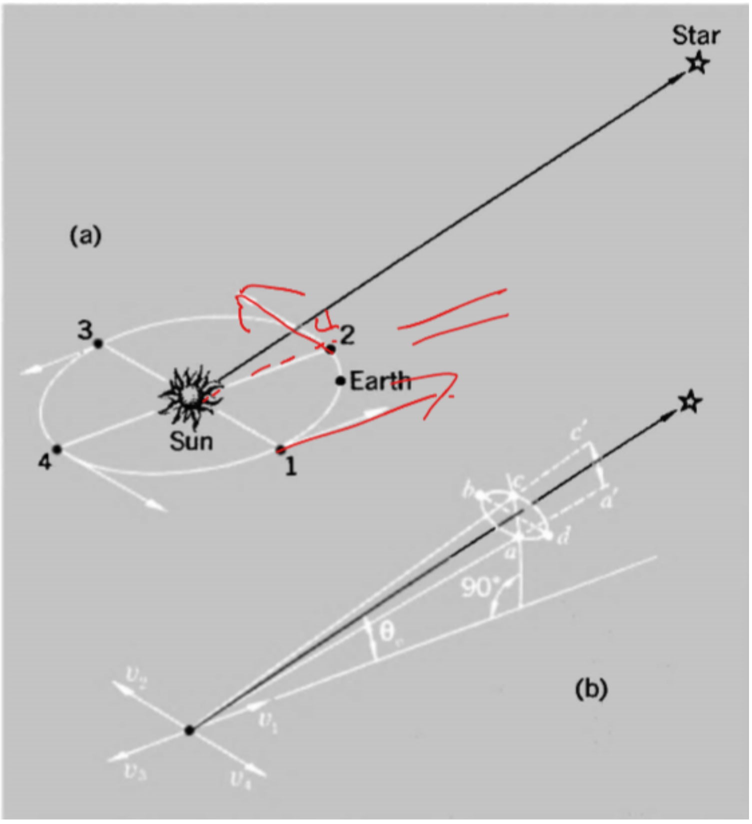
Perplexities in the propagation of light

Stellar aberration



(a) depicts the positions of the earth moves around the sun 1-2-3-4

(b) shows how the apparent positions of the star changes with a elliptical path a-b-c-d (1-2-3-4). The change in position is caused by the velocity of the Earth rather than its position.

If we only consider the position of the earth like (a) does, the altitude of the star should be greatest at 2 and lowest at 4. However, that does not fit the real observation, which indicates the altitude is greatest at 3 and least at 1.

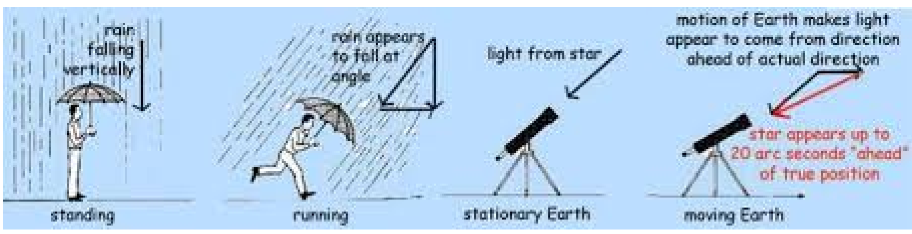


fig 1.2

This phenomenon can be explained the same way as a man running in a rain does. If the man is still relative to the horizontal velocity of the rain, it seems to the man the rain only have a velocity of w pointing downwards. Nevertheless,when man is running at a velocity v to the right. To the coordinate of the man, the rain seems to also have a horizontal velocity of $-v$. Therefore, the rain have a magnitude of $\sqrt{v^2+w^2}$ and direction of $\arctan v/w$ along the straight line inclines to the vertical.

