Testing hierarchical pathway kinetics with residue data on cyantraniliprole

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Contents

Introduction

The purpose of this document is to test demonstrate how nonlinear hierarchical models (NLHM) based on the parent degradation models SFO, FOMC, DFOP and HS, with serial formation of two or more metabolites can be fitted with the mkin package.

It was assembled in the course of work package 1.2 of Project Number 173340 (Application of nonlinear hierarchical models to the kinetic evaluation of chemical degradation data) of the German Environment Agency carried out in 2022 and 2023.

The mkin package is used in version 1.2.3 which is currently under development. The newly introduced functionality that is used here is a simplification of excluding random effects for a set of fits based on a related set of fits with a reduced model, and the documentation of the starting parameters of the fit, so that all starting parameters of `saem` fits are now listed in the summary. The `saemix` package is used as a backend for fitting the NLHM, but is also loaded to make the convergence plot function available.

This document is processed with the `knitr` package, which also provides the `kable` function that is used to improve the display of tabular data in R markdown documents. For parallel processing, the `parallel` package is used.

```r
library(mkin)
library(knitr)
library(saemix)
library(parallel)
n_cores <- detectCores()

# We need to start a new cluster after defining a compiled model that is
# saved as a DLL to the user directory, therefore we define a function
# This is used again after defining the pathway model
start_cluster <- function(n_cores) {
  if (Sys.info()["sysname"] == "Windows") {
    ret <- makePSOCKcluster(n_cores)
  } else {
    ret <- makeForkCluster(n_cores)
  }
  return(ret)
}
cl <- start_cluster(n_cores)
```

Test data

The example data are taken from the final addendum to the DAR from 2014 and are distributed with the mkin package. Residue data and time step normalisation factors are read in using the function `read_spreadsheet` from the mkin package. This function also performs the time step normalisation.

```
data_file <- system.file(
  "testdata", "cyantraniliprole_soil_efsa_2014.xlsx",
  package = "mkin")
cyan_ds <- read_spreadsheet(data_file, parent_only = FALSE)
```

The following tables show the covariate data and the 5 datasets that were read in from the spreadsheet file.

```
pH <- attr(cyan_ds, "covariates")
kable(pH, caption = "Covariate data")
```

Table 1: Covariate data

	pH
Nambsheim	7.90
Tama	6.20
Gross-Umstadt	7.04
Sassafras	4.62
Lleida	8.05

```
for (ds_name in names(cyan_ds)) {
  print(
    kable(mkin_long_to_wide(cyan_ds[[ds_name]]),
      caption = paste("Dataset", ds_name),
      booktabs = TRUE, row.names = FALSE))
    cat("\n\\clearpage\n")
}
```

Table 2: Dataset Nambsheim

time	cyan	JCZ38	J9C38	JSE76	J9Z38
0.000000	105.79	NA	NA	NA	NA
3.210424	77.26	7.92	11.94	5.58	9.12
7.490988	57.13	15.46	16.58	12.59	11.74
17.122259	37.74	15.98	13.36	26.05	10.77
23.543105	31.47	6.05	14.49	34.71	4.96
43.875788	16.74	6.07	7.57	40.38	6.52
67.418893	8.85	10.34	6.39	30.71	8.90
107.014116	5.19	9.61	1.95	20.41	12.93
129.487080	3.45	6.18	1.36	21.78	6.99
195.835832	2.15	9.13	0.95	16.29	7.69
254.693596	1.92	6.92	0.20	13.57	7.16
321.042348	2.26	7.02	NA	11.12	8.66
383.110535	NA	5.05	NA	10.64	5.56
0.000000	105.57	NA	NA	NA	NA
3.210424	78.88	12.77	11.94	5.47	9.12
7.490988	59.94	15.27	16.58	13.60	11.74
17.122259	39.67	14.26	13.36	29.44	10.77
23.543105	30.21	16.07	14.49	35.90	4.96
43.875788	18.06	9.44	7.57	42.30	6.52
67.418893	8.54	5.78	6.39	34.70	8.90
107.014116	7.26	4.54	1.95	23.33	12.93
129.487080	3.60	4.22	1.36	23.56	6.99
195.835832	2.84	3.05	0.95	16.21	7.69
254.693596	2.00	2.90	0.20	15.53	7.16
321.042348	1.79	0.94	NA	9.80	8.66
383.110535	NA	1.82	NA	9.49	5.56

