ELEC 102
Electronic Circuits I

Dec. 21, 1999 (Tuesday), 12:30pm – 3:30pm, Sport Hall
Examiner: Dr. Ki Wing-Hung

Name: Soln   Student ID: ______

Chinese Name: 题解   Signature: ______

Directions:

(1) This is a closed book examination.
(2) Calculators are allowed.
(3) Answer all questions in the space provided. Request for additional sheets from proctors only if necessary.
(4) Show all your calculations. No marks will be given for unjustified answers.
(5) Do your own work. Any form of cheating is a violation of academic integrity, and will be dealt with accordingly.

Question	Maximum Points	Points
1	5	
2	8	
3	15	
4	12	
5	8	
6	8	
7	12	
8	12	
9	20	
Total	100	

Diode: $I_d = I_s(e^{V_d/V_{th}} - 1) \approx I_s e^{V_d/V_{th}}$ for $V_d >> V_{th}$, with $V_{th} = 26mV$ @ 300K

NMOS
$V_{ds} \le V_{gs} - V_T$
$I_D = k[(V_{gs}-V_T)V_{ds} - \frac{1}{2}V_{ds}^2]$

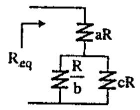
$V_{ds} \ge V_{gs} - V_T$
$I_D = \frac{1}{2}k(V_{gs}-V_T)^2$

PMOS
$|V_{dsp}| \le |V_{gsp}| - |V_{Tp}|$
$|I_{Dp}| = k_p[(|V_{gsp}|-|V_{Tp}|)|V_{dsp}| - \frac{1}{2}|V_{dsp}|^2]$

$|V_{dsp}| \ge |V_{gsp}| - |V_{Tp}|$
$|I_{Dp}| = \frac{1}{2}k(|V_{gsp}| - |V_{Tp}|)^2$

1. Warm up question — 5 points

1a. Let a = 3, b = 7, and c = 16, compute R_{eq} as a rational number (m/n). — 4 points



$$R_{eq} = a + \frac{1}{\frac{1}{1/b} + \frac{1}{c}} R$$

$$= a + \frac{1}{b + \frac{1}{c}} R$$

$$= 3 + \frac{1}{7 + \frac{1}{16}} R$$

$$= 3 + \frac{16}{113} R$$

$$= \frac{355}{113} R \ \#.$$

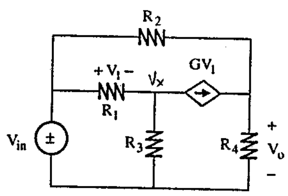
1b. Express this rational number as a real number. What value does it relate to? — 1 point

$R_{eq} = 3.14159 R$

$\doteq \pi R \ \#$

2. Resistive network with dependent source — 8 points

2a. We need to compute the voltage V_o for the following circuit. Use nodal analysis to write equations at appropriate nodes. Determine the condition of G such that no solution exists (i.e., some voltages go to infinity). — 5 points



$V_1 = V_{in} - V_x$

@ V_x: $\frac{V_{in} - V_x}{R_1} = \frac{V_x}{R_3} + GV_1 = \frac{V_x}{R_3} + G(V_{in} - V_x)$

$\Rightarrow V_{in}(\frac{1}{R_1} - G) = V_x(\frac{1}{R_1} + \frac{1}{R_3} - G)$ ①.

@ V_o: $GV_1 + \frac{V_{in} - V_o}{R_2} = \frac{V_o}{R_4} = G(V_{in} - V_x) + \frac{V_{in}}{R_2} - \frac{V_o}{R_2} = \frac{V_o}{R_4}$

$\Rightarrow (G + \frac{1}{R_2})V_{in} - GV_x = V_o(\frac{1}{R_2} + \frac{1}{R_4})$

$\Rightarrow (G + \frac{1}{R_2})V_{in} - G\frac{\frac{1}{R_1} - G}{(\frac{1}{R_1} + \frac{1}{R_3} - G)} V_{in} = V_o(\frac{1}{R_2} + \frac{1}{R_4})$ ②

Clearly, no soln exists if $\frac{1}{R_1} + \frac{1}{R_3} - G = 0$.

