1. **Problem**
For the matrix

$$A = \begin{pmatrix} 16 & 4 & 16 & -4 \\ 4 & 5 & 6 & -9 \\ 16 & 6 & 33 & -28 \\ -4 & -9 & -28 & 58 \end{pmatrix}.$$

compute the matrix $L = (\ell_{ij})_{1 \le i,j \le 4}$ from the Cholesky decomposition $A = LL^\top$.
Which of the following statements are true?

(a) $\ell_{41} = -1$
(b) $\ell_{44} < 4$
(c) $\ell_{22} = -1$
(d) $\ell_{11} > 0$
(e) $\ell_{32} \le 1$

Solution
The decomposition yields

$$L = \begin{pmatrix} 4 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 4 & 1 & 4 & 0 \\ -1 & -4 & -5 & 4 \end{pmatrix}$$

and hence:

(a) True. $\ell_{41} = -1$
(b) False. $\ell_{44} = 4 \not< 4$
(c) False. $\ell_{22} = 2 \neq -1$
(d) True. $\ell_{11} = 4$
(e) True. $\ell_{32} = 1$