Allele sharing for mixtures

Consider marker *i* with mixture $M_i = (A_{i1}, \dots, A_{iI})$ and corresponding allele frequencies $p_{i1}, \dots, p_{iI}$. The number of alleles the defendant shares with the mixture for this marker is denoted Z_i. Let $S_i = p_{i1} + \cdots + p_{iI}$ be the sum of the allele frequencies at marker *i*. Then a direct argument gives (calculations assume HWE and H_d)
$P(Z_i = 0) = (1 - s_i)^2$
$P(Z_i = 1) = 2s_i(1 - s_i)$
$P(Z_i = 2) = s_i^2$

Let $Z = Z_1 + \cdots + Z_I$ be the total number of alleles shared and$\boldsymbol{w} = (w_1, \dots, w_I)$ where $w_i = \{0,1,2\}$ one of these values. Then a for a $k = 0, \dots, I, \dots, 2I$,

$$P(Z = k) = \sum_{all\ permutations\ in\ \boldsymbol{w}: \sum w_i = k} \prod_{i=1}^{I} P(Z_i = w_i)$$

Here "all permutations" means all possible ordered combinations of the elements in the vector $\boldsymbol{w}$. Note here that RMNE simplifies to $P(Z = 2I) = \prod_{i=1}^{I} P(Z_i = 2)$.

Implementation:

Consider that we want to calculate $(Z = 16)$, where $I = 10$

1) First, calculate $P(Z_i = w_i)$ for each locus i and $w_i \in \{0,1,2\}$
2) Let $x = \{w_1, w_2, \dots., w_9, w_{10}\}$ a vector with 0,1 or 2 elements
3) Find all the possible sequences such that $sum(x) = 16$ (by recursion) and $length(x) \leq 10$ by recursion
    a. 247 sequences satisfy this.
4) Find the unique sequences where the loci-order does not matter (combinations), and zero is removed
    a. 3 unique sequences satisfies this:
        i. 1 1 1 1 2 2 2 2 2 2
        ii. 1 1 2 2 2 2 2 2 2
        iii. 2 2 2 2 2 2 2 2
5) For each of the 3 sequences:
    a. Permutate the order of the digits in each of the 3 unique sequences to obtain L permutated sequences.
        i. L=210 for i.
        ii. L=360 for ii.

iii. L=45 for iii.

b. From this we obtain a (L x I) matrix A with elements 0, 1 or 2.

c. Construct another (L x I) matrix B with elements

i. $B_{l,i} = P\left(Z_i = A_{l,i}\right)$

d. From this, calculate $S = \sum_{\mathrm{l}} \prod_i B_{l,i}$

i. Note: $S = \sum_{\mathrm{l}} \exp(\sum_i logB_{l,i})$

6) Sum S together for each of the 3 unique sequences to calculate $P(Z = 16)$