(32)

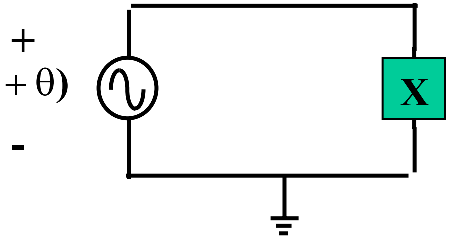
1 In the circuit, X is an unknown element. (32)

(a) If $V_m = 10V$, $I_m = 5A$, $\theta = \phi = 60^o$, find X in Ω, power factor of X, power stored in X, and power absorbed by X.

(b) If $V_m = 8V$, $I_m = 2A$, i(t) leads v(t) by 90^o, find X in Ω, power factor of X, power stored in X, and power absorbed by X.

$\longrightarrow$ $\mathbf{i(t) = I_m \cos(\omega t + \phi)}$

$\mathbf{v(t) = V_m \cos(\omega t + \theta)}$



(a) **V and I in phase**

$$\therefore \mathbf{X = R} = \frac{\mathbf{V_m}}{\mathbf{I_m}} = \frac{10V}{5A} = 2\Omega \qquad \textbf{(4)}$$

$$\mathbf{PF} = 1 \qquad \textbf{(3)}$$

$$\mathbf{P_R = I^2 * R} = (\frac{5A}{\sqrt{2}})^2 * 2\Omega = 25\mathbf{W} \qquad \textbf{(5)}$$

$$\mathbf{Q_R} = 0 \qquad \textbf{(3)}$$

(b) **I leads V**

$$\therefore \mathbf{X = C} = -\mathbf{j}4\Omega \qquad \textbf{(6)}$$

$$\mathbf{PF} = 0 \qquad \textbf{(3)}$$

$$\mathbf{Q_C = I^2 * X} = (\frac{2A}{\sqrt{2}})^2 * 4\Omega = 8\mathbf{VAR(C)} \qquad \textbf{(5)}$$

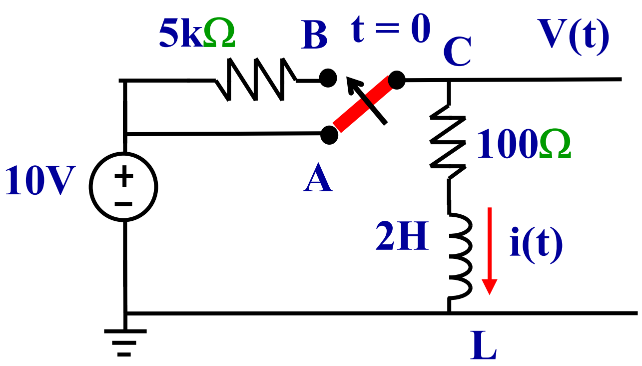
$$\mathbf{P_C} = 0\mathbf{W} \qquad \textbf{(3)}$$

(33)

2

The circuit is at steady state for t < 0. At t = 0, the switch is **switched from A to B** (i.e. BC is shorted). (a) Find i(t) for $t \geq 0$. (b) Find the maximum energy stored in L. (c) Plot V(t) for t < 0 and $t \geq 0$. Label clearly the voltage and time. (33)

Given that $\mathbf{i(t) = i(\infty) + [i(0) - i(\infty)] * e^{-t/\tau}}$ **and** $\boldsymbol{\tau = L/R}$



(a)

$$\therefore \mathbf{i(t=0) = i(t<0) = 0.1A}$$ (4)

$$\therefore \mathbf{i(\infty) = \frac{10V}{5100\Omega} \cong 2mA}$$ (4)

$$\therefore \boldsymbol{\tau = \frac{L}{R} = \frac{2H}{5100\Omega} \cong 0.4ms}$$ (4)

$$\therefore \mathbf{i(t) = i(\infty) + [i(0) - i(\infty)] * e^{-\frac{t}{\tau}}}$$

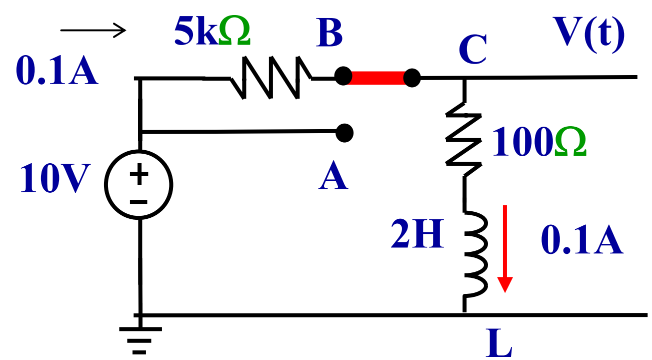
$$\mathbf{\cong 2mA + [100mA - 2mA] * e^{-t/0.4ms}}$$ (3)

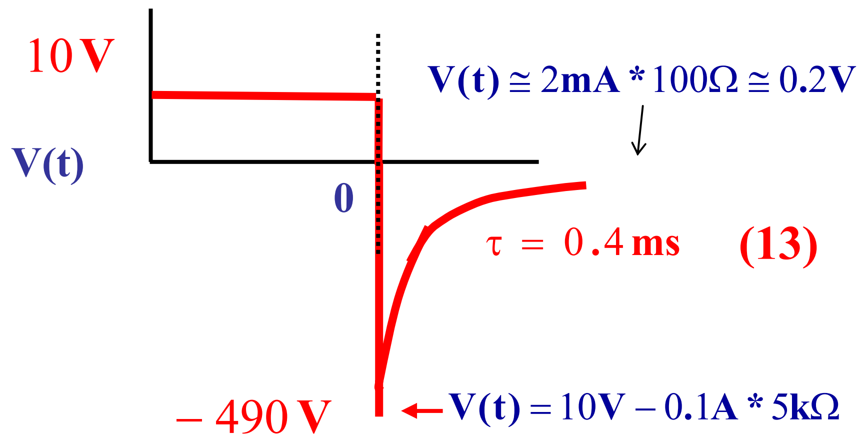
(b)

$$\therefore \mathbf{E_L = \frac{1}{2} L i^2 = \frac{1}{2}(2H)(0.1A)^2 = 10\,mJ}$$

