Electronic Circuits I

Dec., 15, 1998, 12:30pm – 3:30pm, G017

Examiner: Dr. Ki Wing-Hung

Name: Soln (English) Name: 題解 (Chinese)

Student Number: ____________ Signature: ____________

Directions:

(1) This is a closed book examination. No additional sheet is allowed.
(2) Calculators are allowed.
(3) Answer all questions in the space provided. Request for additional sheets from proctors only if necessary.
(4) Show all your calculations. No marks will be given for unjustified answers.
(5) Do your own work. Any form of cheating is a violation of academic integrity, and will be dealt with accordingly.

Question	Maximum Score	Score
1	12	
2	14	
3	8	
4	6	
5	8	
6	12	
7	20	
8	20	
Total	100	

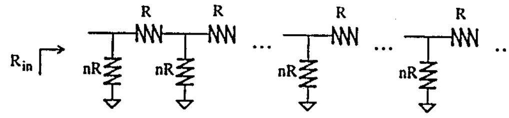
The following equations are provided for your reference. Use them if needed.

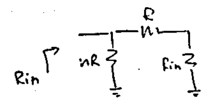
Diode: $I_d = I_s(e^{\frac{v_d}{v_{th}}} - 1)$, $I_s = 2\times10^{-15}$A, $v_{th} = 25$mV

NMOS:
- cutoff: $V_{gs} < V_t$
- linear: $V_{gs}-V_t \gtrless V_{ds}$, $I_d = k_n[(V_{gs}-V_t)V_{ds} - \frac{1}{2}V_{ds}^2]$
- saturation: $V_{gs}-V_t \lessgtr V_{ds}$, $I_d = \frac{1}{2}k_n(V_{gs}-V_t)^2$

PMOS:
- cutoff: $|V_{gs}| < |V_{tp}|$
- linear: $|V_{gs}|-|V_{tp}| \gtrless |V_{ds}|$, $|I_d| = k_p[(|V_{gs}|-|V_{tp}|)|V_{ds}| - \frac{1}{2}|V_{tp}|^2]$
- saturation: $|V_{gs}|-|V_{tp}| \lessgtr |V_{ds}|$, $|I_d| = \frac{1}{2}k_p(|V_{gs}|-|V_{tp}|)^2$

1a) Given an infinite resistive ladder network as shown below. Calculate the input resistance R_{in} if n = 6 and R = 1kΩ. (Hint: Any interesting observation when the array is infinite?) 6 marks





$$R_{in} = nR // (R + R_{in}) = \frac{nR(R+R_{in})}{nR+R+R_{in}}$$

$$\Rightarrow \cancel{nRR_{in}} + RR_{in} + R_{in}^2 = nR^2 + \cancel{nRR_{in}}$$

$$\Rightarrow R_{in}^2 + RR_{in} - nR^2 = 0$$

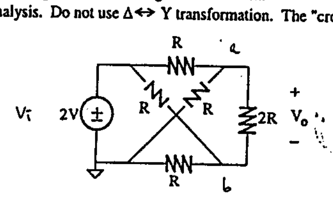
$$\Rightarrow R_{in} = \frac{-R \pm \sqrt{R^2+4nR^2}}{2}$$

$$= \frac{\sqrt{4n+1} - 1}{2} R$$

$$= \frac{5-1}{2} 1K$$

$$= 2K\,\Omega \#$$

b) Calculate the output voltage of the following lattice network. 6 marks
(Hint: Use nodal analysis. Do not use Δ↔ Y transformation. The "cross" is not connected.)



@a: $\dfrac{V_i - V_a}{R} = \dfrac{V_a - V_b}{2R} + \dfrac{V_a}{R}$

$\Rightarrow 2V_i - 2V_a = V_a - V_b + 2V_a$

$\Rightarrow 5V_a - V_b = 2V_i$ (1)

@b: $\dfrac{V_i - V_b}{R} + \dfrac{V_a - V_b}{2R} = \dfrac{V_b}{R}$

$\Rightarrow 2V_i - 2V_b + V_a - V_b = 2V_b$

$\Rightarrow -V_a + 5V_b = 2V_i$ (2)

$\Rightarrow 5V_a - V_b = 2V_i$

$-5V_a + 25V_b = 10V_i$

$24V_b = 12V_i$

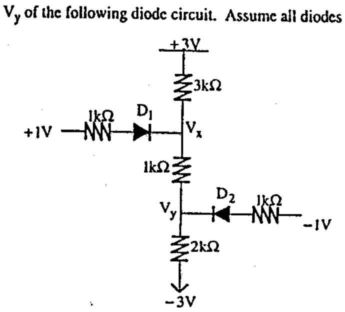
$\Rightarrow V_b = \frac{1}{2}V_i$

$V_a = 5V_b - 2V_i = \frac{5}{2}V_i - 2V_i$

$= \frac{1}{2}V_i$

$V_o = V_a - V_b = 0 \#$

2a) Calculate V_x and V_y of the following diode circuit. Assume all diodes are ideal. 6 marks



If +1V and −1V are not added, then

$V_x' = 3 - 3 = 0$ V

$V_y' = 2 - 3 = -1$V.

