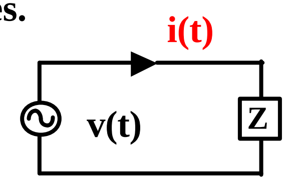
Revision tutorial p.1

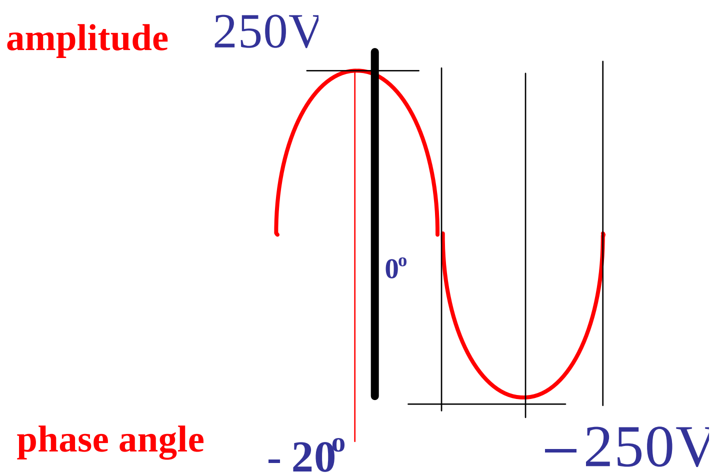
1. In the following circuit, v(t) = 250 cos (100t + 20°) V, i(t) = 10 cos (100t + 40°) A.
(a) Sketch v(t) and i(t) together. Show clearly the phase angles and amplitudes. Does v(t) lead i(t) ?
(b) If Z is two elements in series, find the two elements and the values.
(c) Sketch impedance Z and phasors V, I in a complex plane. (23)



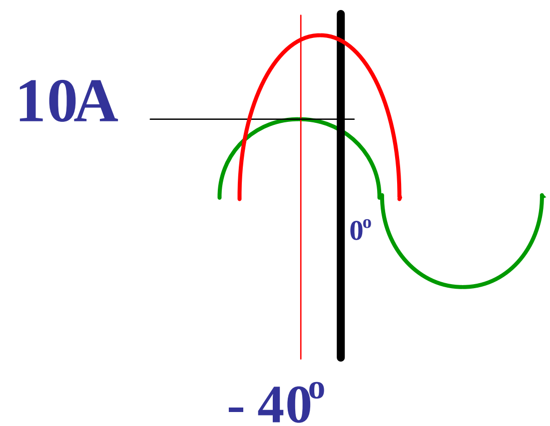
(a) Sketch v(t) and i(t) together.

Show clearly the phase angles and amplitudes.

v(t) = 250 cos (100t + 20°) V



i(t) = 10 cos (100t + 40°) A.



i(t) leads v(t) by 20°

As i(t) peak (10A) is at left side of v(t) peak (250V)

(if at right side, then i lags v)

(b) If Z is two elements in series, find the two elements and the values.

Find Z first, use complex method

Transform v(t) into V phasor

$$v(t) = 250\cos(100t + 20^o)\ V$$

$$\Rightarrow \quad V = 250 \angle 20^o\ V$$

Frequency NOT included in phasor since All current and voltage will have SAME frequency

$$\therefore Z = \frac{V}{I} = \frac{250\angle 20^o\ V}{10\angle 40^o\ A}$$

$$= \frac{250V}{10A}\angle(20^o - 40^o)$$

$$= 25\angle -20^o\ ohm$$

Using Euler equation

$$\therefore Z = 25\ [\cos(-20^o) + j\sin(-20^o)]\ ohm$$

$$= 23.5 - j8.55\ ohm$$

Hence Z is C in series with R

$$\therefore Z = R + \frac{1}{jC(2*pi*f)} = R - \frac{j}{(2*pi*f)C}$$

$$\frac{1}{j} = -j \qquad jj = -1$$

Equate real and j part

$$\therefore R = 23.5\ ohm$$

$$\therefore 8.55 = \frac{1}{(2*pi*f)C}$$

$$\therefore C = \frac{1}{100(8.55)} = 1.17mF$$

Hence Z is 1.17mF in series with 23.5 ohm

(similarly if $Z = 25 \angle 20^o$ ohm

then Z is 23.5 ohm in series with 8.55 ohm L)