(5)

(c)



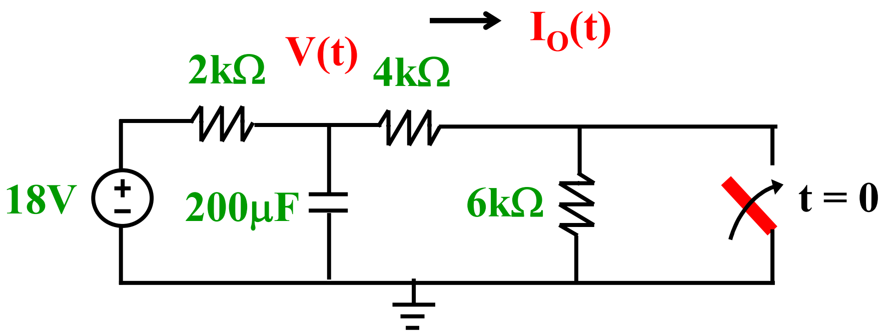


(13)

3

The circuit is at steady state for t < 0. At t = 0, the switch is closed. (a) Find V(0), V(∞) and time constant τ for t > 0. (b) Plot $I_O(t)$ for t < 0 and t ≥ 0. (25)

Given that $\mathbf{V(t) = V(\infty) + [V(0) - V(\infty)] * e^{-t/\tau}}$ **and** $\boldsymbol{\tau = CR}$



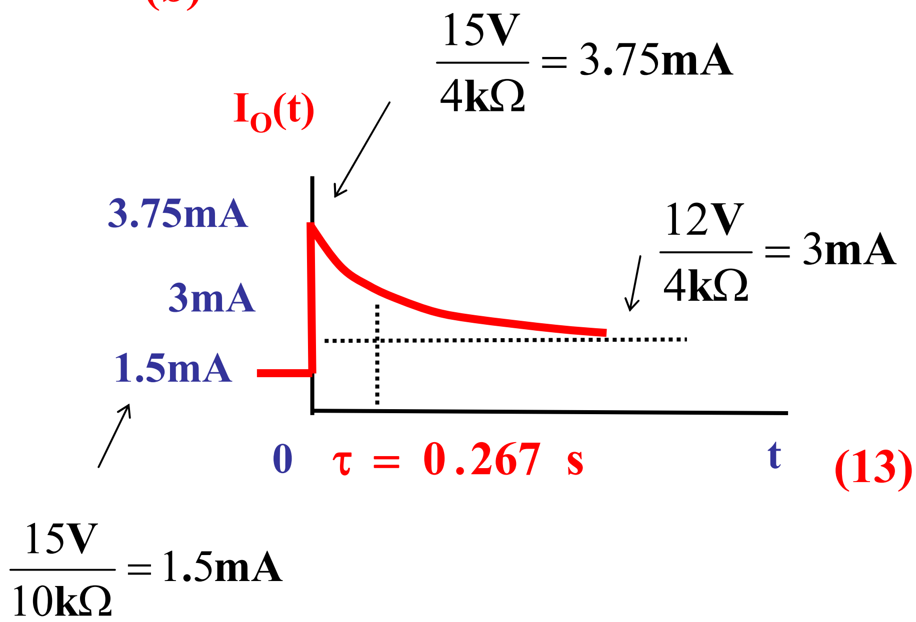
(a)

$$\therefore\ V(0) = V(0-) = 18V\left(\frac{10k\Omega}{12k\Omega}\right) = 15V \qquad (4)$$

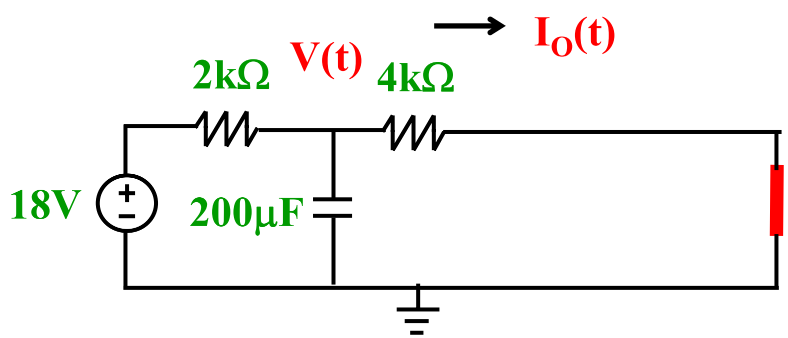
$$\therefore \tau = CR = 200\mu F * (2k\Omega // 4k\Omega) = 0.267s \qquad (4)$$

$$\therefore V(\infty) = 18V * \frac{4k\Omega}{6k\Omega} = 12V \qquad (4)$$

(b)



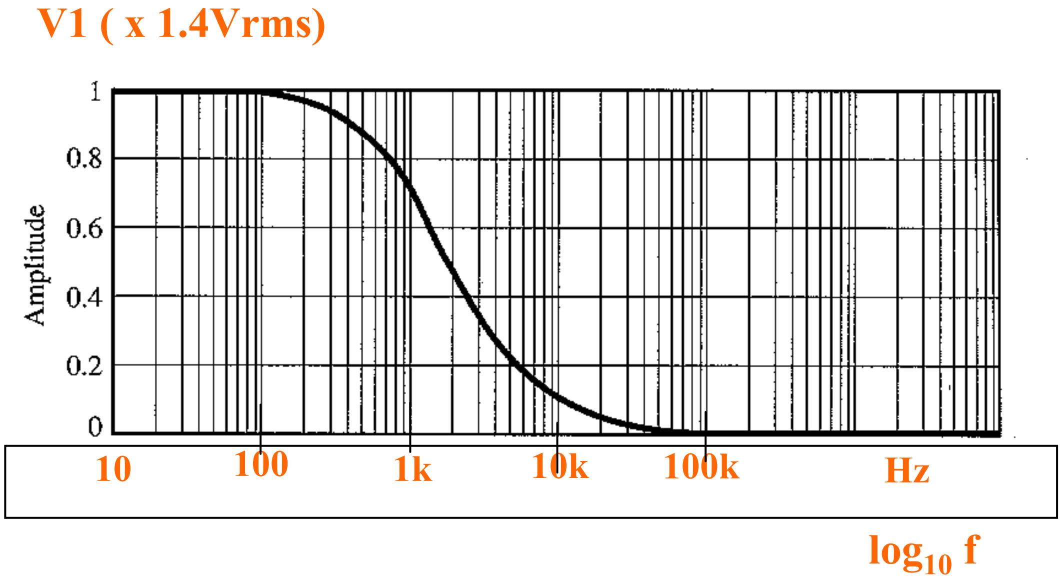
(13)