By adding +1V, D_1 will turn on. The added current increases V_y', thus reverse biasing $D_2 \Rightarrow D_2$ off.

$\therefore \dfrac{1 - V_x}{1k} + \dfrac{3 - V_x}{3k} = \dfrac{V_x - -3}{3k}$

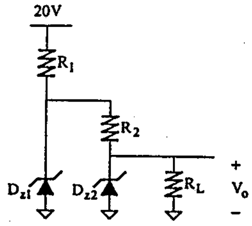
$\Rightarrow \cancel{3} - 3V_x + 3 - V_x = V_x + \cancel{3}$

$\Rightarrow 5V_x = 3 \Rightarrow V_x = \frac{3}{5} = 0.6$ V.

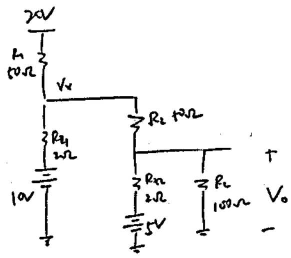
$V_y = \dfrac{2k}{1k+2k}(V_x + 3) - 3 = \frac{2}{3}(\frac{3}{5} + 3) - 3$

$= \frac{2}{3}\cdot\frac{18}{5} - 3 = 2.4 - 3 = -0.6V \#$

2b) With the zener diode models as shown, compute V_o. 8 marks
(That is, each diode D_z is modeled as a battery in series with a resistor.)



$V_{z1o} = 10V$
$V_{z2o} = 5V$
$R_1 = R_2 = 50\Omega$
$R_L = 100\Omega$
$R_{z1} = R_{z2} = 2\Omega$



$$\frac{20-V_x}{50} = \frac{V_x-10}{2} + \frac{V_x-V_o}{50} \Rightarrow 20 - V_x = 25V_x - 250 + V_x - V_o$$

$$\Rightarrow 270 = 27V_x - V_o \quad ①$$

$$\frac{V_x-V_o}{50} = \frac{V_o-5}{2} + \frac{V_o}{100} \Rightarrow 2V_x - 2V_o = 50V_o - 250 + V_o$$

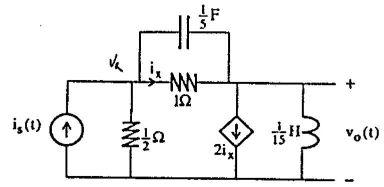
$$\Rightarrow 2V_x = 53V_o - 250 \quad — ②$$

② in ① $\Rightarrow$ $270 = 27\left(\frac{53}{2}V_o - 125\right) - V_o$

$\Rightarrow 714.5V_o = 3645$

$V_o = 5.1\ V_{\#}$

3) Compute the output voltage of the circuit below that operates in the sinusoidal steady state. 8 marks



$i_s(t) = 5\cos(5t + 60^o)$

@ V_a: $\quad i_s = \frac{V_a}{1/2} + \frac{V_a - V_o}{\frac{1}{j5\cdot\frac{1}{5}}} + \frac{V_a - V_o}{1}$

$\Rightarrow i_s = 2V_a + j(V_a - V_o) + V_a - V_o$

$= V_a(3+j) - V_o(1-j) \quad ①$

@ V_o: $\quad \frac{V_a - V_o}{1/j} + \frac{V_a - V_o}{1} = 2\frac{V_a - V_o}{1} + \frac{V_o}{j5\cdot\frac{1}{15}}$

$\Rightarrow jV_a - jV_o + V_a - V_o = 2V_a - 2V_o - j3V_o$

$\Rightarrow V_a(2-1-j) = V_o(2+j3-j-1)$

$\Rightarrow V_a(1-j) = V_o(1+j2) \quad — ②$

② in ① $\Rightarrow$ $\quad i_s = \frac{1+j2}{1-j}V_o(3+j) - V_o(1-j)$

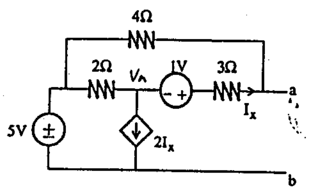
$\Rightarrow 5\angle 60^\circ = \frac{(1+j2)(3+j) - (1-j)(1-j)}{1-j} V_o$

