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A Optimal Power Profiles of Bicycle Time Trial

Summary

For cyclists, a reasonable output power strategy is the key to victory. In this article, we focus on the athletes' own physical characteristics, track environment and the bicycle's dynamics equation, and simulate the real situation based on this information to optimize the study and develop a suitable output power strategy.

For athletes' physical characteristics, we use power curves and define two types of athletes of different genders: endurance type and explosive type. Then, based on years of sports research, we summarized the athletes' physical exertion equation, which provides the basis for the constraints of the simulation optimization in the later section.

For the track environment, we take the venue of the Tokyo Olympic Games time trial as an example, and use the official information provided to fit the curve of the track to obtain the length, elevation, slope, curvature and other data of the track. We also divided the track stages according to the athletes' physical characteristics and simplified the model parameters. This provides a method for iteration in the simulation algorithm.

For the bicycle dynamics model, we mainly consider the way it interacts with the track environment. For example, slope, rolling friction, wind resistance and other factors.

Integrating the above theoretical models, we set up a simulation environment for validation. Firstly, the performance under a fixed power output was studied. Then the power output strategy was optimized with the introduction of athlete strength model constraints, and satisfactory results were obtained. At the same time, we also determine the potential impact of weather conditions and how sensitive the results are to ride deviations from the target power distribution.

Finally, we provide a robust cyclist's race guidance for a Directeur Sportif of a team.

Keywords: Power Output Strategy; Time Trial; Simulation ; Optimization

Contents

1 Overview

1.1 Background

There are many types of bicycle road races including a criterium, a team time trial, and an individual time trial.In an individual time trial, each individual cyclist is expected to ride a fixed course alone, and the winner is the cyclist who does so in the least amount of time. For different types of cyclists, they have different physical characteristics: explosiveness, persistence, recovery, etc. For each individual time trial, it is important to adjust your power output strategy for the course characteristics as well as environmental factors.

1.2 Restatement of the Problem

In solving the power optimization problem of cyclists, we need to consider many factors. For example, the dynamic model of bicycle on straight road and curve road, and for different types of cyclists, they have different physical characteristics: explosiveness, persistence, recovery, etc. For each individual time trial, it is important to adjust your power output strategy for the course characteristics as well as environmental factors. According to the requirements, we must establish at least two types of athletes model, considering the cyclists in these two sections of the speed model. Based on the track of the time trial of the 2020 Tokyo Olympic Games, we carry on the simulation verification to provide the optimal solution of the reasonable distribution of physical strength for the participants. In combination with the actual situation, we also analyze the sensitivity of the model and the influence of the weather and other factors on the model, and finish the discussion of team cooperation in further discussion. Finally, it provides an easy-to-understand guide for Directeur Sportif.

2 Notation

Table 1: Notations

Abbreviation	Value
ρ	Drag coefficient
$1/\mu$	Coefficient of internal resistance
h	Altitude
σ_0	Energy generated per unit time in the body at sea level
$\sigma(h)$	Energy generated per unit time at different altitudes
p	Curvature
T_c	Target time
$f(t)$	Real-time impulse
f	Acceleration stage of the cycle
$E(t)$	The energy stored in the body
E_0	The initial amount of physical ability before the start of a race
E_b	The remaining energy required for the sprint stage
E_{ci}	The amount of energy left at the end of the i th straight
D	The distance left to plan a sprint

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Continued Table

Abbreviation	Value
D_i	The distance of the i th straight
D_b	The distance left to do a sprint
T_{ai}	From the start of the i th straight, the end of the acceleration stage of the cycle.
T_{ci}	From the start of the i th straight, into the next corner of the moment.
T_{di}	The time it takes to make the i th turn.
T_b	Start the clock from entering the last straight, the start of the sprint.
Θ	Maximum angle at which a bicycle will not tip over
α	The inclination of the track and the horizon
μ_s	Coefficient of friction between the track and the bike
k	Human power limiting factor
v_i	The initial velocity of the i th straight
$P(t)$	The real-time power of the acceleration cycle, that is, the maximum real-time power under the condition of maximum power acceleration and physiological limitation
P_{max}	The maximum power of a cyclist
P_0	The power of the cyclist at low power (the energy in the body remains constant)
P_{si}	The ideal uniform speed for cycling on the way in the i th straight,
P_{di}	In the i th Corner, ideal power on the way
P_{ai}	The average maximum acceleration power of the cyclist at each straight acceleration stage
v_{sb}	The straight, the ideal constant speed on the way at the end of the straight
$v(t)$	speed function
w_i	The component of the wind speed in the i th straights
w_b	The component of the wind speed in the last straight
S_i	The length of the i th turn

3 Assumptions and Justifications

According to the analysis of traditional models, road cycling is a comprehensive knowledge of mechanics, biology, aerodynamics, mechanics, etc. , the optimal velocity model is also the organic combination of physics, biology, mathematics, computer and so on. However, the optimal velocity model proposed by T. B. Keller tries to describe the whole process by pure mathematical deduction[1], and finally the realization is reduced because of the complex mathematical method of finding the extreme value by functional, at the same time, the over-theoretical mathematical model is out of line with the reality because it can not take into account many aspects of things. In view of these conditions, we make some valid assumptions on the model, and make the model focus on solving the problem which is more meaningful to the actual competition, so the derivation and practical application of the model will be helpful.

To simplify our problems, we make the following basic assumptions, each of which is properly justified:

1) The amount of wind in the course of the race is proportional to the current speed$(v-w)^2/2$,

with a proportional coefficient of ρ .