by complex method

Transform L into j w L

Transform C into 1 / j w C

For example

If L = 10mH ⇒ Then X_L = j w L

= j (100) 10mH

= j ohm

C = 1mF ⇒ Then X_C = 1 / j w C

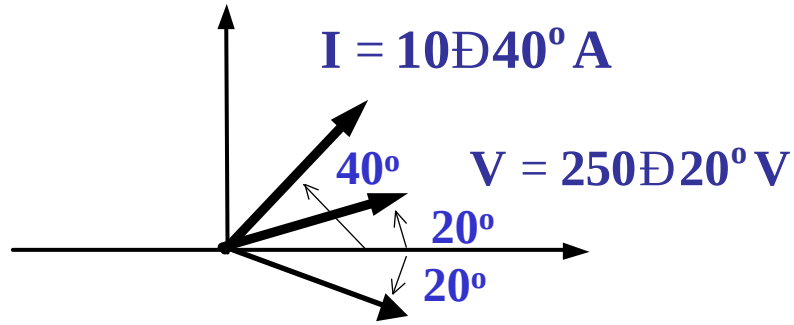
= 1 / j (100) 1mF

= - j10 ohm

X_L : reactance of L

X_C : reactance of C

Sketch impedance Z and phasors V, I in a complex plane or phasor diagram



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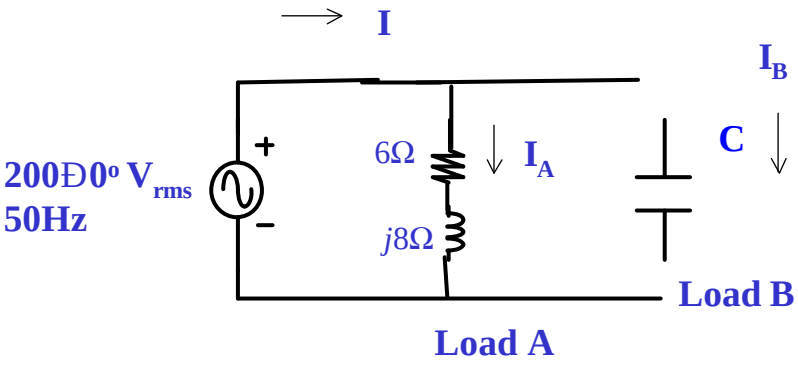
2. An industrial load A is connected in parallel to a 200 Vrms, 50Hz power supply. Load A is composed of 6Ω in series with j8 Ω. (a) Find the complex current flowing in load A I_A. Find also the average power P, reactive power Q, apparent power S, and power factor PF supplied by the power supply. (b) If a load B is connected in parallel to load A to make the total power factor = 1, find the element and value of load B. (c) When PF = 1, find the current delivered by the 200V supply (I_S , in rms) and the current in load B (I_B , in rms). (d) Hence plot I_S, I_B , and I_A in a phasor diagram. **[40]**



Find the complex current flowing in load A I_A

For load A

$$\therefore I_A = \frac{V}{Z}$$

$$= \frac{200\angle 0^o\, V_{rms}}{6 + j8\ ohm}$$

$$= \frac{200\angle 0^o\, V_{rms}}{\sqrt{6^2+8^2}\angle \tan^{-1} 8/6}$$

$$= \frac{200\angle 0^o\, V_{rms}}{10\angle 53.1^o\, ohm}$$

$$= 20\angle -53.1^o\, A_{rms}$$

Find also the average power P, reactive power Q, apparent power S, and power factor PF of load A supplied by the power supply.

Load A is R in series with L, same I, use $P = I^2R$, I in rms

If parallel load, same V, use $P = V^2/R$, V in rms

$$\therefore P = I^2R = 20^2 * 6 = 2400W$$

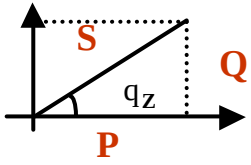
$$\therefore Q = I^2\omega L = 20^2 * 8 = 3200VAR$$

$$\therefore S = \sqrt{P^2 + Q^2} = 4000VA$$

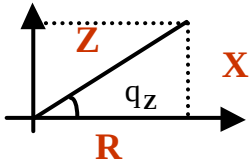
$$\therefore PF = \cos\theta_z = \cos 53.1^o = 0.6 \quad \text{lagging}$$

In L, I lags V

Can remember equations using power triangle



power triangle is similar to impedance triangle



If a load B is connected in parallel to load A to make the total power factor = 1, find the element and value of load B.