$= \frac{3+j+6j-2-(1-2j-1)}{1-j} V_o$

$\therefore V_o = \frac{\sqrt{2}\angle -45^\circ \cdot 5\angle 60^\circ}{1+9j} = \frac{\sqrt{2}\,5\angle -45\angle 60^\circ}{\sqrt{82}\angle 83.66^\circ}$

$= 0.78\angle -68.66^\circ \Rightarrow V_o(t) = 0.78\cos(5t - 68.66^\circ)$

4) Find the Norton equivalent of the following circuit looking into the terminals a and b. 6 marks



V_{oc}: $\quad \frac{5-V_a}{2} = 2I_x + I_x = 3I_x \quad ①$

$\frac{V_a + 1 - V_{oc}}{3} = I_x$

$\Rightarrow \frac{5-V_a}{2} = V_a + 1 - V_{oc} \Rightarrow 5 - V_a = 2V_a + 2 - 2V_{oc}$

$\Rightarrow 2V_{oc} = 3V_a - 3 \quad ②$

$\frac{5-V_{oc}}{4} = -I_x = -\frac{1}{3}(V_a + 1 - V_{oc})$

$15 - 3V_{oc} = -4V_a - 4 + 4V_{oc}$

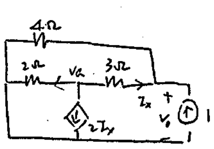
$\Rightarrow 7V_{oc} = 19 + 4V_a \quad —③$

$7V_{oc} = 19 + 4\left(\frac{2}{3}V_{oc} + 1\right) = \frac{8}{3}V_{oc} + 23$

$\Rightarrow \frac{13}{3}V_{oc} = 23$

$V_{oc} = \frac{3\times 23}{13} = \frac{69}{13} = 5.3\ V_{\#}$

R_{th} =

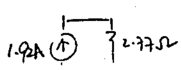


$\frac{V_a}{2} + 3\left(\frac{V_a - V_o}{3}\right) = 0 \quad \frac{V_a}{2} + V_a - V_o = 0$

$\Rightarrow V_o = \frac{3}{2}V_a$

$1 = \frac{V_o}{4} + \frac{V_o - V_a}{3} = \frac{V_o}{4} + \frac{V_o}{3} - \frac{2}{9}V_o$

$\Rightarrow V_o = 2.77 \Rightarrow R_{th} = 2.77\Omega$.

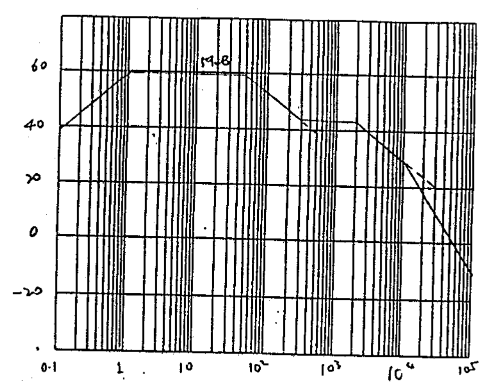


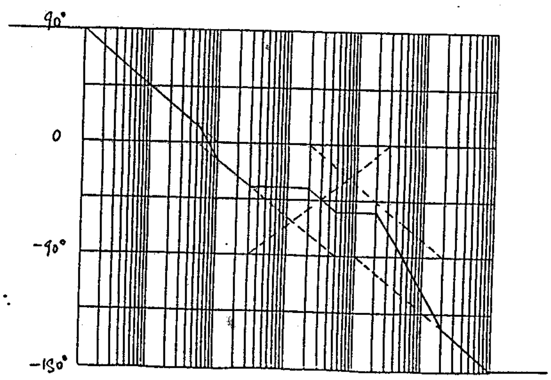
$I_{sc} = \frac{5.3}{2.77} = 1.92A$

5) Sketch the Bode plots of the following transfer function. 8 marks

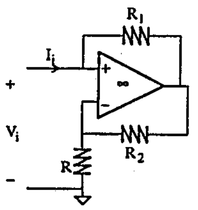
$$H(s) = \frac{3\times10^9 s(s+300)}{(s+1)(s+50)(s+2000)(s+10000)} = \frac{3\times10^9 s(1+\frac{s}{300})\,300}{(1+s)(1+\frac{s}{50})(1+\frac{s}{2000})(1+\frac{s}{10^4})\,50\times2000\times10^4}$$

$$= \frac{900s(1+\frac{s}{300})}{(1+s)(1+\frac{s}{50})(1+\frac{s}{2000})(1+\frac{s}{10000})} \qquad 900 \Rightarrow 59\text{dB}.$$





a) Negative resistance can be obtained by using active devices. For the circuit shown below, compute the negative resistance $R_{in} = V_i/I_i$. 4 marks



