1 Intro

This is a simple version of the model which shall demonstrate the basic economic effects (Please note that sums are written in a simplified way):

$i \in N$	players, firms
$s \in S$	scenarios
$t \in T$	time
K_i^t	available capacity at time t from technology k for firm i
$q_i^{s,t}$	quantity at time t, technology k, firm i, in market state m and scenario s
I_i^t	investment in technology k, at time t from for firm i
p_s	probability of scenario s
α	demand function intercept in market state m
β	demand function slope in market state m
c	variable costs of technology k
Γ	investment costs in technology k
F	scrap values

$$\max \pi_i(q_i^{s,t}, K_i^t, I_i) = (\alpha^0 - \beta \sum_i q_i^0) q_i^0 - c q_i^0 \tag{1}$$

$$+ \sum_s p_s \left[(\alpha^s - \beta \sum_i q_i^{s,1}) q_i^{s,1} - c q_i^{s,1} \right]$$

$$- \Gamma I_i + F I_i$$

$$\text{s.t.: } q_i^{s,t} - K_i^t \leq 0;\ \forall i, t, s \tag{2}$$

$$K_i^2 - K_i^1 - I_i = 0;\ \forall i \tag{3}$$

$$\alpha^s - \beta \sum_i q_i^{s,1} - PC \leq 0; \forall s \tag{4}$$

$$q_i^{s,t}; K_i^t; I_i \geq 0;\ \forall i, s, t$$

This leads to the following Lagrangian / Hamiltonian:

$$\max L_i(q_i^{s,t}, K_i^t, I_i, \lambda_i^{s,t}, u_i, \psi) = (\alpha^0 - \beta \sum_i q_i^0) q_i^0 - c q_i^0 + \tag{5}$$
$$\sum_s p_s \left[(\alpha^s - \beta \sum_i q_i^{s,1}) q_i^{s,1} - c q_i^{s,1} \right]$$
$$-\Gamma I_i + F I_i$$
$$+\lambda_i^0 (q_i^0 - K_i^0) + \lambda_i^{s,1} (q_i^{s,1} - K_i^1)$$
$$u_i (K_i^1 - K_i^0 - I_i)$$
$$\psi_s (\alpha^s - \beta \sum_i q_i^{s,2} - PC)$$

and the following KKT Conditions:

$$\frac{\partial L(\cdot)}{\partial q_i^{s,t}} = p_s \left[\alpha^s - \beta q_i^{s,t} - \beta \sum_i q_i^{s,t} - c \right] - \lambda_i^{s,t} - \beta \psi_s \leq 0 \tag{6}$$
$$\perp q_i^{s,t} \geq 0;\ \forall i, s$$
$$\frac{\partial L(\cdot)}{\partial \lambda_i^{s,t}} = q_i^{s,t} - K_i^t \leq 0 \perp \lambda_i^{s,t} \geq 0; \tag{7}$$
$$\frac{\partial L(\cdot)}{\partial I_{i,k}} = -\Gamma + \varepsilon F + u_i \leq 0 \perp I_i \geq 0; \forall i \tag{8}$$
$$\frac{\partial L(\cdot)}{\partial u} = -K_i^1 + K_i^0 - I_i = 0 \perp u_i\, free;\ \forall i \tag{9}$$
$$\frac{\partial L(\cdot)}{\partial K_i^1} = \sum_s \lambda_i^{s,1} - u_i \leq 0 \perp K_i^2 \geq 0; \forall i, k \tag{10}$$

Under the assumption of perfect Competition, the first FOC looks as follows:

$$\frac{\partial L(\cdot)}{\partial q_i^{s,t}} = p_s \left[\alpha^s - \beta \sum_i q_i^{s,t} - c \right] - \lambda_i^{s,t} - \beta \psi_s \leq 0 \tag{11}$$
$$\perp q_i^{s,t} \geq 0;\ \forall i, s$$
$$\tag{12}$$