Explain in an alternative way

It takes light $t=\frac{L/\cos\alpha}{c+\cos\theta_0 v}$ to travel from the top of the telescope to the bottom. Meanwhile, the telescope travels for $\Delta x = t\sin\theta_0 v$. Therefore $\tan\alpha = \frac{\Delta x}{L/\cos\alpha}$

$$\begin{aligned}
\tan\alpha = \frac{\Delta x}{L/\cos\alpha} &= \frac{L/\cos\alpha}{c+\cos\theta_0 v}\frac{\sin\theta_0 v}{L/\cos\alpha} \\
\because c \gg \cos\theta_0 v \\
\therefore c+\cos\theta_0 v \approx c \\
\tan\alpha = \frac{vsin\theta_0}{c} \\
\because as\ \alpha \to 0\ \tan\alpha \to \alpha \\
\therefore \alpha = \frac{vsin\theta_0}{c}
\end{aligned} \tag{1}$$

At position 2 and 4 the Earth’s velocity is perpendicular to the line from the Sun to the star. Since there is no velocity towards the star, there is no change in altitude. But there do exist a aberration angle that is to b and d. Furthermore, the aberration angle is greatest - $\pm v/c$

When the Earth is at position 1 and 3, the aberration angle is shown by Eq. (1)

Thus the elliptical path have major axis (measured as an angle 2β) equal to $2v/c$ and a minor axis of $2\beta\sin\theta_0$

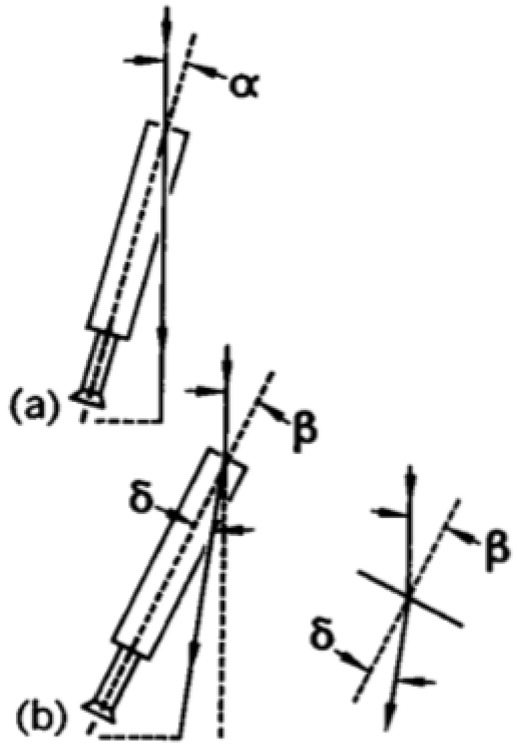
A modified aberration experiment

Suppose that a telescope has been aimed at a star whose true direction is at 90° to the plane of the earth's orbit.

Let the unknown aberration angle to be α, and the speed through the aether to be v. Now assume the telescope is filled with water of reflective index n.

The light travels slower in the water, so the time for the light to travel down the telescope will be lengthened - by the factor of n. Let assume the new aberration angle is β, and the refracted angle to be δ

$$n = \frac{\sin\beta}{\sin\delta} \approx \frac{\beta}{\delta}$$



The light travels downwards with speed of c/n, and the telescope is traveling with speed of v therefore:

$$\delta \approx \frac{v}{c/n} = \frac{nv}{c} \tag{2}$$

Although we don't have the true values of α, β, and δ. but we have:

$$\beta \approx n\delta \approx \frac{n^2 v}{c} \quad \alpha \approx \frac{v}{c}$$

Therefore,

$$\begin{aligned} \beta - \alpha &\approx \frac{n^2 v}{c} - \frac{v}{c} \\ &\approx (n^2 - 1)v/c \end{aligned} \tag{3}$$

therefore there should exist an angle difference between the abbreviation angle with water β and without water α. Nevertheless, the experiment result show a null result, which indicates that $\beta = \alpha$.

Explain by Fresnel's *drag coefficient*

We have the aberration angle $\alpha = \beta = v/c$, the length of the telescope l, the time $t$taken for the light to pass the telescope filled with water $t = nl/c$. The telescope moves a distance of vt. And we suppose that there is a drag coefficient f, for which drags the light with a fraction f of speed v .

To have the light emerge at the center of the eyepiece, the displacement of the telescope should equal to the sum of displacement due to refraction ($l\delta$) and due to dragging effect (fvt)

Hence,

$$
\begin{aligned}
vt &= l\delta + fvt \\
\because l = ct/n \ \ \delta &= \alpha/n = v/nc \\
\therefore vt &= \frac{ct}{n}\frac{v}{nc} + fvt \\
0 &= 1/n^2 + f \\
f &= 1 - 1/n^2
\end{aligned} \tag{4}
$$

