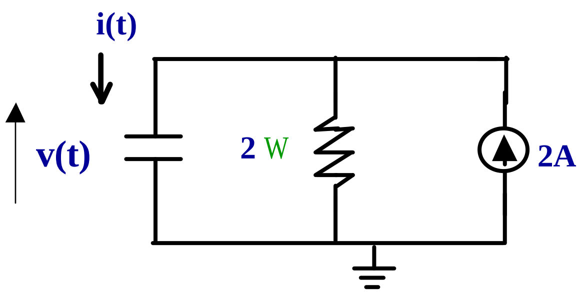
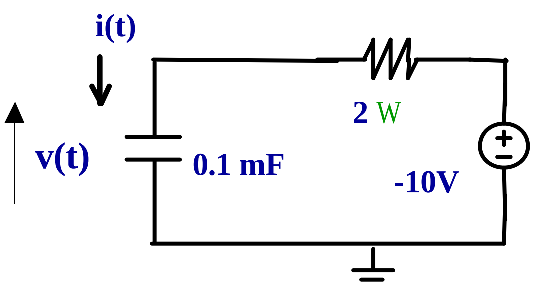
1 **(35)**

t < 0, S is at a, v = 4V



$\therefore v(0) = 4V$

At t = 0, S is switched to b



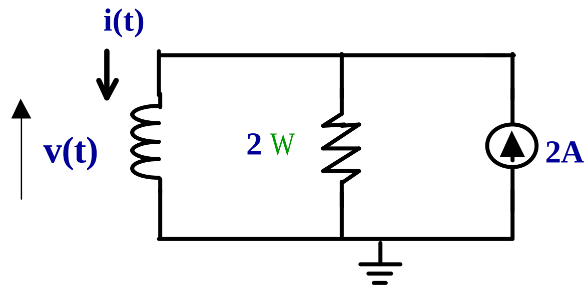
$\therefore v(\infty) = -10V$

$\tau = CR$
$= (0.1m)(2) = 0.2ms$

$$\therefore v(t) = v(\infty) + [v(0) + v(\infty)]\, e^{-t/CR} = -10 + [4 + 10]\, e^{-t/0.2ms} V$$

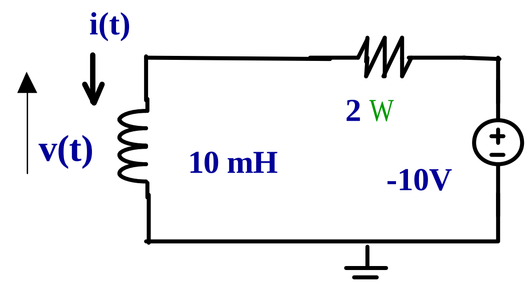
$$\therefore i(t) = \frac{-10 - v(t)}{R} = \frac{-10 + 10 - 14e^{-t/0.2ms}}{2} = -7e^{-t/0.2ms} A$$

t < 0, S is at a, i = 2A



$\therefore i(0) = 2A$

At t = 0, S is switched to b



$\therefore i(\infty) = -5A$

$\tau = L/R$
$= 10m / 2 = 5ms$

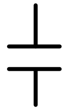
$$\therefore i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau} = -5 + [2 + 5]\, e^{-t/0.2ms} A$$

$$\therefore v(t) = -10 - i(t)2 = -10 - [-5 + 7^{-t/0.2ms}]2 = -14e^{-t/0.2ms} V$$

1. In the circuit, the switch has been at **terminal a for a long time.**
At $t = 0$, the switch is switched to **terminal b.**
(a) If X is a 0.1mF capacitor, find i(t) for $t \geq 0$. Find also the power supplied or absorbed by the –10V source at $t = 0$.
(b) If X is a 10 mH inductor, find v(t) for $t \geq 0$.
Given that $i(t) = i(\infty) + [i(0) - i(\infty)]\, e^{-t/\tau}$ and $v(t) = v(\infty) + [v(0) - v(\infty)]\, e^{-t/\tau}$ (35)

