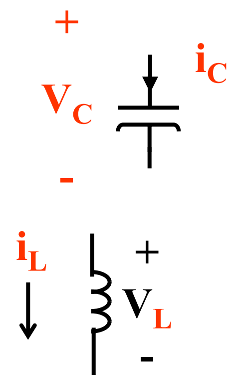
1

(a)

(a) Explain very briefly why voltage in a capacitor and current in an inductor is continuous with time. **(7)**

1 **Energy must be continuous with time.**

Hence E_C and E_L must be continuous with time (unchanged after switching)



$$E_C = \frac{CV_C(t)^2}{2}$$

$$E_L = \frac{Li_L(t)^2}{2}$$

(7)

Hence V_C and i_L must be continuous with time.

---

2 **Another short proof**

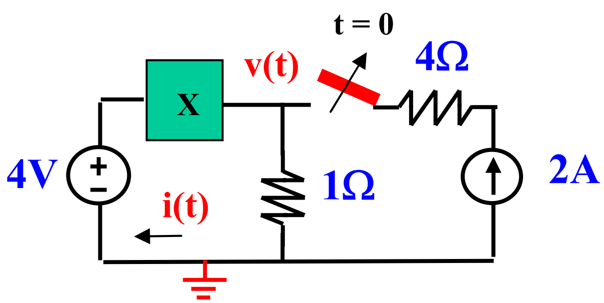
$$i_C = C\frac{dV_C(t)}{dt} \qquad V_L = L\frac{di_L(t)}{dt}$$

If dt is 0, infinite i_C or V_L is required, which is not possible. Hence dV_C/dt and di_L/dt must be finite, or V_C and i_L must be continuous with time .

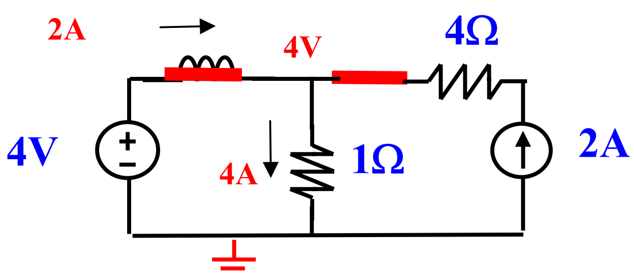
1

(b) The switch has been closed for a long time. At t = 0 second, the switch is opened. (i) If X = 1mH, find i(0) , i(∞), time constant τ (= L/R) , i(t) and v(t) for t t ≥ 0 . Given that i(t) = i(∞) +[i(0) - i(∞)] exp(-t/τ) .
v(t) = v(∞) +[v(0) - v(∞)] exp(-t/τ) . (19)

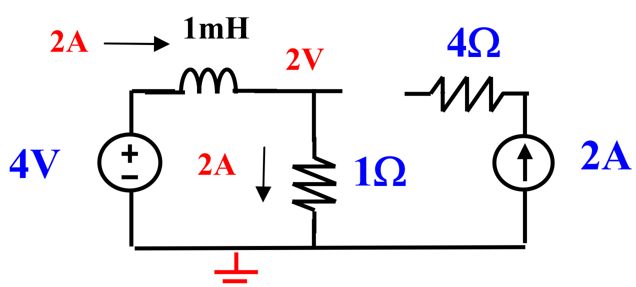
(b)