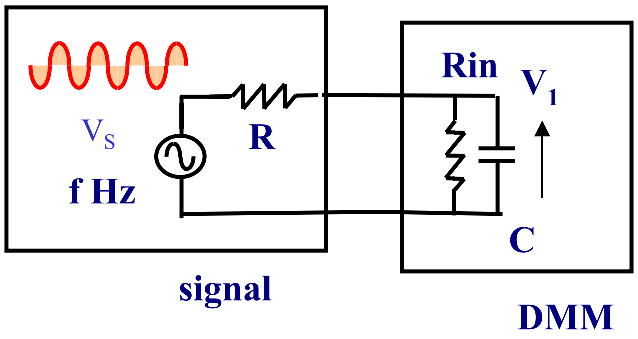


(18)

4

In the circuit, V_1 versus log-frequency f is given as shown.
Find roughly the magnitude of V_S in V, bandwidth of DMM in Hz, and V_1 at bandwidth in Vrms.
Assume Rin >> R. (18)





magnitude Vm ≈ 2V (6)

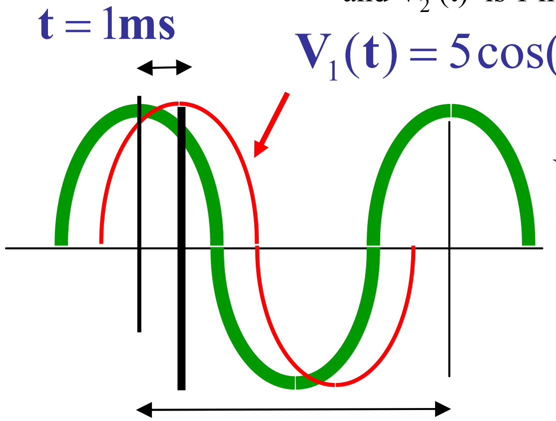
bandwidth ≈ 1kHz (6)

At bandwidth, V1 ≈ 0.7 x 1.4 Vrms (6)

(25)

5

Find ω and θ. Show that V_2 (t) is roughly - 3.5V when t = 2ms. Does V_2 (t) lag V_1 (t) ? Given 2π radian = 360°, period of V_2 (t) is 8ms, and difference between the peaks of V_1 (t) and V_2 (t) is 1 ms. (25)



$$\omega = \frac{2\pi}{T} = \frac{2\pi}{8ms}$$ (5)

$$\theta = \frac{t}{T} * 360^o = \frac{1ms}{8ms} * 360^o = 45^o$$ (6)

$$\therefore V_2(t) = 5\cos\left(\frac{2\pi}{8ms} t + 45^o\right)V$$ (2)

$$\therefore V_2(t) = 5\cos\left(\frac{2\pi}{8ms} * 2ms + 45^o\right)V$$

$$= 5\cos\left(\frac{\pi}{2} + 45^o\right)V = 5\cos(90^o + 45^o)V$$

$$\cong -3.54V$$

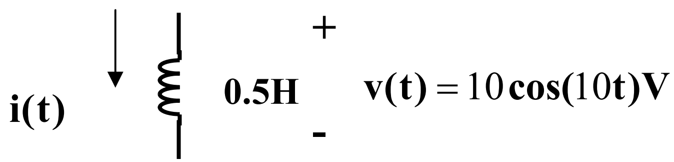
(9)

$V_2(t)$ leads $V_1(t)$ (3)

6

(a) . Find i(t). (12)

(a)



$$\mathbf{i(t)} = \left(\frac{\mathbf{V_m}}{\boldsymbol{\omega}\mathbf{L}}\right)\cos(10\mathbf{t} - 90^{\circ})\mathbf{A}$$

$$= \left(\frac{10\mathbf{V}}{10\mathbf{rad/s} * 0.5\mathbf{H}}\right)\cos(10\mathbf{t} - 90^{\circ})$$

$$= 2\mathbf{A}\cos(10\mathbf{t} - 90^{\circ})$$

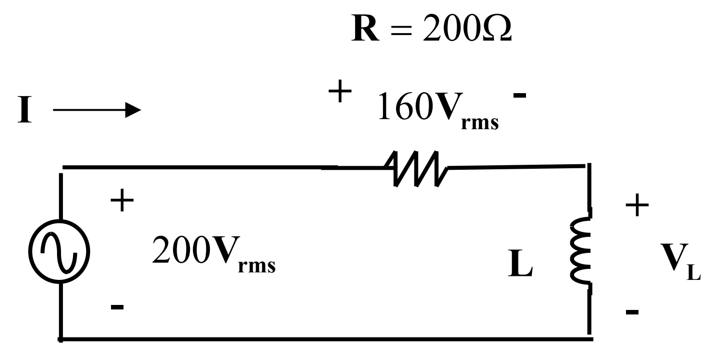
(12)

or $\therefore \mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{10\angle 0^{\circ}\mathbf{V}}{\mathbf{j}\omega\mathbf{L}} = \frac{10\angle 0^{\circ}\mathbf{V}}{\mathbf{j} * 10\mathbf{rad/s} * 0.5\mathbf{H}}$

