(7)

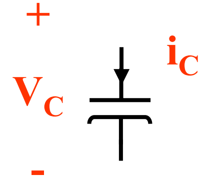
1

(a)

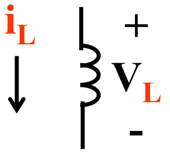
(a) Explain very briefly why current in an inductor and voltage in a capacitor is continuous with time. **(7)**

1 **Energy must be continuous with time.**

Hence E_C and E_L must be continuous with time (unchanged after switching)



$$E_C = \frac{CV_C(t)^2}{2}$$



$$E_L = \frac{Li_L(t)^2}{2}$$

(7)

Hence V_C and i_L must be continuous with time.

---

2 **Another short proof**

$$i_C = C\frac{dV_C(t)}{dt} \qquad V_L = L\frac{di_L(t)}{dt}$$

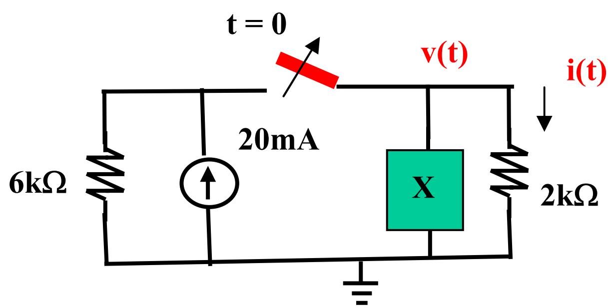
If dt is 0, infinite i_C or V_L is required, which is not possible. Hence dV_C/dt and di_L/dt must be finite, or V_C and i_L must be continuous with time .

(17)

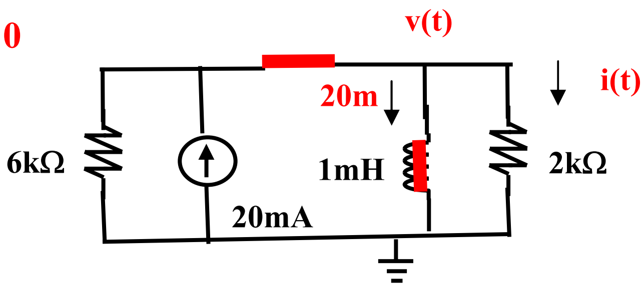
1

(b) The switch has been closed for a long time. At $t = 0$ second, the switch is opened. (i) If $X = 1\text{mH}$, find $i(0)$, $i(\infty)$, time constant τ ($= L/R$), $i(t)$ and $v(t)$ for $t \geq 0$. Given that $i(t) = i(\infty) + [i(0) - i(\infty)] \exp(-t/\tau)$. Given that $v(t) = v(\infty) + [v(0) - v(\infty)] \exp(-t/\tau)$. (17)

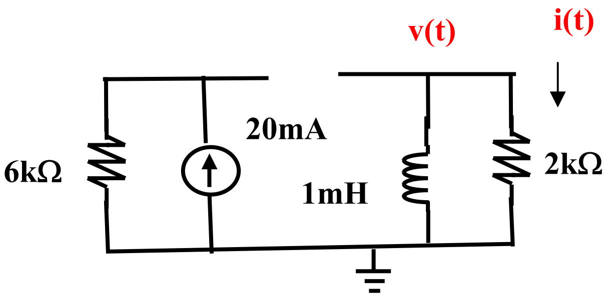
(i)



t < 0



t ≥ 0



$$\therefore i(0) = -20\text{ mA} \qquad \textbf{(4)}$$

$$i(\infty) = 0\text{mA} \qquad \textbf{(3)}$$

$$\tau = \frac{L}{R} = \frac{1\text{mH}}{2\text{k}\Omega} = 0.5\ \mu\text{s} \qquad \textbf{(3)}$$

$$\begin{aligned}\therefore i(t) &= i(\infty) + [i(0) - i(\infty)] * e^{-\frac{t}{\tau}} \\ &= 0\text{ mA} + [-20\text{ mA} - 0] * e^{-t/0.5\mu s} \\ &= -20\text{ mAe}^{-t/0.5\mu s}\end{aligned} \qquad \textbf{(3)}$$

$$v(t) = i(t) * 2\text{k}\Omega = -40\text{ Ve}^{-t/0.5\mu s} \qquad \textbf{(4)}$$