2) The influence of internal factors in the course of cycling race on speed is proportional to speed v, and the proportional coefficient is $1/\mu$. As body temperature is the main index, the correction coefficient of body temperature can be $1/\mu$

3) The amount of energy generated per unit of time during a cycling race is related to oxygen intake, and studies have shown that the amount of energy generated per unit of time during a cycling race depends on the amount of oxygen consumed, for every $1000\,\mathrm{m}$ rise, maximum oxygen update falls by about 6%, a function of altitude h. Let σ_0 be the energy generated per unit time in the body at sea level, then.

$$\sigma(h) = -\frac{6\%}{1000}h + \sigma_0[4]$$

Since the magnitude of the cyclist's initial energy and the energy generated per unit time has little to do with the correctness of the model, the accuracy of the model mainly depends on the "marginal contribution" of the physical fitness model to the different reasons for the definition of appropriate weights;

4) According to the experience, the $P(t)$ range of the power of the cycling race is determined in advance, and the change of the power of the cycling is monitored by the power meter of the terminal, suppose the cyclist accelerates to near maximum impulse power P for a period of time after the start of the ride. When the real-time power $P(t)$ drops to the expected value and reaches a certain speed, the cyclist transitions into the mid-ride stage

5) According to the specific training goal and training plan, has determined in advance each straight time training result, namely determines the target time T_{ci}. Because different training goals have different requirements for cyclists, even the same training goals for different stages of the race are not the same. The general consideration is to distribute the energy evenly among each straight path according to the weight of the straight path length. In the speed optimal model, the "optimal" must be set based on a particular race track. Putting the race aside and talking about the optimal speed only makes the model out of practice and can not be applied in practice;

6) According to the actual experience of the coach combined with the specific circumstances of the cyclist, prior to determine the sprint into the stage of remaining energy E_b. And the distance remaining in the planned sprint, D_b. Or at the start of the sprint, T_b. In the same way, when the cyclist will accelerate in the sprint stage, and at what acceleration should be prepared in advance, depending on the physical condition at the time and the condition of the opponent, it is often counterproductive to introduce the final sprint stage into the model

7) In order to eliminate the influence of the above parameters, which vary from person to person, especially relating to the weight and wind area of the cyclist, we modeled the cyclist's weight per unit in the same race as $m = 1$, windward area $S = 1$.

8) The speed of a turn depends on the radius of curvature of the curve, and we assume that the cyclist will always be able to turn at maximum speed, at which point the cyclist will reduce

speed instantaneously to the maximum safe speed through the turn. That is, the starting speed of a straight is the maximum speed at the start of a curve, except for the straightaway at the end and before the start.

9) After slowing down before the turn and going through the turn, the cyclist can return to the initial maximum power P and enter the acceleration stage at the initial maximum power P on the next straight. That is, the maximum power P of the cyclist's individual straights at the beginning is a constant value.

10) Cornering power can be calculated from the speed and length of the curve, this period of energy consumption in the ideal state is ignored.

4 Model Theory

4.1 General model of road bicycle race

We divide the optimal riding strategy into the optimal speed change strategy and the optimal energy allocation strategy; the definitions of these two strategies are explained below. According to the characteristics of road cycling in [3], we define the change of speed as three stages: "acceleration stage", "middle stage" and "sprint stage" There are two types of track conditions: straight and curve. It is one of the keys to customize the optimal power change strategy that what kind of speed change strategy is carried out in what kind of track condition.

Below we will define several states of energy recovery, as mentioned in the background,"the more power a cyclist produces, the less time the cyclist can maintain that power before having to reduce the amount of power and recover. A cyclist may choose to briefly exceed the limits on their power curve, but the cyclist then requires extra time at a lower power level to recover." Based on this, we establish the relevant energy change model.

In short, there is a low power P_0 , which depends on the amount of energy recovered per unit time $\sigma(h)$, and when the bicycle power is less than P_0 , the energy consumed by the cyclist per unit time is less than the energy recovered by the cyclist per unit time, and the energy stored by the cyclist in the body $E(t)$ is in an increasing state, that is, entering the "energy recovery state"; When the bicycle power is equal to P_0, the energy consumed per unit time is equal to the energy recovered per unit time, and the energy $E(t)$ stored by the cyclist in the body remains the same, that is, the cyclist enters the "energy stable state"; When the bicycle power is greater than P_0, the energy consumed per unit time is greater than the energy recovered per unit time, and the energy stored by the cyclist in the body $E(t)$ will be reduced that is, the rider enters the "Energy consumption stage". Thus, there are two specific moments of energy in the cyclist's body When the energy $E(t)$ in the rider's body is 0, it means that the rider briefly exceeds the limits on their power curve and needs to ride at a power less than P_0 to recover the energy in the body; When the energy $E(t)$ in the rider's body is E_0, it means that the rider has reached the "energy saturation stage", and the power less than P0 will not recover the energy, resulting in energy overflow. Therefore, how to distribute the energy and formulate the corresponding optimal energy change strategy is the key to the competition.

Based on the above analysis and definition, the optimal strategy should be as follows:

In this paper, we study and analyze the optimal running power model proposed by the famous American mathematician T. B. Keller [1], and combine it with the bicycle race model.The optimal power change strategy is as follows: the acceleration stage is usually at the beginning of the straight (including the start of the straight) , the cyclist to the maximum impact of riding a distance; The middle stage is after the acceleration stage, the ideal situation is to ride with uniform power (but according to the actual training situation: the speed and power of the cyclist is not the same throughout the ride stage, will be based on the time of the race and the actual conditions of speed and power changes) . As cyclists enter the turn, they need to slow down to a maximum speed through the turn. Because of the bike's characteristics, the ideal speed is reduced instantaneously and consumes no energy. After the turn, you immediately go into the acceleration stage, and the next straight and turn is the same strategy as before, and so on. Until somewhere in the back end of the final straight, D_b, the cyclist will enter the sprint stage, expending all his energy and relying on inertia to reach the finish.

Taking the four turns as an example, the schematic diagram of the optimal strategy under ideal conditions is as Figure 4.1.

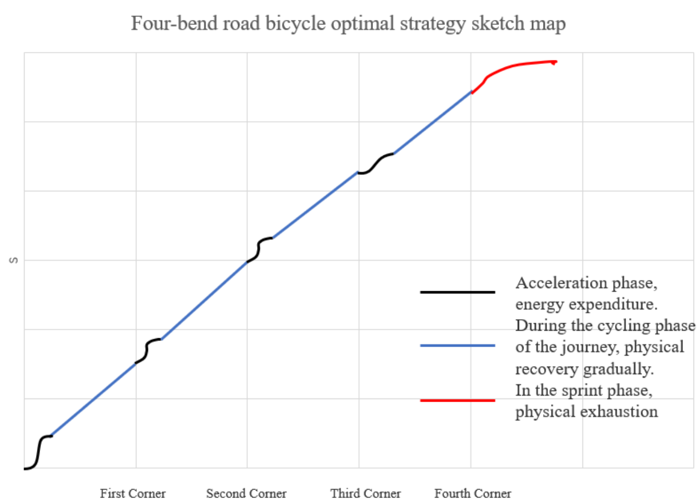


Figure 4.1: Time to exhaustion and power for cyclist [17]

In fact, the middle stage is the key part of the road cycling race, and the cyclist will devote most of his energy to the 90% or more of the road cycling race as a study in [1], at this stage, the demands on the cyclists'physical ability, psychology, techniques and tactics are relatively high. The auxiliary training device will focus on the riding stage on the way, trying to improve the performance of the cyclist at this stage through reasonable speed advice, to be able to reasonably allocate physical strength during the middle stage, to keep the opponent in check, and to lay a solid foundation for the final sprint cycling stage, the above stages according to the mathematical modeling of the model analysis.

