

# Introduction to multivariate analysis — applications in genomics —

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MSc “*Modern epidemiology*”  
22-03-2013

# Outline

Multivariate analysis in a nutshell

Applications to genomic data

Genetic diversity of pathogen populations

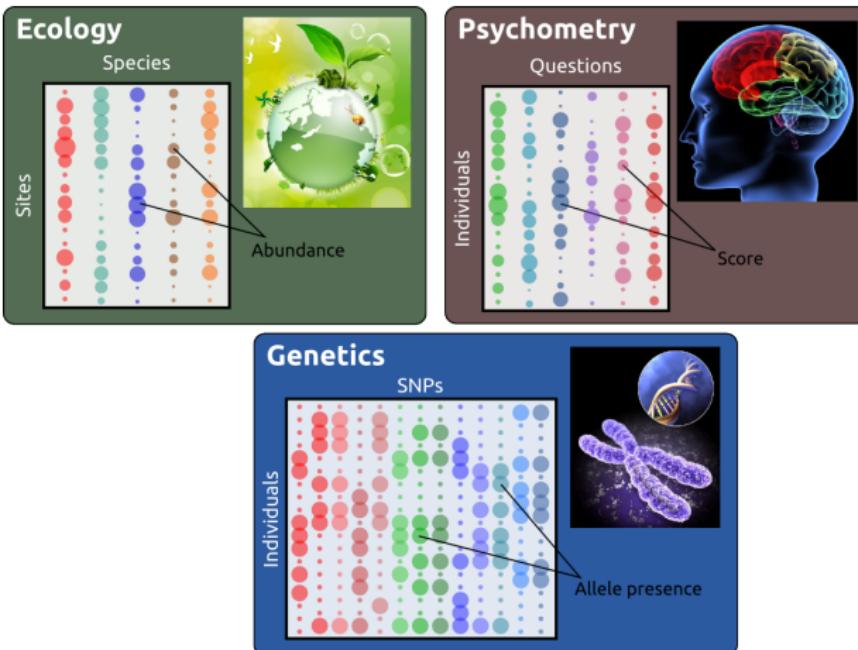
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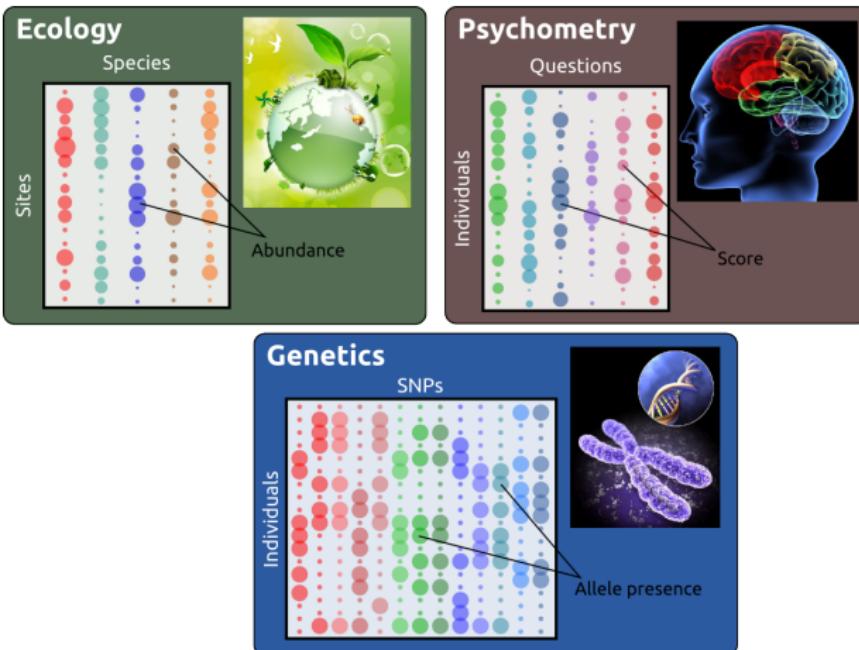
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# Multivariate data: some examples



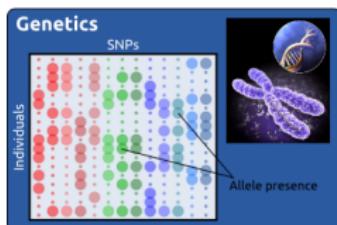
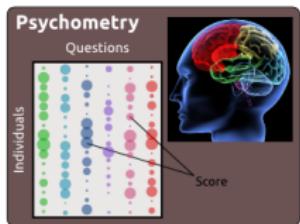
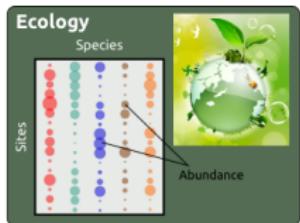
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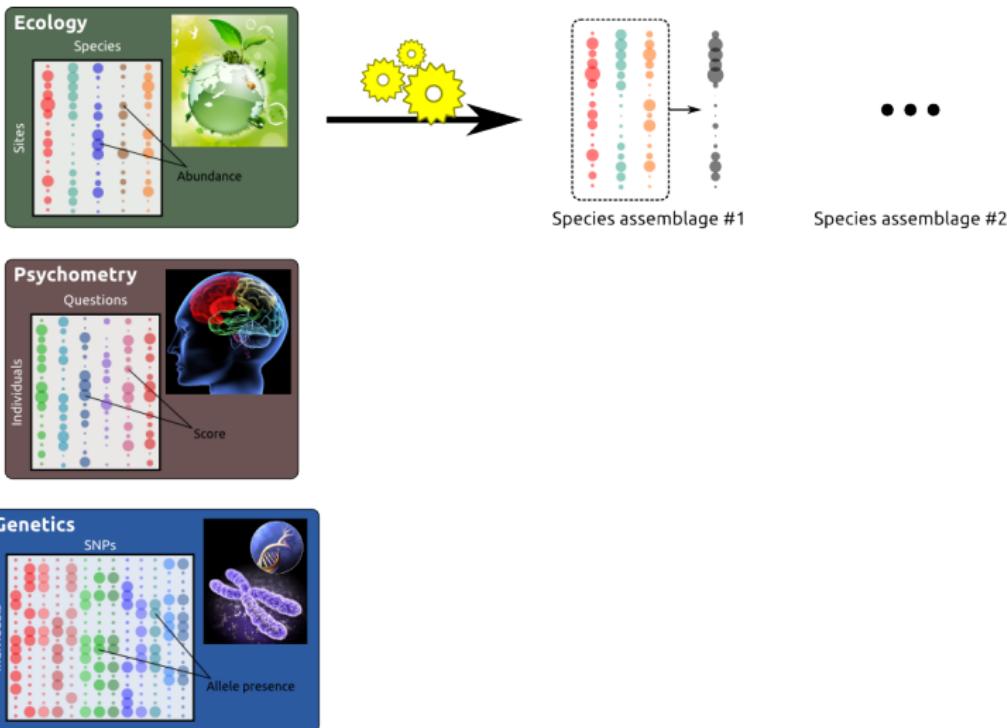


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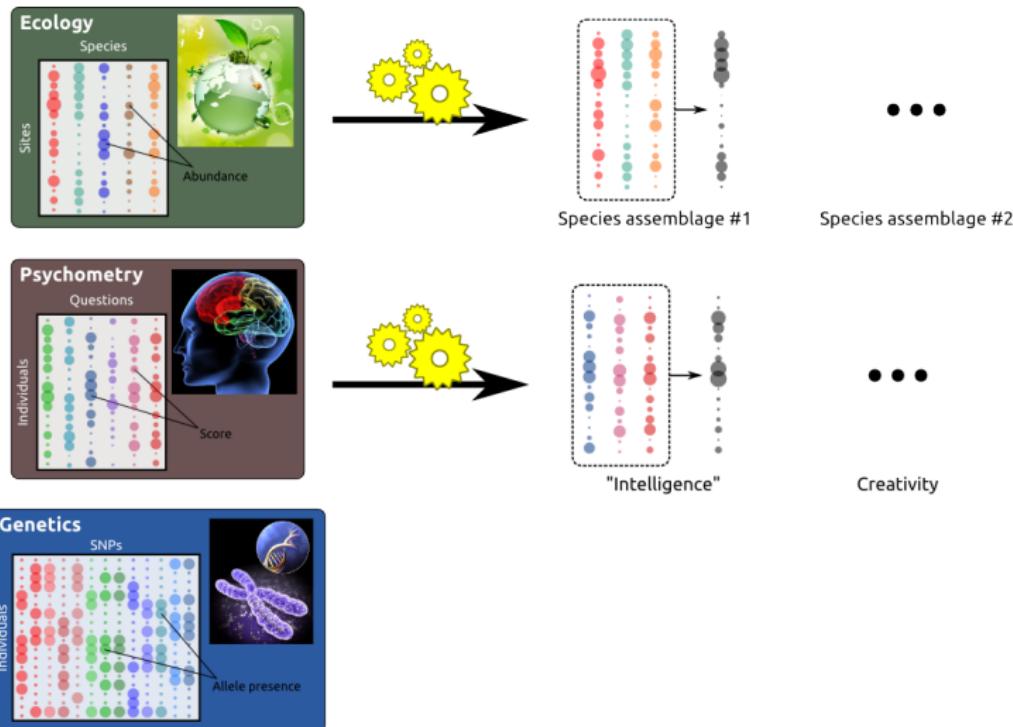
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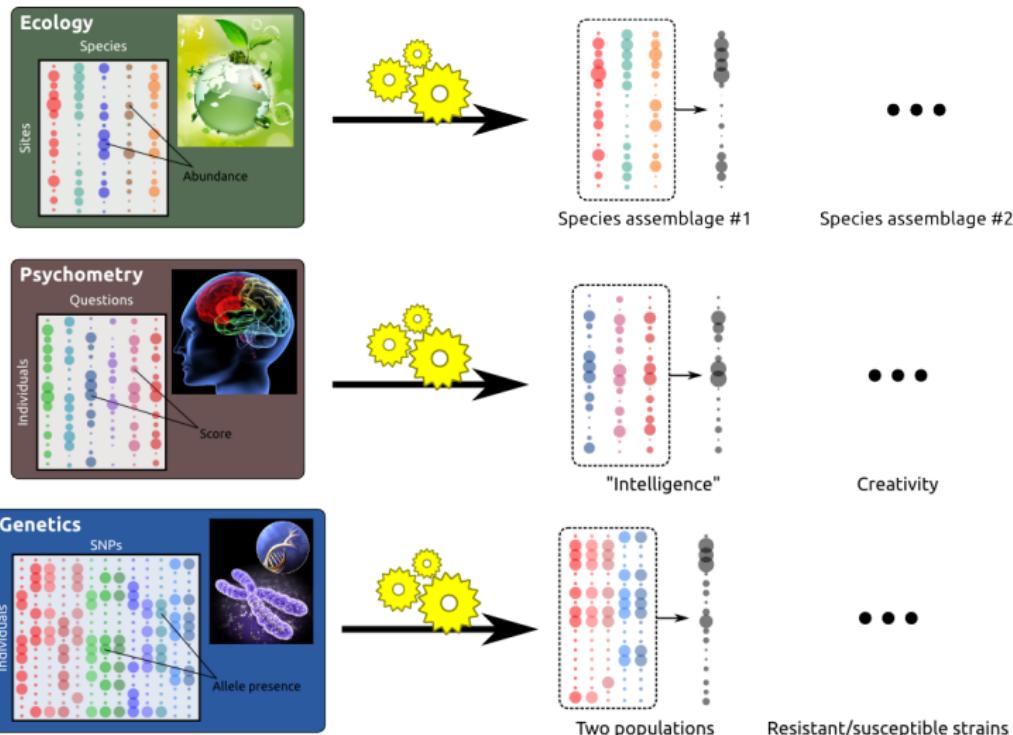
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# Multivariate analysis: an overview