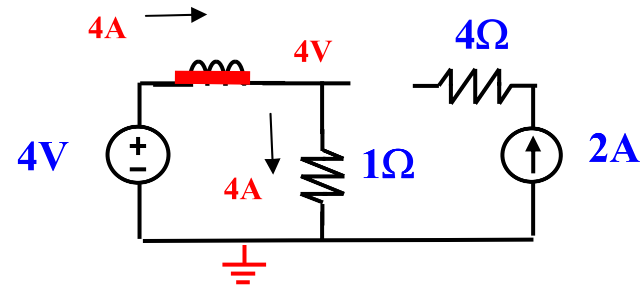


t < 0



t ≥ 0





$\therefore\ i(0) = 2A$ **(5)**

$i(\infty) = 4A$ **(4)**

$\tau = \frac{L}{R} = \frac{1mH}{1\Omega} = 1ms$ **(3)**

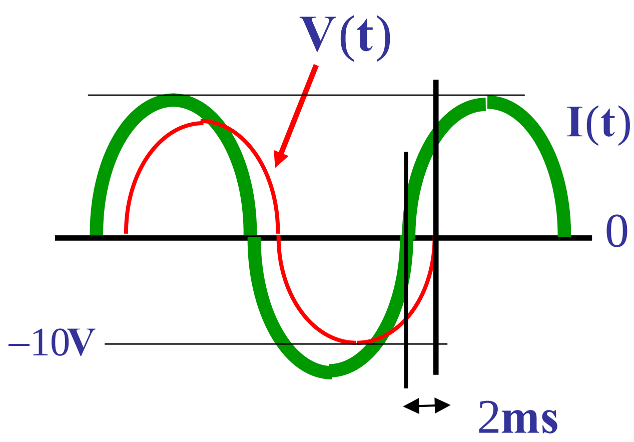
$$\therefore\ i(t) = i(\infty) + [i(0) - i(\infty)] * e^{-\frac{t}{\tau}}$$
$$= 4A + [2A - 4A] * e^{-t/1ms}$$
$$= 4A - 2Ae^{-t/1ms}$$ **(3)**

$v(t) = i(t) * 1\Omega = 4V - 2Ve^{-t/1ms}$ **(4)**

(15)

2

If $\mathbf{I}(\mathbf{t}) = 2\sin(\pi 100 * \mathbf{t} + 5^{o})\ \mathbf{A}$, find $\mathbf{V}(\mathbf{t})$.
Does I(t) lead V((t) (15)



$$\therefore \mathbf{T} = \frac{2\pi}{\omega} = \frac{2\pi}{100\pi} = 20\mathbf{ms}$$ (4)

$$\theta = \frac{\mathbf{t}}{\mathbf{T}} * 360^{o} = \frac{2\mathbf{ms}}{20\mathbf{ms}} * 360^{o} = 36^{o}$$ (4)

$$\therefore \mathbf{V}(\mathbf{t}) = 10\mathbf{V}\sin(\omega\mathbf{t} + \theta)$$

$$= 10\mathbf{V}\sin(100\pi\mathbf{t} + 5^{o} - 36^{o})$$ (5)

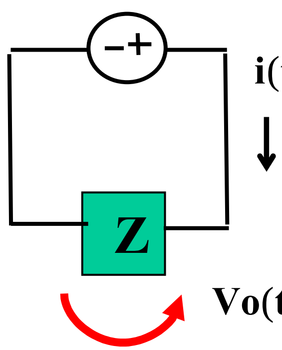
$$= 10\mathbf{V}\sin(100\pi\mathbf{t} - 31^{o})$$

I(t) leads V(t) (2)

(10)

3 (28)

(a) If $i(t) = 8\sin(2kt + 30^o)mA, \quad Z = 2x10^{-6}F,$
find Z in Ω and find Vo(t).



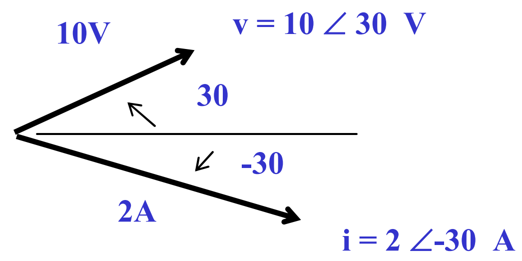
(a) $$Z = \frac{1}{j\omega C} = \frac{1}{j * 2krad/s * 2\mu F} = -j250\Omega$$ (3)

$$Vo(t) = 8mA * 250\Omega \sin(2kt + 30^o - 90^o)$$ (7)

$$= 2V\sin(2kt - 60^o)$$

(18)

(b) If i(t) = 2cos(2kt - 30o)A , v(t) = 10cos(2kt + 30o)V , and Z is R **in parallel** with X. Find Y (= 1/Z) in Ω^{-1} . Find also the element and value of X. **Draw also the phasor diagram of v(t) and v(t).**



(4)

