The testability of maximum magnitude estimates

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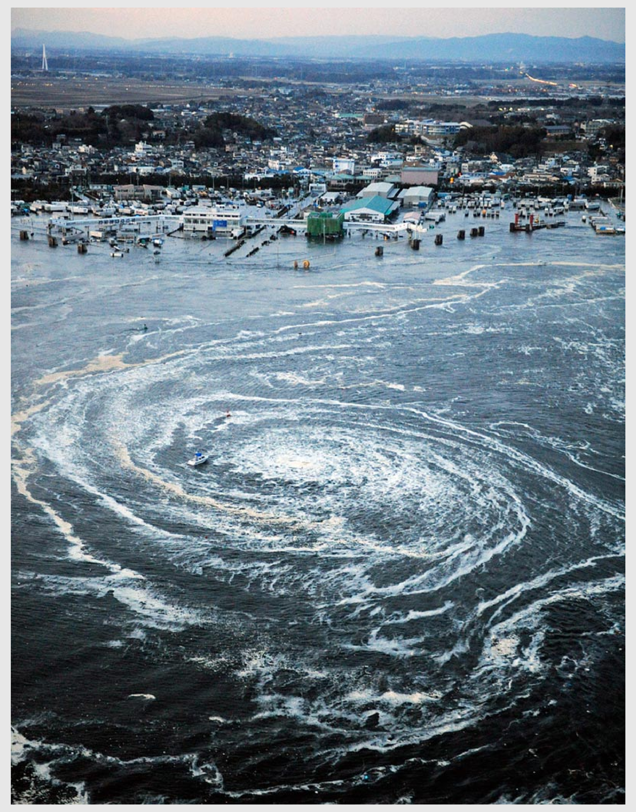
1) We would like to test estimates of M_{max}.

Earthquakes of unexpectedly large magnitude can cause major disasters.

Estimates of the maximum earthquake magnitude (M_{max}) in a region, or for a fault, are crucial for seismic hazard and risk assessment.

Can we statistically and rigorously test them?

They are commonly provided as single values, without a probability distribution. Also, they refer to the maximum possible magnitude ever expected, during an infinite period of time (for an overview, see Wheeler, R.L., 2009, USGS Open-File Report 2009-1018). However, a feasible test of M_{max} can only be performed during a finite period.



Tsunami whirlpool near Oarai City, Japan, after the M=9 Tohoku earthquake.

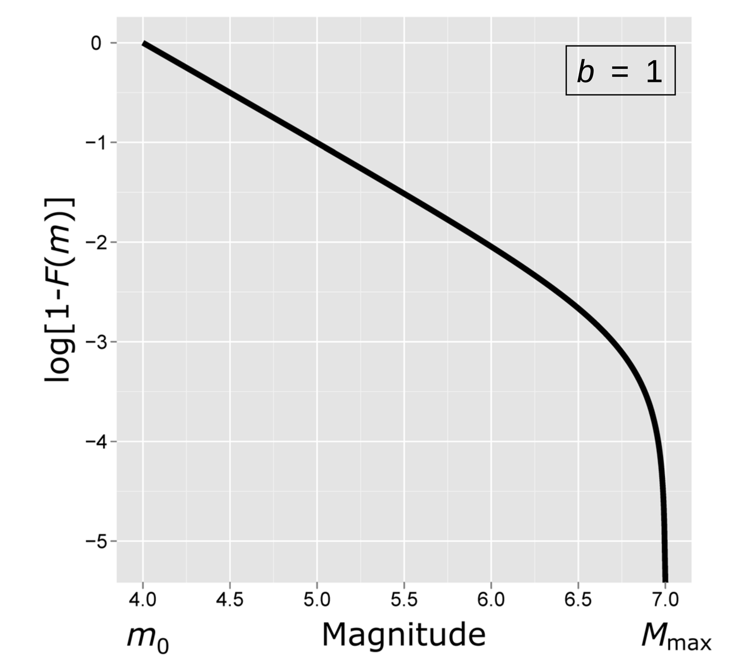
2) We will assume that earthquake magnitudes follow a doubly truncated Gutenberg-Richter distribution.

$$F(m) = \frac{e^{-\beta m_0} - e^{-\beta m}}{e^{-\beta m_0} - e^{-\beta \hat{M}_{max}}}$$

Where:

- m is magnitude,
- β is $b\log(10)$, where b is the b-value of the Gutenberg-Richter distribution,
- m_0 is the magnitude of completeness, and
- $\hat{M}_{max}$ is the upper truncation, that is, a given estimate of M_{max}.