$R_1 = R_2 = R = 10k\Omega$

$$V_- = \frac{R}{R+R_2}V_x = V_i$$

$$I_i = \frac{V_i - V_x}{R_1} = \frac{V_i - \frac{R+R_2}{R}V_i}{R_1}$$

$$I_i = -\frac{R_2}{R_1}\frac{1}{R}V_i$$

$$R_{in} = \frac{V_i}{I_i} = -\frac{R_1}{R_2}R \quad \#$$

$$= -10k\Omega \#$$

6b) The op-amp is not ideal, and $A(s) = \frac{A_o}{1+\frac{s}{\omega_b}}$, with $A_o = 500$, and $\omega_b = 2000$. 8 marks

Compute the pole and zero of the negative impedance.

$$V_- = \frac{R}{R+R_2}V_x$$

$$I_i = \frac{V_i - V_x}{R_1}$$

$$V_x = A(V_i - V_-) \Rightarrow V_x = AV_i - \frac{AR}{R+R_2}V_x$$

$$V_x\left(1 + A\frac{R}{R+R_2}\right) = AV_i$$

$$\therefore I_i = \frac{V_i - \frac{A}{1+A\frac{R}{R+R_2}}V_i}{R_1} = \frac{1}{R_1}\left(1 - \frac{A}{1+A\frac{R}{R+R_2}}\right)V_i$$

$$\frac{V_i}{I_i} = R_{in} = \frac{R_1}{1 - \frac{A}{1+A\frac{R}{R+R_2}}} = \frac{R_1\left(1+A\frac{R}{R+R_2}\right)}{1+A\frac{R}{R+R_2} - A}$$

$$= \frac{R_1 A\frac{R}{R+R_2}\left(1+\frac{1}{A}\frac{R+R_2}{R}\right)}{1 - \frac{R_2}{R+R_2}A} = \frac{RR_1A}{R+R_2}\frac{\left(1+\frac{1}{A}\left(1+\frac{R_2}{R}\right)\right)}{-\frac{R_2}{R+R_2}A\left(1-\frac{1}{A}\left(1+\frac{R_2}{R}\right)\right)}$$

$$= -\frac{R_1}{R_2}R\,\frac{1+\frac{1+\frac{s}{\omega_b}}{A_o}\left(1+\frac{R_2}{R}\right)}{1-\frac{1+\frac{s}{\omega_b}}{A_o}\left(1+\frac{R_2}{R}\right)}$$

$$\hat{=} -\frac{R_1}{R_2}R\,\frac{1+\left(1+\frac{R_2}{R}\right)\frac{s}{\omega_t}}{1-\left(1+\frac{R_2}{R}\right)\frac{s}{\omega_t}}$$

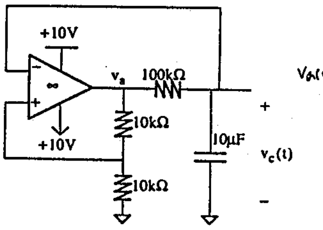
$A_o\omega_b = \omega_t$
$= 500 \times 2000$
$= 10^6$

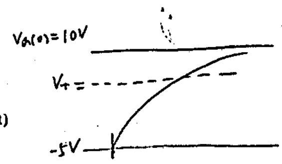
pole $\quad s_p = -\frac{\omega_t}{1+R_2/R} = -\frac{10^6}{2} = -5\times10^5 \text{ rad s}^{-1}$ //

zero $\quad s_z = +\frac{\omega_t}{1+R_2/R} = +5\times10^5 \text{ rad s}^{-1}$ //

7a) Let the output of the op-amp $v_a(0)$ be at 10V initially, and $v_c(0) = -5V$. Assume ideal op-amp with power supply of +10V and -10V. Calculate the time needed for v_a to change state. 8 marks

Hint: Perform transient analysis (with solution in the form of $k_1+k_2e^{-t/\tau}$). Note that $v_+ \neq v_-$, except at change of state.