## Multivariate analysis, a.k.a:

- “*dimension reduction techniques*”
- “*ordinations in reduced space*”
- “*factorial methods*”

## Purposes:

- summarize diversity amongst observations
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# Most common methods

Differences lie in input data:

- quantitative/binary variables: *Principal Component Analysis* (PCA)
- 2 categorical variables: *Correspondance Analysis* (CA)
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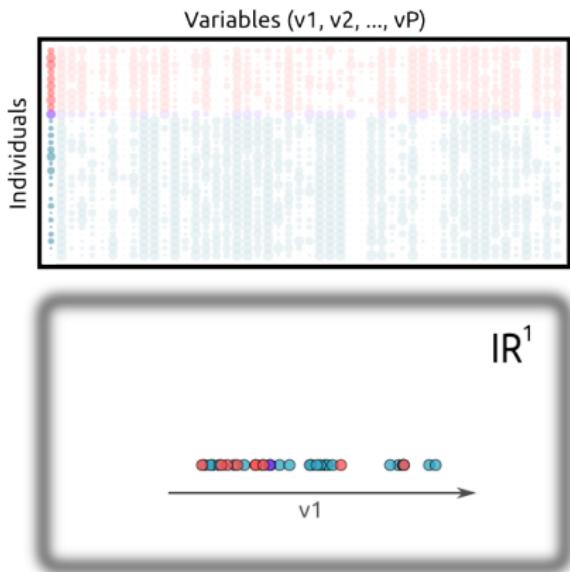
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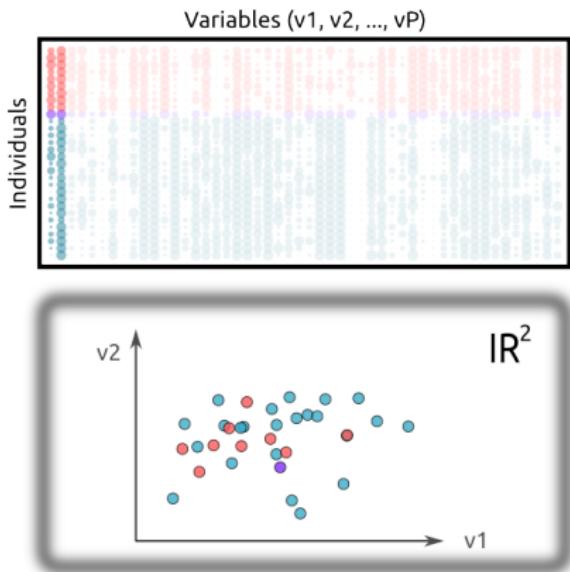
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# 1 dimension, 2 dimensions, $P$ dimensions



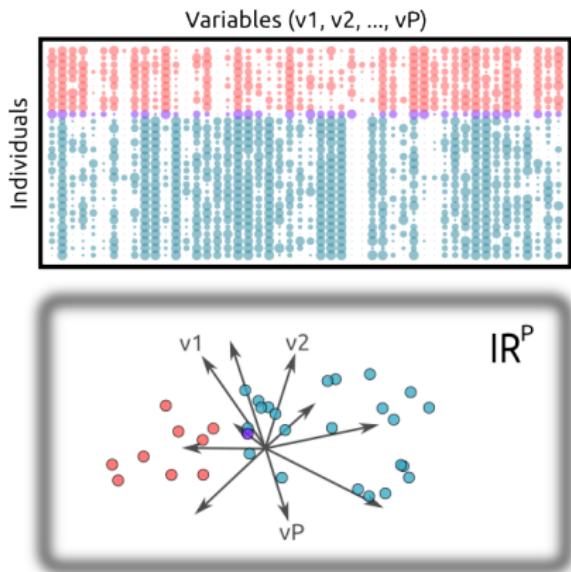
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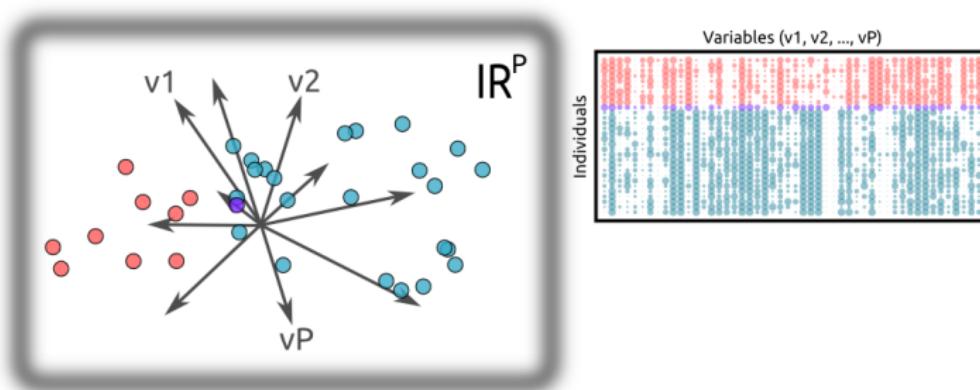
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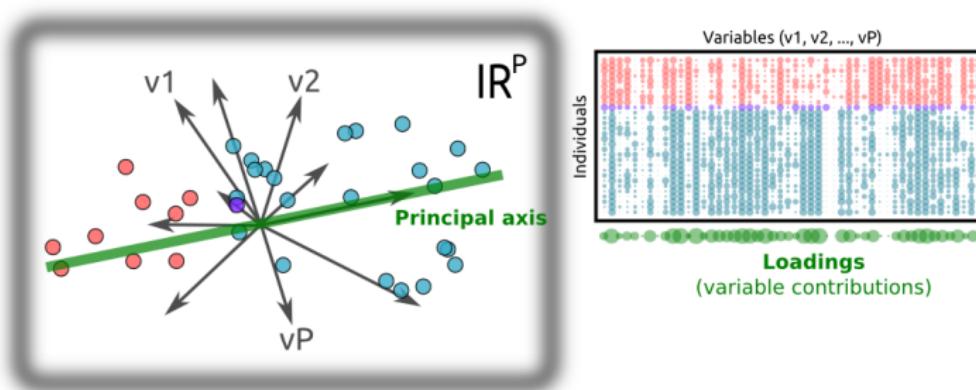
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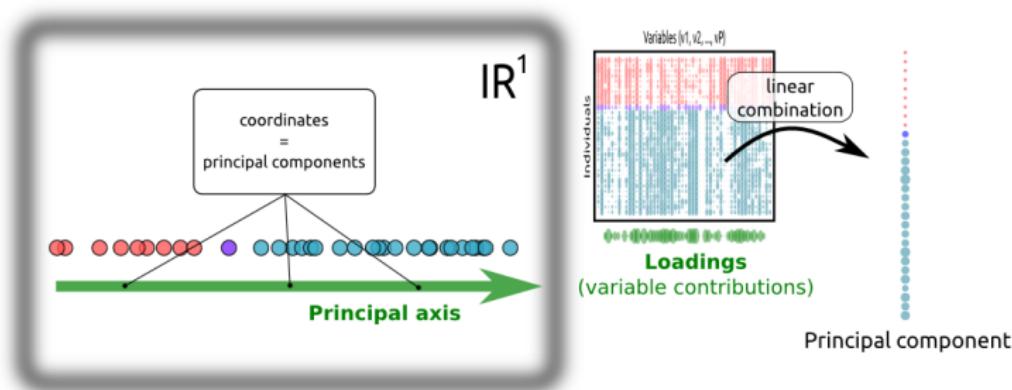
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  - $\mathbf{u} \in \mathbb{R}^P; \mathbf{u} = [u_1, \dots, u_P]$ : **principal axis**  
( $\|\mathbf{u}\|^2 = \sum_{j=1}^P u_j^2 = 1$ )
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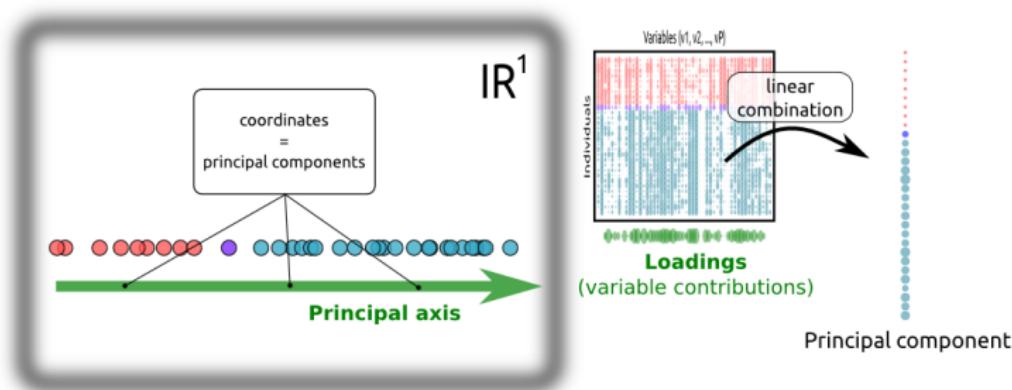
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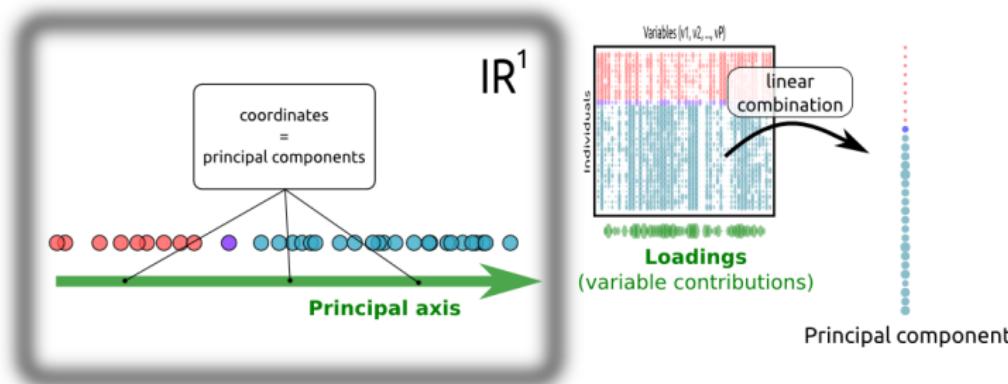
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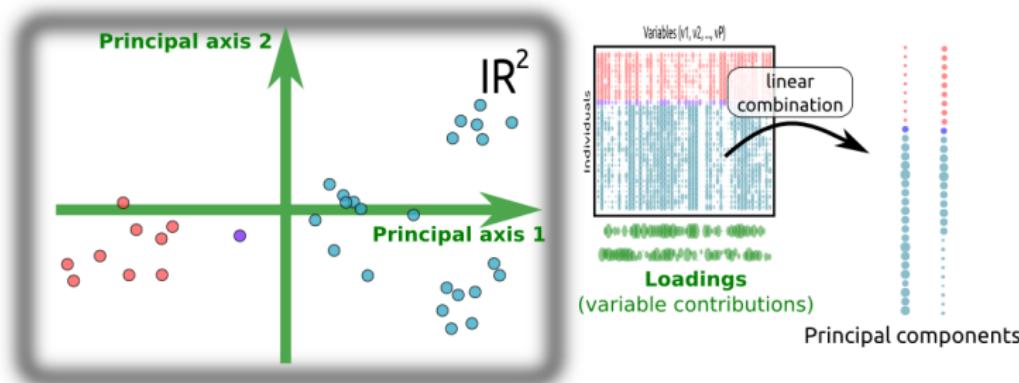


