

Econometrics - TA Session 1

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Programming

Let's wait and see what we can achieve today. Maybe next Tuesday.

Something to say in Problem Set 1

1. Denote the subscript when you have multiple pdf: $f_X(x)$, $f_Y(y)$, $f_{XY}(x, y)$
2. It is better to write $\int_{x \in \Omega}$ (or simply \int_X) instead of $\int_{-\infty}^{\infty}$ if the support is not given.
3. Assign the **seed** beforehand when you do simulation. (very useful when you conduct research)

Linear Algebra and derivatives

$$\frac{\partial}{\partial \beta} \alpha' \beta = \alpha, \quad \frac{\partial}{\partial \beta} \beta' \alpha = \alpha$$

$$\frac{\partial}{\partial \beta} \beta' A \beta = (A + A') \beta$$

Why the following is correct?

$$(X'X)^{-1}X'\varepsilon = \sum_{i=1}^n (x'_i x_i) \left(\sum_{j=1}^n x_j \varepsilon_j \right)$$

Proof:

$$X'X = \begin{pmatrix} x_{11} & \cdots & x_{N1} \\ \vdots & & \vdots \\ x_{1K} & \cdots & x_{NK} \end{pmatrix}_{K \times N} \begin{pmatrix} x_{11} & \cdots & x_{1K} \\ \vdots & & \vdots \\ x_{N1} & \cdots & x_{NK} \end{pmatrix}_{N \times K}$$

$$\Rightarrow (X'_{K \times N} X_{N \times K})_{ij} = \sum_{n=1}^N (X')_{in} (X)_{nj} = \sum_{n=1}^N (X)_{ni} (X)_{nj} = \sum_{n=1}^N x_{ni} x_{nj}$$

The dimension of N is absorbed through summation.

$$\sum_{i=1}^N x'_i x_i = \sum_i \begin{pmatrix} x_{i1} \\ \vdots \\ x_{iK} \end{pmatrix} (x_{i1}, \dots, x_{iK}) = \sum_i \begin{pmatrix} x_{i1} x_{i1} & \cdots & x_{i1} x_{iK} \\ \vdots & & \vdots \\ x_{iK} x_{i1} & \cdots & x_{iK} x_{iK} \end{pmatrix}$$

$$\Rightarrow \left(\sum_{n=1}^N x'_n x_n \right)_{ij} = \sum_{n=1}^N x_{ni} x_{nj}$$

Variance-Covariance Matrix

$$\text{Var}[X'\varepsilon] = E[X'\varepsilon(X'\varepsilon)'] = E[X'\varepsilon\varepsilon'x]$$

Proof for Chebyshev's Inequality

$$P\{|X| \geq \varepsilon\} \leq \frac{E|X|^k}{\varepsilon^k}, \forall \varepsilon > 0, \forall k > 0.$$

Proof:

$$\mathbf{1}\{|X| \geq \varepsilon\} \leq \left| \frac{X}{\varepsilon} \right|^k$$

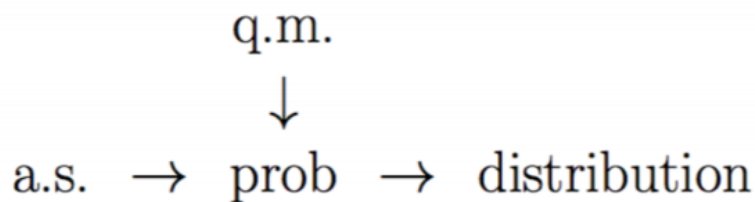
$$\Rightarrow P(\{|X| \geq \varepsilon\}) = \mathbf{E}[\mathbf{1}\{|X| \geq \varepsilon\}] \leq \frac{E|X|^k}{\varepsilon^k}$$

Convergence Mode

The most important two convergence mode is \xrightarrow{p} and \xrightarrow{d} .

Q1: Is convergence in quadratic mean the same as convergence in probability?

A1: No.



LLN & CLT

LLN: diff between them

The oldest version: Bernoulli LLN

$$P(X = 1) = p, P(X = 0) = 1 - p,$$

Weak LLN

$$\begin{aligned} \{x_i\}_{i=1}^n, x_i \stackrel{i.i.d}{\sim} \mathcal{F}, \mu < \infty, \sigma^2 < \infty, \\ \Rightarrow \bar{x}_n \xrightarrow{p} \mu \end{aligned}$$

Khinchin's WLLN

$$\begin{aligned} \{x_i\}_{i=1}^n, x_i \stackrel{i.i.d}{\sim} \mathcal{F}, \mu < \infty, \\ \Rightarrow \bar{x}_n \xrightarrow{p} \mu \end{aligned}$$

Chebyshev's WLLN

$$\{x_i\}_{i=1}^n, E[x_i] = \mu_i < \infty, Var[x_i] = \sigma^2 < \infty, \text{ still independent}$$

Assume

$$\bar{\sigma}_n^2 = n^{-1} \sum_i \sigma_i^2 \xrightarrow{p \rightarrow \infty} 0$$

$$\Rightarrow \text{plim}(\bar{x}_n - \bar{\mu}_n) = 0$$

CLT: def & diff

Lindeberg-Levy CLT

$$\{x_i\}_{i=1}^n, x_i \stackrel{i.i.d}{\sim} \mathcal{F}, \mu < \infty, \sigma^2 < \infty$$

$$\Rightarrow \sqrt{n}(\bar{x}_n - \mu) \xrightarrow{d} N(0, \sigma^2)$$

Lindeberg-Feller CLT

$$\{x_i\}_{i=1}^n, x_i \sim F(\mu_i, \sigma_i^2), \text{ still independent}$$

Assume

$$\lim \frac{\max(\sigma_i^2)}{n\bar{\sigma}_n^2} = 0, \lim \bar{\sigma}_n^2 = \bar{\sigma}^2,$$

then

$$\sqrt{n}(\bar{x}_n - \mu) \xrightarrow{d} N(0, \bar{\sigma}_n^2)$$

Why this course more mathematic than economic?