Table 3: Dataset Tama

time	cyan	JCZ38	J9Z38	JSE76
0.000000	106.14	NA	NA	NA
2.400833	93.47	6.46	2.85	NA
5.601943	88.39	10.86	4.65	3.85
12.804442	72.29	11.97	4.91	11.24
17.606108	65.79	13.11	6.63	13.79
32.811382	53.16	11.24	8.90	23.40
50.417490	44.01	11.34	9.98	29.56
80.027761	33.23	8.82	11.31	35.63
96.833591	40.68	5.94	8.32	29.09
146.450803	20.65	4.49	8.72	36.88
190.466072	17.71	4.66	11.10	40.97
240.083284	14.86	2.27	11.62	40.11
286.499386	12.02	NA	10.73	42.58
0.000000	109.11	NA	NA	NA
2.400833	96.84	5.52	2.04	2.02
5.601943	85.29	9.65	2.99	4.39
12.804442	73.68	12.48	5.05	11.47
17.606108	64.89	12.44	6.29	15.00
32.811382	52.27	10.86	7.65	23.30
50.417490	42.61	10.54	9.37	31.06
80.027761	34.29	10.02	9.04	37.87
96.833591	30.50	6.34	8.14	33.97
146.450803	19.21	6.29	8.52	26.15
190.466072	17.55	5.81	9.89	32.08
240.083284	13.22	5.99	10.79	40.66
286.499386	11.09	6.05	8.82	42.90

Table 4: Dataset Gross-Umstadt

time	cyan	JCZ38	J9Z38	JSE76
0.0000000	103.03	NA	NA	NA
2.1014681	87.85	4.79	3.26	0.62
4.9034255	77.35	8.05	9.89	1.32
10.5073404	69.33	9.74	12.32	4.74
21.0146807	55.65	14.57	13.59	9.84
31.5220211	49.03	14.66	16.71	12.32
42.0293615	41.86	15.97	13.64	15.53
63.0440422	34.88	18.20	14.12	22.02
84.0587230	28.26	15.64	14.06	25.60
0.0000000	104.05	NA	NA	NA
2.1014681	85.25	2.68	7.32	0.69
4.9034255	77.22	7.28	8.37	1.45
10.5073404	65.23	10.73	10.93	4.74
21.0146807	57.78	12.29	14.80	9.05
31.5220211	54.83	14.05	12.01	11.05
42.0293615	45.17	12.12	17.89	15.71
63.0440422	34.83	12.90	15.86	22.52
84.0587230	26.59	14.28	14.91	28.48
0.0000000	104.62	NA	NA	NA
0.8145225	97.21	NA	4.00	NA
1.9005525	89.64	3.59	5.24	NA
4.0726125	87.90	4.10	9.58	NA
8.1452251	86.90	5.96	9.45	NA
12.2178376	74.74	7.83	15.03	5.33
16.2904502	74.13	8.84	14.41	5.10
24.4356753	65.26	11.84	18.33	6.71
32.5809004	57.70	12.74	19.93	9.74
0.0000000	101.94	NA	NA	NA
0.8145225	99.94	NA	NA	NA
1.9005525	94.87	NA	4.56	NA
4.0726125	86.96	6.75	6.90	NA
8.1452251	80.51	10.68	7.43	2.58
12.2178376	78.38	10.35	9.46	3.69
16.2904502	70.05	13.73	9.27	7.18
24.4356753	61.28	12.57	13.28	13.19
32.5809004	52.85	12.67	12.95	13.69

