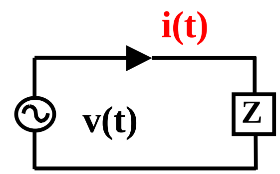
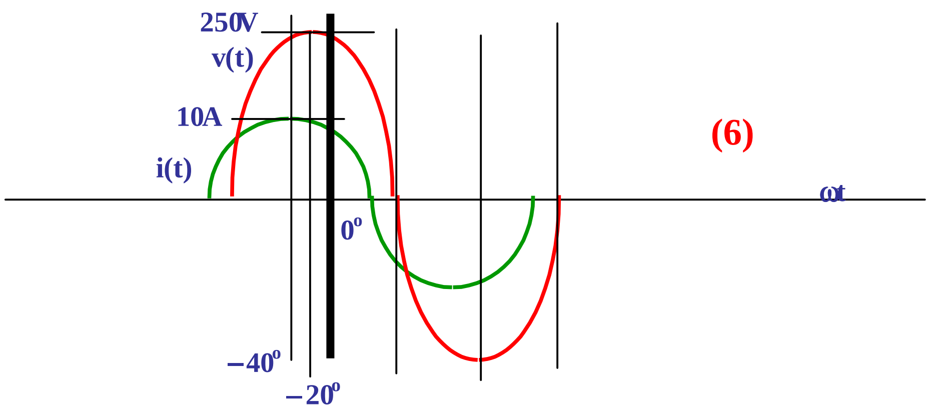
(23)

1



$v(t) = 250\cos(100t + 20^o)$ V

$i(t) = 10\cos(100t + 40^o)$ A



(6)

i(t) leads v(t) (2)

Capacitive element (C in series with R)

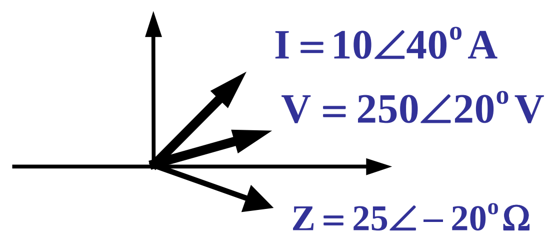
$$\therefore\ Z = \frac{V}{I} = \frac{250\angle 20^o\,V}{10\angle 40^o\,A} = 25\angle -20^o\,\Omega$$

(3)

$\therefore\ Z = 25(\cos -20^o + j\sin -20^o)\ \Omega$
$= 23.5 - j8.55\ \Omega$
$= R - j(1/\omega C)$

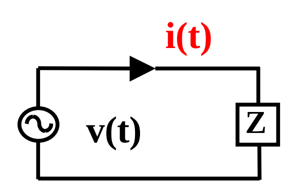
$\therefore\ R = 23.5\ \Omega$ (4)

$$\therefore C = \frac{1}{8.55\omega} = \frac{1}{8.55(100)} = 1.17\text{mF}$$ (5)



(3)

1. In the following circuit, v(t) = 250 cos (100t + 20°) V, i(t) = 10 cos (100t + 40°) A.
(a) Sketch v(t) and i(t) together. Show clearly the phase angles and amplitudes. Does v(t) lead i(t) ?
(b) If Z is two elements in series, find the two elements and the values.
(c) Sketch impedance Z and phasors V, I in a complex plane. (23)



(31)

$\boxed{2}$

$$\therefore P = VI\cos\theta = (250)(\frac{10}{2})\cos(20) = 1174.62W$$ (3)

$$\therefore Q = VI\sin\theta = (250)(\frac{10}{2})\sin(-20) = -427.53VAR$$ (3)

$$\therefore S = VI = (\frac{10}{2})(250) = 1250VA$$ (3)

$$\therefore PF = \cos(-20^o) = 0.94 \text{ leading}$$ (3)

P = real (or average) power dissipated by load Z

S = power supplied by source to load Z

Q = maximum reactive power stored in Z (4)

Connect L in parallel to improve PF = 1

$$\therefore L = \frac{V^2}{\omega_O Q} = \frac{(250/\sqrt{2})^2}{100(427.53)} = 0.73H$$ (5)

