Market Risk Beta Estimation using Adaptive Kalman Filter

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Abstract

Market risk of an asset or portfolio is recognized through beta in Capital Asset Pricing Model (CAPM). Traditional estimation techniques emerge poor results when beta in CAPM assumed to be dynamic and follows auto regressive model. Kalman Filter (KF) can optimally estimate dynamic beta where measurement noise covariance and state noise covariance are assumed to be known in a state-space framework. This paper applied Adaptive Kalman Filter (AKF) for beta estimation when the above covariances are not known and estimated dynamically. The technique is first characterized through simulation study and then applied to empirical data from Indian security market. A modification of the used AKF is also proposed to take care of the problems of AKF implementation on beta estimation and simulations show that modified method improves the performance of the filter measured by RMSE.

Keywords: market risk, beta, estimation, Adaptive Kalman Filter.

1. Introduction

According to Capital Asset Pricing Model (CAPM) [1] if the market portfolio is efficient, then the i-th asset return is described by $r_i = \alpha_i + \beta_i r_m + \varepsilon_i$ where r_i and r_m is the returns of the i th asset and market index respectively, α_i is the risk free rate of return (risk free interest), ε_i is the random error term (with variance σ_ε^2), β_i is the relationship of the asset return with market index return, or in other words, it is the sensitivity of the i-th asset return with respect to the market. β_i can be expressed as a ratio of covariance (σ_{im}) between specific asset returns and market index returns and variance (σ_m^2) of market index returns. i.e. $\beta_i = \dfrac{\sigma_{im}}{\sigma_m^2}$ [2, 3]. If asset returns is completely uncorrelated to the market returns i.e. $\beta_i = 0$, then according to CAPM asset return will be equal to the risk-free rate of return. The CAPM changes concept of the risk from the volatility to beta. The portfolio beta (i.e β_p) is calculated as weighed average of betas of the individual assets in the portfolio where weights being identical to those that define the portfolio [2].

Good number of techniques are already in place for estimating beta assuming it to be a constant and have been discussed in the section of beta estimation techniques. That section also illustrates the dynamic beta estimation literature. The next challenge is to identify the appropriate autoregressive model of time varying beta and has been explained in the section models for beta dynamics. Next task is to estimate beta which has been addressed by [4, 5] using KF [6, 7, 4]. KF application for beta estimation assumes the state noise covariance and measurement noise covariance to be known. However in reality these are not ensued always. To deal with such situations when those parameters are not known and estimated during filtering is addressed by [8] together with its references and citations. The techniques are popularly known as Adaptive Kalman Filter (AKF). [9] has recently proposed an AKF technique which has been used and modified in this work for dynamic beta estimation. Application of AKF for beta estimation emerge measurement covariance to be negative for small true values of the measurement covariance together with some relation with state noise covariance. This work investigates the possibility of modifying the AKF by reinitializing the measurement noise covariance when it becomes negative. Simulations are carried out to confirm the efficiency of the modified technique.

The rest of the paper is organized as follows. The next section briefly described the possible models for describing beta dynamics in the literature. The third section presents the beta estimation techniques in the literature as well as those which have been characterized. The fourth section illustrates the simulation and empirical investigation techniques. The results of the investigations are articulated in the fifth section. The paper is ended with a conclusion.

2. Models for Beta Dynamics

[10, 11 12, 4, 13, 14] and many others references there in studied the nature of beta with respect to time evolution. Empirical investigations in the above literatures proved that beta is not a constant as assumed in the CAPM. They tested the fact in the developed markets like UK, USA Japan etc. as well as in the developing markets like India, China etc. Some of these papers argued GARCH models [13, 15, 14] and some advocated stochastic [10, 13] nature (models) for characterizing time-varying beta.

[16 & 17] proved that structural models are superior and most accurate among the complex forecasting methods especially for long forecasting horizons (like annually, quarterly and monthly) and seasonal data. [18, 19] argued that autoregressive process is indeed an appropriate and parsimonious model of beta variations with evidence from Australian and Indian equity return data respectively. It has been observed [12] that there are four admired stochastic model for explaining the dynamics of beta: random coefficient model (attributed to [20] and explained in [36 & 37]) expressed as $\beta = \overline{\beta} + \eta_t$, random walk model (attributed to [21, 22]) expressed as $\beta_t = \beta_{t-1} + \eta_t$, ARMA (1,1) (auto regressive moving average) beta model [12] expressed as $\beta_t = \phi\beta_{t-1} + \eta_t - \theta\eta_{t-1}$, and mean reverting model (attributed to [23, 11, 13, 15]) expressed as $\beta_t = \overline{\beta} + \phi(\beta_{t-1} - \overline{\beta}) + \eta_t$, where ϕ and θ is the speed parameter, β tends to go back (revert) towards $\overline{\beta}$ and η_t is the noise term (with variance σ_η^2). [24 & 25] extended the mean reverting model with name moving mean model by adding a new constraint given by $\overline{\beta}_t = \overline{\beta}_{t-1} + \gamma_t$ where γ_t is the noise component with variance σ_γ^2. [12, 4 and 26 have shown that mean reverting model outperform over others comparatively simple models with suitable assumptions where as [15] proved that mean reverting model is superior than GARCH beta models through simulation study.