Table 5: Dataset Sassafras

time	cyan	JCZ38	J9Z38	JSE76
0.000000	102.17	NA	NA	NA
2.216719	95.49	1.11	0.10	0.83
5.172343	83.35	6.43	2.89	3.30
11.083593	78.18	10.00	5.59	0.81
22.167186	70.44	17.21	4.23	1.09
33.250779	68.00	20.45	5.86	1.17
44.334371	59.64	24.64	3.17	2.72
66.501557	50.73	27.50	6.19	1.27
88.668742	45.65	32.77	5.69	4.54
0.000000	100.43	NA	NA	NA
2.216719	95.34	3.21	0.14	0.46
5.172343	84.38	5.73	4.75	0.62
11.083593	78.50	11.89	3.99	0.73
22.167186	71.17	17.28	4.39	0.66
33.250779	59.41	18.73	11.85	2.65
44.334371	64.57	22.93	5.13	2.01
66.501557	49.08	33.39	5.67	3.63
88.668742	40.41	39.60	5.93	6.17

Table 6: Dataset Lleida

time	cyan	JCZ38	J9Z38	JSE76
0.000000	102.71	NA	NA	NA
2.821051	79.11	5.70	8.07	0.97
6.582451	70.03	7.17	11.31	4.72
14.105253	50.93	10.25	14.84	9.95
28.210505	33.43	10.40	14.82	24.06
42.315758	24.69	9.75	16.38	29.38
56.421010	22.99	10.06	15.51	29.25
84.631516	14.63	5.63	14.74	31.04
112.842021	12.43	4.17	13.53	33.28
0.000000	99.31	NA	NA	NA
2.821051	82.07	6.55	5.60	1.12
6.582451	70.65	7.61	8.01	3.21
14.105253	53.52	11.48	10.82	12.24
28.210505	35.60	11.19	15.43	23.53
42.315758	34.26	11.09	13.26	27.42
56.421010	21.79	4.80	18.30	30.20
84.631516	14.06	6.30	16.35	32.32
112.842021	11.51	5.57	12.64	32.51

Parent only evaluations

As the pathway fits have very long run times, evaluations of the parent data are performed first, in order to determine for each hierarchical parent degradation model which random effects on the degradation model parameters are ill-defined.

```
cyan_sep_const <- mmkin(c("SFO", "FOMC", "DFOP", "SFORB", "HS"),
  cyan_ds, quiet = TRUE, cores = n_cores)
cyan_sep_tc <- update(cyan_sep_const, error_model = "tc")
cyan_saem_full <- mhmkin(list(cyan_sep_const, cyan_sep_tc))
status(cyan_saem_full) |> kable()
```

	const	tc
SFO	OK	OK
FOMC	OK	OK
DFOP	OK	OK
SFORB	OK	OK
HS	OK	OK

All fits converged successfully.

```
illparms(cyan_saem_full) |> kable()
```

	const	tc
SFO	sd(cyan_0)	sd(cyan_0)
FOMC	sd(log_beta)	sd(cyan_0)
DFOP	sd(cyan_0)	sd(cyan_0), sd(log_k1)
SFORB	sd(cyan_free_0)	sd(cyan_free_0), sd(log_k_cyan_free_bound)
HS	sd(cyan_0)	sd(cyan_0)

In almost all models, the random effect for the initial concentration of the parent compound is ill-defined. For the biexponential models DFOP and SFORB, the random effect of one additional parameter is ill-defined when the two-component error model is used.

```
anova(cyan_saem_full) |> kable(digits = 1)
```

	npar	AIC	BIC	Lik
SFO const	5	833.9	832.0	-412.0
SFO tc	6	831.6	829.3	-409.8
FOMC const	7	709.1	706.4	-347.6
FOMC tc	8	689.2	686.1	-336.6
DFOP const	9	703.0	699.5	-342.5
SFORB const	9	701.3	697.8	-341.7
HS const	9	718.6	715.1	-350.3
DFOP tc	10	703.1	699.2	-341.6
SFORB tc	10	700.1	696.2	-340.1
HS tc	10	716.7	712.8	-348.3

