are eigenstates of S_z with eigenvalues $+\hbar/2$ and $-\hbar/2$, respectively, and satisfy

$$\langle +|+\rangle = \langle -|-\rangle = 1, \qquad \langle +|-\rangle = 0.$$

1. Normalization: We compute

$$\langle \psi|\psi\rangle = \frac{1}{2}\big(\langle +|+\rangle + \langle -|-\rangle\big) = \frac{1}{2}(1+1) = 1.$$

Thus $|\psi\rangle$ is normalized, and statement (1) is false.

2 − 3. Measurement of S_x: The eigenstates of S_x are

$$|+\rangle_x = \frac{1}{\sqrt{2}}\left(|+\rangle + |-\rangle\right), \qquad |-\rangle_x = \frac{1}{\sqrt{2}}\left(|+\rangle - |-\rangle\right).$$

The corresponding probability amplitudes are

$$\langle +_x|\psi\rangle = \frac{1}{\sqrt{2}}\left(\langle +| + \langle -|\right)\frac{e^{i\theta}}{\sqrt{2}}\left(|+\rangle + e^{i\varphi}|-\rangle\right) = \frac{e^{i\theta}}{2}\left(1+e^{i\varphi}\right),$$
$$\langle -_x|\psi\rangle = \frac{1}{\sqrt{2}}\left(\langle +| - \langle -|\right)\frac{e^{i\theta}}{\sqrt{2}}\left(|+\rangle + e^{i\varphi}|-\rangle\right) = \frac{e^{i\theta}}{2}\left(1-e^{i\varphi}\right).$$

Therefore, the measurement probabilities are

$$\mathbb{P}(+_x) = |\langle +_x|\psi\rangle|^2 = \frac{1}{4}\left|1+e^{i\varphi}\right|^2 = \frac{1}{2}\left(1+\cos\varphi\right) = \cos^2\frac{\varphi}{2},$$
$$\mathbb{P}(-_x) = |\langle -_x|\psi\rangle|^2 = \frac{1}{4}\left|1-e^{i\varphi}\right|^2 = \frac{1}{2}\left(1-\cos\varphi\right) = \sin^2\frac{\varphi}{2}.$$

Since these probabilities depend on φ, statement (3) is false. The global phase factor $e^{i\theta}$ cancels in the probabilities, so they are independent of θ, and statement (4) is true.

2. Eigenstate condition: The state $|\psi\rangle$ is an eigenstate of S_x only if one of the probabilities above equals 1 and the other equals 0. This occurs only for special values of φ (e.g. $\varphi = 0$ or π), and not for arbitrary φ. Hence statement (2) is false.

Conclusion: The only statement that is always true is **4**.