

# Using Propensity Scores

Lucy D'Agostino McGowan

Wake Forest University

# Propensity scores

Matching

Weighting

Stratification

Direct Adjustment

...



#### Ingredients

150g unsalted butter, plus extra for greasing  
150g plain chocolate, broken into pieces  
150g plain flour  
½ tsp baking powder  
½ tsp bicarbonate of soda  
200g light muscovado sugar  
2 large eggs

#### Method

1. Heat the oven to 160C/140C fan/gas 3. Grease and base line a 1 litre heatproof glass pudding basin and a 450g loaf tin with baking parchment.
2. Put the butter and chocolate into a saucepan and melt over a low heat, stirring. When the chocolate has all melted remove from the heat.



estimand

estimator

estimate

# Propensity scores

Matching

Weighting

Stratification

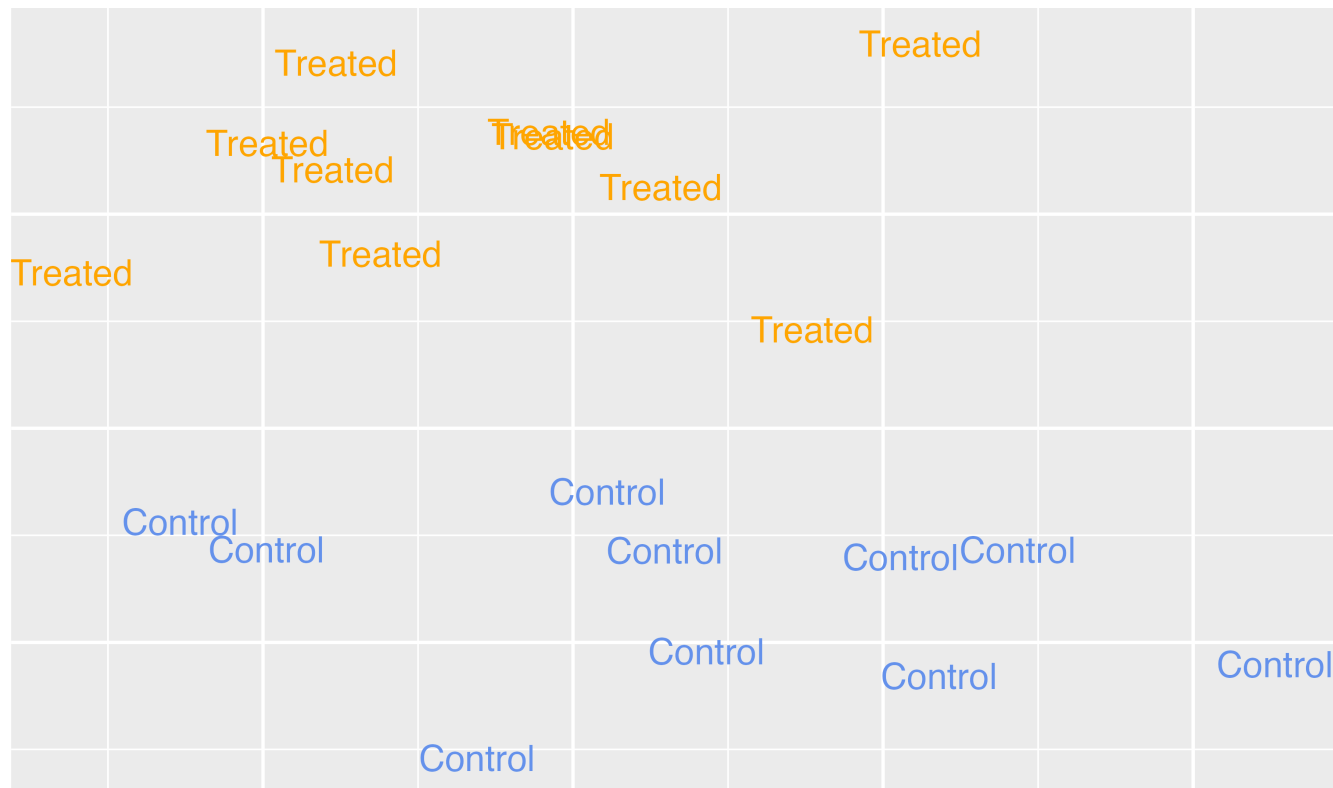
Direct Adjustment

...