(b)

$$\therefore \mathbf{Y} = \frac{\mathbf{I}}{\mathbf{V}} = \frac{2\angle -30^{\circ}\,\mathbf{A}}{10\angle +30^{\circ}\,\mathbf{V}} = 0.2\angle -60^{\circ}\ \Omega^{-1}$$

$$= 0.2\cos(-60^{\circ}) + 0.2\mathbf{j}\sin(-60^{\circ})$$

$$= 0.1\mathbf{S} - \mathbf{j}0.1732\mathbf{S}$$

$$= \frac{1}{\mathbf{R}} + \frac{1}{\mathbf{jX}}$$

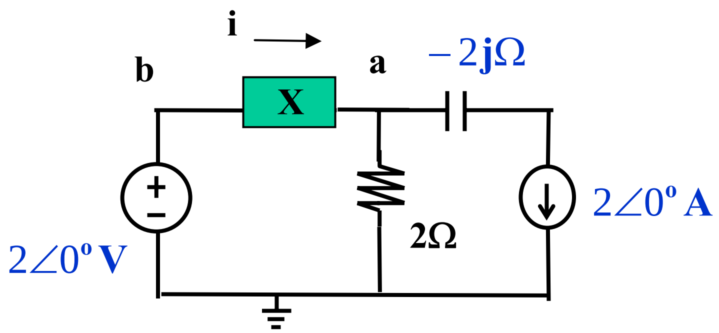
(10)

$$\therefore \mathbf{X} = \omega\mathbf{L} \Rightarrow \mathbf{L} = \frac{\mathbf{X}}{\omega} = \frac{1}{0.1732 * 2\mathbf{k}} \cong 2.89\mathbf{mH}$$

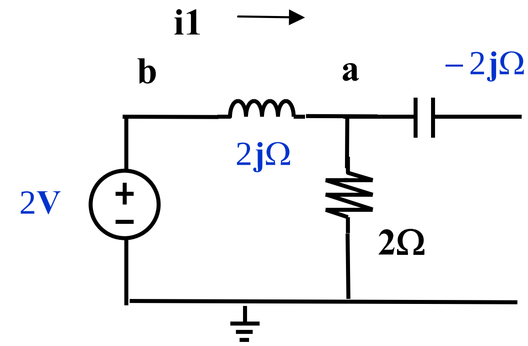
(4)

4

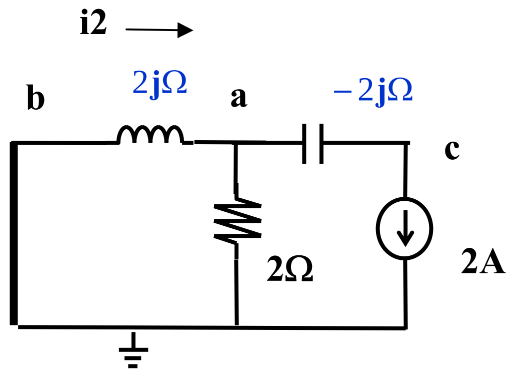
If X = 2jΩ, find i using **superposition**.
Find also the **Thevenin impedance** at terminals ab and hence find the **Thevenin voltage** at ab . (24)



(a)



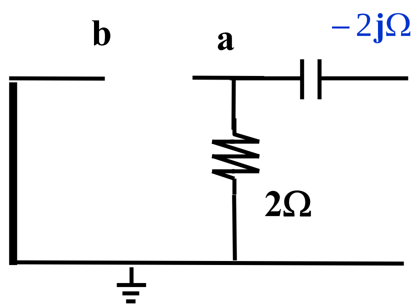
$$i1 = \frac{2V}{2\Omega + 2j\Omega} = \frac{1A}{1+j} \qquad (5)$$



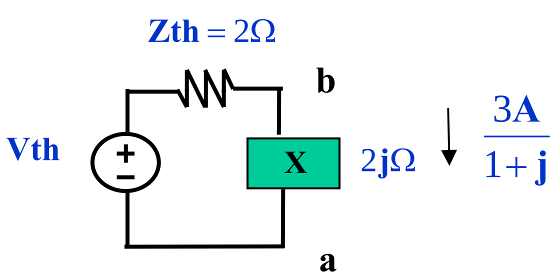
$$i2 = 2A * \frac{2\Omega}{2\Omega + 2j\Omega} = \frac{2A}{1+j} \qquad (5)$$

$$\therefore \mathbf{i} = \mathbf{i}1 + \mathbf{i}2 = \frac{3A}{1+\mathbf{j}}$$ **(2)**