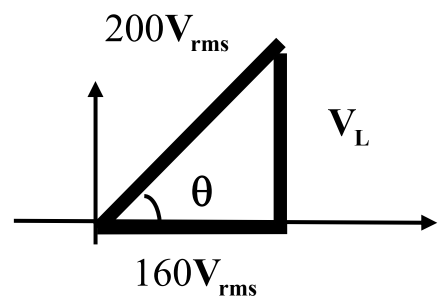
$$= \frac{10\angle 0^{\circ}\mathbf{V}}{5\mathbf{j}\Omega} = 2\angle -90^{\circ}\mathbf{A}$$

(b)

(b). Find V_L in V_{rms} and I in A_{rms}. (15)



$$200\mathbf{V_{rms}}\angle\theta = \mathbf{I}(\mathbf{R} + \mathbf{j}\omega\mathbf{L}) = 160\mathbf{V_{rms}} + \mathbf{j}\mathbf{V_L}$$



$$\mathbf{V_L} = \sqrt{200\mathbf{V_{rms}}^2 - 160\mathbf{V_{rms}}^2} = 120\mathbf{V_{rms}}$$

(10)

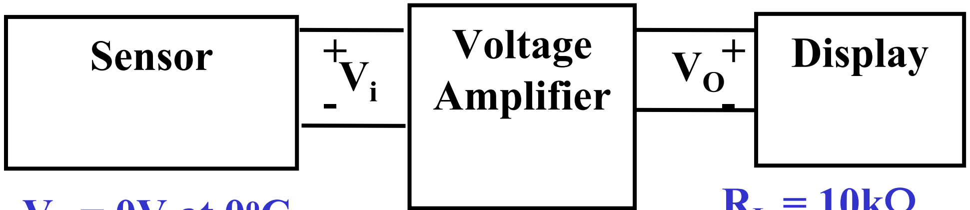
$$\mathbf{I} = \frac{160\mathbf{V_{rms}}}{200\Omega} = 0.8\mathbf{A_{rms}}$$

(5)

(22)

7

7. A voltage amplifier is used to amplify the sensor signal (0 to 100mV) to drive the display (0 to 2V) as shown. Draw the **circuit model** and then find the voltage gain of the voltage amplifier. (22).



$V_S = 0V$ at 0^oC $+1mV/^oC$

$R_S = 300\Omega$

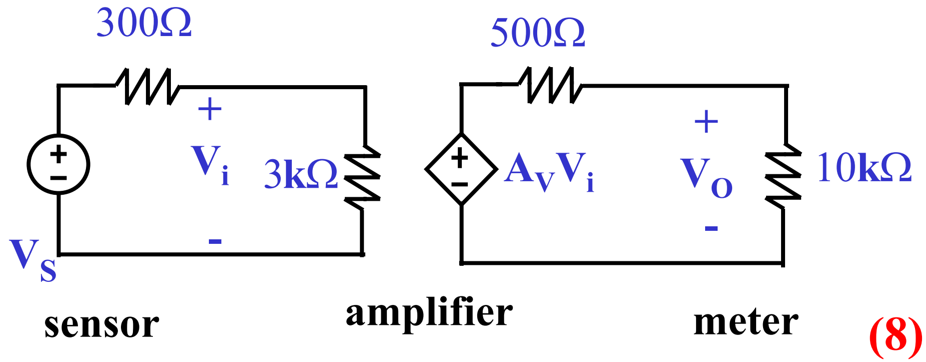
$R_i = 3k\Omega$

$R_O = 500\Omega$

$R_L = 10k\Omega$

$V_O = 2V$ when $V_S = 100mV$

Circuit Model



(8)

Use voltage divider

$$V_O = A_V * V_i * \frac{10k\Omega}{10k\Omega + 500\Omega}$$

$$= A_V * V_S * \frac{3k\Omega}{3k\Omega + 300\Omega} * \frac{10k\Omega}{10k\Omega + 500\Omega}$$

(8)

$$2V = A_V * 100mV * \frac{3k\Omega}{3.3k\Omega} * \frac{10k\Omega}{10.5k\Omega}$$

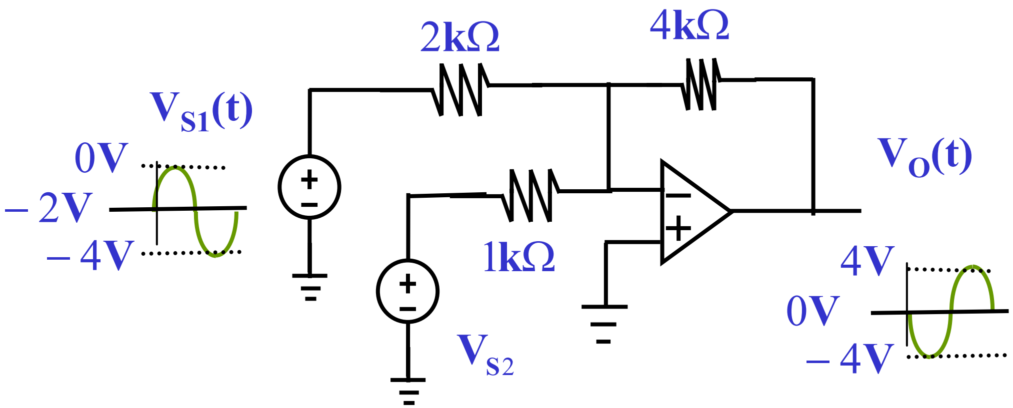
$$\therefore A_V = 23.1$$

(6)

(30)

8

(a). Find the value of V_{S2} . Assume op amp is ideal. (18)



(a)

$$V_O(t) = -\frac{4k\Omega}{2k\Omega}V_{S1}(t) - \frac{4k\Omega}{1k\Omega}V_{S2}$$ (7)

$$= -2V_{S1}(t) - 4V_{S2}$$ (2)

$$4V = -2 * (-4V) - 4V_{S2}$$

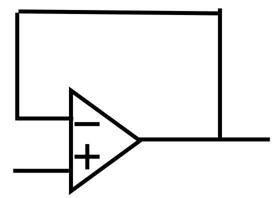
$$\therefore V_{S2} = 1V$$ (9)

$$0V = -2 * (-2V) - 4V_{S2}$$
$$\therefore V_{S2} = 1V$$

$$-4V = -2 * (0V) - 4V_{S2}$$
$$\therefore V_{S2} = 1V$$

(b) Draw the circuit of an op amp **voltage follower** and name two advantages of the circuit. (12)

