HW2

Question1.

(a).

$$P(C|x) = \frac{P(x|C)P(C)}{P(x)} = \frac{P(x|C)P(C)}{\sum P(x|C_i)P(C_i)}$$

I choose to use log odds to determine which class x belongs to.

$$\log \frac{P(C_1|x)}{P(C_2|x)} = \log P(x|C_1) + \log P(C_1) - \log P(x|C_2) - \log P(C_2)$$

Since

$$P(x|C) = p(x=0|C)^{1-x}(1-p(x=0|C))^x$$

So we have

$$f(x) = \log \frac{P(C_1|x)}{P(C_2|x)} = (1-x)\log p_1 + x\log(1-p_1) - (1-x)\log p_2 - x\log(1-p_2) + \log P(C_1) - \log P(C_2)$$

(b).

$$P(x|C_i) = \prod p_{ij}^{1-x_j}(1-p_{ij})^{x_j}$$

$$P(C|x) = \frac{P(x|C)P(C)}{P(x)} = \frac{P(x|C)P(C)}{\sum P(x|C_i)P(C_i)}$$

I still choose log odds here.

$$f(x) = \log \frac{P(C_1|x)}{P(C_2|x)} = \log P(x|C_1) + \log P(C_1) - \log P(x|C_2) - \log P(C_2)$$

$$f(x) = \sum (1-x_j)\log p_{1j} + \sum x_j \log(1-p_{1j}) - \sum (1-x_j)\log p_{2j} - \sum x_j \log(1-p_{2j}) + \log(P(C1)) - \log(P(C2))$$

(c).

$$P(C|x) = \frac{P(x|C)P(C)}{P(x)} = \frac{P(x|C)P(C)}{\sum P(x|C_i)P(C_i)}$$

$$P(0,0|C_1) = 0.6 * 0.1 = 0.06$$
$$P(0,1|C_1) = 0.6 * 0.9 = 0.54$$
$$P(1,0|C_1) = 0.4 * 0.1 = 0.04$$
$$P(1,1|C_1) = 0.4 * 0.9 = 0.36$$

$P(0,0 \mid C_2) = 0.6 * 0.9 = 0.54$
$P(0,1 \mid C_2) = 0.6 * 0.1 = 0.06$
$P(1,0 \mid C_2) = 0.4 * 0.9 = 0.36$
$P(1,1 \mid C_2) = 0.4 * 0.1 = 0.04$

	P(C1) = 0.2	P(C1) = 0.6	P(C1) = 0.8
P(0,0)	0.06*0.2 + 0.54*0.8 = 0.444	0.06*0.6 + 0.54*0.4 = 0.252	0.06*0.8 + 0.54*0.2 = 0.156
P(0,1)	0.54*0.2 + 0.06*0.8 = 0.156	0.54*0.6 + 0.06*0.4 = 0.348	0.54*0.8 + 0.06*0.2 = 0.444
P(1,0)	0.04*0.2 + 0.36*0.8 = 0. 296	0.04*0.6+ 0.36*0.4 = 0. 168	0.04*0.8 + 0.36*0.2 = 0. 104
P(1,1)	0.36*0.2 + 0.04*0.8 = 0. 104	0.36*0.6 + 0.04*0.4 = 0. 232	0.36*0.8 + 0.04*0.2 = 0. 296

$P(C_1) = 0.2$
$P(C_1 \mid 0,0) =$ 0.06*0.2/0.444 = 0.027
$P(C_1 \mid 0,1) =$ 0.54*0.2/0.156 = 0.6923
$P(C_1 \mid 1,0) =$ 0.04*0.2/0.296 = 0.027
$P(C_1 \mid 1,1) =$ 0.36*0.2/0.104 = 0.6923
$P(C_2 \mid 0,0) =$ 0.54*0.8/0.444 = 0.973
$P(C_2 \mid 0,1) =$ 0.06*0.8/0.156 = 0.3077
$P(C_2 \mid 1,0) =$ 0.36*0.8/0.296 = 0.973
$P(C_2 \mid 1,1) =$ 0.04*0.8/0.104 = 0.3077