The Fizeau's measurement of the drag coefficient

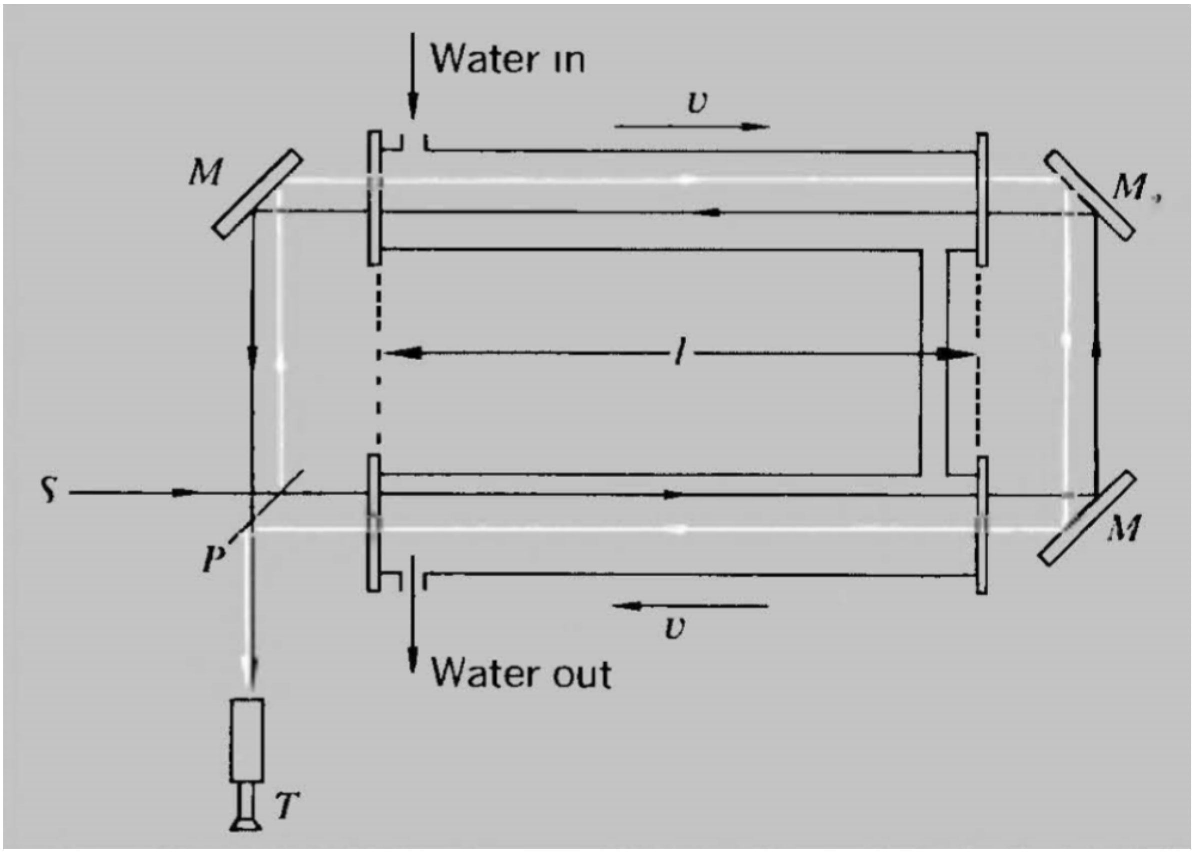


fig. from P47 fig 2-5

In the Fizeau's experiment, one semitransparent glasses(P) and three mirror(M) are used. When a light shoot on the semitransparent glass, half is reflected and take the white path, the other half passes through the glass and follows the black. As we can see from the graph, the white path always have the same direction as the water flow, while the black path always have the opposite direction as the water flow. If the drag coefficient does exists, a path difference should be observed, by a shift in interference pattern in the telescope (T) before the present of the water flow and after it.