Q is 3200 VAR, Q is QL

Hence QC of 3200 VAR is required to cancel the QL (such that NET Q = 0)

$$\because Q_C = \frac{V^2}{1/Cw}$$

$$\therefore C = \frac{Q_C}{V^2 w} = \frac{3200}{200^2 (2pi50)} = 0.255mF$$

Hence load B is a 0.255 mF capacitor

When PF = 1, find the current delivered by the 200V supply (I_S , in rms) and the current in load B (I_B , in rms).

$$\therefore I_B = \frac{V}{1/Cw} = VCw = 200(2pi50 * 0.255m) = 16A_{rms}$$

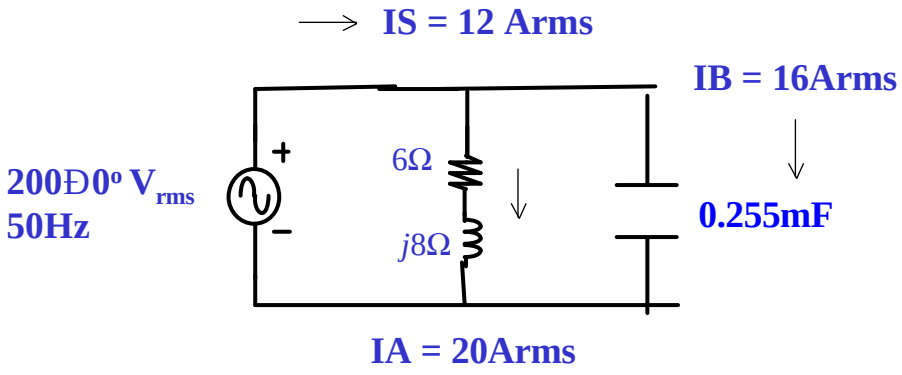
Hence current in load B is 16 A rms

When PF = 1, net Q of load A and load B = 0

S = P of combined load = 2400VA

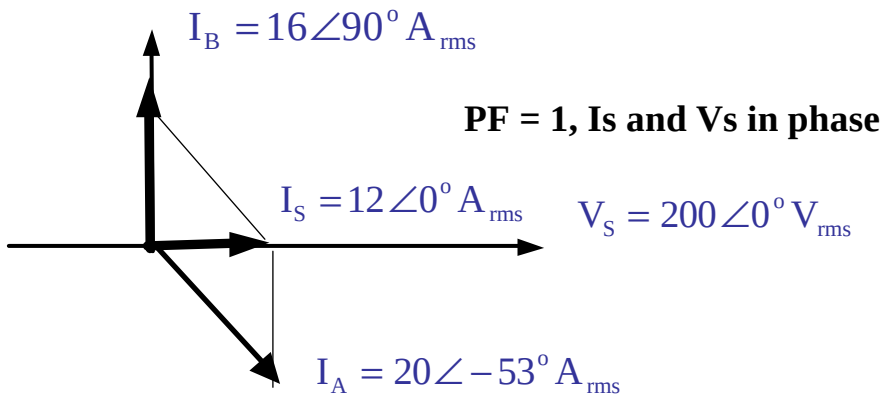
$$\therefore I_S = \frac{S}{V} = \frac{2400VA}{200V} = 12A_{rms}$$

Hence source current is 12 A rms



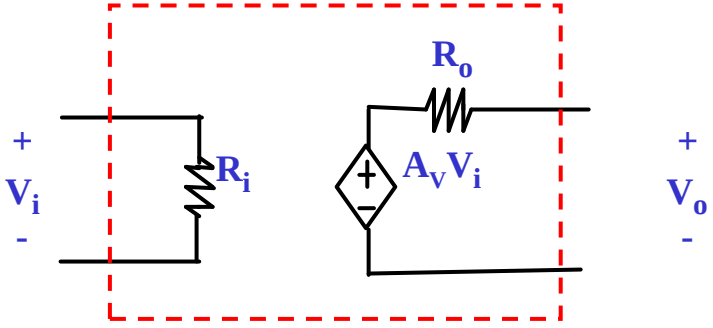
Hence plot I_S, I_B , and I_A in a <u>phasor diagram</u>

In C, I leads V by 90º



5. (a) Sketch the equivalent circuit of an ideal voltage amplifier. What are the values of the input and output resistances.
(b) In the following ideal op amp circuit, find Vo when Vi = 2V. (26)

Sketch the equivalent circuit of an ideal voltage amplifier.