$P(C_1) = 0.6$
$P(C_1 \mid 0,0) =$ 0.06*0.6/0.252 = 0.1429
$P(C_1 \mid 0,1) =$ 0.54*0.6/0.348 = 0. 9310
$P(C_1 \mid 1,0) =$ 0.04*0.6/0.168 = 0.1429
$P(C_1 \mid 1,1) =$ 0.36*0.6/0.232 = 0.9310
$P(C_2 \mid 0,0) =$ 0.54*0.4/0.252 = 0.8571
$P(C_2 \mid 0,1) =$ 0.06*0.4/0.348 = 0.069
$P(C_2 \mid 1,0) =$ 0.36*0.4/0.168 = 0.8571
$P(C_2 \mid 1,1) =$ 0.04*0.4/0.232 = 0.069

$P(C_1) = 0.8$
$P(C_1 \mid 0,0) =$ 0.06*0.8/0.156 = 0.3077
$P(C_1 \mid 0,1) =$ 0.54*0.8/0.444 = 0.973
$P(C_1 \mid 1,0) =$ 0.04*0.8/0.104 = 0.3077

$P(C_1 \mid 1{,}1)$ = 0.36*0.8/0.296 = 0.973
$P(C_2 \mid 0{,}0)$ = 0.54*0.2/0.156 = 0.6923
$P(C_2 \mid 0{,}1)$ = 0.06*0.2/0.444 = 0.027
$P(C_2 \mid 1{,}0)$ = 0.36*0.2/0.104 = 0.6923
$P(C_2 \mid 1{,}1)$ = 0.04*0.2/0.296 = 0.027

(d).

See script_1d.m

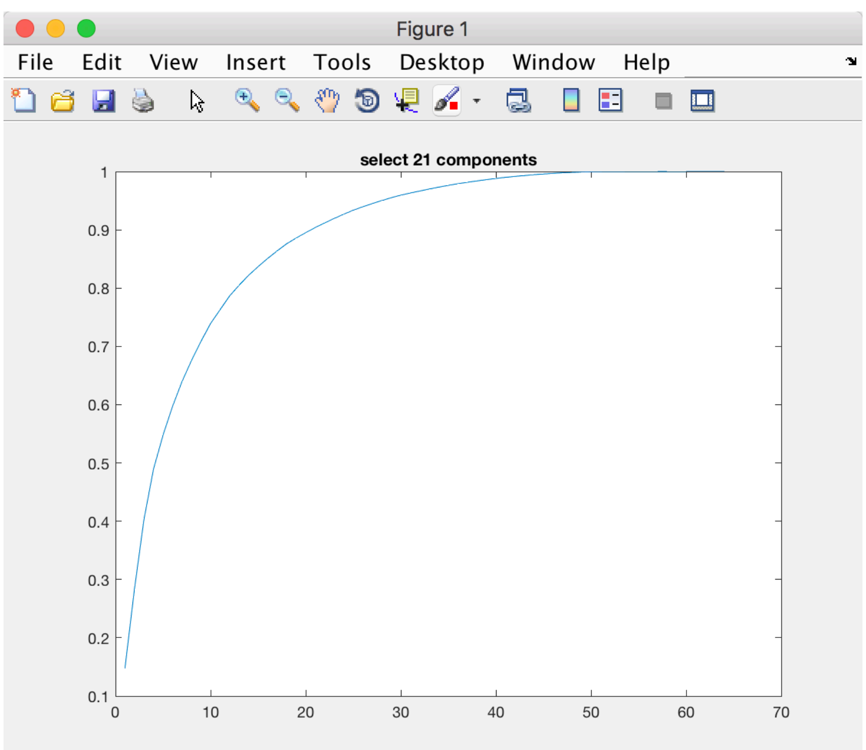
'error rate of each prior 0.49438 0.48315 0.47191 0.43820 0.40449 0.31461 0.29213 0.22472 0.22472 0.19101 0.24719'
err_rate = 0.17978

Problem 2

(a).
'k = 1, err_rate = 0.05387'
'k = 3, err_rate = 0.04040'
'k = 5, err_rate = 0.04377'
'k = 7, err_rate = 0.05387’

