

per hour, prevailed, after which the force of the wind rapidly increased, shifting to W.N.W. until about 2 o'clock when it became a perfect hurricane, accompanied by a storm of snow and sleet. The velocity of the wind must have been betwixt 40 and 50 miles per hour, and partly from heavy black clouds covering the sky, and partly from the dense sheet of snow which was drifting along, everything became obscured.

From the point where I stood (at the door of my house, the Old Observatory, Calton Hill) there appeared two currents of snow-drift, one on each side, which meeting each other about 20 feet in front of me, after a severe struggle coalesced and shot upwards with excessive velocity slantingly, and towards E.S.E. As that upshot was going or gone, but when the storm was still at its maximum, a loud crash was heard, and a vivid flash of light simultaneously seen, much as one may suppose would be the effect of the bursting of a bomb-shell within a few feet of you.

Almost immediately after this the storm began to abate, and the remainder of the day was comparatively calm. Altogether the storm did not last above ten minutes, and during that time the barometer fell $\frac{2}{10}$ ths of an inch, but rose again immediately afterwards to its former height.

4. On the Colouring of Maps. By Professor Tait.

(*Abstract.*)

Some years ago, while I was still working at knots, Professor Cayley told me of De Morgan's statement that four colours had been found by experience to be sufficient for the purpose of completely distinguishing from one another the various districts on a map.

I had previously shown that if an even number of boundaries meet at each point on a diagram, two colours (as on a chess-board) will suffice for the purpose. But in a map, boundaries usually meet in threes.

I replied to Professor Cayley that I thought the proof might be made to depend upon the obvious proposition that not more than four points in a plane can be joined two and two by non-intersecting lines. Here points were made to stand for districts. When two such points are joined by a line they must have different colour-titles. I

did not at the time pursue the subject, as I found that it was more complex than it appeared at first.

Mr Kempe's paper in *Nature* (February 26, 1880) has recalled my attention to the subject, and some simple modes of treating the question have occurred to me. The germs of them are in what I have said above, and they show one easily how to proceed to colour any map. A sketch only of one of them is now given.

Begin by making as above stated a companion diagram, putting points for districts, and lines joining them for common boundaries. Then by introducing (in any way) as many new joining lines as possible (but so that no two intersect) the diagram is divided into three-sided compartments.

Next, make all of these compartments four-sided by taking a number of new points, each on a joining line. The whole set of points can now be lettered A and B alternately, because two colours suffice for a map whose boundaries meet in fours. But let the intruded points be lettered a and b , instead of A and B respectively.

Now perform the same operation in a second way, differing everywhere from the first, and call the newly intruded points α and β instead of A and B.

Rules are laid down for carrying out these operations; but they require too many illustrative cuts to be given here.

Then any one triangular compartment will appear in two essentially different forms: for instance, with its intruded points it may read (in the two cases, taking the corners in the same order) B, a , B, A and B, A, β , A. Now superpose the two figures, lettering included, and attend to the order of the two letters at the same point. We have, from the instance above, the compound reading (attending now to the corners only) BB, BA, AA, of which the separate terms are necessarily different. Hence every point in the figure is lettered differently from all that are joined to it, and only four designations can occur, viz.: AA, AB, BA, BB. This proves the proposition, and gives one mode of colouring the original map. For the *erasure* of joining lines (such as were originally introduced to divide the whole into three-sided compartments) does not necessitate any change of lettering.

This mode of treating the question shows incidentally that in a map where only three boundaries meet at each point, the boundaries

may be coloured with *three* colours, so that no two of the same colour are conterminous.

This particular process essentially introduces four different colours, and therefore does not necessarily give the simplest way of colouring a map. Another method, quite different from this, but involving virtually the same principles, is next given. Then come two other processes, different in form from that of Mr Kempe, but based like it ultimately on the fact that only $3(n - 2)$ non-intersecting lines can be drawn (except for $n = 2$) joining n points in a plane.