CSE 523 Project Report

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1 Introduction

Given a set of points in the interval $(-1.0, 1.0)$ given in set S, with center of mass at 0, the aim is to find an ordering of points P, such that the ordering is "good". Let c_i be, the center of mass of P after the i^{th} point is placed. Some definitions of good are, ordering points in P such that the range of c_i is minimized or the total distance c_i moves as each point is placed, is minimized. These are the two factors considered for evaluating any ordering in this project.

The problem can be defined in two dimensional space if we allow the set S to be defined in the domain $R \times R$. We have looked at a variant of the problem where we are allowed to pick at most k points to be placed in set P instead of exactly one in the original problem description. In this case, if k points are picked during the i^{th} attempt, we add k points to P and calculate center of mass c_i on P. It has the effect of adding the center of mass of k points to P as a single point. Here we consider the case in which $k = 2$.

2 Greedy Range Minimization Heuristic

2.1 Definition

The heuristic aims to minimize the difference between the left most and right most values that c_i can take. That is, minimize the value of $max(c_i) - min(c_i)$. The algorithm achieves this approximately by selecting a point such that c_i is closest to c_n for $i > 0$.

2.2 Property

Let c_{min} and c_{max} represent the least and highest values of c_i at any time. Then Greedy Range Minimization Heuristic ensures that no point outside the range (c_{min}, c_{max}) is in selected in the set P before all points within the range are selected.

Consider two points X and Y such that both points are not selected in P till step $i - 1$. Let Y be outside (c_{min}, c_{max}) and X be inside this range. Let the center of mass values at this stage be be, c_{i-1}. If any point within (c_{min}, c_{max}) is picked in P, it will make c_i closer to c_n than any point outside the range. Thus X will always be picked ahead of Y and this choice is the one that minimizes the range of center of mass and hence the property holds.

Thus we can establish a relationship between reducing the interval of center of mass and the method of picking a point so that c_i is closest to c_n. We can see that the heuristic takes a greedy approach to minimizing the range of c_i, by how it selects the next point to include in P.

3 Multiple Selections

Instead of choosing a single point, we can choose exactly 2 points at every attempt except the last one (we choose 1 point or 2 points).

From a set of n points let us pick m sets of points P_{k1}, P_{k2}, ... , P_{km}. $|P_{ki}| = k$, if $1 \leq i < m$, and $|P_{km}| \leq k$. The unordered set of points in each P_{ki} are assumed to be placed at the same time. After each P_{ki} is placed the center of mass changes to c_i.

3.1 Property

Let P $= P_{k1}, P_{k2}, ..., P_{km}$ be the set of points picked during m attempts by heuristic H_k, where each set has at most k points as described above. Let $c'_1, ..., c'_m$ be the center of masses after each attempt. If some heuristic $H1$, which picks a single point at a time, picks point in the order $Q_1, Q_2, ..., Q_n$, such that it is some ordering of points in P where each point in P_{ki-1} is listed before any point in P_{ki}, for $ki > 0$. The range of center of mass of $H_k \leq$ range of center of mass of H_1. $(c'_{max} - c'_{min}) \leq c_{max} - c_{min}$.

The property holds because in H_k we end up selecting every k^{th} c_i value from H_1. The center of masses selected in H_k is a subset of those selected in $H1$. Thus we may miss some values that are representative of the extreme points c_{max} or c_{min}. Thus $c'_{max} \leq c_{max}$ and $c'_{min} \geq c_{min}$. Hence the range of $H_k \leq$ range of H_1.

4 Heuristics

We have compared 7 heuristics during the test runs. They appear with the heuristic number in source code.

- Heuristic 0: Greedy Movement Minimization: Keep c_i closest to c_{i-1}, picking one point a time. Greedily tries to minimize the total movement of c_i.
- Heuristic 1: Greedy Range Minimization: Keep c_i closest to c_n, picking one point at a time.
- Heuristic 2: Greedy Balancing Pair (Two simultaneous selections): Pick two points at a time such that their average is closest to c_n.

- Heuristic 3: Greedy Range Minimization: (Two simultaneous selections): Pick two points at a time such that c_i after picking two points at the i^{th} attempt is closest to c_n.