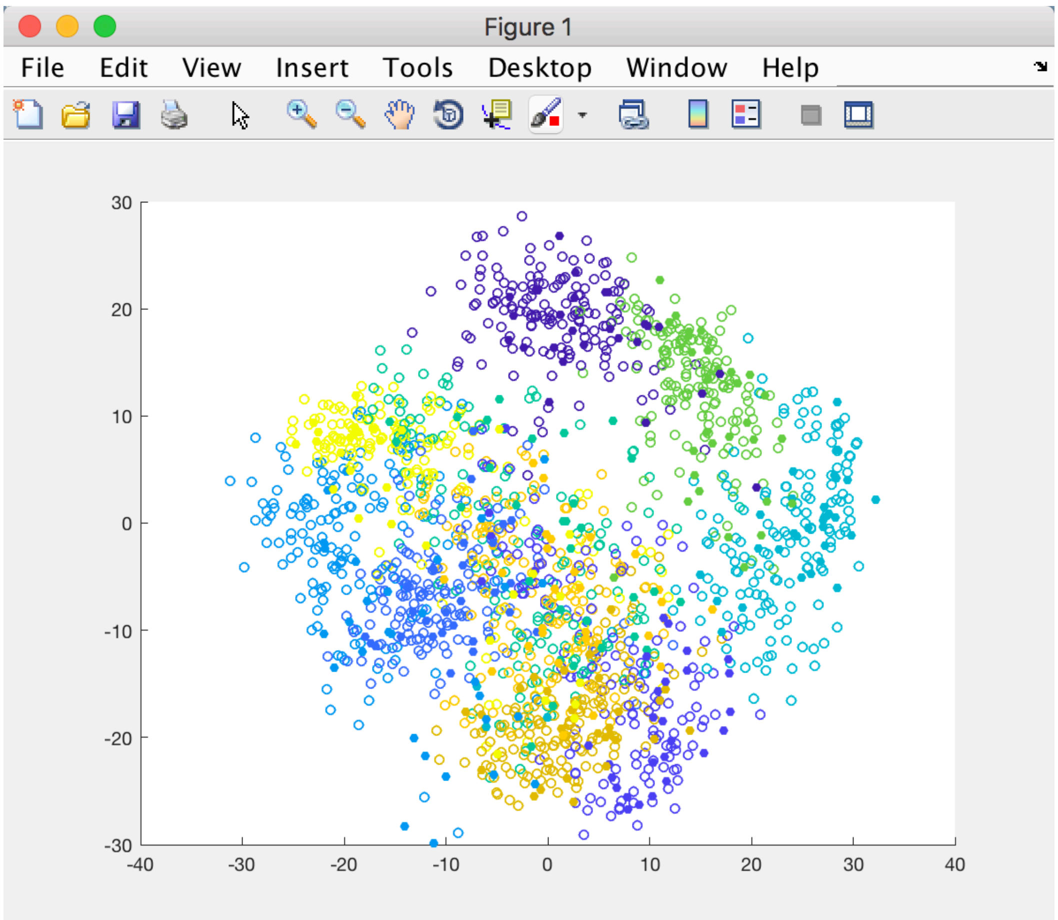
(b).