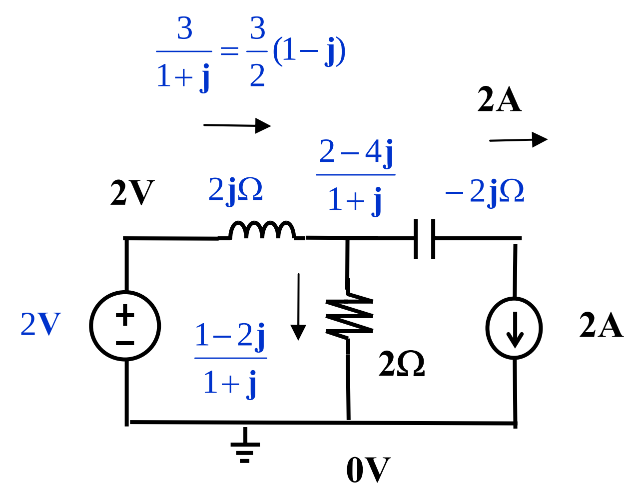
(b)



$$\mathbf{Zth} = 2\Omega$$ **(4)**

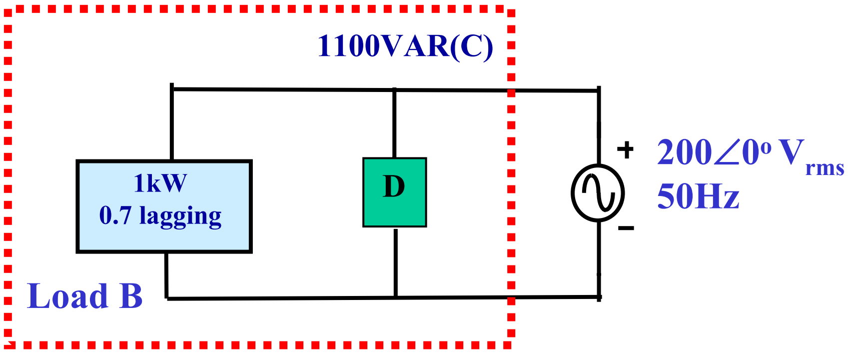


$$\therefore \mathbf{Vth} = \frac{3A}{1+\mathbf{j}} * (2+2\mathbf{j})\Omega = 6\mathbf{V}$$ **(8)**

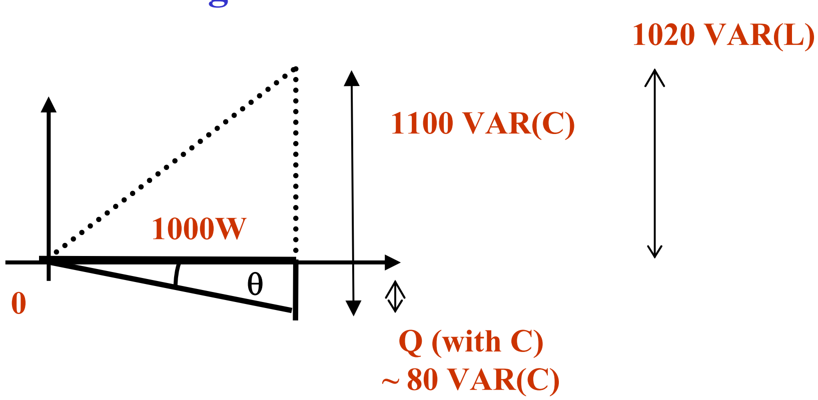


5

A load D with 1100VAR (C) is connected in parallel to load B. Find the power factor of the combined load. Find also the element and value of load D. (19)



Power triangle



$$\therefore \textbf{New Q} = 1\textbf{kW}\tan(\cos^{-1}0.7) - 1100$$
$$\cong 1020.2 - 1100 = 79.8\textbf{VAR(C)}$$

(7)

Power factor of combined load

$$= \cos(\tan^{-1}\frac{79.8}{1000}) \cong \cos(4.56^{o}) \cong 0.997 \textbf{ leading}$$

(6)

