Robustness of the algorithm of identification of the type of dynamic object found at the finite sequence of 2D background frames of the optoelectron device

A. N. Katulev,[1,][*]   A. N. Sotnikov,[2,][**]
V. K. Kemaikin,[3,][***]   and I. V. Kozhukhin[3,][****]

[1]*Central Research Institute of Aerospace Defense Forces of Ministry of Defense of the Russian Federation, Afanasiy Nikitin Embankment, building 32, Tver, 170026, Russia*
[2]*Joint Supercomputer Center of the Russian Academy of Sciences - branch of Scientific Research Institute of System Analysis of the Russian Academy of Sciences, Leninsky prospect 32a, Moscow, 119334, Russia*
[3]*State Technical University, Lenin Avenue, building 25, Tver, 170023, Russia*

Abstract—Here are proposed robustness characteristics of the algorithm of identification of the type of the dynamic object (DO) and the law of probability distribution of identification sufficient statistics, formed by the algorithm under prior uncertainty. The law is applied for verification and validation of the algorithm. Wavelet fractal correlation algorithm (WFCA) implements vectorial criterion of ratio of likelihood functions of simple alternative hypotheses - types of DOs, this criterion being invariant to specific features of DO motion trajectories. The likelihood functions are reconstructed by simulation according to sufficiently representative complexes of implementations of fractal dimensions, energies, wavelet spectra and maximum eigenvalues of biased correlation matrices as functional of the measured coordinates of spatial attitude of various types of real DOs located by the optoelectron device (OED). The simulation proved robustness and high efficiency of the algorithm of identification of the type of DOs.

Keywords and phrases: *identification algorithm, dynamical object, optoelectronic device*

1. RELEVANCE

In [1], the solution of dynamic object type identification is proposed to implement automatically with the wavelet-fractal-correlation algorithm for processing samples of measurements of the coordinates for the elevation angle, azimuth and range of the detected DO optoelectron device (OED) in the final sliding time interval with a priori uncertainty about the actual current phono-target situation in the device control zone. During the movement, the DO can maneuver and be observed by OED from different angles. As a result, the intensity of the DO optical radiation in the direction of the OED will represent a non-stationary space-time random process with an unknown probability distribution law. In samples of measurements of DO coordinates, "outliers" and fluctuations can appear. In Fig. 1, Fig. 2, Fig. 3 shows such samples obtained by real DO on various trajectories of their movement in the zone of control of OED (the abscissa axis in the figures shows the number of phono-target frames formed by the OED, that is, the sample sizes, and the ordinate axis is kilometers and degrees).

Obviously, this will determine the different significance of individual measurements and the effect on the result reliability producing by the recognition algorithm of the detected DO type. It is not possible to reduce the influence through the implementation of the adaptation principle due to a

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[*]   E-mail: `katuleva@mail.ru`
[**] E-mail: `asotnikov@jscc.ru`
[***] E-mail: `vk-kem@mail.ru`
[****]E-mail: `kozhukhin@mail.ru`

priori uncertainty regarding the nonstationarity of the current phono-target situation in the zone of OED control.

Because of this, there arises an objective necessity to build an algorythm of dynamic object type identification that is invariant to the features of the DO trajectory and robust (stable) to various elements – "outliers" and fluctuations in measurement samples.

The stability should be quantified and include the initial data for the algorithm, the algorithm of DO type identification and the result of the processing of the original measurement samples by the algorithm.

In the contemporary literature on digital processing of signals and images generated by an OED, the stability of DO automatic detection and DO tracking algorithms is not affected. Optimization and research of algorithms is performed from deterministic positions by assessing the accuracy of solving the corresponding problems and convergence of the solution to the exact, corresponding stationary conditions for the functioning of the OED. These conditions we receive from samples of stationary processes from OED [2–6].

However, it is obvious that for real non-stationary conditions for the functioning of OED and a priori uncertainty about the state of the current phono-target situation, it is advisable to optimize and evaluate the algorithm of DO type identification from the standpoint of robustness – statistical stability.

2. THE OBJECTIVE OF ARTICLE

Substantiation of the stability property of the initial data as sufficient statistics for WFCA identification of DO type, the stability of the identification statistics generated by the algorithm and the stability of the algorithm itself. Identifying the probability distribution law for recognition statistics and its stability properties. Estimation of efficiency indicators for the DO type identification with a priori uncertainty regarding the phono-target situation in the zone of control of the OED.

3. WAVELET-FRACTAL-CORRELATION ALGORITHM: THE STRUCTURE AND PROPERTIES

The structure and computational operations of VFKA are completely determined by expressions of DO type identification criteria based on sufficient statistics: fractal dimensions – $d(\cdot)$, wavelet spectral energies – $u(\cdot)$ and maximum eigenvalues – $\lambda(\cdot)$ for shifted correlation matrices of samples of elevation coordinates measurements for elevation angle – θ, azimuth – ϕ and elevation angle – D of DO locating OED (the dot $\cdot$ is a measured coordinate, three types of DO have nine statistics).

The sufficiency of each of these statistics is physically determined by the fact that they are calculated directly from samples of DO coordinate measurements as minimal sufficient statistics, and formally it is established by the well-known Rao-Blequella-Kolmogorov theorem: optimal estimates (subject to their existence) are functions of sufficient statistics [7]. In this case, the validity of the application of the theorem is obvious. Additionally, we note that in [1], the authors confirmed the sufficiency property based on the analysis of the likelihood functions of obtaining measurement samples under the condition that they are located in the zone of control of the OED for DO corresponding type which we should recognize (as an option it is considered transport aircraft, fighter and helicopter [8]). Identification criteria are written as [1]

$$\frac{f_d(d(\theta)|p_j,q_j,d(\theta_j),s_j)}{f_d(d(\theta)|p_i,q_i,d(\theta_i),s_i)} \geq 1, \frac{f_d(d(\phi)|p_j,q_j,d(\phi_j),s_j)}{f_d(d(\phi)|p_i,q_i,d(\phi_i),s_i)} \geq 1, \frac{f_d(d(D)|p_j,q_j,d(D_j),s_j)}{f_d(d(D)|p_i,q_i,d(D_i),s_i)} \geq 1,$$

$$\frac{f_u(u(w(\theta))|p_j,q_j,u(w_j),s_j)}{f_u(u(w(\theta))|p_i,q_i,u(w_i),s_i)} \geq 1, \frac{f_u(u(w(\phi))|p_j,q_j,u(w_j),s_j)}{f_u(u(w(\phi))|p_i,q_i,u(w_i),s_i)} \geq 1, \frac{f_u(u(w(D))|p_j,q_j,u(w_j),s_j)}{f_u(u(w(D))|p_i,q_i,u(w_i),s_i)} \geq 1,$$

