Applied Time Series

STA 137

Homework 5

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1. (a) With the given $ARMA(1,1)$ model

$$X_t - \mu = \phi(X_{t-1} - \mu) + \varepsilon_t + \theta\varepsilon_{t-1}$$

The series has $\{X_1, \cdots, X_n\}, n = 80$ and $X_n = 2.5$. In this section, the following are also given, $\mu = 0, \phi = 0.6, \theta = 0.8$ and $\hat{\varepsilon}_n = 0.5$.

$$\hat{X}_{n+1} = 0 + 0.6(2.5 - 0) + 0.8 \times 0.5 = 1.9$$

$$\hat{\varepsilon}_{n+1} = \hat{X}_{n+1} - \hat{X}_{n+1} = 0$$

$$\hat{X}_{n+2} = 0 + 0.6(1.9 - 0) + 0.8 \times 0 = 1.14$$

$$\hat{\varepsilon}_{n+2} = \hat{X}_{n+2} - \hat{X}_{n+2} = 0$$

$$\hat{X}_{n+3} = 0 + 0.6(1.14 - 0) + 0.8 \times 0 = 0.684$$

1. (b) With the given $ARMA(1,1)$ model

$$X_t - \mu = \phi(X_{t-1} - \mu) + \varepsilon_t + \theta\varepsilon_{t-1}$$

The series has $\{X_1, \cdots, X_n\}, n = 80$ and $X_n = 2.5$. In this section, the following are also given, $\mu = 15, \phi = 0.6, \theta = 0.8$ and $\hat{\varepsilon}_n = 0.5$.

$$\hat{X}_{n+1} = 15 + 0.6(2.5 - 15) + 0.8 \times 0.5 = 7.9$$

$$\hat{\varepsilon}_{n+1} = \hat{X}_{n+1} - \hat{X}_{n+1} = 0$$

$$\hat{X}_{n+2} = 15 + 0.6(7.9 - 15) + 0.8 \times 0 = 10.74$$

$$\hat{\varepsilon}_{n+2} = \hat{X}_{n+2} - \hat{X}_{n+2} = 0$$

$$\hat{X}_{n+3} = 15 + 0.6(10.74 - 15) + 0.8 \times 0 = 12.444$$

1. (c) In this section, X_t comes from a differenced series Y_t with $Y_{n-1} = 5.0$ and $Y_n = 7.5$. The mean of X_t is 0.01 and $\hat{\varepsilon}_n = 0.5$.

$$X_t = Y_t - Y_{t-1}$$

$$X_n = Y_n - Y_{n-1} = 7.5 - 5.0 = 2.5$$

$$\hat{X}_{n+1} = 0.01 + 0.6(2.5 - 0.01) + 0.8 \times 0.5 = 1.904$$

$$\hat{\varepsilon}_{n+1} = \hat{X}_{n+1} - \hat{X}_{n+1} = 0$$

$$\Rightarrow \hat{Y}_{n+1} = \hat{X}_{n+1} + Y_n = 1.904 + 7.5 = 9.404$$

$$\hat{X}_{n+2} = 0.01 + 0.6(1.904 - 0.01) + 0.8 \times 0 = 1.1464$$

$$\hat{\varepsilon}_{n+2} = \hat{X}_{n+2} - \hat{X}_{n+2} = 0$$

$$\Rightarrow \hat{Y}_{n+2} = \hat{X}_{n+2} + \hat{Y}_{n+1} = 1.1464 + 9.404 = 10.5504$$

$$\hat{X}_{n+3} = 0.01 + 0.6(1.1464 - 0.01) + 0.8 \times 0 = 0.69184$$

$$\hat{\varepsilon}_{n+3} = \hat{X}_{n+3} - \hat{X}_{n+3} = 0$$

$$\Rightarrow \hat{Y}_{n+3} = \hat{X}_{n+3} + \hat{Y}_{n+2} = 0.69184 + 10.5504 = 11.24224$$

2. (a) With the given $ARMA(1,2)$ model

$$X_t - \mu = \phi(X_{t-1} - \mu) + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2}$$

The series has $\{X_1, \cdots, X_n\}, n = 80$ and $X_n = 2.5$. In this section, the following are also given, $\mu = 0, \phi = 0.6, \theta_1 = -0.8, \theta_2 = 0.5, \hat{\varepsilon}_{n-1} = 0.5$ and $\hat{\varepsilon}_n = -0.3$.