# Target estimands

## *Average Treatment Effect (ATE)*

$$\tau = E[Y(1) - Y(0)]$$



# Target estimands

| Estimand | Target population | Example Research Question   |
|----------|-------------------|---|
| ATE      | Full population   | <i>Should we decide whether to have extra magic hours all mornings to change the wait time for Seven Dwarfs Mine Train between 9-10 AM?</i><br><br><i>Should a specific policy be applied to all eligible observations?</i> |

# Target estimands

## Average Treatment Effect among the Treated (ATT)

$$\tau = E[Y(1) - Y(0) | Z = 1]$$

# Target estimands

| Estimand | Target population              | Example Research Question  |
|----------|--------------------------------|--|
| ATT      | Exposed (treated) observations | <i>Should we stop extra magic hours to change the wait time for Seven Dwarfs Mine Train between 9-10 AMpm?</i><br><br><i>Should we stop our marketing campaign to those currently receiving it?</i><br><br><i>Should medical providers stop recommending treatment for those currently receiving it?</i> |



# Matching in R (ATT)

```
1 library(MatchIt)
2 m <- matchit(
3   qsmk ~ sex +
4     race + age + I(age^2) + education +
5     smokeintensity + I(smokeintensity^2) +
6     smokeyrs + I(smokeyrs^2) + exercise +
7     active + wt71 + I(wt71^2),
8   data = nhfs_complete
9 )
10 m
```

A 'matchit' object

- method: 1:1 nearest neighbor matching without replacement
- distance: Propensity score
  - estimated with logistic regression
- number of obs.: 1566 (original), 806 (matched)
- target estimand: ATT
- covariates: sex, race, age, I(age^2), education, smokeintensity, I(smokeintensity^2), smokeyrs, I(smokeyrs^2), exercise, active, wt71, I(wt71^2)

# Matching in R (ATT)

```
1 matched_data <- get_matches(m, id = "i")
2 as_tibble(matched_data)
```

```
# A tibble: 806 × 71
```

|    | i     | subclass | weights | seqn  | qsmk  | death | yrdth | modth |
|----|-------|----------|---------|-------|-------|-------|-------|-------|
|    | <chr> | <fct>    | <dbl>   | <dbl> | <dbl> | <dbl> | <dbl> | <dbl> |
| 1  | 11    | 1        | 1       | 428   | 1     | 0     | NA    | NA    |
| 2  | 1220  | 1        | 1       | 23045 | 0     | 0     | NA    | NA    |
| 3  | 15    | 2        | 1       | 446   | 1     | 1     | 88    | 1     |
| 4  | 1082  | 2        | 1       | 22294 | 0     | 0     | NA    | NA    |
| 5  | 18    | 3        | 1       | 596   | 1     | 0     | NA    | NA    |
| 6  | 534   | 3        | 1       | 14088 | 0     | 0     | NA    | NA    |
| 7  | 23    | 4        | 1       | 618   | 1     | 0     | NA    | NA    |
| 8  | 697   | 4        | 1       | 18085 | 0     | 0     | NA    | NA    |
| 9  | 27    | 5        | 1       | 806   | 1     | 0     | NA    | NA    |
| 10 | 879   | 5        | 1       | 21128 | 0     | 0     | NA    | NA    |

```
" - 706 - - - - -
```

# Target estimands

*Average Treatment Effect among the Controls  
(ATC)*

$$\tau = E[Y(1) - Y(0)|Z = 0]$$

# Target estimands

| Estimand | Target population                | Example Research Question  |
|----------|----------------------------------|--|
| ATU      | Unexposed (control) observations | <i>Should we add extra magic hours for all days to change the wait time for Seven Dwarfs Mine Train between 9-10 AMpm?</i><br><br><i>Should we extend our marketing campaign to those not receiving it?</i><br><br><i>Should medical providers extend treatment to those not currently receiving it?</i> |

# Matching in R (ATC)

```
1 m <- matchit(  
2   qsmk ~ sex +  
3     race + age + I(age^2) + education +  
4     smokeintensity + I(smokeintensity^2) +  
5     smokeyrs + I(smokeyrs^2) + exercise +  
6     active + wt71 + I(wt71^2),  
7   data = nhefs_complete,  
8   estimand = "ATC"  
9 )  
10 m
```

A `matchit` object

- method: 1:1 nearest neighbor matching without replacement
- distance: Propensity score
  - estimated with logistic regression
- number of obs.: 1566 (original), 806 (matched)
- target estimand: ATC
- covariates: sex, race, age, I(age^2), education, smokeintensity, I(smokeintensity^2), smokeyrs, I(smokeyrs^2), exercise, active, wt71, I(wt71^2)

# Target estimands

*Average Treatment Effect among the Matched (ATM)*

# Target estimands

| Estimand | Target population | Example Research Question   |
|----------|-------------------|---|
| ATM      | Evenly matchable  | <i>Are there some days we should change whether we are offering extra magic hours in order to change the wait time for Seven Dwarfs Mine Train between 9-10 AMpm?</i><br><br><i>Is there an effect of the exposure for some observations?</i><br><br><i>Should those at clinical equipoise receive treatment?</i> |