2b. With $R_1 = R_2 = R_3 = R_4 = R$, and $G = 3/R$. Compute V_o. — 3 points

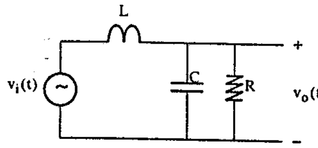
② $\Rightarrow (\frac{3}{R} + \frac{1}{R})V_{in} - \frac{3}{R}\frac{\frac{1}{R} - \frac{3}{R}}{\frac{1}{R} + \frac{1}{R} - \frac{3}{R}} V_{in} = V_o(\frac{1}{R} + \frac{1}{R})$.

$\Rightarrow \frac{4}{R}V_{in} - \frac{3}{R}\frac{-2}{-1}V_{in} = \frac{2}{R}V_o$

$\Rightarrow V_o = 2V_{in} - 3V_{in}$

$= -V_{in} \ \#$.

3. We want to use two approaches to find the output voltage $v_o(t)$ of the following circuit. 15 points



$L = 10\mu H$
$C = 1\mu F$
$R = \frac{10}{11}\Omega$

3a. The first method is to use phasor. With $v_i(t) = 10\cos(3\times10^5 t)$, find $v_o(t)$. 6 points

$$\frac{V_o(s)}{V_i(s)} = \frac{R /\!/ \frac{1}{sC}}{sL + R /\!/ \frac{1}{sC}} = \frac{\frac{R}{1+sCR}}{sL + \frac{R}{1+sCR}} = \frac{R}{sL + s^2LCR + R}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{1 + s\frac{L}{R} + s^2LC}$$

$$\Rightarrow V_o(j3\times10^5) = \frac{1}{1 + j3\times10^5 \frac{10\times10^{-6}}{10/11} - (3\times10^5)^2\, 10\times10^{-6}\times10^{-6}} \times 10$$

$$= \frac{1}{1 - 9\times10^{10}\times10^{-11} + j33\times10^{-1}} \times 10$$

$$= \frac{1}{1 - 0.9 + 3.3j} \times 10$$

$$= \frac{1}{0.1 + 3.3j} \times 10$$

$$= \frac{1}{3.3\angle 88.3^\circ} \times 10$$

$$= 3.03\angle -88.3^\circ$$

$$\Rightarrow V_o(t) = 3.03\cos(3\times10^5 t - 88.3^\circ) \quad \#$$

3b. The second method makes use of Bode plots. Derive the transfer function $H(s) = V_o(s)/V_i(s)$ of the LCR circuit. Hint: The denominator can be factorized into two first order terms. 3 points

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{1}{1 + \frac{sL}{R} + s^2LC} = \frac{1}{1 + s\frac{10\times10^{-6}}{10/11} + s^2\, 10\times10^{-6}\times10^{-6}}$$

