PHYS234: Quantum Physics 1 (Winter 2026)

Test 1 Formula Sheet

- Basic states and operators expressed in the S_z basis.

$$|+\rangle = \begin{pmatrix}1\\0\end{pmatrix}, \quad |-\rangle = \begin{pmatrix}0\\1\end{pmatrix}$$

$$|+\rangle_x = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}, \quad |-\rangle_x = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$$

$$|+\rangle_y = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\i\end{pmatrix}, \quad |-\rangle_y = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-i\end{pmatrix}$$

$$|+\rangle_n = \begin{pmatrix}\cos(\theta/2)\\ \sin(\theta/2)e^{i\phi}\end{pmatrix}, \quad |-\rangle_n = \begin{pmatrix}\sin(\theta/2)\\ -\cos(\theta/2)e^{i\phi}\end{pmatrix}$$

- Spin-$\frac{1}{2}$ operators:

$$S_x = \frac{\hbar}{2}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}; \quad S_y = \frac{\hbar}{2}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}; \quad S_z = \frac{\hbar}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

$$S_n = \frac{\hbar}{2}\begin{pmatrix}\cos\theta & \sin\theta\, e^{-i\phi}\\ \sin\theta\, e^{i\phi} & -\cos\theta\end{pmatrix}$$

- Other relations that may be useful:

$$\Delta A = \sqrt{\langle A^2\rangle - \langle A\rangle^2}$$

$$\Delta A \Delta B \geq \frac{1}{2}|\langle [A,B]\rangle|$$

$$\langle A\rangle = \langle\psi|A|\psi\rangle$$

$$|\psi'\rangle = \frac{\hat{P}_n\,|\psi\rangle}{\sqrt{\langle\psi|\hat{P}_n|\psi\rangle}}$$

- Identities you may need:

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

$$\cos u \cos v = \tfrac{1}{2}(\cos(u+v) + \cos(u-v))$$

$$\sin u \sin v = \tfrac{1}{2}(\cos(u-v) - \cos(u+v))$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}, \qquad \sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

- Power series expansions of commonly-used functions:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \approx 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$\sin(x) \approx x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\cos(x) \approx 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$