2

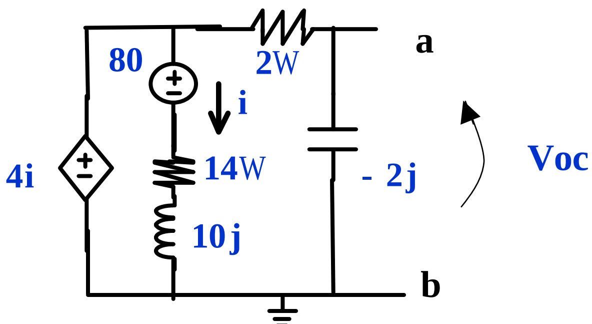
(33)



$$\frac{1}{j\omega C} = \frac{1}{j(1k)\frac{1}{2}m} = -2j$$

$$j\omega L = j(1k)10m = 10j$$

$$v(t) = 80\cos(1kt)V \Rightarrow V = 80$$



$$\because 4i = 80 + i(14 + 10j)$$

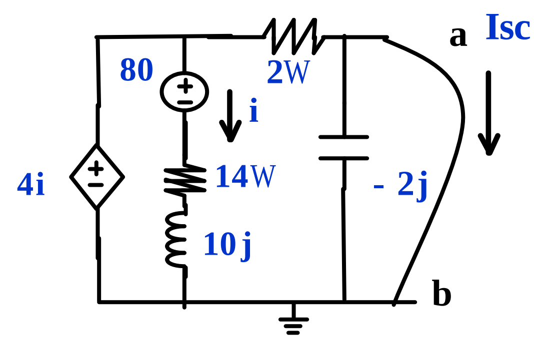
$$\therefore i = \frac{-80}{10 + 10j} = \frac{-8}{1 + j}$$

$$\therefore Voc = 4i\frac{-2j}{2-2j}$$

$$= 4i\frac{-j}{1-j} = 4\frac{-8}{1+j}\frac{-j}{1-j} = \frac{32j}{2} = 16j$$

$$\therefore Voc(t) = 16\cos(1000t + 90^o)V$$

Hence Voc(t) leads v(t)



$$\therefore Isc = \frac{4i}{2} = 2i$$

$$= 2\frac{-8}{1+j} = -16\frac{1}{1+j}\frac{1-j}{1-j}$$

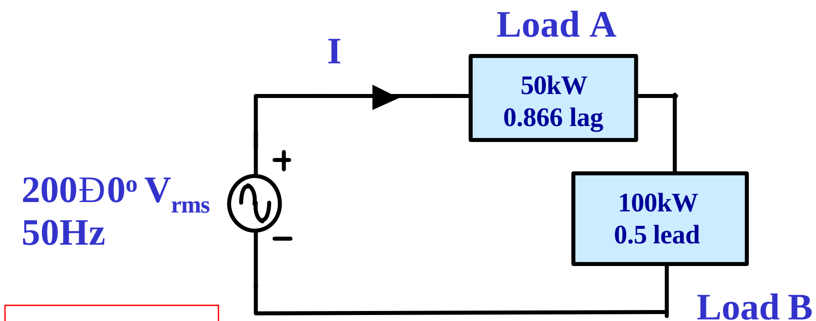
$$= -16\frac{1-j}{2} = -8(1-j) = -8\frac{\angle -45}{\sqrt{2}}$$

$$\therefore Isc(t) = \frac{-8}{\sqrt{2}}\cos(1000t - 45^o)A$$

2. In the circuit, v(t) = 80 cos (1k t) V. Find Voc(t) and Isc(t) at **terminals ab.** Does Voc(t) lead V(t)? (33)

3 **(33)**

3. In the circuit, load A is 50 kW at 0.866 lagging power factor. Load B is 100 kW at 0.5 leading power factor.
(a) Find the total apparent power S, reactive power Q, average power P and power factor PF of the combined load (load A and B).
(b) If a load X is connected to terminals ab to make the total power factor = 1, find the element and value of load X. Find also the new load current I in Arms. (33)



For load A

$$P = 50\text{kW}$$

$$Q = P\tan\theta = 50\text{k}\tan(\cos^{-1}0.866) = 28.87\text{kVAR (L)}$$

For load B

$$P = 100\text{kW}$$

$$Q = P\tan\theta = 100\text{k}\tan(\cos^{-1}0.5) = 173.2\text{kVAR(C)}$$

For load A and B

$$\text{total } P = 50\text{k} + 100\text{k} = 150\text{kW}$$

$$\text{total } Q = 28.87\text{k} - 173.2\text{k} = -144.33 \text{ kVAR}$$

$$\text{total } S = \sqrt{P^2 + Q^2} = \sqrt{150\text{k}^2 + 144.33\text{k}^2} = 208.16\text{kVA}$$

$$\text{total PF} = \frac{P}{S} = \frac{150\text{k}}{208.16\text{k}} = 0.72\text{leading}$$

Add Load X (inductance L)

$$\therefore Q_L = \frac{V^2}{\omega L}$$

$$\therefore L = \frac{V^2}{\omega Q_L} = \frac{200^2}{2\pi 50(144.33\text{k})} = 0.88\text{mH}$$