4.2 The mathematical model and guiding strategy of the acceleration stage

The acceleration stage is always at the beginning of each straights and refers to the stage where time is $0 < t < T_{ai}$(t counts from entering the i th straights) . The cyclist should accelerate quickly during the acceleration stage to take advantage of the transition to the middle stage, when the cyclist is pedaling the distance with his or her maximum momentum. Since the cyclist is in the slow stage of the acceleration cycle before entering the curve and in the constant speed cycle before entering the curve, we can assume that the physiological level of the cyclist has initially recovered before then, that is to say, each time before entering the straight, the cyclist can accelerate the ride at the initial maximum power of P.

In the competition, the cyclist because of the physiological condition limit, after achieves certain high speed, is unable to continuously display own maximum power. Suppose $P(t)$ is the maximum power of the bicycle at time t, and the power that the cyclist can exert after overcoming the physiological limit $P(t)$ satisfies $\dot{P}(t)/P(t) = -1/k$, where k is the impulse limiting coefficient, so the maximum power $P(t)$ of the bicycle is a function of time t. By solving the differential equation, you can see that the cyclist is riding at full power at all times during the acceleration stage, when real time power meets the following conditions:

$$P(t) = P_{max}e^{-\frac{t}{k}} \tag{1}$$

Let v_i be the speed of entry into the i th straight. Since let $m = 1$ for all the cyclists and $s = 1$ for the wind,according to Newton's second law: [2]

$$\begin{cases} \dot{v}(t) + \frac{v(t)}{\mu} + \frac{\rho(v(t)-\omega_i)^2}{2} + g\cos\alpha + \mu_s g\sin\alpha = \frac{P(t)}{v(t)} = P_{max}e^{-\frac{t}{k}} \quad 0 \le t \le T_{ai} \\ v(0) = v_i \end{cases} \tag{2}$$

4.3 The mathematical model and guiding strategy of the riding stage

The middle stage is the stage after each straight road acceleration stage, and ideally requires constant power for the cyclist, so the most important thing in this model is the best power suggestion. In order to ensure the implementation of the optimal energy allocation strategy, that is, under the same straights, the energy consumed during the acceleration cycle is equal to the energy recovered during the middle stage cycle. So the amount of power at this point depends on the change in energy.

The following is a mathematical model of the energy change, which is consistent with the above definition of the energy change strategy: when the output power is less than P_0, the energy in the body is in the $E(t)$ recovery stage; otherwise, the $E(t)$ will be reduced, at the same time guarantee the restriction of $E(t)$, so there is the following formula:

$$\begin{cases} E(t) = \int (P_0 - P(t))\, dt + E_0 \\ \int (P_0)\, dt = \sigma(h) \\ \sigma(h) = -\frac{6\%}{1000}h + \sigma_0 \\ 0 < E(t) < E_0 \end{cases} \tag{3}$$

In order to accelerate the energy consumed in the cycling stage equal to the energy recovered during the middle stage, let P_{si} be the ideal power for the cycling stage of the i th straightaway, defined by the formula (3) and the average maximum acceleration power P_{ai}, we have:

$$\left\{ \begin{array}{c} \int_0^{T_{ai}} (P_0 - P_{ai})\,dt + \int_{T_{ai}}^{T_{ci}} (P_0 - P_{Si})\,dt = 0 \\ P_{ai}t = \int_0^{T_{ai}} P(t)dt \end{array} \right. \tag{4}$$

Based on the relationship between speed and the length of the straight, D_i is the length of the i th straight. We have:

$$\int_0^{T_{ci}} v(t)dt = D_i \tag{5}$$

From the formula (2)(3)(4)(5) , the power of the first straight path, the time T_{ai} at the end of the acceleration stage and the time T_{ci} at the end of the uniform speed stage can be obtained.

The guiding significance of the above strategy lies in the determination of P_{si}, the P_{si} in Formula (4) is the ideal power, because the power can not be increased indefinitely, will be bound by physical strength and momentum. The practical significance of this power is that if an athlete uses P_{si} to cycle the entire length of the road cycle, the overall energy consumption for the rest of the way can be more evenly distributed, the power change is more in line with the ideal curve, which makes the whole course of the race reach the predetermined goal.

4.4 The mathematical model and guiding strategy of the curve stage

Next, we need to model the turning stage of the bicycle. When the cyclist enters the curve from the middle stage at the end of the last straight road, under ideal conditions, he should quickly reduce to the maximum safe speed when crossing the curve, and the cyclist hardly consumes energy in this process [5].

When the bicycle turns, the pedestrian vehicle system needs to be inclined, and the included angle between the body plane and the vertical plane is θ,The stress of the pedestrian vehicle system is: gravity G, ground support force N and static friction force f.If the vehicle does not tilt when turning, set m, v and p as the total mass, speed and curvature radius of the pedestrian vehicle respectively.It can be seen that the larger v or the smaller p, the greater the required static friction f between the ground and the vehicle, and the maximum static friction$f_{\max} = \mu_s mg$.

Therefore, through mechanical analysis, the conditions under which the vehicle does not tip laterally when turning are as follows:

$$\left\{ \begin{array}{l} N = mg \\ f = mv^2/p \end{array} \right. \tag{6}$$

and

$$\mu_s mg \geqslant mv^2/p \tag{7}$$

The maximum speed of the cyclist crossing the corner can be solved, and the initial speed of each straight road can also be calculated:

$$v_{di} = \left\{ \begin{array}{c} \sqrt{p_{\max}\mu_s g}(i = 2, 3, 4 \ldots) \\ 0(i = 1) \end{array} \right. \tag{8}$$

assuming m = 1, the power and curve crossing time during curve stage can be calculated:

$$P_{d_i} = v_i f = \sqrt{p_{\max} \mu_s g} \cdot \mu_s g \tag{9}$$

$$T_{di} = \frac{S_i}{\sqrt{\mathrm{p_{max}} / \mu_s g}} \tag{10}$$

On the other hand ,in order to guide the driver to pass the curve in the safest position. we should calculate the maximum inclination angle of the bicycle .From the moment balance formula, it can be calculated that the vehicle at the above rate is at the radius of curvature . The moment of the pedestrian vehicle system to the center of mass G is:

$$M = f \cdot GA \cdot \cos\theta - N \cdot GA \cdot \sin\theta \tag{11}$$

Where GA is the distance from the center of mass to the contact point between the rear wheel and the ground, which is solved by $M = 0$

$$\operatorname{tg}\theta = f/N = \mu_s mg/(mg) = \mu_s \tag{12}$$

The guiding significance of the above results is that [6], The smaller the μ_s , the smaller the maximum inclination angle θ allowed when turning, which is the reason why cycling on ice and snow or smooth roads is most likely to fall. If you want to ride in a straight line as far as possible without falling down, even if you walk in a curve, you must keep the inclination angle θ very small and the radius of curvature p very large.

