1. Problem

Theory: Consider a linear regression of y on x. It is usually estimated with which estimation technique (three-letter abbreviation)?

This estimator yields the best linear unbiased estimator (BLUE) under the assumptions of the Gauss-Markov theorem. Which of the following properties are required for the errors of the linear regression model under these assumptions?

 $independent \ / \ zero \ expectation \ / \ normally \ distributed \ / \ identically \ distributed \ / \ homoscedastic$

Application: Using the data provided in linreg.csv estimate a linear regression of y on x. What are the estimated parameters?

Intercept:

Slope:

In terms of significance at 5% level:

 ${\tt x}$ and ${\tt y}$ are not significantly correlated / ${\tt y}$ increases significantly with ${\tt x}$ / ${\tt y}$ decreases significantly with ${\tt x}$

Interpretation: Consider various diagnostic plots for the fitted linear regression model. Do you think the assumptions of the Gauss-Markov theorem are fulfilled? What are the consequences?

Code: Please upload your code script that reads the data, fits the regression model, extracts the quantities of interest, and generates the diagnostic plots.

Solution

Theory: Linear regression models are typically estimated by ordinary least squares (OLS). The Gauss-Markov theorem establishes certain optimality properties: Namely, if the errors have expectation zero, constant variance (homoscedastic), no autocorrelation and the regressors are exogenous and not linearly dependent, the OLS estimator is the best linear unbiased estimator (BLUE).

Application: The estimated coefficients along with their significances are reported in the summary of the fitted regression model, showing that x and y are not significantly correlated (at 5% level).

```
Call:
lm(formula = y ~ x, data = d)
```

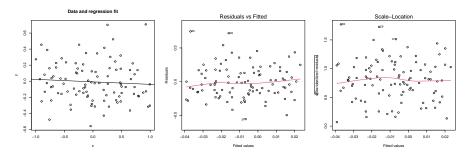
Residuals:

```
Min 1Q Median 3Q Max -0.55258 -0.15907 -0.02757 0.15782 0.74504
```

Coefficients:

```
Residual standard error: 0.2425 on 98 degrees of freedom Multiple R-squared: 0.004811, Adjusted R-squared: -0.005344 F-statistic: 0.4738 on 1 and 98 DF, p-value: 0.4929
```

Interpretation: Considering the visualization of the data along with the diagnostic plots suggests that the assumptions of the Gauss-Markov theorem are reasonably well fulfilled.



Code: The analysis can be replicated in R using the following code.

```
## data
d <- read.csv("linreg.csv")
## regression
m <- lm(y ~ x, data = d)
summary(m)
## visualization
plot(y ~ x, data = d)
abline(m)
## diagnostic plots
plot(m)</pre>
```