3. Beta Estimation Techniques

Literature shows that there have been quite a number of techniques for beta estimation: OLS [5, 4, 15], GLS [27], KF [6, 12, 4, 26, 14], Adaptive KF [28]. [27] compared GLS and KF for purpose of estimation of non-stationary beta parameter in time varying extended CAPM. [19, 29, 30] used modified KF for estimating daily betas with high frequency Indian data exhibiting significant non-Gaussianity in the distribution of beta. [31 & 32] approached to estimate time varying beta using KF and Quadratic Filter. [33] integrated realized beta as function of realized volatility [34, 35] in a unified framework. The present work first applied an AKF for the dynamic beta estimation using state space model where measurement noise covariance and state noise covariance are not known and estimated during filtering. A small modification of the AKF is proposed to take care of the situations arises by characterizing AKF for beta estimation.

The Kalman filter is based on the representation of the dynamic system with a state space regression modeling the beta dynamics through an autoregressive process. The state-space representation of the dynamics of the Sharpe Diagonal Model is given by the following system of equations

$$R_{i,t} = \alpha_i + \beta_i R_{m,t} + \varepsilon_{i,t} \tag{1a}$$

$$\beta_{i,t} = T_t \beta_{i,t-1} + \xi_{i,t} \tag{1b}$$

The above system can be represented in a more general way as

$$y_t = H_t x_t + d_t + w_t \tag{2a}$$

$$x_t = T_t x_{t-1} + c_t + v_t \tag{2b}$$

H_t is known constant or time varying coefficients, the return of index stock for our application, T_t is the state transition matrix and d_t is known. y_t is the return of the stock and x_t is the vector of state variables. Finally w_t is a vector of serially uncorrelated disturbances with mean zero and covariance R_t and v_t is identified with a vector of serially uncorrelated disturbances with mean zero and covariance Q_t. In our application the system matrices R, T, d and c are all independent of time and so can be written without a subscript.

If it is assumed that the parameters of the above system of equations are known then KF algorithm [6, 7, 4] can estimate the dynamic beta (state). [9] proposed a improved AKF technique over [8] for estimation of states when Q and R are unknown and assumed to be dynamic over time known as Adaptive Kalman Filter for GPS/INS navigation problems. In this method innovation sequence v_t (Algo 3.1 Step 4) is used for calculation of Q and R in the following way:

$$R_{t+1} = \frac{1}{m}\sum_{i=0}^{m-1} v_{t-i}v_{t-i}^{'} - H_t P_{t|t-1} H_t^{'} \tag{3}$$

where m is the estimation window size and $Q_t = Q_{t-1}\sqrt{\lambda}$ where $\lambda = \dfrac{\frac{1}{m}\sum_{i=0}^{m-1} v_{t-i}v_{t-i}^{'} - R_t}{H_t P_{t|t-1} H_t^{'}}$ (4)

3.1 AKF Algorithm:

Step 1: Set the values of T & H and initial value of x, P, Q & R.
Step 2: Calculate State Prediction: $x_{t|t-1} = Tx_t + c$

Step 3: Calculate State Covariance Prediction: $P_{t|t-1} = TP_tT^{'} + Q_t$

Step 4: Calculate Innovation: $v_t = y_t - H_t x_{t|t-1} - d$

Step 5: Calculate Measurement Noise Covariance Update: $F_t = H_t P_{t|t-1} H_t^{'} + R_t$

Step 6: Calculate State update: $x_t = x_{t|t-1} + \dfrac{P_{t|t-1}H_t^{'}v_t}{F_t}$

Step 7: Calculate State Covariance update: $P_t = P_{t|t-1} - \dfrac{P_{t|t-1}H_t H_t^{'} P_{t|t-1}}{F_t}$

Step 8: Update Measurement Noise Covariance: $R_{t+1} = \frac{1}{m}\sum_{i=0}^{m-1} v_{t-i}v_{t-i}^{'} - H_t P_{t|t-1} H_t^{'}$, where m is the estimation window size

Step 9: Update State Noise Covariance: $Q_{t+1} = Q_t\sqrt{\lambda}$ where $\lambda = \dfrac{\frac{1}{m}\sum_{i=0}^{m-1} v_{t-i}v_{t-i}^{'} - R_t}{H_t P_{t|t-1} H_t^{'}}$

3.2 Modified AKF Algorithm:

Step 1: Replicate Step 1 to 8 of the above algorithm.
Step 2: Check whether measurement noise covariance R is negative.
Step 3: If R found negative set the value of R to its initial value.

