Relativistic Doppler effect

In classical mechanics

In high school, we have learnt about Doppler effect of sound wave, skip to xxx if you think you are fine with it.

Since sound travels at a much lower speed relative to light, therefore relativistic effect is negligible. First let us consider that the sender moving at a speed of u towards the receiver. The speed of sound is v, and the sent and received frequency are f_s and f_r, respectively.

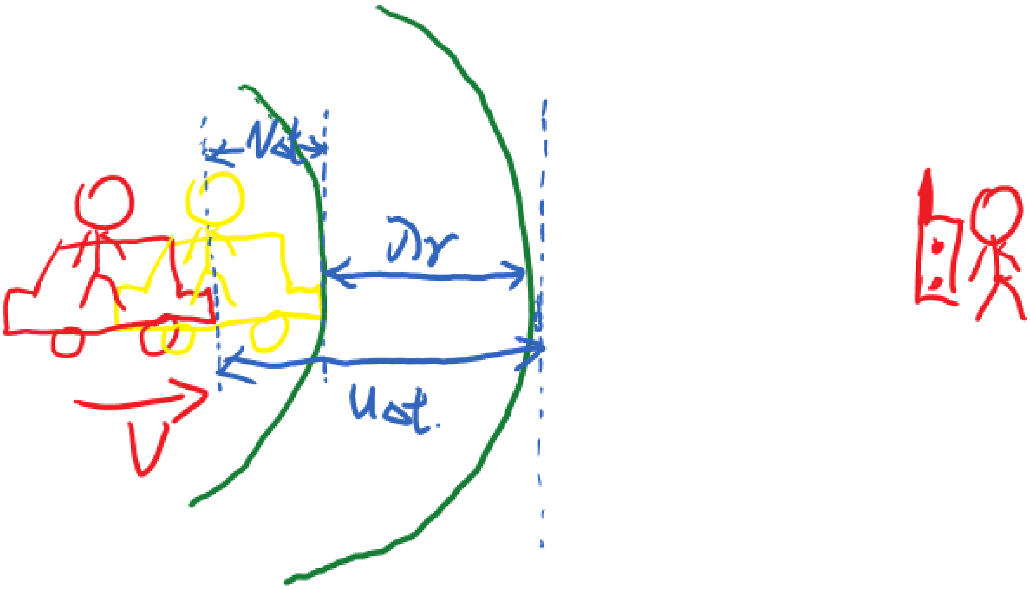

fig. 1

For $\Delta t = \frac{\lambda}{v}$:

$$\begin{align} distance\ traveled\ by\ sound &= v\Delta t \tag{1} \\ distance\ traveled\ by\ sender &= u\Delta t \tag{2} \\ distance\ between\ the\ 1st\ and\ 2nd\ wavefront &= \Delta t(v-u) \tag{3} \end{align}$$

Since the distance between two consecutive wavefronts is the wavelength of the wave, therefor the apparent wavelength for the receiver would be $\Delta t(v-u) = \frac{v-u}{v}\lambda$

fig.2

Following the same method, you can get formula for other circumstances like moving receiver. Eventually, we can get the following formula that contain all the circumstances

$$f = \frac{c \pm v_r}{c \pm v_s} f_0 \tag{4}$$

Relativistic Doppler effect

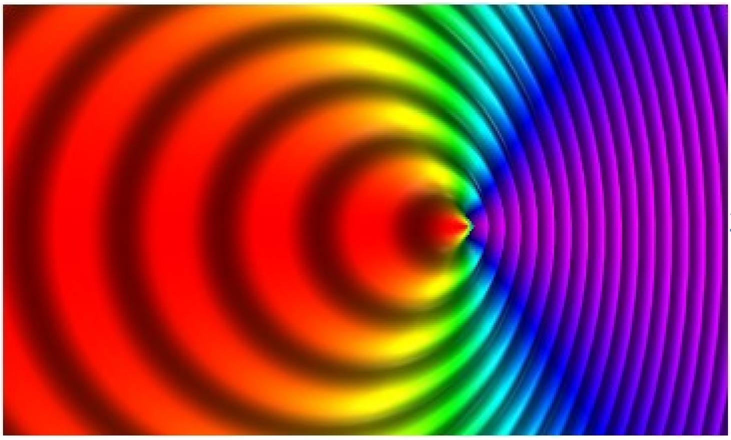

fig.3

Longitudinal

We ignored relativistic effects in our acoustic example, but for light considering relativistic effect is compulsory. We first assume that the sender is moving **towards** the receiver.

Meaning of symbols

v : the velocity of the sender moving relative to the receiver

c: The velocity of light ☀

λ_s: wavelength observed by sender

f_s: frequency observed by sender

λ_r: wavelength observed by receiver

f_r: frequency observed by receiver

Δt_s: the time taken for the light to travel one wavelength from the sender's view

Δt_r: the time taken for the light to travel one wavelength from the receiver's view

$\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$

$\beta = v/c$

Consider from the sender's perspective

This argument involved for the light circumstance is fundamentally the same. We just need to add the relativistic factor to the distance. I am going to discuss the distance travelled by light wavefronts from both perspectives of sender and receiver.

Sender

The speed of light is constant - c. For the same reason as acoustic, the apparent wavelength would be $\Delta t(c-v) = \frac{c-v}{c}\lambda$

Receiver

Since the receiver moving at a speed v relative to the sender, considering relativistic effect by multiplying the factor $\frac{1}{\sqrt{1-v^2/c^2}}$.

However, this way of approaching is actually wrong, because in relativity, the speed of light should be constant for every inertial frame, if we assume that the distance between two wavefronts changes from the perspective of the sender, we are actually assuming that the speed of light can be added or substracted.

Hence, instead of considering relativistic distance, we focus on time.

From the perspective of receiver

In this very case, from the receiver's view, the sender is moving at a speed of v, relative to light. Under such a circumstance, we compare the time for the light to travel one wavelength.

$\Delta t_s = \frac{\lambda_s}{c}$

$\Delta t_r = \gamma \Delta t_s$

$$\begin{aligned} \lambda_r = \Delta t_r (c-v) = \gamma \Delta t_s (c-v) &= \frac{\lambda_s}{c} \frac{c-v}{\sqrt{1-v^2/c^2}} = \lambda_s \frac{1-\beta}{(1-\beta)^{-1/2}(1+\beta)^{-1/2}} \\ &= \lambda_s \sqrt{\frac{1-\beta}{1+\beta}} \\ \therefore f_r &= f_s \sqrt{\frac{1+\beta}{1-\beta}} \end{aligned} \tag{5}$$

If they are **moving away** from each other, then:

$$f_r = f_s \sqrt{\frac{1-\beta}{1+\beta}} \tag{6}$$

Reference

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