Dynamic Estimation of the Zero-Coupon Yield Curve

Research Seminar Statistics

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June 5, 2008

Outline

Multistage Model

- stochastic dynamic equilibrium model
- long-run economic effects of strategic investment decisions
- oligopolistic market structure

- multi-stage stochastic program with recourse
- uncertainty is reflected in future level of demand and strategic investment decisions
- solution as mixed complementarity problem (MCP) with GAMS and PATH solver
- S-adapted open-loop information structure leads to Nash equilibria in each node

Scenario Tree

0.60 nodesep=0pt,labelsep=1pt,nrot=:U [colsep=3.0cm,rowsep=0.0cm] Hn_7

Dn_3

In_8

Bn_1

Jn_9

En_4

Kn_{10}

An_0

Ln_{11}

Fn_5

Mn_{12}

Cn_2

Nn_{13}

Gn_6

On_{14}

$t = 0 \quad t = 1 \quad t = 2 \quad t = 3$

[linewidth=1.5pt]AB50% [linewidth=1.5pt]AC50% [linewidth=1.5pt]BD25%
[linewidth=1.5pt]BE25% [linewidth=1.5pt]CF25% [linewidth=1.5pt]CG25%
[linewidth=1.5pt]DH12.5% [linewidth=1.5pt]DI12.5%

Notation

$i \in \mathcal{I}$	players, firms
$j \in \mathcal{J}$	available technologies
$m \in \mathcal{M}$	market states
p_n	probability to reach a certain node
p^m	probability of market state m
$q_{i,n}^{j,m}$	production quantities
$I_{i,n}^{j}$	investment quantities, for $n \in \{n_0\} \cup \mathcal{T}$
$K_{i,n}^{j}$	available capacity
F_n^j	investment cost
$P_n^m = \alpha_n^m - \beta^m \sum_{i\in\mathcal{I}} \sum_{j\in\mathcal{J}} q_{i,n}^{j,m}$	market equilibrium price
α_n^m	intercept of demand function
β^m	slope of demand function
c_j	variable costs
δ_n	discount factor
ν	salvage value parameter
ρ	depreciation rate

Optimization

$$\max_{q_{i,n}^{j,m}, I_{i,n}^{j}} \sum_{n \in \mathcal{N}} p_n \delta_n \Pi_{i,n} \left(q_{i,n}^{j,m}, I_{i,n}^{j}, K_{i,n}^{j}, Q_n^m \right) + \sum_{n \in \mathcal{S}} p_n \, \delta_n \sum_{j \in \mathcal{J}} K_{i,n}^{j} F_n^j \nu \qquad (1)$$

subject to

$$\begin{aligned}
q_{i,n}^{j,m} - K_{i,n}^{j} &\leq 0 \quad \forall i, j, m, n && (2)\\
Q_n^m - \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} q_{i,n}^{j,m} &= 0 \quad \forall m, n && (3)\\
K_{i,n}^{j} - (1-\rho) K_{i,a(n)}^{j} - I_{i,a(n)}^{j} &= 0 \quad \forall i, j, n && (4)\\
q_{i,n}^{j,m}, K_{i,n}^{j}, I_{i,n}^{j}, Q_n^m &\geq 0 \quad \forall i, j, m, n && (5)
\end{aligned}$$

Installed Capacities

Source: International Energy Agency (2007)

Electricity Load

Source: UCTE (2006)

Electricity Prices

Source: EEX (2006); UCTE (2006)

Market Segments

Price intervals	Average price (Euro/MWh)	Average quantity (MWh)	Number of prices	Percentage of of total prices
$0 \leq p < 20$	12.67	46,111.63	611	6.98%
$20 \leq p < 40$	31.35	54,103.50	3,003	34.28%
$40 \leq p < 60$	49.00	64,806.04	2,626	29.98%
$60 \leq p < 80$	68.46	72,385.56	1,588	18.13%
$80 \leq p < 100$	88.40	75,991.21	665	7.59%
$100 \leq p < \infty$	176.06	76,482.34	266	3.04%

Marginal and Investment costs

	Variable costs (Euro/MWh)	Investment costs (Euro/GW)
Hydro	7.6	3,500
Nuclear	9.5	1,841
Lignite	10.6	1,074
Hard coal	16.1	971
Gas (CCGT)	33.5	460
Oil	44	n/a
Pump	80	n/a

Source: Auer et al. (2006)

Scenario Generation

Production in the initial node $q_{i,0}^{m}$ (in MWh)

	$m=1$	$m=2$	$m=3$	$m=4$	$m=5$	$m=6$
Rwe	10,945.21	13,590.03	17,150.64	20,586.19	21,872.07	21,984.52
EON	10,945.21	13,592.14	17,150.64	20,586.09	21,871.99	21,984.48
Vattenfall	10,940.85	13,590.03	15,273.00	15,273.00	15,273.00	15,273.00
EnBW	10,528.00	10,528.00	10,528.00	10,528.00	10,528.00	10,528.00

Sensitivity Analysis

- Investment quantities in the initial node $I_{i,0}$ (in MW) with $\rho = 0.025$

	$\nu = 0.99$	$\nu = 0.95$	$\nu = 0.9$	$\nu = 0.85$
Vattenfall	5,015.93	4,832.23	4,626.18	4,516.54
EnBW	9,582.64	9,405.55	9,229.64	9,120.00

- Investment quantities in the initial node $I_{i,0}$ (in MW) with $\nu = 0.95$

	$\rho = 0.025$	$\rho = 0.03$	$\rho = 0.035$	$\rho = 0.04$
Vattenfall	4,832.23	4,908.60	4,984.96	5,061.33
EnBW	9,405.55	9,458.19	9,510.83	9,563.47

Motivation

- €€€
- dynamic forecast of zero-coupon yield curves
- useful for bond portfolio management, hedging interest rate risk, identify arbitrage opportunities, ...
- further step towards arbitrage-free model with joint estimation procedure