4.5 The mathematical model and guiding strategy of the sprint stage

When the race enters the sprint stage, the cyclist can use the maximum power at each moment $P(t)$Ride the whole journey, which is bound to achieve the best results.As for how long the race distance can enter the sprint stage, it should be based on the energy stored in the body $E(t)$ .Not less than zero as the limit, and finally reach the end energy as small as possible.

Similarly, it is assumed that the cyclist can exert the maximum power after overcoming the physiological constraints $P(t)$ satisfy $\dot{P}(t)/P(t) = -1/k$, of which k is the power limit coefficient. It can be obtained from formula (1):$P(t) = P_{max}e^{-\frac{t}{k}}$

Because the duration of the sprint is short and the temperature change in the cyclist's body can be ignored, the internal resistance at this time can be regarded as approximately zero. At the same time, there is only external resistance in the final sprint stage.$v_{sb}$is the speed of uniform riding in the previous acceleration stage. Since it is assumed that the mass of all cyclists is $m = 1$ and the contact area of wind is $s = 1$, it is obtained by Newton's second law, Thus we get change function of v_t:

$$\begin{cases} \dot{v}(t) + \frac{\rho(v(t)-\omega_b)^2}{2} + g\cos\alpha + \mu_s g \sin\alpha = \frac{P(t)}{v(t)} = \frac{P_{max}e^{-\frac{t}{K}}}{v(t)} \\ v(0) = v_{sb} \\ 0 \le P(t) \le P_{max} \end{cases} \tag{13}$$

Set up E_b is the energy remaining in the middle stage on the last straight road, and then consider the energy in the cyclist's body $E(t)$ by formula (3)

$$\int (P_0 - P(t))\, dt + E_b = 0 \tag{14}$$

Combined with formula (13) (14), the elapsed time t and real-time power after starting the sprint can be solved $P(t)$ change function of.

To sum up, in the process of road bicycle race, the timing starts from entering the straight road, the time on each straight road is less than T_{ai} ,when the athlete can compete according to the maximum power; the time on each straight road reaches T_{ai} when adjust the speed to the current speed and keep moving at a constant speed and power until entering the curve and decelerating to the maximum safe speed v_i Make the final sprint at the distance D_b arranged by the coach before the game, try the best to consume all the energy, so that cyclist can achieve the best results in theory and obtain the relationship between power and position [7].

4.6 Cyclist's Model

4.6.1 General modeling of power curve

The above analysis is the mathematical modeling of bicycle road individual time trial under general conditions, the cyclist's model of different types of players need to be considered below. The cyclist's model is mainly composed of the following two constraints: explosive power and endurance which are determined by the power curve.

The oxygen uptake and heart rate have commonly been used to assess athletes' explosive power performance [11][12][13]. The oxygen consumption rate in [13] illustrates that, when the cyclists' output power increases, oxygen consumption also increases. This is consistent with the fact that oxygen uptake and mechanical power output is roughly linearly related [14], except for high-power cycling when a significant fraction of the power is produced anaerobically [11, 15]. In addition, the map never reaches zero because humans still consume oxygen at rest to support basal metabolism.

Per unit oxygen consumption is in Figure 4.6.1 (modified from [10]). The most efficient cycling is to pedal at 70-100 rpm and 40-50 Nm, which corresponds to producing 400 watts of power. However, this power level is high and not sustainable for amateur cyclists. According to the NASA testing quoted in [16], a healthy male can maintain cycling at 200 watts for an hour, but can only last one minute at 400 watts. In other words, the oxygen uptake reflects the power generated by the cyclist, but does not reflect the exhaustion level of the cyclist. Therefore another factor, stamina consumption, must be considered.

To capture the explosive power level and exhaustion level of the cyclist, we need the power curve to describe it. Power curve is a visual representation of the maximum power a cyclist can maintain for a particular length of time, different types of cyclists should have different power curves.

So we adopt the time to exhaustion and power curve in [17]. As shown in Figure 4.6.1, it represents the phenomena that people are exhausted by a short burst of aggressive exercise but can endure a long low-powerexercise, even though the total oxygen consumption may be similar.

By analyzing the power curve, we can know the explosive power and endurance of a cyclist. On one hand, the cyclist's endurance level decreases drastically as the output power increases; on the other hand, the maximum output power of a cyclist, which is also the P_0 (the power of the cyclist at low power (the energy in the body remains constant)) we defined, represents his explosive power level; the minimum output power, which is also the P_{max} (the cyclist maximum power)we defined, represents the level of energy recovery of the cyclist.

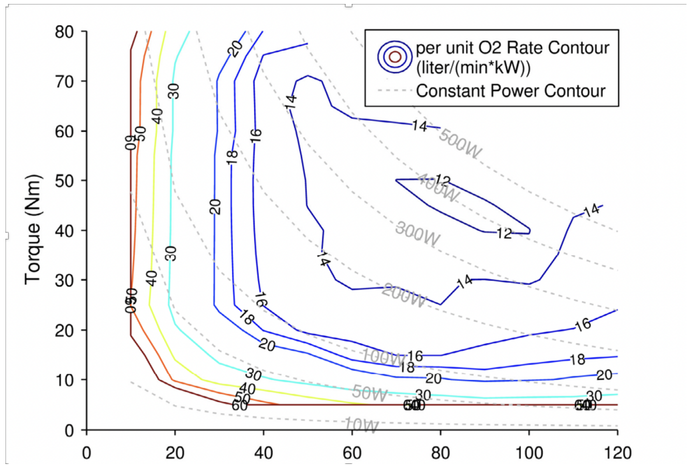


Figure 4.2: Per unit oxygen consumption rate (modified from [10])