(b)



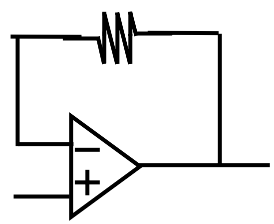
voltage follower

(6)

1. Very high input resistance
2. Very low output resistance

(6)

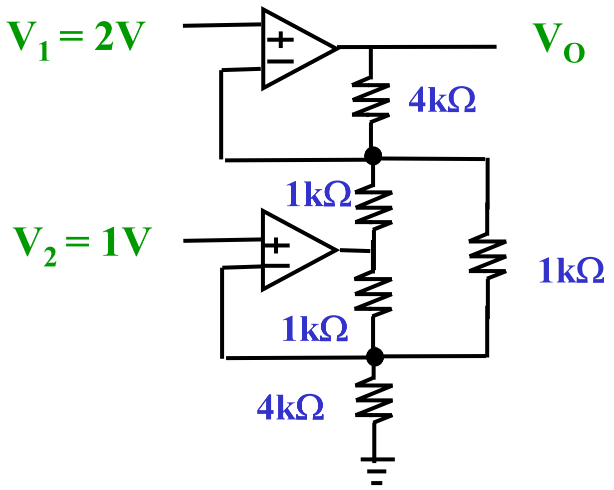
or

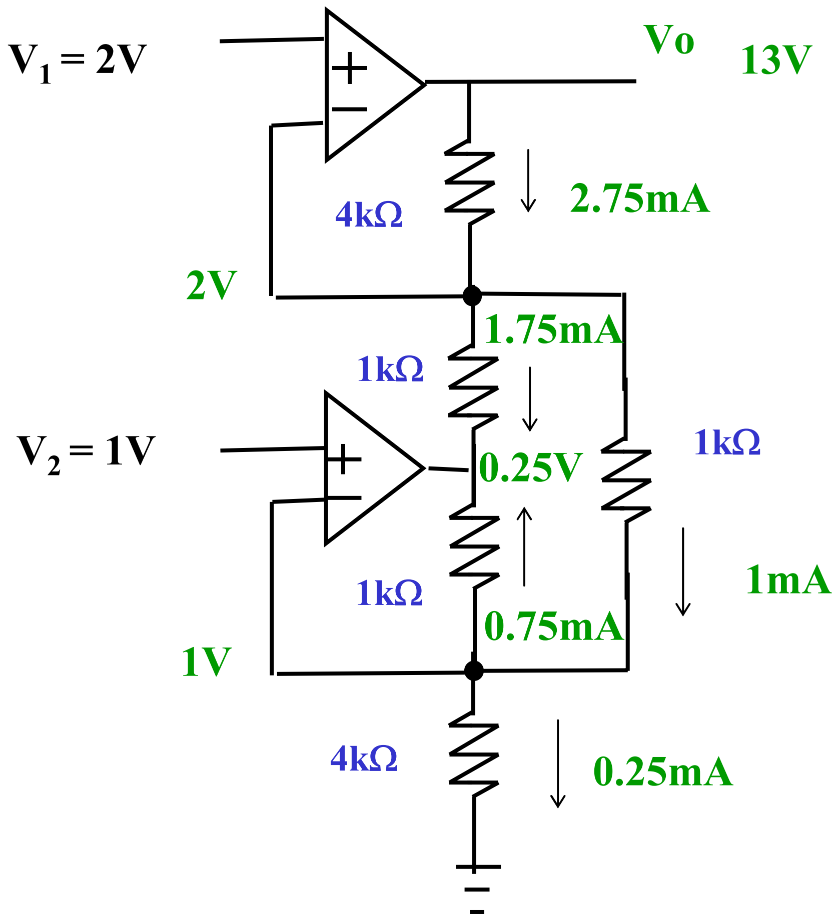


(30)

9

Find V_O. Show clearly all your steps. Assume op amp is ideal. (30)



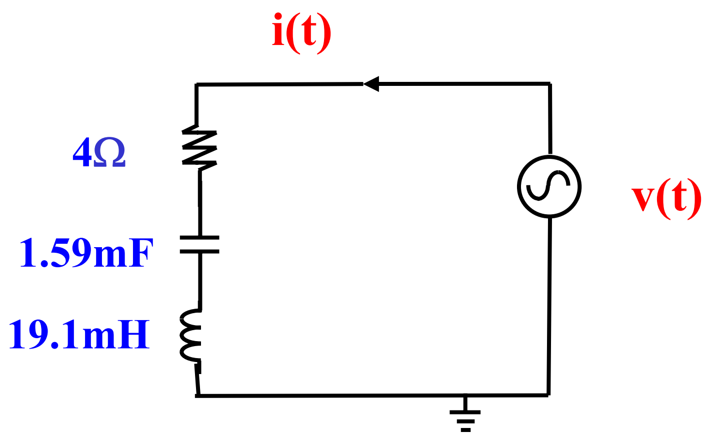


$V_O = 13V$ **(30)**

10

In the circuit, $\mathbf{v(t)} = 240\sqrt{2}\cos(2\pi 50\mathbf{t})\mathbf{V}$.
Find the frequency and period of v(t) , and the V phasor of v(t). Show that $\mathbf{i(t)} \cong 60\cos(2\pi 50\mathbf{t} - 45^{\mathbf{o}})\mathbf{A}$.

