PHYS 234: Quantum Physics 1 (Winter 2026)
Tutorial 1: Review of Linear Algebra

The goal of this tutorial session is to review some of the basic linear algebra needed in quantum physics.

Consider the matrices given below:

$$M_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad M_2 = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}, \qquad M_3 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}, \qquad M_4 = \begin{pmatrix} i & 1 \\ 0 & 0 \end{pmatrix}$$

1. Diagonalize the above matrices by finding their eigenvalues and (normalized) eigenvectors. If your matrix is real, check if the calculated eigenvalues and eigenvectors are consistent with the geometric interpretation of the matrix as a linear map on $\mathbb{R}^2$.

2. Find the inner product between the eigenvectors of each matrix. Notice how the result is zero in some cases. Show that in these cases, the matrix commutes with its Hermitian adjoint, i.e. $M^\dagger M = MM^\dagger$. Recall that the Hermitian adjoint is the same as the conjugate transpose, i.e. $M^\dagger = (M^T)^*$.

   [Matrices with this property are called *normal* matrices and their eigenvectors form an orthogonal basis for the underlying vector space ($\mathbb{C}^2$ in this example). Most of the matrices encountered in quantum mechanics are normal matrices.]

Something to think about: All of the above matrices have two linearly independent eigenvectors, i.e. their eigenvectors span $\mathbb{C}^2$. Can you come up with a 2×2 matrix that doesn't?