Continuous exposures with propensity scores

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Warning! **Propensity score weights are sensitive to positivity violations for continuous exposures.**

The story so far

Propensity score weighting

❶ Fit a propensity model predicting exposure x, x + z where z is all covariates

❷ Calculate weights

❸ Fit an outcome model estimating the effect of x on y weighted by the propensity score

Continous exposures

1. Use a model like `lm(x ~ z)` for the propensity score model.
2. Use `wt_ate()` with `.fitted` and `.sigma`; transforms using `dnorm()` to get on probability-like scale.
3. Apply the weights to the outcome model as normal!

Alternative: quantile binning

1. Bin the continuous exposure into quantiles and use categorical regression like a multinomial model to calculate probabilities.
2. Calculate the weights where the propensity score is the probability you fall into the quantile you actually fell into. Same as the binary ATE!
3. Same workflow for the outcome model

1. Fit a model for `exposure ~ confounders`

```r
model <- lm(
  exposure ~ confounder_1 + confounder_2,
  data = df
)
```

2. Calculate the weights with `wt_ate()`

```r
model |>
  augment(data = df) |>
  mutate(wts = wt_ate(
    exposure,
    .fitted,
    # .sigma is from augment()
    .sigma = .sigma
  ))
```

Does change in smoking intensity (smkintensity82_71) affect weight gain among lighter smokers?

```r
nhefs_light_smokers <- nhefs_complete |>
  filter(smokeintensity <= 25)
```

1. Fit a model for `exposure ~ confounders`

```r
nhefs_model <- lm(
  smkintensity82_71 ~ sex + race + age + I(age^2) +
    education + smokeintensity + I(smokeintensity^2) +
    smokeyrs + I(smokeyrs^2) + exercise + active +
    wt71 + I(wt71^2),
  data = nhefs_light_smokers
)
```

2. Calculate the weights with `wt_ate()`

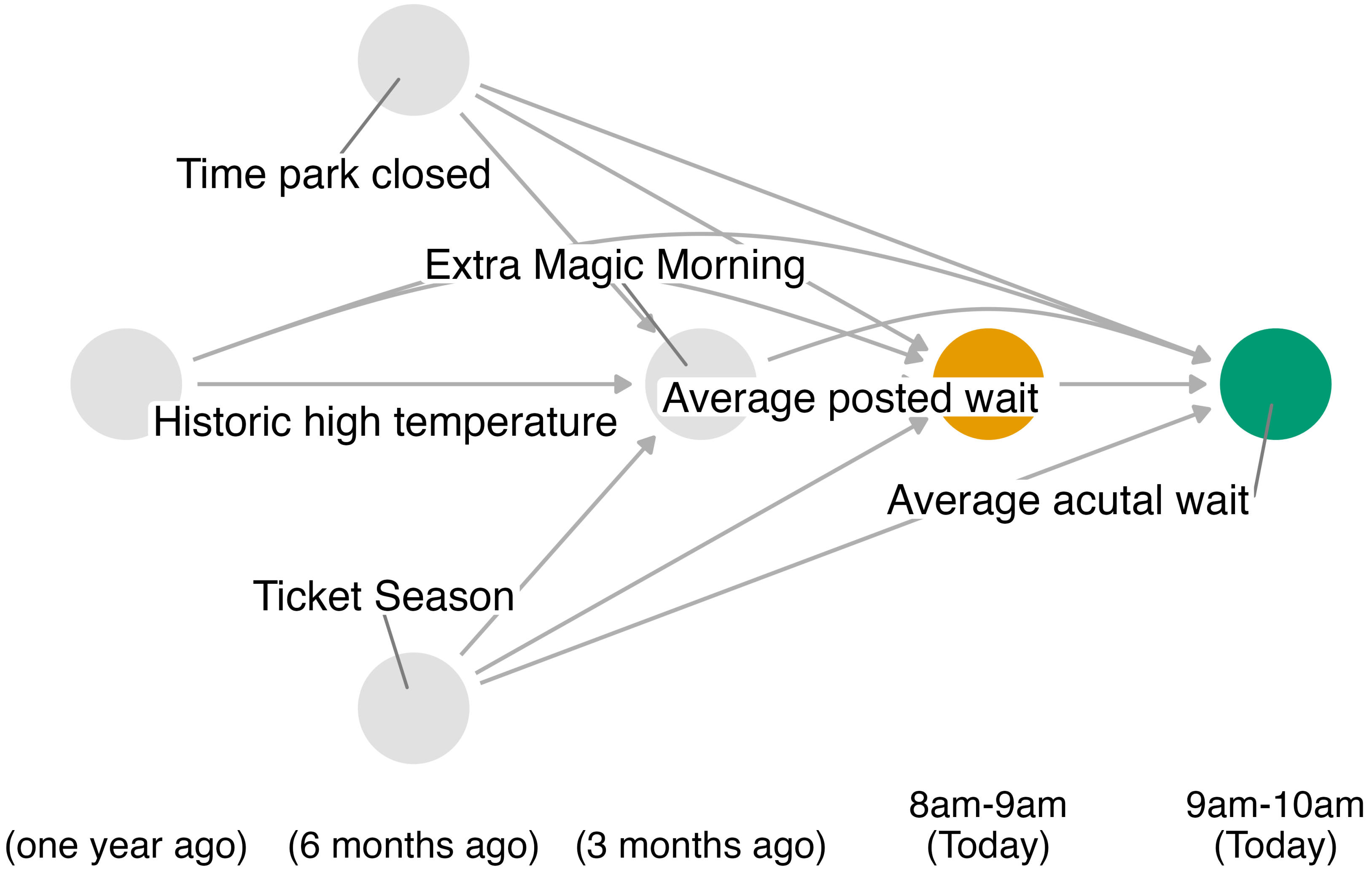
```r
nhefs_wts <- nhefs_model |>
  augment(data = nhefs_light_smokers) |>
  mutate(wts = wt_ate(
    smkintensity82_71,
    .fitted,
    .sigma = .sigma
  ))
```