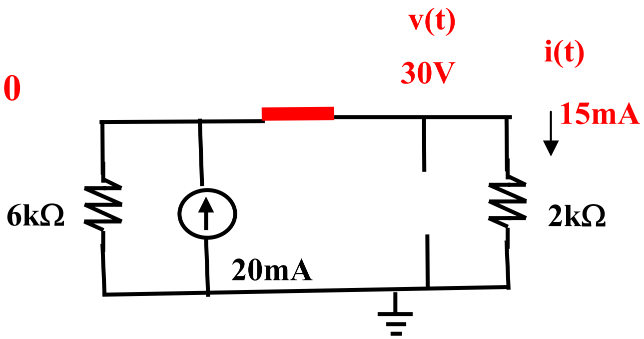
(7)

1

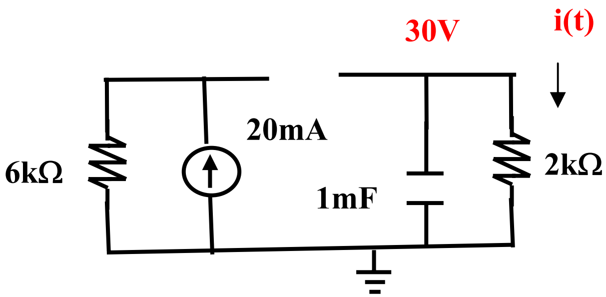
(ii) If X = 1mF, find v(0) , i(0) for $t \geq 0$. Given that $i(t) = i(\infty) + [i(0) - i(\infty)] \exp(-t/\tau)$. (7)

(ii)

t < 0



t ≥ 0



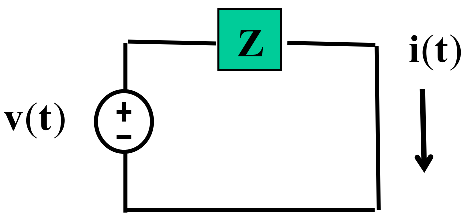
$\therefore\ v(0) = 30\ V$ (4)

$i(0) = 15\ mA$ (3)

2 (24)

(a) If $\mathbf{i}(\mathbf{t}) = 10\sin(100\mathbf{t} + 30^\circ)\mathbf{A}$, $\mathbf{Z} = 2\Omega$, **find** $\mathbf{v}(\mathbf{t})$.

(b) If $\mathbf{v}(\mathbf{t}) = 8\sin(2\mathbf{kt} + 30^\circ)\mathbf{V}$, $\mathbf{Z} = 2\mathbf{mH}$,
find Z in Ω **and find** $\mathbf{i}(\mathbf{t})$.



(a) $\mathbf{v}(\mathbf{t}) = 20\sin(100\mathbf{t} + 30^\circ)\mathbf{V}$ (3)

(b) $\mathbf{Z} = \mathbf{j}\omega\mathbf{L} = \mathbf{j}(2\mathbf{krad/s})2\mathbf{mH} = 4\mathbf{j}\Omega$ (2)

$$\mathbf{i}(\mathbf{t}) = \frac{8\mathbf{V}}{4\Omega}\sin(2\mathbf{kt} + 30^\circ - 90^\circ)\mathbf{A}$$
$$= 2\mathbf{A}\sin(2\mathbf{kt} - 60^\circ)$$
(6)

(c) If i(t) = 5cos(1kt + 30o)A , v(t) = 20cos(1kt - 30o)V , and Z is R in series with X. Find Z in Ω . Find also the element and value of X. Does i(t) lag v(t) ?

$$\therefore \mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{20\angle -30\mathbf{V}}{5\angle 30\mathbf{A}} = 4\angle -60^{o}\Omega$$

$$= 4\cos(-60^{o}) + 4\mathbf{j}\sin(-60^{o})$$

$$= 2\Omega - \mathbf{j}3.464\Omega$$

$$= \mathbf{R} - \mathbf{j}\mathbf{X}$$

(7)

$$\therefore \mathbf{X} = \frac{1}{\omega \mathbf{C}} \Rightarrow \mathbf{C} = \frac{1}{\mathbf{X}\omega} = \frac{1}{3.464 * 1\mathbf{k}} \cong 0.289\mathbf{mF}$$