4. Investigations

Simulation Investigation: Simulation experiments are conducted to realize the optimal window size m of adaptive estimation of Q and R. The simulation also characterizes the system with synthetic data. The following assumptions are imposed while experimenting: Truth is generated with the following: T=1, β_0=0.5, α_i=0.02, $r_{mt} = [1,1,...,1]_{5000x1}$; Three types of situations are considered: Q is constant, Q is exponentially increasing and Q is exponentially decreasing while generating the truth. Several initial guess values of the x, Q and R of the filter are considered and estimation performances are reported in the result section.

Empirical Investigations: Daily closing values of three popular NSE (of India) indices are used to characterize the system's empirical behavior. The time period considered is 1st January, 2001 to 31st December, 2007 (total of 1757 days data). NSE Nifty values are used as market portfolio values which is used for calculating the market return (r_{mt}). Dynamic beta values of IT and Bank indices of NSE are separately estimated. These two indices values are taken in to account to calculate portfolio return (r_{it}) because these two indices are suitably designed portfolio of IT and Bank equities respectively. According to the algorithmic demands no zero returns are allowed. A small number 0.00001 replaces the zero values when zero return appeared.

5. Results

5.1 Results of the Simulation Experiments:

To find the optimal window size m:
The following table 1 gives the RMSE of beta estimation for different values of m with initial guess of R=0.9, Q=0.1.

Table 1: RMSE of beta estimation

m	50	100	200	300	400	500
RMSE	0.7151	0.6871	0.6959	0.7180	0.7044	0.7020

The following table 2 gives the RMSE of Q estimation for different values of m with initial guess of R=0.9, Q=0.1.

Table 2: RMSE of Q estimation

m	50	100	200	300	400	500
RMSE	0.3531	0.2202	0.1559	0.1548	0.1657	0.1673

To find the optimal and feasible value of m with initial guess R=0.9 the following table 3 gives the RMSE of beta estimation for different values of m

Table 3: RMSE of beta estimation for different constant True Q

m	50	100	200	300	400	500	1000
$Q_0=0.05$	NA	NA	NA	0.5038	0.4983	0.5025	0.5254
$Q_0=0.5$	NA	NA	0.4887	0.4892	0.4643	0.4787	0.4929
$Q_0=5$	NA	NA	NA	0.5128	0.5023	0.5153	0.5507

The following table 4 gives the RMSE of Q estimation for different values of m

Table 4: RMSE of Q estimation for different constant True Q

M	50	100	200	300	400	500	1000
$Q_0=0.05$	NA	NA	NA	0.1137	0.1316	0.1349	0.1620
$Q_0=0.5$	NA	NA	0.0826	0.0675	0.0549	0.0493	0.1034
$Q_0=5$	NA	NA	NA	1.1215	1.2999	1.4606	2.0964

To check the Q and beta estimation performance the following figures 1 & 2 are generated with window size 300. The initial guess supplied to the filter is Q=5.

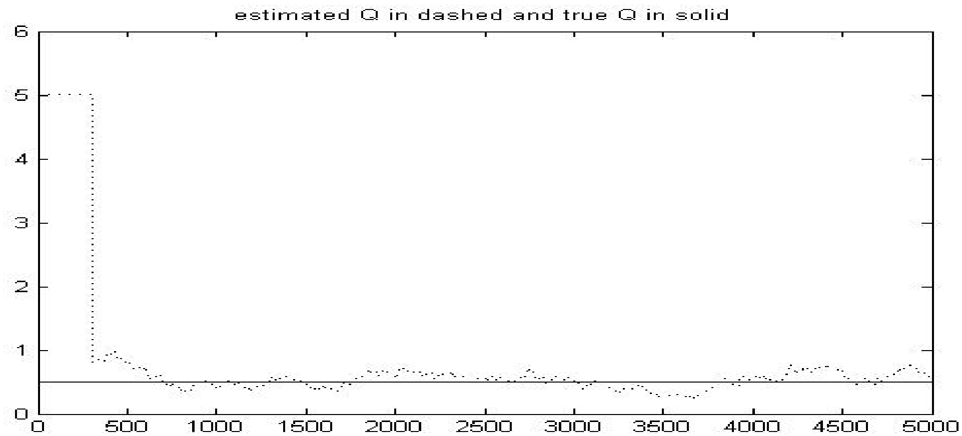

Fig. 1: Q estimation performance where truth is generated with Q=0.5 (Constant) and, and initial guess Q=5, R=0.9, RMSE=1.1140