New I

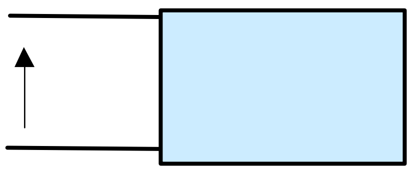
$$I = \frac{S}{V} = \frac{150\text{k}}{200} = 750A_{rms}$$

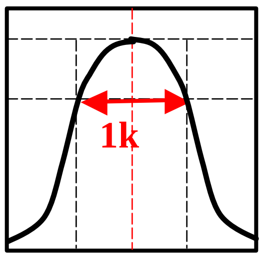
4

(30)

$i(t) = \sqrt{2}\cos\omega t\,\text{mA}$

$V_o(t)$





$$\omega_o = 120\text{krad/s}$$

$$BW = 1\text{krad/s}$$

$$\omega_2 = \omega_o + \frac{BW}{2} = 120.5\text{krad/s}$$

$$\omega_1 = \omega_o - \frac{BW}{2} = 119.5\text{krad/s}$$

$$Q = \frac{\omega_o}{BW} = \frac{120k}{1k} = 120$$

Vo is maximum at resonance, hence network is parallel LCR

$$\therefore Q = \frac{R}{\omega_o L}$$

$$\therefore R = Q\omega_o L = 120(120k)0.4m = 5.76k\Omega$$

$$\therefore C = \frac{1}{\omega_o^2 L} = \frac{1}{(120k)^2(0.4m)} = 0.174\mu F$$

$$\therefore \max V_o(t) = i(t)R$$

$$= \sqrt{2}\cos(\omega_o t)\text{mA} * 5.76k\Omega = 5.76\sqrt{2}\cos(\omega_o t)V$$

$$\therefore \max i_L = \frac{\max V_o}{j\omega_o L} = \frac{IR}{j\omega_o L} = \frac{QI}{j}$$

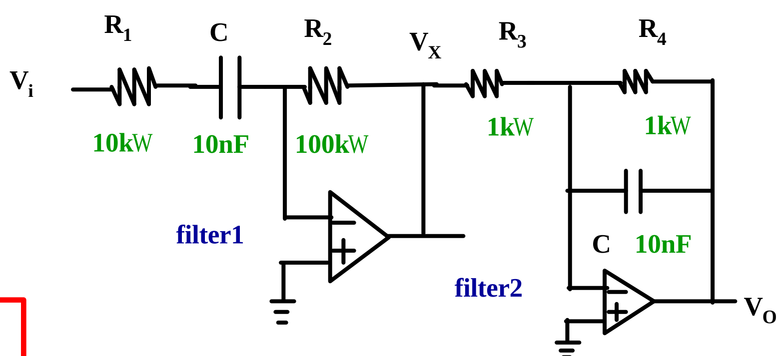
$$\therefore \max i_L(t) = 120\sqrt{2}\cos(\omega_o t - 90^o)\text{mA}$$

4. A LCR circuit (band pass filter) has the following frequency response curve (magnitude of Vo(t) versus frequency ω).
(a) Find the resonant frequency, bandwidth (BW), upper and lower cut-off frequencies (in rad/s), and the Q-factor of the LCR circuit.
(b) If L = 0.4mH, find the values of C and R. Find also the maximum Vo(t) and maximum current flowing in L ($i_L(t)$). (30)

5

(30)

5. An ideal op amp filter circuit is composed of filter 1 in series with filter 2.
(a) Obtain the complex transfer function H (=Vo/Vi) in terms of $j\omega$, C, R_1, R_2, R_3 and R_4.
(b) Obtain the cut-off frequency (in rad/s) for filter1 and filter 2.
(c) Sketch |Vo/Vi| in dB versus angular frequency ω. Label clearly all intercepts.
(d) What type of filter is it? (30)



a

$$G_1 = \frac{Vx}{Vi} = -\frac{R_2}{R_1 + 1/jwC} = -\frac{R_2}{R_1}\frac{1}{1 - j/wCR_1}$$

$$G_2 = \frac{Vo}{Vx} = -\frac{R_4 // \frac{1}{jwC}}{R_3} = -\frac{R_4(\frac{1}{jwC})}{R_3\,(R_4 + \frac{1}{jwC})} = -\frac{R_4}{R_3}\frac{1}{(1 + jwCR_4)}$$

b

$$\backslash\ \frac{Vo}{Vi} = \frac{Vo}{Vx}\frac{Vx}{Vi} = \frac{R_2}{R_1}\frac{R_4}{R_3}\frac{1}{(1 + jwCR_4)}\frac{1}{(1 - j/wCR_3)}$$

