1. Problem

What is the derivative of $f(x) = x^7 e^{3.9x}$, evaluated at x = 0.51?

Solution

Using the product rule for $f(x) = g(x) \cdot h(x)$, where $g(x) := x^7$ and $h(x) := e^{3.9x}$, we obtain

$$f'(x) = [g(x) \cdot h(x)]' = g'(x) \cdot h(x) + g(x) \cdot h'(x)$$

$$= 7x^{7-1} \cdot e^{3.9x} + x^7 \cdot e^{3.9x} \cdot 3.9$$

$$= e^{3.9x} \cdot (7x^6 + 3.9x^7)$$

$$= e^{3.9x} \cdot x^6 \cdot (7 + 3.9x).$$

Evaluated at x = 0.51, the answer is

$$e^{3.9 \cdot 0.51} \cdot 0.51^6 \cdot (7 + 3.9 \cdot 0.51) = 1.155964.$$

Thus, rounded to two digits we have f'(0.51) = 1.16.