When L is connected (PF = 1),
S = P = 1174.62VA, hence

$$\therefore I = \frac{S}{V} = \frac{1174.62}{250/\sqrt{2}} = 4.7\sqrt{2} \text{ Arms}$$ (5)

If V is 500V (2x of original V)
I is 2x of original I
but R and C are unchanged
hence P and Q ($\propto I^2$) are 4x of original P and Q
Hence new L $\propto V^2 / Q$ = original L = 0.73H

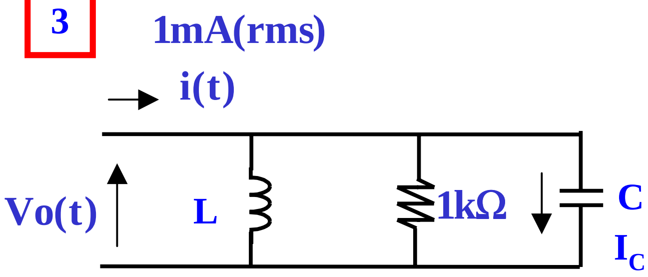
(5)

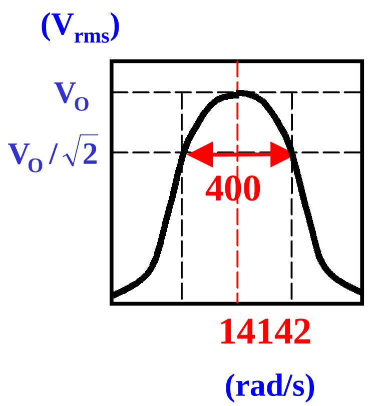
2. Using the same circuit in question (1),
(a) find the apparent power S, reactive power Q, average power P and power factor PF of load Z.
(b) What are the physical meanings of S, P, Q?
(c) A load B is now connected in parallel to Z to make the total power factor = 1. Find the load B and the value. Find also the current (in rms) supplied by v(t) when PF is 1.
(d) If v(t) is now changed to 500 cos (100t + 20^o) V, find the new load B required.

(31)

(37)

3





$$\therefore \omega_O = 14142 \text{rad/s}$$

$$f_O = \frac{\omega_O}{2\pi} = 2250.8\text{Hz} \qquad (3)$$

$$\therefore BW = 400 \text{rad/s} \qquad (2)$$

$$\therefore \omega_2 = \omega_O + \frac{BW}{2} = 14142 + 200 = 14342 \text{rad/s} \qquad (2)$$

$$\therefore \omega_1 = \omega_O - \frac{BW}{2} = 14142 - 200 = 13942 \text{rad/s} \qquad (2)$$

$$\therefore Q = \frac{\omega_O}{BW} = \frac{14142}{400} = 35.355 \qquad (3)$$

Q > 10, good resonant circuit (2)

Vo is max at resonance, hence circuit is parallel LCR (2)

$$\therefore L = \frac{R}{\omega_O Q} = \frac{1k}{14142(35.355)} = 2\text{mH} \qquad (4)$$

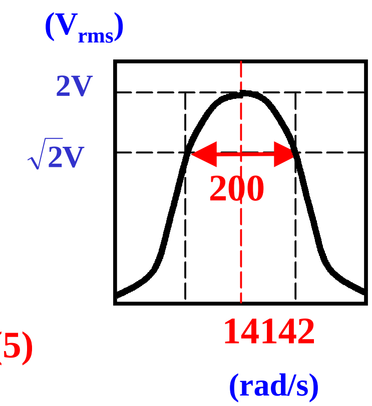
$$\therefore C = \frac{Q}{\omega_O R} = \frac{35.355}{14142(1k)} = 2.5\mu F \qquad (3)$$

$$\therefore \max V_O = iR = 1m(1k) = 1\text{Vrms} \qquad (3)$$

$$\therefore \max i_C = \frac{\max V_O}{1/j\omega_O C} = IR(j\omega_O C) = jIQ$$

$$\therefore \max i_C(t) = 35.355(\sqrt{2})\cos(14142t + 90^o)\text{mA} \qquad (6)$$