$$\frac{f_\lambda(\lambda(\theta)|p_j,q_j,\lambda(\theta_j),s_j)}{f_\lambda(\lambda(\theta)|p_i,q_i,\lambda(\theta_i),s_i)} \geq 1, \frac{f_\lambda(\lambda(\phi)|p_j,q_j,\lambda(\phi_j),s_j)}{f_\lambda(\lambda(\phi)|p_i,q_i,\lambda(\phi_i),s_i)} \geq 1, \frac{f_\lambda(\lambda(D)|p_j,q_j,\lambda(D_j),s_j)}{f_\lambda(\lambda(D)|p_i,q_i,\lambda(D_i),s_i)} \geq 1,$$
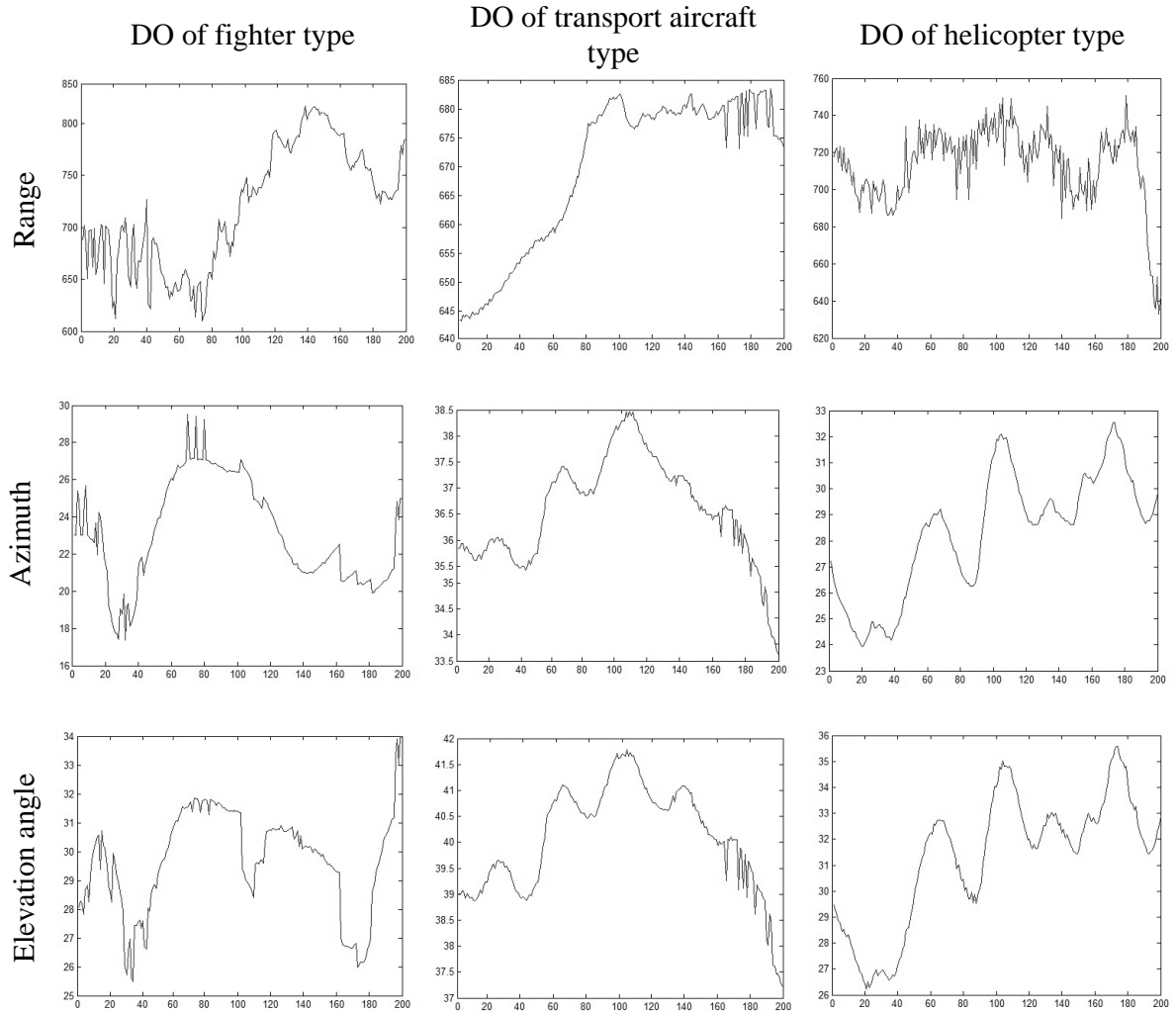
$$(1)$$

Figure 1. The samples of DO coordinate measuring for different angles of standard flight trajectory their observations by OED. Option 1.

where $f_d(\cdot|\cdot)$, $f_u(\cdot|\cdot)$, $f_\lambda(\cdot|\cdot)$ are beta-likelihood functions of samples, provided that they are in the control zone of OED for DO type s_j, $j, i = 1, 2, \ldots, M$, $j \neq i$, $p_j > 0$, $q_i > 0$ – beta-functions parameters.

Criteria (1) is a vector of independent subtests (independence is determined by the independence of the statistics $d(\theta)$, $d(\phi)$, $d(D)$, $u(w(\theta))$, $u(w(\phi))$, $u(w(D))$, $\lambda(\theta)$, $\lambda(\phi)$, $\lambda(D)$).

Directly from (11 it follows that the structures of the criteria are similar, trans-form after logarithmation to the same type of computational expression and completely determine the sequence of computational operations of the algorithm of DO type identification.

Write this expression, for example, for an algorithm that implements the first criterion from (1):
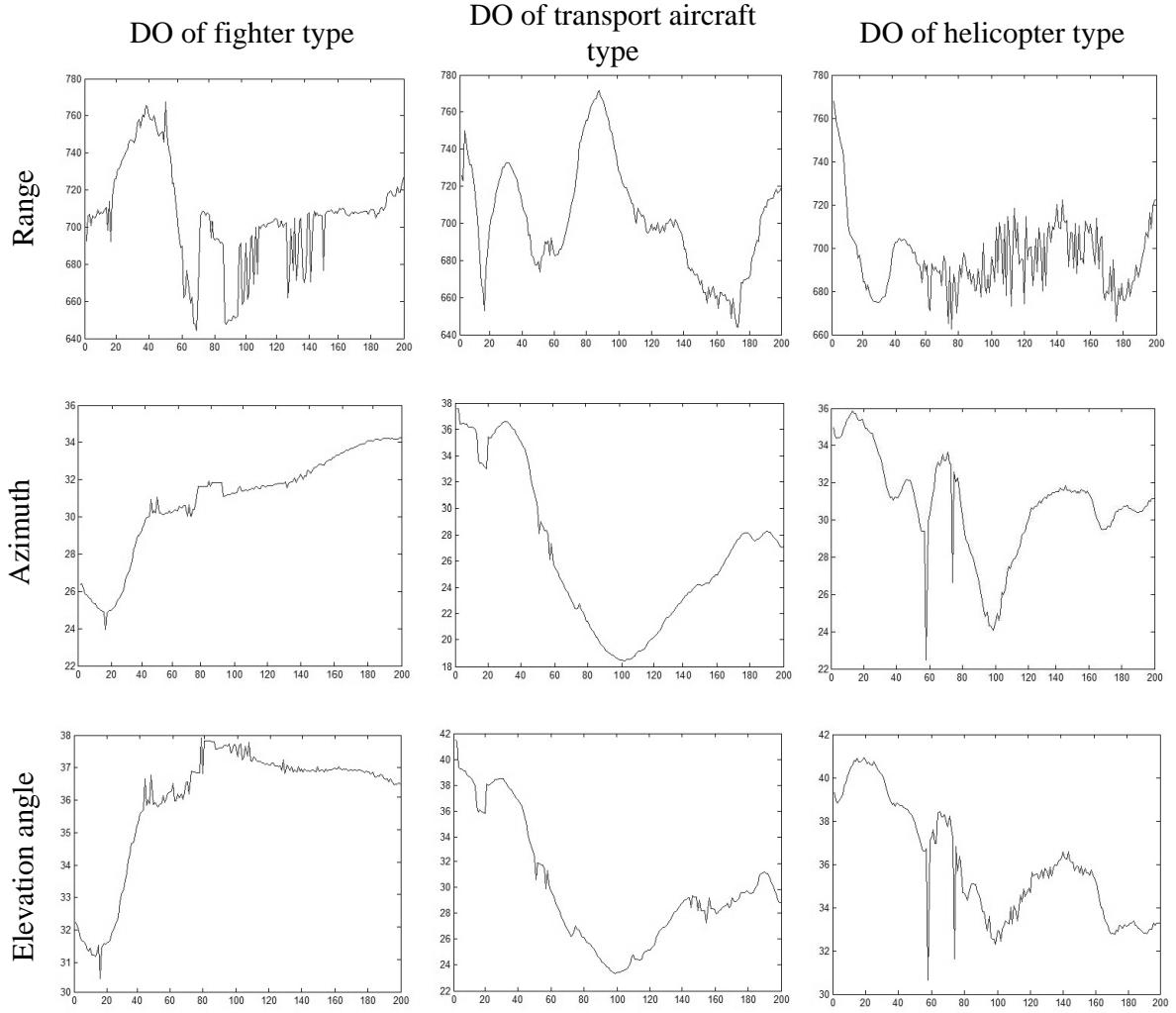
Figure 2. The samples of DO coordinate measuring for different angles of standard flight trajectory their observations by OED. Option 2.

$$(p_j - 1)\ln(u(w(\theta)) - \mu_{j0}) + (q_j - 1)\ln(\mu_{j1} - u(w(\theta)))$$

$$-(p_i - 1)\ln(u(w(\theta)) - \mu_{i0}) - (q_i - 1)\ln(\mu_{i1} - u(w(\theta))) \geq$$

$$\geq -\ln\frac{1}{\mu_{j1} - \mu_{j0}} - \ln\frac{\Gamma(p_j + q_j)}{\Gamma(p_j)\Gamma(q_j)} + (p_j - 1)\ln(\mu_{j1} - \mu_{j0}) + (q_j - 1)\ln(\mu_{j1} - \mu_{j0}) \qquad (2)$$

$$+\ln\frac{1}{\mu_{i1} - \mu_{i0}} + \ln\frac{\Gamma(p_i + q_i)}{\Gamma(p_i)\Gamma(q_i)} - (p_i - 1)\ln(\mu_{i1} - \mu_{i0}) + (q_i - 1)\ln(\mu_{i1} - \mu_{i0})$$

where $\mu_{j0} \leq u(w(\theta)) \leq \mu_{j1}$, $j, i = 1, 2, \ldots, M$, $j \neq i$, μ_{j0}, μ_{j1} are left and right limits of range for values of sample sufficient statistics for identification algorithm. They are given in [1].

