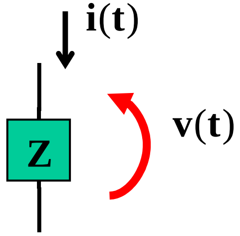
1 (26)

(a) If $v(t) = 10\cos(1kt + 30^o)V$, $Z = 2\Omega$, find $i(t)$.
Find also the power factor of Z, and power stored in Z.

(b) If $i(t) = 5\cos(1kt + 30^o)A$, $Z = 2mF$,
find Z in Ω and find $v(t)$.

(c) If $i(t) = 5\cos(1kt - 30^o)A$, $v(t) = 10\cos(1kt + 30^o)V$,
Z is R in series with X. Find Z in Ω and find R.
Find also the apparent power S of Z.



(a) $i(t) = 5\cos(1kt + 30^o)A$ **(3)**

$PF = 1$ **(2)**

$Q_R = 0$ **(2)**

(b) $Z = \dfrac{1}{j\omega C} = \dfrac{1}{j(1k)2mF} = -0.5j\Omega$ **(2)**

$v(t) = 2.5\cos(1kt - 60^o)V$ **(5)**

(12)

(c)

$$\therefore \mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{10\angle 30\mathbf{V}}{5\angle -30\mathbf{A}} = 2\angle 60^{\circ}\,\Omega$$

(5)

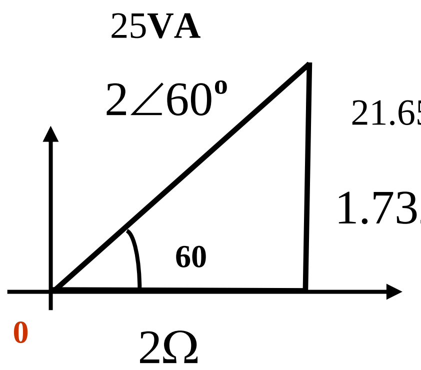
$$\mathbf{or} = 2\cos 60^{\circ} + 2\mathbf{j}\sin 60^{\circ} = 1\Omega + \mathbf{j}1.732\Omega$$

$$\therefore \mathbf{R} = 1\Omega$$

(2)

$$\therefore \mathbf{S} = \mathbf{V} * \mathbf{I} = \frac{10\mathbf{V}}{\sqrt{2}} * \frac{5\mathbf{A}}{\sqrt{2}} = 25\mathbf{VA}$$

(5)



$$21.65\mathbf{VAR}(\mathbf{L}) = \left(\frac{5}{\sqrt{2}}\right)^2 * 1.732$$

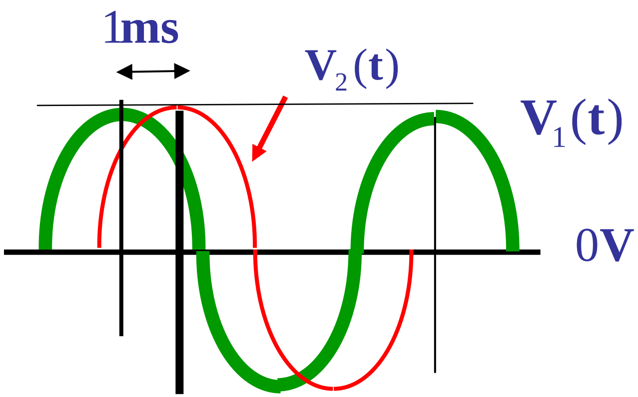
$$12.5\mathbf{W} = \left(\frac{5}{\sqrt{2}}\right)^2 * 1\Omega$$

(13)

2

If $\mathbf{V}_2(\mathbf{t}) = 5\cos 400\pi\mathbf{t}\ \mathbf{V}$, find $\mathbf{V}_1(\mathbf{t})$.