21 Componensts are selected



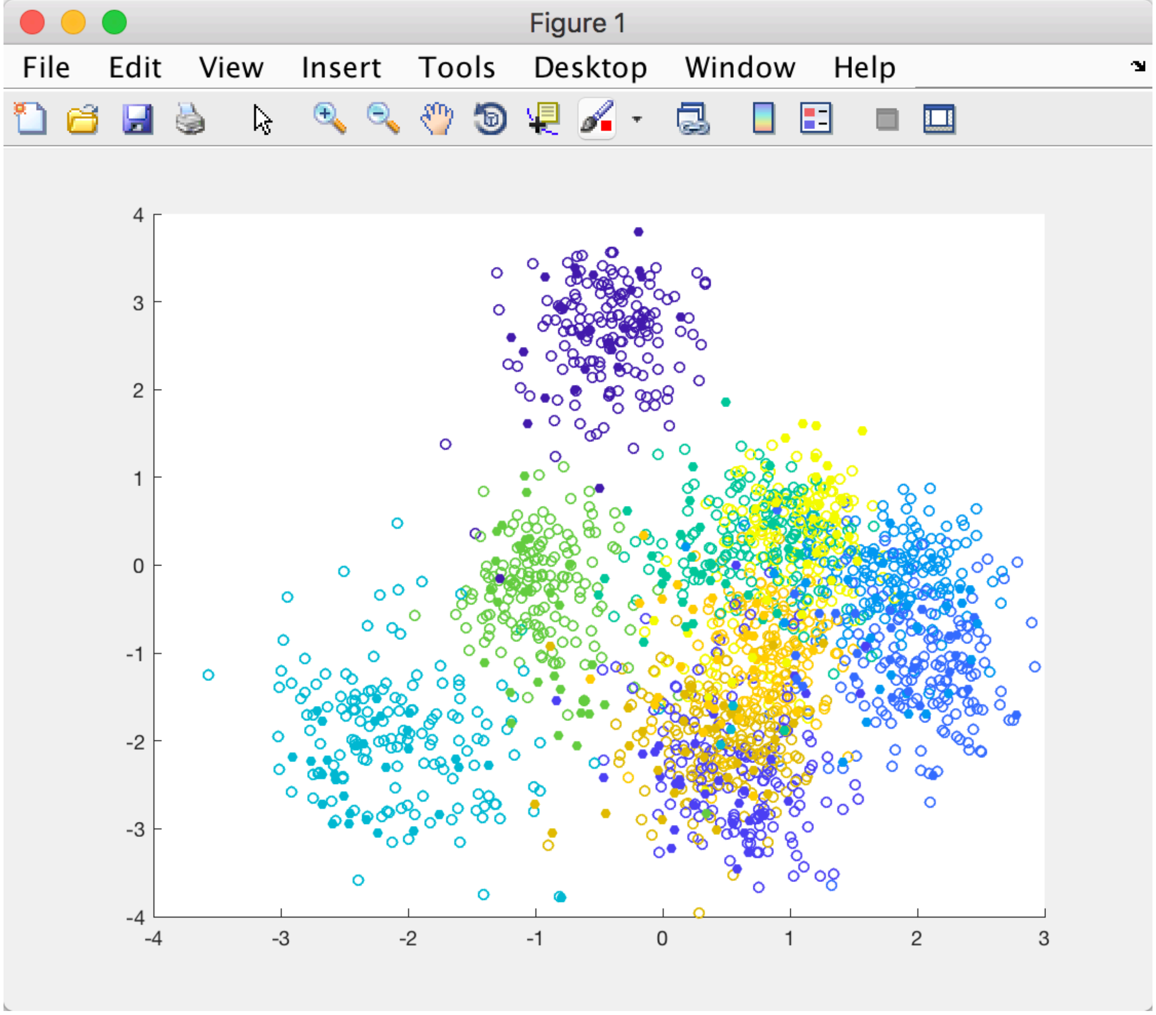
’k = 1, err_rate = 0.04714'
'k = 3, err_rate = 0.04714'
'k = 5, err_rate = 0.05387'
'k = 7, err_rate = 0.05387’

(c).
Empty circle -> test
Filled circle -> train



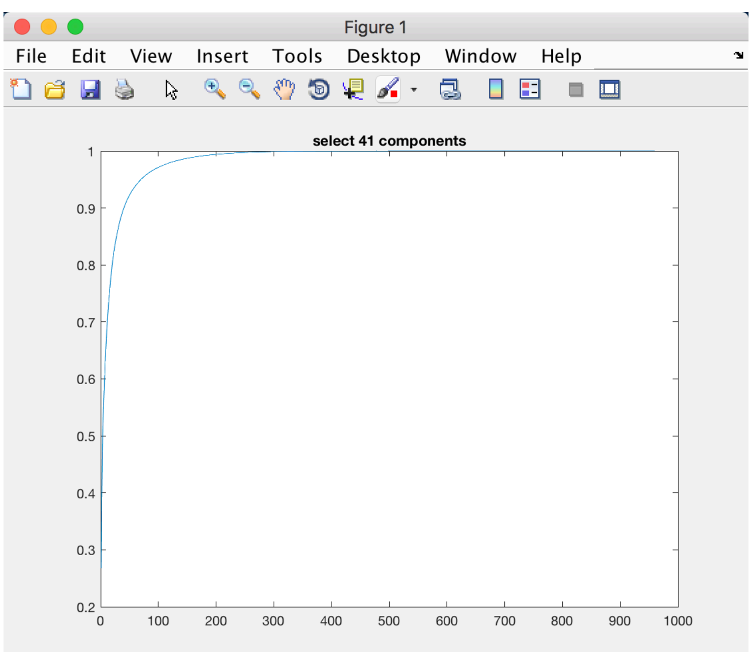
(d).
'k = 1, L = 2, err_rate = 0.46801'
'k = 3, L = 2, err_rate = 0.42424'
'k = 5, L = 2, err_rate = 0.39394'
'k = 1, L = 4, err_rate = 0.19192'
'k = 3, L = 4, err_rate = 0.19192'
'k = 5, L = 4, err_rate = 0.16835'
'k = 1, L = 9, err_rate = 0.09428'
'k = 3, L = 9, err_rate = 0.07744'
'k = 5, L = 9, err_rate = 0.08418’