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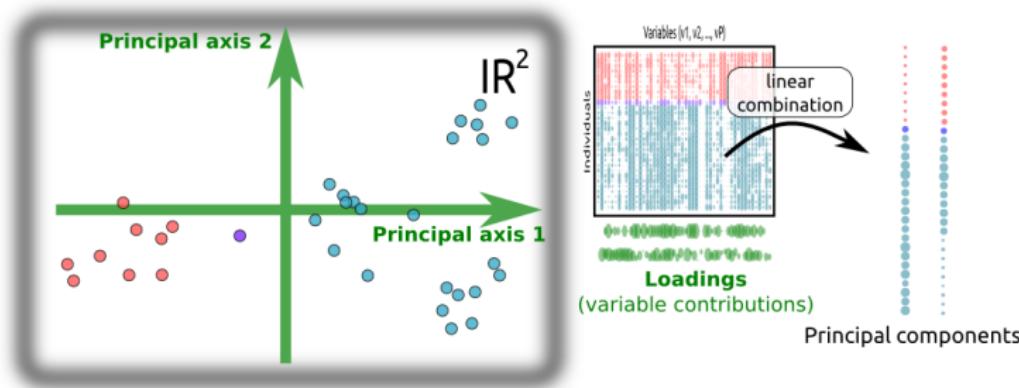


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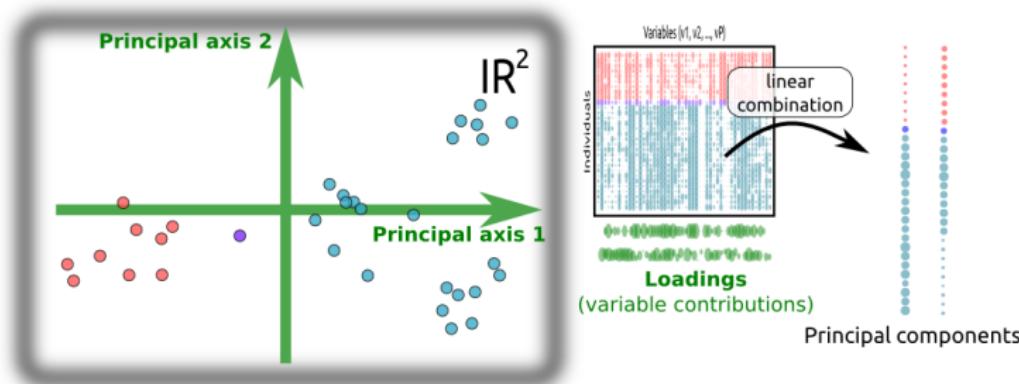


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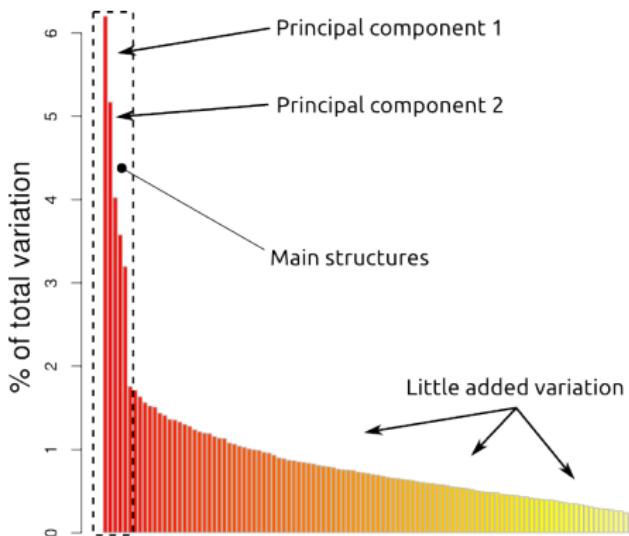
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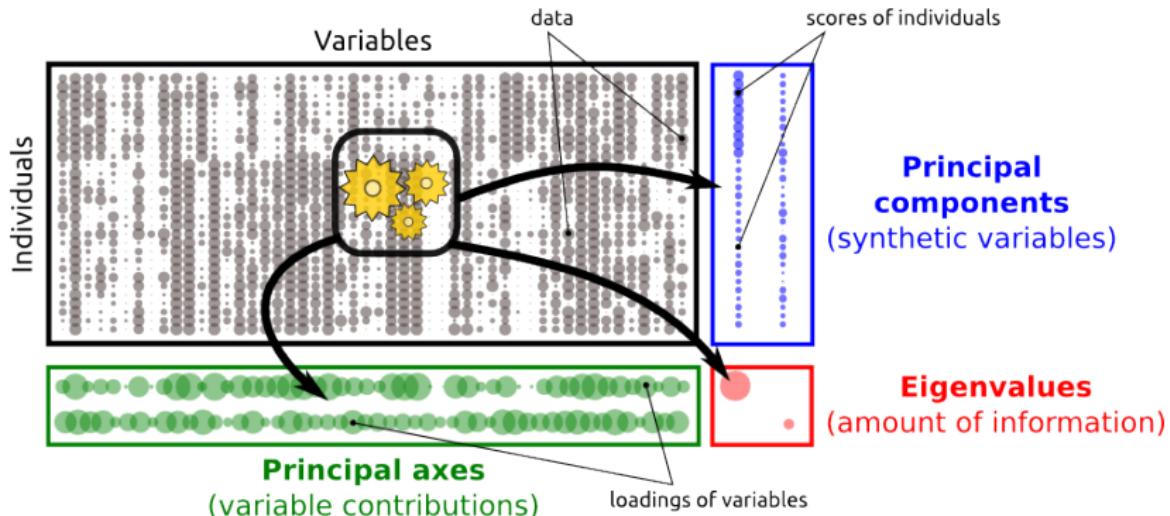
# How many principal components to retain?

Choice based on “**screeplot**”: barplot of eigenvalues



Retain only “significant” structures... but not trivial ones.

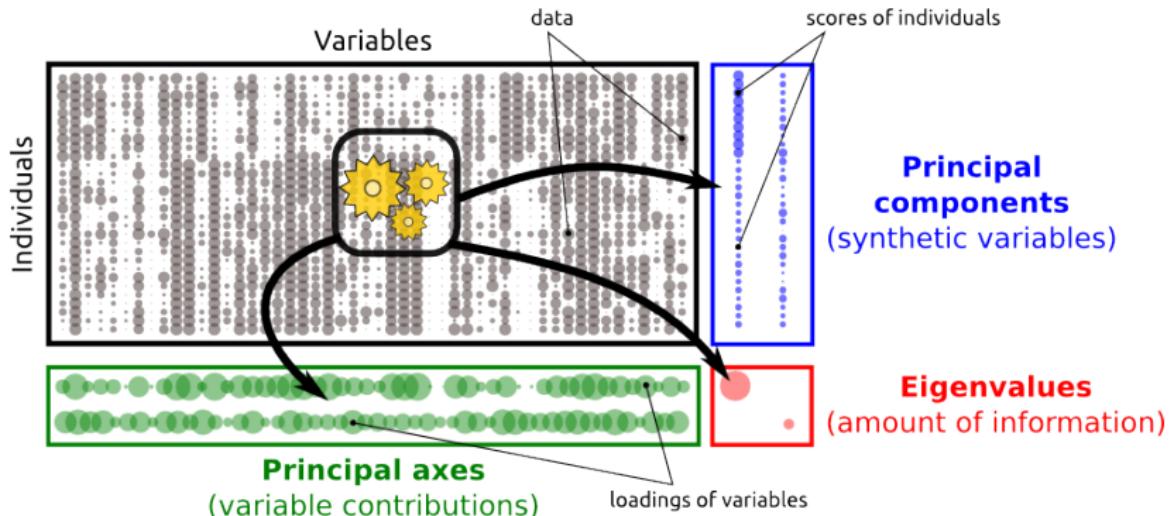
# Outputs of multivariate analyses: an overview



Main outputs:

- **principal components:** diversity amongst individuals
- **principal axes:** nature of the structures
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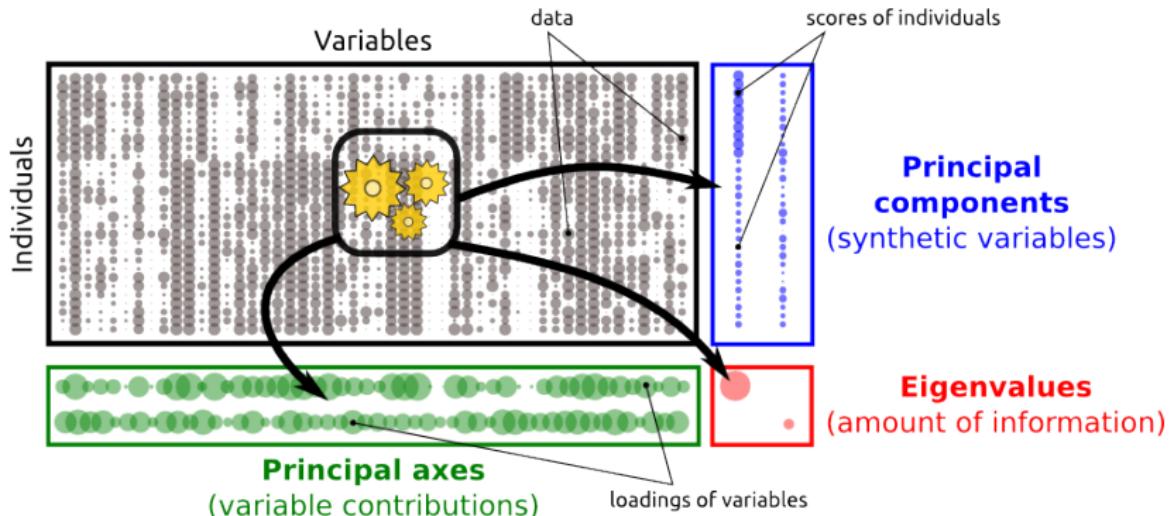
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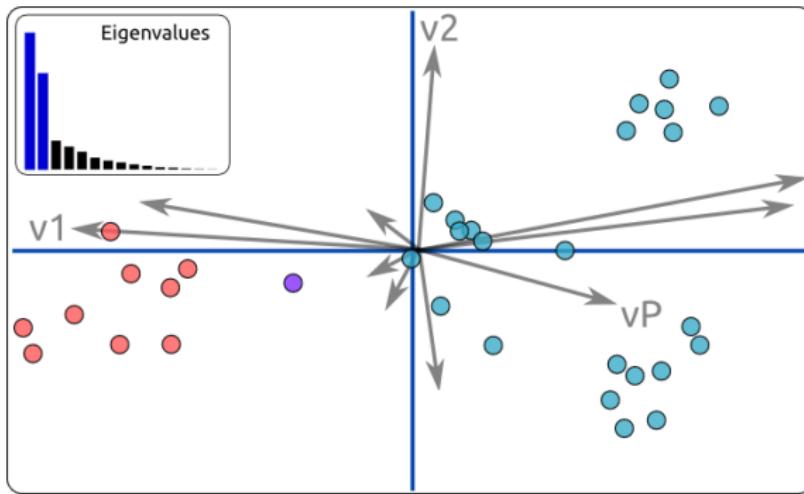
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## Usual summary of an analysis: the biplot



Biplot: principal components (points) + loadings (arrows)

- groups of individuals
- discriminating variables (longest arrows)
- magnitude of the structures

# Multivariate analysis in a nutshell

- **variety of methods** for different types of variables
- **principal components** (PCs) summarize diversity
- **variable loadings** identify discriminating variables
- other uses of PCs: **maps** (spatial structures), **models** (response variables or predictors), ...