In this inequality expression, the right-hand side is the threshold level of decision making based on subset DO type identification criterion, and the left side is the recognition statistics as a function of a random variable – sufficient statistics – on the wavelet-spectrum energy of a measurements sample for the elevation angle of detecting DO in the current time interval.

From (2) it can be seen that the recognition statistics is represented by a linear combination of logarithmic functions of sufficient statistics $u(w(\theta))$.
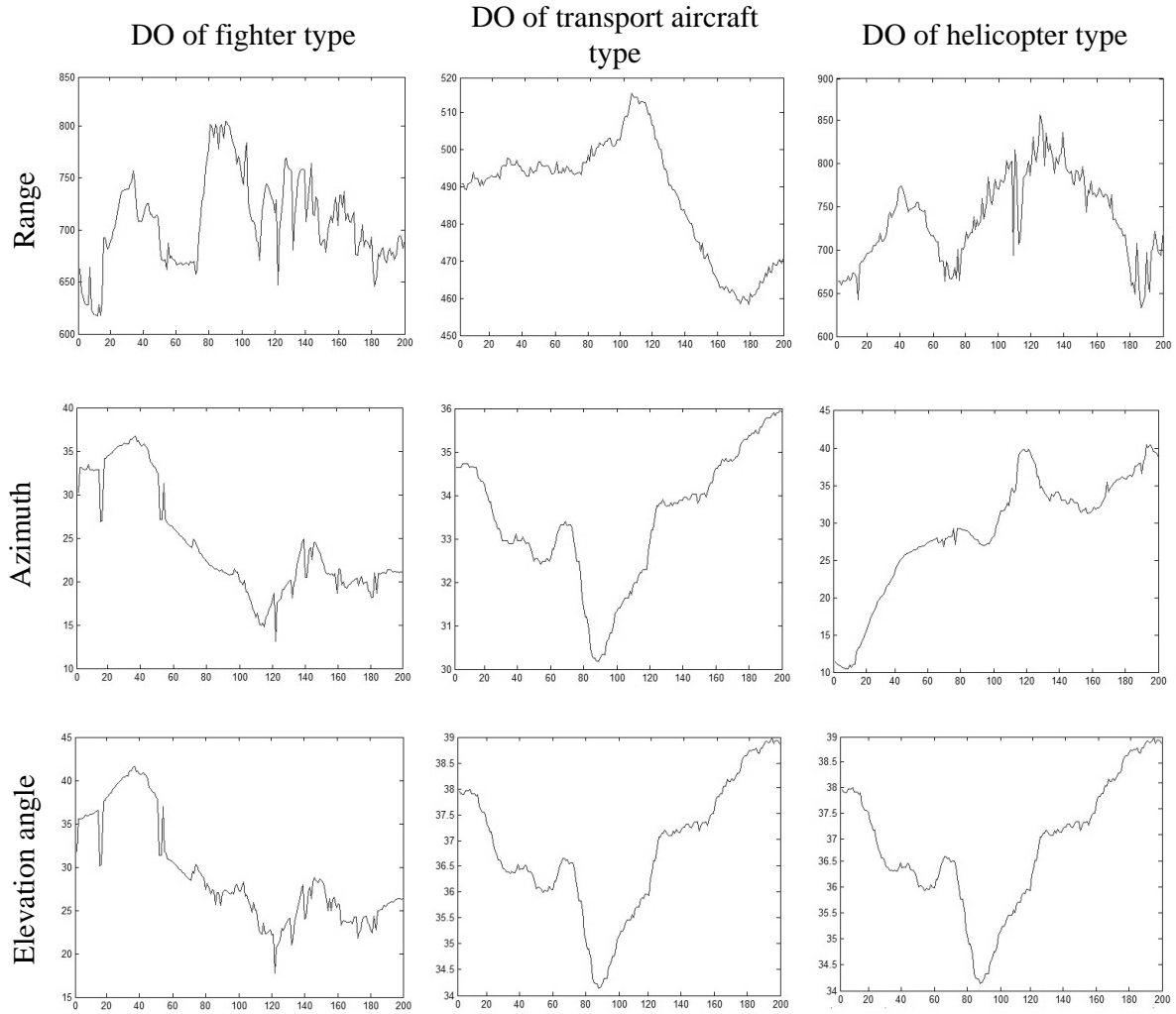
Figure 3. The samples of DO coordinate measuring for different angles of standard flight trajectory their observations by OED. Option 3.

Such functions are monotonous, they do not distort the sufficiency of the transformed statistics $u(w(\theta))$ [9, 10]. This means that each component of a linear combination is sufficient statistics as a function of sufficient statistics $u(w(\theta))$ and that a linear combination as a whole as statistics of DO type identification for a particular criterion is sufficient statistics, naturally, random, since the sample is finite. At the same time, we also note that the sufficiency of statistics is a consequence of the sufficiency properties of each initially considered likelihood ratio [9] and that with a sufficiently large sample, the likelihood ratio statistics are consistent and robust [10].

The established properties of the statistics of the left-hand side of (2) are sufficient conditions for assessing the stability of a particular DO type identification algorithm.

The necessary conditions are the stability of each initial sufficient statistics for the algorithm: fractal dimension $- d(\cdot)$, energy of the wavelet spectrum $- u(\cdot)$, maximum eigenvalue $- \lambda(\cdot)$ as a function of a sample of OED measurements of position for identifying DO, otherwise, as functions of minimal sufficient statistics obtained from the DO detection time interval.

The stability of private algorithms and the initial sufficient statistics for them together determine the stability property of DO type identification algorithm that implements a vector of criteria (1).

4. THE STABILITY OF THE INITIAL SUFFICIENT STATISTICS FOR THE DO TYPE IDENTIFICATION ALGORITHM

Taking into account that the initial data: the fractal dimension, the energy of the wavelet spectrum and the maximum eigenvalue generated for the DO type identification algorithm are sufficient statistics, here we will show that they and the algorithms for their formation are stable.

To that end, in the statistical approach, we use the law of large numbers and the central limit theorem as a theoretical basis [11] for determining stability and algorithms for estimating sufficient statistics for DO type identification algorithm, and the statistics itself. The validity of the approach comes from the fact that

– OED measurement sample for each coordinate of the DO detected position is a set of realizations of a sufficiently large volume of independent equally distributed random variables according to the same law, a priori unknown;

– sufficient statistics for each particular algorithm of DO type identification are estimates of the expectation for the fractal dimension, the energy of the wavelet spectrum and the maximum eigenvalue. They are determined by empirical variances. The evaluation of the third and fourth moments are not affected, since they are completely and unambiguously determined by empirical estimates of the expectation and variance for the beta distribution.

Now, to determine the stability index of the first two moments for sufficient statistics, we use the fundamental theory of confidence estimation of the expectation and variance, which asserts a unique functional relationship among the sample size, the number of observations, the accuracy of the estimated parameter as a measure of the minimum length of the confidence interval of the parameter estimate and the acceptable (required) confidence level proximity parameter estimates to the true (unknown) value.

The measure of localization of the estimated parameter by the confidence interval at a given confidence level and the final sample size is an indicator of the stability of the assessment.