$$= \frac{1}{1 + s\,11\times10^{-6} + s^2 10^{-11}}$$

$$= \frac{1}{1 + (10^{-5} + 10^{-6})s + (10^{-5})(10^{-6})s^2}$$

$$= \frac{1}{(1 + \frac{s}{10^5})(1 + \frac{s}{10^6})}$$

3c. Sketch the Bode plots of H(s) (graphs on next page). 4 points

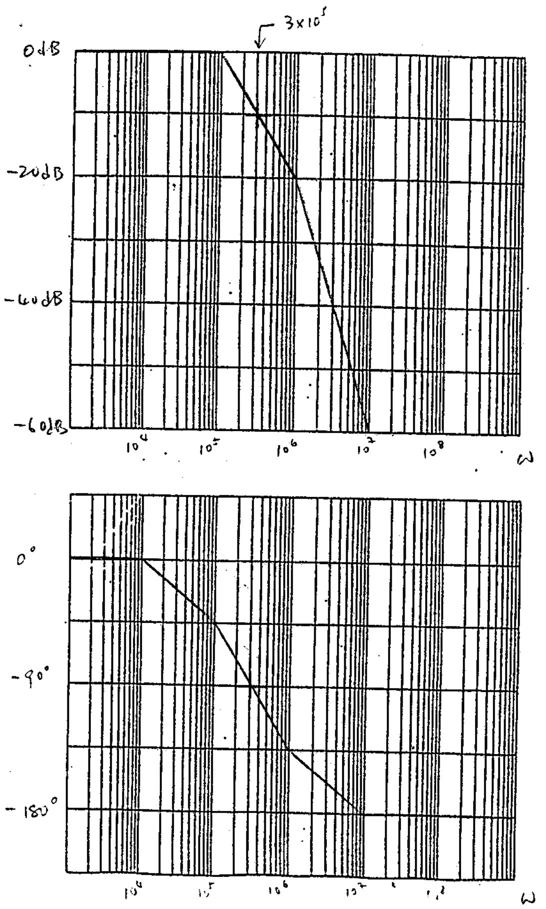
3d. Read the magnitude of $20\log|H|$ and phase of $\angle H$ at $\omega = 3\times10^5$. Thus, find $v_o(t)$ accordingly. (The answer differs slightly from that of 3a.) 2 marks

at $\omega = 3\times10^5$, $20\log|H| = -10\,dB \Rightarrow 0.316$

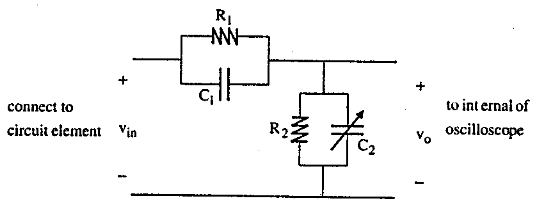
$\angle H = -90^\circ$

$$\therefore V_o(t) = 10\times0.316\cos(3\times10^5 t - 90^\circ)$$

$$= 3.16\cos(3\times10^5 t - 90^\circ) \quad \#$$



4. The internal structure of an oscilloscope 10X probe is shown below. 12 points



4a. Derive the corresponding transfer function $V_o(s)/V_i(s)$. 6 points

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{R_2 /\!/ \frac{1}{sC_2}}{R_1 /\!/ \frac{1}{sC_1} + R_2 /\!/ \frac{1}{sC_2}} = \frac{\frac{R_2}{1+sC_2R_2}}{\frac{R_1}{1+sC_1R_1} + \frac{R_2}{1+sC_2R_2}}$$

$$= \frac{R_2(1 + sC_1R_1)}{R_1 + sC_2R_1R_2 + R_2 + sC_1R_1R_2}$$

$$= \frac{R_2(1 + sC_1R_1)}{R_1 + R_2 + s(C_1 + C_2)R_1R_2}$$

$$= \frac{R_2}{R_1 + R_2} \frac{(1 + sC_1R_1)}{1 + s(C_1 + C_2)(R_1 /\!/ R_2)} \quad \#$$

4b. The capacitor C_2 is a variable capacitor that can be adjusted. What value should you pick such that the transfer function is a constant? **4 points**

For $H(s) = \text{constant} \Rightarrow C_1R_1 = (C_1+C_2)\, R_1\|R_2$

$\Rightarrow \frac{C_1}{C_1+C_2} = \frac{R_1R_2}{R_1+R_2}\frac{1}{R_1} = \frac{R_2}{R_1+R_2}$

$\Rightarrow C_1R_1 + C_1R_2 = C_1R_2 + C_2R_2$

$\Rightarrow C_1R_1 = C_2R_2$

$\Rightarrow$

$$C_2 = \frac{R_1}{R_2}C_1$$

4c. To achieve the 10X reduction, if R_2 is $1M\Omega$, what should R_1 be? **2 points**

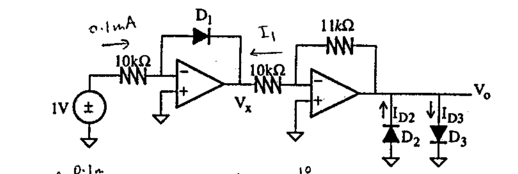
with $\frac{C_2}{C_1} = \frac{R_2}{R_1}$

$H(s) = \frac{R_2}{R_1+R_2}$.