If R is changed to 2k

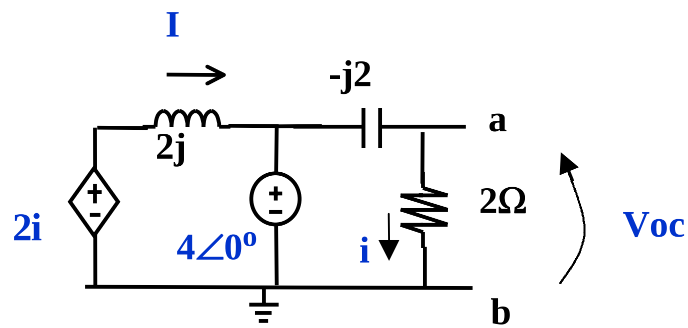


(5)

3. A LCR circuit has the following resonance curve (magnitude of Vo(t) versus frequency ω).
(a) Find the resonant frequency (in rad/s and in Hz), bandwidth (BW), upper and lower cut-off frequencies (in rad/s), and the Q-factor of the LCR circuit. Is the circuit a good resonant circuit and why?
(b) If R = 1kΩ, find the values of C and L. Find also the maximum Vo (in rms) and maximum current flowing in C ($i_C(t)$).
(c) If R is now changed to 2kΩ (C and L are unchanged), sketch the new resonance curve. Show clearly all intercepts. (37)

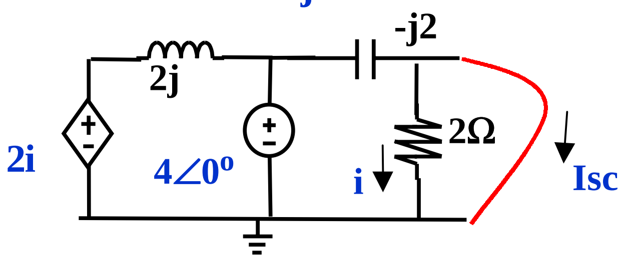
(29)

4



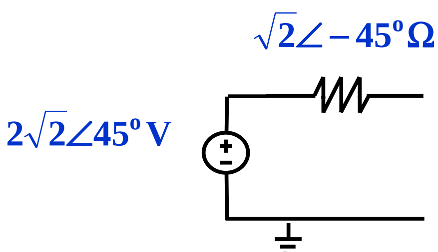
$$i=\frac{4}{2-2j}=\frac{2}{1-j}=\frac{2}{\sqrt{2}\angle-45^o}=\sqrt{2}\angle 45^o\,A$$

$$\therefore Voc = 2i=\frac{4}{1-j}=2\sqrt{2}\angle 45^o\,V \qquad (6)$$



$$Isc=\frac{4}{-2j}=2j=2\angle 90^o\,A \qquad (5)$$

$$\therefore Zth=\frac{Voc}{Isc}=\frac{2\sqrt{2}\angle 45^o}{2\angle 90^o}=1-j=\sqrt{2}\angle-45^o\,\Omega \qquad (3)$$

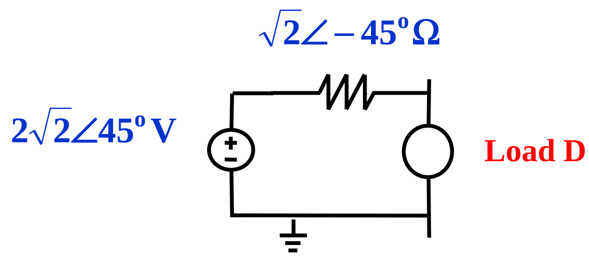


(2)

use KVL:

2i = I(2j) + i(2 – 2j)

I = i (5)



(3)

$$\text{load } D=\sqrt{2}\angle 45^o\,\Omega$$

$$I=\frac{Voc}{Z}=\frac{\frac{4}{1-j}}{1-j+D}=\frac{4(1+j)}{1-j+D}=2 \qquad (5)$$

$$\therefore \text{load } D=1+3j$$

4. In the following circuit,
(a) find the complex open circuit voltage and the short circuit current at terminals ab. Hence sketch the Thevenin equivalent at ab.
(b) Find I in terms of i.
(c) A load D is now connected across ab. Find load D (i) if it has maximum power dissipation, (ii) if current in load D is $2\angle 0^o$ A. (29)

