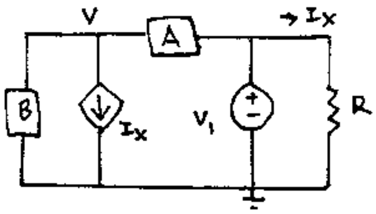
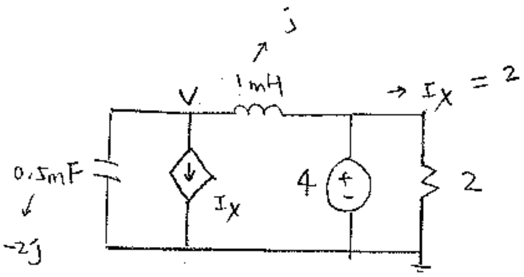
1. In the following circuit, $V_1(t) = 4\cos(1000t)$ V.
(a) If element A = 1mH, element B = 0.5mF, R = 2 Ω, show that $V(t) = \mathbf{4\sqrt{5}\cos(1000t - 26.6^\circ)V}$.
(b) Does V(t) lead $V_1(t)$?
(c) If R = 2 Ω, V(t) = 1 cos (1000t) V, suggest the elements and values of A and B.

(29 marks)





(a)

$j\omega L = j(1k)(1m) = j$

$\frac{1}{j\omega C} = \frac{1}{j(1k)(0.5m)} = -2j$

$4\cos 1kt \rightarrow 4\angle 0 \rightarrow 4V$

$I_X = 2\angle 0 = 2A$ 8

Using KCL : (node V)

$\frac{4 - V}{j} = I_X + \frac{V}{-2j}$

$\therefore 8 - 2V = 2(2j) - V$

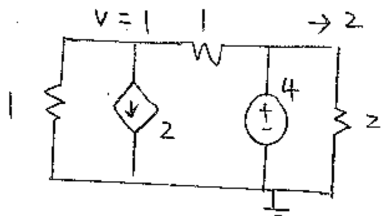
$V = 8 - 4j = 4(2 - j)$

$= 4\sqrt{5}\angle -26.6^\circ$

$\therefore V(t) = 4\sqrt{5}\cos(1000t - 26.6^\circ)$ V 11

(b) V(t) lags $V_1(t)$ 2

(c)



$\therefore A = 1\Omega$

$B = 1\Omega$ 8

2. In the following parallel LCR circuit, $i(t) = \sqrt{2}\cos\omega t$ mA, L = 50 mH, C = 10 μF, R = 10kΩ.

(a) Find the resonant frequency in rad/s.

(b) Find the Q factor.

(c) Show that the bandwidth is $\frac{10}{2\pi}$ ~~2π10~~ Hz .

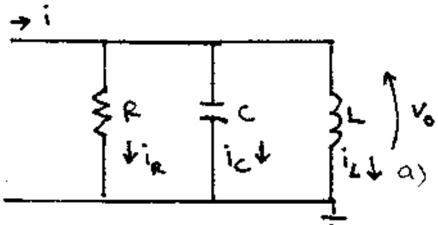
(d) Find the upper and lower cut-off frequencies.

(e) Find the maximum Vo(t).

(f) Sketch Vo (in Vrms) versus ω. Label clearly all intercepts

(g) Find the maximum i_C (in mA rms).

(h) Explain briefly two advantages if R is changed to 20kΩ.