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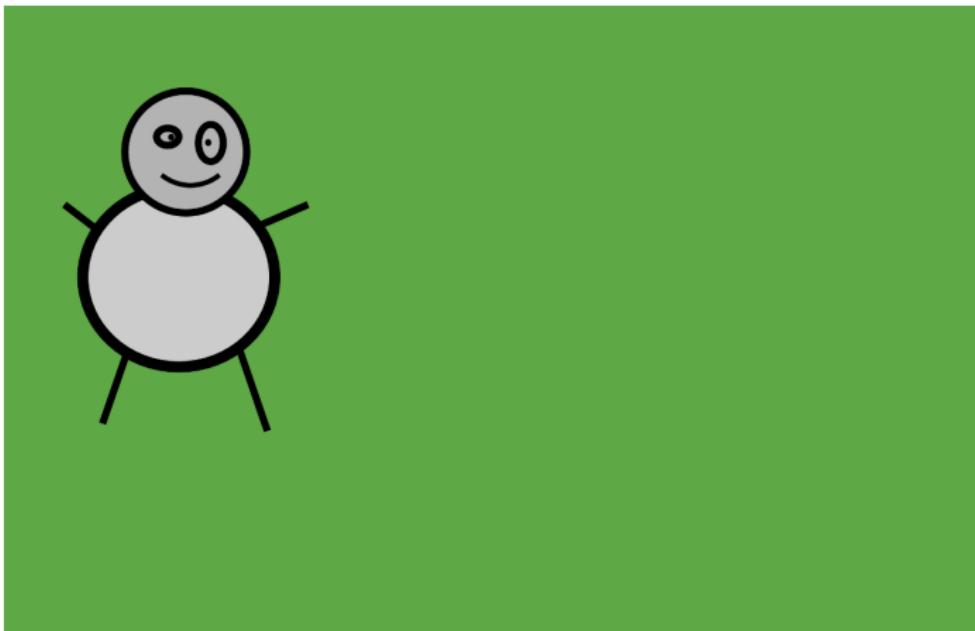
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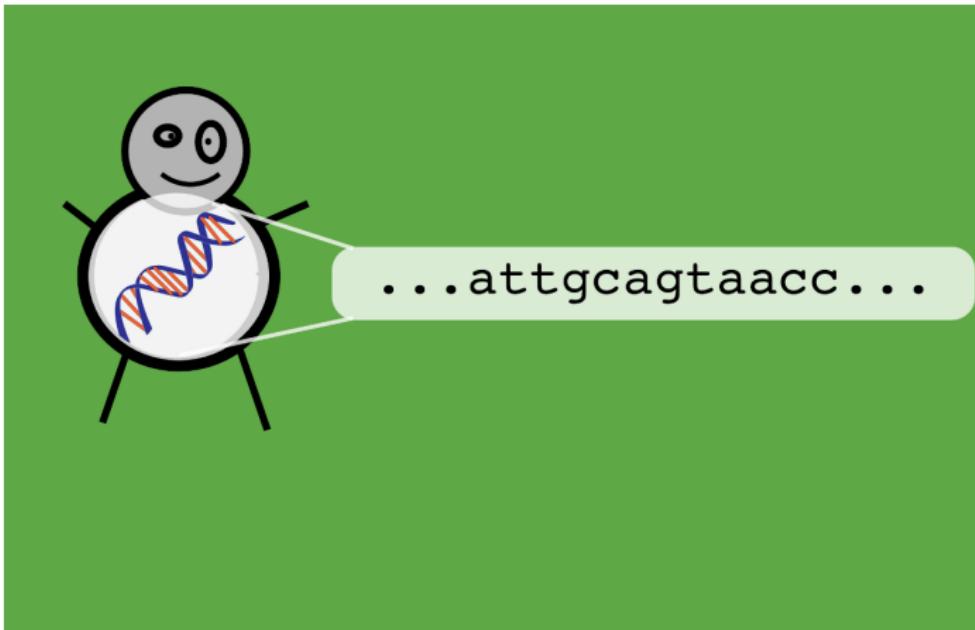
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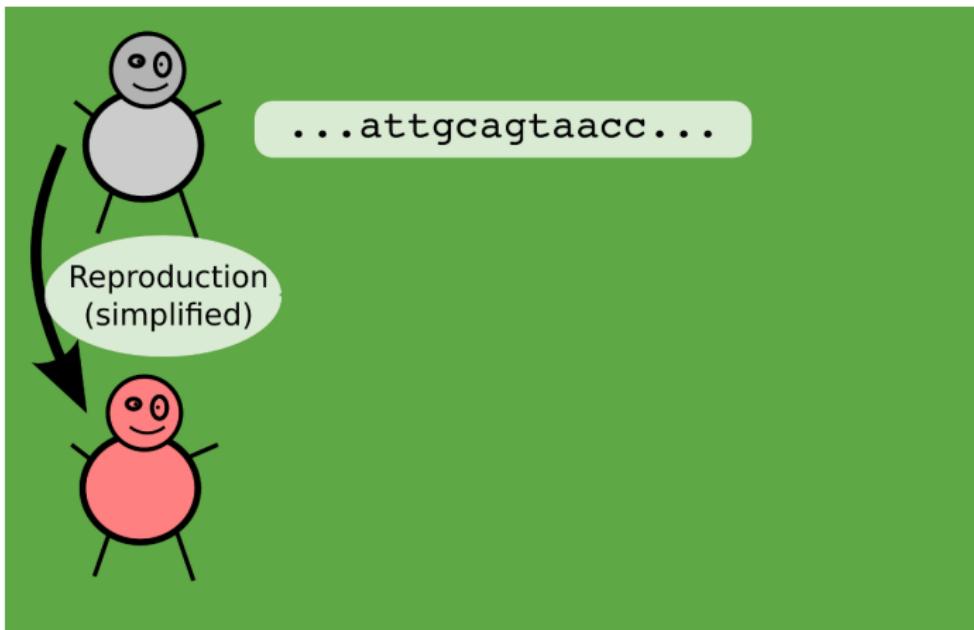
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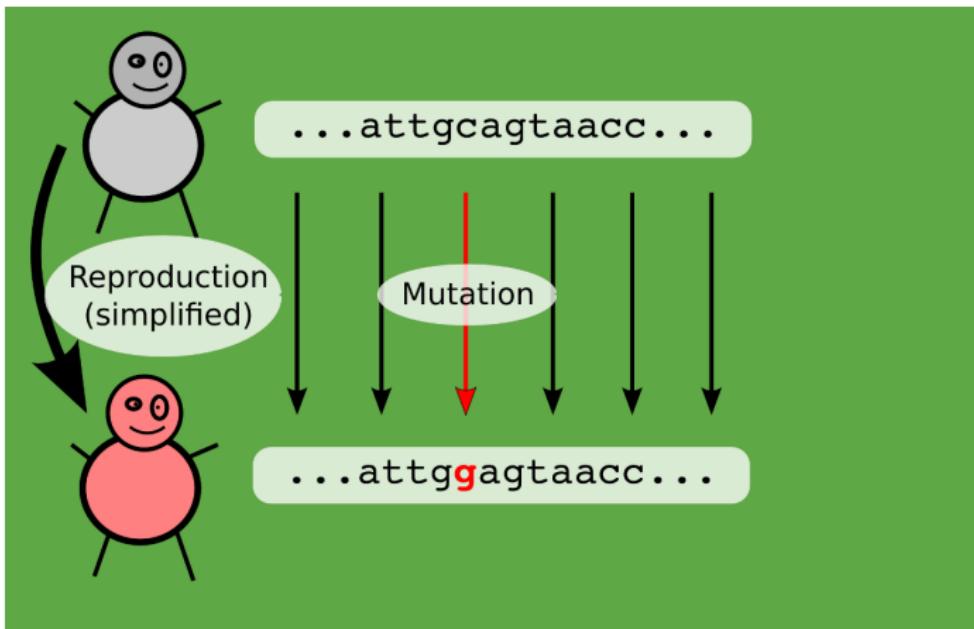
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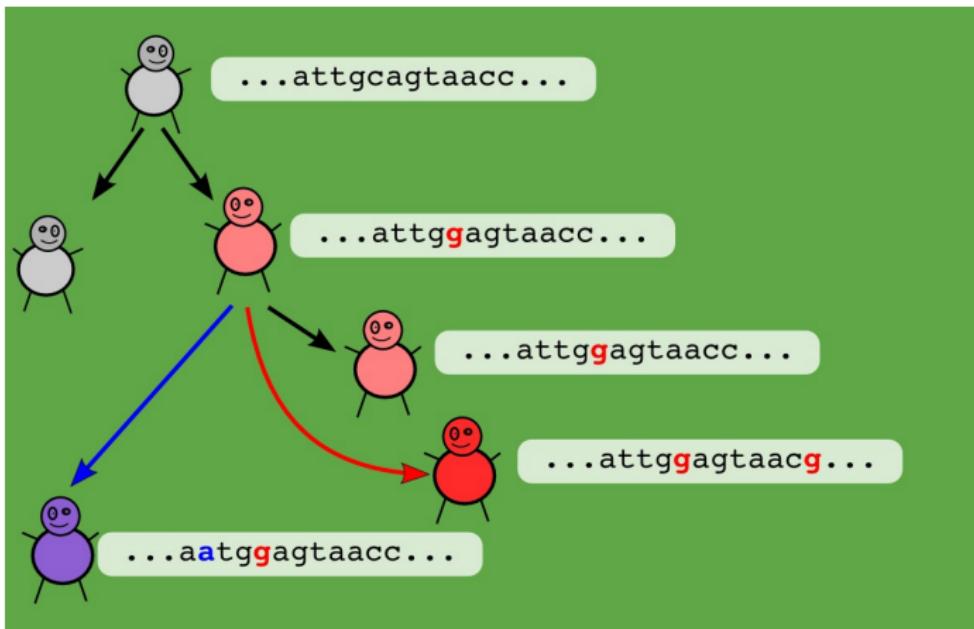
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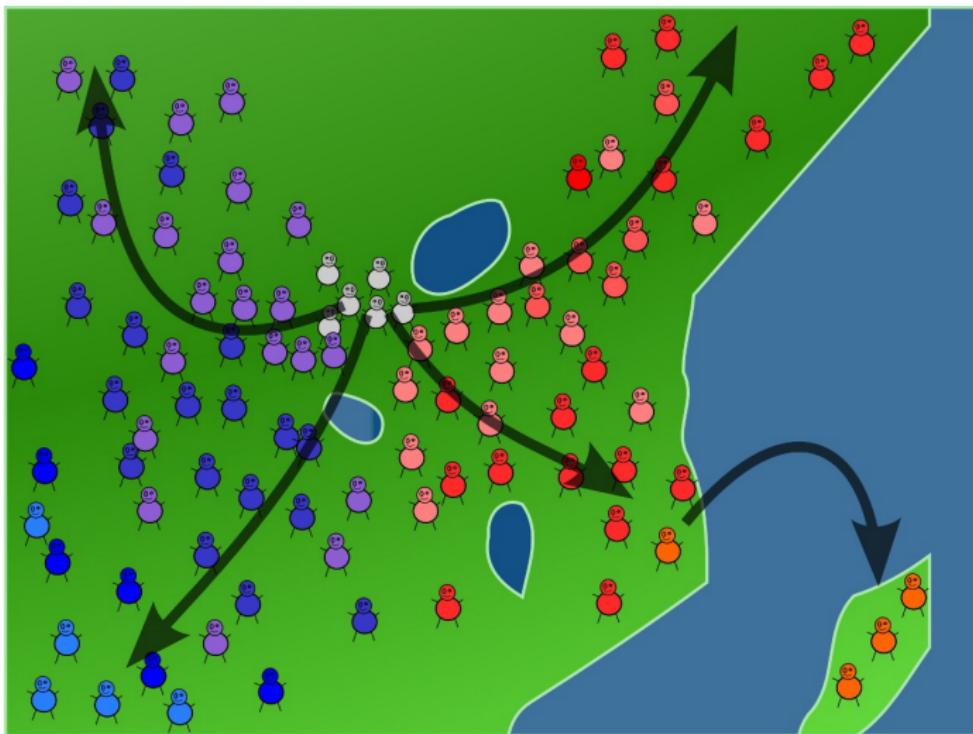
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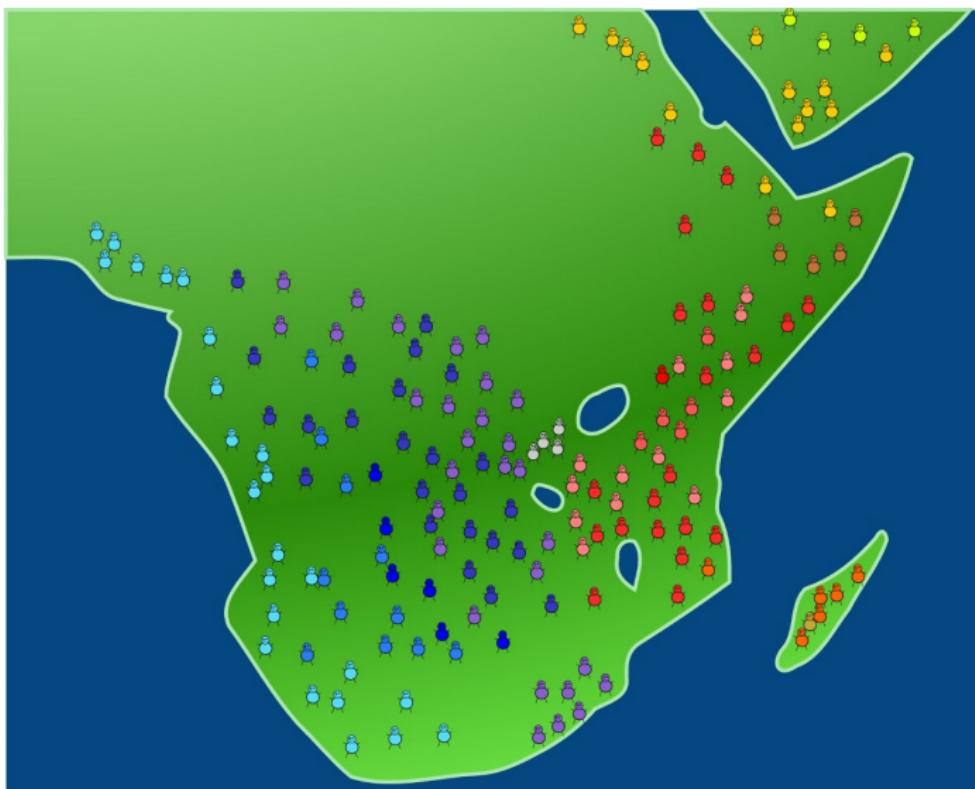
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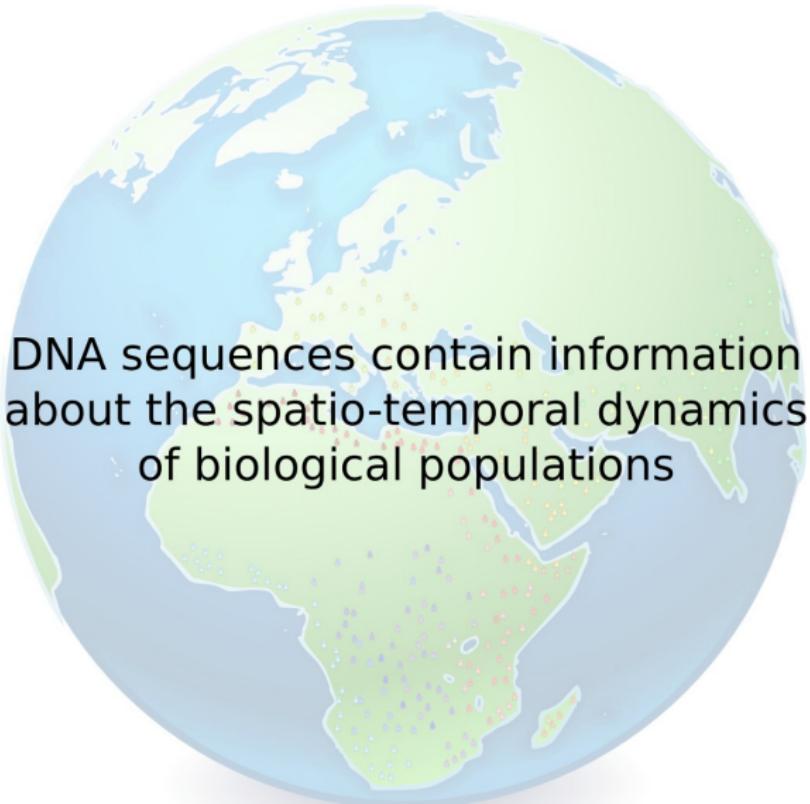
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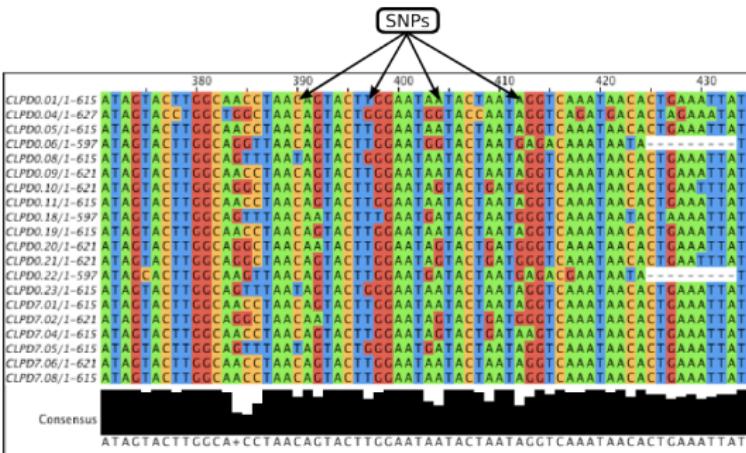


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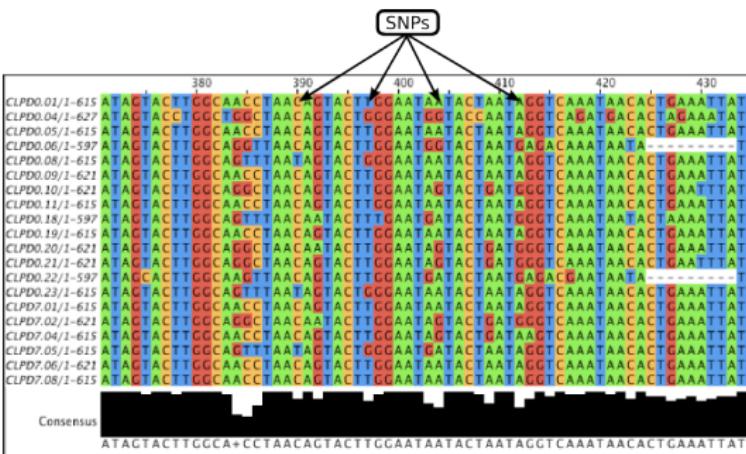
DNA sequences contain information about the spatio-temporal dynamics of biological populations