(13)



$$\mathbf{V}_2(\mathbf{t}) = 5\cos 2\pi\mathbf{f} * \mathbf{t}\ \mathbf{V} = 5\cos 2 * \pi * 200\mathbf{t}\ \mathbf{V}$$

$$\therefore \mathbf{T} = \frac{1}{\mathbf{f}} = \frac{1}{200} = 5\mathbf{ms}$$ **(6)**

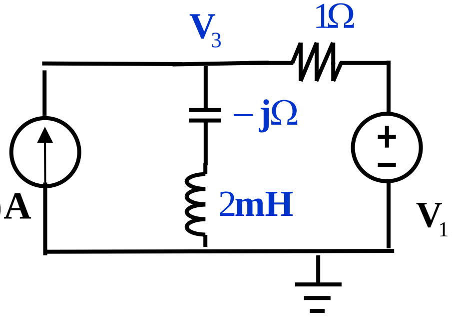
$$\theta = \frac{\mathbf{t}}{\mathbf{T}} * 360^{\mathbf{o}} = \frac{1\mathbf{ms}}{5\mathbf{ms}} * 360^{\mathbf{o}} = 72^{\mathbf{o}}$$ **(4)**

$$\therefore \mathbf{V}_1(\mathbf{t}) = 5\cos(\omega\mathbf{t} + \theta)\mathbf{V} = 5\cos(400\pi\mathbf{t} + 72^{\mathbf{o}})\mathbf{V}$$

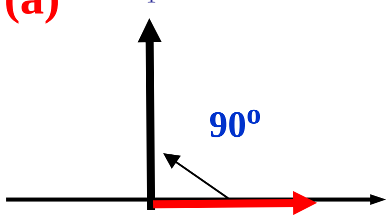
(3)

3

(a) Draw V1 and I1 in a **phaser diagram**.
(b) Use superposition to find the phasor V3. (25)



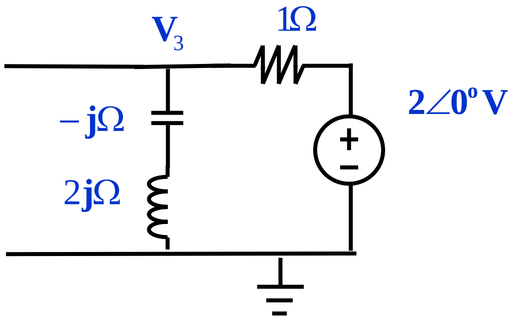
(a)



(4)

(b) $j\omega L = j(1k)2mH = 2j\Omega$ (2)

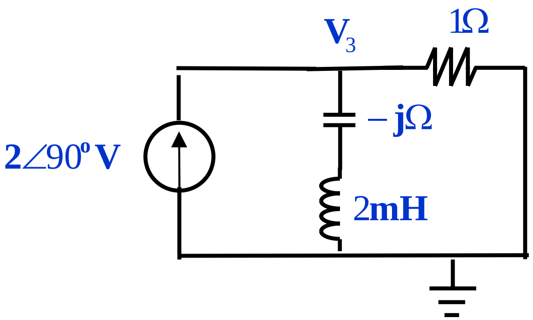
Use superposition



(2)

$$\therefore V_{3,2V} = 2\angle 0^o V * \frac{j\Omega}{1\Omega + j\Omega} = \frac{2j}{1+j} V$$ (5)

(12)



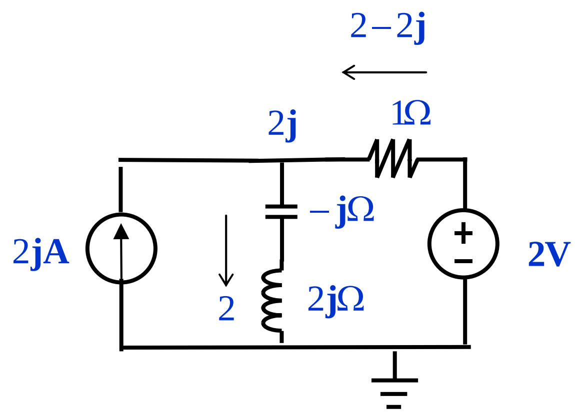
(2)

$$\therefore \mathbf{V}_{3,2\mathbf{A}} = 2\angle 90^{\circ}\,\mathbf{A} * \frac{1\Omega * \mathbf{j}\Omega}{1+\mathbf{j}\Omega} = \frac{2\mathbf{j} * \mathbf{j}}{1+\mathbf{j}}\mathbf{V}$$

(5)

$$\therefore \mathbf{V}_3 = \frac{2\mathbf{j}}{1+\mathbf{j}} + \frac{2\mathbf{j} * \mathbf{j}}{1+\mathbf{j}} = \frac{2\mathbf{j}(1+\mathbf{j})}{1+\mathbf{j}} = 2\mathbf{j} = 2\angle 90^{\circ}\,\mathbf{V}$$

