

Homework 3 Code

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# 1. For each of the following models, generate a random series of length  $n = 250$ 
# and  $\sigma^2 = 1$ , plot the data and obtain the ACF (with 20 lags). Comment on the results
# in each case.
# (a) MA(1),  $\theta_1 = -0.7$ 
set.seed(137)
simMA1 <- arima.sim(n = 250, model = list(ma = c(-0.7)))
simMA1b <- arima.sim(n = 250, model = list(ma = c(0.7)))

plot(simMA1, main = 'MA(1)  $\theta_1 = -0.7$ ', ylab = 'MA(1)')
acfMA1 <- acf(simMA1, lag.max = 20, main = 'MA(1)  $\theta_1 = -0.7$ ')

#theoretical value
 $\theta_1 = -0.7$ 
 $\theta_1 / (1 + \theta_1^2)$ 

# (b) MA(2),  $\theta_1 = 1$ ,  $\theta_2 = 0.8$ 
set.seed(123)
simMA2 <- arima.sim(n = 250, model = list(ma = c(1, 0.8)))
plot(simMA2, main = 'MA(2)  $\theta_1 = 1$ ,  $\theta_2 = 0.8$ ', ylab = 'MA(2)')
acfMA2 <- acf(simMA2, lag.max = 20, main = 'MA(2)  $\theta_1 = 1$ ,  $\theta_2 = 0.8$ ')

# theoretical value
 $\theta_1 = 1$ ;  $\theta_2 = 0.8$ 
 $\theta_1(1 + \theta_2) / (1 + \theta_1^2 + \theta_2^2)$ 
 $\theta_2 / (1 + \theta_1^2 + \theta_2^2)$ 

# 2. Let  $\bar{X}$  be the sample mean of an MA(1) sequence. For the parts below, assume that  $\sigma^2 = 9$ 
# and the sample size  $n = 90$ .
# a
#  $j=0$ 
 $9 + ((-0.7)^2) * 9$  # 13.41

#  $j=1$ 
 $-0.7 * 9$  # -6.3

#  $\rho(1)$ 
 $-0.7 * 9 / (9 + ((-0.7)^2) * 9)$  # -0.4697987

# b / c
 $(13.41 + 2 * ((1 - 1/90) * (-6.3))) / 90$  #  $\theta_1 = -0.7$ 
 $(13.41 + 2 * ((1 - 1/90) * (6.3))) / 90$  #  $\theta_1 = 0.7$ 

 $35.2 - (1.96 * \sqrt{(13.41 + 2 * ((1 - 1/90) * (6.3)))) / \sqrt{90}$ 
 $35.2 + (1.96 * \sqrt{(13.41 + 2 * ((1 - 1/90) * (6.3)))) / \sqrt{90}$ 

# 3. Let  $X_t$  be an AR(1) sequence with  $\sigma^2 = 9$ . Let  $\bar{X}$  be the sample mean
# with the sample size  $n = 90$ 
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# a
(9 / (1+0.7)^2) / 90

# b
(9 / (1-0.7)^2) / 90
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