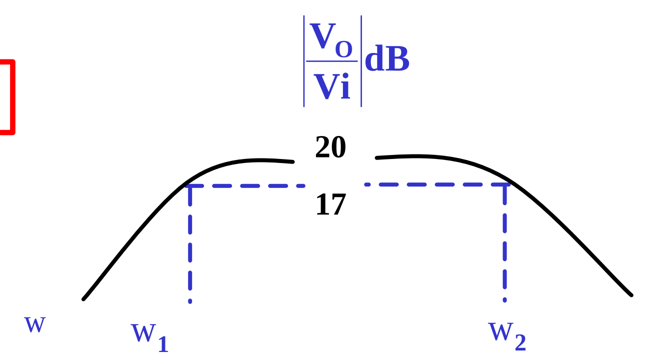
$$\backslash\ w_1 = \frac{1}{CR_1} = \frac{1}{10n(10k)} = 10krad/s$$

$$\backslash\ w_2 = \frac{1}{CR_4} = \frac{1}{10n(1k)} = 100krad/s$$

$$|G_1| = \left|\frac{R_2}{R_1}\right| = \frac{100k}{10k} = 10 \qquad 20\log 10 = 20dB$$

$$|G_2| = \left|\frac{R_4}{R_3}\right| = \frac{1k}{1k} = 1$$

c

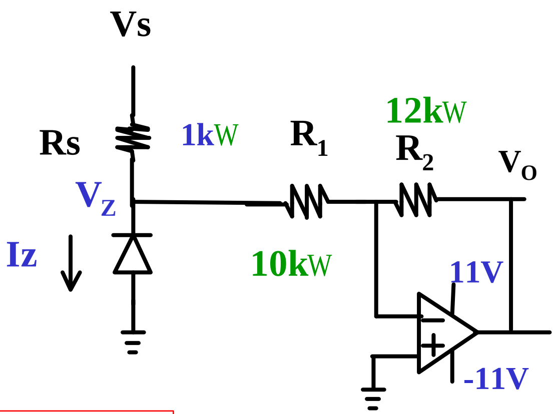


d

Circuit is a band pass filter

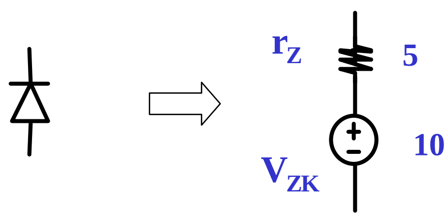
6

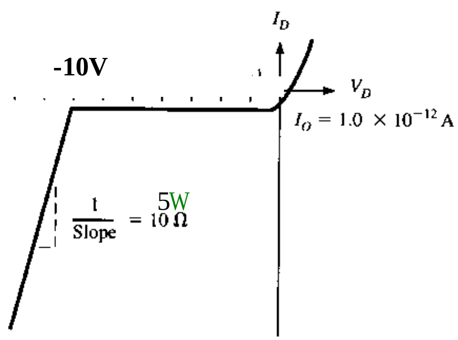
(41)



a

Model of zener diode





b

Vs in terms of Iz

$$\because \frac{V_S - V_Z}{R_S} = \frac{V_Z}{R_1} + I_Z$$

$$\therefore \frac{V_S}{R_S} = V_Z\left(\frac{1}{R_1} + \frac{1}{R_S}\right) + I_Z = (V_{ZK} + I_Z r_Z)\left(\frac{1}{R_1} + \frac{1}{R_S}\right) + I_Z$$

$$\therefore V_S = (V_{ZK} + I_Z r_Z)\left(\frac{R_S}{R_1} + 1\right) + I_Z R_S$$

$$= (10 + I_Z 5)\left(\frac{1k}{10k} + 1\right) + I_Z 1k = 11 + I_Z(1005.5)$$

c

Vz = - 0.5V Zener is a on diode

$$\therefore Iz = -Io\exp\left(\frac{V_Z}{25m}\right) = -(1p)\exp\left(\frac{700}{25}\right) = -0.49mA$$

d (i)

Vs = 10V Zener is not breakdown and is an off diode (open)

$$\therefore Vo = -\frac{R_2}{R_1 + R_S} Vs = -\frac{12k}{10k + 1k} 10 = -10.91V$$

d (ii)