- Heuristic 4: Greedy Range Minimization within δ distance: Pick two points at a time such that, if there are two points within δ distance of each other and c_i after picking two points at the i^{th} attempt is closest to c_n, the two points are picked. If no two points are within δ distance, pick two points without the distance restriction such that, c_i after picking the two points is closest to c_n. The value of δ can be set upto the minimum range of (c_{max}, c_{min}) found using Heuristic 1.

  The motivation for this heuristic is that Greedy Range Minimization may pick extreme points (both may be away from c_n), such that a heuristic which picks up points in same order but a single point at a time, may result in poor results for the range/ interval of center of mass. By trying to pick the early points selected in P, within a short interval, the heuristic reflects a good ordering if we evaluate the selected points as being picked one at a time.

- Heuristic 5: Movement Minimization with δ distance: Pick one point at a time such that, the point is within δ distance of c_n, and c_i after picking two points at the i^{th} attempt is closest to c_{i+1} (Similar to Heuristic 0). If no two points are within δ distance and c_i after picking a pair of remaining points is closest to c_{i+1}, pick those two points. The value of δ can be upto the minimum range of (c_{max}, c_{min}) found using Heuristic 1. (In the source code, Heuristic 5 is implemented as a special case of Heuristic 0.)

- Heuristic 6: Movement Minimization: (Two simultaneous selections): Pick two point at a time such that, the center of mass after the i^{th} attempt c_{i-1} is closest to c_i.

5 Results

For each graph show below, the experiment was run for 10 random points in the closed interval $(-1.0, 1.0)$. In each case the range (interval) or the movement of center of mass is compared with respect to the corresponding optimal value. The points are sorted based on their values, as a result this does not bring down the range value when we evaluate two points placed simultaneously (c_i would have been decreased if we sorted points based on distance from c_n).

The figure below shows the comparison of the range of center of mass with respect to the optimal value. Greedy Range Minimization Heuristic (H1) performs better than other heuristics. The interval value from H1 is always within two times the optimal value for all test runs. For heuristics that pick 2 points at a time, we evaluate range/interval value for Fig 1. as if the pair of points were picked one after the other. In Fig 2. we evaluate the Movement of Center of Mass, where Greedy Movement Minimization (H0 and H5 perform equally well) outperforms other heuristics.

