Econometrics - TA Session 1

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Programming

Let's wait and see what we can achieve today. Maybe next Tuesday.

Something to say in Problem Set 1

1. Denote the subscript when you have multiple pdf: $f_X(x), f_Y(y), f_{XY}(x, y)$
2. It is better to write $\int_{x\in\Omega}$ (or simply $\int_X$) instead of $\int_{-\infty}^{\infty}$ if the support is not given.
3. Assign the **seed** beforehand when you do simulation. (very useful when you conduct research)

Linear Algebra and derivatives

> $$\frac{\partial}{\partial \beta}\alpha'\beta = \alpha, \quad \frac{\partial}{\partial \beta}\beta'\alpha = \alpha$$
>
> $$\frac{\partial}{\partial \beta}\beta' A\beta = (A + A')\beta$$

Why the following is correct?

$$(X'X)^{-1}X'\varepsilon = \sum_{i=1}^{n}(x_i'x_i)(\sum_{j=1}^{n}x_j\varepsilon_j)$$

Proof:

$$X'X = \begin{pmatrix} x_{11} & \cdots & x_{N1} \\ \vdots & & \vdots \\ x_{1K} & \cdots & x_{NK} \end{pmatrix}_{K\times N} \begin{pmatrix} x_{11} & \cdots & x_{1K} \\ \vdots & & \vdots \\ x_{N1} & \cdots & x_{NK} \end{pmatrix}_{N\times K}$$

$$\Rightarrow (X'_{K\times N}X_{N\times K})_{ij} = \sum_{n=1}^{N}(X')_{in}(X)_{nj} = \sum_{n=1}^{N}(X)_{ni}(X)_{nj} = \sum_{n=1}^{N}x_{ni}x_{nj}$$

The dimension of N is absorbed through summation.

$$\sum_{i=1}^{N}x_i'x_i = \sum_i \begin{pmatrix} x_{i1} \\ \vdots \\ x_{iK} \end{pmatrix}(x_{i1}, ..., x_{iK}) = \sum_i \begin{pmatrix} x_{i1}x_{i1} & \cdots & x_{i1}x_{iK} \\ \vdots & & \vdots \\ x_{iK}x_{i1} & \cdots & x_{iK}x_{iK} \end{pmatrix}$$

$$\Rightarrow (\sum_{n=1}^{N}x_n'x_n)_{ij} = \sum_{n=1}^{N}x_{ni}x_{nj}$$

Variance-Covariance Matrix

$$Var[X'\varepsilon] = E[X'\varepsilon(X'\varepsilon)'] = E[X'\varepsilon\varepsilon' x]$$

Proof for Chebyshev's Inequality

$$P\{|X| \geq \varepsilon\} \leq \frac{E|X|^k}{\varepsilon^k}, \forall\varepsilon > 0, \forall k > 0.$$

Proof:

$$\mathbf{1}\{|X| \geq \varepsilon\} \leq \left|\frac{X}{\varepsilon}\right|^k$$

$$\Rightarrow \quad P(\{|X| \geq \varepsilon\}) = \mathbf{E}[1\{|X| \geq \varepsilon\}] \leq \frac{E|X|^k}{\varepsilon^k}$$

Convergence Mode

The most important two convergence mode is $\xrightarrow{p}$ and $\xrightarrow{d}$.

> Q1: Is convergence in quadratic mean the same as convergence in probability?
>
> A1: No.

$$\begin{array}{ccccc} & & \text{q.m.} & & \\ & & \downarrow & & \\ \text{a.s.} & \rightarrow & \text{prob} & \rightarrow & \text{distribution} \end{array}$$

LLN & CLT

LLN: diff between them

The oldest version: Bernoulli LLN

$$P(X=1) = p, P(X=0) = 1-p,$$

Weak LLN

$$\{x_i\}_{i=1}^n, x_i \overset{i.i.d}{\sim} \mathcal{F}, \mu < \infty, \sigma^2 < \infty,$$

$$\Rightarrow \bar{x}_n \xrightarrow{p} \mu$$

Khinchin's WLLN

$$\{x_i\}_{i=1}^n, x_i \overset{i.i.d}{\sim} \mathcal{F}, \mu < \infty,$$

$$\Rightarrow \bar{x}_n \xrightarrow{p} \mu$$

Chebyshev's WLLN

$$\{x_i\}_{i=1}^n, E[x_i] = \mu_i < \infty, Var[x_i] = \sigma^2 < \infty,\ still\ independent$$

Assume

$$\bar{\sigma}_n^2 = n^{-1} \sum_i \sigma_i^2 \overset{p \to \infty}{\to} 0$$

$$\Rightarrow \operatorname{plim}(\bar{x}_n - \bar{\mu}_n) = 0$$

CLT: def & diff

Lindeberg-Levy CLT

$$\{x_i\}_{i=1}^n, x_i \overset{i.i.d}{\sim} \mathcal{F}, \mu < \infty, \sigma^2 < \infty$$

$$\Rightarrow \sqrt{n}(\bar{x}_n - \mu) \overset{d}{\to} N(0, \sigma^2)$$

Lindeberg-Feller CLT

$$\{x_i\}_{i=1}^n, x_i \sim F(\mu_i, \sigma_i^2),\ still\ independent$$

Assume

$$\lim \frac{\max(\sigma_i^2)}{n\bar{\sigma}_n^2} = 0, \lim \bar{\sigma}_n^2 = \bar{\sigma}^2,$$

then

$$\sqrt{n}(\bar{x}_n - \mu) \overset{d}{\to} N(0, \bar{\sigma}_n^2)$$

Why this course more mathematic than economic?