Vs = 25V Zener is breakdown

$$\therefore V_Z = V_{ZK} + I_Z r_Z = 10 + I_Z(5)$$

$$= 10 + \frac{V_S - 11}{1005.5}(5) = 10 + \frac{25 - 11}{1005.5}(5) = 10.07V$$

$$\therefore Vo = -\frac{R_2}{R_1} V_Z = -\frac{12k}{10k}(10.07) = -12.1V \Rightarrow -11V$$

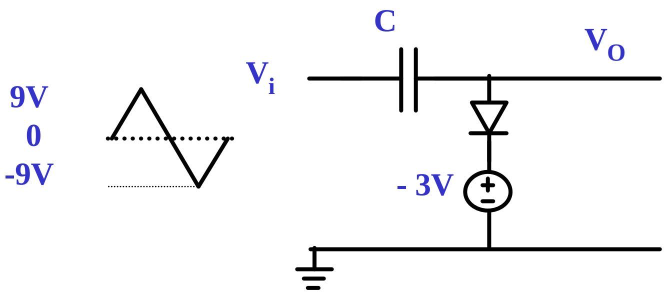
6. In the ideal op amp circuit, the diode has the reverse characteristics as shown. The diode equation is .

$$I_D = I_O \exp\frac{V_D}{25mV}$$

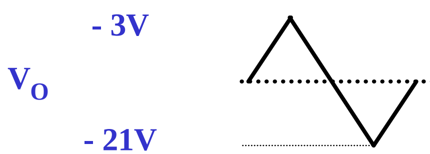
(a) Sketch the model of the diode at breakdown.
(b) When diode is at breakdown, find Vs in terms of Iz only.
(c) Find Iz if Vz = - 0.5V.
(d) Find the output voltage Vo if Vs = (i) 10V, (ii) 25V. (41)

7 **(30)**

a

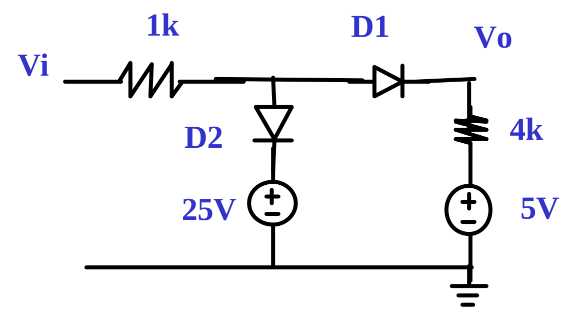


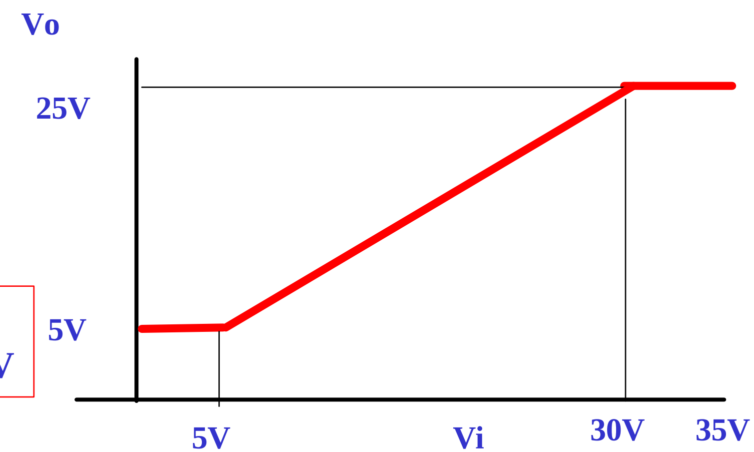
Vo = Vi + Vc = Vi - 12



7. In the ideal diode circuit, sketch Vo(t). Show clearly the voltages in your sketch. (12)
(b) In the ideal diode circuit, plot Vo versus Vi for 0V ≤ Vi ≤ 35V. Show clearly all voltages in your sketch. (18)

b





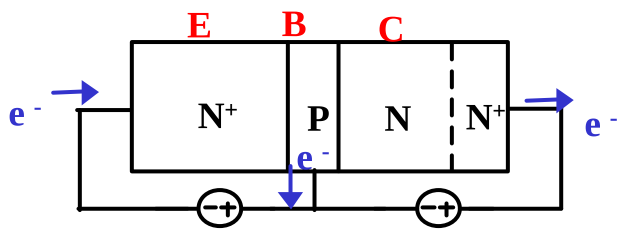
Vi < 5V, D1 and D2 OFF, Vo = 5V

Vi > 5V, D1 ON,
Vo = 5 + I(4k)
= 5 + (Vi - 5)4k/(4k + 1k)
= 5 + (Vi - 5)4/5
Vi = 30, Vo = 5 + 25*4/5 = 25

Vi = 30V, Vo = 25, D2 ON, Vo is fixed at 25V

8 **(20)**

a



1. EB Junction is a <u>forward</u> (on) diode and BC is reverse (off) diode
2. E is made very heavily doped (N + for NPN)
   Hence I_E mainly are electrons diffusing from E to B.
3. But B is made very thin and has a wide depletion region near C.
4. Hence most electrons will arrive C.