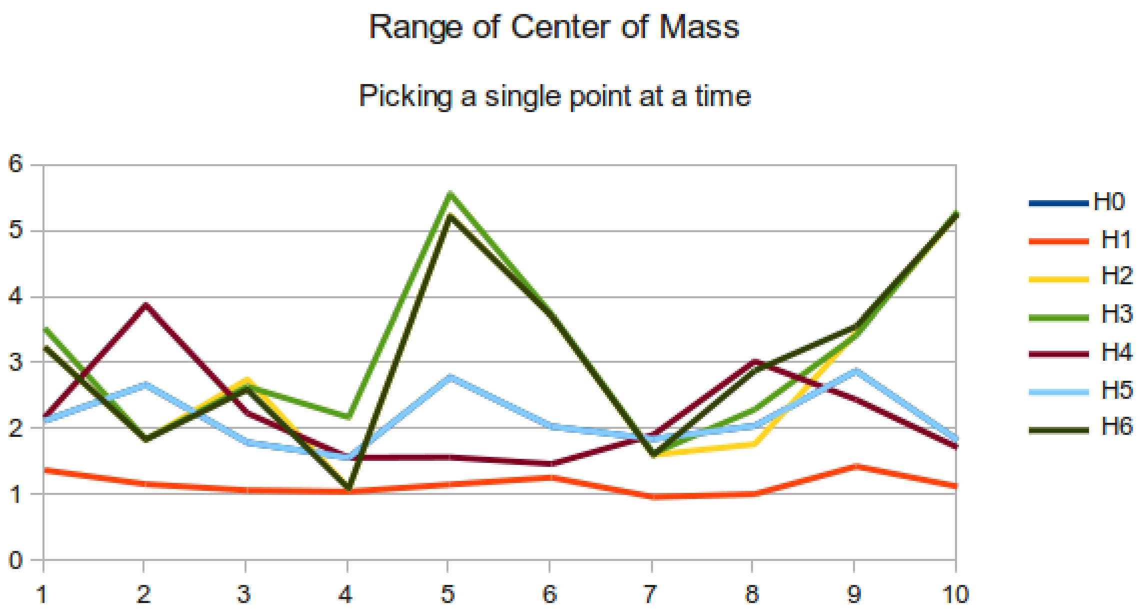

Fig 1. Range of Center of Mass

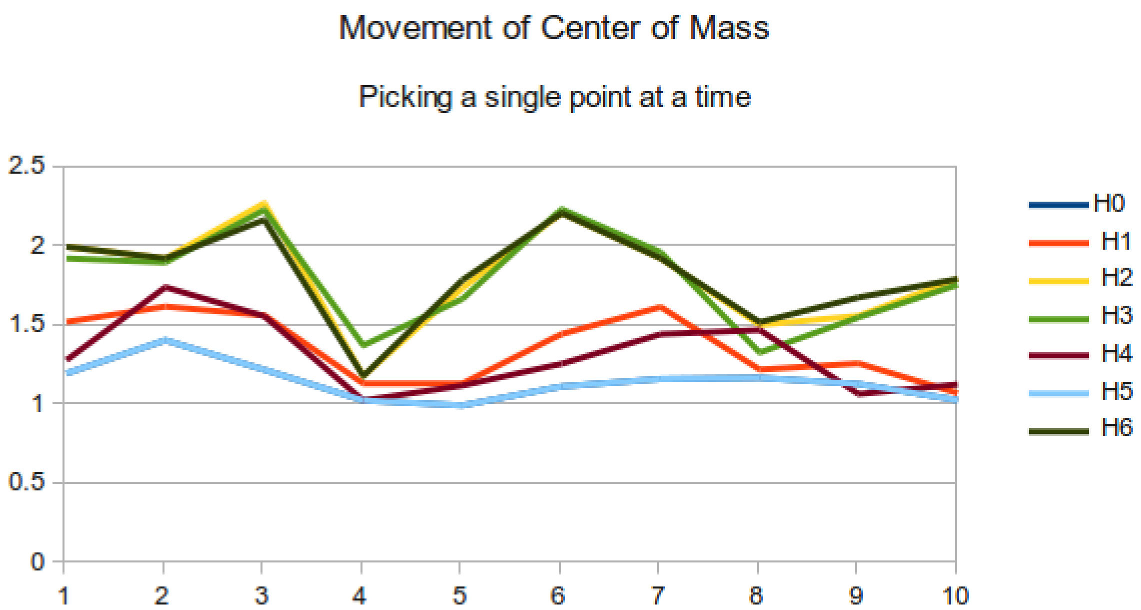

Fig 2. Movement of Center of Mass

Fig 3. and Fig 4. show the evaluation of parameters for heuristics that simultaneously pick two points. A difference between picking (picking two points at a time) and ordering (consider as if pairs of points were placed one after other in some order) is made for the sake of evaluation. The figures compare two evaluation parameters - 2 Pick strategy and 1 Pick strategy. This shows which heuristics are better for ordering a single point at a time, though they happen to pick two points at a time.

A motivation for considering this evaluation is that even if there is a method to place two points simultaneously on a surface, there may be small timing errors due to which points are placed one after another in some order. The evaluation tries to account for this.