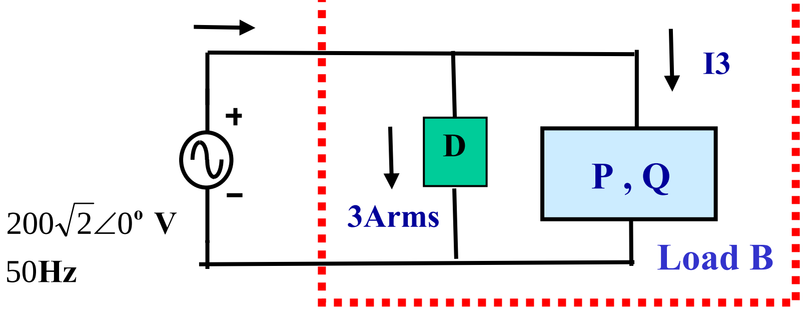
$$\therefore \mathbf{C} = \frac{|\mathbf{Qc}|}{\mathbf{V}^2\omega} \cong \frac{1100\textbf{VAR(C)}}{(200^2)2\pi 50} \cong 87.5\mu\mathbf{F}$$

(6)

6

(29) (a) Will electricity fee be reduced if the power factor of a load is improved to 1? Why?

(b) Power factor of Load A is 1 . D is a capacitor. Find the average power P, reactive power Q , and I3 in phasor form .



Load A

(a) Electric fees is related to P only
P is unchanged hence fees
will not be reduced (5)

(b) $$\mathbf{P} = \mathbf{VI} = 200\mathbf{V}_{\mathbf{rms}} * 2\mathbf{A}_{\mathbf{rms}} = 400\mathbf{W}$$ (6)

$$\mathbf{Q} = \mathbf{VI} = 200\mathbf{V}_{\mathbf{rms}} * 3\mathbf{A}_{\mathbf{rms}} = 600\mathbf{VAR(L)}$$ (6)

$$\mathbf{I3} = \frac{\mathbf{S}}{\mathbf{V}} = \frac{\sqrt{400^2 + 600^2}}{200} \cong 3.6\mathbf{A}_{\mathbf{rms}}$$ (6)

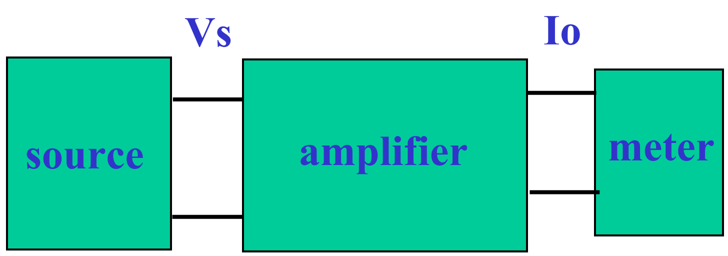
$$\therefore \mathbf{I3} \cong 3.6\angle -\tan^{-1}\frac{\mathbf{Q}}{\mathbf{P}}$$

$$= 3.6\angle -\tan^{-1}\frac{600}{400} \cong 3.6\angle -56.3^o\,\mathbf{A}_{\mathbf{rms}}$$ (6)

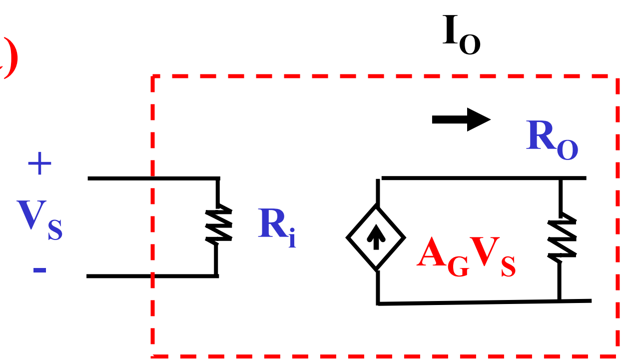
7

(a)

An amplifier is used to amplify the voltage Vs of a source and display the current Io with a meter. Draw the circuit model of the amplifier. What is the ideal value of the output resistance of the amplifier. (8)



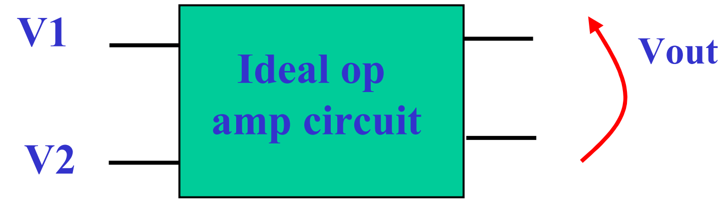
(a)