10X reduction $\Rightarrow H(s) = \frac{1}{10} = \frac{R_2}{R_1+R_2}$

$\Rightarrow R_1 = 9\,M\Omega$ #.

5. Let the op-amps be ideal in the following circuit. With $I_s = 2fA = 2\times10^{-15}A$, calculate V_x, V_o, I_{D2} and I_{D3}. Hint: No iteration is needed. Follow the diode equation. **8 points**



$V_{o1} = V_{th}\ln\frac{0.1m}{2\times10^{-15}} = 26m\ln 5\times10^{10}$

$= 26m \times 24.635$

$= 0.6405V$

$\Rightarrow V_x = -V_{o1} = -0.6405V$.

$I_1 = \frac{V_x}{10k} = \frac{0.6405}{10k} = 64.05\mu A$

$V_o = 11k \times I_1 = 0.7046V$

$\Rightarrow D_3$ will be ON. and $I_{D3} = 2\times10^{-15}e^{0.7046/26m}$

$= 2\times10^{-15}\times 5.88\times10^{11}$

$= 1.176mA$ #.

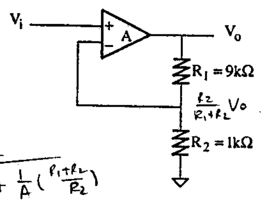
D_2 reverse biased $\Rightarrow$ off.

$I_{D2} = -I_s = -2\times10^{-15}A$ #

or 0 #

6. Nonlinear effect due to op-amp gain **8 points**

6a. For the non-inverting amplifier below, derive the gain $G = V_o/V_i$ for an arbitrary A. **3 points**



$\frac{R_2}{R_1+R_2}V_o$

$A(V_i - \frac{R_2}{R_1+R_2}V_o) = V_o$

$\Rightarrow AV_i = V_o(A\frac{R_2}{R_1+R_2} + 1)$

$\Rightarrow G = \frac{V_o}{V_i} = \frac{A}{A\frac{R_2}{R_1+R_2}+1} = \left(\frac{R_1+R_2}{R_2}\right)\frac{1}{1+\frac{1}{A}\left(\frac{R_1+R_2}{R_2}\right)}$

$G = 10\frac{1}{1+\frac{10}{A}}$

6b. Calculate the gain G for A = 9,000, 10,000 and 11,000. **3 points**

$G(A=9000) = 10\frac{1}{1+\frac{10}{9000}} = 9.98890$

$G(A=10000) = 10\frac{1}{1+\frac{10}{10000}} = 9.99001$

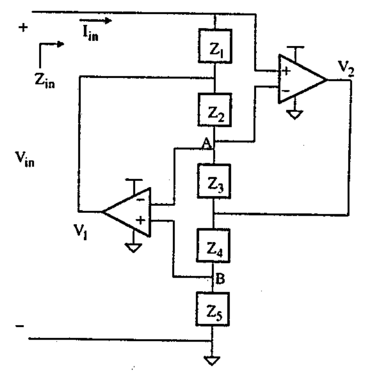
$G(A=11000) = 10\frac{1}{1+\frac{10}{11000}} = 9.99092$

6c. The sensitivity of the gain w.r.t. the op-amp gain is defined as $S^G_A = \frac{\partial G/\partial A}{G/A} \approx \frac{\Delta G/\Delta A}{G/A}$. Use result of (6b) to compute S^G_A at A = 10,000. **2 points**