Plot also phasor V and I in a <u>phasor diagram</u>. (33)



Find frequency f of v(t)

$$\mathbf{f} = 50\,\mathbf{Hz}$$ (2)

Find period of v(t)

$$\mathbf{T} = \frac{1}{\mathbf{f}} = \frac{1}{50\,\mathbf{Hz}} = 20\,\mathbf{ms}$$ (3)

Find V phasor

$$\mathbf{V} = 240\sqrt{2}\angle 0^{\mathbf{o}}\,\mathbf{V}$$ (2)

Find impedance Z

$$\frac{1}{j\omega C} = \frac{1}{j(2\pi 50)(1.59\,mF)} \cong -2j\Omega \qquad \textbf{(3)}$$

$$j\omega L = j * 2\pi 50 * 19.1mH \cong 6j\Omega \qquad \textbf{(3)}$$

$$Z = R + j\omega L + \frac{1}{j\omega C} = 4 + j6 - j2\Omega$$

$$= 4 + j4\,\Omega$$

$$= 4\sqrt{2}\angle 45^o\,\Omega \qquad \textbf{(6)}$$

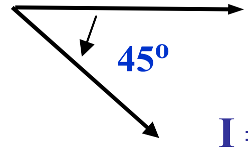
Find I

$$\therefore I = \frac{V}{Z} = \frac{240\sqrt{2}\angle 0^o\,V}{4\sqrt{2}\angle 45^o\,\Omega} \qquad \textbf{(4)}$$

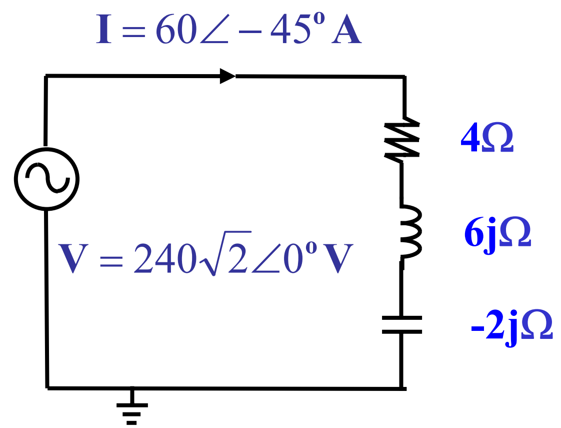
$$= 60\angle -45^o\,A$$

$$\therefore i(t) = 60\cos(2\pi 50t - 45^o)\,A \qquad \textbf{(4)}$$

$$V = 240\sqrt{2}\angle 0^o\,V \qquad \textbf{(6)}$$



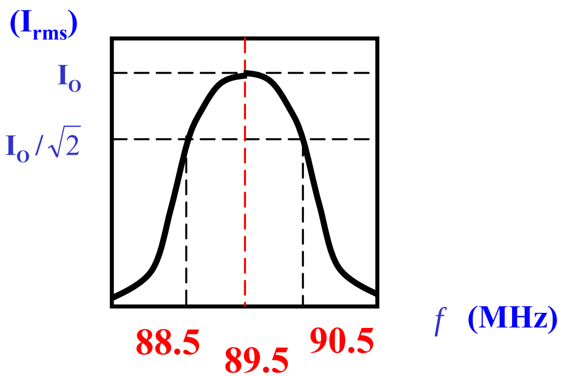
$$I = 60\angle -45^o\,A$$

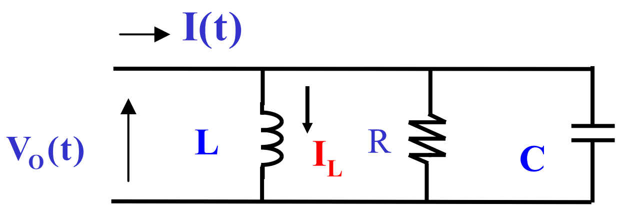


(32)

11

A parallel LCR tuner circuit is used to receive radio stations as shown in the resonance curve.
(a) Find in Hz the resonant frequency (fo), bandwidth (BW), upper and lower frequencies of the tuner. Show also the quality factor (QF) is 44.75. (b) If maximum Vo of the tuner is 1Vrms and maximum current in the inductor I_L is 4.475mArms, show that R is 10kΩ and L is about 0.4×10^{-6} H. Find also the value of C of the tuner circuit. Given that QF = fo/BW and QF = R/ωoL. (32)





(a)

$f_o = 89.5\text{MHz}$ **(2)**

$BW = 2\text{MHz}$ **(2)**

$f_{uppper} = 90.5\text{MHz}$ **(2)**

$f_{lower} = 88.5\text{MHz}$ **(2)**

$$QF = \frac{f_o}{BW} = \frac{89.5\text{MHz}}{2\text{MHz}} = 44.75 \quad \textbf{(4)}$$

(b)

$$\therefore I = \frac{I_L}{QF} = \frac{4.475\text{mA}}{44.75} = 0.1\text{mA} \quad \textbf{(5)}$$

$$\therefore R = \frac{V_o}{I} = \frac{1V_{rms}}{0.1\text{mA}_{rms}} = 10\text{k}\Omega \quad \textbf{(5)}$$

$$\therefore L = \frac{R}{QF * \omega_o} = \frac{10\text{k}\Omega}{44.75 * 89.5\text{MHz} * 2\pi} \cong 0.397\mu\text{H} \quad \textbf{(5)}$$

$$\therefore C = \frac{1}{\omega_o{}^2 L} \cong \frac{1}{(2\pi * 89.5\text{M})^2 * 0.397\mu\text{H}} \cong 8\text{pF} \quad \textbf{(5)}$$

(27)

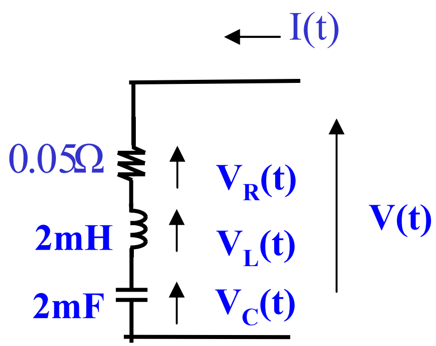
12

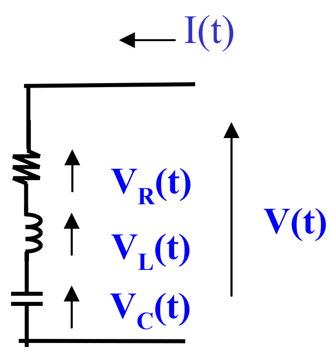
The series LCR circuit is in resonance. If $\mathbf{V_R(t)} \cong 0.28\cos(500\mathbf{t})\mathbf{V}$