(4)

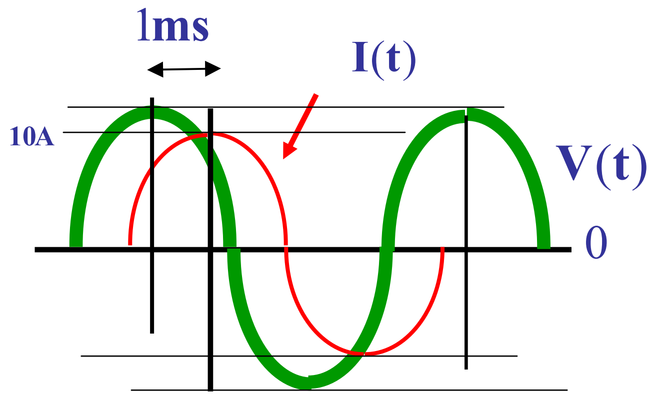
i(t) leads v(t) **(2)**

(17)

3

(a)

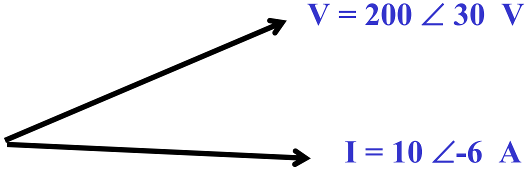
(a) If $V(t) = 200\cos(200\pi * t + 30^{o})\ V$, find $I(t)$.
Draw also the phasor diagram of V(t) and I(t). (17)



$$\therefore T = \frac{2\pi}{\omega} = \frac{2\pi}{200\pi} = 10ms$$ (4)

$$\theta = \frac{t}{T} * 360^{o} = \frac{1ms}{10ms} * 360^{o} = 36^{o}$$ (4)

$$\therefore I(t) = 10A\cos(\omega t + \theta)V$$
$$= 10A\cos(200\pi t + 30^{o} - 36^{o})$$
$$= 10A\cos(200\pi t - 6^{o})$$ (5)

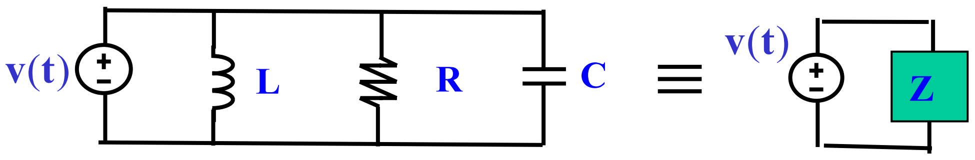


(4)

(6)

3 **(b)**

(b) If v(t) = Vm cos ωt, express the impedance Z in terms of R, L , C and ω . (6)



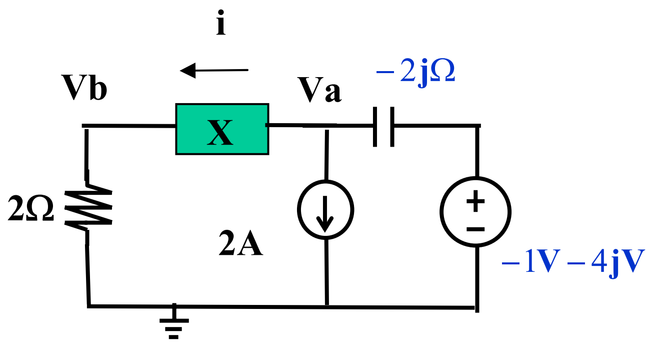
$$Z = \frac{1}{Y} = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}}$$

(6)

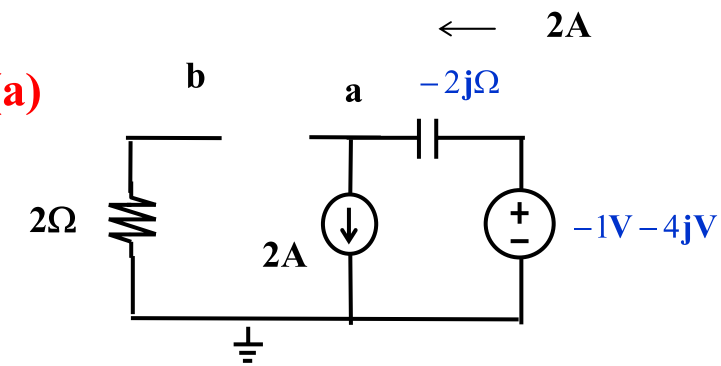
(21)

