Chapter 8: Experiments with two factors

Introductory Statistics for Engineering Experimentation

Peter R. Nelson, Marie Coffin and Karen A.F. Copeland

Slides by Douglas Bates

Outline

Chapter 8: Experiments with two factors

- Recall from chapter 3 that when we have a response on a continuous scale and two categorical covariates, we can consider the observations as being in the *cells* of a *two-way layout* determined by the levels of the two factors.
- There will always be one experimental factor whose levels are of interest in themselves. The second factor may be a *blocking factor*: a known source of variability for which we are controlling.
- If the factors are `A` and `B` we write the number of levels of the factors as I and J, respectively, and the number of observations in the (i, j)th cell as $n_{ij}, i = 1, \ldots, I; j = 1, \ldots, J$.
- In a *balanced* experiment all the n_{ij} are equal so we simply write the number of observations per cell as n.
- If none of the n_{ij} are zero then we have a *complete layout*, which is a type of *full factorial design*.
- When there is at least one of the $n_{ij} > 1$ the design is said to

Overview of techniques

- We use interaction plots to assess the changes due to the levels of the factors and due to possible interaction.
- In chapter 3 we discussed fitting models of the form

$$\mathcal{Y}_{ijk} = \mu + \alpha_i + \beta_j + \alpha\beta_{ij} + \epsilon_{ijk}, \quad i = 1, \ldots, I;\ j = 1, \ldots, J;\ k = 1, \ldots$$

  when we have replicates. This model allows for interactions (the $\alpha\beta_{ij}$ terms).
- Our first test is on the null hypothesis that all the interactions are zero. If we do not reject this hypothesis then we simplify the model to the *additive model*

$$\mathcal{Y}_{ijk} = \mu + \alpha_i + \beta_j + \epsilon_{ijk}, \quad i = 1, \ldots, I;\ j = 1, \ldots, J;\ k = 1, \ldots, n_{ij}$$

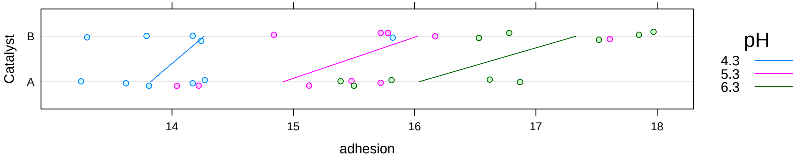
- For an additive model we can perform multiple comparisons on the levels of the factors separately (but only for experimental factors, not for blocking factors) For a model with significant interactions we cannot separate the effects of the factors.
- For an unreplicated design we can fit a model with interactions but cannot do any further analysis. We must fit the additive

R functions used in this chapter

- Interaction plots are obtained with `dotplot`.
- We use `aov` to fit the models, `summary` or `anova` to obtain the analysis of variance table, and `model.tables` to obtain estimates of the cell means or the effects.
- We use `TukeyHSD` to perform multiple comparisons when appropriate.
- We assess residual plots obtained with `plot(fm, which = 1)` and `plot(fm, which = 2)`

Section 8.1, Interaction

- Recall that in an interaction plot a lack of interaction is indicated by more-or-less parallel lines joining the averages.




```
> summary(fm1 <- aov(adhesion ~ cat * factor(pH), adhesion2))
                Df Sum Sq Mean Sq F value    Pr(>F)
cat              1  6.712   6.712 11.4424   0.00246
factor(pH)       2 34.924  17.462 29.7694 3.161e-07
cat:factor(pH)   2  1.003   0.502  0.8553   0.43772
Residuals       24 14.078   0.587
```