(5)



$$\frac{2-\mathbf{V}2}{1\Omega} + 2\mathbf{j} = \frac{\mathbf{V}2}{\mathbf{j}}$$

$$\therefore \mathbf{j}(2-\mathbf{V}2) - 2 = \mathbf{V}2$$

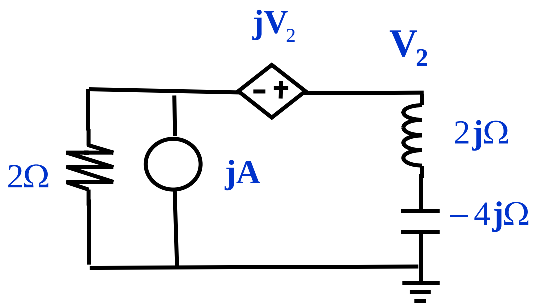
$$\therefore \mathbf{V}2 = \frac{2-2\mathbf{j}}{-\mathbf{j}-1} = 2\mathbf{j}$$

$$\therefore \mathbf{V}_3(\mathbf{t}) = 2\cos(1\mathbf{k}\mathbf{t} + 90^{\circ})\mathbf{V}$$

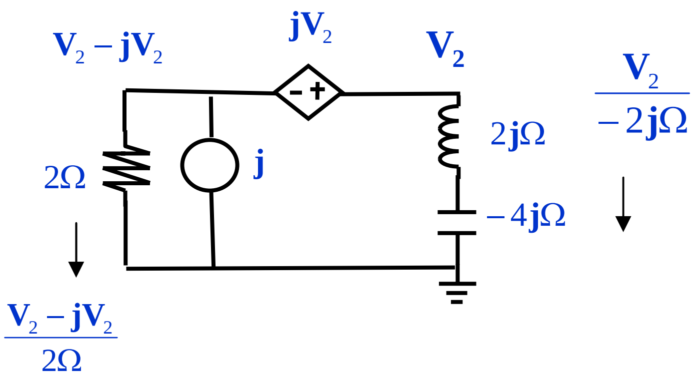
(14)

4

Find V2 . The voltage dependent voltage source is in volt and equal to jV2. (14)



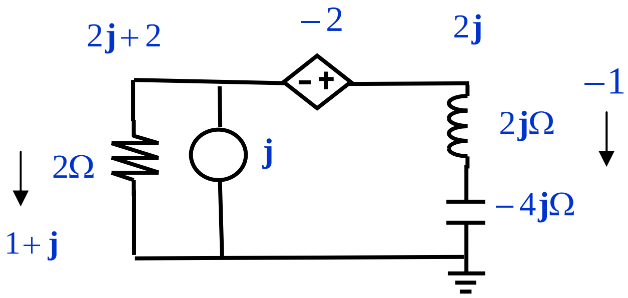
Use KCL



$$\mathbf{j} = \frac{V_2}{-2j\Omega} + \frac{V_2 - jV_2}{2\Omega} \qquad \textbf{(7)}$$

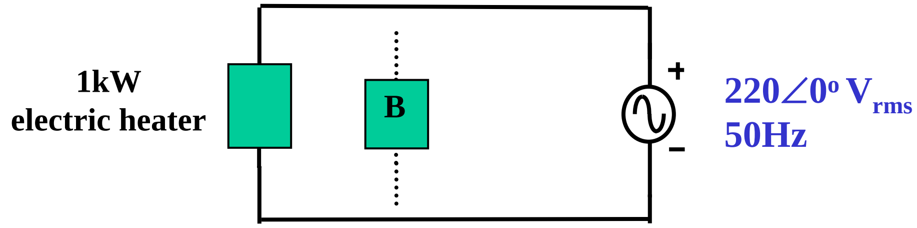
$$\mathbf{j} = \frac{V_2}{-2j} + \frac{V_2}{2} - \frac{jV_2}{2} = \frac{V_2}{2}$$

$$\therefore V_2 = 2jV \qquad \textbf{(7)}$$



5

Reactive power Q of the 1kW electric heater is 750VAR(L) . (a) Find the power factor PF of the heater. (b) If load B is connected in parallel to the heater such that the power factor of the combined load is 0.95 lagging, find the element and value of load B.
(21)



(a)