$$\hat{X}_{n+1} = 0 + 0.6(2.5 - 0) + (-0.8)(-0.3) + 0.5(0.5) = 1.99$$

$$\hat{\varepsilon}_{n+1} = \hat{X}_{n+1} - \hat{X}_{n+1} = 0$$

$$\hat{X}_{n+2} = 0 + 0.6(1.99 - 0) + (-0.8)(0) + 0.5(-0.3) = 1.044$$

$$\hat{\varepsilon}_{n+2} = \hat{X}_{n+2} - \hat{X}_{n+2} = 0$$

$$\hat{X}_{n+3} = 0 + 0.6(1.044 - 0) + (-0.8)(0) + 0.5(0) = 0.6264$$

2. (b) In this section, the following are also given, $\mu = 15, \phi = 0.6, \theta_1 = -0.8, \theta_2 = 0.5, \hat{\varepsilon}_{n-1} = 0.5$ and $\hat{\varepsilon}_n = -0.3$.

$$\hat{X}_{n+1} = 15 + 0.6(2.5 - 15) + (-0.8)(-0.3) + 0.5(0.5) = 7.99$$

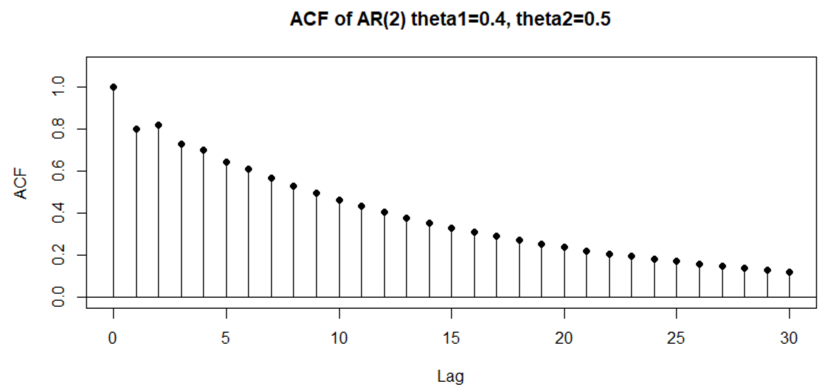
$$\hat{\varepsilon}_{n+1} = \hat{X}_{n+1} - \hat{X}_{n+1} = 0$$

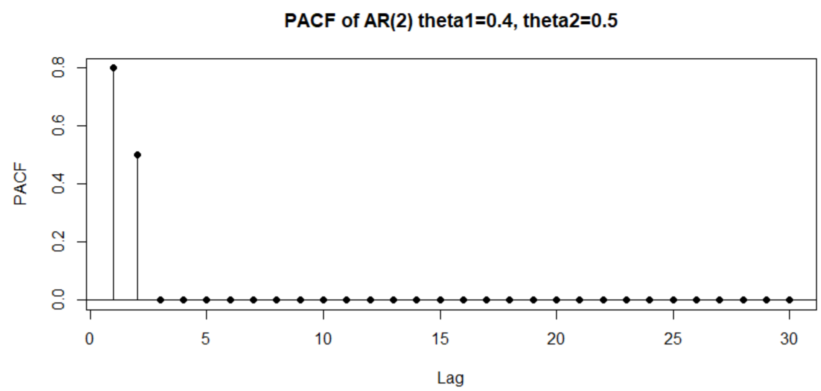
$$\hat{X}_{n+2} = 15 + 0.6(7.99 - 15) + (-0.8)(0) + 0.5(-0.3) = 10.644$$
$$\hat{\varepsilon}_{n+2} = \hat{X}_{n+2} - \hat{X}_{n+2} = 0$$
$$\hat{X}_{n+3} = 15 + 0.6(11.284 - 15) + (-0.8)(0) + 0.5(0) = 12.3864$$

2. (c) In this section, X_t comes from a differenced series Y_t with $Y_{n-1} = 5.0$ and $Y_n = 7.5$. The mean of X_t is 0.01, $\hat{\varepsilon}_{n-1} = 0.5$, and $\hat{\varepsilon}_n = -0.3$.