In the previous section we have the frag coefficient equals to $f = 1 - 1/n^2$. The light speed in still water is $c_{water} = c/n$. Taking account of f, for white path $c_w = c/n + vf$, for black path $c_b = c/n - vf$. The time difference created is:

$$
\begin{aligned}
\Delta t &= 2l/c_b - 2l/c_w \\
&= \frac{2l}{c/n - vf} - \frac{2l}{c/n + vf} \\
&= \frac{2l \times 2vf}{(c/n)^2 - f^2v^2} \\
&= \frac{4lvf}{c^2(1/n^2 - f^2v^2/c^2)} \\
&= \frac{4lvfn^2}{c^2(1 - f^2v^2n^2/c^2)} \\
&\because fvn \ll c \\
&\therefore 1 - f^2v^2n^2/c^2 \approx 1 \\
\Delta t &\approx \frac{4n^2fvl}{c^2}
\end{aligned} \tag{5}
$$

This indicates that there is a light path difference of $c\Delta t$. Which can be expressed as a multiple δ of the wavelength λ of the light:

$$\delta = \frac{4n^2fvl}{c\lambda} \tag{6}$$

Therefore, the Fizeau's experiment suggests that if there exist moving aether, then the light must gain a part (f) of the velocity of the aether.

THE MICHELSON-MORLEY EXPERIMENT

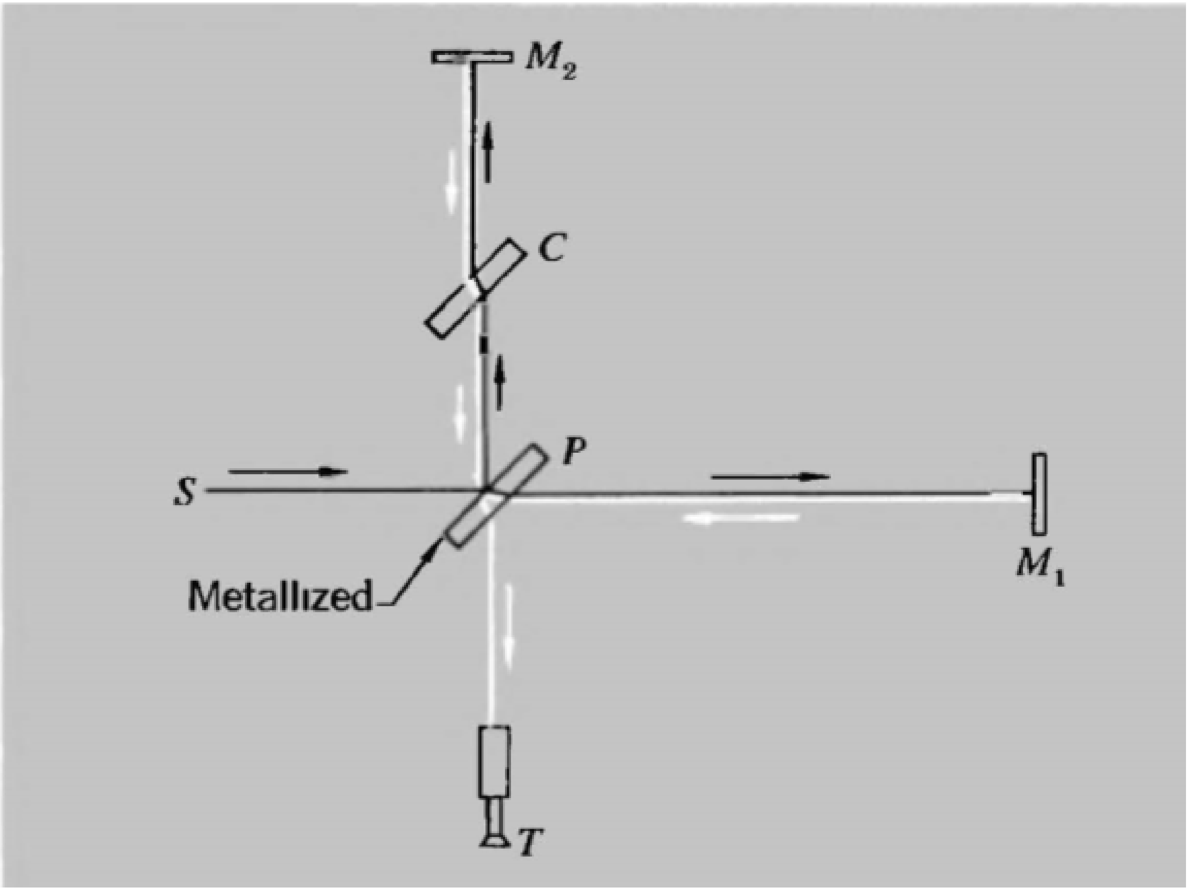


fig. from P51 fig 2-8

The light from S falls on the semi-transparent glass P which have a metal paint on the front side. Half of the light passes through directly and follows the path $P \to M_1 \to P \to T$. While the other half follow the path of $P \to C \to M_2 \to C \to P \to T$. C, which have the same thickness as P, is needed to compensate the path difference created by passing though the glass.