2. Calculate the weights with `wt_ate()`

```r
nhefs_wts
```

```
# A tibble: 1,162 × 74
    seqn  qsmk death yrdth modth dadth   sbp   dbp sex  
   <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <fct>
 1   235     0     0    NA    NA    NA   123    80 0    
 2   244     0     0    NA    NA    NA   115    75 1    
 3   245     0     1    85     2    14   148    78 0    
 4   252     0     0    NA    NA    NA   118    77 0    
 5   257     0     0    NA    NA    NA   141    83 1    
 6   262     0     0    NA    NA    NA   132    69 1    
 7   266     0     0    NA    NA    NA   100    53 1    
 8   419     0     1    84    10    13   163    79 0    
 9   420     0     1    86    10    17   184   106 0    
10   434     0     0    NA    NA    NA   127    80 1    
```

Do *posted* wait times at 8 am affect *actual* wait times at 9 am?



Your Turn 1

Fit a model using `lm()` with `wait_minutes_posted_avg` as the outcome and the confounders identified in the DAG.

Use `augment()` to add model predictions to the data frame

In `wt_ate()`, calculate the weights using `wait_minutes_posted_avg`, `.fitted`, and `.sigma`

05:00

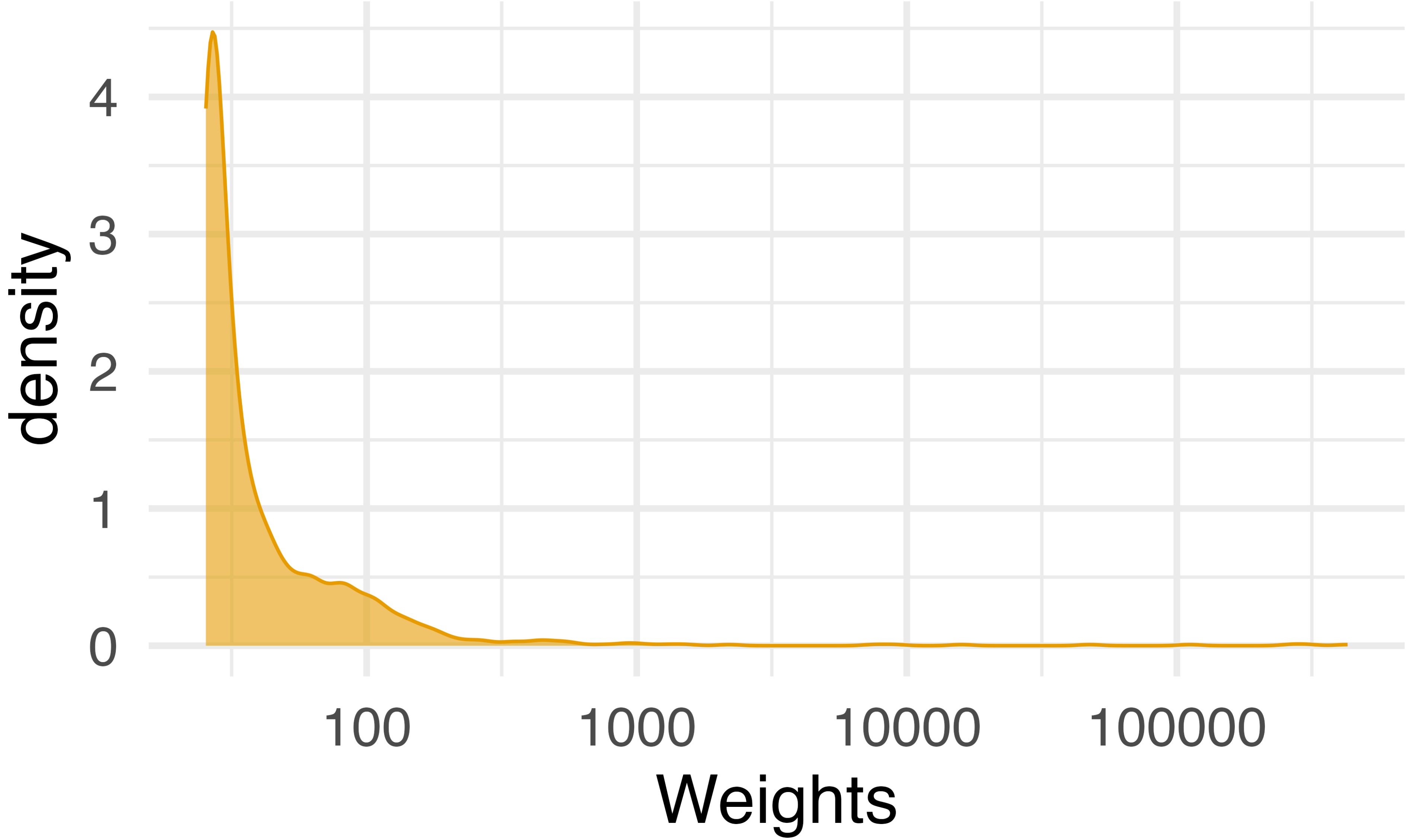
Your Turn 1

```r
post_time_model <- lm(
  wait_minutes_posted_avg ~
    park_close + park_extra_magic_morning +
    park_temperature_high + park_ticket_season,
  data = wait_times
)
```

Your Turn 1

```r
wait_times_wts <- post_time_model |>
  augment(data = wait_times) |>
  mutate(wts = wt_ate(
    wait_minutes_posted_avg, .fitted, .sigma = .sigma
  ))
```

