

1. **Problem**

Compute the Hessian of the function

$$f(x_1, x_2) = 7x_1^2 + 5x_1x_2 + 3x_2^2$$

at $(x_1, x_2) = (1, 4)$. What is the value of the upper left element?

- (a) 6
- (b) 7
- (c) 14
- (d) 5
- (e) -19

Solution

The first-order partial derivatives are

$$f'_1(x_1, x_2) = 14x_1 + 5x_2$$

$$f'_2(x_1, x_2) = 5x_1 + 6x_2$$

and the second-order partial derivatives are

$$f''_{11}(x_1, x_2) = 14$$

$$f''_{12}(x_1, x_2) = 5$$

$$f''_{21}(x_1, x_2) = 5$$

$$f''_{22}(x_1, x_2) = 6$$

Therefore the Hessian is

$$f''(x_1, x_2) = \begin{pmatrix} 14 & 5 \\ 5 & 6 \end{pmatrix}$$

independent of x_1 and x_2 . Thus, the upper left element is: $f''_{11}(1, 4) = 14$.

- (a) False
- (b) False
- (c) True
- (d) False
- (e) False