Log survival function of the truncated Gutenberg-Richter distribution



3) This implies that we can expect to observe certain M_{max} values in a finite time period...

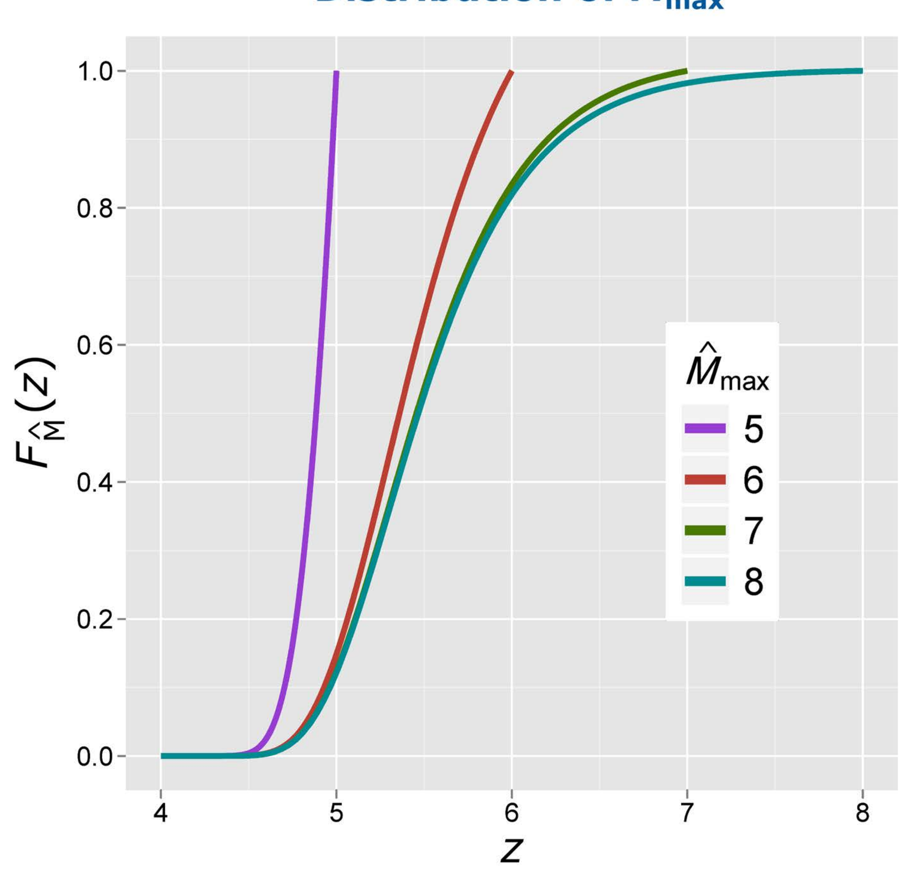
Let us say that during a finite time period, T, a total of n earthquakes are observed above m_0.

The magnitude of the largest one is a random variable, denoted M^T_{max}. The longer the T, the larger n can be, and the more likely it is to observe a larger M^T_{max}.

The corresponding probability distribution of M^T_{max} is given, generically, by

$F_{\hat{M}}(z) = P(M^T_{max} < z) = [F(m)]^n$.