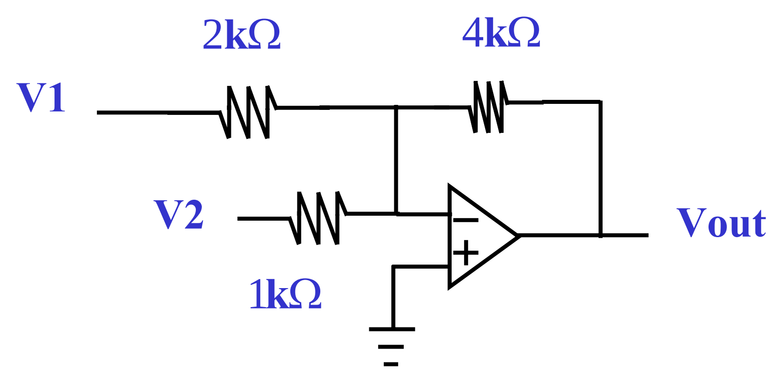


(6)

Ro = ∞Ω **(2)**

(b) Design the ideal op amp circuit below if Vout = - 2V1 – 4V2 . Use 1k – 10kΩ resistors in your design. (9)



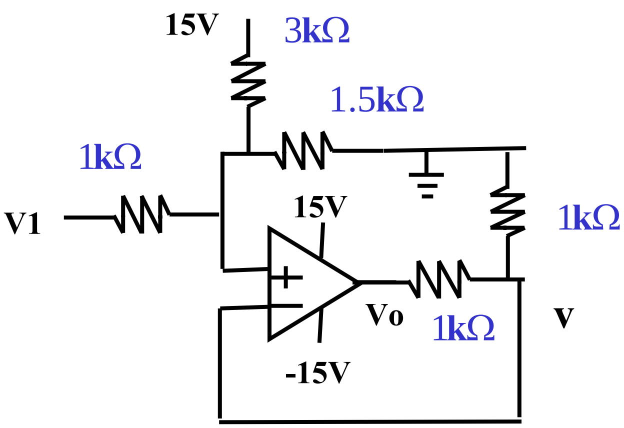


(9)

(15)

8

If v = 3V, find Vo and V1. Assume the op amp is ideal . (15)



$$4mA = \frac{15V - 3V}{3k\Omega}$$

$$2mA = \frac{3V - 0V}{1.5k\Omega}$$

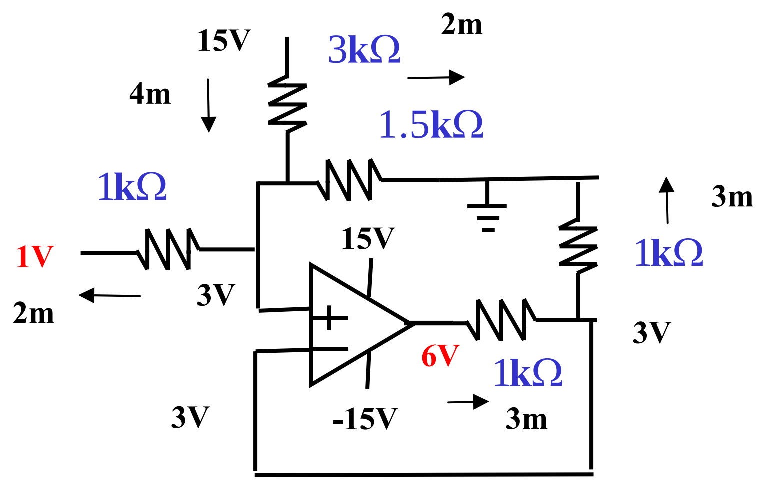
$$2mA = 4mA - 2mA$$

$$\text{Hence}\quad V1 = 3V - 2mA * 1k\Omega = 1V$$

(10)

$$Vo = 3V + \frac{3V - 0V}{1k\Omega} * 1k\Omega = 6V$$

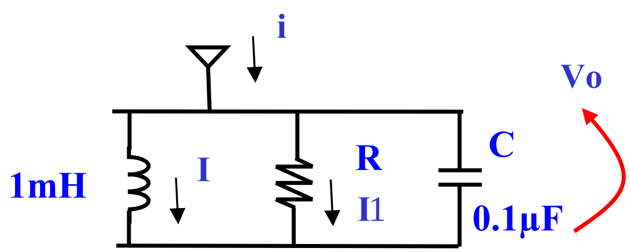
(5)