If the aether is still, then from the perspective of the Earth, there should be a 'aether wind' blowing pass the apparatus.

So, assume that the apparatus is moving with a speed v, to the direction of PM_1. To simplify the calculation, assuming that the glass P have zero thickness. As the result, we can remove the glass C too, and the apparatus turns to be like fig 2-10.

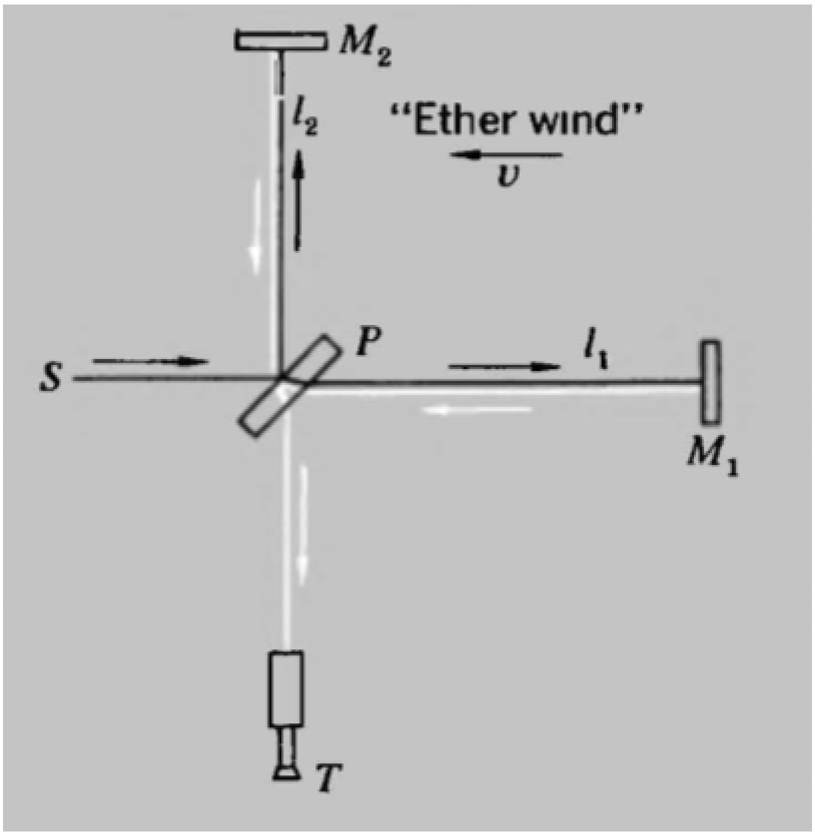


fig. from P55 fig 2-10

(we uses classical mechanics here)

First, we calculate the time needed for the light to travel from P to M_1 and back to P. Since the light should be still relative to the aether, so we have $c - v$ for P to M_1 and $c + v$ for M_1 to P. Setting this time to be t_1:

$$\begin{aligned} t_1 &= \frac{l_1}{c-v} + \frac{l_1}{c+v} \\ &= \frac{2l_1 c}{c^2 - v^2} = \frac{2l_1/c}{1 - v^2/c^2} \end{aligned} \tag{7}$$

For the path P to M_2. To gain a vertical path, The light must have a fraction of its velocity travel opposite to v. As the graph shows, the velocity of the light travelling along the PM_2 direction is only $\sqrt{c^2 - v^2}$

