HW2

Question1.

(a).

$$P(C \mid x) = \frac{P(x \mid C)P(C)}{P(x)} = \frac{P(x \mid C)P(C)}{\sum P(x \mid C_i)P(C_i)}$$

I choose to use log odds to determine which class x belongs to.

$$\log \frac{P(C_1 \mid x)}{P(C_2 \mid x)} = \log P(x \mid C_1) + \log P(C_1) - \log P(x \mid C_2) - \log P(C_2)$$

Since

$$P(x \mid C) = p(x = 0 \mid C)^{1-x}(1 - p(x = 0 \mid C))^x$$

So we have

$$f(x) = \log \frac{P(C_1 \mid x)}{P(C_2 \mid x)} = (1 - x)\log p_1 + x \log(1 - p_1) - (1 - x)\log p_2 + x \log(1 - p_2) + \log P(C_1) - \log P(C_2)$$

(b).

$$P(x \mid C_i) = \prod p_{ij}^{1-x_j}(1 - p_{ij})^{x_j}$$

$$P(C \mid x) = \frac{P(x \mid C)P(C)}{P(x)} = \frac{P(x \mid C)P(C)}{\sum P(x \mid C_i)P(C_i)}$$

I still choose log odds here.

$$f(x) = \log \frac{P(C_1 \mid x)}{P(C_2 \mid x)} = \log P(x \mid C_1) + \log P(C_1) - \log P(x \mid C_2) - \log P(C_2)$$

$$f(x) = \sum (1 - x_j)\log p_{1j} + \sum x_j \log(1 - p_{1j}) - \sum (1 - x_j)\log p_{2j} - \sum x_j \log(1 - p_{2j}) + \log(P(C1)) - \log(P(C2))$$

(c).

$$P(C \mid x) = \frac{P(x \mid C)P(C)}{P(x)} = \frac{P(x \mid C)P(C)}{\sum P(x \mid C_i)P(C_i)}$$

$$P(0,0 \mid C_1) = 0.6 * 0.1 = 0.06$$
$$P(0,1 \mid C_1) = 0.6 * 0.9 = 0.54$$
$$P(1,0 \mid C_1) = 0.4 * 0.1 = 0.04$$
$$P(1,1 \mid C_1) = 0.4 * 0.9 = 0.36$$

$P(0,0 \mid C_2) = 0.6 * 0.9 = 0.54$
$P(0,1 \mid C_2) = 0.6 * 0.1 = 0.06$
$P(1,0 \mid C_2) = 0.4 * 0.9 = 0.36$
$P(1,1 \mid C_2) = 0.4 * 0.1 = 0.04$

$P(0,0) = 0.6$
$P(0,1) = 0.6$
$P(1,0) = 0.4$
$P(1,1) = 0.4$

$P(C_1) = 0.2$
$P(C_1 \mid 0,0) =$ 0.06*0.2/0.6 = 0.02
$P(C_1 \mid 0,1) =$ 0.54*0.2/0.6 = 0.18
$P(C_1 \mid 1,0) =$ 0.04*0.2/0.4 = 0.02
$P(C_1 \mid 1,1) =$ 0.36*0.2/0.4 = 0.18
$P(C_2 \mid 0,0) =$ 0.54*0.8/0.6 = 0.72
$P(C_2 \mid 0,1) =$ 0.06*0.8/0.6 = 0.08
$P(C_2 \mid 1,0) =$ 0.36*0.8/0.4 = 0.72
$P(C_2 \mid 1,1) =$ 0.04*0.8/0.4 = 0.08

$P(C_1) = 0.6$
$P(C_1 \mid 0,0) =$ 0.06*0.6/0.6 = 0.06
$P(C_1 \mid 0,1) =$ 0.54*0.6/0.6 = 0.54
$P(C_1 \mid 1,0) =$ 0.04*0.6/0.4 = 0.06
$P(C_1 \mid 1,1) =$ 0.36*0.6/0.4 = 0.54
$P(C_2 \mid 0,0) =$ 0.54*0.4/0.6 = 0.36
$P(C_2 \mid 0,1) =$ 0.06*0.4/0.6 = 0.04
$P(C_2 \mid 1,0) =$ 0.36*0.4/0.4 = 0.36
$P(C_2 \mid 1,1) =$ 0.04*0.4/0.4 = 0.04

$P(C_1) = 0.8$
$P(C_1 \mid 0,0) =$ 0.06*0.8/0.6 = 0.08
$P(C_1 \mid 0,1) =$ 0.54*0.8/0.6 = 0.72
$P(C_1 \mid 1,0) =$ 0.04*0.8/0.4 = 0.08
$P(C_1 \mid 1,1) =$ 0.36*0.8/0.4 = 0.72
$P(C_2 \mid 0,0) =$ 0.54*0.2/0.6 = 0.18
$P(C_2 \mid 0,1) =$ 0.06*0.2/0.6 = 0.02
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