$$X_t = Y_t - Y_{t-1}$$
$$X_n = Y_n - Y_{n-1} = 7.5 - 5.0 = 2.5$$
$$\hat{X}_{n+1} = 0.01 + 0.6(2.5 - 0.01) + (-0.8)(-0.3) + 0.5(0.5) = 1.994$$
$$\hat{\varepsilon}_{n+1} = \hat{X}_{n+1} - \hat{X}_{n+1} = 0$$
$$\Rightarrow \hat{Y}_{n+1} = \hat{X}_{n+1} + Y_n = 1.994 + 7.5 = 9.494$$
$$\hat{X}_{n+2} = 0.01 + 0.6(1.994 - 0.01) + (-0.8)0 + 0.5(-0.3) = 1.0504$$
$$\hat{\varepsilon}_{n+2} = \hat{X}_{n+2} - \hat{X}_{n+2} = 0$$
$$\Rightarrow \hat{Y}_{n+2} = \hat{X}_{n+2} + \hat{Y}_{n+1} = 1.0504 + 9.494 = 10.5444$$
$$\hat{X}_{n+3} = 0.01 + 0.6(1.0504 - 0.01) + (-0.8)0 + 0.5(0) = 0.63424$$
$$\hat{\varepsilon}_{n+3} = \hat{X}_{n+3} - \hat{X}_{n+3} = 0$$
$$\Rightarrow \hat{Y}_{n+3} = \hat{X}_{n+3} + \hat{Y}_{n+2} = 0.63424 + 10.5444 = 11.17864$$

3. (a) The following are an ACF and PACF plot of $AR(2), \phi_1 = 0.4, \phi_2 = 0.5$

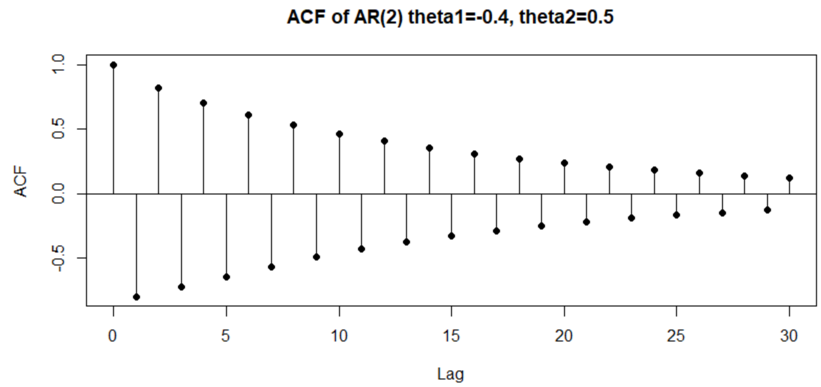


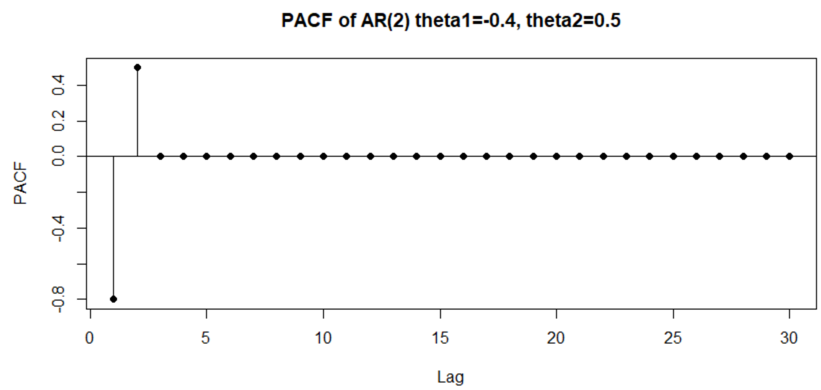


It seems that with the ACF plot, the lags seem to tail off slowly from 0 all the way until they start to appear insignificant. This makes sense, considering that it is a plot of $AR(p)$ data, and so the ACF plot should not be able to detect what sort of model it is. The PACF plot below it shows two clear lags at 1 and 2, which coincides with the model and is expected. The rest of the lags drop off to 0.

Also, given that the first coefficient is 0.4 and the second 0.5, then the plot fits into area 1 of the triangle from handout 11. Therefore, both the ACF and PACF plots appear the way they do, showing the typical pattern of any $AR(2)$ process in area 1.

3. (b) The following are an ACF and PACF plot of $AR(2), \phi_1 = -0.4, \phi_2 = 0.5$