Distribution of M^T_{max}

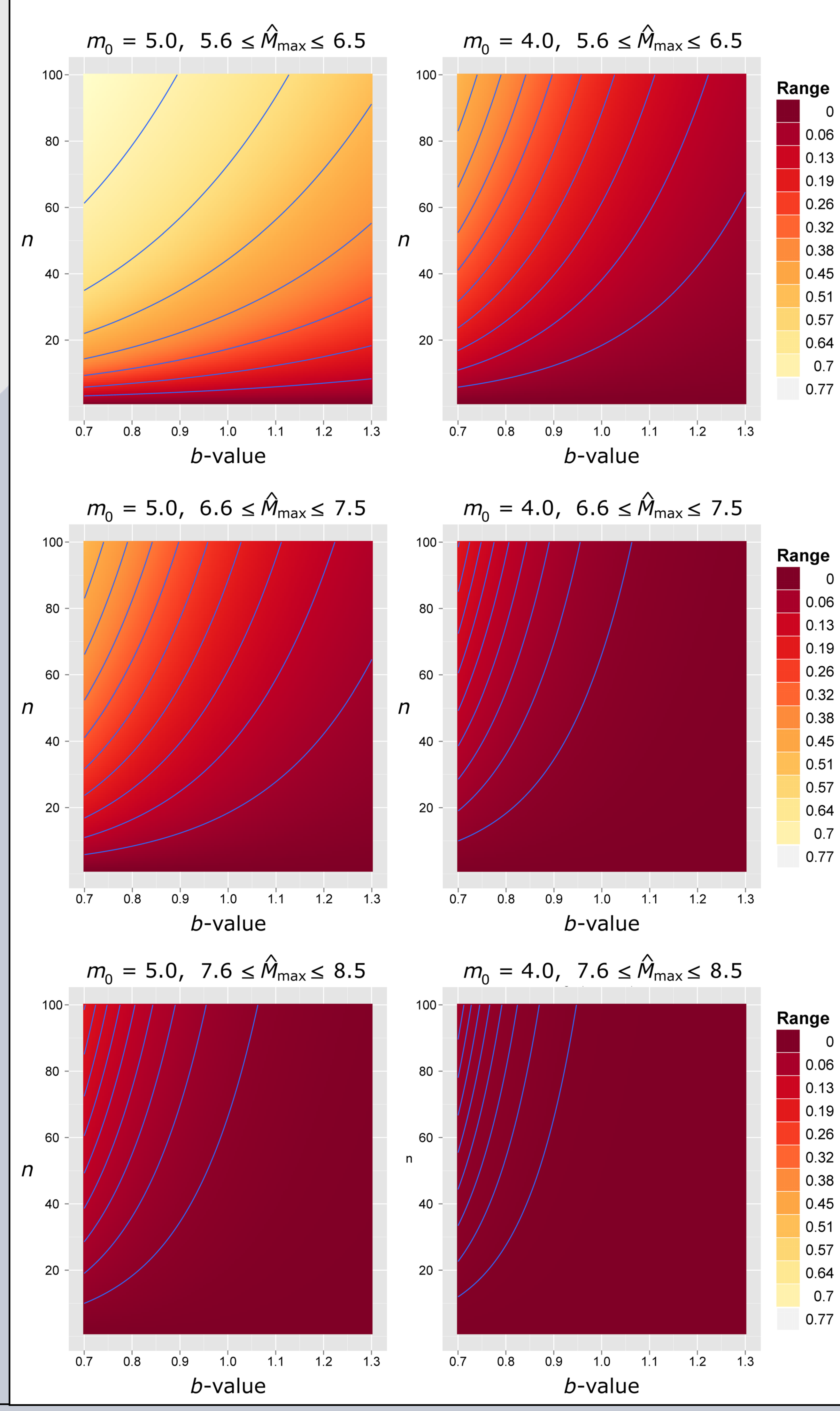


4) ...which are not very sensitive, in the short term, to the long-term estimate of M_{max}.

Let m^* be the critical value, i.e., the value at which $P(M^T_{max} < m^*) \le 0.05$. We measure the change in m^* with respect to $\hat{M}_{max}$, for all combinations of $0.7 \le b \le 1.3$, $5.6 \le \hat{M}_{max} \le 8.5$, $1 \le n \le 100$, and $m_0 \in \{4,5\}$.