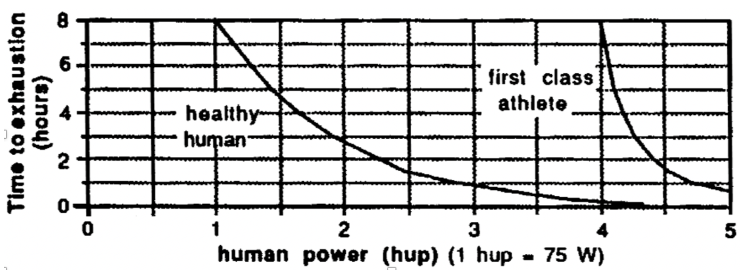


Figure 4.3: Time to exhaustion and power for cyclist [17]

4.6.2 Power curves of different types of cyclist

As mentioned in the background of the problem, in the bicycle road time trial, there are many different types of cyclists. In addition to the time trial specialist, there are mainly climbers, puncheur , rouleur, sprinter and other types of cyclists. Similarly, gender factors also affect the player's power curve. The power curves of different types of cyclists are different.

For those different types of cyclists , they all have different levels of endurance and explosiveness. These two factors determine the shape of their power curve. Cyclists with strong endurance the time to exhaustion is longer when they consume the same power in low and medium power state; Cyclists with strong explosive power have higher the cyclist maximum power P and can continue to ride longer at high power. We discussed the power curves of time trial specialist and climbers, and considered gender factors. The four types of power curves are shown in Figure 4.6.2. (Climber is a cyclist that specializes in races that have multiple long climbs and time trial Specialist: a cyclist that specializes in the individual time trial events.)

As shown in Figure 4.6.2, the red and green curves are the power curves of climbers. They have stronger endurance so that they can climb the mountain longer, so they can last longer under the condition of low and medium power. However, their explosive power level is not as good as that of time trial cyclists, because time trial cyclists are good at sprint and more lasting acceleration, so time trial cyclists have higher maximum power P and can persist in high-power state for a longer time. For gender factors, because women's explosive power and endurance are generally lower than those of men of the same type, women's overall power level and corresponding duration are lower than men. Power levels among women are still affected by explosiveness and endurance. Other types of players can be inferred in the same way.

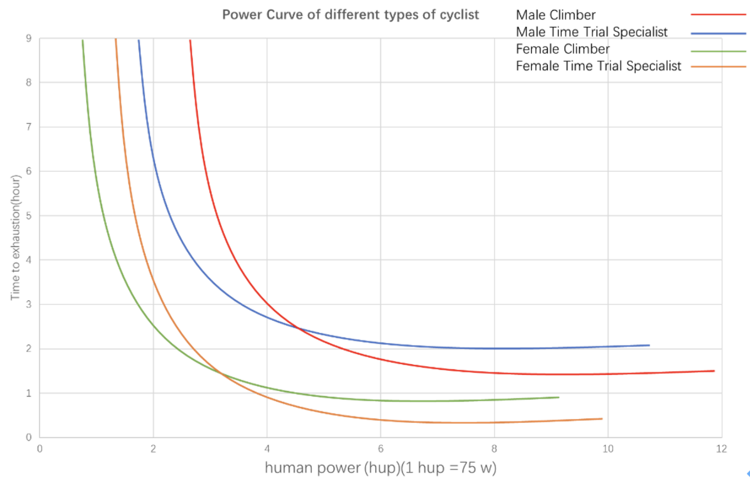
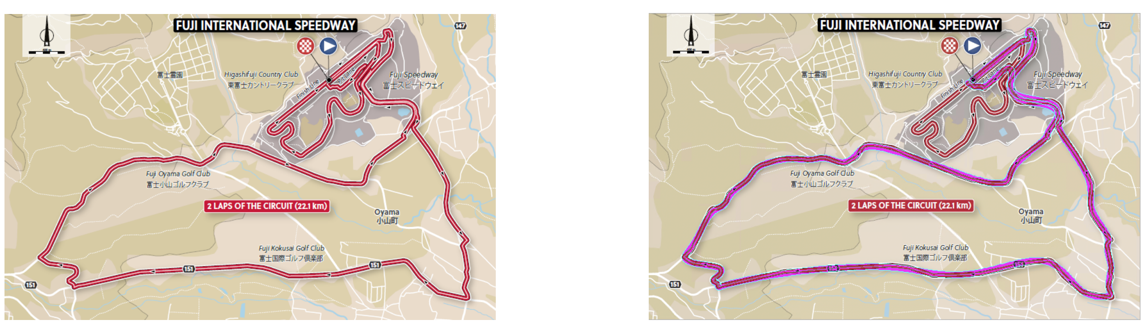


Figure 4.4: Time to exhaustion and power for cyclist [17]

5 Model Implementation and Results

5.1 Track data processing

As shown in Figure 5.1, we downloaded the map data of the road race from the Tokyo Olympic Games website, which contains the length of the course, the curvature of the corners, the elevation and other information.



(a) Original track (b) Processed track by ImageJ

Figure 5.1: The track of 2021 Olympic Time Trial course in Tokyo [8]

Using the Kappa plug-in of ImageJ software, we can fit the curve of the full track and get the curvature information of each point of the track with a suitable scale set.

As we mentioned above, the maximum speed of a bike through a corner can be calculated based on the curvature of the track. And we believe that the maximum speed of a cyclist in a time trial will not exceed $25\,\mathrm{m/s}$, so we truncate the calculated maximum speed as in Figure 5.1. The method for calculating the maximum speed is the previously mentioned formula $v = \sqrt{\mu g p}$. Every downward spike means a bend in the road. Following this approach, we can automatically divide the entire track into 14 straights and 13 corners.

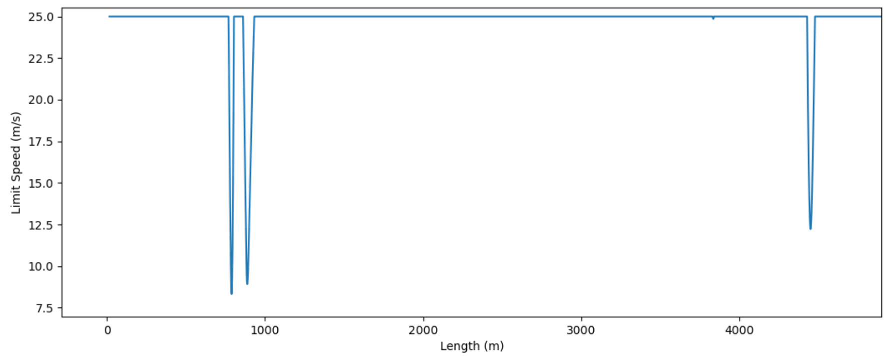


Figure 5.2: Maximum speed of the track

Finally, we need to calculate the average slope using the provided height map and apply it to the different stages of the division.