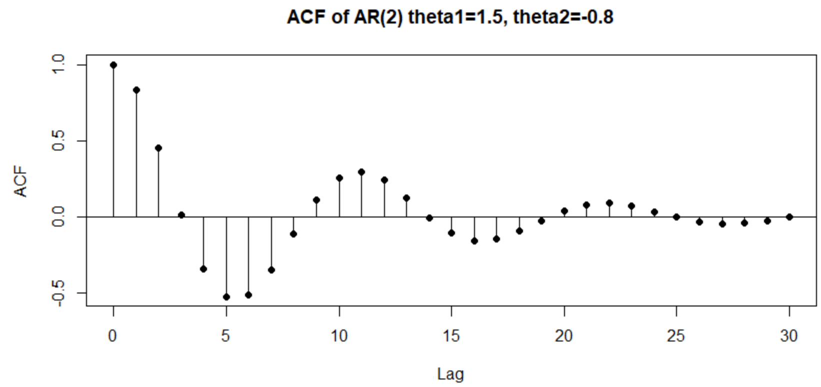


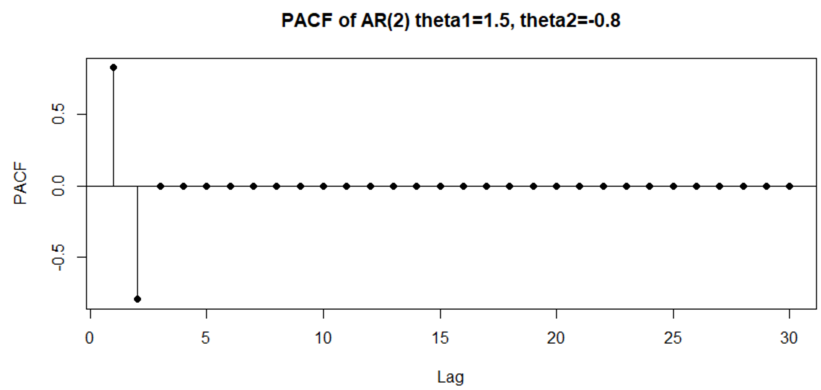


It seems that with the ACF plot, the lags slowly tail off, switching from positive to negative until they get closer to 0. This pattern is to be expected, considering that the model is an $AR(2)$ series, and the plot is an ACF. The second PACF plot shows two sharp spikes at lags 1 and 2. This is to be expected considering that the PACF is supposed to be able to identify $AR(p)$ models. The negative value followed by a positive value on the vertical axis can be indicative of the negative ϕ_1 followed by a positive ϕ_2. The rest of the lags are all at 0. This makes sense since the PACF plot is expected to identify an $AR(p)$ model.

Also, given that the first coefficient is -0.4 and the second 0.5, then the plot fits into area 2 of the triangle from handout 11. Therefore, both the ACF and PACF plots appear the way they do, showing the typical pattern of any $AR(2)$ process in area 2.

3. (c) The following are an ACF and PACF plot of $AR(2), \phi_1 = 1.5, \phi_2 = -0.8$

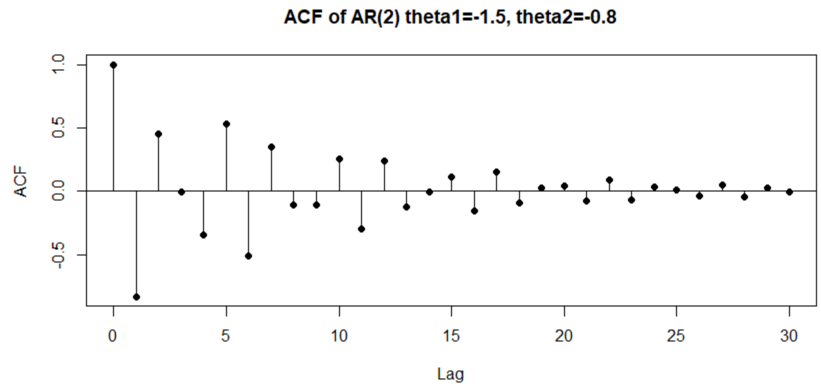


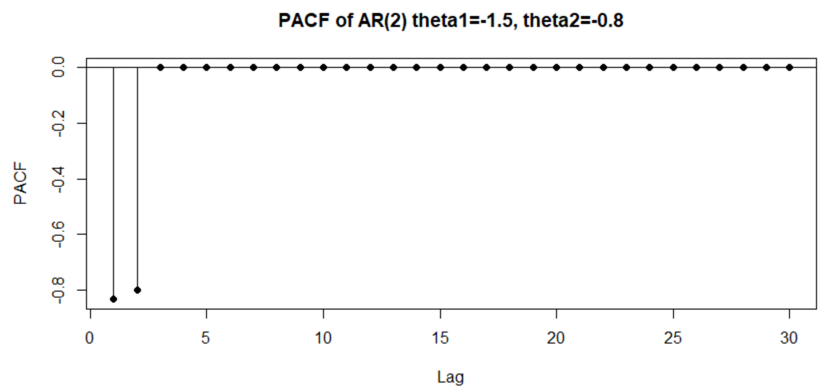


It seems that with the ACF plot, the lags seem to start positive, and then slowly become negative, then positive again, going about the 0-axis. This sort of tailing off is expected again, considering that it is an $AR(2)$ model being plotted in an ACF plot. So, the plot shouldn't be able to properly identify the type of model. The PACF plot has two large spikes at lags 1 and 2. The first is positive, and the second is negative, this is a pattern which matches with what was seen before. The difference is that this time ϕ_1 is positive and ϕ_2 is negative. The rest of the lags cut off and go to 0. This makes sense since the PACF plot is expected to identify an $AR(p)$ model.