4

(a) If X = ∞Ω, find Va – Vb .
(b) If X = 0Ω, find i.
(c) Find the Thevenin impedance and hence the Thevenin equivalent at terminals ab.   (21)



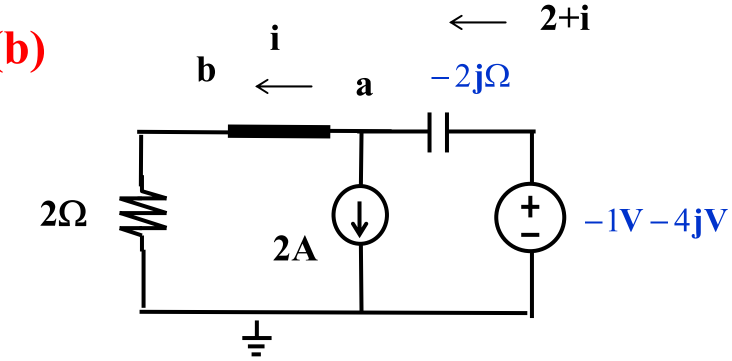
(a)



$$Voc = Vab = (-1-4j) - (2A * -2j\Omega)$$
$$= -1 - 4j + 4j = -1V$$

(6)

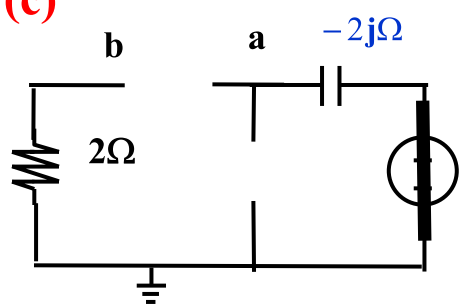
(b)



$$-1V - 4jV - (2+i)*(-2j\Omega) - i*2\Omega = 0$$
$$-1 - 4j + 4j + 2ij - 2i = 0$$
$$i = \frac{1}{2(j-1)} A$$

(8)

(c)

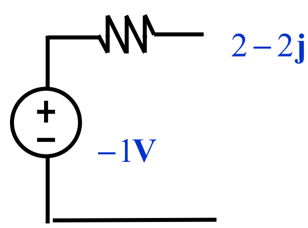


$$Zth = 2\Omega - 2j\Omega$$

or

$$Zth = \frac{Vab}{i} = 2 - 2j$$

(4)

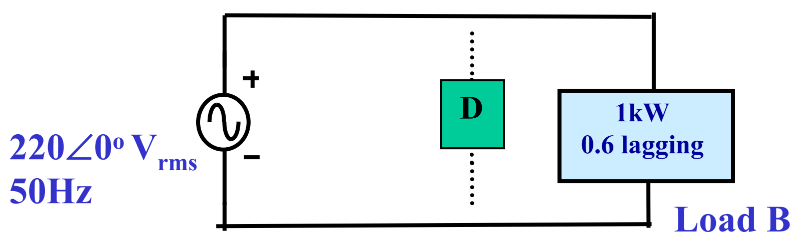


(3)

(15)

5 If a load D is connected in parallel to load B to make the total power factor = 0.9 lagging, find the element and value of load D. (15)

(a)



$$\therefore Q_C \text{ required}$$

$$= 1\mathbf{kW}\tan(\cos^{-1}0.6) - 1\mathbf{kW}\tan(\cos^{-1}0.9)$$

$$\cong 849\mathbf{VAR(C)}$$

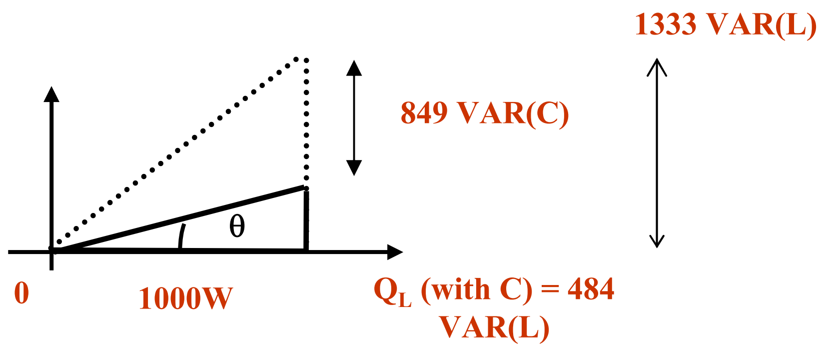
(9)