$F_{\hat{M}}(z)$ is most sensitive in regions of high earthquake frequency and low b-values, because in these regions there is a greater chance of observing larger earthquakes (closer to the long-term M_{max}) in a given period.

Ranges of critical values m^* for different values of m_0 and M_{max}



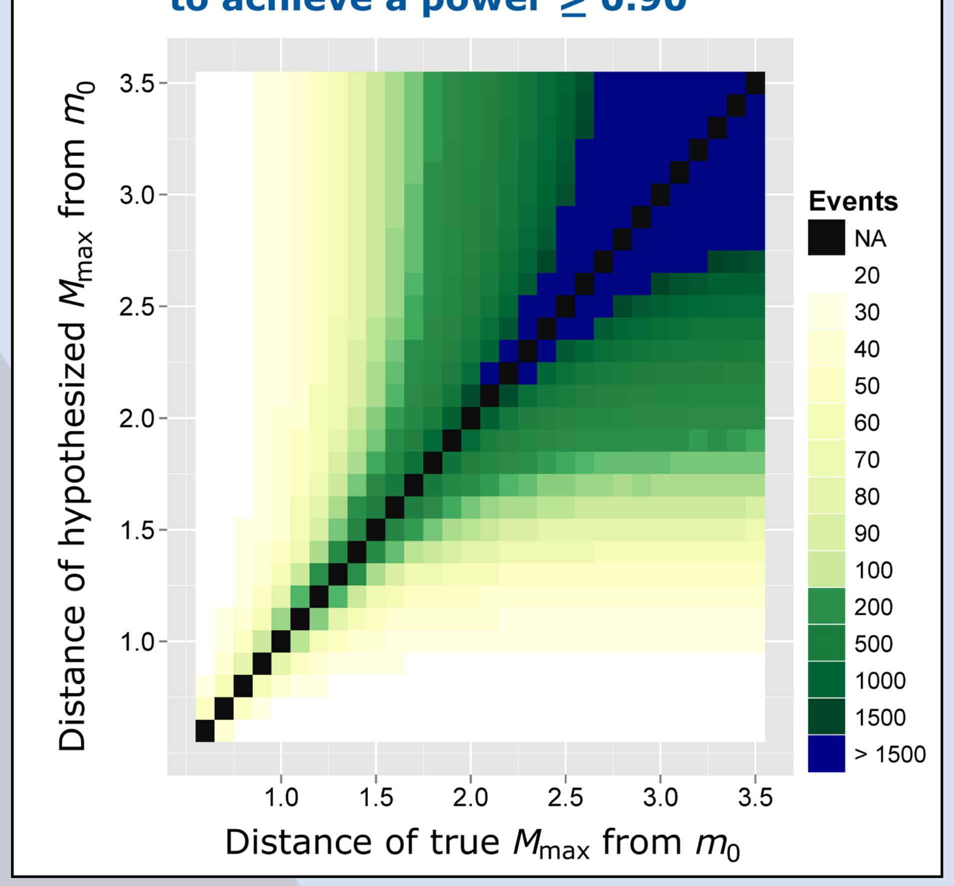
5) The minimum number of earthquakes required for a powerful test depends on the difference between M_{max} and the completeness magnitude.

If the observed maximum magnitude is too unlikely, given the assumed earthquake magnitude distribution, we can reject it.

We want to know how many earthquakes must occur for our test to correctly reject a false $\hat{M}_{max}$ with high probability (at least 0.90).

Using a simulation study, we notice that as the true M_{max} gets further away from m_0, we will likely never have enough earthquakes in a reasonable time period to test $\hat{M}_{max}$, unless $\hat{M}_{max}$ is grossly underestimated.

Number of earthquakes needed to achieve a power $\geq$ 0.90



6) Conclusions:

Testing M_{max} in a reasonable time period may not be possible in most regions.

Using our approach, estimates of M_{max} that are provided without a probability distribution, and for long time periods, are testable only in certain ideal situations (high earthquake frequency, low b-value, and a relatively low M_{max}).

In the future, it would be beneficial to further examine the impact of M_{max} on the expected ground motions, to see to what degree hazard assessment is affected by this apparently non-testable parameter.

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