Show that the quality factor (QF) is 20. Show also

$\mathbf{V}(\mathbf{t}) = 0.2\sqrt{2}\cos(500\mathbf{t})\mathbf{V}$

Find also V_L in Vrms and I in Arms. Given that QF = ωoL/R . (27)





$$j\omega_O L = j*500\,rad/s*2mH = j\Omega$$

$$\frac{1}{j\omega_O C} = \frac{1}{j*500\,rad/s*2mF} = -j\Omega$$

$$\therefore \omega_O = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2mH*2mF}} = 500\,rad/s \qquad (5)$$

$$\therefore QF = \frac{\omega_O L}{R} = \frac{500\,rad/s*2mH}{0.05\Omega} = 20 \qquad (5)$$

Circuit is in resonance

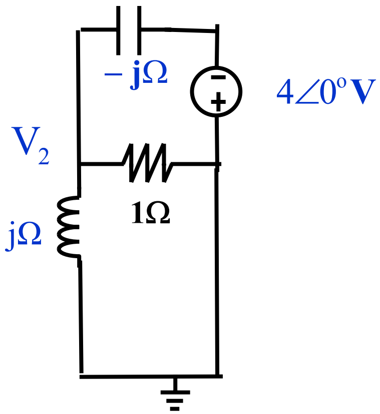
$$\therefore \mathbf{V_R(t) = V(t)}$$

$$\therefore \mathbf{V(t)} = 0.2\sqrt{2}\cos(500\mathbf{t})\ \ \mathbf{V} \qquad (5)$$

$$\therefore \mathbf{V_L} = 20*0.2\ \mathbf{V_{rms}} \qquad (6)$$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{R}} = \frac{0.2\mathbf{V_{rms}}}{0.05\Omega} = 4\mathbf{A_{rms}} \qquad (6)$$

13 In the circuit, find V_2 . (17)

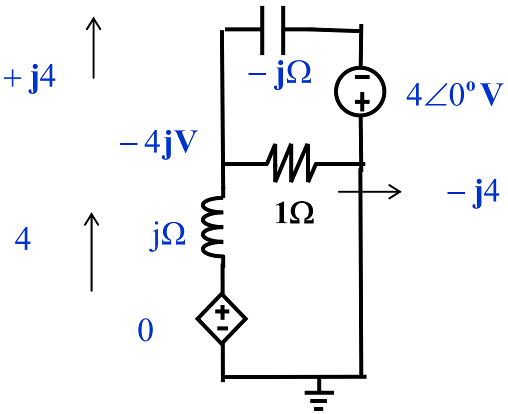


$$\frac{0-\mathbf{V}_2}{\mathbf{j}\Omega}=\frac{\mathbf{V}_2}{1\Omega}+\frac{\mathbf{V}_2+4}{-\mathbf{j}\Omega}$$ **(9)**

$$-\mathbf{V}_2=\mathbf{j}\mathbf{V}_2-(\mathbf{V}_2+4)$$ **(8)**

$$\mathbf{V}_2=-\mathbf{j}4\mathbf{V}$$

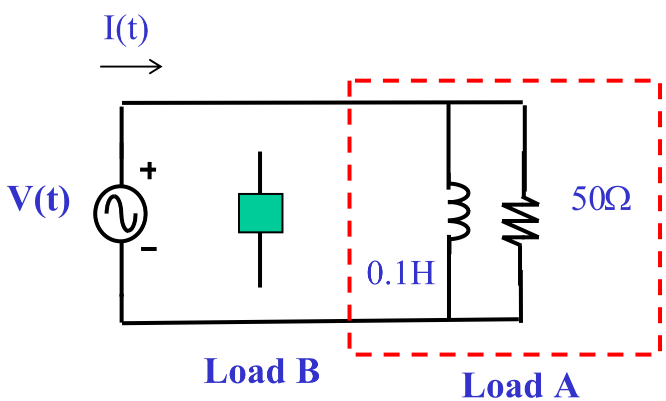
$$\frac{-j4+4}{-j}=4+j4$$



14

Load A is connected to V(t) as shown. Given $\mathbf{V(t)} = 200\sqrt{2}\,\mathbf{cos}(2\pi 50\mathbf{t})\mathbf{V}$
(a) Show that the power absorbed by load A is 800W. Find also the power stored by load A.
(b) Show that the power supplied (S) by V(t) is roughly 1504VA . Find also the power factor of load A. (25)

(c) Show that I is about 7.5Arms.
(d) If load B is connected in parallel to load A such that the power factor (PF) of the combined load is 1, find the element and value of load B. Show also the new I is about 4 Arms.
(e) Will electric fees be reduced if the power factor is improved to 1 ? Why ? Name also two advantages of improving the power factor. (33) Given that S = VI. and P = S * PF



(a)

power absorbed by load A = P **(2)**

$$\mathbf{P} = \frac{\mathbf{V}^2}{\mathbf{R}} = \frac{(200\mathbf{V_{rms}})^2}{50\Omega} = 800\mathbf{W} \quad \mathbf{(4)}$$

power stored by load A = Q **(2)**

$$\mathbf{Q} = \frac{\mathbf{V}^2}{\boldsymbol{\omega}\mathbf{L}} = \frac{(200\mathbf{V_{rms}})^2}{2\boldsymbol{\pi}(50\mathbf{Hz})0.1\mathbf{H}} \cong 1273.2\mathbf{VAR(L)} \quad \mathbf{(4)}$$

(b)

power supplied by v(t) = S **(2)**

$$\therefore \mathbf{S} = \sqrt{\mathbf{P}^2 + \mathbf{Q}^2} \cong \sqrt{800^2 + 1273.2^2} \cong 1503.7\mathbf{VA} \quad \mathbf{(5)}$$

power factor of load A = PF

$$\therefore \mathbf{PF} = \frac{\mathbf{P}}{\mathbf{S}} = \frac{800}{1503.7} \cong 0.532 \textbf{ lagging}$$ **(6)**