In the problem of estimating the first moment – the mathematical expectation with unknown true variance l – the measure of localization of the estimate by the confidence interval, the confidence probability – β, the estimate of the standard deviation s and the sample size – n are related by the expression $l = 2sC_\beta/\sqrt{n}$, where C_β is the required (admissible given) confidence probability β from the condition: Laplace function $\Phi_n(C_\beta) = \beta$ with a sufficiently large sample n [13, 14].

In the problem of estimating the second moment – the standard deviation with an unknown expectation, $l_\sigma = |s - \sigma|$ – the localization measure – s the standard deviation, the sample size – n and the confidence level – β are related $l_\sigma = st_\beta/\sqrt{n}$, where t_β is the value β of the condition that (for large n) the Laplace function $\Phi_n(t_\beta)$.

So, in the case $C_\beta = 1.6$, defined in [1] by the "no-overrunning" condition of selective sufficient statistics beyond the unit interval and corresponding to a confidence level of 0.89 when approximating the probability distribution law of the deviation of the expectation estimate from the true value by a normal law [13], the localization measure $l = 2sC_\beta/\sqrt{n} = 2 \cdot 0.36 \cdot 1.6/10 = 0.1192$ for $n = 100$ and estimate of standard deviation $s = 0.36$.

It can be seen that in this case there is a very good approximation of the value of the estimated parameter – the mathematical expectation to the true one, and the estimate can be considered consistent, that is, stable. The algorithm for generating an estimate as sufficient statistics is also stable. Similar holds for estimating standard deviation.

Additionally, under conditions of a priori uncertainty, the adoption of a normal approximation of an unknown law is also based on the well-known [16] property of a normal law: the entropy of a random variable has the largest value with the same standard deviation if the probability distribution of a random variable is normal.

Thus, sufficient statistics: the estimates of the first two moments of the fractal dimension, the energy of the wavelet spectrum, and the maximum eigenvalue, which are generated for particular DO type indefication algorithms, and the algorithms for cal-culating them are stable. At the same time, l and l_σ are measures of localization of estimates in the vicinity of their true values were found as the least favorable – guaranteed for an acceptable level of confidence probability: without performing the operation of minimizing the interval.

If we now take into account [1] that the mentioned sufficient statistics take only positive values and only from finite intervals of their values, then the following becomes true: the initial sufficient

statistics for (1 are subject to probability distributions from the class of beta distributions. The latter, as is known [12], are uniquely determined by only two parameters $p_j > 0$, $q_j > 0$ – the DO type index. These parameters, as noted, in turn, are uniquely determined by estimates of mathematical expectations – m_j^* and variances – $s^2 = \sigma_j^{*2}$ sufficient statistics (in this case, guaranteed estimates): the parameters are one-to-one associated with these estimates by rational functions of the form $p_j = (m_j^* - \sigma_j^{*2})/[(\sigma_j^{*2}/m_j^*) - m_j^*]$ and $q_j = (p_j/m_j^*) - p_j$.

With such communication functions, the parameters, p_j, q_j, by virtue of the stability m_j^* and σ_j^{*2} and assurance of their values, are stable [16]. From this it follows that in this sense, the beta laws of the probability distribution of sufficient statistics of the fractal dimension, the energy of the wavelet spectrum, and the maximum eigenvalue are stable as laws that are completely determined by their stable parameters $p_j > 0$, $q_j > 0$. The laws are represented by power functions from sufficient statistics and are determined from samples of a sufficiently large amount of measurements of the coordinates of the position of the de-tected DO by OED with different features of the flight trajectory, sighting DO and background conditions (Fig. 1, Fig. 2, Fig. 3).

5. THE STABILITY OF THE RECOGNITION ALGORITHM TYPE

Theorem. Algorithm of the type (1, 2) for DO type identification is stable.

Evidence. Directly follows from the following facts:

– The coordinates of the elevation angle, azimuth and range of the detected DO measured by OED are initially considered to be minimal sufficient statistics. Measurements are carried out on the final sliding period of time. Collections of dimensions are represented by vectors of finite-dimensional spaces.

– Samples of measurements of coordinates on the goal-like images generated by the OED are filtered and not transmitted to the DO recognition algorithm [1].

– Minimum sufficient statistics are independent both in their totality and in pairs on each of the time intervals for detecting DO, subject to the same law of probability distribution.

– The sets of minimal sufficient statistics make up samples of a sufficiently large volume, they are converted into sufficient statistics as input data for the recognition algorithm of the type of detected DO. Such statistics for the algorithm, as noted above, are: the fractal dimension of the sample, the energy of the wavelet spectrum of the sample and the maximum eigenvalue of the sample offset correlation matrix.

– The property of sufficiency of statistics for the identification algorithm is due to the fact that their calculation is performed by algorithmically implemented linear functionals and operators [1].

Thus, the calculation of the fractal dimension is fundamentally based on a power law by minimizing the linear functional, represented by the sum of the squared deviations of the sample data from the true ones.

The calculation of the maximum eigenvalue is performed on the basis of the solution of the characteristic equation for the sample offset correlation matrix as a linear operator.

The energy of the wavelet spectrum of the sample is a linear transformation in the form of a sum of squares of the decomposition wavelet coefficients for measurement samples according to basic orthogonal wavelets with the corresponding shift and scale parameters, while the wavelet coefficients are determined by scalar products (linear functionals) of the sampling process on basic orthogonal wavelets.

– DL type recognition is carried out by a team of likelihood ratio criteria and is reduced to performing a comparison operation of the statistics generated by it with optimally established threshold levels of the form of the right-hand side (2).

– DO type recognition statistics are sufficient: they are functionals of likelihood ratios of simple DL type hypotheses, directly realized by the collective by the recognition algorithm. Likelihood functions are beta distributions, their parameters are stable.

– Likelihood functionals are defined in finite-dimensional space, so they are bounded and continuous, that is, the likelihood functionals are linear [17].

In this regard, as proved [11], the algorithm that implements linear functionals is stable.

This statement completes the proof of the stability of the DO type identification algorithm – the algorithm for implementing the collective criteria (1).

6. THE PROBABILITY DISTRIBUTION LAW OF STATISTICS RECOGNITION OF DO TYPE. STABILITY PROPERTIES.

Definition: DO type identification statistics is a random variable of the type of the left part of the rule (2).

To evaluate the efficiency of the algorithm, it is naturally necessary to know the law of probability distribution of the recognition statistics, the threshold level for deciding on the type of the detected DO and the stability properties of the law.

Obviously, from the structure of the left-hand side of (2), the law can be established subject to the known laws of the distribution of the probabilities of each of its terms as sufficient statistics. The laws of the terms are conditional: they are established provided that DO are located in the OED control zone for each alternative type.

The terms are the functions of samples of independent measurements of the coordinates of the elevation angle of either the azimuth or the distance in the time interval for detecting the DO by OED.

Under such circumstances, a formal solution to the problem of restoring the desired law of statistics of the form of the left part (2) as a linear combination is reduced to performing the sequence of the following operations for the measurement samples and the functions of them:

Under such circumstances, a formal solution to the problem of restoring the desired law of statistics of the form of the left part (2) as a linear combination is reduced to performing the sequence of the following operations for the measurement samples and the functions of them:

1. Conversion of samples to the form of primary sufficient statistics: fractal dimensions $d(\theta)$, $d(\varphi)$, $d(D)$, wavelet spectral energy $u(w(\theta))$, $u(w(\varphi))$, $u(w(D))$ and maximum eigenvalues $\lambda(\theta)$, $\lambda(\varphi)$, $\lambda(D)$. The transformation is carried out by algorithmically realized linear functionals and operators, their substantive essence is disclosed in the previous subsection.

2. Calculation of the laws of probability distribution of sufficient statistics $d(\theta)$, $d(\varphi)$, $d(D)$, $u(w(\theta))$, $u(w(\varphi))$, $\lambda(\theta)$, $\lambda(\varphi)$, $\lambda(D)$.

The type of laws for various conditions - the types of DO, observed by the ODE, is set by modeling, their mathematical description is presented in [1]. These are beta distributions with appropriate parameters for alternative DO types.