Add load D to make PF = 0.9 lagging (capacitance C)

$$\therefore C = \frac{|Qc|}{V^2\omega} \cong \frac{849\mathbf{VAR(C)}}{(220^2)2\pi 50} \cong 55.84\mu\mathbf{F}$$

(6)

Power triangle



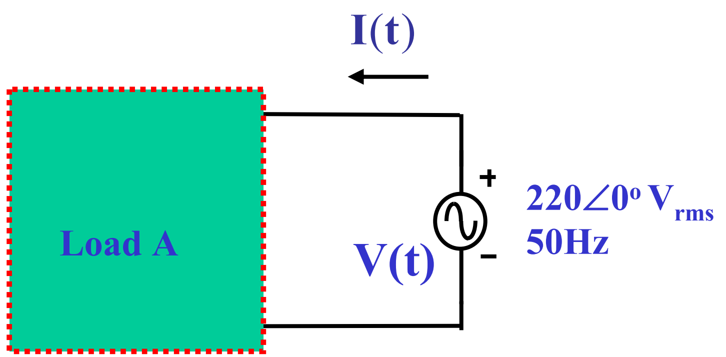
(13)

5

(b)

For load A, the apparent power S is 2200VA, and the reactive power Q is $2200 * \frac{\sqrt{3}}{2} \mathbf{VAR(L)}$

Find the average power P, power factor PF of load A, and I(t). (13)

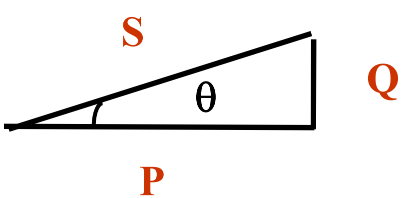


$$\sin\theta = \frac{\mathbf{Q}}{\mathbf{S}} = \frac{2200\sqrt{3}/2}{2200} = \frac{\sqrt{3}}{2}$$

$$\therefore \mathbf{PF} = \cos\theta = \cos(\sin^{-1}\frac{\sqrt{3}}{2})$$

$$= \cos 60^o = 0.5\mathbf{lagging}$$ **(9)**

$$\mathbf{P} = \mathbf{S}\cos\theta = 2200\mathbf{VA} * \cos(60^o) = 1100\mathbf{W}$$ **(4)**

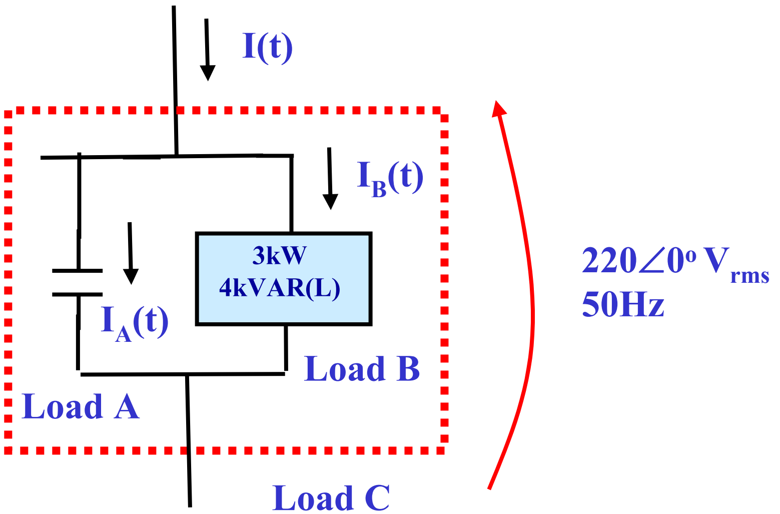


(21)

6

(a) Will electricity fee be reduced if the power factor is improved to 1? Why? (21)

(b) The power factor of Load C is 1, find I in Arms . If load B is R in series with X. Find R in Ω .



