A model may be fitted to data to estimate parameters of interest that can´t be derived empirically, e.g. mortality from catch data, or just as importantly to look at residuals in order to explore patterns and trends in the data. In this regard simple methods can be as useful as more complicated ones, since they are easier to apply, explain to others and can quickly provide insight into the data and processes being modeled. Important steps before setting up and running a more complicated model.

For example in stock assessment, mortality is an important parameter of interest including how it may vary over time and by age. A simple method to explore such patterns in total mortality (Z) is to look at the log catch ratios. Catch-at-age can be derived from numbers.at-age assuming that a constant proportion s caught depending on the fishing effort i.e.

$$C_t = qEN_t$$

number-at-age next year are given by

$$N_{t+1} = N_t\, e^{-Zt}$$

and catch-at-age next year by

$$C_{t+1} = qEN_t\, e^{-Zt}$$

taking the log catch ratio allows Z to be estimated

$$log(C_{t+1}/C_{t+1}) = -Z_t$$

This assumes that q and effort is the same both at age and year; more than likely violated but hopefully not by too much so that any signals are obscured.

```
load("C:\\Stuff_\\Papers\\Submitted\\MovingTargets\\data\\albFLStock.RData")
pc       <-as.data.frame(catch.n(alb)[,-15])[,c("age","year","data")]
pc$cohort<-pc$year-pc$age
pc       <-pc[,c("age","year","data")]
pc2      <-pc
pc2$age  <-pc2$age-1
names(pc2)[3]<-"data2"
pc       <-merge(pc,pc2)
pc$Z     <-log(pc$data/pc$data2)
pc$decade<-decade(pc$year)


ggplot(pc)+geom_point(aes(age,Z))+stat_smooth(aes(age,Z))+facet_wrap(~decade)

p<-ggplot(as.data.frame(harvest(alb)))
p$data$decade<-decade(p$data$year)
p+geom_point(aes(age,data))+stat_smooth(aes(age,data))+facet_wrap(~decade)
```

Catch curves, where log(catch) is plotted against age, are a simple method to explore patterns in total mortality (Z). It is assumed that

$$N_t = N_0\, e^{-Zt}$$

Where N_0 is the initial population size of a cohort N_t are numbers at time or age t and Z is the instantaneous mortality rate in each time step.

Catch is then a function of numbers assuming that a constant proportion of the population (q_a) is vulnerable to the fishery

$$C_t = q_t N_0\, e^{-Zt}$$

Taking logs of each side gives a linear relationship if catchability (q) is constant over ages and time

$$log(C_t) = log(EqN_0) – Zt$$

The slope (Z) therefore provides an estimate of total mortality-at-age within a year class.

A similar relationship can also be derived for catch per unit effort (U) since if catchability is assumed constant then a constant proportion of the stock is being caught i.e.

$$C_t = Eq_t N_0\, e^{-Zt}$$

$$log(C_t/E) = log(qN_0) – Zt$$

However, catchability is unlikely to be constant, for example it may increase initially as fish recruit to the fishery. In this case the selection pattern will be flat topped and the catch curve will show an ascending left limb, a domed middle portion, and a descending right limb. The ascending left limb represents age-classes of fish that are not fully recruited to the fishery and the width of the domed middle section provides an indication of the rate of recruitment to the fishery. Once the right hand limb is identified its slope can provide an estimate of Z.

```
Age <-0:15
E   <-0.2
M   <-0.1

## constant q
q   <-rep(1,16)
F   <-E*q
Z   <-M+F
N   <-1000*exp(-cumsum(Z))
C   <-N*F/Z*(1-exp(-Z))

plot(Age,log(C),type="b")
lm(log(C)~Age)

## Flat topped
q   <-c(seq(0.1,1.0,.2),rep(1.0,11))
F   <-E*q
Z   <-M+F
N   <-1000*exp(-cumsum(Z))
C   <-N*F/Z*(1-exp(-Z))

plot(Age,log(C),type="b")
lm(log(C)[-(1:5)]~T[-(1:5)])

## Domed
q   <-c(seq(0.1,1.0,.1),rep(1.0,6),seq(1.0,.1,-.1))
F   <-E*q
Z   <-M+F
N   <-1000*exp(-cumsum(Z))
C   <-N*F/Z*(1-exp(-Z))
```

```
plot(Age,log(C),type="b")
```

If the analysis is done by cohort then trends over time may be evaluated. However, changes in catchability-at-age and effort between years may obscure trends.

Cotter (2004) introduced the term ‘year-class curve’ to distinguish between a ‘catch curve’ which is applied to a single catch or to the total catch from a single season.

$$log(U_{a,c}) = ln(k)R_{c,0}+Za+e_{a,c}$$

Year-class curves do allow you to estimate variation in recruitment by cohort for example but not absolute recruitment if you don´t know what units k is in. Additional terms may be included to allow fishing mortality to vary by age and year.

```
ggplot(c.[c.$age<10,])+geom_point(aes(age,log(data)))+stat_smooth(aes(age,log(data)))

coef(lm(log(data)~-1+as.factor(cohort)+age,data=c.[c.$age %in% 4:8,]))
```

Examples of such terms given by Cotter et al. (2004) are

i) Varying selectivity or discarding-at-age can be modelled using a quadratic term (a^2) or a log transformation (log[a+1]), thereby allowing higher or lower Z at older ages respectively.

ii) One off events, such as a change in mesh size regulations can be modelled by the addition of a factor identifying the period before and after the change.

iii) Changes of Z over time, these are slightly trickier since year, cohort and age are related you can´t fit year and age unless the model is constrained in some way or you have multiple CPUE within a year. A simple trick is rather than using year use blocks of years.

Models can be used in two main way, i.e. to estimate a quantity of interested such as Z or else to remove the obvious quantities and investigate further the things that the model doesn´t explain. For example the plot in figure x, suggests that fishing mortality at ages 4 to 8 is similar; therefore if you wanted an index of F then you could use the slope for ages 4 to 8. Also if you wished to fit a separable VPA or a VPA-ADAPT then the plot gives you a good indication of the selection pattern, i.e. constant selectivity after age 4 with an increase at older ages. Fitting an additional term isn´t really needed. While if there had been a mesh change then plotting the residuals by age, for a common slope would be expected to show a pattern confirming that the regulation had worked. To estimate changes in Z over time a factor corresponding to pairs of years can be fitted e.g.

```
ggplot(c.[c.$age<10,])+geom_point(aes(age,log(data)))+stat_smooth(aes(age,log(data)))
+facet_wrap(~decade(year))

biYr<-function(year){
            res<-year - (year %% 2)
            return(factor(res,seq(min(res),max(res),2)))}

minMax<-function(x) (x-min(x))/diff(range(x))

t.<-coef(lm(log(data)~-1+as.factor(cohort)+age:biYr(year),data=c.[c.$age %in% 4:8,]))

range(ple4,c("minfbar","maxfbar"))<-c(4,8)
plot(seq(1956,2008,2),minMax(-t.[57:83]))
lines(as.numeric(dimnames(fbar(ple4))$year),minMax(c(fbar(ple4))))
```

This shows the trend in F estimated from the year class curve is similar to the VPA results, while the plot in figure x, allows you to estimate when fish are fully recruited