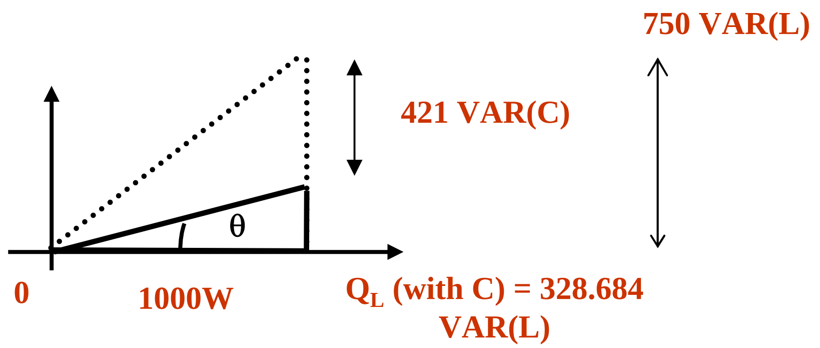
$$\tan\theta = \frac{\mathbf{Q}}{\mathbf{P}} = \frac{750\mathbf{VAR(L)}}{1\mathbf{kW}} = 0.75$$

$$\therefore \mathbf{PF} = \cos\theta = \cos(\tan^{-1} 0.75) = 0.8\ \mathbf{lagging}$$ (7)

(b)

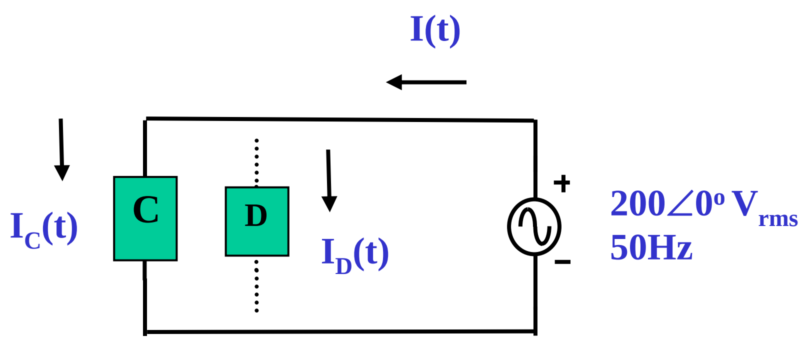
$$\therefore \mathbf{Q_C\ required} = 750 - 1000\tan(\cos^{-1} 0.95)$$
$$\cong 421.316\mathbf{VAR(C)}$$
(8)

$$\therefore \mathbf{C} = \frac{|\mathbf{Qc}|}{\mathbf{V}^2\omega} \cong \frac{421.316}{(220^2)2\pi 50} \cong 27.71\mu\mathbf{F}$$ (6)



6

Load C has 12kW and 5kVAR(L) . (a) Find the apparent power S of load C. Find also IC in Arms.
(b) If load D is connected in parallel to load C such that the power factor of the combined load is 1, find I_D in Arms.
Find also I(t). (22)



(a)

$$S = \sqrt{P^2 + Q^2} = \sqrt{12k^2 + 5k^2} = 13kVA$$ **(5)**

$$I_C = \frac{S}{V} = \frac{13kVA}{200V_{rms}} = 65A_{rms}$$ **(5)**

(b)

$$I_D = \frac{Q}{V} = \frac{5kVAR(L)}{200V_{rms}} = 25A_{rms}$$ **(5)**

$$I = \frac{P}{V} = \frac{12k}{200} = 60A_{rms}$$

$$I(t) = 60\sqrt{2}\cos(2\pi 50t)A$$ **(8)**

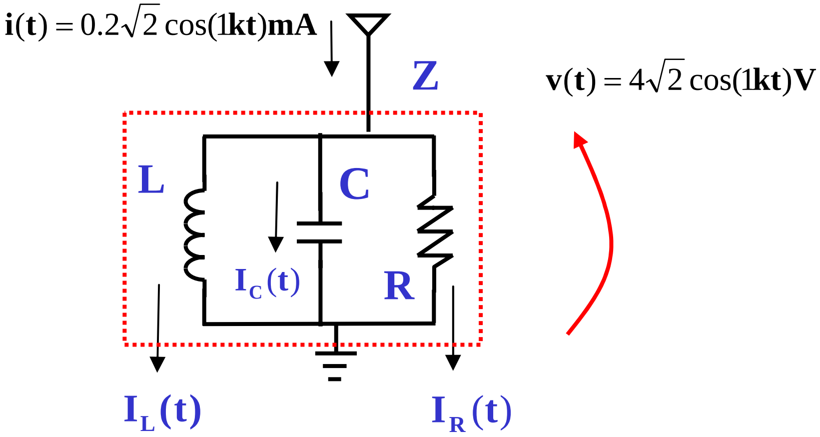
PF = 1, hence I(t) and source are in phase