(a) **Electric fees is related to P only P is unchanged hence fees will not be reduced** **(5)**

(b)

$$I = \frac{P}{V} = \frac{3kW}{220V_{rms}} \cong 13.64A_{rms} \qquad \textbf{(6)}$$

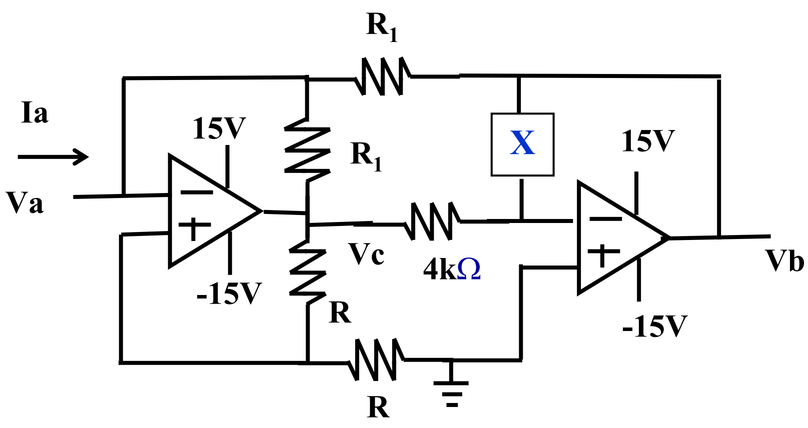
$$I_B = \frac{S}{V} = \frac{\sqrt{3k^2 + 4k^2}}{V} = \frac{5kVA}{220V_{rms}} \cong 22.73A_{rms} \qquad \textbf{(6)}$$

$$R = \frac{P}{I_B^{\,2}} \cong \frac{3kW}{(22.73A_{rms})^2} \cong 5.8\Omega \qquad \textbf{(4)}$$

(23)

7

(a) If Va = 1V, find Vb , Vc and Ia .
(b) If Va = 10V, find Vc .
Given X = 8kΩ , R1 = 1kΩ , R = 2kΩ .
Assume the op amps are ideal . (23)



(a)

$$Vc = 2Va = 2V$$ (5)

$$\therefore Vb = -\frac{X}{4k}Vc = -\frac{8k}{4k}Vc$$

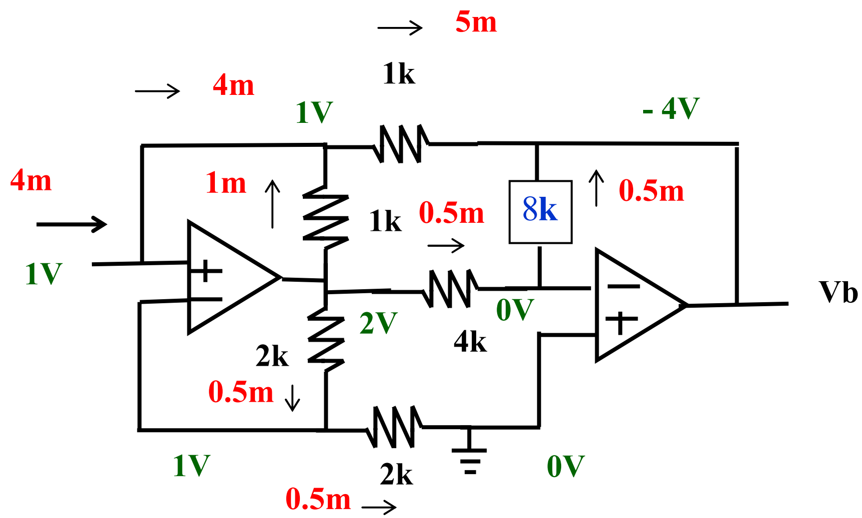
$$= -2Vc = -4Va = -4V$$ (6)

$$\therefore Ia = \frac{Va - Vb}{R_1} + \frac{Va - Vc}{R_1}$$

$$= \frac{1}{R_1}[5Va - Va]$$

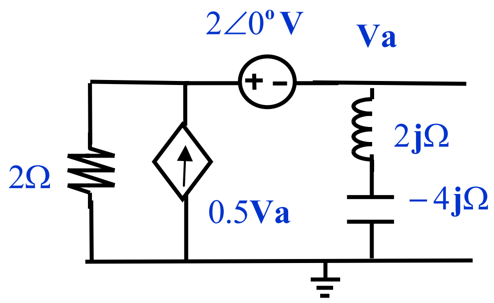
$$= \frac{4Va}{R_1} = \frac{4V}{1k} = 4mA$$ (7)