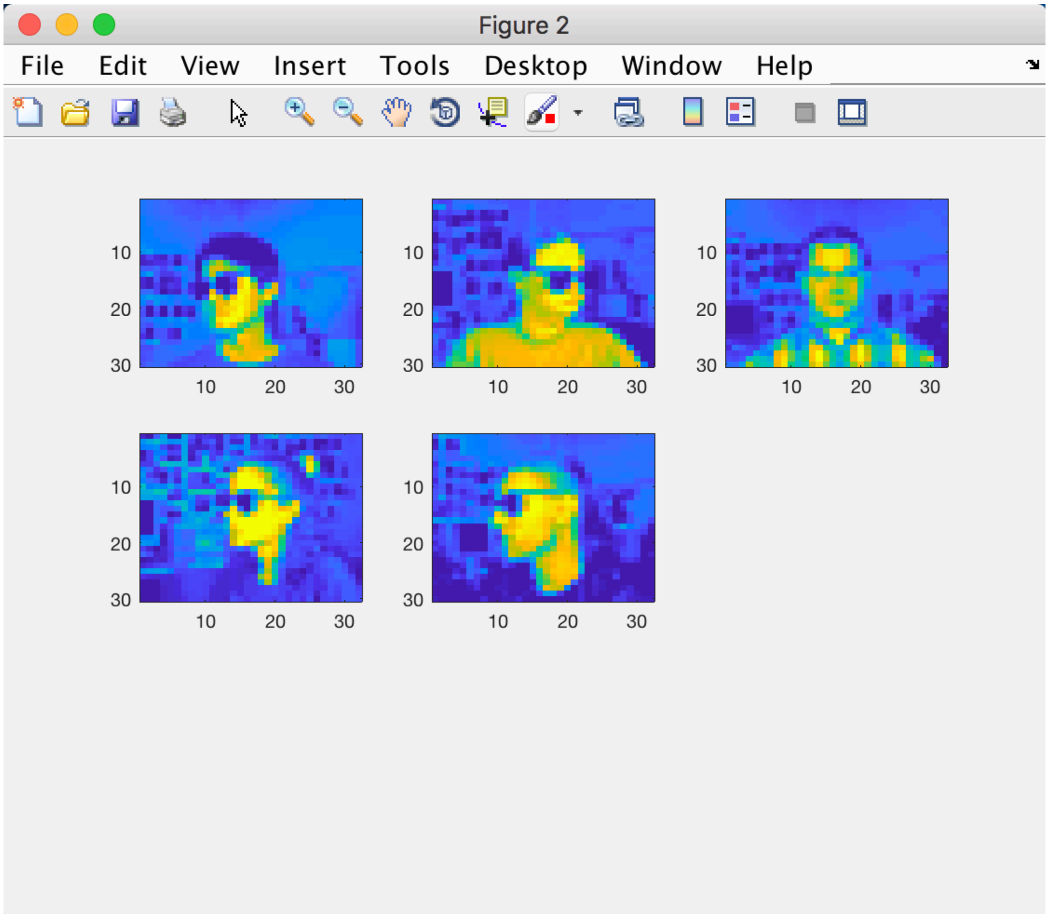
(e).



Problem 3

(a)
41 components are selected





(b).

'k = 1, err_rate = 0.00000'
'k = 3, err_rate = 0.06800'
'k = 5, err_rate = 0.14000'
'k = 7, err_rate = 0.26400'

(c).
From subplot(2,2,1) ->(2,2,4) I have origin, k = 10, k = 50, k = 100

I can see the more Eigenvector I use for reconstruction, the more similar to the original picture. i.e. less reconstruction error.