$I = 60\angle 0o$

(15)

7

In the circuit, L = 0.5H, C = 2μF. (a) Find the resonant frequency of Z. Show that IR(t) = i(t). (b) Find also R, power consumed by Z, quality factor QF, and power stored in C . Given that QF = R/X. (26)



(a)

$$\because \omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.5H * 2\mu F}} = \frac{1}{\sqrt{1*10^{-6}}} = 1krad/s$$

(4)

ω(=1k) = ωo (=1k) , Z is in resonance, Z = R

$$\therefore I_R(t) = i(t) = 0.2\sqrt{2}\cos(1kt)mA$$

(4)

(b)

$$\therefore R = \frac{v}{I_R} = \frac{4V_{rms}}{0.2mA_{rms}} = 20k\Omega$$

(4)

$$\therefore P = I^2R = (0.2mA_{rms})^2 * 20k\Omega = 0.8mW$$

(4)

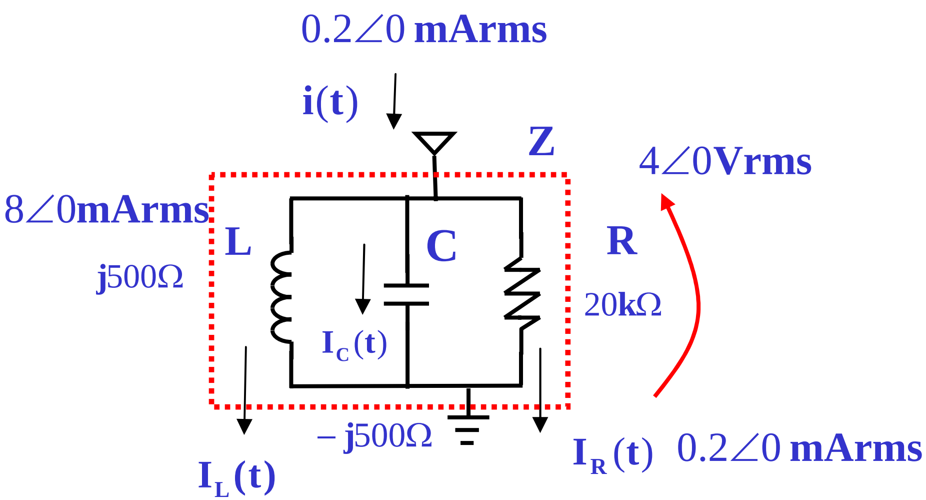
$$\text{or} \quad P = VI = 4V_{rms} * 0.2mA_{rms} = 0.8mW$$

$$QF = \frac{R}{\omega_O L} = \frac{20k\Omega}{1k*0.5H} = 40 = \frac{Q_C}{P}$$ **(4)**

$$\therefore Q_C = P*QF = 0.8mW*40 = 32mVAR(C)$$

(6)

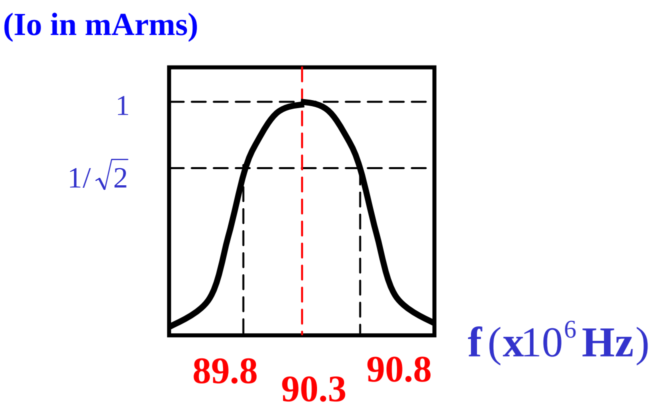
$$\therefore Q_C = VI = 4V_{rms}*0.2mA_{rms}*40$$
$$= 32mVAR(C)$$