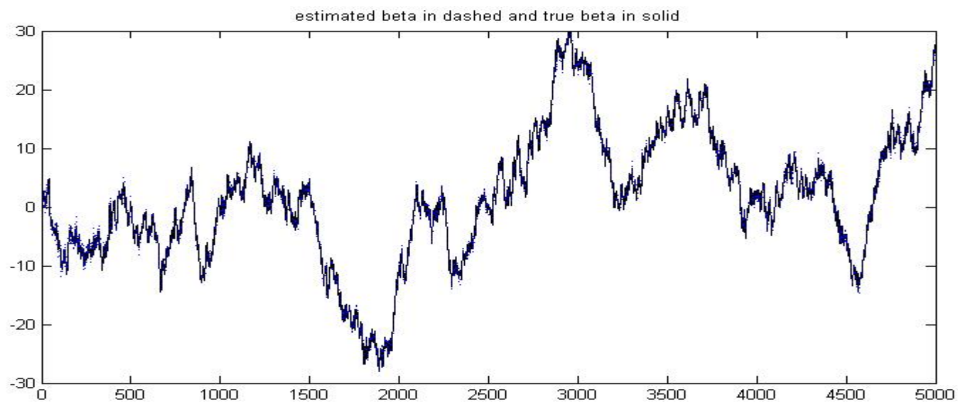

Fig. 2: Beta estimation performance where truth is generated with Q=0.5 (Constant) and initial guess Q=5, R=0.9, RMSE=0.7049.

Figure 3 and 4 illustrate the Q and beta estimation performance with initial guess Q=0.05 and R=0.9.

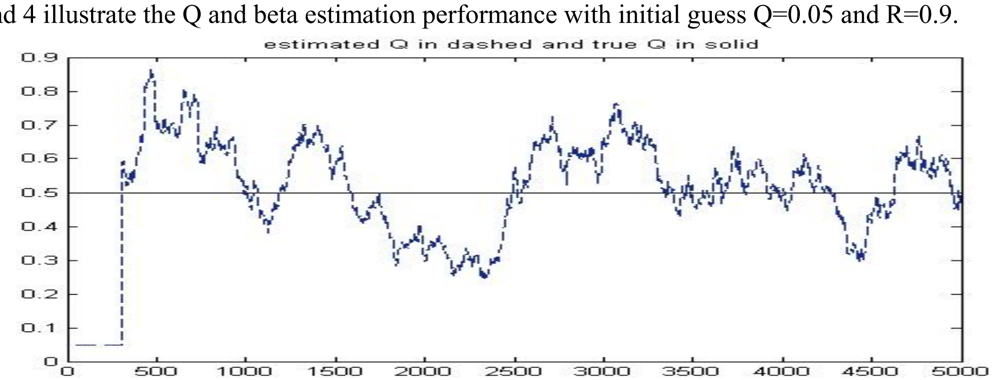

Fig. 3: Q estimation performance where truth is generated with Q=0.5 (Constant) and R=0.9, and initial guess Q=0.05, RMSE=0.1597.

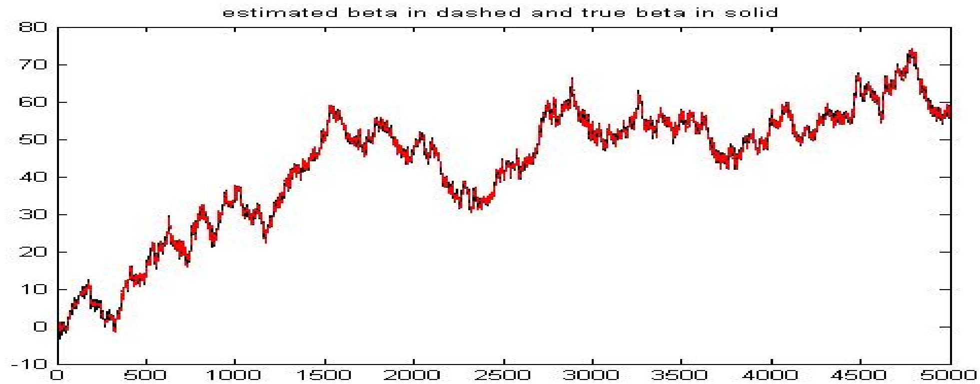

Fig. 4: Beta estimation performance where truth is generated with Q=0.5 (Constant), R=0.9 and initial guess of Q=0.05, RMSE=0.6958.

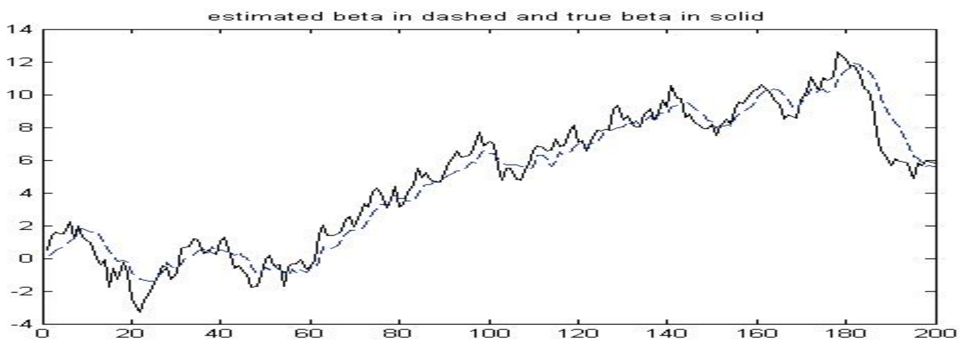

Fig. 5: Enlarged view of Beta estimation performance where truth is generated with Q=0.5 (Constant), R=0.9, and initial guess Q=0.05, RMSE=0.6958.