# Matching in R (ATM)

```
1 m <- matchit(  
2   qsmk ~ sex +  
3     race + age + I(age^2) + education +  
4     smokeintensity + I(smokeintensity^2) +  
5     smokeyrs + I(smokeyrs^2) + exercise +  
6     active + wt71 + I(wt71^2),  
7   data = nhefs_complete,  
8   link = "linear.logit",  
9   caliper = 0.1  
10 )  
11 m
```

Observations with propensity scores (on the linear logit scale) within 0.1 standard errors (the caliper) will be discarded



# Matching in R (ATM)

A `'matchit'` object

- method: 1:1 nearest neighbor matching without replacement
- distance: Propensity score [caliper]
  - estimated with logistic regression and linearized
- caliper: <distance> (0.063)
- number of obs.: 1566 (original), 780 (matched)
- target estimand: ATT
- covariates: sex, race, age, I(age^2), education, smokeintensity, I(smokeintensity^2), smokeyrs, I(smokeyrs^2), exercise, active, wt71, I(wt71^2)

# Matching in R (ATM)

```
1 matched_data <- get_matches(m, id = "i")
2 as_tibble(matched_data)
```

# A tibble: 780 × 71

|    | i     | subclass | weights | seqn  | qsmk  | death | yrdth | modth |
|----|-------|----------|---------|-------|-------|-------|-------|-------|
|    | <chr> | <fct>    | <dbl>   | <dbl> | <dbl> | <dbl> | <dbl> | <dbl> |
| 1  | 11    | 1        | 1       | 428   | 1     | 0     | NA    | NA    |
| 2  | 1220  | 1        | 1       | 23045 | 0     | 0     | NA    | NA    |
| 3  | 15    | 2        | 1       | 446   | 1     | 1     | 88    | 1     |
| 4  | 1082  | 2        | 1       | 22294 | 0     | 0     | NA    | NA    |
| 5  | 18    | 3        | 1       | 596   | 1     | 0     | NA    | NA    |
| 6  | 534   | 3        | 1       | 14088 | 0     | 0     | NA    | NA    |
| 7  | 23    | 4        | 1       | 618   | 1     | 0     | NA    | NA    |
| 8  | 697   | 4        | 1       | 18085 | 0     | 0     | NA    | NA    |
| 9  | 27    | 5        | 1       | 806   | 1     | 0     | NA    | NA    |
| 10 | 879   | 5        | 1       | 21128 | 0     | 0     | NA    | NA    |

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## ***Your Turn 1***

**Using the propensity scores you created in the previous exercise, create a “matched” data set using the ATM method with a caliper of 0.2.**