3. Calculation of probability distribution laws for function statistics

$$(p_j - 1)\ln(d(\theta) - \mu_{j0}),\ \dots\ (q_j - 1)\ln(\mu_{j1} - d(\theta)),$$

$$(p_j - 1)\ln(d(\varphi) - \mu_{j0}),\ \dots\ (q_j - 1)\ln(\mu_{j1} - d(\varphi)),$$

$$(p_j - 1)\ln(d(D) - \mu_{j0}),\ \dots\ (q_j - 1)\ln(\mu_{j1} - d(D)),$$

$$(p_j - 1)\ln(u(w(\theta)) - \mu_{j0}),\ \dots\ (q_j - 1)\ln(\mu_{j1} - u(w(\theta))),$$

$$(p_j - 1)\ln(u(w(\varphi)) - \mu_{j0}),\ \dots\ (q_j - 1)\ln(\mu_{j1} - u(w(\varphi))),$$

$$(p_j - 1)\ln(u(w(D)) - \mu_{j0}),\ \dots\ (q_j - 1)\ln(\mu_{j1} - u(w(D))),$$

$$(p_j - 1)\ln(\lambda(\theta)) - \mu_{j0}),\ \dots\ (q_j - 1)\ln(\mu_{j1} - \lambda(\theta)),$$

$$(p_j - 1)\ln(\lambda(\varphi)) - \mu_{j0}),\ \dots\ (q_j - 1)\ln(\mu_{j1} - \lambda(\varphi)),$$

$$(p_j - 1)\ln(\lambda(D)) - \mu_{j0}),\ ...\ (q_j - 1)\ln(\mu_{j1} - \lambda(D)).$$

The laws are calculated using the well-known formula $\psi(y) = f(\phi(y))\left|\phi'(y)\right|$, where $y = \xi(x)$, ξ is a differentiable monotonic function, φ is an inverse function with respect to a function ξ, is a density of a random variable y, $f(x|s_j)$ a density of a random variable subject to the condition s_j of the DO type.

In the case, for the component, $(p_j - 1)\ln[u(w(\theta)) - \mu_{j0}]$ taking into account the notation in the formula for $\psi(y)$ have $y = (p_j - 1)\ln(u(w(\theta)) - \mu_{j0})$ or this $x = u(w(\theta)) = \exp\{y/(p_j - 1)\} + \mu_{j0} = \phi(y)$ is a random variable with a beta-distribution density (θ - designation of the non-random elevation position) and the probability distribution density of a random variable has the form.

$$\psi(y|s_j) = f(\phi(y|s_j)\left|\phi'(y)\right| = f(\exp\{y/(p_j - 1)\} + \mu_{i0})\left|(1/(p_j - 1))\exp\{y/(p_j - 1)\}\right| =$$

$$= \frac{|(1/(p_j - 1))|\exp\{y/(p_j - 1)\}}{(\mu_{j1} - \mu_{j0})}\frac{\Gamma(p_j + q_j)}{\Gamma(p_j)\Gamma(q_j)}\times$$

$$\times\left(\frac{\exp\{y/(p_j - 1)\} + \mu_{i0}}{\mu_{j1} - \mu_{j0}}\right)^{p_j - 1}\left(1 - \frac{\exp\{y/(p_j - 1)\} + \mu_{j0}}{\mu_{j1} - \mu_{j0}}\right)^{q_j - 1}$$

Below we will consider the option $u(w(\theta)) \in [\mu_{j0} = 0, \mu_{j1} = 1]$. The density $\psi(y|s_j)$ will be determined only on the negative semi-axis of the values, that is, on the interval $- \infty \leq y = |p_j - 1|\ln(u(w(\theta))) \leq 0$,

$$\psi(y|s_j) = (1/\left|p_j - 1\right|)\exp\{y/\left|p_j - 1\right|\}\frac{\Gamma(p_j + q_j)}{\Gamma(p_j)\Gamma(q_j)}(\exp\{y/\left|p_j - 1\right|\})^{p_j - 1}(1 - \exp\{y/\left|p_j - 1\right|\})^{q_j - 1}$$

and we can see all other random variables type $|p_i - 1|\ln u(w(\theta))$ will have the same type of density, derived from (1).

Acting in a similar way, we obtain the conditional probability density distribution of random variables of the type $\nu = |q_j - 1|\ln[1 - u(w(\theta))]$; density is written as

$$\psi(\nu|s_j) = f(\phi(\nu|s_j)\left|\phi'(\nu)\right| = f(1 - \exp\{\nu/|q_j - 1|\})\left|(1/|q_j - 1|)\exp\{\nu/|q_j - 1|\}\right|$$

and given that $f(\mu_{j1} - \exp\{\nu/(q_j - 1)\})$ is a beta distribution, in the form

$$\psi(\nu|s_j) = (1/\left|q_j - 1\right|)\exp\{\nu/\left|q_j - 1\right|\}\frac{\Gamma(p_j + q_j)}{\Gamma(p_j)\Gamma(q_j)}\times$$

$$\times(1 - \exp\{\nu/\left|q_j - 1\right|\})^{p_j - 1}(\exp\{\nu/\left|q_j - 1\right|\})^{q_j - 1}.$$

and note that the laws obtained $\psi(y|s_j)$ and $\psi(\nu|s_j)$ do not preserve the form of the beta law: they describe random variables y and ν, taking values from an interval $(-\infty, 0)$.

4. Calculation of probability distribution laws for statistics of functions of the form

$$(p_j - 1)\ln(d(\theta) - \mu_{j0}) - (p_i - 1)\ln(d(\theta) - \mu_{i0}),\ ...\ (q_j - 1)\ln(\mu_{j1} - d(\theta)) - (q_i - 1)\ln(\mu_{i1} - d(\theta))$$

$$(p_j - 1)\ln(d(\varphi) - \mu_{j0}) - (p_i - 1)\ln(d(\varphi) - \mu_{i0}),\ ...\ (q_j - 1)\ln(\mu_{j1} - d(\varphi)) - (q_i - 1)\ln(\mu_{i1} - d(\varphi))$$

$$(p_j - 1)\ln(d(D) - \mu_{j0}) - (p_i - 1)\ln(d(D) - \mu_{i0}),\ ...\ (q_j - 1)\ln(\mu_{j1} - d(D)) - (q_i - 1)\ln(\mu_{i1} - d(D))$$

$$(p_j - 1)\ln(u(w(\theta)) - \mu_{j0}) - (p_i - 1)\ln(u(w(\theta)) - \mu_{i0}), \ \dots \ (q_j - 1)\ln(\mu_{j1} - u(w(\theta))) - $$
$$-(q_i - 1)\ln(\mu_{i1} - u(w(\theta)))$$

$$(p_j - 1)\ln(u(w(\varphi)) - \mu_{j0}) - (p_i - 1)\ln(u(w(\varphi)) - \mu_{i0}), \ \dots \ (q_j - 1)\ln(\mu_{j1} - u(w(\varphi))) - $$
$$-(q_i - 1)\ln(\mu_{i1} - u(w(\varphi)))$$

$$(p_j - 1)\ln(u(w(D)) - \mu_{j0}) - (p_i - 1)\ln(u(w(D)) - \mu_{i0}), \ \dots \ (q_j - 1)\ln(\mu_{j1} - u(w(D))) - $$
$$-(q_i - 1)\ln(\mu_{i1} - u(w(D)))$$

$$(p_j - 1)\ln(\lambda(\theta) - \mu_{j0}) - (p_i - 1)\ln(\lambda(\theta) - \mu_{i0}), \ \dots \ (q_j - 1)\ln(\mu_{j1} - \lambda(\theta)) - (q_i - 1)\ln(\mu_{i1} - \lambda(\theta))$$

$$(p_j - 1)\ln(\lambda(\varphi) - \mu_{j0}) - (p_i - 1)\ln(\lambda(\varphi) - \mu_{i0}), \ \dots \ (q_j - 1)\ln(\mu_{j1} - \lambda(\varphi)) - (q_i - 1)\ln(\mu_{i1} - \lambda(\varphi))$$

$$(p_j - 1)\ln(\lambda(D) - \mu_{j0}) - (p_i - 1)\ln(\lambda(D) - \mu_{i0}), \ \dots \ (q_j - 1)\ln(\mu_{j1} - \lambda(D)) - (q_i - 1)\ln(\mu_{i1} - \lambda(D))$$

and establishing the properties of the distribution laws of these statistics for $\mu_{j0} = 0$, $\mu_{j1} = 1$.

It can be seen that the probability densities of differences of equally distributed random variables should be calculated, compared with (1) for the alternative DO types of s_j and s_i. To do this, we use the formula of the composition of the laws of the distribution of independent random variables. $|p_j - 1|\ln u(w(\theta))$ and $|p_i - 1|\ln u(w(\theta))$.