Next truth is generated with the exponentially decreasing Q as $Q_{t+1} = \gamma Q_t$, γ =0.999, Q_0 =0.5, R=0.9. The following figures 6 and 7 gives the performance of Q estimation for higher and lower initial values respectively.

Initial guess: Q_0 =0.7; m=300.

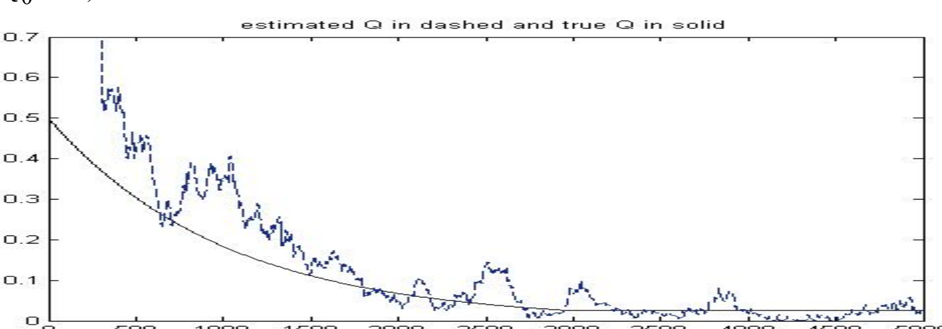

Fig. 6: Q estimation performance where truth is generated by exponentially decreasing Q and R=0.9, and initial guess Q_0 =0.7, RMSE= 0.0939.

RMSE of beta estimation in this case is 0.4948.

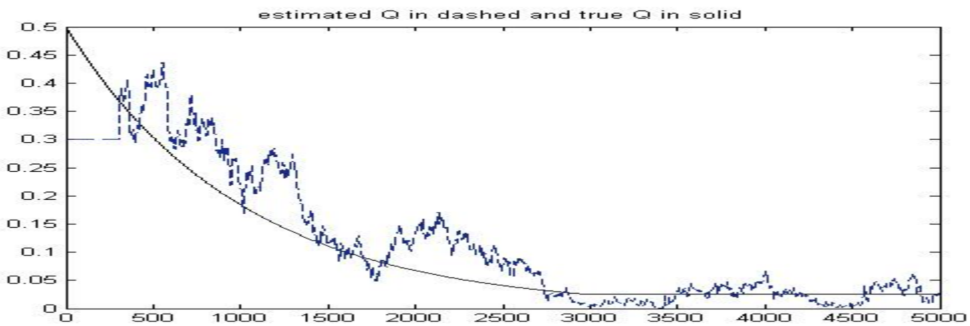

Fig. 7: Q estimation performance where truth is generated with exponentially decreasing Q and, and initial guess Q_0 =0.3, RMSE= 0.0571.

RMSE of beta estimation in this case is 0.4703.

Next truth is generated with the exponentially increasing Q as $Q_{t+1} = \gamma Q_t$, $\gamma = 1.001$, Q_0 =0.5, R=0.9. The following figures 8 and 10 gives the performance of Q estimation for higher and lower initial guess values respectively.

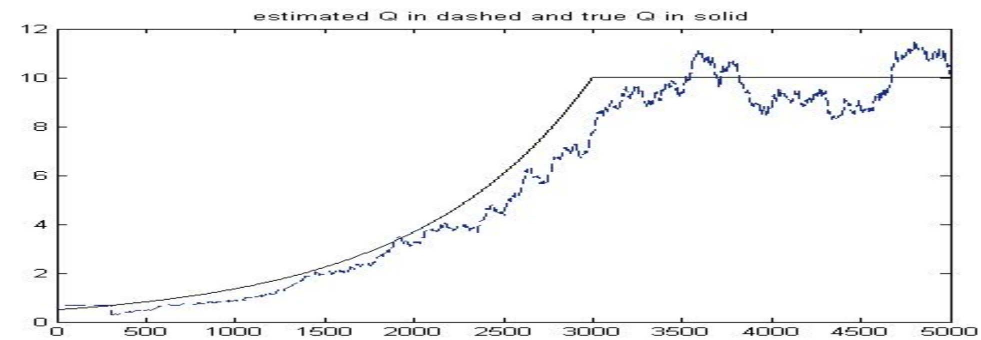

Fig. 8: Q estimation performance where truth is generated with exponentially increasing Q, and initial guess Q_0 =0.7, RMSE= 0.8919.