$S^G_A\Big|_{A=10000} = \frac{(9.99092 - 9.98890)/(11000-9000)}{10/10000}$

$= \frac{2.02\times10^{-3}/2000}{10/10^{-5}} = 1.01\times10^{-3}$

7. In circuit design, we avoid using physical inductors because of their large size and weight. If needed, we may use circuit method to "obtain" an inductor from other circuit elements. One such method is shown below. **12 points**



7a. Let all op-amps be ideal. What are the voltages at node A and node B (V_A and V_B)? **2 points**

$V_A = V_{in}$

$V_B = V_{in}$

7b. Write nodal equations at node A and node B, and solve V_1 and V_2 in terms of V_{in}. **4 points**

@ A: $\frac{V_1 - V_{in}}{Z_2} = \frac{V_{in} - V_2}{Z_3} \Rightarrow \frac{V_1}{Z_2} = (\frac{1}{Z_2}+\frac{1}{Z_3})V_{in} - \frac{V_2}{Z_3}$ ①

@ B: $\frac{V_2 - V_{in}}{Z_4} = \frac{V_{in}}{Z_5} \Rightarrow \frac{V_2}{Z_4} = (\frac{1}{Z_4}+\frac{1}{Z_5})V_{in}$

$\Rightarrow V_2 = \frac{Z_4+Z_5}{Z_5}V_{in} = (1+\frac{Z_4}{Z_5})V_{in}$ # ②

② in ① $\Rightarrow V_1 = (1+\frac{Z_2}{Z_3})V_{in} - \frac{Z_2}{Z_3}(1+\frac{Z_4}{Z_5})V_{in}$

$= (1+\frac{Z_2}{Z_3} - \frac{Z_2}{Z_3} - \frac{Z_2Z_4}{Z_3Z_5})V_{in} \Rightarrow V_1 = (1 - \frac{Z_2Z_4}{Z_3Z_5})V_{in}$

7c. Consider the input current going into Z_1, find I_{in} and Z_{in} (= V_{in}/I_{in}). 4 points

$$I_{in} = \frac{V_{in} - V_1}{Z_1} = \frac{V_{in} - \left(1 - \frac{Z_2 Z_4}{Z_3 Z_5}\right) V_{in}}{Z_1}$$

$$= \frac{Z_2 Z_4}{Z_1 Z_3 Z_5} V_{in}$$

$$\Rightarrow Z_{in} = \frac{V_{in}}{I_{in}} = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4} \#$$

7d. With the result of (7c), show how an inductor can be obtained using 4 resistors and 1 capacitor. 2 points

For Z_{in} to be inductive $\Rightarrow Z_{in} = sL$

$\Rightarrow$ choose either Z_2 or Z_4 to be $1/sC$.

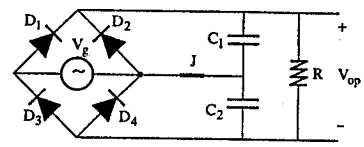
e.g. let $Z_1 = Z_2 = Z_3 = Z_5 = R$; $Z_4 = 1/sC$

$$\therefore Z_{in} = sL = \frac{R\,R\,R}{R\ 1/sC} = sCR^2.$$

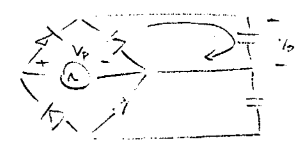
or $L = CR^2 \#$

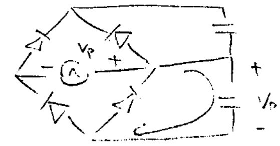
8. Modified full wave rectifier 12 points

8a. The figure below shows a modified full wave rectifier. 4 points
The jumper (switch) J is closed as shown. Let $V_g(t) = V_p\cos(2\pi 50t)$.
With $V_p = 110\sqrt{2}$ V and $R = \infty$, what is the DC output voltage V_{op}?
Show your argument by sketching the current flow in the diagram.
All the diodes are ideal diodes.