The desired density is

$$g_{y-\nu}(z_1|s_j, s_i) = \int\limits_{\min\{y\}}^{\max\{y\}} \psi_Y(y|s_j)\psi_\nu((y - z_1)|s_i)dy$$

where $z_1 = y - \nu = |p_j - 1|\ln u(w(\theta)) - |p_i - 1|\ln u(w(\theta))$ and for $\mu_{j0} = 0$, $\mu_{j1} = 1$ and $\mu_{i0} = 0$, $\mu_{i1} = 1$ $\quad u(w(\theta) \in [0, 1]$, density of sufficient statistics y and ν for DO types $j, i = 1, 2, \dots, M$, $j \neq i$, are not equal to zero in the intervals $-\infty \leq |p_j - 1|\ln u(w(\theta)) \leq 0$, $-\infty \leq |p_i - 1|\ln u(w(\theta)) \leq 0$ for each j, i. In this case, the density $\psi_y(y|s_j)$ is nonzero at $-\infty \leq y \leq 0$, and the density $\psi_\nu((y - z_1)|s_i)$ is nonzero at $y - z_1 \leq 0$ or $y \leq z_1$, $-\infty \leq z_1 \leq \infty$, the statistics is unlimited and the required density is written in such convolutions:

as $z_1 \leq 0$ $g_{y-\nu}(z_1|s_j, s_i) = \int\limits_{-\infty_1}^{-z_1} \psi_Y(y|s_j)\psi_\nu((y - z_1)|s_i)dy$,

and as $z_1 > 0$ $g_{y-\nu}(z_1|s_j, s_i) = \int\limits_{-\infty}^{0} \psi_Y(y|s_j)\psi_\nu((y - z_1)|s_i)dy$,

where

$$\psi_Y(y|s_j)\psi_\nu((y - z_1)|s_i) = $$
$$= \left|\frac{1}{p_j - 1}\right| \exp\{y/|p_j - 1|\} \frac{\Gamma(p_j + q_j)}{\Gamma(p_j)\Gamma(q_j)} (\exp\{y/|p_j - 1|\})^{p_j - 1} (1 - \exp\{y/|p_j - 1|\})^{q_j - 1} \times$$
$$\times \left|\frac{1}{p_i - 1}\right| \exp\{(y - z_1)/|p_i - 1|\} \frac{\Gamma(p_i + q_i)}{\Gamma(p_i)\Gamma(q_i)} (1 - \exp\{(y - z_1)/|p_i - 1|\})^{p_i - 1} (\exp\{(y - z_1)/|p_i - 1|\})^{q_i - 1}$$

Convolutions of the same type of distribution. It is known (for example, [18]) that such convolutions lead to the same type of distribution.

Table 1. Parameter values $\{p_j, q_j\}$

Measured coordinates	DO type	Parameters for the β - distribution of fractal dimension		Parameters for the β - distribution of maximum eigenvalue		Parameters for the β - distribution of energy of wavelet spectrum	
		p	q	p	q	p	q
Range	1	0.64	0.88	0.64	0.64	0.73	1.47
	2	0.73	1.47	1.08	4.32	0.46	0.46
	3	0.46	0.46	1.00	5.65	0.76	2.29
Azimuth	1	1.02	3.58	0.64	0.64	0.92	2.01
	2	1.01	2.63	1.09	3.27	0.83	1.38
	3	0.60	0.60	1.05	2.45	0.57	0.57
Elevation angle	1	1.13	5.07	1.08	4.32	1.13	5.07
	2	1.07	2.30	1.00	5.65	0.90	1.29
	3	0.66	0.66	0.64	0.64	0.66	0.66

Due to the complexity of the expression for $\psi_Y(y|s_j)\psi_\nu((y - z_1)|s_i)$ obtaining an analytical solution, convolutions are practically difficult. The Laplace transform is also difficult to compute for the analytical expression of the desired law. Therefore, we use numerical integration using the MatLab trapz function. In Fig. 4, Fig. 5, Fig. 6, a graphic representation of the laws numerically obtained for different pairs of parameters are given $\{p_j, q_j\}$; the latter are given in table 1 [1].

From the representations of the laws in Fig. 4, Fig. 5, Fig. 6 that they are characterized by the following properties: they are continuous, single-vertex, have a single (each its own) mode, are differentiable everywhere except for (for some densities) a single point - the argument corresponding to the top of the law that the derivatives of the laws do not decrease to the left of the arguments vertices and do not increase to the right. The laws are defined on areas as areas of attraction, they are symmetric (the asymmetry of some graphs is due to the nonoptimality of the choice of the integration step in the Matlab system), they have finite dispersions and can be approximated only by the density of the Laplace law. The latter are infinitely divisible and limit for the sum of independent identically distributed random variables [12, 17].

In this case, such a sum is the left side of expression (2). According to these features, the properties of the density of the laws of probability distribution of recognition statistics, are stable [18].

Similar graphical representations and properties determine the laws of the statistics of the form $(q_j - 1)\ln(\mu_{j1} - d(\theta)) - (q_i - 1)\ln(\mu_{i1} - d(\theta))$, that is, the statistics functions of the fractal dimension, the maximum eigenvalue and the wavelet spectrum of measurements of the coordinates of the elevation angle, azimuth and distance of the detected DO by OED.

Such statistics are taken into account in the next subparagraph.

5. Calculation of probability distribution laws for statistics sums

$$(p_j - 1)\ln(d(\theta) - \mu_{j0}) - (p_i - 1)\ln(d(\theta) - \mu_{i0}) + (q_j - 1)\ln(\mu_{j1} - d(\theta)) - (q_i - 1)\ln(\mu_{i1} - d(\theta))$$

$$(p_j - 1)\ln(d(\varphi) - \mu_{j0}) - (p_i - 1)\ln(d(\varphi) - \mu_{i0}) + (q_j - 1)\ln(\mu_{j1} - d(\varphi)) - (q_i - 1)\ln(\mu_{i1} - d(\varphi))$$

$$(p_j - 1)\ln(d(D) - \mu_{j0}) - (p_i - 1)\ln(d(D) - \mu_{i0}) + (q_j - 1)\ln(\mu_{j1} - d(D)) - (q_i - 1)\ln(\mu_{i1} - d(D))$$

$$(p_j - 1)\ln(u(w(\theta)) - \mu_{j0}) - (p_i - 1)\ln(u(w(\theta)) - \mu_{i0}) + (q_j - 1)\ln(\mu_{j1} - u(w(\theta))) -$$
$$-(q_i - 1)\ln(\mu_{i1} - u(w(\theta)))$$

$$(p_j - 1)\ln(u(w(\varphi)) - \mu_{j0}) - (p_i - 1)\ln(u(w(\varphi)) - \mu_{i0}) + (q_j - 1)\ln(\mu_{j1} - u(w(\varphi))) -$$
$$-(q_i - 1)\ln(\mu_{i1} - u(w(\varphi)))$$

$$(p_j - 1)\ln(u(w(D)) - \mu_{j0}) - (p_i - 1)\ln(u(w(D)) - \mu_{i0}) + (q_j - 1)\ln(\mu_{j1} - u(w(D))) -$$
$$-(q_i - 1)\ln(\mu_{i1} - u(w(D)))$$

$$(p_j - 1)\ln(\lambda(\theta) - \mu_{j0}) - (p_i - 1)\ln(\lambda(\theta) - \mu_{i0}) + (q_j - 1)\ln(\mu_{j1} - \lambda(\theta)) - (q_i - 1)\ln(\mu_{i1} - \lambda(\theta))$$

$$(p_j - 1)\ln(\lambda(\varphi) - \mu_{j0}) - (p_i - 1)\ln(\lambda(\varphi) - \mu_{i0}) + (q_j - 1)\ln(\mu_{j1} - \lambda(\varphi)) - (q_i - 1)\ln(\mu_{i1} - \lambda(\varphi))$$

$$(p_j - 1)\ln(\lambda(D) - \mu_{j0}) - (p_i - 1)\ln(\lambda(D) - \mu_{i0}) + (q_j - 1)\ln(\mu_{j1} - \lambda(D)) - (q_i - 1)\ln(\mu_{i1} - \lambda(D))$$

The calculation of the laws for the written amounts of random variables is also established using the composition theorem [14] of the laws of distribution of the differences of random variables, compiled in the previous subparagraph 4.