(14)

9

In the parallel LCR circuit, $Vo(t) = 2\sqrt{2}\cos(100kt)V$
R = 10kΩ. Show that I = I1 and hence find i(t) .
(14)



$$\omega L = 100krad/s * 1mH = \frac{1}{100krad/s * 0.1\mu F} = \frac{1}{\omega C} = 100\Omega \quad \textbf{(5)}$$

$$\omega L // \frac{1}{\omega C} = \infty$$ **Hence LCR is equivalent to R** **(3)**

$$\therefore i(t) = I1(t) = \frac{Vo(t)}{10k\Omega} = \frac{2\sqrt{2}\cos 1ktV}{10k\Omega} = 200\sqrt{2}\cos 1kt\ \mu A \quad \textbf{(6)}$$

(26)

10

A series LCR circuit has a resonant frequency of 100MHz . R = 5Ω .

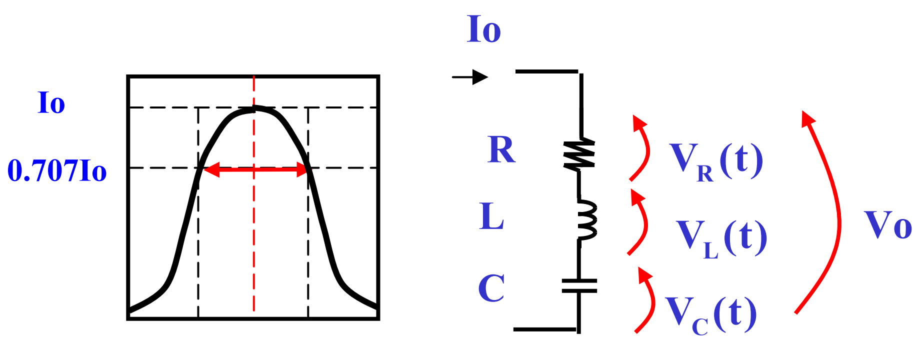
(a) If the quality factor (QF) is 100 , find the bandwidth .

(b) Find C and L .

(c) If at resonance, Io = 2cos(ωt) mA , find Vo(t) .

(d) Find the new bandwidth and QF if R is changed to 2.5Ω.

Given that QF = fo/BW = X/R . (26)



(a)

$$BW = \frac{fo}{QF} = \frac{100M}{100} = 1MHz$$ **(3)**

(b)

$$\therefore QF = \frac{\omega_O L}{R}$$

$$\therefore L = \frac{QF * R}{\omega_O} = \frac{100*5}{2\pi * 100M} \cong 0.8\mu H$$

(5)

$$\omega_O = \frac{1}{\sqrt{LC}}$$

$$\therefore C = \frac{1}{\omega_O^2 L} \cong \frac{1}{(2\pi 100MHz)^2 0.8\mu H} \cong 3.17x10^{-12} F$$ **(5)**

(c)

$$\therefore Vo(t) = Io(t) * R = 2mA * 5\Omega \cos(2\pi 100Mt)$$
$$\cong 10mV \cos 2\pi 100Mt$$ **(7)**

(d)

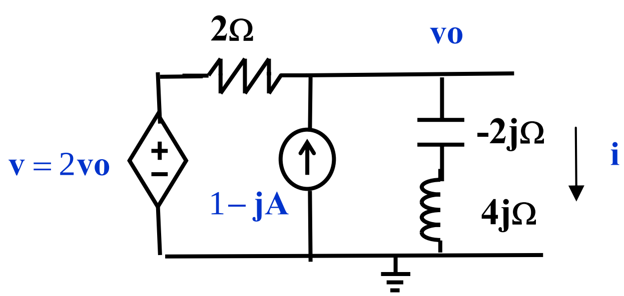
QF = X/R = 200

BW = fo/QF = 500kHz

(6)

11

In the circuit, the dependent source ($v = 2vo$) is a **voltage controlled voltage source** and unit is volt. **Find i in phasor form.**
(14)



$$\frac{2vo - vo}{2\Omega} + 1 - j = \frac{vo}{2j\Omega} \quad (7)$$

$$jvo + 2j(1-j) = vo$$

$$\therefore vo = \frac{2j(1-j)}{1-j} = 2jV \quad (5)$$

$$i = \frac{vo}{2j\Omega} = 1A \quad (2)$$