Also, given that the first coefficient is 1.5 and the second -0.8, then the plot fits into area 4 of the triangle from handout 11. Therefore, both the ACF and PACF plots appear the way they do, showing the typical pattern of any $AR(2)$ process in area 4. The random cyclical pattern is therefore evident with the complex roots.

3. (d) The following are an ACF and PACF plot of $AR(2), \phi_1 = -1.5, \phi_2 = -0.8$

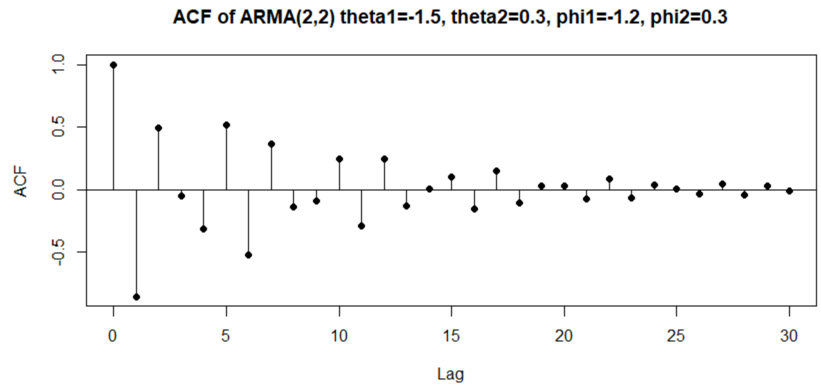


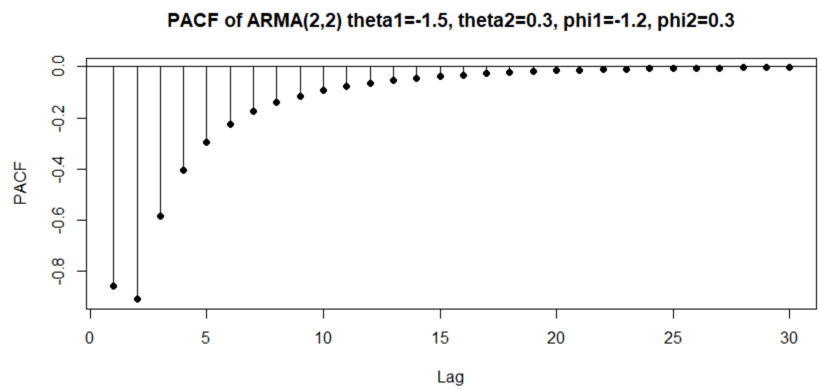


It seems from the ACF plot that there is a positive and negative oscillation going around the 0-axis. This tailing off is to be expected considering that it is an $AR(2)$ series being examined by an ACF plot. The second plot shows two long tails at lags 1 and 2, indicating an $AR(2)$ model. This is to be expected since it is an $AR(2)$ model being examined under a PACF plot. The tails are also negative, this is due to the coefficients both being negative.

Also, given that the first coefficient is -1.5 and the second -0.8, then the plot fits into area 3 of the triangle from handout 11. Therefore, both the ACF and PACF plots appear the way they do, showing the typical pattern of any $AR(2)$ process in area 3. The random cyclical pattern is therefore evident with the complex roots.

3. (e) The following are an ACF and PACF plot of $ARMA(2,2), \phi_1 = -1.5, \phi_2 = -0.8, \theta_1 = -1.2, \theta_2 = 0.3$





The ACF plot shows the oscillation of positive to negative about the 0-axis. The pattern then continues until the tails come close to 0. This type of pattern is expected since it is an $ARMA(2,2)$ model, under the examination of an ACF plot. The ACF plot is not capable of identifying exactly such a series, so only the tailing off is expected. The same happens in the PACF plot. There are some large tails, and then a slow tailing off from the negative side. This pattern is also expected, since a PACF plot can't identify an $ARMA(p, q)$ model. Since it is an $ARMA(2,2)$ model, this tailing off in the ACF and PACF plots are expected.

Since this is an $ARMA(2,2)$ series rather than an $AR(2)$ series, it is not possible to see this exact pattern in the triangle from handout 11. Although the ACF may appear like area 2, the PACF is definitely different since the lags don't drop off immediately after 2.

Reference

https://stat.ethz.ch/R-manual/R-devel/library/stats/html/ARMAacf.html