5.2 Simulation environment parameters

Listed in Table 2 are some of the parameters used to simulate the ride. Among them, the difference in physical characteristics can be adjusted according to the gender of the different athletes, the mass of the bike and the athlete.

The parameter P_{a_i} is the focus of our attention because it directly determines the power distribution of the athlete in different stages, which is the parameter we need to optimize.

Table 2: Simulation environment parameters [9]

Abbreviation	Value	Unit	Description
m_a	$50 \sim 70$	kg	Mass of the cyclist
m_b	$7 \sim 10$	kg	Mass of the bicycle
g	9.8	$\mathrm{m/s^2}$	Gravitational acceleration
C_{rr}	0.005	-	Rolling friction coefficient
μ	0.7	-	Sliding friction coefficient
Δt	0.1	s	Simulation time interval
$C_d A$	0.26	$\mathrm{m^2}$	Air-resistance coefficient
ρ	1.225	$\mathrm{kg/m^3}$	Air density
P_{a_i}	$200 \sim 900$	W	Power output parameters in each straightaway stage

5.3 Flow of the simulation program

The flow chart of the simulation program is given in Figure 5.3.

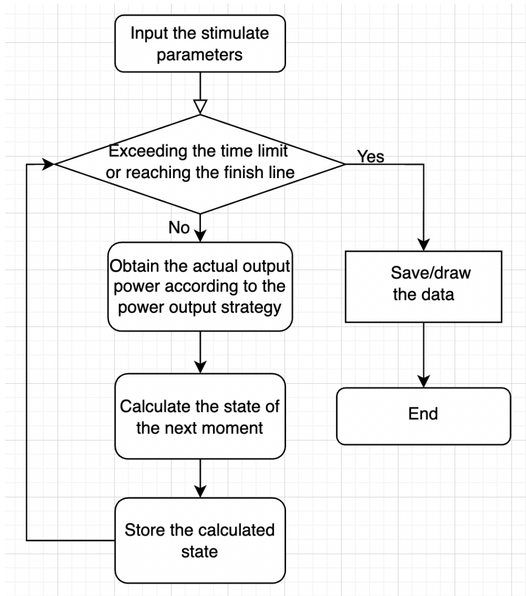


Figure 5.3: Flow of the simulation program

For any moment t in the simulation, there is a current velocity v, a current distance x. Then for a particular power output parameter in one simulation there is $P = \text{policy}(P_a, x)$, we can calculate:

$$a \leftarrow \frac{\frac{P}{v} - mg\sin\theta - C_{rr}mg\cos\theta - \frac{1}{2}\rho C_d A(v+w)^2}{m} [9]$$

Then it is possible to calculate:

$$v \leftarrow v + a\Delta t, x \leftarrow x + v\Delta t$$

5.4 Analysis with constant power output

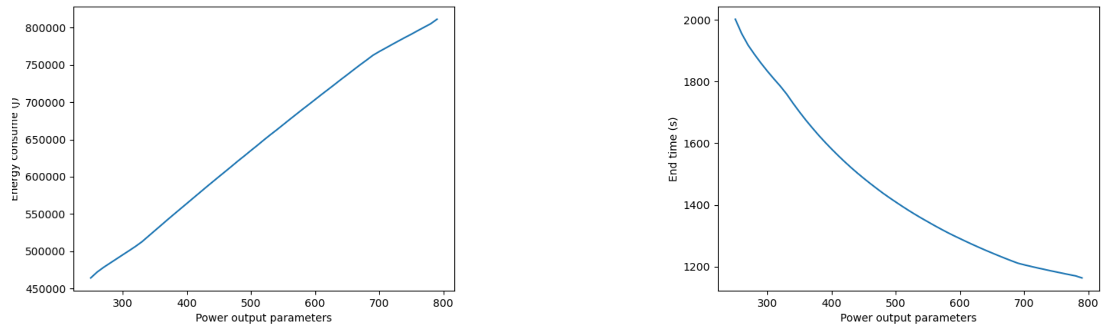
In this section, we set the power output parameter (P_{a_i}) for all straight stages to a constant value of P_o. The relationship between P_o and final energy consumption (E_c), and race performance (end time t), is investigated.

As can be seen from Figure 1, the energy consumption shows a linear increase as the overall power output parameter increases, while the final time deceleration decreases. In other words, for a stage, the average speed increase from a large increase in energy consumption is limited, so optimizing the power distribution can improve the final performance with limited energy.

5.5 Optimize power parameters

We selected $P_o = 400$ as the baseline for the optimization and ran the optimization program to obtain the following data. The energy consumption $E = 600\,197$ J, and the end time $t_e = 1486.4$ s

According to the mentioned simulation process, our optimization objectives:



(a) Energy Consumption (b) End Time

Figure 5.4: Relationship between power parameters and energy consumption, time consumption

$T = \sum_1^N \Delta t$,N is the total number of time intervals experienced by the simulation.

Constraints:

$$0 \leq P_a \leq P_{max}$$
$$\sum_1^N \mathsf{Policy}(P_a, x)\Delta t \leq E_0 + P_0 T$$

Since it is difficult to find the gradient of the objective function with respect to the parameters, we use a particle swarm algorithm. PSO is a metaheuristic as it makes few or no assumptions about the problem being optimized and can search very large spaces of candidate solutions. Also, PSO does not use the gradient of the problem being optimized, which means PSO does not require that the optimization problem be differentiable as is required by classic optimization methods such as gradient descent and quasi-newton methods. However, metaheuristics such as PSO do not guarantee an optimal solution is ever found. And because of the computational complexity of the problem, it takes a lot of time to run the particle swarm algorithm once and requires constant exploration of the best parameter choices, each of which should be run many times to ensure that the global optimal solution is found.

Table 3 shows the results of our optimization

Table 3: Simulation environment parameters

E_0 (J)	P_{max} (W)	T (s)
600000	800	1379.6
600000	850	1376.8
600000	900	1373.
650000	800	1294.8

Continued on next page

Continued Table

E_0 (J)	P_{max} (W)	T (s)
650000	850	1290.7
650000	900	1288.

Compared to the baseline, our optimized result was able to get a 110s improvement in performance with a very close energy consumption.