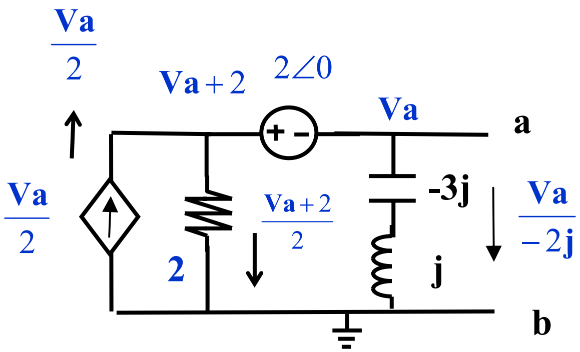
(b) If Va is changed to 10V,
Vc = 15V (op amp saturated) (5)



8

Find Va . The voltage dependent current source is in ampere and equal to 0.5Va. (14)

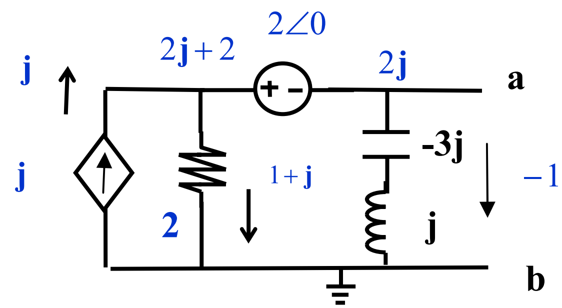




$$\frac{Va}{2\Omega} = \frac{Va+2V}{2\Omega} + \frac{Va}{-2j\Omega} \qquad (8)$$

$$0 = 1 - \frac{Va}{2j\Omega}$$

$$\therefore Va = 2jV \qquad (6)$$



(25)

9

A parallel LCR radio tuner circuit is used to receive HK Radio Station 5 as shown in the resonance curve .

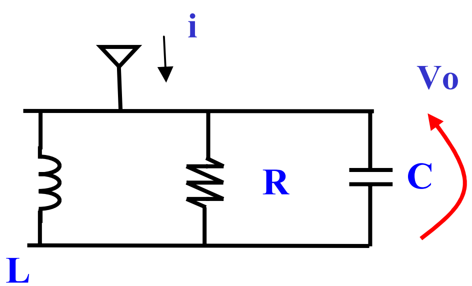
(a) Show that the quality factor (QF) is 50 .

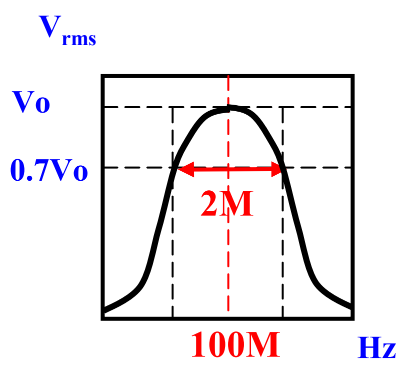
(b) If L = 1μH, find C and R .

(c) If at resonance, i = 1∠0o mArms , find Vo(t) at resonance .

(d) Suggest a method to improve the bandwidth to 1 MHz .

