The testability of maximum magnitude

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1) We would like to statistically test the maximum magnitude (*M*).

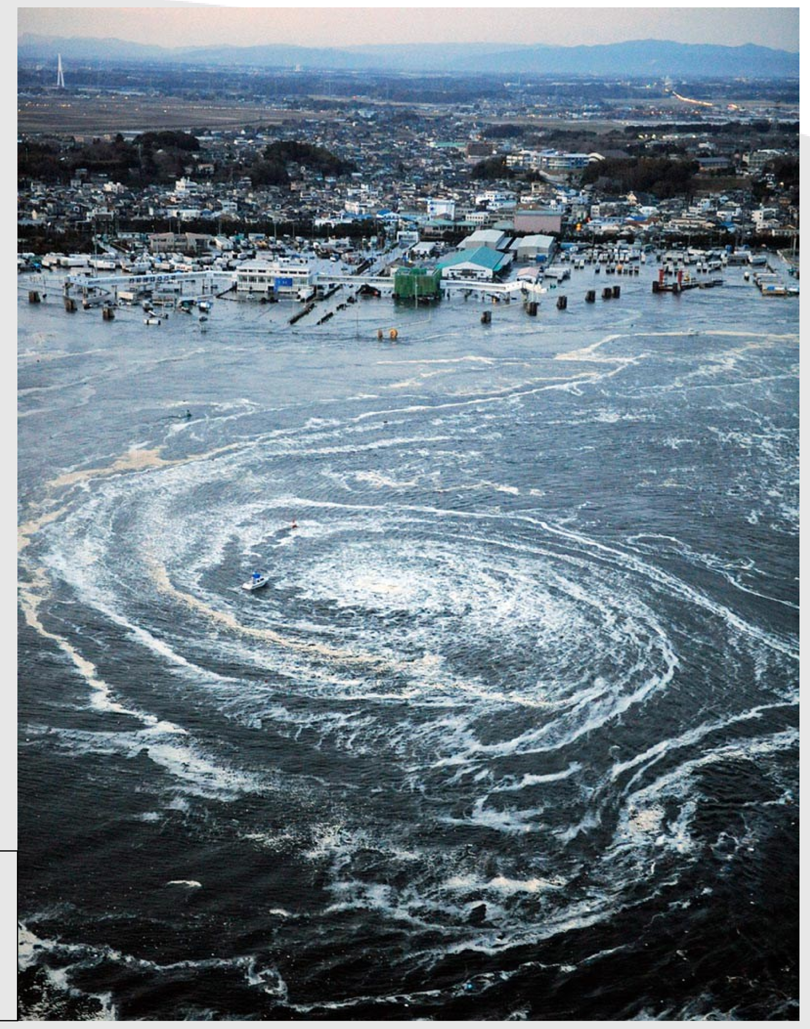
Earthquakes of unexpectedly large magnitude can cause major disasters.

Estimates of the maximum earthquake magnitude (*M*) in a region, or for a fault, are crucial for seismic hazard and risk assessment.

Can we statistically and rigorously test *M*?

The estimates are commonly provided as single values, referring to the maximum possible magnitude ever expected, during an infinite period of time (for an overview, see Wheeler, R.L., 2009, USGS Open-File Report 2009-1018).

However, a feasible test of *M* can only be performed with a finite sample of earthquakes.



Tsunami whirpool near Oarai City, Japan, after the M=9 Tohoku earthquake.