(c)

$$\therefore \theta \cong \cos^{-1} 0.532 \cong 57.9^{\mathrm{o}}$$

$$\mathbf{I} = \frac{\mathbf{S}}{\mathbf{V}} = \frac{1503.7\mathbf{VA}}{200\mathbf{V}_{\mathbf{rms}}} \cong 7.52\mathbf{A}_{\mathbf{rms}}$$ **(6)**

(d)

load B is capacitance **(2)**

$$\mathbf{Q} = \mathbf{V}^2 \omega \mathbf{C}$$

$$\therefore \mathbf{C} = \frac{\mathbf{Q}}{\mathbf{V}^2 \omega} = \frac{1273.2\mathbf{VAR}}{200^2(2\pi 50)} \cong 101.4\mu\mathbf{F}$$ **(7)**

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{R}} = \frac{200\mathbf{V}_{\mathbf{rms}}}{50\Omega} = 4\mathbf{A}_{\mathbf{rms}}$$ **(6)**

(e) **Electric fees is related to P only**
P is unchanged hence fees
will not be reduced **(6)**

Any two : **Lower line current from source**
Lower line loss
Source can supply more load **(6)**

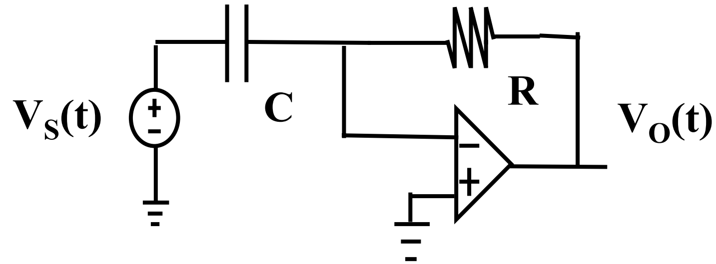
(27)

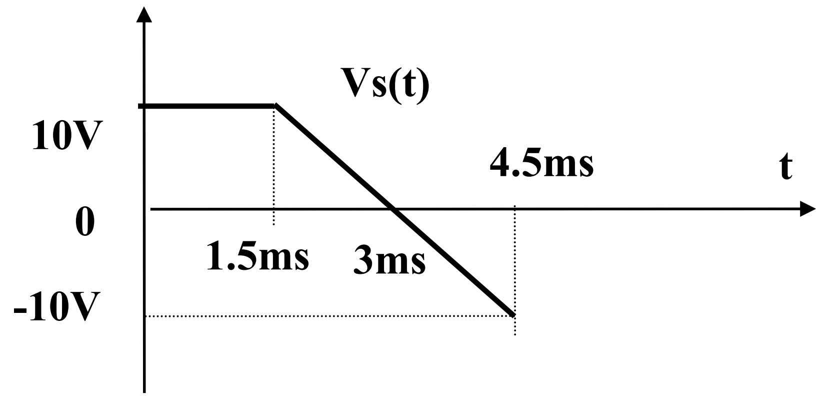
15

15. (a) Show that the ideal op amp circuit acts as a differentiator, that is

$$V_O(t) = -CR\frac{dV_S(t)}{dt}$$

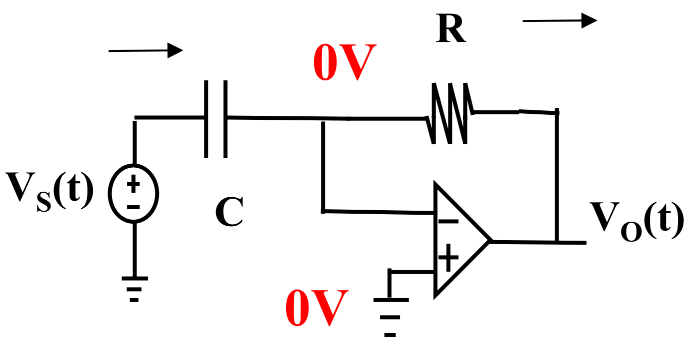
(b) Plot the output voltage for the circuit, $v_O(t)$, as a function of time. Label clearly the voltage and time. Given R = 10kΩ, C = 0.1μF, and op amp is ideal. (27)





(a)

$$C\frac{d(V_S - 0)}{dt} \qquad \frac{0 - V_O}{R}$$



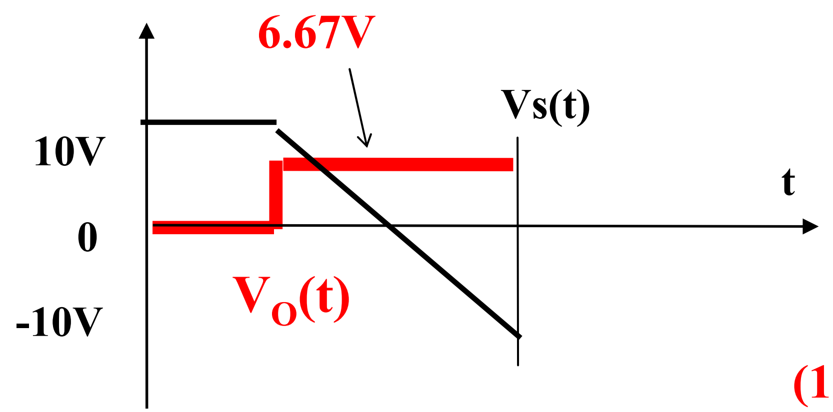
(9)

$$\text{apply KCL} \Rightarrow C\frac{dV_S}{dt} \cong \frac{-V_O}{R} \qquad \therefore V_O \cong -CR\frac{dV_S}{dt}$$

(b)

$$CR = 0.1\mu F * 10k\Omega = 1ms$$

$$\therefore V_O \cong -CR\frac{dV_S}{dt} = -1ms * \frac{-10V - 10V}{3ms} \cong 6.67V$$



(18)