With the same energy consumption, increasing the power output capability helps improve performance, but it is very insignificant. After boosting the total energy output of the athlete, increasing the maximum power output is the only way to get a significant performance improvement.

6 Sensitivity Analysis

6.1 Wind direction and force

To simplify the model, the wind speed and direction discussed in this section do not change throughout the race. We choose the optimal parameter of $P_m = 850\,\mathrm{W}$ as the baseline in the above section, and add the effect of wind in the simulated environment to test the final performance of the runners, as well as the energy consumption.

Due to the long east-west length of the Olympic track we used, we chose a due east direction with a due west wind in order to be able to highlight the effect of wind speed.

As shown in Figure 6.1, for the original power output parameters, the time consumed always increases with increasing wind speed for eastward winds. For the westward wind, the consumption time slightly decreases at wind speeds less than $3\,\mathrm{m/s}$, and then it also starts to increase. And for the consumed energy, the westward wind always increases, while the eastward wind is decreasing until $4\,\mathrm{m/s}$ and then rises.

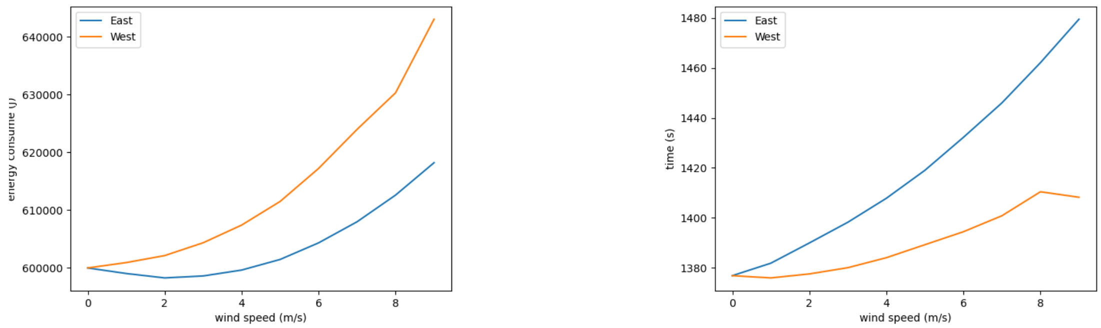
The reason for this situation is that the track we have chosen is approximately closed, that is, for two sets of parallel and opposite winds, the projection of the distance affected is always equal. Then the factors that affect the final time are the complex terrain undulation data, the current speed of the athlete, the position of the turn, etc.

If we analyze the absolute magnitude of the resistance we can see that the wind resistance has the greatest influence, which also coincides with the degree of influence in Figure 6. In conclusion, wind, wind direction is crucial to the impact of athletes, and the wind information should be obtained in time before the race and brought back into the model to calculate the optimal solution.

6.2 Deviations from the target power

In the same way as the wind discussion, we still use the optimal parameter of $P_m = 850\,\mathrm{W}$ as the baseline in the above section. Then we add some Gaussian noise to these power parameters, adjust the mean value of Gaussian noise, and explore the influence of move near the power parameters on the final time.

As shown in 6.2, if we add-50 to 50 noise, the final result will be significantly affected, with a



(a) Energy Consume (b) End Time

Figure 6.1: The effect of wind power on energy consumption and time consumption

minimum of about 15 seconds and a maximum of about 30 seconds.

The reason for this may lie in the cumulative effect of adjustments to all parameters. That is, if the athlete fails to reach the target power curve many times, the error of the final result will gradually accumulate.

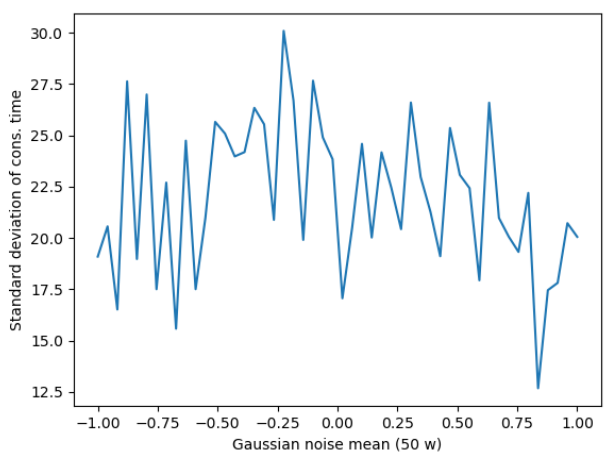


Figure 6.2: the standard deviation of different mean noise

7 Further Discussion

7.1 Team cycling

Team cycling is a type of time trial, which is a good reflection of the teamwork between teams. In the competition, each team of 6 athletes participate in the competition, between the team and the team from 2 to 3 minutes away from the start. During the race, six athletes rode at high speed in formation according to the wind direction. Each person in turn in front of the leader about 200 meters down to the end of the team, the exchange of lead riding. At the end of the race, the team's time of arrival of the fourth player shall be taken as the team's result. The selection of team members and the Order of cycling play a key role in the outcome of the race.

1. According to the aerodynamics, when players vary greatly in height and size, the air can become turbulent and turbulent, affecting the execution of tactics. So choose athletes who are as similar in size as possible.

2. Use the lead cyclist in conjunction with the tactics to overcome the effects of wind on the team. The lead cyclist is on the upwind side of the team. His role is to minimize wind resistance to the other cyclists. Usually the team members take turns as the lead cyclist. In this model, we discuss the riding characteristics of two types of cyclists, one is explosive cyclists, they can produce a large instantaneous power, in a short period of time has a very good performance, but the disadvantage is that high power output requires a long recovery time; one type of endurance players, they produce instantaneous power is not as big as the explosive players, but can be sustained and stable output power. For their lead horse arrangement to win the race.

7.2 Strategy suggestions

- For the acceleration stage of the race, it is necessary for the burst cyclist to lead the horse in the front, so as to motivate the team in a shorter period of time, to reach a predetermined cruising speed ahead of time, and then to quickly push the burst cyclist back, give him a long rest.

- For the straight leg of the race, the endurance cyclist can lead the horse, so that the team to maintain a relatively stable power output, this time allows the explosive player in the small wind resistance to restore physical strength, for the subsequent stages of preparation.

- For the multi-curve stage of the race, it is best not to use the explosive cyclist to lead the horse. The reduction of the output power in corners is a great waste of the explosive cyclist's physical strength and needs to be adjusted in time.