For ideal V amplifier,

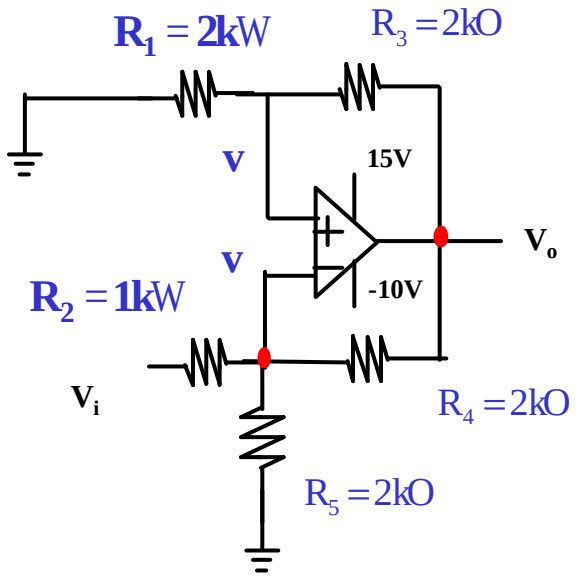
input resistance :

Ri = ¥

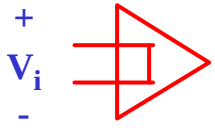
output resistance :

Ro = 0

In the following ideal op amp circuit, find Vo when Vi = 2V.



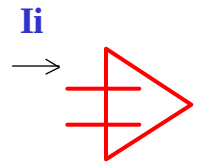
ideal op amp **Vi = 0 (virtual short)**



$$\because V_- = V_+ = v$$

$$\therefore V_+ = V_O \frac{2k}{2k+2k} = \frac{V_O}{2} = v = V_-$$

Ii = 0 (virtual open)



KCL

$$\therefore \frac{Vi - v}{1k} = \frac{v}{2k} + \frac{v - V_O}{2k}$$

$$\therefore 2V_i - 2v = v + v - V_O$$

$$\therefore 2V_i = 4v - V_O = V_O$$

$$\because v = \frac{V_O}{2}$$

$$\therefore A_V = \frac{V_O}{V_i} = \frac{2V_i}{V_i} = 2$$

If Vi = 2V, Vo = 4V

2

$$\therefore P = I^2R \cong \left(\frac{10A}{\sqrt{2}}\right)^2 * 23.5\Omega = 1175W$$

$$\therefore Q = I^2\frac{1}{\omega C} \cong \left(\frac{10A}{\sqrt{2}}\right)^2 * 8.55\Omega = 427.5VAR\,(C)$$

$$\therefore S = \sqrt{P^2 + Q^2} = \sqrt{1175^2 + 427.5^2} = 1250VA$$

$$\therefore PF = \cos(-20^o) = 0.94 \text{ leading}$$

P = real (or average) power dissipated by load Z

S = power supplied by source to load Z

Q = maximum reactive power stored in Z

Connect L in parallel to improve PF = 1

$$\therefore L = \frac{V^2}{w_o Q} = \frac{(250/\sqrt{2})^2}{100(427.53)} = 0.73H$$

When L is connected (PF = 1),
S = P = 1174.62VA, hence

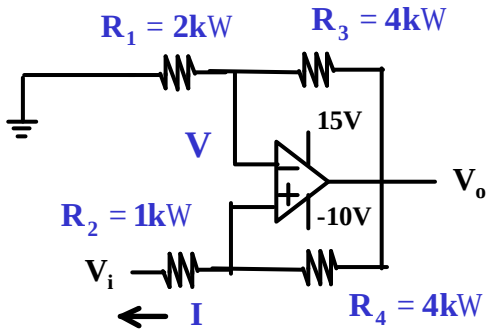
$$\therefore I = \frac{S}{V} = \frac{1174.62}{250/\sqrt{2}} = 4.7\sqrt{2} \text{ Arms}$$

If V is 500V (2x of original V)
I is 2x of original I
but R and C are unchanged
hence P and Q (μI^2) are 4x of original P and Q
Hence new L μ V^2 / Q = original L = 0.73H

2. Using the same circuit in question (1),
(a) find the apparent power S, reactive power Q, average power P and power factor PF of load Z.
(b) What are the physical meanings of S, P, Q?
(c) A load B is now connected in parallel to Z to make the total power factor = 1. Find the load B and the value. Find also the current (in rms) supplied by v(t) when PF is 1.
(d) If v(t) is now changed to 500 cos (100t + 20^o) V, find the new load B required.

(31)

5. (d) In the following circuit, assume ideal op amp, (i) find Vo if Vi = 3V, (ii) find I if Vi = - 1V. (40)



$\because V_- = V_+ = V$

$\therefore V_- = V_O \frac{2k}{2k + 4k} = \frac{V_O}{3}$

$\therefore \frac{Vi - V}{1k} = \frac{V - V_O}{4k}$

$\therefore 4V_i - 4V = V - V_O \quad \therefore 4V_i = \frac{2}{3}V_O$

$\therefore A_V = \frac{V_O}{V_i} = 6$

If Vi = 3V, Vo = 15V

If Vi = -1V, Vo = -6V

$\therefore I = \frac{Vo - Vi}{5k} = \frac{-6 - -1}{5k} = -1mA$