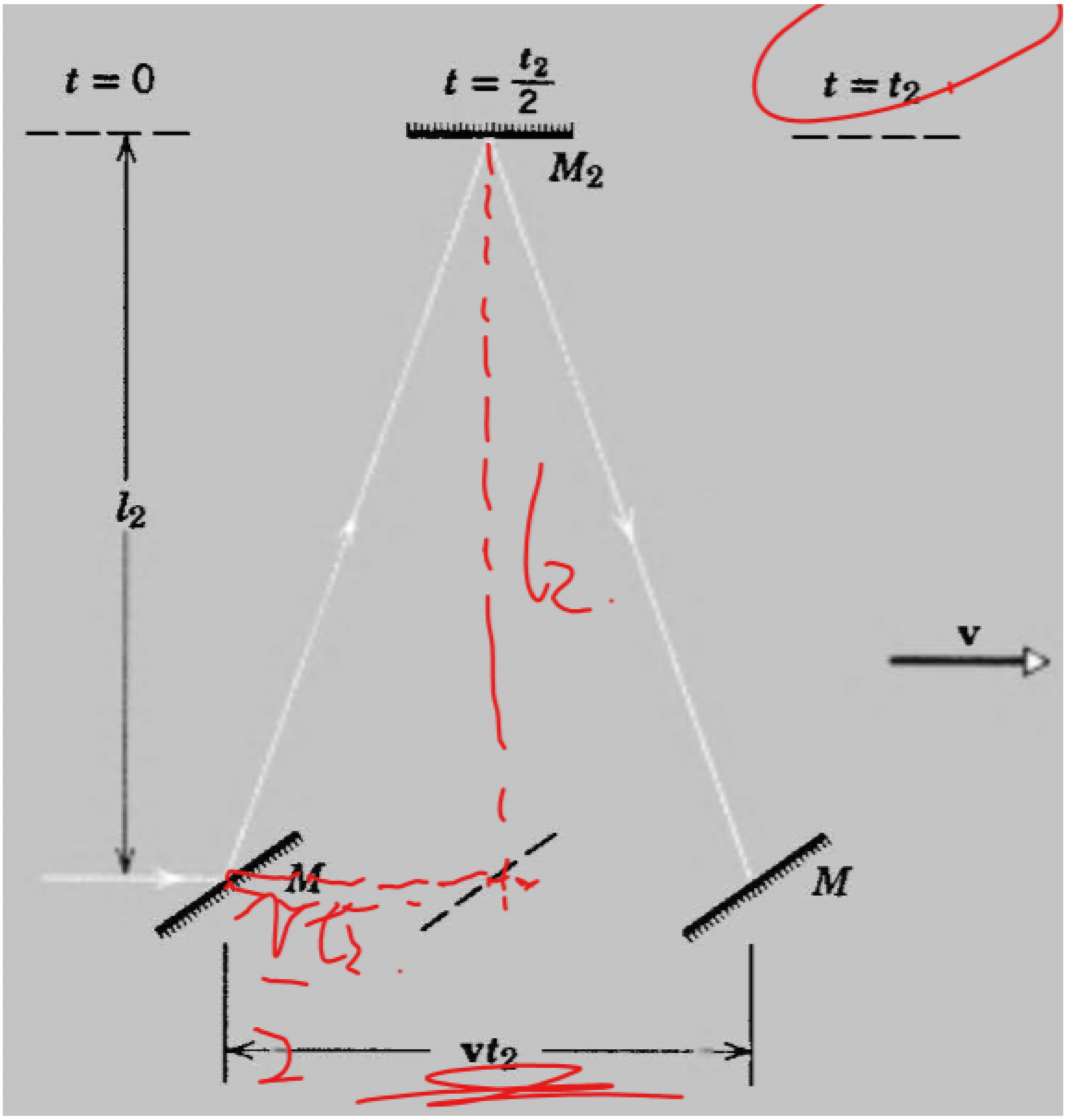


Fig 1-7 Introduction to special relativity, by Rensick Ch1, P22.

Therefore:

$$t_2 = \frac{2l_2}{\sqrt{c^2 - v^2}} = \frac{2l_2/c}{\sqrt{1 - v^2/c^2}} \tag{8}$$

From t_1 and t_2, we can deduce their time difference is given by:

$$\begin{aligned} \Delta t &= t_1 - t_2 \\ &= \frac{2l_1/c}{1 - v^2/c^2} - \frac{2l_2/c}{\sqrt{1 - v^2/c^2}} \end{aligned} \tag{9}$$

Then we turn the apparatus for 90°, so that PM_2 becomes parallel to the direction of the aether wind. Applying the same calculation as the previous circumstance, we can get:

$$\begin{aligned} t_1' &= \frac{2l_1/c}{\sqrt{1 - v^2/c^2}} \\ t_2' &= \frac{2l_2/c}{1 - v^2/c^2} \end{aligned}$$

$$\begin{aligned} \Delta t' &= t_1' - t_2' \\ &= \frac{2l_1/c}{\sqrt{1 - v^2/c^2}} - \frac{2l_2/c}{1 - v^2/c^2} \end{aligned} \tag{10}$$

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