Given that QF = fo/BW = R/X . (25)





(a)

$$QF = \frac{fo}{BW} = \frac{100M}{2M} = 50$$

(3)

(b)

$$\omega_o = \frac{1}{\sqrt{LC}}$$

$$\therefore C = \frac{1}{\omega_o^2 L} = \frac{1}{(2\pi 100MHz)^2 1\mu H} \cong 2.53x10^{-12} F$$

(5)

$$\therefore QF = \frac{R}{\omega_o L}$$

$$\therefore R = \omega_o L * QF = 2\pi * 100MHz * 1\mu H * 50 \cong 31416\Omega$$

(5)

(c)

$$\therefore V_o(t) = i(t) * R \cong (1mA\sqrt{2}\cos 2\pi 100Mt) * 31416\Omega$$

$$\cong 31.4\sqrt{2}V\cos 2\pi 100Mt$$

(7)

(d) Change R to 2R **(5)**

(13)

$\boxed{10}$

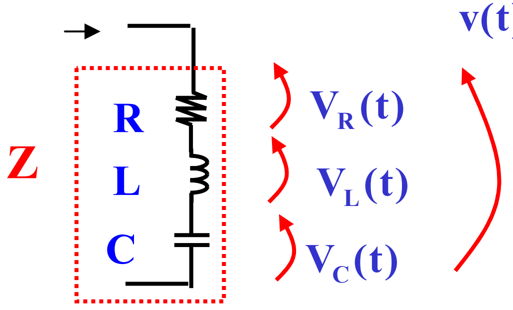
The series LCR circuit is in resonance . R is 0.05Ω and

$$V_R(t) = 0.2\sqrt{2}\cos(500t)\,V$$

Find Z, resonant frequency fo in Hz, and v(t). (13)

$$i(t) = I_m \cos(\omega t + \theta_I)$$

$$v(t) = V_m \cos(\omega t + \theta_V)$$



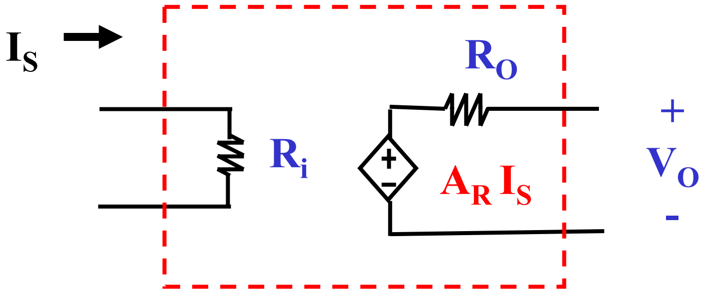
$$Z = R = 0.05\Omega \qquad (4)$$

$$\therefore f_o = \frac{\omega_o}{2\pi} = \frac{500}{2\pi}\,\text{Hz} \qquad (4)$$

$$\therefore v(t) = 0.2\sqrt{2}\cos(500t) \quad V \qquad (5)$$

8

(a)
Draw the circuit model for a <u>current to voltage</u> (resistance) amplifier. What is the ideal value of the input resistance. (8)

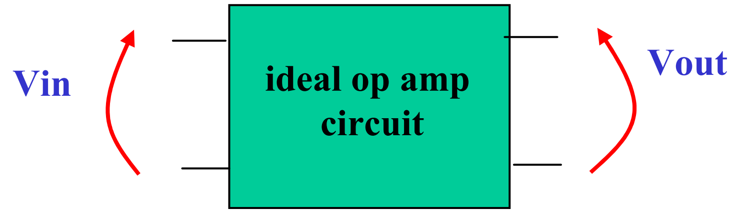


(7)

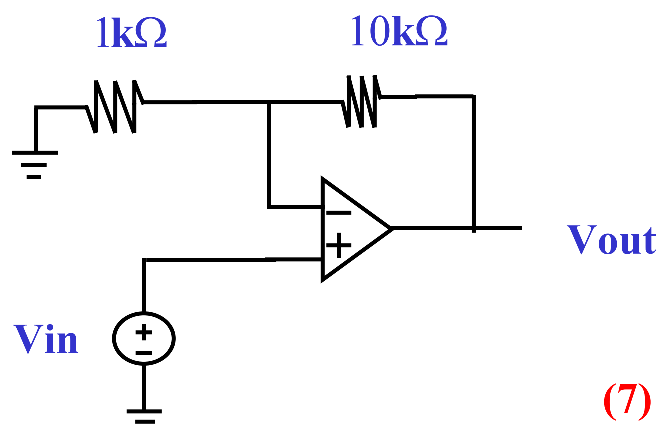
Ri = 0Ω **(2)**

(b)

Design the ideal op amp circuit below if Vout = 11 Vin . Use 1k – 10kΩ resistors in your design. (8)



(a)



(7)