Model comparison based on AIC and BIC indicates that the two-component error model is preferable for all parent models with the exception of DFOP. The lowest AIC and BIC values are are obtained with the FOMC model, followed by SFORB and DFOP.

```
stopCluster(cl)
```

Pathway fits

Evaluations with pathway established previously

To test the technical feasibility of coupling the relevant parent degradation models with different transformation pathway models, a list of `mkinmod` models is set up below. As in the EU evaluation, parallel formation of metabolites JCZ38 and J9Z38 and secondary formation of metabolite JSE76 from JCZ38 is used.

```r
if (!dir.exists("cyan_dlls")) dir.create("cyan_dlls")
cyan_path_1 <- list(
  sfo_path_1 = mkinmod(
    cyan = mkinsub("SFO", c("JCZ38", "J9Z38")),
    JCZ38 = mkinsub("SFO", "JSE76"),
    J9Z38 = mkinsub("SFO"),
    JSE76 = mkinsub("SFO"), quiet = TRUE,
    name = "sfo_path_1", dll_dir = "cyan_dlls", overwrite = TRUE),
  fomc_path_1 = mkinmod(
    cyan = mkinsub("FOMC", c("JCZ38", "J9Z38")),
    JCZ38 = mkinsub("SFO", "JSE76"),
    J9Z38 = mkinsub("SFO"),
    JSE76 = mkinsub("SFO"), quiet = TRUE,
    name = "fomc_path_1", dll_dir = "cyan_dlls", overwrite = TRUE),
  dfop_path_1 = mkinmod(
    cyan = mkinsub("DFOP", c("JCZ38", "J9Z38")),
    JCZ38 = mkinsub("SFO", "JSE76"),
    J9Z38 = mkinsub("SFO"),
    JSE76 = mkinsub("SFO"), quiet = TRUE,
    name = "dfop_path_1", dll_dir = "cyan_dlls", overwrite = TRUE),
  sforb_path_1 = mkinmod(
    cyan = mkinsub("SFORB", c("JCZ38", "J9Z38")),
    JCZ38 = mkinsub("SFO", "JSE76"),
    J9Z38 = mkinsub("SFO"),
    JSE76 = mkinsub("SFO"), quiet = TRUE,
    name = "sforb_path_1", dll_dir = "cyan_dlls", overwrite = TRUE),
  hs_path_1 = mkinmod(
    cyan = mkinsub("HS", c("JCZ38", "J9Z38")),
    JCZ38 = mkinsub("SFO", "JSE76"),
    J9Z38 = mkinsub("SFO"),
    JSE76 = mkinsub("SFO"), quiet = TRUE,
    name = "hs_path_1", dll_dir = "cyan_dlls", overwrite = TRUE)
)
cl_path_1 <- start_cluster(n_cores)
```

To obtain suitable starting values for the NLHM fits, separate pathway fits are performed for all datasets.

```r
f_sep_1_const <- mmkin(
  cyan_path_1,
  cyan_ds,
  error_model = "const",
  cluster = cl_path_1,
  quiet = TRUE)
status(f_sep_1_const) |> kable()
```

	Nambsheim	Tama	Gross-Umstadt	Sassafras	Lleida
sfo_path_1	OK	OK	OK	OK	OK
fomc_path_1	OK	OK	OK	OK	OK
dfop_path_1	OK	OK	OK	OK	OK

	Nambsheim	Tama	Gross-Umstadt	Sassafras	Lleida
sforb_path_1	OK	OK	OK	OK	OK
hs_path_1	C	C	C	C	C

```
f_sep_1_tc <- update(f_sep_1_const, error_model = "tc")
status(f_sep_1_tc) |> kable()
```

	Nambsheim	Tama	Gross-Umstadt	Sassafras	Lleida
sfo_path_1	OK	OK	OK	OK	OK
fomc_path_1	OK	OK	OK	OK	C
dfop_path_1	OK	OK	OK	OK	OK
sforb_path_1	OK	C	OK	OK	OK
hs_path_1	C	OK	C	OK	OK

Most separate fits converged successfully. The biggest convergence problems are seen when using the HS model with constant variance.