So, for the sum $z = z_1 + z_2$, in which with $\mu_{j0} = 0$, $\mu_{j1} = 1$ and $\mu_{i0} = 0$, $\mu_{i1} = 1$ with the terms $z_1 = |p_j - 1|\ln u(w(\theta)) - |p_i - 1|\ln u(w(\theta))$ and $z_2 = |q_j - 1|\ln u(w(\theta)) - |q_i - 1|\ln u(w(\theta))$, and for similar other sums of random variables, as can be seen directly from the above expressions, the distribution law is established by the formula.

$$g_{z_1+z_2}(z|s_j, s_i) = \int\limits_{\min\{z_1\}}^{\max\{z_1\}} \psi_{z_1}(z_1|s_j, s_i)\psi_{z_2}(z - z_1|s_j, s_i)dz_1$$

where the type of laws $\psi_{z_1}(z_1|s_j, s_i)$ and $\psi_{z_2}(z - z_1|s_j, s_i)$ is defined in subparagraph 4 and presented in Fig. 4, and the limits of integration can be calculated by the intervals of values , of random variables, as indicated in the figures.

The laws of the form $g_{z_1+z_2}(z|s_j, s_i)$ are convolutions of stable single-type distributions (Fig. 4), they are reduced to the same form and have the same properties as the laws of random variables z_1 and z_2.

Hence the conclusion: the law of probability distribution of recognition statistics of DO type according to the rule of the form (2), which implements the corresponding particular algorithm (1), has the properties of stable distributions.

7. PERFORMANCE INDICATORS OF DO TYPE IDENTIFICATION ALGORITHM

The randomness of the sufficient statistics obtained in (2) can lead to correct and incorrect recognition of DO type for each particular algorithm, and as a whole by the team of DO type identification algorithms.

The quantitative measure of the identification result is the probability of correct and incorrect recognition of DO type ofthe performance indicators for each particular algorithm and the algorithm as a whole.

Due to the impossibility of recovering analytical expressions of the probability distribution laws of sufficient statistics for DO type identification, private algorithms for evaluating, their performance indicators are obtained here by modeling under various conditions of functioning OED (Fig. 1, Fig. 2, Fig. 3 illustrate such conditions).
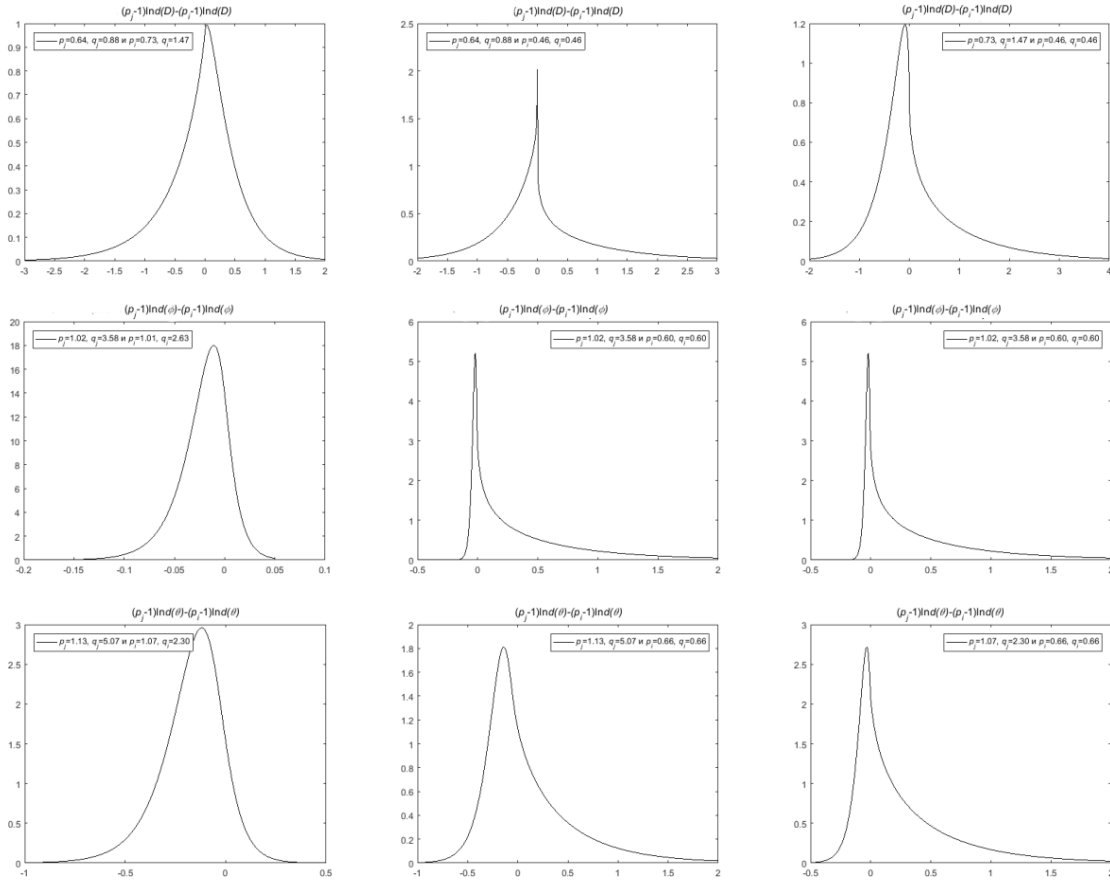
Figure 4. The laws of the distribution of statistics functions of fractal dimension.

Private algorithms operate independently of one another. The output of each of them is represented in the form $\delta_j = 1$ or $\delta_j = 0$, respectively, when recognizing DO type j or making a decision on recognizing DO of another type. At the same time, the performance indicators in general of the DO type recognition algorithm j will be determined by the binomial distribution, which describes that, of the full team of private algorithms, "voted" at least to private algorithms "for" DO type.

Estimates of the probability of correct identification of DO type by a particular algorithm are obtained by modeling for each sufficiently separately considered statistics; their values $P(s_j|s_j) \cong 0.67$, $j = 1, 2, \ldots, 9$, (s_j – DO type notation j, $P(s_j|s_j)$ – is the probability that the private algorithm makes decisions about the detection of the DO of type s_j, when the DO of s_j type really does exist in the control zone of the OED).

The performance indicators of the team of recognition algorithms of the type of detectable DO by OED: probability of correct recognition of DO type

$$P(s_j) = \sum_{l=6}^{l=9} C_9^l P^l(s_j|s_j)(1 - P(s_j|s_j))^{9-l} \cong 0.6 \qquad (3)$$

where $P(s_j|s_j) \cong 0.67$, $j = 1, 2, \ldots, 9$, and the threshold level of making the right decision by a team of algorithms, as established in [1], is equal to 6;