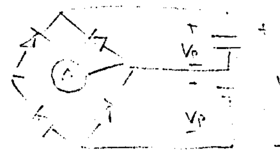


With $R = \infty$, we have



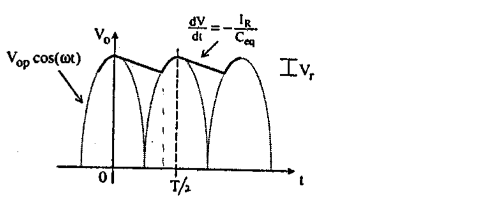


Hence



$V_{op} = 2V_p = 220\sqrt{2}$ V

8b. The exact charging and discharging mechanism is a little bit complicated. 8 points
The simplified mechanism is as shown below.

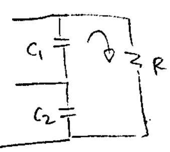


You may assume a constant load current of $I_R = V_{op}/R$ discharging the circuit.
Also, from $C_{eq}\frac{dV}{dt} = I_R$, the output voltage drops at a rate of $\frac{dV}{dt} = -I_R/C_{eq}$
(C_{eq} is the equivalent capacitance). With $R = 100\Omega$, and $C_1 = C_2 = C$,
Find C such that the ripple voltage V_r is 1% of the peak DC voltage V_{op}.

$$I_R = \frac{V_{op}}{R} = \frac{2V_p}{R}$$

For most of the time, the circuit is discharging.

$\Rightarrow$



$\Rightarrow C_{eq} = C_1 // C_2 = \frac{C}{2}.$

$$\therefore \frac{dV}{dt} = -\frac{I_R}{C_{eq}} = -\frac{2V_p}{R}\frac{1}{C/2} = -\frac{4V_p}{CR}$$

Since $V_r = 0.01 V_{op}$, we may assume $\Delta t \ll T/2$

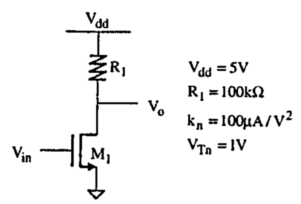
$T = \frac{1}{50} = 20$ms

$$\text{so } \Delta V = V_r = \frac{I_R}{C_{eq}} T/2 = \frac{4V_p}{CR} T/2$$

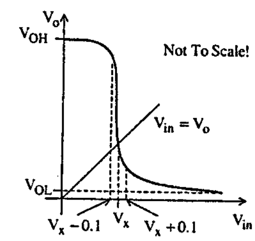
$$\frac{V_r}{V_{op}} = 0.01 = \frac{4V_p T/2}{CR\,2V_p} = \frac{T}{CR}$$

$$\therefore C = \frac{20m}{100 \times 0.01} = 20mF \#.$$

9. CMOS circuits: Consider a simple RTL inverter below. 20 points



$V_{dd} = 5V$
$R_1 = 100k\Omega$
$k_n = 100\mu A/V^2$
$V_{Tn} = 1V$



9a. Calculate V_{OH} (i.e., when $V_{in} = 0V$) and V_{OL} (i.e., when $V_{in} = V_{dd}$). 4 points
Hint: Make sure the transistor operates in the appropriate region.

when $V_{in} = 0 \Rightarrow V_{GS1} = 0 \Rightarrow M_1$ cut-off $\Rightarrow V_{OH} = V_{dd} = 5V \#$

when $V_{in} = 5V$, V_o should be small, $\Rightarrow M_1$ in linear region.