Also, the more eigenvector I use, the less “marginal benefit”. Because the contribution of each eigenvector is given by its eigenvalue, so the first some of the eigenvectors captured most of the information.

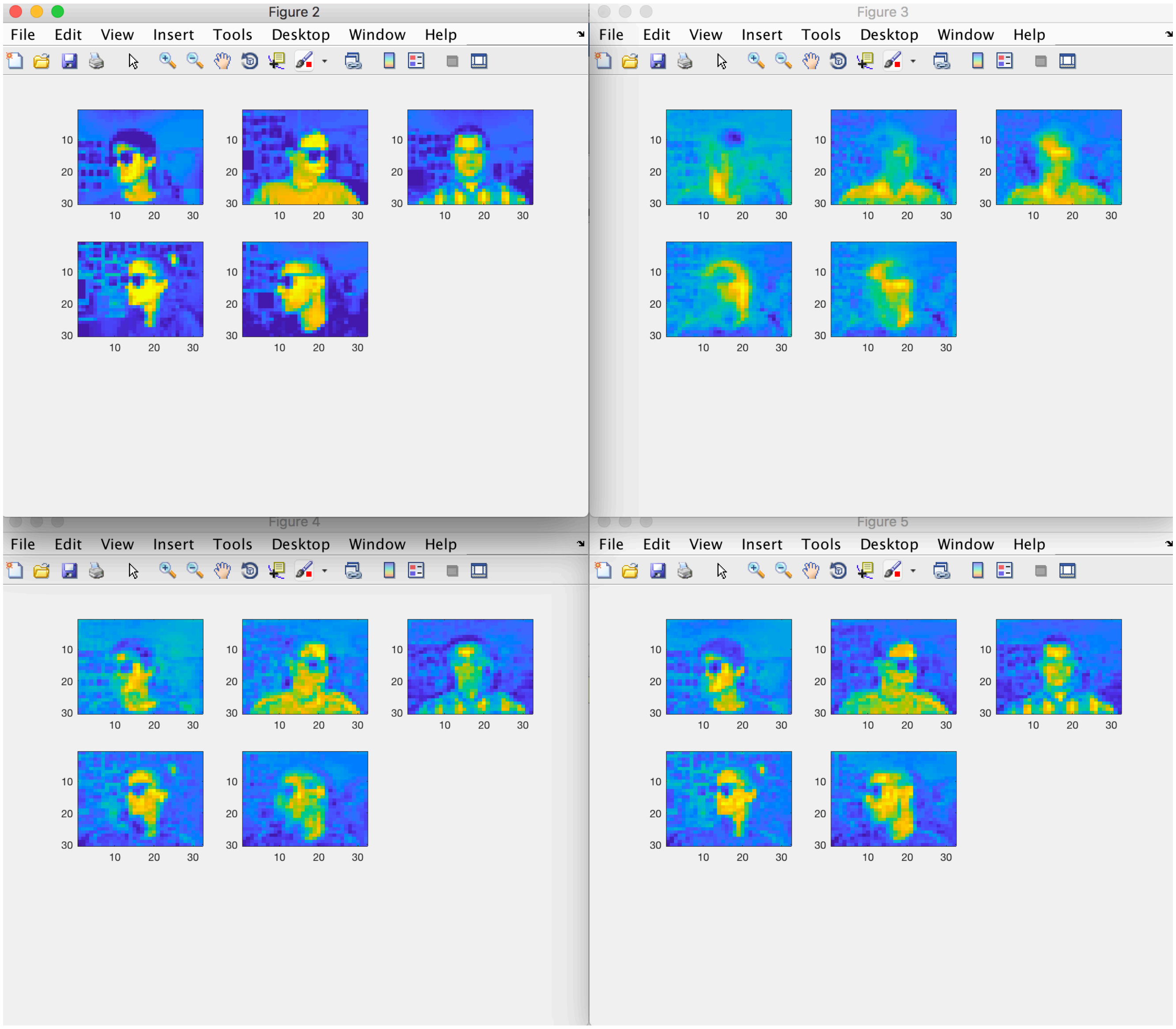


Figure 2
File Edit View Insert Tools Desktop Window Help
Figure 3
File Edit View Insert Tools Desktop Window Help
Figure 4
File Edit View Insert Tools Desktop Window Help
Figure 5
File Edit View Insert Tools Desktop Window Help