# Propensity scores

Matching

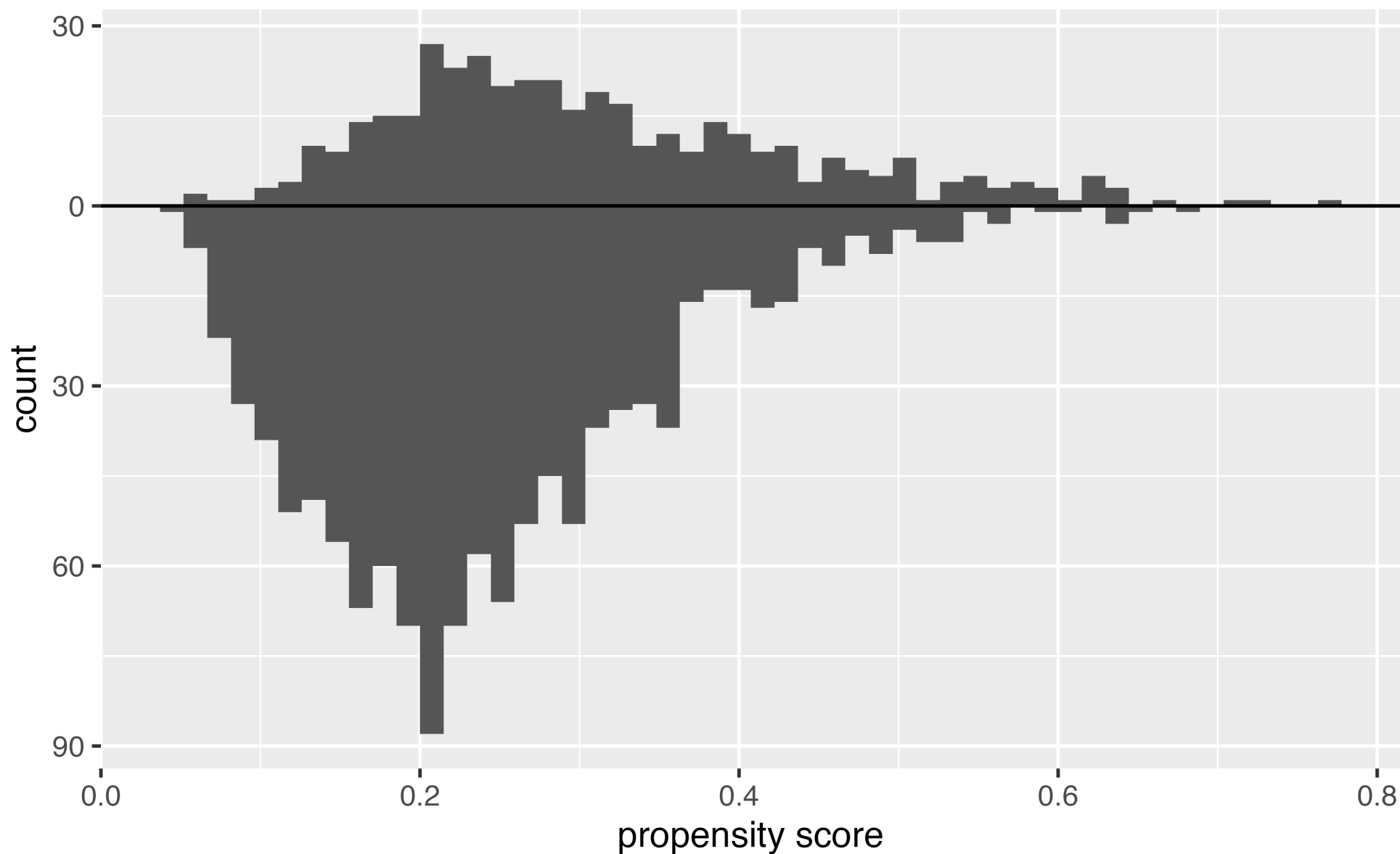
**Weighting**

Stratification

Direct Adjustment

...

