JHU Engineering for Professionals
Applied and Computational Mathematics
Data Mining: 625.740

Homework for Module on Multilayer Neural Networks

In problems 1 through 5, the perceptrons and neural networks sought each have three inputs: x_1, x_2, x_3. Let p_k represent the parity function of k inputs:
$p_2 = x_1 \,\texttt{XOR}\, x_2$ and $p_3 = x_1 \,\texttt{XOR}\, x_2 \,\texttt{XOR}\, x_3$.

1. Show a perceptron that calculates $\overline{x}_1 x_2 \overline{x}_3$.

2. Show a perceptron that calculates $\overline{x}_2 x_3$.

3. Show a neural network that calculates $\overline{x}_1 \overline{x}_2 \overline{x}_3 + x_1 x_2 x_3$.

4. Show a neural network that calculates $\overline{p}_1 + x_1 x_2 x_3$.

5. Show a neural network that calculates $p_2(x_1, x_2) \cdot p_3(x_1, x_2, x_3)$.

6. Generate two-dimensional samples for each of two Gaussians, $p(\mathbf{x}|\omega_i) \sim N(\boldsymbol{\mu}_i, \Sigma_i)$ with

$$\boldsymbol{\mu}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \boldsymbol{\mu}_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \Sigma_1 = \begin{pmatrix} 0.2 & 0 \\ 0 & 0.2 \end{pmatrix} = \Sigma_2.$$

   Produce 100 samples from each distribution, but to guarantee linear separability, produce a replacement for a Class 1 vector whenever $x_1 - x_2 < 1$ and produce a replacement for a Class 2 vector whenever $x_1 - x_2 > 1$.

   Use these vectors to design a linear classifier using the perceptron algorithm. After convergence, draw the decision boundary.