For the hierarchical pathway fits, those random effects that could not be quantified in the corresponding parent data analyses are excluded.

In the code below, the output of the `illparms` function for the parent only fits is used as an argument `no_random_effect` to the `mhmkin` function. The possibility to do so was introduced in mkin version `1.2.2` which is currently under development.

```
f_saem_1 <- mhmkin(list(f_sep_1_const, f_sep_1_tc),
  no_random_effect = illparms(cyan_saem_full),
  cluster = cl_path_1)
```

```
status(f_saem_1) |> kable()
```

	const	tc
sfo_path_1	Fth, FO	Fth, FO
fomc_path_1	OK	Fth, FO
dfop_path_1	Fth, FO	Fth, FO
sforb_path_1	Fth, FO	Fth, FO
hs_path_1	Fth, FO	Fth, FO

The status information from the individual fits shows that all fits completed successfully. The matrix entries Fth and FO indicate that the Fisher Information Matrix could not be inverted for the fixed effects (theta) and the random effects (Omega), respectively. For the affected fits, ill-defined parameters cannot be determined using the `illparms` function, because it relies on the Fisher Information Matrix.

```
illparms(f_saem_1) |> kable()
```

	const	tc
sfo_path_1	NA	NA
fomc_path_1	sd(log_k_J9Z38), sd(f_cyan_ilr_2), sd(f_JCZ38_qlogis)	NA
dfop_path_1	NA	NA
sforb_path_1	NA	NA
hs_path_1	NA	NA

The model comparison below suggests that the pathway fits using DFOP or SFORB for the parent compound provide the best fit.

```r
anova(f_saem_1) |> kable(digits = 1)
```

	npar	AIC	BIC	Lik
sfo_path_1 const	16	2692.8	2686.6	-1330.4
sfo_path_1 tc	17	2657.7	2651.1	-1311.9
fomc_path_1 const	18	2427.8	2420.8	-1195.9
fomc_path_1 tc	19	2423.4	2416.0	-1192.7
dfop_path_1 const	20	2403.2	2395.4	-1181.6
sforb_path_1 const	20	2401.4	2393.6	-1180.7
hs_path_1 const	20	2427.3	2419.5	-1193.7
dfop_path_1 tc	20	2398.0	2390.2	-1179.0
sforb_path_1 tc	20	2399.8	2392.0	-1179.9
hs_path_1 tc	21	2422.3	2414.1	-1190.2