\ I_C » $a_F I_E$

Since $I_E = I_B + I_C$

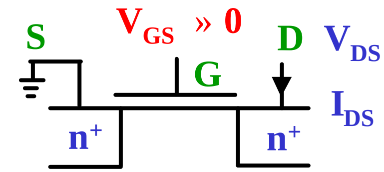
Hence I_C » $b_F I_B$, where $b_F = \frac{a_F}{1 - a_F}$

8. (a) Sketch the cross section of a NPN transistor operated in the active region, describe the movement of electrons and explain briefly why $I_C \sim \alpha I_E$ and $I_C \sim \beta I_B$.
(b) Sketch the cross section of an enhancement NMOSFET, describe the movement of electrons and the change of the channel, and explain the linear and saturation regions. (20)

b

Cut-off

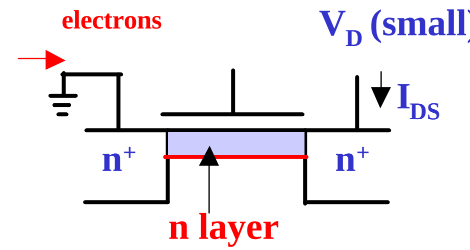
$V_{GS} < V_T$



triode

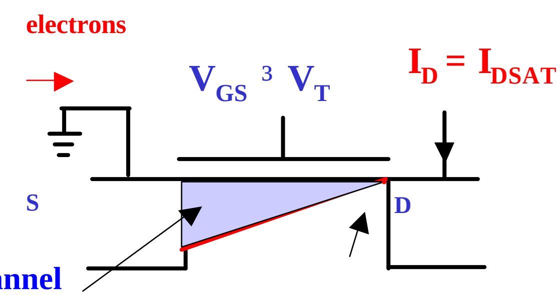
V_{GS} 3 V_T

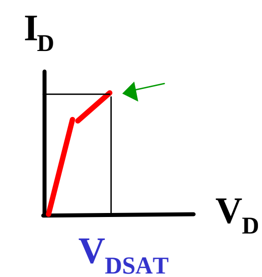
$V_{DS} << V_{GS} - V_T$



saturation

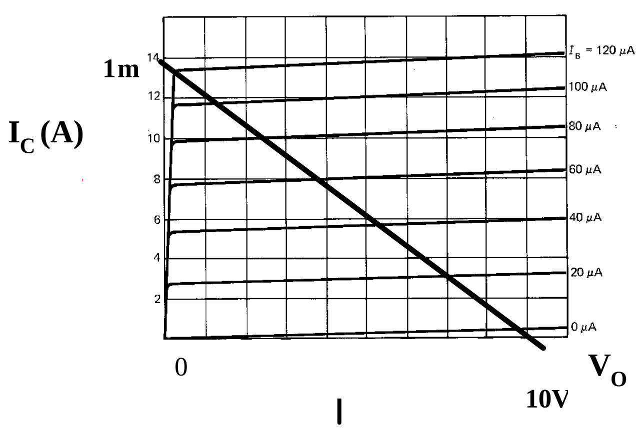
V_{DS} 3 $V_{GS} - V_T$



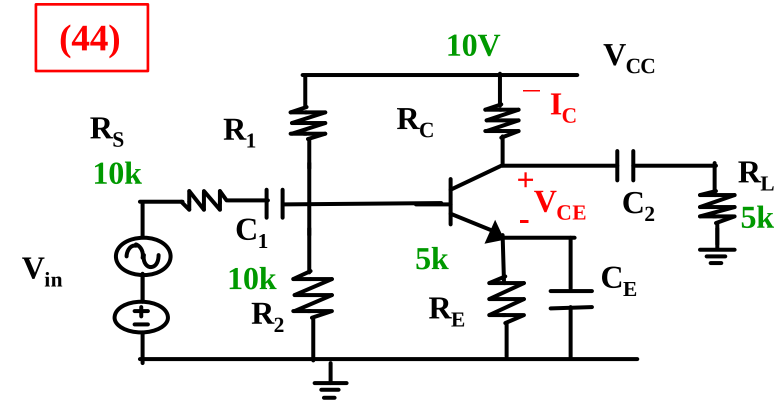


P: pinch-off point

9. The BJT amplifier has the output characteristic curves and DC load line as shown. (a) Assume $I_C \sim I_E$, write the load line equation, find V_{CC} and show that $R_C = 5k\Omega$. (b). If the amplifier is to have maximum symmetrical output, find the value of the Q-point and sketch the Q point on the load line.
(c) At the Q-point in (b), find the value of R_1 if $R_2 = 10k\Omega$. You can assume $I_B \sim 0$ in your calculation. (d) Sketch the AC equivalent circuit and find the voltage gain A_V (= ΔVo /ΔVin) of the amplifier. Given that for the BJT, $V_{BE} = 0.7V$, $r_\pi = 5\ k\Omega$, ro = ∞, $\beta = 100$. (44)



9 **(44)**