# Histogram of propensity scores



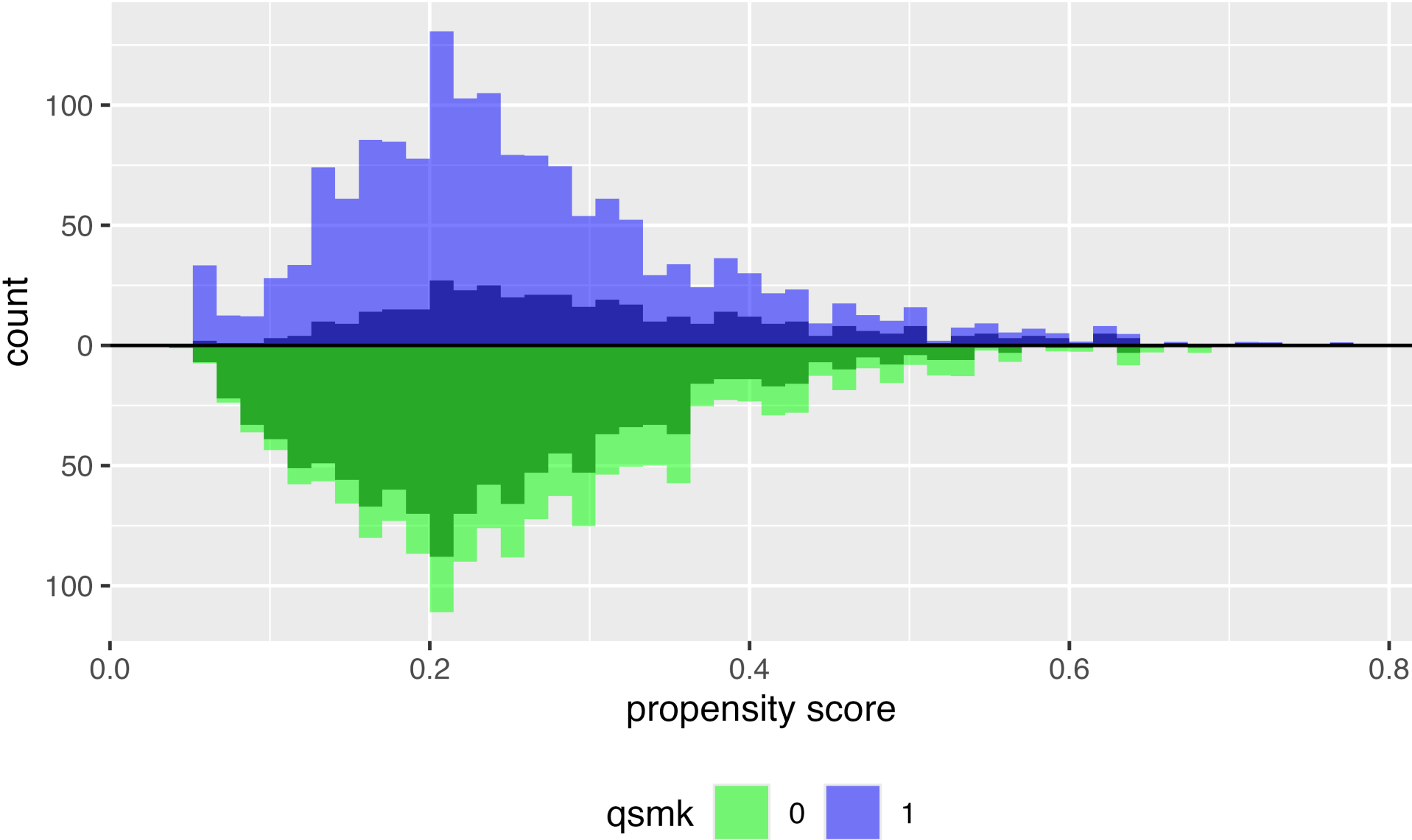
# Target estimands: ATE

Average Treatment Effect (ATE)

$$w_{ATE} = \frac{Z_i}{p_i} + \frac{1 - Z_i}{1 - p_i}$$

$$1 \cdot (Z / p) + ((1 - Z) / (1 - p))$$

# ATE



# Target estimands: ATT & ATC

Average Treatment Effect Among the Treated (ATT)