We may approximate M_1 as a resistor

$$I_D = k_n\left[(V_{GS} - V_T)V_{DS} - \tfrac{1}{2}V_{DS}^2\right] \triangleq k_n(V_{GS} - V_T)V_{DS}$$

$$R_{ds} = \frac{V_{DS}}{I_D} = \frac{1}{k_n(V_{GS} - V_T)} = \frac{1}{100\mu \times (5-1)} = 2.5K\Omega$$

$$\therefore V_o = \frac{R_{ds}}{R_{ds} + R_1} V_{dd} = \frac{2.5K}{100k + 2.5k} V_{dd} = 0.122V \#$$

(small V_o $\Rightarrow$ approx. OK)

9b. Calculate V_x at which the output voltage is equal to the input voltage.. 4 points

when $V_{in} = V_o \Rightarrow V_{GS} = V_{DS} \Rightarrow V_{DS} > V_{GS} - V_T$

$\Rightarrow M_1$ in saturation

$$\Rightarrow I_D = \frac{1}{2}k_n(V_{GS} - V_T)^2 = \frac{V_{dd} - V_o}{R_1}$$

$$\Rightarrow \frac{1}{2}k_n(V_o - 1)^2 = \frac{5 - V_o}{R_1}$$

$$\Rightarrow \frac{100\mu \times 100k}{2}(V_o^2 - 2V_o + 1) = 5 - V_o$$

$$\Rightarrow 5V_o^2 - 10V_o + 5 = 5 - V_o \Rightarrow 5V_o^2 = 9V_o \Rightarrow V_o = 1.8V \#$$

$V_x = 1.8V \#$

9c. Finding the points with unity slope is very difficult. Here we approximate $V_{IL} = V_x - 0.1V$, and $V_{IH} = V_x + 0.1V$. With these values, find the low and high noise margins N_{ML} and N_{MH}. **2 points**

$\therefore V_{IL} = 1.8 - 0.1 = 1.7V.$

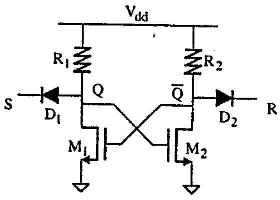
$V_{IH} = 1.8 + 0.1 = 1.9V.$

$NM_L = V_{IL} - V_{OL} = 1.7 - 0.12 = 1.58V$ #

$NM_H = V_{OH} - V_{IH} = 5 - 1.9 = 3.1V$ #

9d. Next, we form a SR latch (whatever this means) using the inverter discussed above. With $S = V_{dd}$ and $R = 0V$, write down V_Q and $V_{\bar{Q}}$. **2 points**

Hint: Use previous results.



$V_{dd} = 5V$
$R_1 = R_2 = 100k\Omega$
$k_n = 100$...
$V_{Tn} = 1V$
$V_{D(on)} = 0.7V$

$S = V_{dd} \Rightarrow D_1$ is OFF.
$R = 0V \Rightarrow D_2$ pulls $\bar{Q}$ low, cutting off $M_1 \Rightarrow V_Q = V_{dd} = 5V$ #.
V_Q turns on M_2 and $V_{\bar{Q}} = 0.122V$ #

9e. Calculate the power consumed by the circuit in this state. **2 points**

only R_2 and M_2 is conducting current

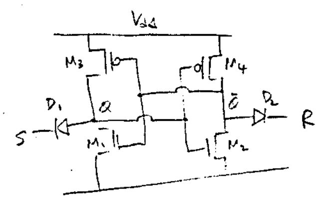
and $I_{R2} = \frac{V_{DD} - 0.122}{R_2} = 48.78\mu A$.

$P = V_{dd} I_{R2} = 243.9\mu W$ #

9f. Next, change of state happens, with R changes to V_{dd}, and S changes to 0V. Describe briefly the actions to follow, and the subsequent voltages of V_Q and $V_{\bar{Q}}$. **4 points**

when S changes to 0V, D_1 turns on, pulling V_Q down to 0.7V (initially)
$\Rightarrow M_2$ then cuts off $\Rightarrow V_{\bar{Q}} = V_{dd} = 5V$ #.
with $V_{\bar{Q}} = 5V \Rightarrow M_1$ turns on, driving V_Q down to
$V_Q = 0.122V$ # and D_1 then turns off.

9g. By replacing the resistors with PMOS transistors, the power consumption can be reduced to 0. Draw the corresponding logic gate below. Hint: How should the gate of each PMOS be connected? **2 points**



Merry X'mas