Given Ic = 0.5mA, β = 100, V_{BE} = 0.7V, r_π = 5k

a

Find R_C

$$\therefore V_{CC} \approx V_{CE} + I_C R_C + I_E R_E = 10$$

Since $I_C \sim I_E = 1mA$ when $V_{CE} = 0$

$$\therefore R_C = \frac{V_{CC}}{I_C} - R_E = \frac{10}{1m} - 5k = 5k\Omega$$

9

b

Q-point at middle of load line :
I_C ~ 0.5mA, V_{CE} ~ 5V

c

$$\backslash\ V_B = V_{BE} + I_B R_B + I_E R_E$$

$$\backslash\ V_{CC}\frac{R_2}{R_1 + R_2} = 0.7 + \frac{I_C}{b}\frac{R_1 R_2}{R_1 + R_2} + 0.5m(5k)$$

$$\backslash\ 10\frac{R_2}{R_1 + R_2} = 0.7 + \frac{0.5m}{100}\frac{R_1 R_2}{R_1 + R_2} + 0.5m(5k)$$

$$\backslash\ 10R_2 = 0.7(R_1 + R_2) + 5m(R_1 R_2) + 2.5(R_1 + R_2)$$

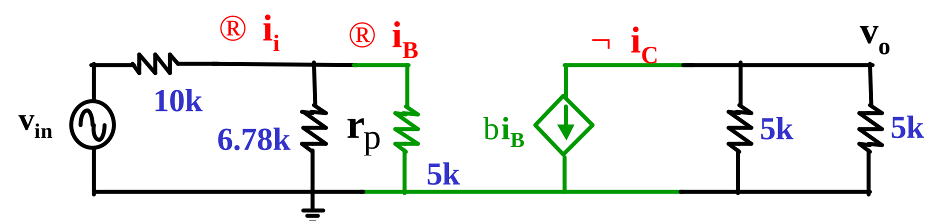
$$\backslash\ R_2(10 - 0.7 - 2.5) = R_1(0.7 + 2.5) + 5mR_1 R_2$$

$$\backslash\ R_1 = \frac{R_2(6.8)}{3.2 + 5mR_2} = \frac{10k(6.8)}{3.2 + 5m10k} = 21kW$$

d

$$\backslash\ R_B = 10k // 21k = 6.78kW$$

$$\backslash\ i_i = \frac{i_B r_p}{r_p // R_B} = i_B\frac{5k}{5k // 6.78k} = i_B\frac{5k}{2.88k} = 1.74 i_B$$



voltage gain

$$A_V = \frac{v_o}{v_{in}} = \frac{-b\, i_B(R_C // R_L)}{i_B r_p + i_i R_S}$$

$$= \frac{-b\, i_B(5k // 5k)}{i_B 5k + 1.74 i_B 10k}$$

$$= \frac{-100\ (2.5k)}{22.4k} = -11.16$$

10. Using the same BJT circuit in question (9),
(a) Explain briefly why the circuit can have stable I_C .
(b) Explain briefly the importance of having a stable Q-point .
Explain also briefly the difference between normal β and forced β.
. (18)

10

a

(18)

Circuit can maintain stable I_C (Q-point).