$$w_{ATT} = \frac{p_i Z_i}{p_i} + \frac{p_i(1 - Z_i)}{1 - p_i}$$

$$1 \cdot ((p * Z) / p) + ((p * (1 - Z)) / (1 - p))$$



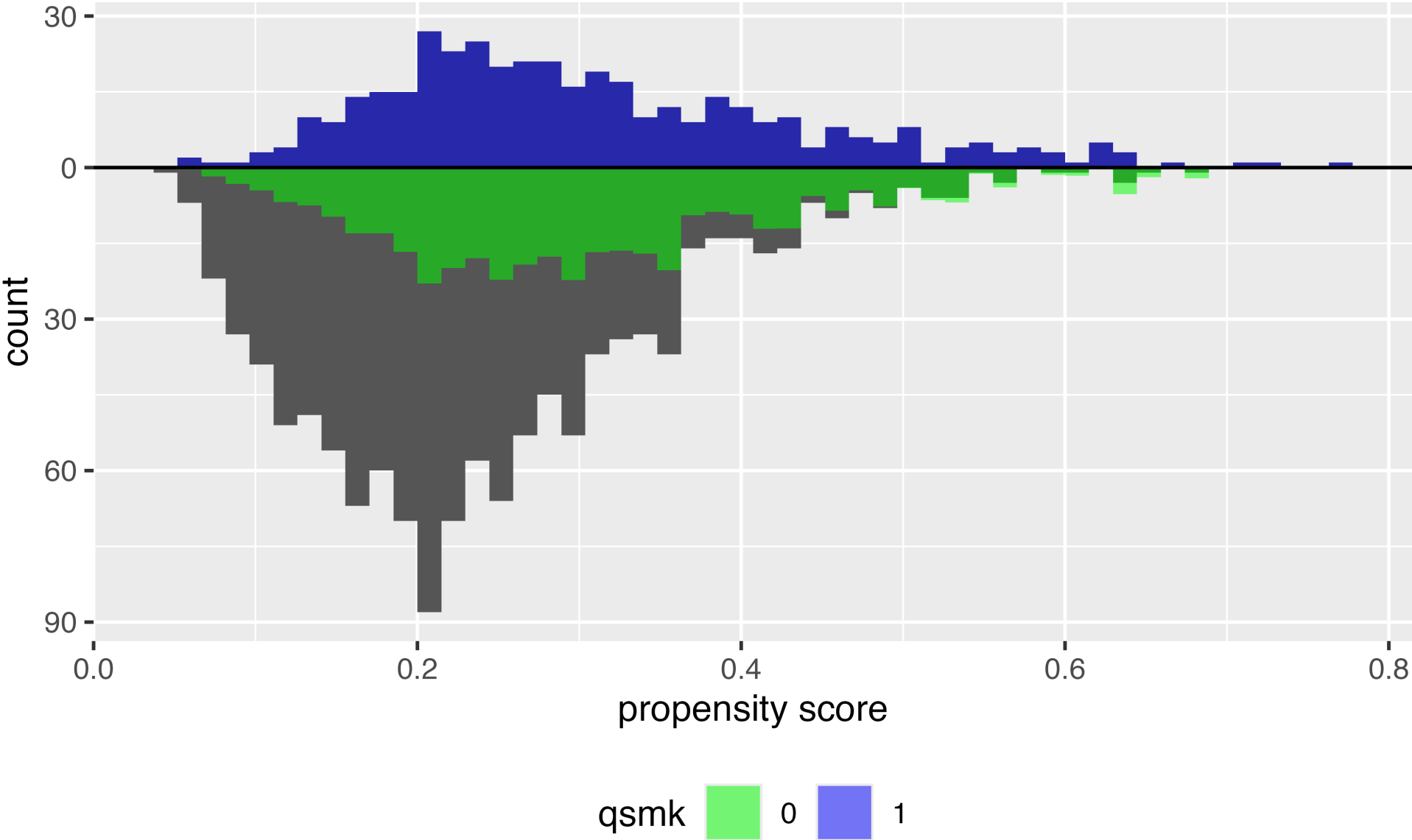
# Target estimands: ATT & ATC

Average Treatment Effect Among the Controls (ATC)

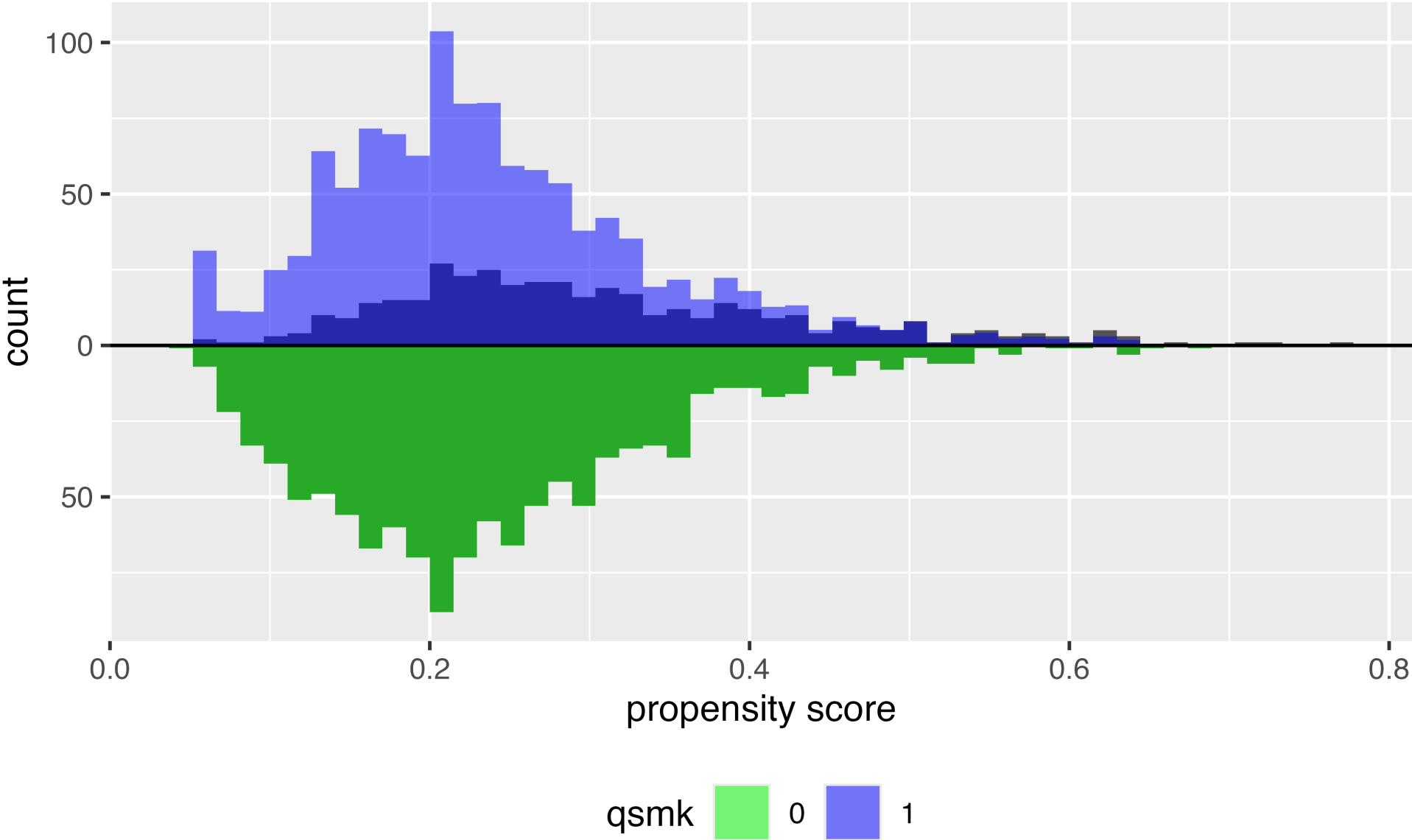
$$w_{ATC} = \frac{(1 - p_i)Z_i}{D_i} +$$

$$1 \left( \left( (1 - p) * Z \right) / p \right) + \left( \left( (1 - p) * (1 - Z) \right) / (1 - p) \right)$$

