JHU Engineering for Professionals
Applied and Computational Mathematics
Data Mining: 625.740

Homework 6 Solutions (Part I)

1. Exercise 1: See figure 1(α).

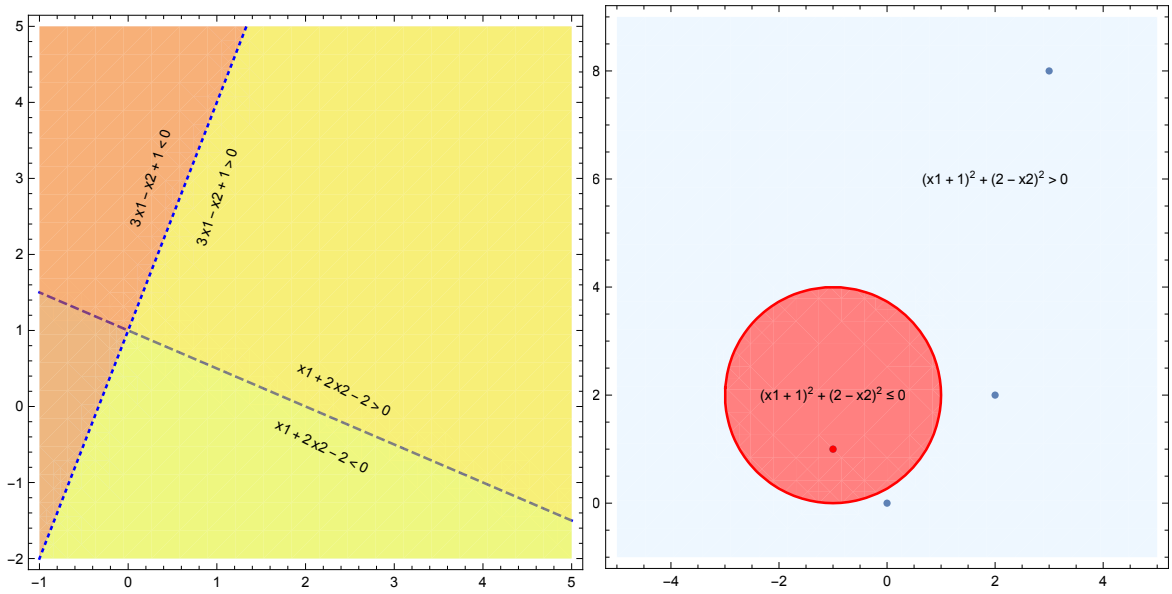

Figure 1. (α) Exercise 1: Hyperplanes in two dimensions. Also known as lines. (β) Exercise 2: A circle.

2. Exercise 2: See figure 1(β).
   (d) Let $x_3 = x_1^2$ and $x_4 = x_2^2$. The boundary curve can be written as: $1+2x_1-4x_2+x_3+x_4 = 0$ which is linear in the variables $\{x_1, x_2, x_3, x_4\}$.

3. Exercise 3: See figure 2(α). The parameters, $\beta_0 = \frac{\sqrt{2}}{4}, \beta_1 = -\frac{\sqrt{2}}{2}, \beta_2 = \frac{\sqrt{2}}{2}$, are found by solving this maximization problem:

$$\begin{aligned} &\text{Maximize } M, \text{ s.t.} \\ &\beta_1^2 + \beta_2^2 = 1 \\ \beta_0 + 2\beta_1 + 2\beta_2 &\geq M \\ \beta_0 + 4\beta_1 + 4\beta_2 &\geq M \\ \beta_0 + 2\beta_1 + \beta_2 &\leq -M \\ \beta_0 + 4\beta_1 + 3\beta_2 &\leq -M \end{aligned}$$

   Classify a point as "red" if $1 - 2x_1 + 2x_2 \geq 0$ and as "blue" otherwise.

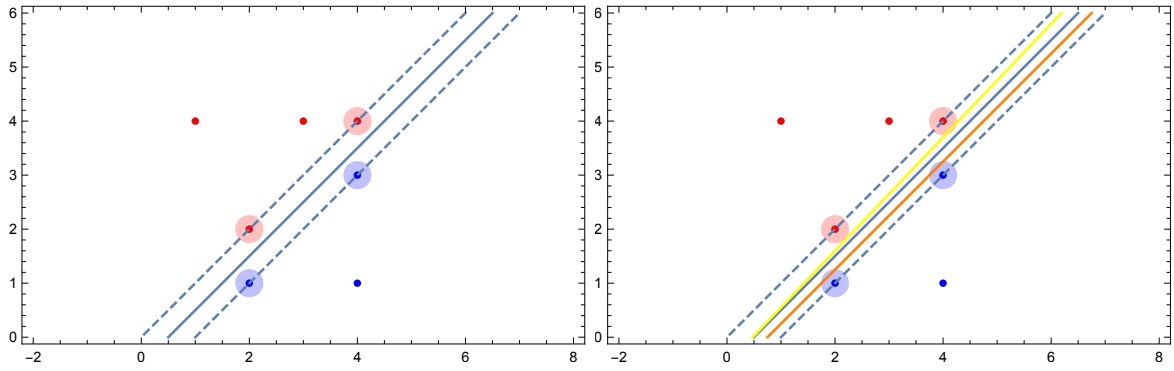

Figure 2. Exercise 3: (α) The red and blue points, and the separating line: $1 - 2x_1 + 2x_2 = 0$. The margin, $\frac{\sqrt{2}}{4} \approx 0.353553$, is the distance from the boundary line to each of the dashed lines. Red or blue disks (according to class) surround each support vector. (β) Added to the plot are a couple of lines that separate the data, but are not optimal (they do not maximize the margin). The yellow line is a plot of the line $1 - 2.1x_1 + 2x_2 = 0$. The orange line is a plot of $1.5 - 2x_1 + 2x_2 = 0$. There are many others, infinitely many!

(f) The seventh point is far enough away from the margin, that a slight disturbance will not change anything about the classifier. Only when it moves enough to be inside the margin (and thus a support vector) will it have any effect.
(g) Any other line will do! Or are the textbook authors asking for another separating line? In that case, moving the line either to the right or left a bit will work. Also, rotating the line slightly, etc. Please see figure 2(β) for a couple of examples.
(h) Draw a blue point at $(2, 4)$ or a red point at $(6, 1)$ or $\cdots$