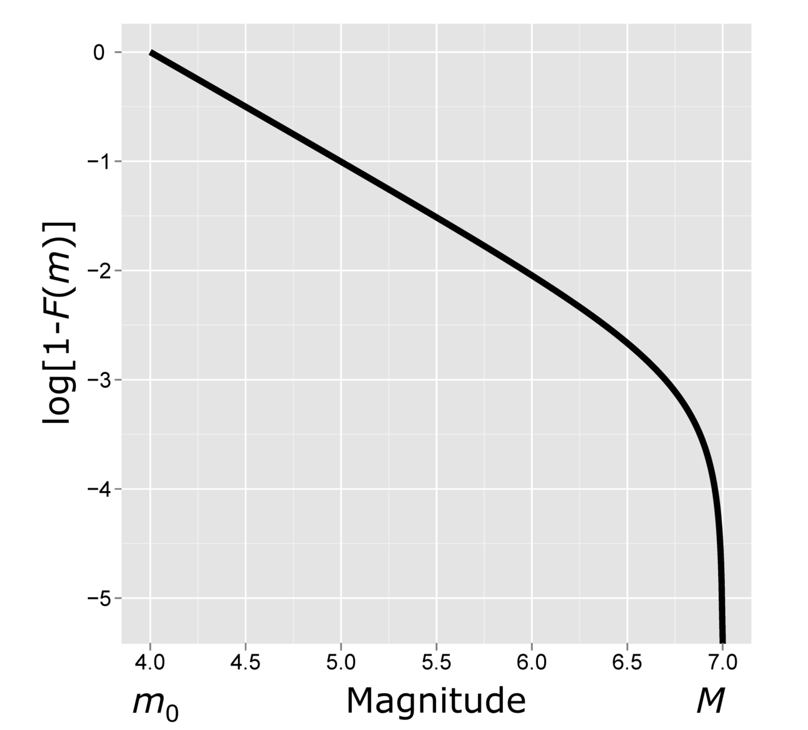
2) We will assume that earthquake magnitudes follow a doubly truncated Gutenberg-Richter distribution.

$$F(m) = \frac{e^{-\beta m_0} - e^{-\beta m}}{e^{-\beta m_0} - e^{-\beta \hat{M}}}$$

Where:

- m is magnitude,
- β is $b\log(10)$, where b is the b-value of the Gutenberg-Richter distribution,
- m_0 is the magnitude of completeness, and
- $\hat{M}$ is the upper truncation, that is, a given estimate of M.

Log survival function of the truncated Gutenberg-Richter distribution



3) This implies that we can expect to observe a certain maximum magnitude in a finite sample of earthquakes...

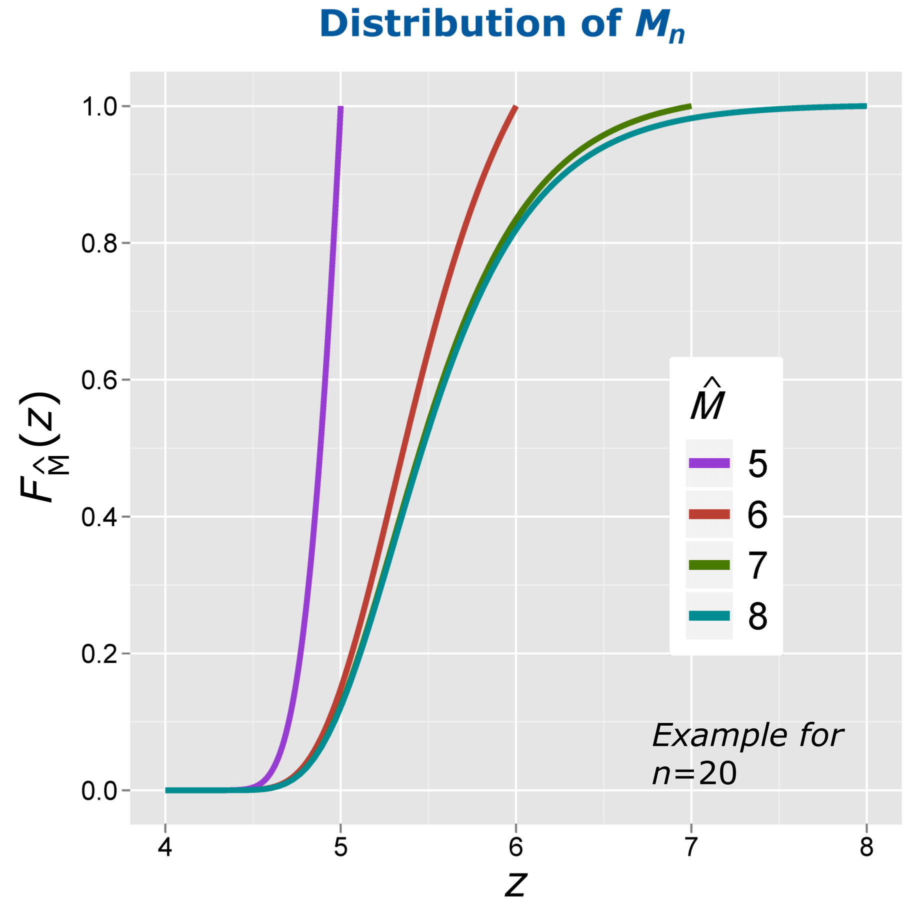
Let us say that a total of n earthquakes above m_0 are sampled.