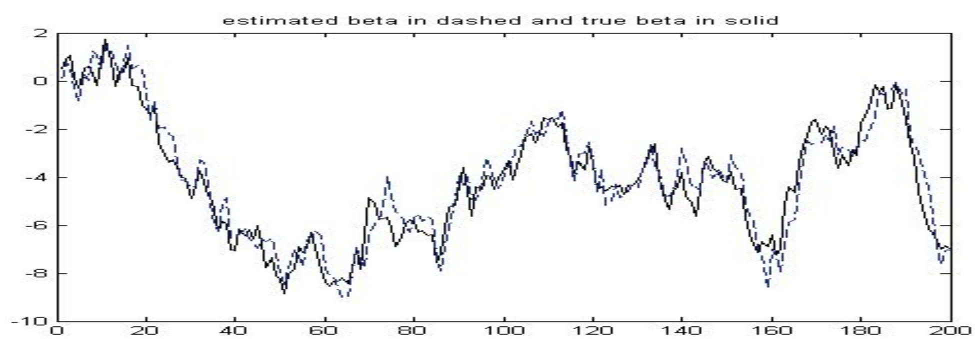

Fig. 9: Enlarged view of beta estimation performance where truth is generated with exponentially increasing Q and initial guess Q_0 =0.7, RMSE=0.8556.

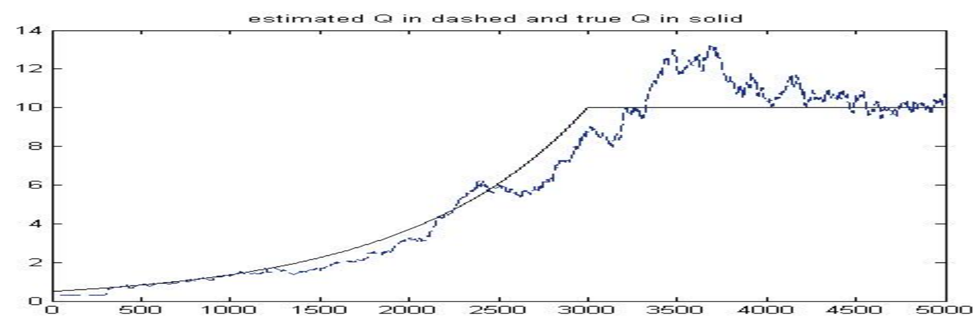

Fig. 10: Q estimation performance where truth is generated exponentially increasing Q, and initial guess Q_0 =0.3, RMSE=0.9714.

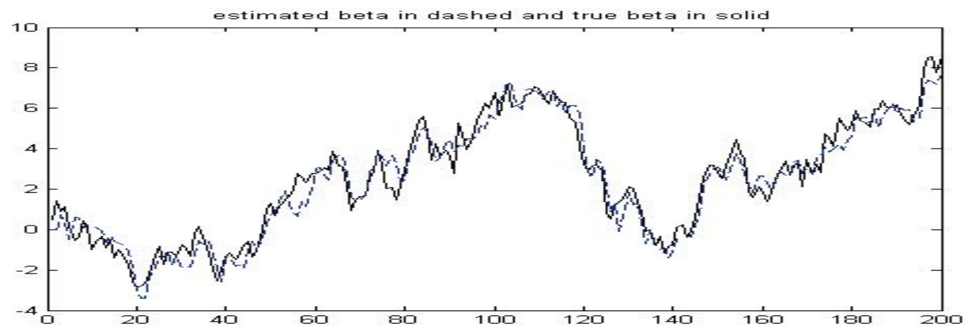

Fig. 11: Enlarged view of beta estimation performance where truth is generated with exponentially increasing Q and initial guess Q_0 =0.3, RMSE=0.8594.

To find the optimal value of window size m when Q is of exponentially changing nature the following table 5 & 6 is constructed where initial guess of R=0.9.

Table 5: RMSE of beta estimation for exponentially decreasing Q

m	**50**	**100**	**200**	**300**	**400**	**500**	**1000**
Q_0 =0.05	0.8518	0.8852	0.8757	0.8697	0.8914	0.8961	0.9634
Q_0 =0.5	0.8506	0.8614	0.8512	0.8499	0.8651	0.8653	0.8749
Q_0 =5	0.8701	0.8494	0.8678	0.8498	0.8507	0.8638	0.8827

Table 6: RMSE of beta estimation for exponentially increasing Q

m	**50**	**100**	**200**	**300**	**400**	**500**	**1000**
Q_0 =0.05	1.6005	1.4220	0.7716	0.7338	0.7024	0.9614	1.3986
Q_0 =0.5	1.6132	1.2983	0.7921	0.7788	0.7347	0.9086	1.2919
Q_0 =5	1.8404	1.4160	1.1124	1.2656	1.6856	1.7089	2.2345

For m greater than 60 the estimator is working without problem but for m=50 and less some time it is not working in the above case. So it seems while using such estimator the chosen window size should preferably be greater than 60 and should not be chosen less than 50.

