PHYS234: Quantum Physics 1 (Winter 2026)
Assignment 4

1. Given the density operator

$$\hat{\rho} = \tfrac{3}{4}\left|+\right\rangle\!\left\langle+\right| + \tfrac{1}{4}\left|-\right\rangle\!\left\langle-\right|$$

   (a) Construct the density matrix.

   (b) Show that this is the density operator for a mixed state.

   (c) Determine $\langle S_x \rangle$, $\langle S_y \rangle$, and $\langle S_z \rangle$ for this state.

   (d) Find states $|\psi_1\rangle$ and $|\psi_2\rangle$ for which the density operator can be expressed in the form

$$\hat{\rho} = \tfrac{1}{2}\left|\psi_1\right\rangle\!\left\langle\psi_1\right| + \tfrac{1}{2}\left|\psi_2\right\rangle\!\left\langle\psi_2\right|$$

2. The density matrix for an ensemble of a spin-1/2 particle in the S_z basis is

$$\hat{\rho} \xrightarrow{S_z\,\text{basis}} \begin{pmatrix} \frac{1}{4} & n \\ n^* & p \end{pmatrix}$$

   (a) What value must p have? Why?

   (b) What value(s) must n have for the density matrix to represent a pure state?

   (c) What pure state is represented when n takes its maximum possible real value? Express your answer in terms of the state $|+\rangle_n$

$$|+\rangle_n = \cos\left(\tfrac{\theta}{2}\right)|+\rangle + e^{i\phi}\sin\left(\tfrac{\theta}{2}\right)|-\rangle$$