# DNA sequences: a rich source of information



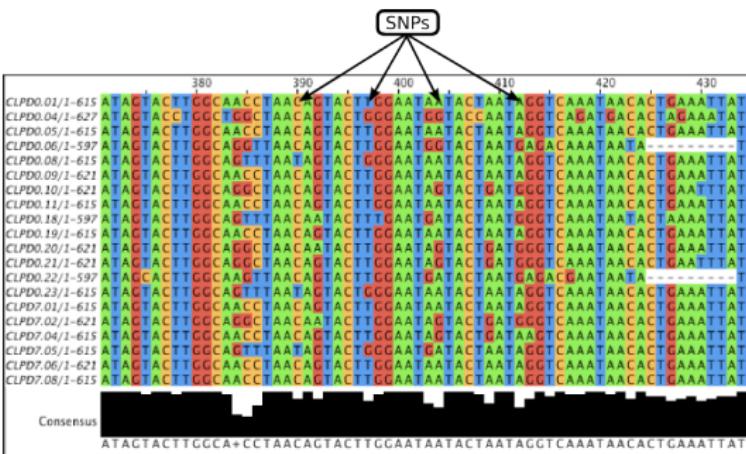
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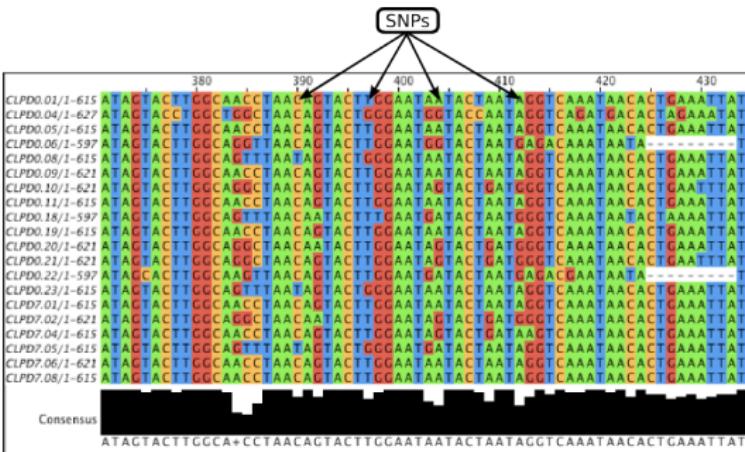
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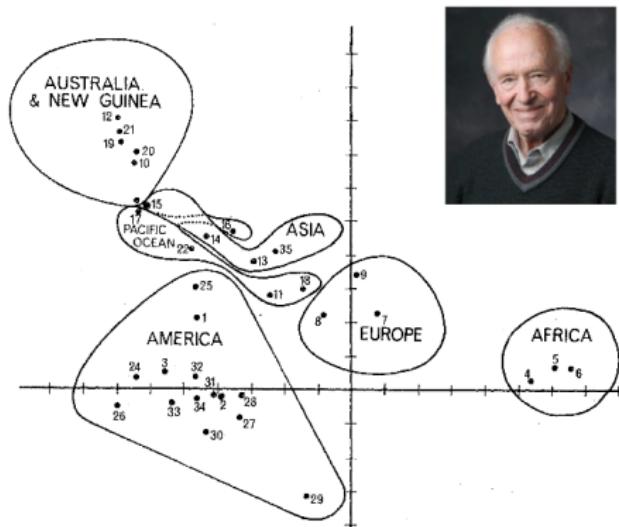
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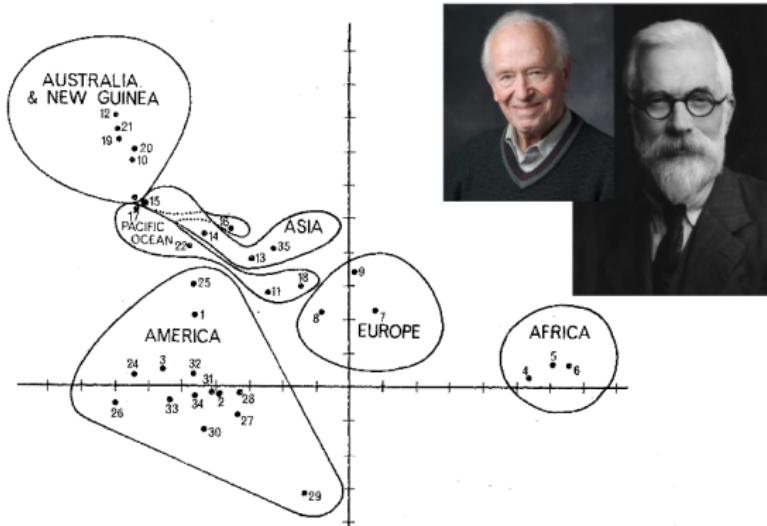
PCA of genetic data, native human populations (Cavalli-Sforza 1966, *Proc B*)



First 2 principal components separate populations into continents.

# First application of multivariate analysis in genetics

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First 2 principal components separate populations into continents.

## Applications: some examples

PCA of genetic data + colored maps of principal components

(Cavalli-Sforza et al. 1993, *Science*)



Signatures of Human expansion out-of-Africa.

# Since then...

## Multivariate methods used in genetics

- Principal Component Analysis (PCA)
- Principal Coordinates Analysis (PCoA) / Metric Multidimensional Scaling (MDS)
- Correspondance Analysis (CA)
- Discriminant Analysis (DA)
- Canonical Correlation Analysis (CCA)
- ...

## Since then...

### Applications

- reveal spatial structures (historical spread)
- explore genetic diversity
- identify cryptic species
- discover genotype-phenotype association
- ...
- review in Jombart et al. 2009, *Heredity* **102**: 330-341

Applications in genetics of pathogen populations.

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# Why investigate the diversity of pathogen populations?

Genetic data: increasingly important in infectious disease epidemiology

## Purposes

- classify pathogens, describe their relationships
- assess the spatio-temporal dynamics of infectious diseases
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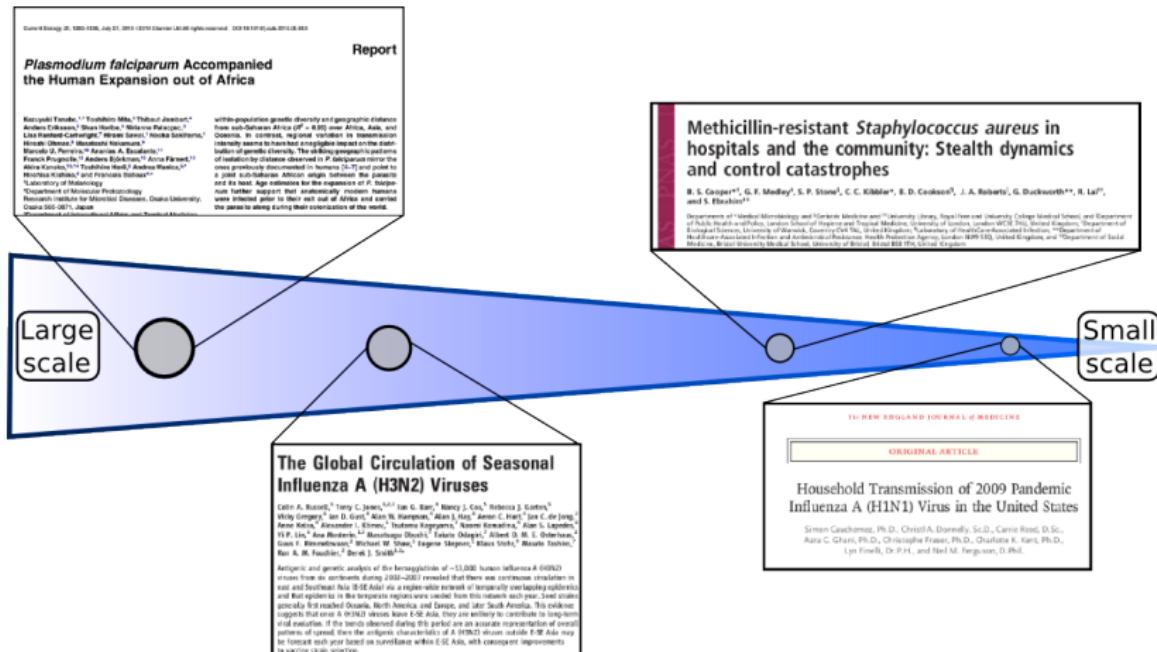
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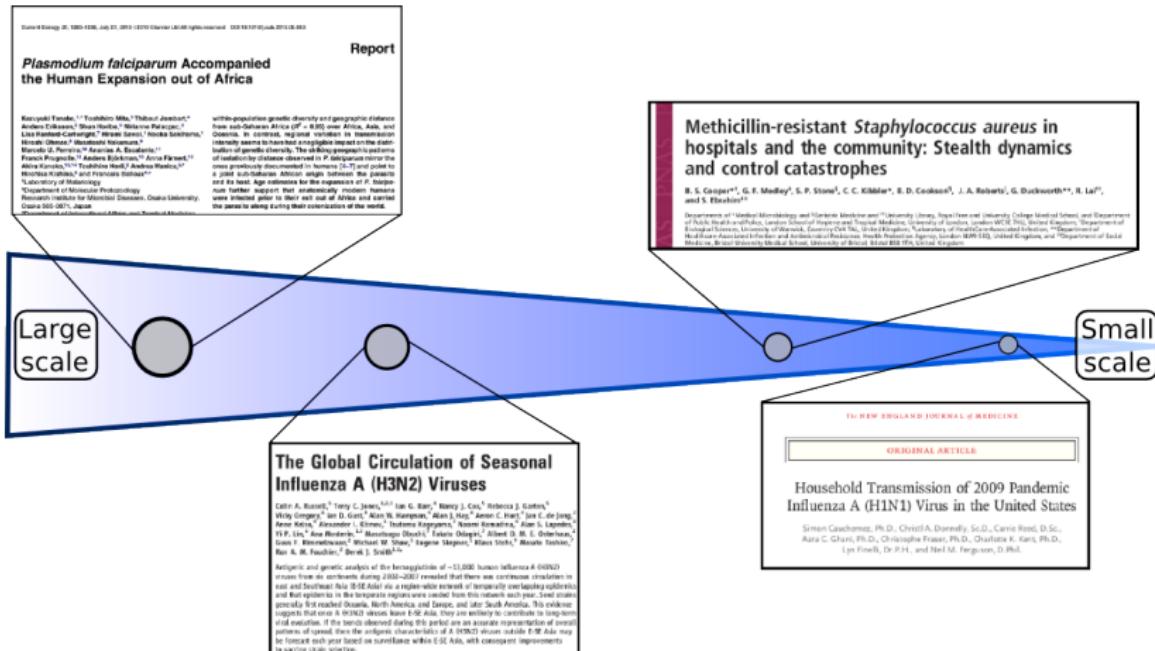


## Different questions at different scales



Where and how can multivariate analysis of pathogen genetic data be useful?