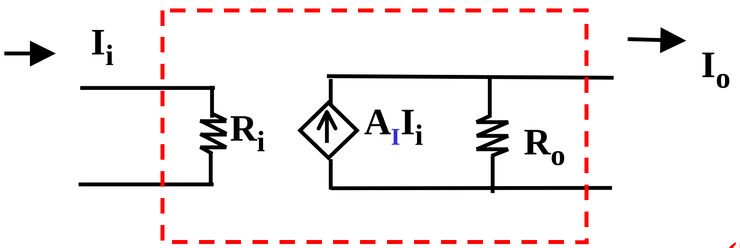
(40)

5

$\because\ A \sim \infty \qquad \therefore V_i = \dfrac{V_o}{A} \sim 0$

(5)

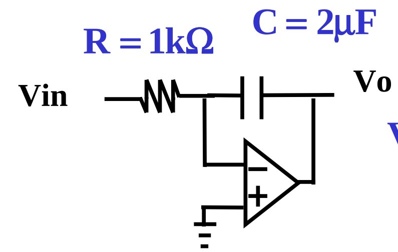
$\because\ R_i \sim \infty \qquad \therefore I_i = \dfrac{V_i}{R_i} \sim 0$



(5)

ideal I amplifier, Ri = 0 Ro = ∞

Vin: +/- 6V,
500Hz sq wave



$V_O(0) = 0V$

(3)

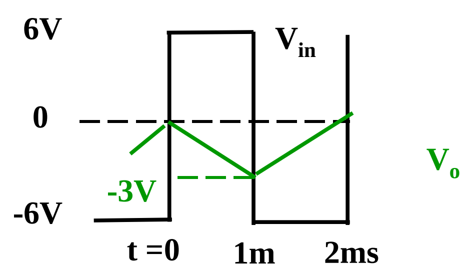
$$\therefore \frac{V_{in} - 0}{R} = -C\frac{dV_o}{dt}$$

$$\therefore \frac{dV_o}{dt} = -\frac{V_{in}}{RC}$$

(3)

$$= \frac{-6V}{2ms} = -3V/ms$$

(2)



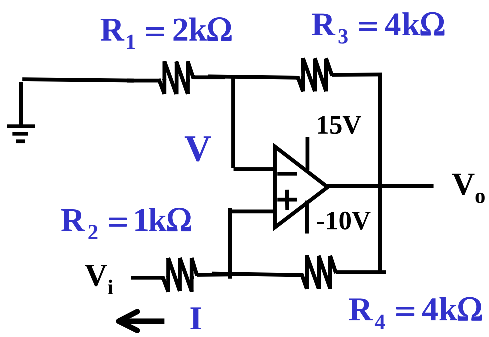
(6)

5. (a) An ideal op amp assumes Vi = 0 and Ii = 0 (voltage across and current flow into input terminals are zero). Explain briefly the reasons.

(b) Sketch the equivalent circuit of an ideal current amplifier. What are the values of the input and output resistances?

(c) Sketch the circuit of an ideal op amp RC integrator. If C = 2µF, R = 1kΩ, Vin(t) is a +/- 6V 500Hz square wave, sketch Vout(t) and Vin(t) together. Show clearly all voltage and time. Given at t = 0, Vin = 6V, Vout = 0V.

(d) In the following circuit, assume ideal op amp, (i) find Vo if Vi = 3V, (ii) find I if Vi = - 1V. (40)



$$\because V_- = V_+ = V$$

$$\therefore V_- = V_O \frac{2k}{2k+4k} = \frac{V_O}{3}$$

$$\therefore \frac{Vi - V}{1k} = \frac{V - V_O}{4k}$$

$$\therefore 4V_i - 4V = V - V_O \quad \therefore 4V_i = \frac{2}{3} V_O$$

$$\therefore A_V = \frac{V_O}{V_i} = 6 \qquad \text{(8)}$$

If Vi = 3V, Vo = 15V (3)

If Vi = -1V, Vo = -6V (2)

$$\therefore I = \frac{Vo - Vi}{5k} = \frac{-6 - -1}{5k} = -1mA \qquad \text{(3)}$$