Energy, momentum, and mass

For a photon, we have that

$$E = cp \tag{1}$$

$$E = mc^2 \tag{2}$$

combining the two, we have

$$m = \frac{p}{c} \tag{3}$$

if we consider the Newtonian mechanics, we have,

$$m = \frac{p}{v} \tag{4}$$

if we suppose that Eq.(2) describes a universal equivalence of energy and inertial mass, by combine Eqs. (2) and (4), we have:

$$E = \frac{c^2 p}{v} \tag{5}$$

the increment of kinetic energy is given by

$$dE = Fdx = \frac{dp}{dt}dx \tag{6}$$

i.e.

$$dE = vdp \tag{7}$$

by $E = cp$, and the speed of photon is c, we get

$$E\,dE = c^2 p\,dp \tag{8}$$

integrate both side,

$$\begin{gathered} \int E\,dE = c^2 \int p\,dp \\ \frac{E^2}{2} = \frac{c^2 p^2}{2} + c \end{gathered} \tag{9}$$

$$E^2 = c^2 p^2 + E_0^2 \tag{10}$$

where E_0^2 is a constant of integration, written as the square of some constant energy.

By substituting Eq (5) into Eq (10), it leads to:

$$\because E = \frac{c^2 p}{v}$$
$$\therefore p = \frac{Ev}{c^2}$$
$$\therefore E^2 = c^2 \frac{E^2 v^2}{c^4} + E_0^2$$
$$E^2 = \frac{E^2 v^2}{c^2} + E_0^2$$
$$E^2 (1 - \frac{v^2}{c^2}) = E_0^2$$

$$E(v) = \frac{E_0}{(1 - \frac{v^2}{c^2})^{1/2}} \tag{11}$$

We can apply binomial expansion when $v \ll c$, neglecting terms of higher order than v^2/c^2:

$$E(p) \approx E_0 + \frac{1}{2}(\frac{E_0}{c^2})v^2 \tag{12}$$

If Eq. (11) is to harmonise the Newtonian mechanics at low velocity, $\frac{E_0}{c^2}$ must be identified by the classical inertial mass of a particle, which can be denoted by m_0. Furthermore, if substitute $E = mc^2$ into Eq. (11), we have:

$$m(v) = \frac{m_0}{(1 - \frac{v^2}{c^2})^{1/2}} \tag{13}$$

the quantity m_0, which in Newtonian mechanics is the *inertial mass* of a body, now assume a new role as the **rest mass** of the body, and at any other speed the inertial mass is greater, and given by Eq. (13).

Eqs. (10) and (13) are two of the central results of the new dynamics. Eq. (10) gives the relation ship between energy and momentum. The difference between the total energy E and the rest energy E_0:

$$K = m_0 c^2 [\frac{1}{(1 - \frac{v^2}{c^2})^{1/2}} - 1] \tag{14}$$

Also by Eq. (13), we can get the new form of p and E, for a body moving at velocity $\mathbf{v}$

$$\mathbf{p} = m(v)\mathbf{v} \tag{15}$$

$$\mathbf{E} = m(v)c^2 \tag{16}$$

The denominator $\frac{1}{(1-\frac{v^2}{c^2})^{1/2}}$ appears so often in special relativity, a single symbol γ

$$\begin{aligned} m &= \gamma m_0 \\ p &= \gamma m_0 v \\ E &= \gamma m_0 c^2 \end{aligned} \tag{17}$$