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**Population genetics:** identify populations of organisms and describe their relationships

What is a population?

- *Usual definition:* set of organisms mating at random
- *Problem:* no “mating” in most pathogens (e.g. viruses, bacteria)
- **Genetic clusters:** set of genetically related pathogens (e.g. same outbreak, same epidemic).

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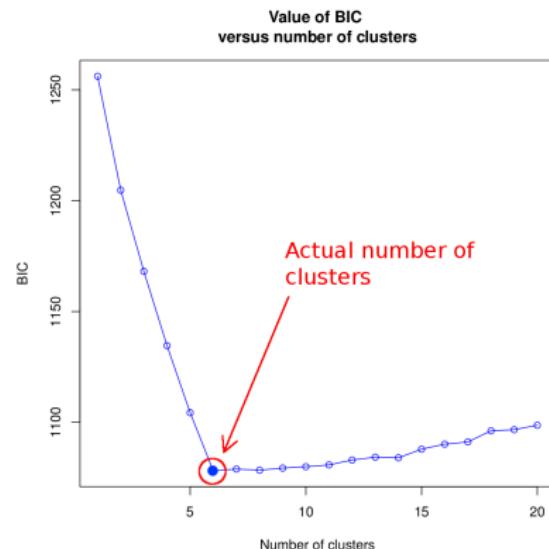
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# Genetic clustering using K-means & BIC

(Jombart et al. 2010, *BMC Genetics*)



Variance partitioning model (ANOVA):

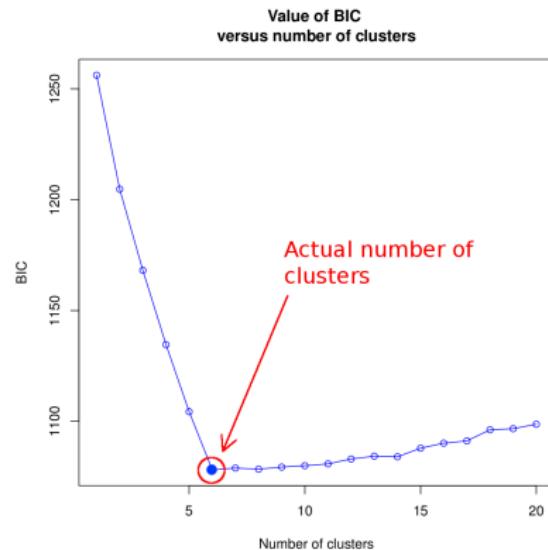
$$\text{tot. variance} = (\text{bet. groups}) + (\text{wit. groups})$$

Performances:

- K-means  $\geq$  STRUCTURE on simulated data (various island and stepping stone models)
- orders of magnitude faster (seconds vs hours/days)

# Genetic clustering using K-means & BIC

(Jombart et al. 2010, *BMC Genetics*)



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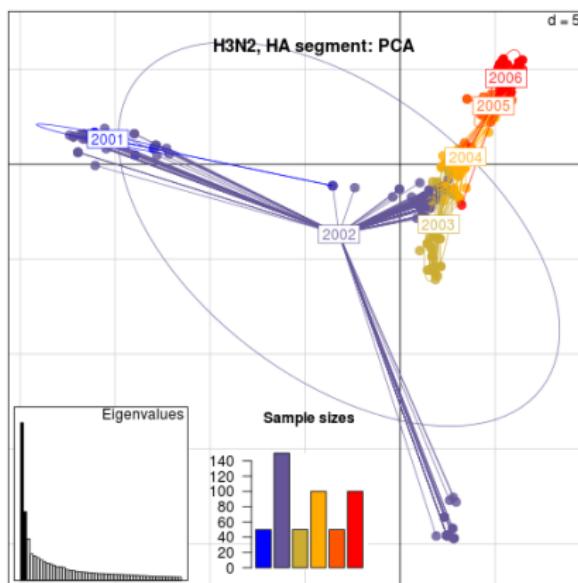
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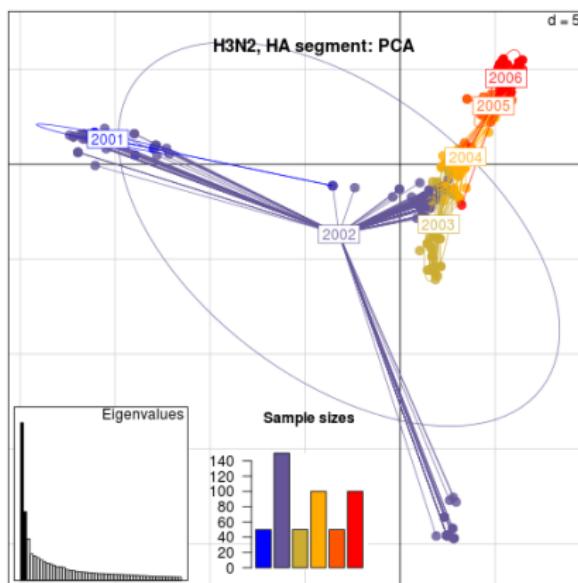
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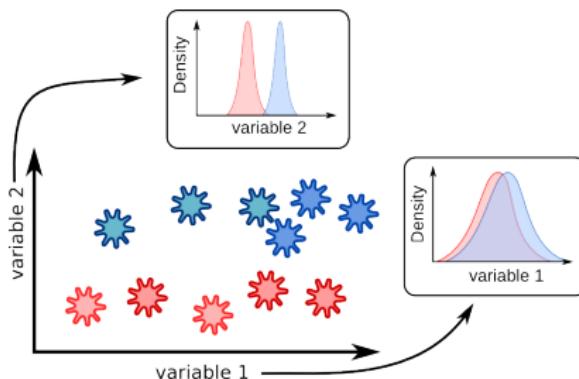
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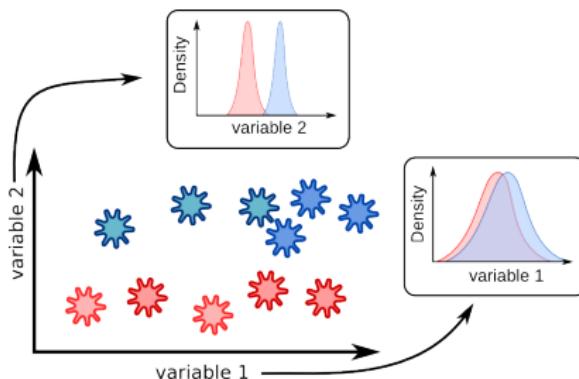
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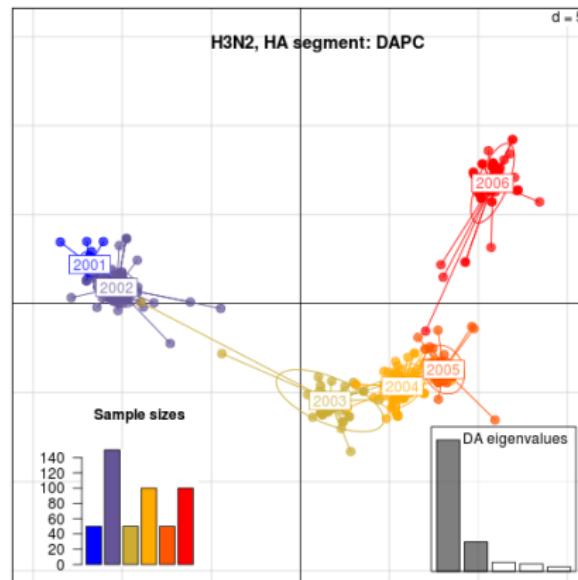