For these two parent model, successful fits are shown below. Plots of the fits with the other parent models are shown in the Appendix.

```r
plot(f_saem_1[["dfop_path_1", "tc"]])
```
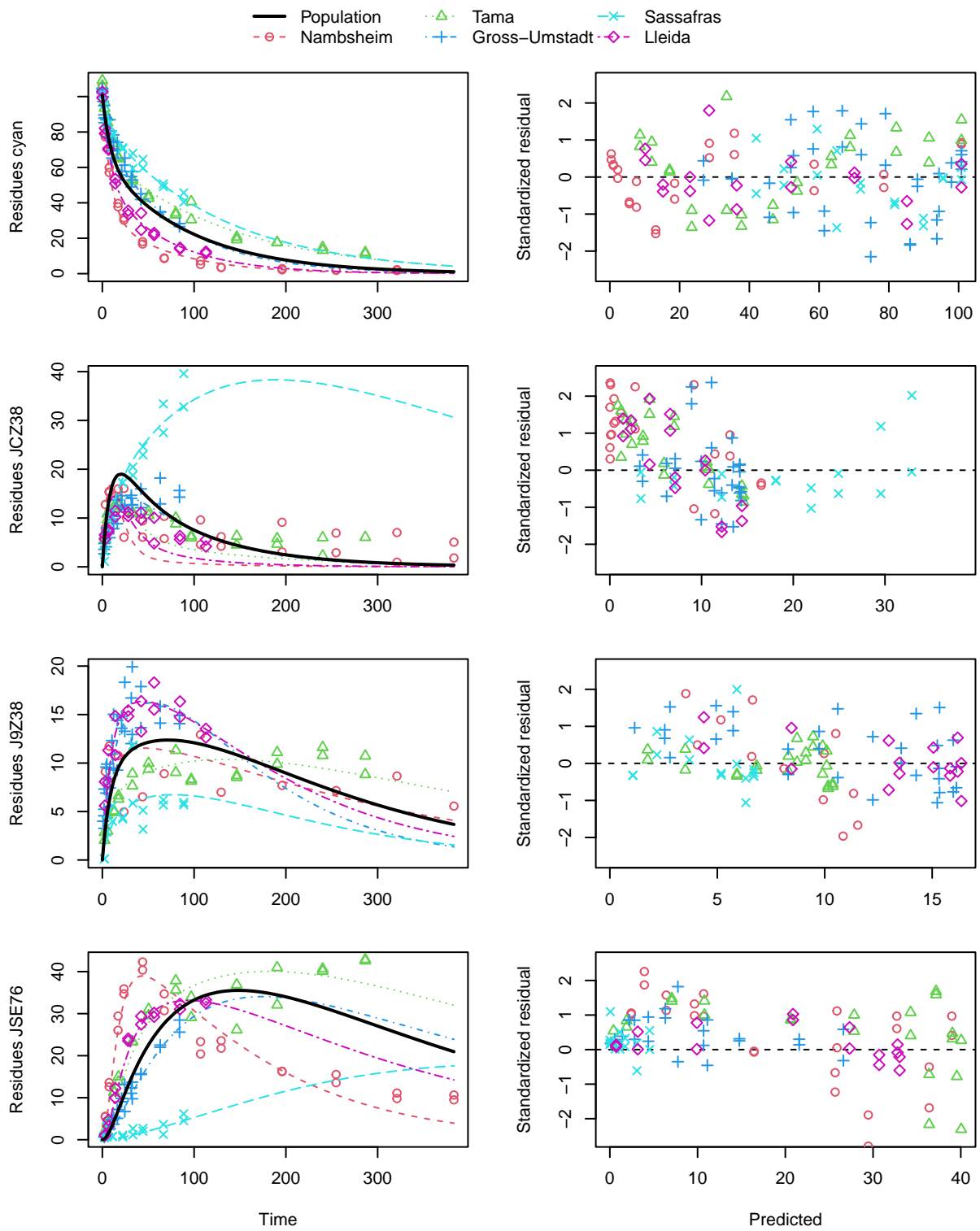
Figure 1: DFOP pathway fit with two-component error

```
plot(f_saem_1[["sforb_path_1", "tc"]])
```
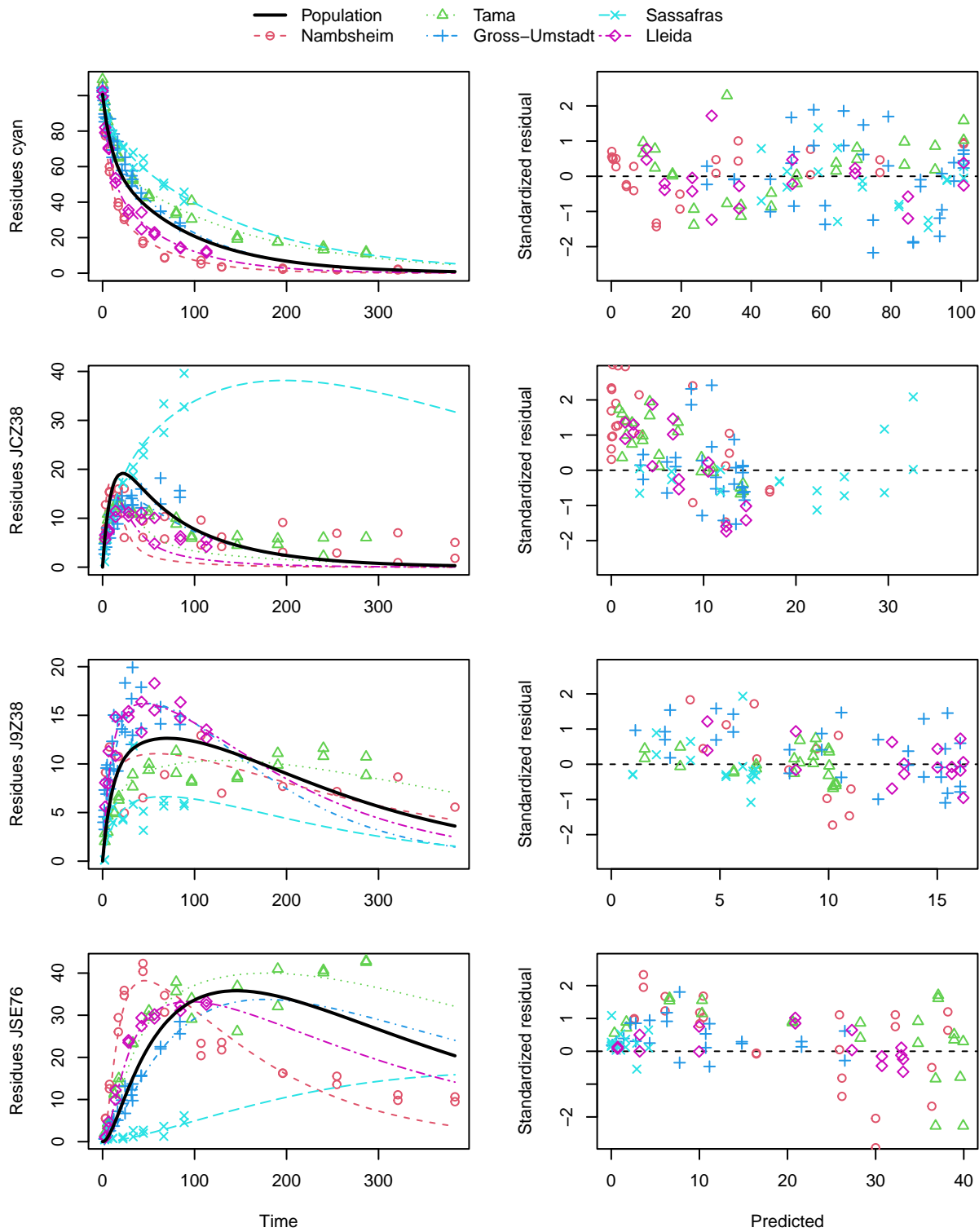