The following table 7, 8 and 9 illustrate the performance of modified AKF in comparison to AKF where true R is 0.01.

Table 7: RMSE of beta estimation using AKF and modified AKF where initial guess for R=0.01

Q	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
AKF	0.1551	0.5854	0.5752	0.4381	0.8300	0.8258	1.2283	0.8234	2.4752	2.6655
Modified AKF	0.0980	0.1004	0.1008	0.1023	0.1069	0.1045	0.1068	0.1041	0.1040	0.1135

Table 8: RMSE of beta estimation using AKF and modified AKF where initial guess for R=0.1

Q	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
AKF	0.1502	0.2720	0.3474	0.4853	0.3369	0.4525	0.5387	1.2618	1.0602	1.7679
Modified AKF	0.1316	0.1370	0.1356	0.1298	0.1337	0.1273	0.1282	0.1262	0.1232	0.1246

Table 9: RMSE of beta estimation using AKF and modified AKF where initial guess for R=0.001

Q	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
AKF	0.1362	0.1800	0.3361	0.6986	0.5252	0.8263	0.4304	0.6017	1.8913	1.4180
Modified AKF	0.0973	0.1022	0.1036	0.1038	0.1053	0.1031	0.1105	0.1129	0.1116	0.1131

5.2 Results of Empirical Investigations

5.2.1 Investigation with IT Index of NSE India:

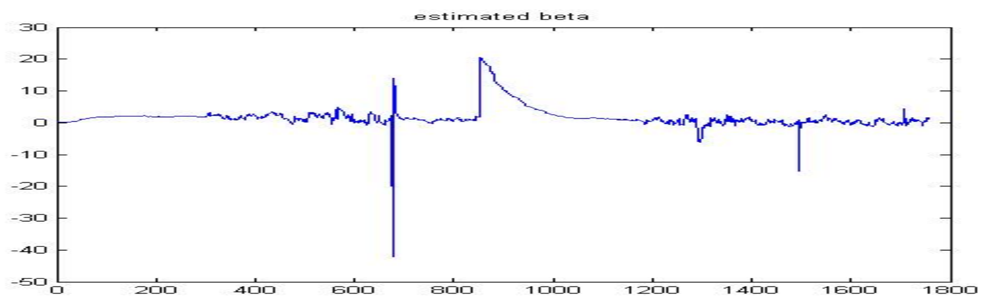

Fig. 12: Estimated beta for the IT index of NSE-India with window size 300

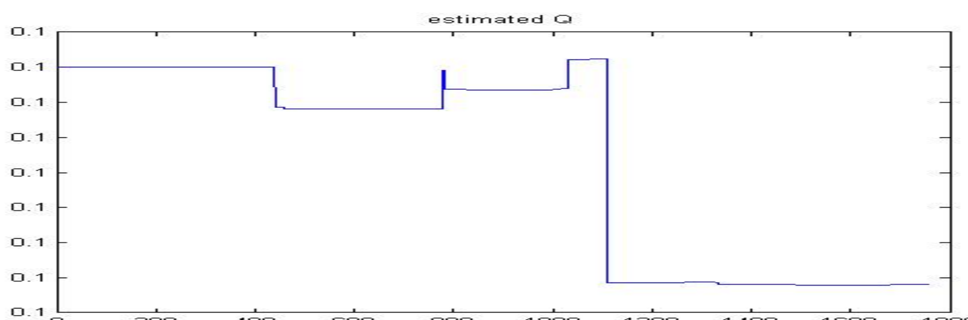

Fig. 13: Estimated Q for the IT index of NSE-India with window size 300 and initial guess for Q=0.1

It has been noticed that changes (or adaptation) take place at least at the 12th decimal places in the sequence of estimated Q values.

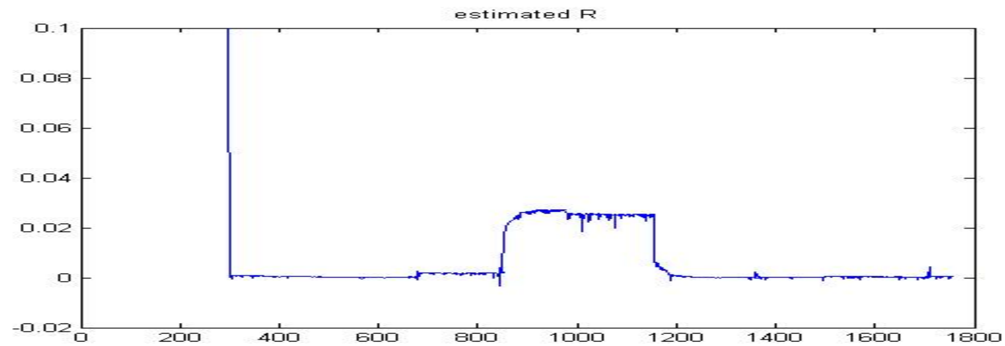

Fig. 14: Estimated R for the IT index of NSE-India with window size 300 and initial guess for R=0.1