Stabilizing extreme weights



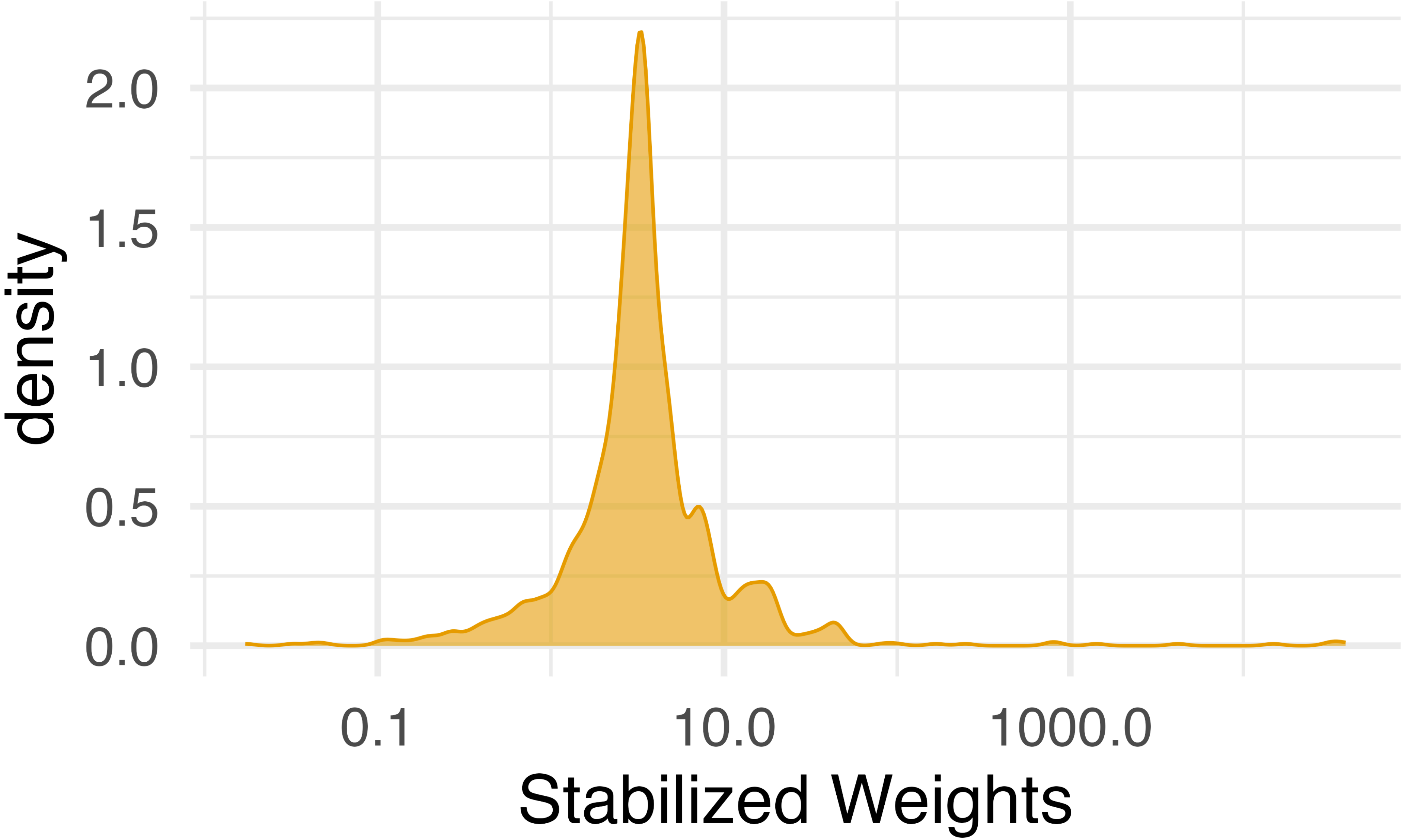
Stabilizing extreme weights

1. Fit an intercept-only model (e.g. `lm(x ~ 1)`) or use mean and SD of `x`
2. Calculate weights from this model.
3. Divide these weights by the propensity score weights. `wt_ate(.., stabilize = TRUE)` does this all!

Calculate stabilized weights

```r
nhefs_swts <- nhefs_model |>
  augment(data = nhefs_light_smokers) |>
  mutate(swts = wt_ate(
    smkintensity82_71,
    .fitted,
    .sigma = .sigma,
    stabilize = TRUE
  ))
```

Stabilizing extreme weights



Your Turn 2

Re-fit the above using stabilized weights

03:00

Your Turn 2

```r
wait_times_swts <- post_time_model |>
  augment(data = wait_times) |>
  mutate(swts = wt_ate(
    wait_minutes_posted_avg,
    .fitted,
    .sigma = .sigma,
    stabilize = TRUE
  ))
```

Fitting the outcome model

1. Use the stabilized weights in the outcome model. Nothing new here!

```r
lm(
  wt82_71 ~ smkintensity82_71,
  weights = swts,
  data = nhefs_swts
) |>
  tidy() |>
  filter(term == "smkintensity82_71") |>
  mutate(estimate = estimate * -10)
```

```
# A tibble: 1 × 5
  term              estimate std.error statistic  p.value
  <chr>                <dbl>     <dbl>     <dbl>    <dbl>
1 smkintensity82_71     1.99    0.0316     -6.30 4.33e-10
```

Your Turn 3

Estimate the relationship between posted wait times and actual wait times using the stabilized weights we just created.

03:00

Your Turn 3

```r
lm(
  wait_minutes_actual_avg ~ wait_minutes_posted_avg,
  weights = swts,
  data = wait_times_swts
) |>
  tidy() |>
  filter(term == "wait_minutes_posted_avg") |>
  mutate(estimate = estimate * 10)
```

```
# A tibble: 1 × 5
  term                      estimate std.error statistic p.value
  <chr>                        <dbl>     <dbl>     <dbl>   <dbl>
1 wait_minutes_posted_…         2.39    0.0659      3.63 4.93e-4
```

Diagnosing issues

1. Extreme weights even after stabilization
2. Bootstrap: non-normal distribution
3. Bootstrap: estimate different from original model

More info

https://github.com/LucyMcGowan/writing-positivity-continous-ps