The magnitude of the largest one is a random variable, denoted M_n. The larger the n, the more likely it is to observe a larger M_n.

The corresponding probability distribution of M_n is given, generically, by

$F_{\hat{M}}(z) = P(M_n < z) = [F(m)]^n$.

Distribution of M_n



4) ...which is not very sensitive to the long-term estimate of *M*.

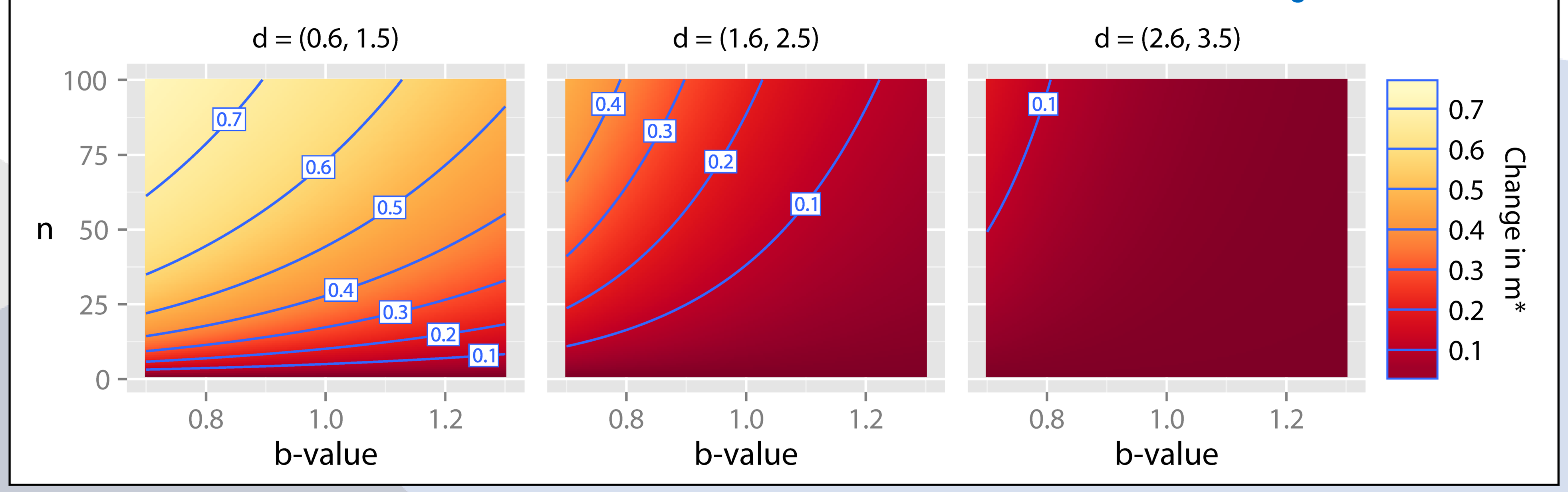
To measure the sensitivity of $F_{\hat{M}}(z)$ to the different parameters, we use the critical value m^*, for which $P(M_n < m^*) \leq 0.05$.

We measure the change in m^* with respect to $\hat{M}$, for combinations of:

- $0.7 \leq b \leq 1.3$
- $0.6 \leq d \leq 3.5$, where $d=\hat{M}-m_0$, and
- $1 \leq n \leq 100$.

$F_{\hat{M}}(z)$ is most sensitive in regions of high earthquake frequency above the completeness threshold, low b-values and relatively small d.