$V_a(0) = 10V \Rightarrow V_+ = 5V$

now, capacitor charges towards 10V. i.e.

$$V_c(t) = k_1 + k_2e^{-t/\tau}$$

$$\tau = RC = 100k \times 10\mu = 1 \text{ sec.}$$

$$V_c(0) = -5 \Rightarrow k_1 + k_2 = -5$$

$$V_c(\infty) = 10 \Rightarrow k_1 = 10 \Rightarrow k_2 = -15.$$

$$\therefore V_c(t) = 10 - 15e^{-t}.$$

Op amp change state when $V_c(t) = 5V \Rightarrow 5 = 10 - 15e^{-t}$

$\Rightarrow e^{-t} = 1/3 \quad t_1 = 1.099s.$

b) Let t_1 be the time the op-amp changes its state. What are $v_a(t_1)$ and $v_c(t_1)$? 4 marks

at t_1, op-amp changes state,

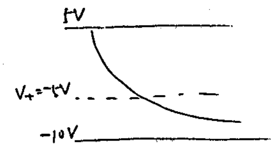
$\therefore V_a(t_1) = -10V.$

and $V_c(t_1) = 5V.$

7c) Calculate the time elapsed for the op-amp to change to its initial state. 4 marks

now, the capacitor starts to discharge towards -10V.

i.e.,



which the threshold is now $V_+ = \frac{1}{2}(-10) = -5V$

$$V_c(t) = k_1 + k_2e^{-t/\tau}. \quad \tau = 1$$

$$V_c(0) = 5V \Rightarrow k_1 + k_2 = 5$$

$$V_c(\infty) = -10 \Rightarrow k_1 = -10 \Rightarrow k_2 = 15$$

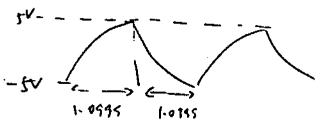
$$\therefore V_c(t) = -10 + 15e^{-t}$$

$$V_c(t') = -5 = -10 + 15e^{-t'}$$

$$\Rightarrow \frac{1}{3} = e^{-t'}$$

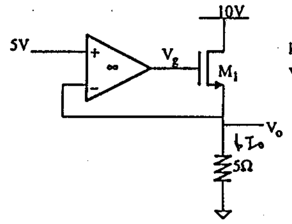
$$\Rightarrow t' = 1.099s.$$

7d) The circuit of (7) is an oscillator. From (7a) and (7c), calculate the oscillation frequency. 4 marks



$$\Rightarrow f = \frac{1}{T} = \frac{1}{1.099+1.099} = 0.455 \text{ Hz} \#$$

8a) An NMOS transistor is used to construct a voltage regulator. With an ideal op-amp, calculate the voltage at the gate V_g. 6 marks



$k_n = 500mA/V^2$
$V_t = 1V$

Ideal op-amp $\Rightarrow V_o = V_- = V_+ = 5V$.

$I_o = \frac{5V}{5\Omega} = 1A \Rightarrow I_d = 1A$.

Assume M_1 in saturation, $I_d = 1A = \frac{1}{2}(0.5)(V_{GS} - V_T)^2$

$\Rightarrow (V_{GS} - V_T)^2 = 4$

$\Rightarrow V_{GS} - V_T = 2$

$\Rightarrow V_g - V_o - 1 = 2$

$\Rightarrow V_g = 2 + 5 + 1 = 8V$ #.

check: $V_{gs} - V_T = 2$, $V_{ds} = 10 - 5 = 5V$

$\therefore V_{gs} - V_t < V_{ds} \Rightarrow$ saturation.

b) Is the NMOS transistor operating in the saturation or linear region? 2 marks

Saturation

8c) Now, the op-amp is not ideal, but with a DC gain of $A_o = 500$. Assume V_o is approximately 5V, recalculate V_g, and thus obtain a more accurate V_o. 4 marks

For $A_o = 500$,

$V_g = (5 - V_-)500 = 8$

$\Rightarrow 5 - V_- = \frac{8}{500} = 16m$

$\Rightarrow V_- = 5 - 16m$

$= 4.984V$

$V_o = V_- = 4.984V$ #.

8d) Next, with the above non-ideal op-amp, the load resistor is changed to 2.5Ω. Calculate the new V_o. 6 marks

assume $V_o = 5V$, $I_d = 2A = \frac{1}{2}(\frac{1}{2})(V_{GS} - V_T)^2$

$\Rightarrow V_{GS} - V_T = 2\sqrt{2} = 2.828$

$\Rightarrow V_g - 5 - 1 = 2.828$

$\Rightarrow V_g \doteq 8.828$

$= (5 - V_-)500$

$\Rightarrow V_- = 5 - \frac{8.828}{500} = 5 - 0.0177 = 4.9813V$ #.

8e) With the result obtained in (8c) and (8d), calculate the load regulation $\Delta V_o / \Delta I_o$. 2 marks

$\frac{\Delta V_o}{\Delta I_o} = \frac{4.984V - 4.9813V}{1A - 2A} = -2.7mV/A$ #.