probability of type confusion for DO type $P(s_j) = 1 - P(s_j|s_j) \cong 0.4$
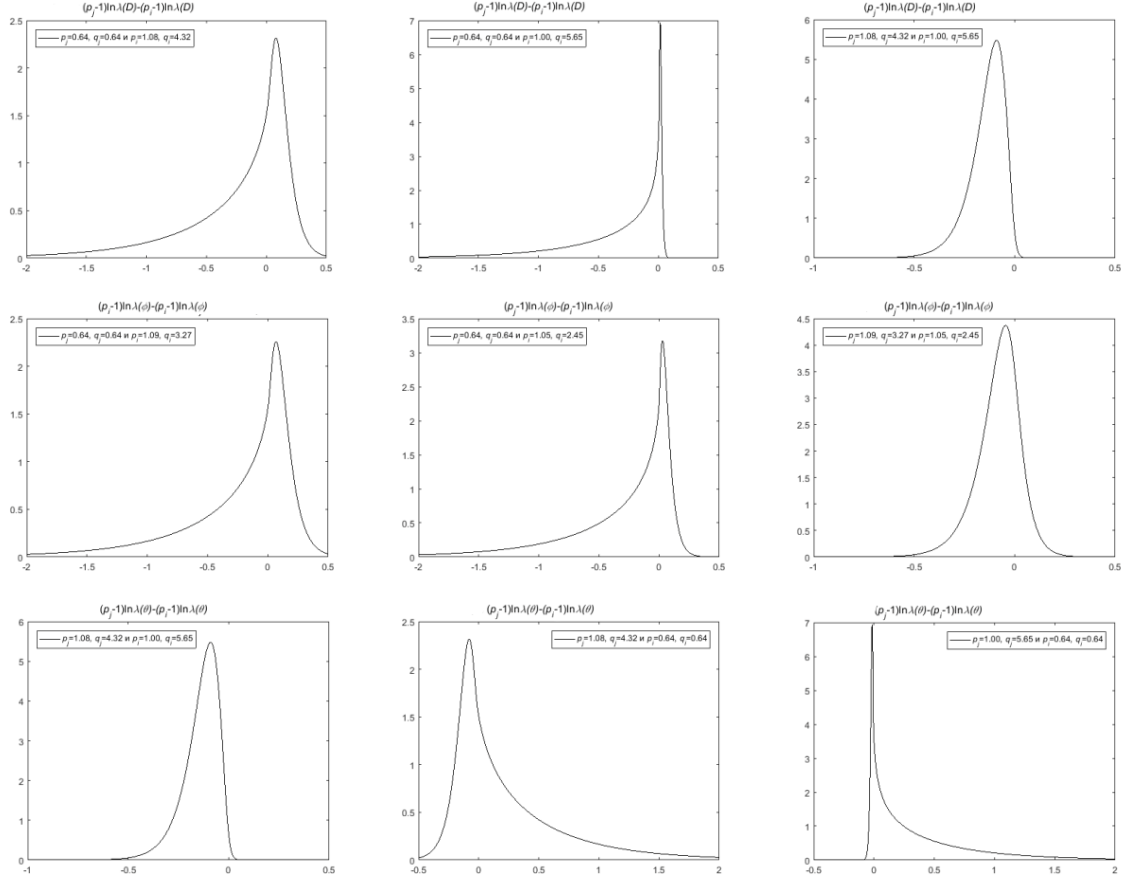
Figure 5. The laws of the distribution of statistics functions of maximum eigenvalue.

The probability value $P(s_j)$, as noted above, can be increased either by using a priori information about the current phono-target situation in the zone of control of the OED, but only by relatively simple stationary conditions, or by accumulating the results of "voting" by the team of private algorithms at regular time intervals of DO type indefication, implementation of the recognition criterion "at least once k from l" for n intervals [20, 21] in any in real phono-target conditions.

Here, the second approach and the "$k = 3$ from $l = 5$" criterion are adopted, as the closest to the optimal "3 out of 4" [22]; it is implemented by the authors in [1].

The decision on DO type is estimated by cumulative probability by the recurrent expression [22] $P(3, 5; n) = P(3, 5; n - 1) + f(3, 5; n)$, where $P(3, 5; n)$ is the probability of the "3 out of 5" criterion at least once at consecutive intervals, $P(3, 5; n - 1)$ – the same probability over $(n - 1)$ the intervals, $f(3, 5; n)$ the probability of the 3 out of 5 criterion execution for the first time on n time interval.

The calculation $P(\cdot)$ and $f(3, 5; n)$ is performed on guaranteed values of the probabilities $P(s_j) \triangleq p_n = P_{pr.rasp} = 0.6$ and $P(s_j) \triangleq q_n = P_{per} = 0.4$ at successive intervals, starting with the first, $n = 1$, then for $n = 2, 3, 4, \ldots$.

The results are as follows: with $n = 1$ probability $P(3, 5; n = 1) = 0$, with $n = 2$ $P(3, 5; n = 2) = P_{pr.rasp} \cdot P_{pr.pasr} = 0$, with $n = 3$ $P(3, 5; n = 3) = 0.6^3$, with $n = 4$ $P(3, 5; n = 4) = 0.64$, with $n = 5$ $P(3, 5; n = 5) = 0.84$, with $n > 5$ $P(3, 5; n > 5) > 0.9$ and $P_{per} < 0.1$.

Estimates of the probability of correct recognition of DO type are lower guaranteed, the probability of entanglement is upper guaranteed. Note that in [22] the value of the probability of correct recognition of DO type was obtained in the range of $0.708 - 0.976$ with the introduction of a priori formed standards of the contours of the detected DO with a simple background.
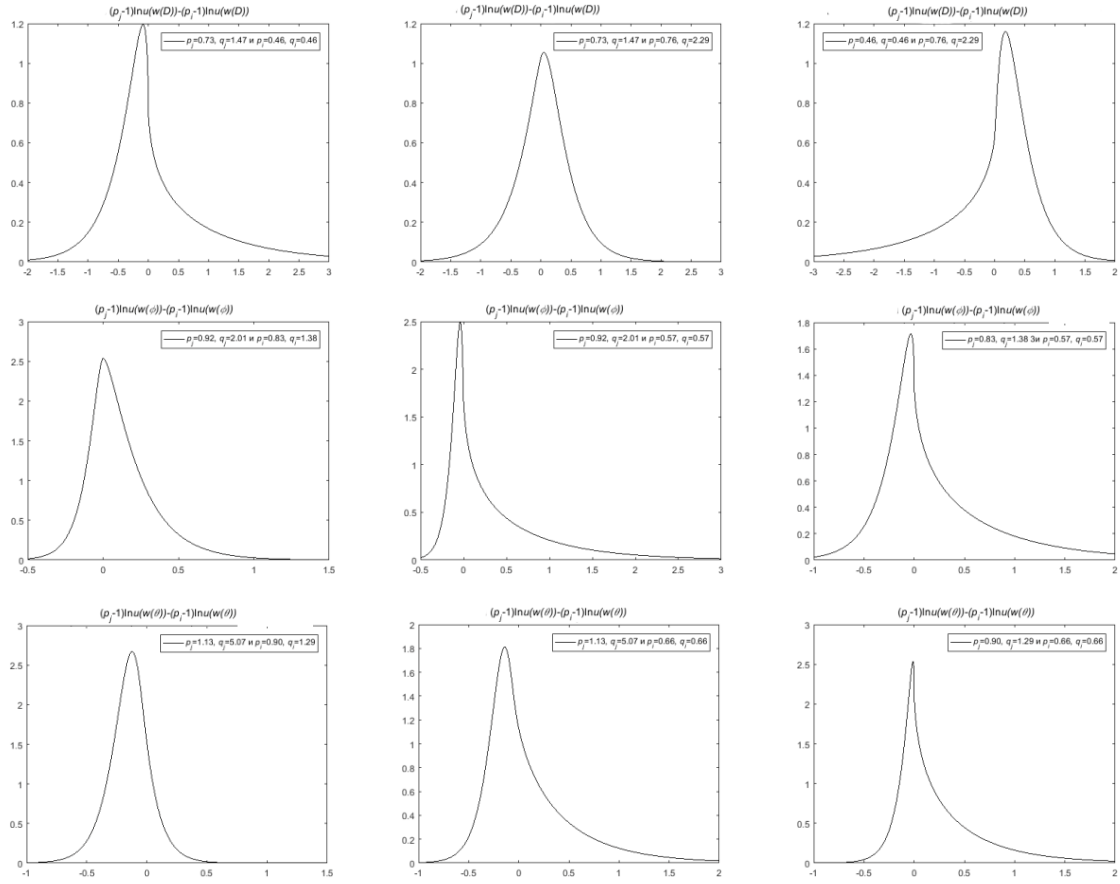
Figure 6. The laws of the distribution of the statistics functions of the energy of the wavelet spectrum.

8.   CONCLUSION

It is established the invariance of the structure of the wavelet-fractal-correlation DO type identification algorithm to the peculiarities of the current phono-target environment controlled by OED.

The statistical stability of the initial sufficient statistics for the algorithm and the statistical stability of DO type identification algorithm itself are proved for various features of its flight path and sight.

The performance indicators of the wavelet-fractal-correlation type recognition algorithm are estimated for various real-world operating conditions of the OED as the primary sensor of the sets of measurements of the coordinates of the elevation angle, azimuth and range of the detected DO at each finite current time interval and the stable performance of the algorithm in both simple and complex phono-target conditions for the functioning of the OED, including during intensive maneuvering of DO.

Set forth above it represents the development of the theory of methods for assessing the stability of the functioning of algorithms in the conditions of statistical and tactical uncertainty about the current phono-target situation in the zone of control of OED.

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