(33 marks)

a) $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{50m \cdot 10\mu}} = 1414$ rad/s 4

b) $Q = \frac{R}{\omega_0 L} = \frac{10K}{1414(50m)} = 141.4$ 4

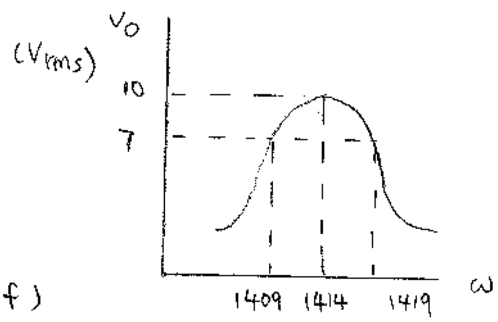
c) $BW = \frac{\omega_0}{Q} = \frac{1414}{141.4} = 10 = \frac{10}{2\pi}$ Hz 3

d) $\omega_2 = \omega_0 + \frac{BW}{2} = 1414 + 5$ rad/s

$\omega_1 = \omega_0 - \frac{BW}{2} = 1414 - 5$ rad/s 4

e) max $V_0(t) = iR = 10\sqrt{2}\cos 1414t$ V 5

f)



5

g) max $i_C = Q\,i$

$= 141.4$ mA rms 4

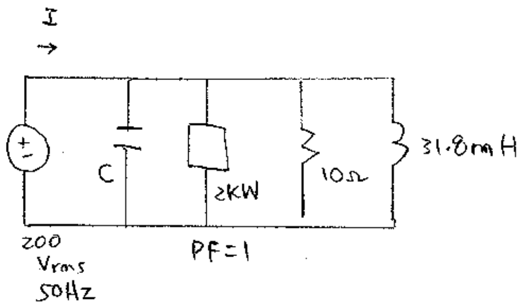
h) R = 20K

Q increases

BW decreases 4

3. (a) Load A is connected in parallel to a 200 $\angle 0°$ Vrms 50Hz power supply. If load A is a 10 Ω resistor in parallel with a 31.8 mH inductor, find the apparent power S, reactive power Q, average power P and power factor PF of load A.
(b) A load B of 2kW and PF = 1 is connected in parallel with load A, find the total Q of the combined load.
(c) A load C is now connected in parallel to the combined load in (b) to make the total power factor = 1, show that load C is a 0.318 mF capacitor. Find also the apparent power and current (in Arms) supplied by the power supply.

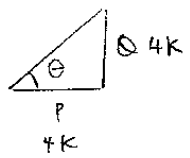
(35 marks)



(a)

$$P = \frac{V^2}{R} = \frac{200^2}{10} = 4\text{KW} \qquad 4$$

$$Q = \frac{V^2}{\omega L} = \frac{200^2}{2\pi 50 (31.8m)} = 4\text{kVAR} \qquad 4$$



$$S = \sqrt{P^2 + Q^2} = 4\sqrt{2}\ \text{KVA} \qquad 4$$

$$PF = \cos\theta = \cos 45° = 0.707 \text{ lagging} \qquad 4$$

(b) total $Q = 4\text{kVAR}$ 4

(c)

$$Q = \frac{V^2}{\frac{1}{\omega C}} = V^2 \omega C$$

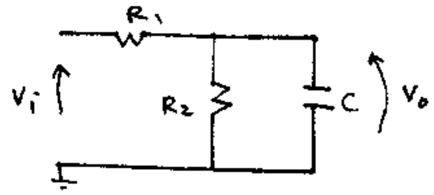
$$\therefore C = \frac{Q}{V^2 \omega} = \frac{4k}{200^2\ 2\pi 50} = 0.318\text{mF} \qquad 6$$

$$S = 2k + 4k = 6k\text{VA} \qquad 5$$

$$I = \frac{S}{V} = \frac{6k}{200} = 30\,A_{rms} \qquad 4$$

(4) In the following circuit, $R_1 = R_2 = R$.
(a) Find the complex transfer function G (= Vo/Vi) as a function of jω, C and R.
(b) Find the pole and zero of G.
(c) If C = 2mF and R = 1kΩ, find the cut-off frequency.
(d) Sketch the magnitude of G versus angular frequency ω. Label clearly the intercepts.
(e) Find the magnitude of G in dB at the half-power frequency.
(f) What is the type and order of the filter?