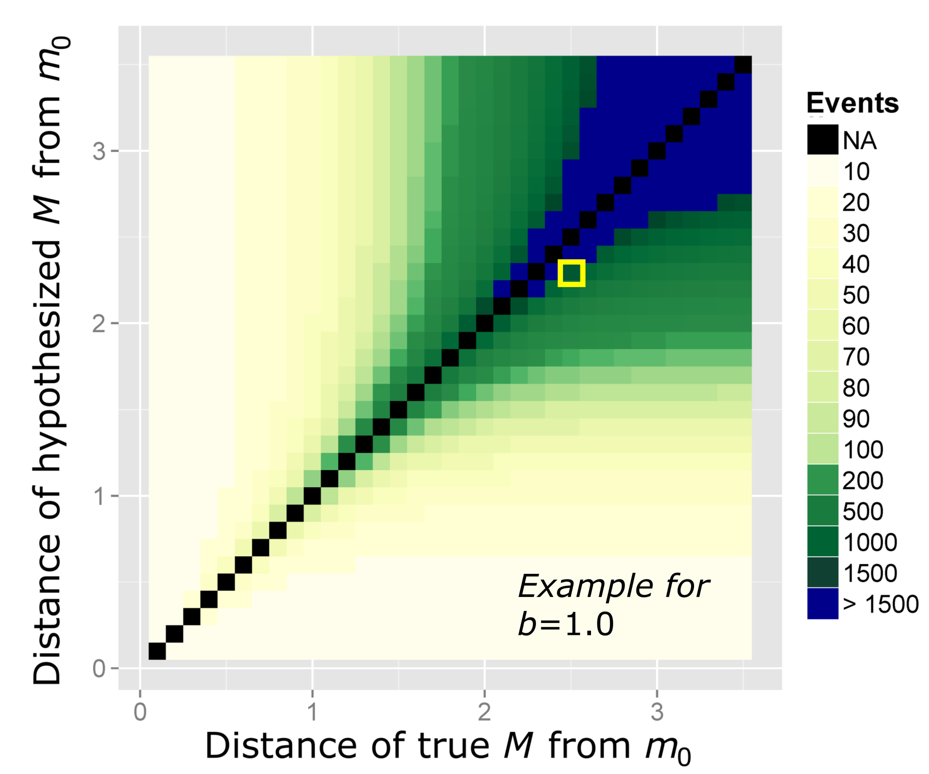
Ranges of critical values m^* for different distances $d=\hat{M}-m_0$



The light colors show the parameters for which a statistical test of *M* can be more powerful: where m^* changes the most, indicating that $F_{\hat{M}}(z)$ is most sensitive.

5) The minimum number of earthquakes required for a powerful test depends on the difference between *M* and the completeness magnitude.

Number of earthquakes needed to achieve a power ≥ 0.90



We reject the estimate of M in two cases, namely if the observed maximum magnitude:

- is larger than $\hat{M}$, or
- is unlikely low ($< m^*$),

given the assumed earthquake magnitude distribution.

We want to know how many earthquakes must occur for our test to correctly reject a false $\hat{M}$ with high probability (at least 0.90).

We notice that as the true M gets further away from m_0, we will likely never have enough earthquakes in a reasonable sample size to test M, unless M is grossly underestimated.

Example (pixel marked in yellow in the plot):

- The largest recorded earthquake (Chile, 1960) had m=9.6.
- Assuming b=1, if we hypothesize a worldwide $\hat{M}$=9.8, but actually M=10, then 1220 earthquakes with $m \geq 7.5$ would be needed to correctly reject 9.8 as a too low estimate with a power of 90%.
- But there is a complete record of only ~ 450 earthquakes with m_0=7.5 since the year 1900!

6) Conclusions: Testing the maximum magnitude *M* may not be possible in most regions.

Using our approach, estimates of the long-term *M* are testable only in certain ideal situations (high earthquake frequency above the magnitude of completeness in the catalog, low *b*-value, and a relatively low *M*).

In any hazard assessment, it would be necessary to carefully examine the impact of the estimates of *M* on the expected ground motions.

Extreme care should be exercised if the hazard substantially depends on the estimates of *M*, since these seem practically untestable in a statistically sound way.

Hazard assessment should clearly state this source of uncertainty.

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