# ATT



# ATC



# Target estimands: ATM & ATO

Average Treatment Effect Among the Evenly Matchable (ATM)

$$w_{ATM} = \frac{\min\{p_i, 1 - p_i\}}{Z_i p_i + (1 - Z_i)(1 - p_i)}$$

```
1 pmin(p, 1 - p) / (Z * p + (1 - Z) * (1 - p))
```

# Target estimands: ATM & ATO

Average Treatment Effect Among the Overlap Population

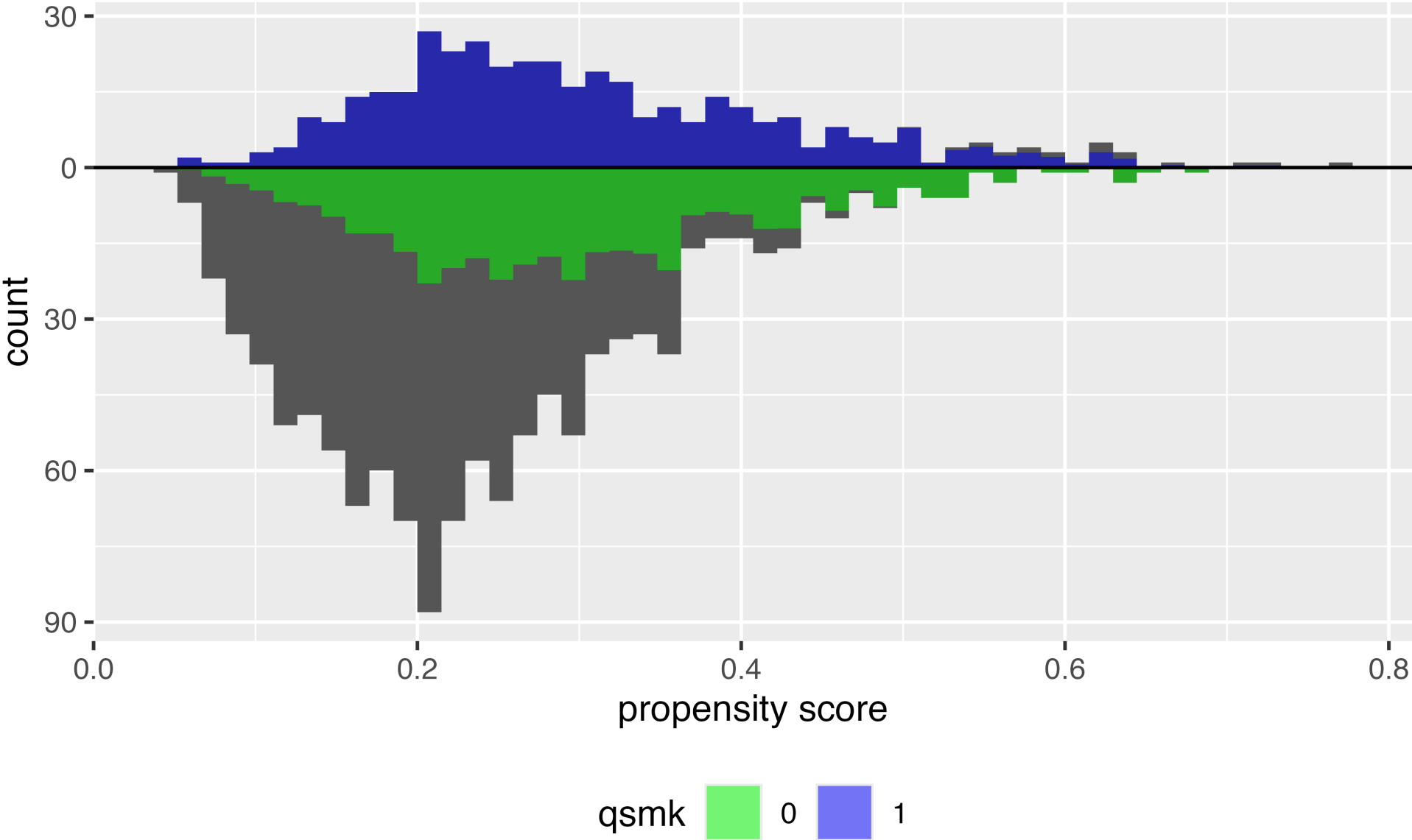
$$w_{\Delta T \cap} = (1 - p_i)Z_i + p_i(1 - Z_i)$$