5.2.2 Investigation with Bank Index of NSE India:

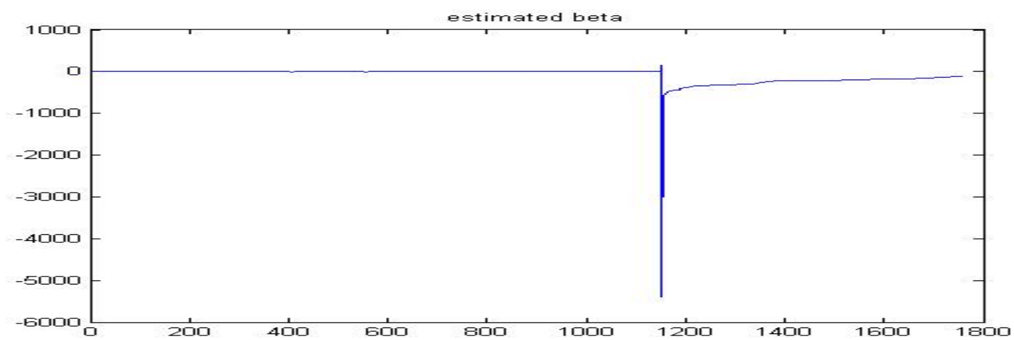

Fig. 15: Estimated beta of the Bank index of NSE-India with window size 300

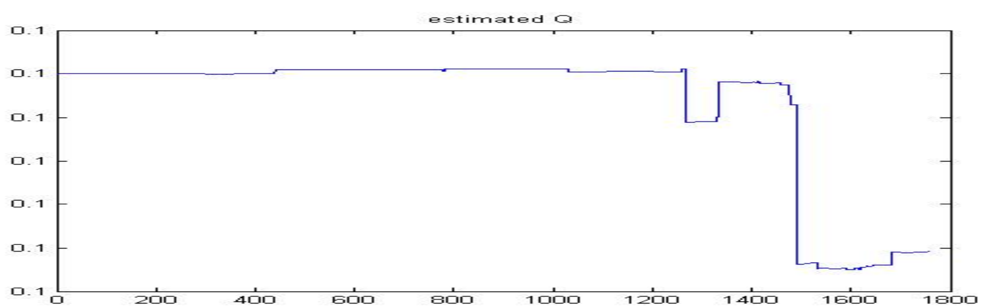

Fig. 16: Estimated Q for the Bank index of NSE-India with window size 300 and initial guess for Q=0.1

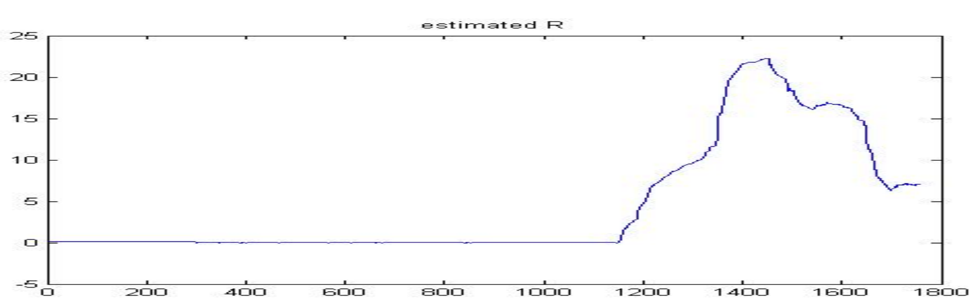

Fig. 17: Estimated R for the Bank index of NSE-India with window size 300 and initial guess for R=0.1

Interestingly the Q adaptation takes place at around 12th decimal place even though Q is less than 100000. For initial value Q> 100000 almost, Q did not change over time. Though R started from may be 100, it actually concentrates around 0.01 for first 1200 days. Later it suddenly jumped upward.

4. Conclusions

This work characterizes the AKF technique for beta estimation. The characterization is first carried out through simulation study. It has been found that measurement noise covariance (R) become negative for some combination of true Q (around 1) and true R (small like 0.01). The AKF technique is modified to take care of such situations. The simulation experiments show that modified AKF can efficiently estimate the beta and Q as exhibited by the RMSE results. Another challenge which has been addressed is identifying the optimal window size for adaptive estimation of beta. The simulation experiments revealed that estimation window size should preferably be of 300 time steps.

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