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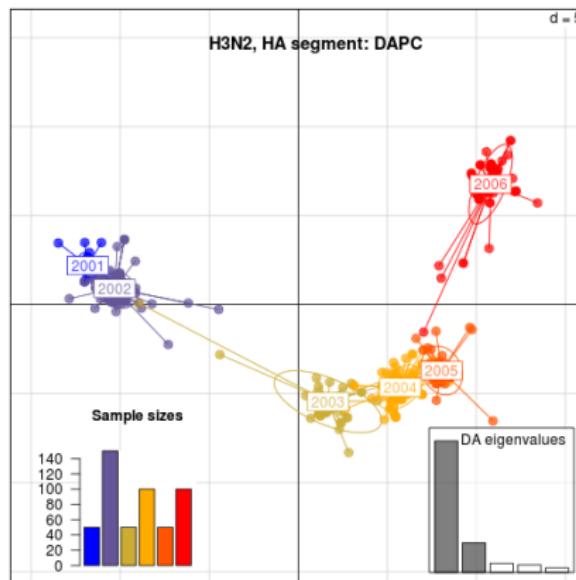
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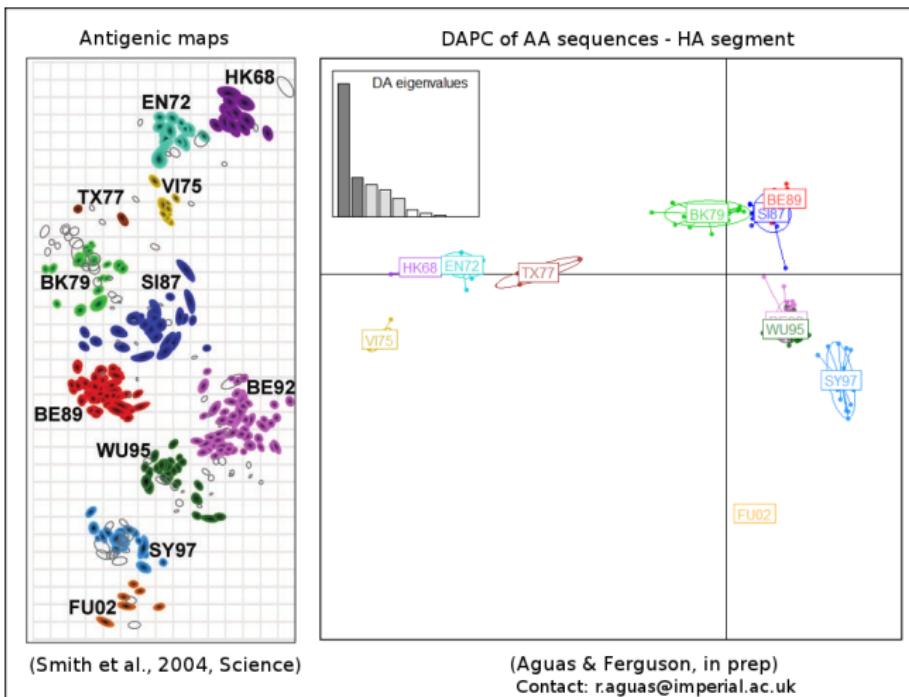
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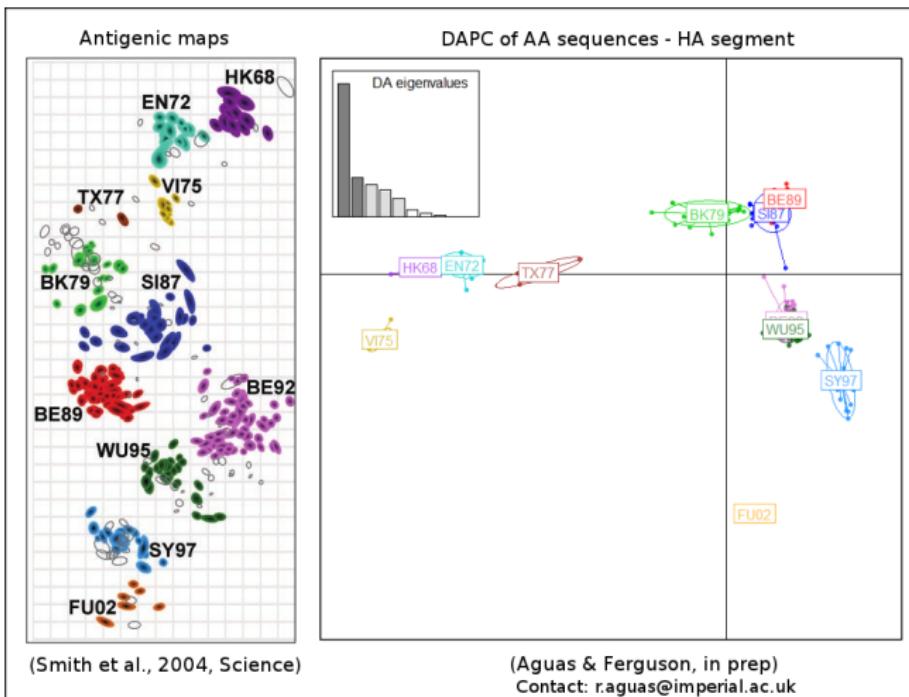
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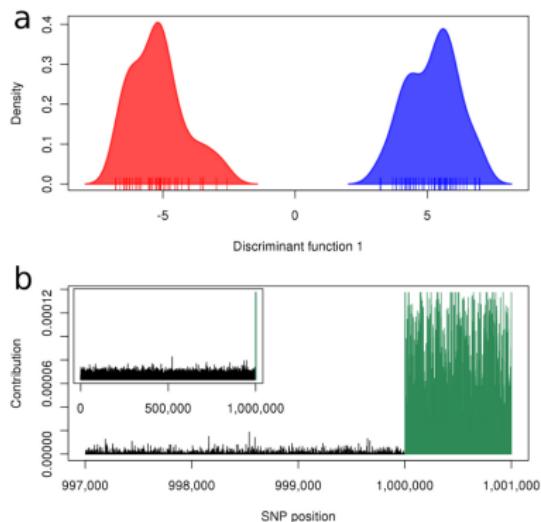
# DAPC to identify structuring alleles

DAPC finds combinations of alleles most differing between groups.

## Simulated data:

(Jombart & Ahmed 2011, *Bioinformatics*)

- 2 clusters, 50 isolates each
- 1,000,000 non structured SNPs
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(i.e. different frequencies between groups)



Possible applications to pathogen GWAS (e.g. SNPs related to antibiotic resistance in bacteria).

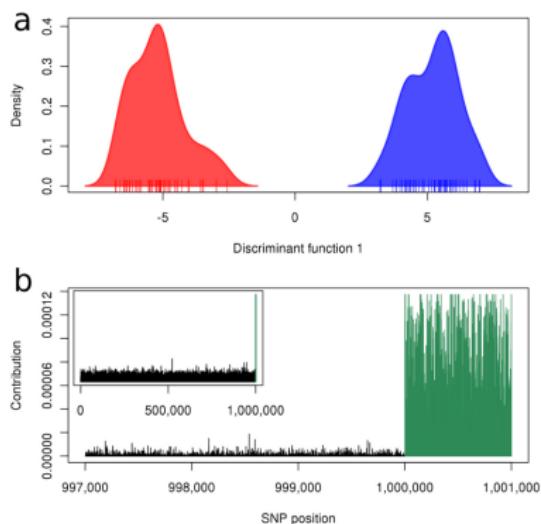
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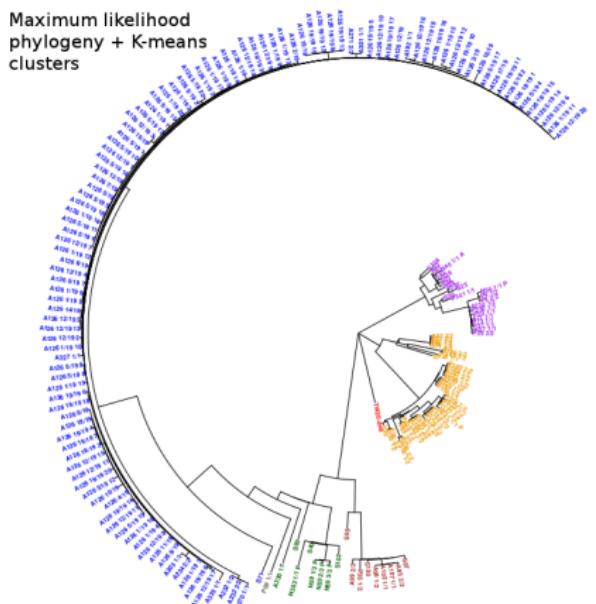
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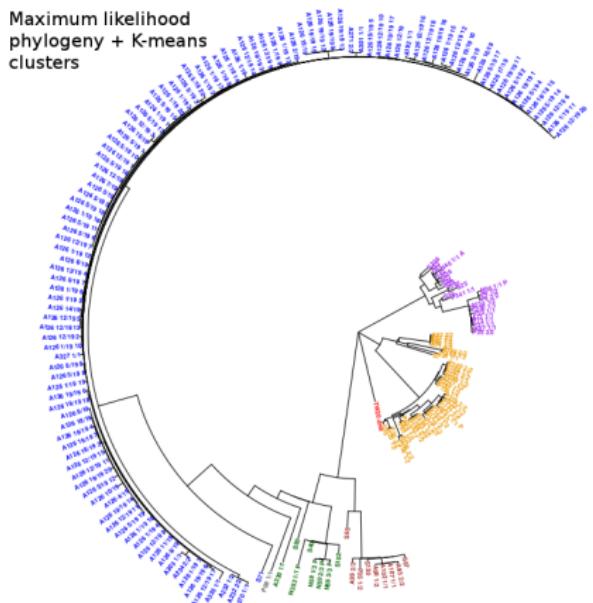


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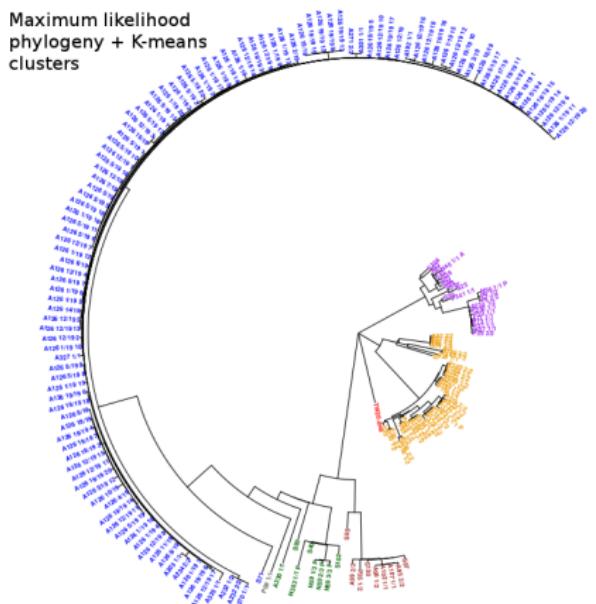


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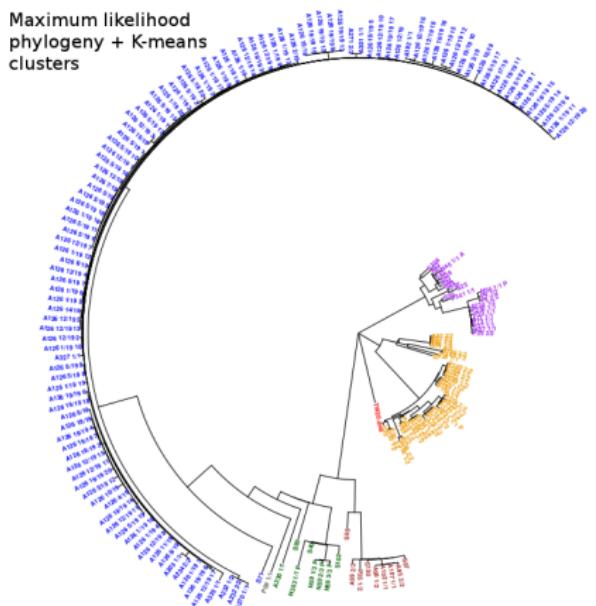


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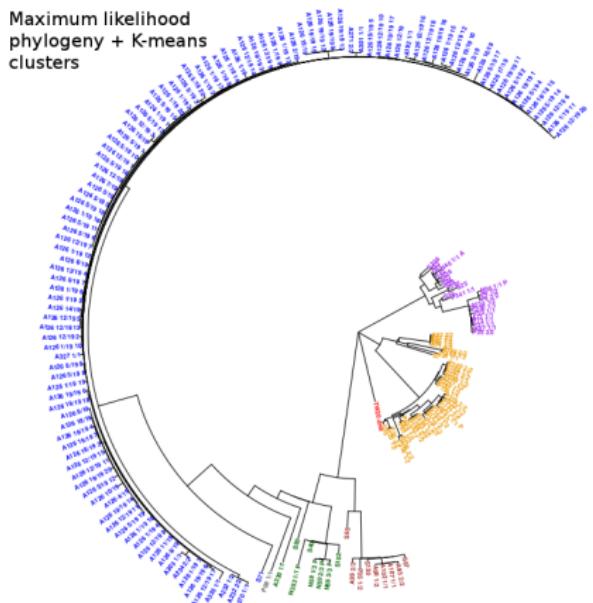


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Multivariate analysis usually not informative on small-scale processes.

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- multivariate analysis used for ~ 50 years in genetics, still an active field for methodological development
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