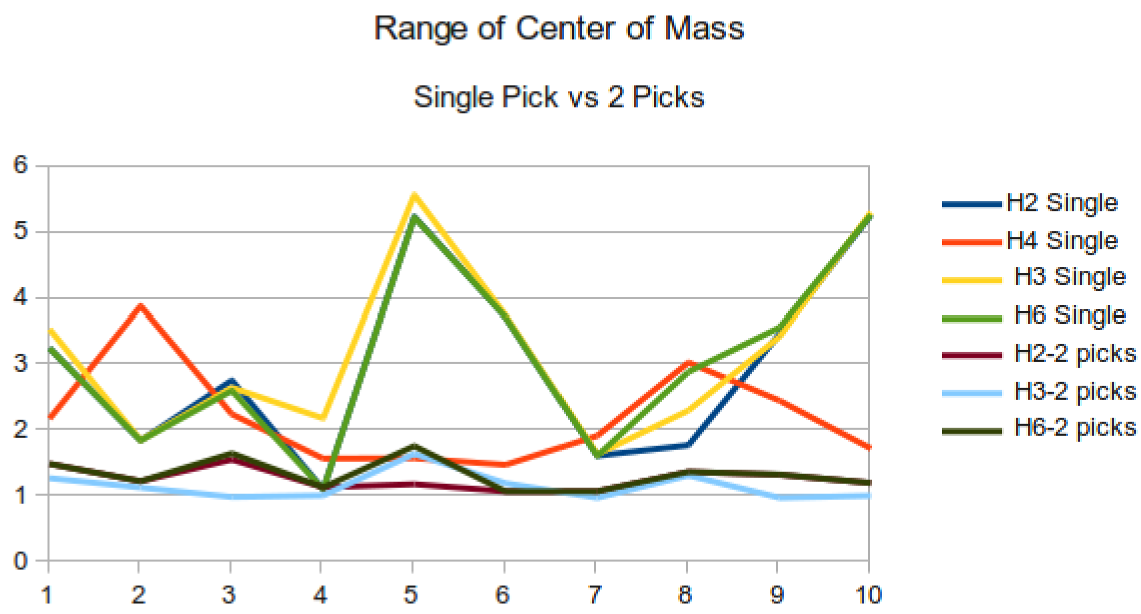

Fig 3. Range of Center of Mass

Heuristic 4 (Greedy Range Minimization within δ distance) aims to keep the range of center of mass minimum but picks points within δ distance of each other first. By setting a suitable value of δ equal to the optimal values for interval (OPT interval value) and distance moved (OPT distance moved), we see that the single pick evaluation betters the two simultaneous points (2 pick) evaluation for Heuristic 4.

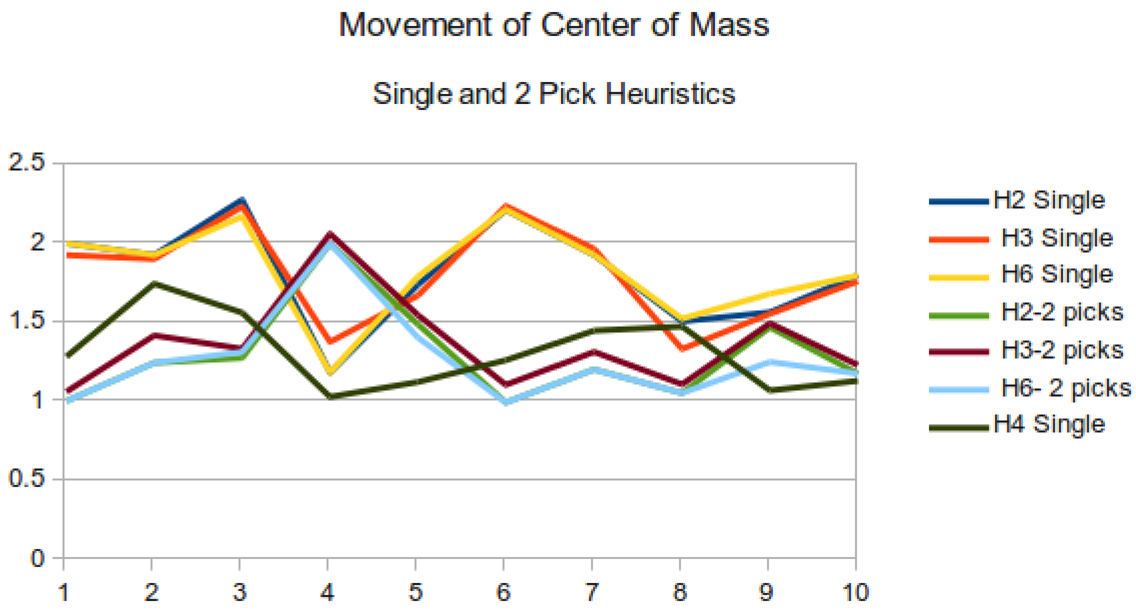

Fig 4. Movement of Center of Mass

In Fig 4. we see that H4 partitions the single pick and 2 pick evaluations for other heuristics (H2, H3 and H6). However the 2 pick evaluation for H4 (not show in the figures, but listed in results directories) has high values compared to 1 pick evaluation, though it is similar to H3 (which appears best for the interval of center of mass). H3 has the best results for 2 pick evaluation, while H4 has the worst results.