- On the downhill leg, let the explosive cyclist lead, and use the potential energy to lead the team to speed advantage.

- At the end of the sprint, let the burst cyclist take the lead. If necessary, the burst cyclist and the faster endurance cyclist can break away from the pack and ride at full speed with maximum power output, reach your burn-out goal at the finish line.

8 Strengths and Weaknesses

8.1 Strengths

- Our model can be easily applied to other power output modes without modifying the overall simulation logic.

- Our model can easily analyze data from other tracks. Based on the track maps and elevation information provided by the organizers, the same analysis methods can be applied to obtain relatively accurate track information for use in the model.
- Our track data comes from official Olympic documents and the data has a high degree of credibility.
- Our model has a relatively simple division of the track, which makes it easy for the athletes to follow the power output strategy.

8.2 Weaknesses

- Our model has a high requirement for the accuracy of the power output of the athlete, otherwise there will be a decrease in the effect. This is more difficult to achieve for the average athlete.
- Our model is a bit rough for the modeling of the track, not modeling the elevation data of each point of the real, but calculating the angle of the ramp in stages, making the model and reality have a certain pian li
- Some parameters in our model are taken from the general literature and are not really measured and may differ from reality.

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Appendices

Appendix A cyclist's race guidance for a Directeur Sportif.

Dear Directeur Sportif

I am a college student participating in the 2022 MCM and am honored to be your team's technical advisor, providing reference and guidance for the physical fitness distribution of your team from a mathematical model perspective.

First of all, we need to understand that we can divide the race into the acceleration stage, the halfway stage and the sprint stage according to the driving speed. Depending on the type of track, we can be divided into straight and curve sections. In addition, we have summed up two types of strength for athletes. One is the burst type, where they can cycle at a high intensity for a short period of time, but have a longer recovery time. The other is the endurance type, where they can consistently deliver a moderate intensity of power and have a shorter recovery time. Finally, the gender difference also has the certain influence, the male athlete's explosive power is strong, the endurance is good, the female athlete must be slightly weaker.

After understanding the above, we will introduce the cyclists in the straight race. Changes in physical energy in the process. At the start, the athletes themselves have a certain physical storage, at this time the physical storage reached the upper limit. During the acceleration stage, the output power is maintained at a high level. The body expends physical energy, and the physical energy stored in the body is less than the upper limit. During the en route stage, the intensity of the ride is reduced, the power output is reduced, and there is a brief rebound, culminating in an upper limit. In the sprint stage, the output power reaches maximum, to achieve higher speed, at the end of the physical exhaustion. For the curve, the driver needs to brake to bring the speed down to a lower level (too much speed can cause a fall) . But in the curve section physical ability will not have the obvious change.

So, the best strategy is to speed up on the first stretch of the straight, with the cyclist riding for the length of the stretch with maximum impact, and then accelerate on the way, the ideal condition should be the uniform power ride. As you enter the turn, you need to slow down to the maximum speed you can go around the corner to prevent a fall, without using any energy. After the turn, you immediately go into the acceleration stage, and the next straight and turn is the same strategy as before, and so on. At some point in the back of the final straight, the cyclist will enter the sprint stage, where the cyclist expends all his energy and uses all his inertia to get across the finish line.

Below, I'll give you some helpful tips on how to train your team using the model we've built:

1. We modeled burst and endurance players and found that burst players didn't have a clear advantage in the time trial, short-term high-intensity power output can not offset the impact of long-term physical recovery. Therefore, in the selection and development of athletes, to choose those endurance cyclists, rather than strong explosive.

2. Our team added wind power to the model and found that players who were able to adjust to changes in the direction of the wind did better.

3. cyclists who have a better grasp of the timing and pace of the transition between the Acceleration stage and the midcourse stage are better able to complete the race.

4, in the training team players, the first to choose the height, build similar players to the group can largely avoid the impact of turbulence. Secondly, the choice of the lead cyclist and the Order of the lead cyclist in the race should be carefully selected.

5. During the training, through the specialized equipment and personnel to the athlete's oxygen consumption, the riding power carries on the detailed measurement, then understands the athlete's strength type, formulates the more specialized training plan.

6. Before a race, if possible, ask experts to model the course as best they can. The terrain parameters such as the length of the track, the position of the curve, the radius of curvature, the coefficient of friction with the road surface, the temperature and humidity of the air are obtained.

Sincerely yours,

Your friends

Appendix B Part of the simulation code

```
if __name__ == '__main__':
    secs, straight_count, curve_count = load_track_data("../track3.csv")
    print("straight: {}, curve: {}".format(straight_count, curve_count))
    for s in secs:
        print(s)

    secs_degree = [-1.88] * 4 + [1.06] * 4 + \
        [2.34] * 10 + [-1.938] * 2 + [2.57] * 7

    # straight parameters
    params = [372.41699569, 0.,      581.5762856, 750.,      750.,
              31.69604852, 750.,    750.,       750.,          0.,
              662.52916961, 572.97421331, 387.78628176, 680.46269502]
    ps.append(params[0])

    for i in range(int(alltime / intern)):
        if x > secs[-1].end_x:
            break

        # find section index
        index = 0
        for index in range(len(secs)):
            if (secs[index].isInSection(x)):
                break
        sec = secs[index]
        theta = secs_degree[index] / 180 * math.pi
        if sec.type == 0:
            # straight
            Pout = calc_straight_pout(params[int(index / 2)], a, v)
            a = (Pout / v - G * math.sin(theta) - Crr *
                 G * math.cos(theta) - 1/2*k*v*v) / m
            v = v + a * intern
            x = x + v * intern
            vs.append(v)
            xs.append(x)
            ts.append(ts[-1] + intern)
            ass.append(a)
            ps.append(Pout)
        else:
            # curve
            if (v <= sec.max_speed):
                Pout = calc_curve_pout(v)
                a = 0
                v = v
                x = x + v * intern
                vs.append(v)
                xs.append(x)
                ts.append(ts[-1] + intern)
                ass.append(a)
                ps.append(Pout)
```

```
        else:
            Pout = calc_curve_pout(sec.max_speed)
            a = 0
            v = sec.max_speed
            x = x + v * intern
            vs.append(v)
            xs.append(x)
            ts.append(ts[-1] + intern)
            ass.append(a)
            ps.append(Pout)
        pass
    # calc the energy consume
    E_consume = 0
    for p in ps:
        E_consume += p * intern
    print("E_consume: {}".format(E_consume))
    print("time_consume:{}".format(ts[-1]))
```