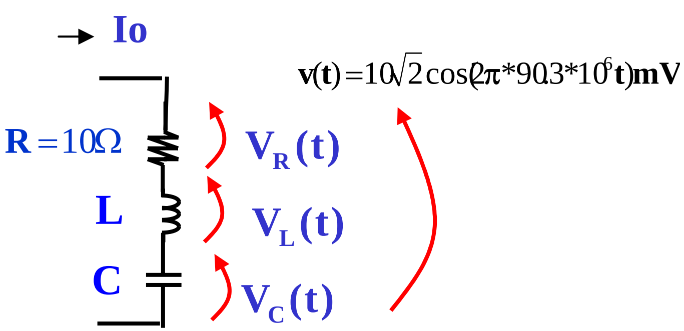


$$\therefore I_C(t) = 0.2*40\sqrt{2}\cos(1kt+90^o)mA$$

(30)

8 A series LCR tuner circuit is used to receive radio stations as shown in the resonance curve. (a) Find in Hz the resonant frequency fo and bandwidth BW. Find also the quality factor QF of the tuner. (b) Find the value of L and find Vc in Vrms. (c) If R is changed from 10Ω **to** 5Ω **(**with L , C and v(t) unchanged), find the new BW and new maximum Io in mArms. Given that QF = X/R = fo/BW. (30)





(a)

$$f_o = 90.3MHz \qquad (2)$$

$$BW = 1MHz \qquad (3)$$

$$QF = \frac{f_o}{BW} = \frac{90.3MHz}{1MHz} = 90.3 \qquad (3)$$

(b)

$$L = \frac{QF * R}{\omega_o} = \frac{90.3 * 10\Omega}{2\pi * 90.3MHz} \cong 1.59\mu H \qquad (5)$$

$$Vc = Io * X_C = Io * QF * R$$
$$= 1mA * 90.3 * 10\Omega = 903mVrms \qquad (7)$$

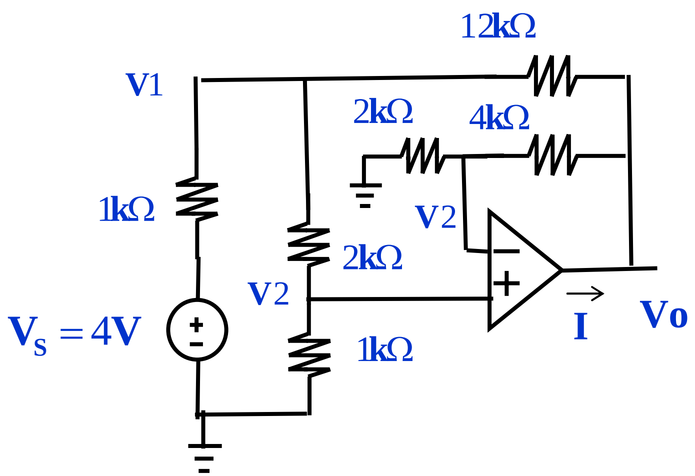
(c)

QF = X/R = <u>180.6</u> = fo/BW

$$\therefore BW = 0.5MHz \qquad (5)$$

$$\max Io = \frac{10V_{rms}}{5\Omega} = 2mA_{rms} \qquad (5)$$

(20)



$$\therefore V2 = \frac{Vo}{2k\Omega + 4k\Omega} * 2k\Omega = \frac{Vo}{3} \qquad \textbf{(4)}$$

$$\therefore V1 = \frac{V2}{1k\Omega} * 3k\Omega = 3V2 = Vo \qquad \textbf{(4)}$$

$$\therefore \frac{Vs - V1}{1k\Omega} = \frac{V1 - V2}{2k\Omega} \qquad \textbf{(4)}$$