$$1 - (1 - p) * Z + p * (1 - Z)$$

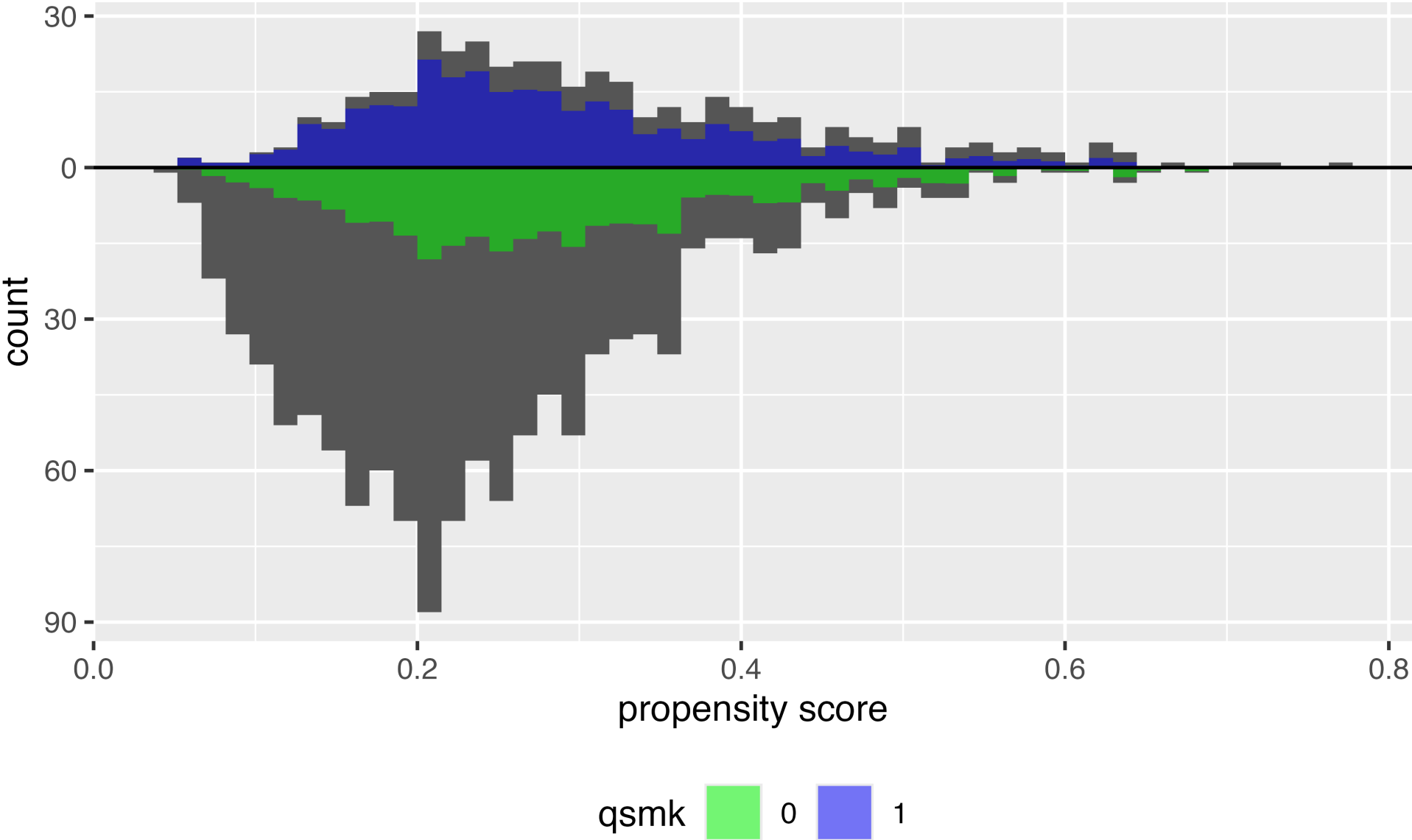
# Target estimands

| Estimand | Target population  | Example Research Question |
|----------|--------------------|---------------------------|
| ATO      | Overlap population | <i>Same as ATM</i>        |

# ATM



# ATO





# ATE in R



Average Treatment Effect (ATE)  $w_{ATE} = \frac{Z_i}{p_i} + \frac{1-Z_i}{1-p_i}$

```
1 library(propensity)
2 df <- propensity_model |>
3   augment(type.predict = "response", data = nhefs_complete) |>
4   mutate(w_ate = wt_ate(.fitted, qsmk))
```

## ***Your Turn 2***

**Using the propensity scores you created in the previous exercise, add the ATE weights to your data frame**

***Stretch:* Using the same propensity scores, create ATM weights**