(31 marks)



$$R//C = \frac{R\left(\frac{1}{j\omega C}\right)}{R + \frac{1}{j\omega C}} = \frac{R}{1 + j\omega CR}$$

a) $\therefore \quad G = \frac{V_o}{V_i} = \frac{\frac{R}{1+j\omega CR}}{R + \frac{R}{1+j\omega CR}} = \frac{1}{1 + 1 + j\omega CR}$

$$= \frac{1}{2 + j\omega CR}$$

8

b) No zero

pole : $2 + j\omega CR = 0$

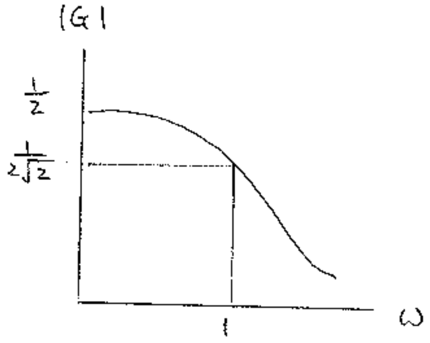
$j\omega = \frac{-2}{CR}$ 5

c) cut off : $2 + j\omega CR = 2 + 2j$

$\omega CR = 2$ 5

$\therefore \omega_{co} = \frac{2}{CR} = \frac{2}{2m \cdot 1k} = 1 \text{ rad/s}$

d)



$|G| = \frac{1}{\sqrt{4 + (\omega CR)^2}}$ 5

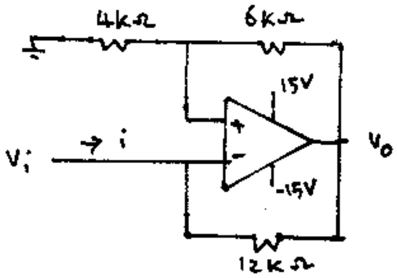
e) $20 \log \frac{1}{2\sqrt{2}} = -9 \text{ dB}$ 4

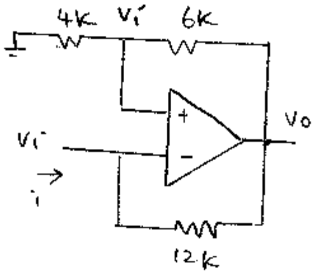
f) 1st order low pass filter 4

5. (a) In the following circuit, assume op amp is ideal. (i) Show that i = - 0.5mA if Vi = 4V. (ii) Find i if Vi = - 8V.

(b) A voltage amplifier (with input resistance R_{in}, voltage gain A and output resistance R_{out}) is connected between a source (with voltage V_s and source resistance R_s) and a load (R_L). Sketch the circuit model of the whole circuit and explain briefly why an ideal voltage amplifier should have infinite input resistance and zero output resistance.

(24 marks)





(a)

$$V_o = V_i + \frac{V_i}{4K} 6K$$

$$= 4 + 6 = 10\text{ V}$$

$$\therefore\ i = \frac{V_i - V_o}{12K} = \frac{4-10}{12K}$$

$$= -0.5\text{ mA}$$

9

$$V_o = V_i + \frac{V_i}{4K} 6K$$

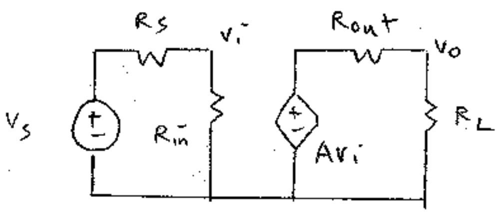
$$= -8 - 12 = -20\text{V}$$ op amp saturates!

$$\therefore\ V_o = -15\text{ V}$$

$$\therefore\ i = \frac{V_i - V_o}{12K} = \frac{-8+15}{12K} = \frac{7}{12}\text{ mA}$$

6

(b)



$R_{in} \to \infty \qquad V_i \to V_s$

$R_{out} \to 0 \qquad V_o \to AV_i \to AV_s$

9