I_C -
(∵ b **or T -)**

V_E -
(∵ $V_E = I_E R_E$
~ $I_C R_E$)

V_{BE} –
(∵ $V_{BE} = V_B - V_E$)

I_B –

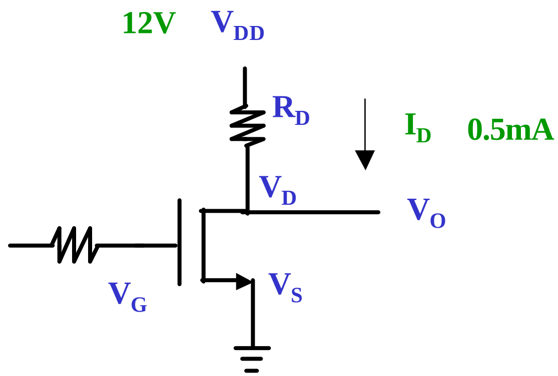
I_C –
(∵ I_C = b I_B)

b

Stable Q-point can maintain stable output

Forced beta is dI_C / dI_B at saturation region
Beta is dI_C / dI_B at active region

11 **(30)**



Given
$V_T = 1V$
$K = 0.02mA/V^2$

1 **Find R_D such that Vi = V_O**

When Vi = V_O ,
i.e. $V_{GS} = V_{DS}$
NMOS is saturate

$$\because I_{DS} = K\,(V_{GS} - V_T)^2 = 0.02m(V_{GS} - 1)^2 = 0.5mA$$

$$\backslash\ V_{GS} = \sqrt{\frac{0.5m}{0.02m}} + 1 = 6V$$

$$\because V_{DD} = I_{DS}R_D + V_{DS}$$

$$\backslash\ R_D = \frac{V_{DD} - V_{DS}}{I_{DS}} = \frac{12 - 6}{0.5m} = 12kW$$

2 **Find V_O such that Vi = V_{DD}**

When Vi = V_{DD}
i.e. $V_{GS} = 12V$,
hence $V_{GS} - V_T > V_{DS}$ and NMOS is in triode mode

$$\backslash\ I_{DS} = 2K[(V_{GS} - V_T)V_{DS} - \frac{V_{DS}^{\ 2}}{2}]$$

$$= 2(0.02m)[(12 - 1)V_{DS} - \frac{V_{DS}^{\ 2}}{2}]$$

$$\backslash\ I_{DS} = \frac{V_{DD} - V_{DS}}{R_D} = \frac{12 - V_{DS}}{12.5k} = 0.04m(11V_{DS} - \frac{V_{DS}^{\ 2}}{2})$$

$$\backslash\ 12 - V_{DS} = 0.5[11V_{DS} - \frac{V_{DS}^{\ 2}}{2}]$$

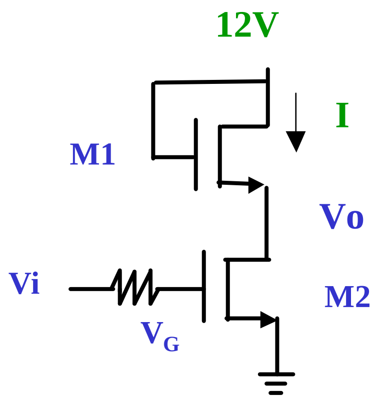
$$\backslash\ 12 - 6.5V_{DS} + 0.25V_{DS}^{\ 2} = 0$$

$$\backslash\ V_{DS}^{\ 2} - 26V_{DS} + 48 = 0$$

$$\backslash\ V_{DS} = 24V \quad \text{or} \quad 2V$$

hence $V_O = V_{DS} = 2V$
($V_O = 24\ V > V_{DS}$ is impossible)

12 **(20)**



Given
$V_T = 1V$
$K = 0.02mA/V^2$

Find the maximum Vi such that M2 still operates in saturation region

1 **M1 is also saturate, since**

$V_{DS} = V_{GS}$
$V_{DS} > V_{GS} - V_T$

\ $I_{DS} = K\,(V_{GS1} - V_T)^2 = K\,(V_{GS2} - V_T)^2$

\ $(V_{GS1} - V_T) = (V_{GS2} - V_T)$

\ $V_{GS1} = V_{GS2} = Vi$

2 **If M2 is saturate, then**

$V_{DS}\ {}^{3}\ V_{GS} - V_T$

When Vi (= V_G) is maximum and M2 is still saturate, then

$V_{DS} = V_{GS} - V_T$

\ $V_{DS2} = V_{GS2} - V_T$

\ $V_{DD} - V_{GS1} = V_{GS2} - V_T$

\ $V_{DD} + V_T = V_{GS2} + V_{GS1} = 2V_{GS2} = 2Vi$

\ $Vi = \dfrac{V_{DD} + V_T}{2} = \dfrac{12+1}{2} = 6.5V$