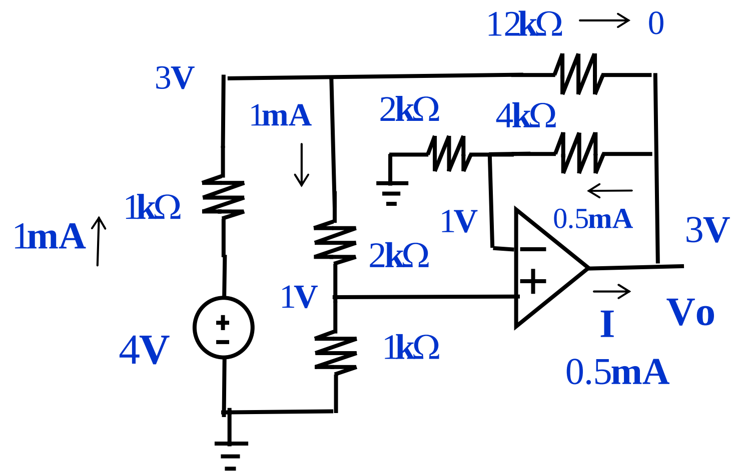
$$\frac{4V - 3V2}{1k\Omega} = \frac{2V2}{2k\Omega}$$

$$\therefore V2 = 1V$$

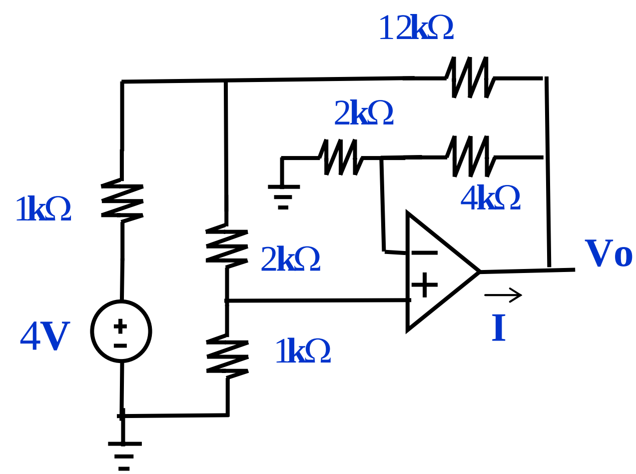
$$\textbf{(5)}$$

$$\therefore Vo = 3V$$

$$I = 0.5mA \qquad \textbf{(3)}$$



9 Find Vo and I. Assume the op amp is ideal. (20)

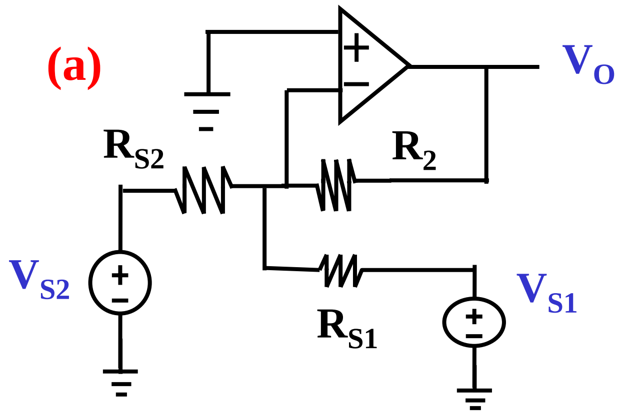


(18)

10

(a) Find the equation relating Vo, VS1, VS2, RS1, RS2, and R2. Assume the op amp is ideal. (9)

(a)

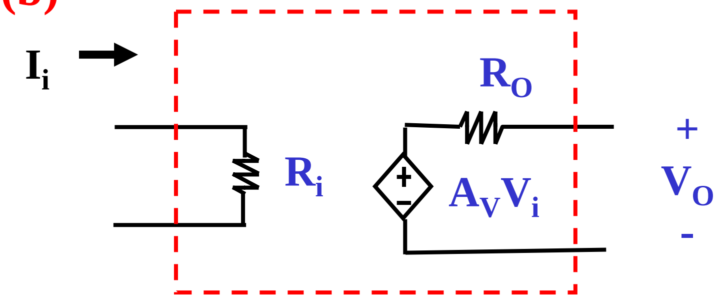


(a)

$$\therefore \mathbf{KCL} \Rightarrow \frac{V_{S1}-0}{R_{S1}} + \frac{V_{S2}-0}{R_{S2}} = \frac{0-V_O}{R_2} \quad \textbf{(7)}$$

$$V_O \cong -\frac{R_2}{R_{S1}} V_{S1} - \frac{R_2}{R_{S2}} V_{S2} \quad \textbf{(2)}$$

(b)



(7)

ideal I to V amplifier, Ri = 0 **(2)**

(b)
Draw the circuit model for a **resistance (I to V) amplifier**.
What is the ideal value of the input resistance of the amplifier ? (9)