Figure 2: SFORB pathway fit with two-component error

A closer graphical analysis of these Figures shows that the residues of transformation product JCZ38 in the soils Tama and Nambsheim observed at later time points are strongly and systematically underestimated.

```
stopCluster(cl_path_1)
```

Alternative pathway fits

To improve the fit for JCZ38, a back-reaction from JSE76 to JCZ38 was introduced in an alternative version of the transformation pathway, in analogy to the back-reaction from K5A78 to K5A77. Both pairs of transformation products are pairs of an organic acid with its corresponding amide (Addendum 2014, p. 109). As FOMC provided the best fit for the parent, and the biexponential models DFOP and SFORB provided the best initial pathway fits, these three parent models are used in the alternative pathway fits.

```
cyan_path_2 <- list(
  fomc_path_2 = mkinmod(
    cyan = mkinsub("FOMC", c("JCZ38", "J9Z38")),
    JCZ38 = mkinsub("SFO", "JSE76"),
    J9Z38 = mkinsub("SFO"),
    JSE76 = mkinsub("SFO", "JCZ38"),
    name = "fomc_path_2", quiet = TRUE,
    dll_dir = "cyan_dlls",
    overwrite = TRUE
  ),
  dfop_path_2 = mkinmod(
    cyan = mkinsub("DFOP", c("JCZ38", "J9Z38")),
    JCZ38 = mkinsub("SFO", "JSE76"),
    J9Z38 = mkinsub("SFO"),
    JSE76 = mkinsub("SFO", "JCZ38"),
    name = "dfop_path_2", quiet = TRUE,
    dll_dir = "cyan_dlls",
    overwrite = TRUE
  ),
  sforb_path_2 = mkinmod(
    cyan = mkinsub("SFORB", c("JCZ38", "J9Z38")),
    JCZ38 = mkinsub("SFO", "JSE76"),
    J9Z38 = mkinsub("SFO"),
    JSE76 = mkinsub("SFO", "JCZ38"),
    name = "sforb_path_2", quiet = TRUE,
    dll_dir = "cyan_dlls",
    overwrite = TRUE
  )
)

cl_path_2 <- start_cluster(n_cores)
f_sep_2_const <- mmkin(
  cyan_path_2,
  cyan_ds,
  error_model = "const",
  cluster = cl_path_2,
  quiet = TRUE)

status(f_sep_2_const) |> kable()
```

	Nambsheim	Tama	Gross-Umstadt	Sassafras	Lleida
fomc_path_2	OK	OK	OK	C	OK
dfop_path_2	OK	OK	OK	C	OK
sforb_path_2	OK	OK	OK	C	OK

Using constant variance, separate fits converge with the exception of the fits to the Sassafras soil data.

```r
f_sep_2_tc <- update(f_sep_2_const, error_model = "tc")
status(f_sep_2_tc) |> kable()
```

	Nambsheim	Tama	Gross-Umstadt	Sassafras	Lleida
fomc_path_2	OK	C	OK	C	OK
dfop_path_2	OK	OK	OK	C	OK
sforb_path_2	OK	OK	OK	OK	OK

Using the two-component error model, all separate fits converge with the exception of the alternative pathway fit with DFOP used for the parent and the Sassafras dataset.

```r
f_saem_2 <- mhmkin(list(f_sep_2_const, f_sep_2_tc),
  no_random_effect = illparms(cyan_saem_full[2:4, ]),
  cluster = cl_path_2)
```

```r
status(f_saem_2) |> kable()
```

	const	tc
fomc_path_2	OK	FO
dfop_path_2	OK	OK
sforb_path_2	OK	OK

The hierarchical fits for the alternative pathway completed successfully.

```r
illparms(f_saem_2) |> kable()
```

	const	tc
fomc_path_2	sd(f_JCZ38_qlogis), sd(f_JSE76_qlogis)	NA
dfop_path_2	sd(f_JCZ38_qlogis), sd(f_JSE76_qlogis)	sd(f_JCZ38_qlogis), sd(f_JSE76_qlogis)
sforb_path_2	sd(f_JCZ38_qlogis), sd(f_JSE76_qlogis)	sd(f_JCZ38_qlogis), sd(f_JSE76_qlogis)

In both fits, the random effects for the formation fractions for the pathways from JCZ38 to JSE76, and for the reverse pathway from JSE76 to JCZ38 are ill-defined.

```r
anova(f_saem_2) |> kable(digits = 1)
```

	npar	AIC	BIC	Lik
fomc_path_2 const	20	2308.3	2300.5	-1134.2
fomc_path_2 tc	21	2248.3	2240.1	-1103.2
dfop_path_2 const	22	2289.6	2281.0	-1122.8
sforb_path_2 const	22	2284.1	2275.5	-1120.0
dfop_path_2 tc	22	2234.4	2225.8	-1095.2
sforb_path_2 tc	22	2240.4	2231.8	-1098.2

The variants using the biexponential models DFOP and SFORB for the parent compound and the two-component error model give the lowest AIC and BIC values and are plotted below. Compared with the original pathway, the AIC and BIC values indicate a large improvement. This is confirmed by the plots, which show that the metabolite JCZ38 is fitted much better with this model.

```
plot(f_saem_2[["fomc_path_2", "tc"]])
```
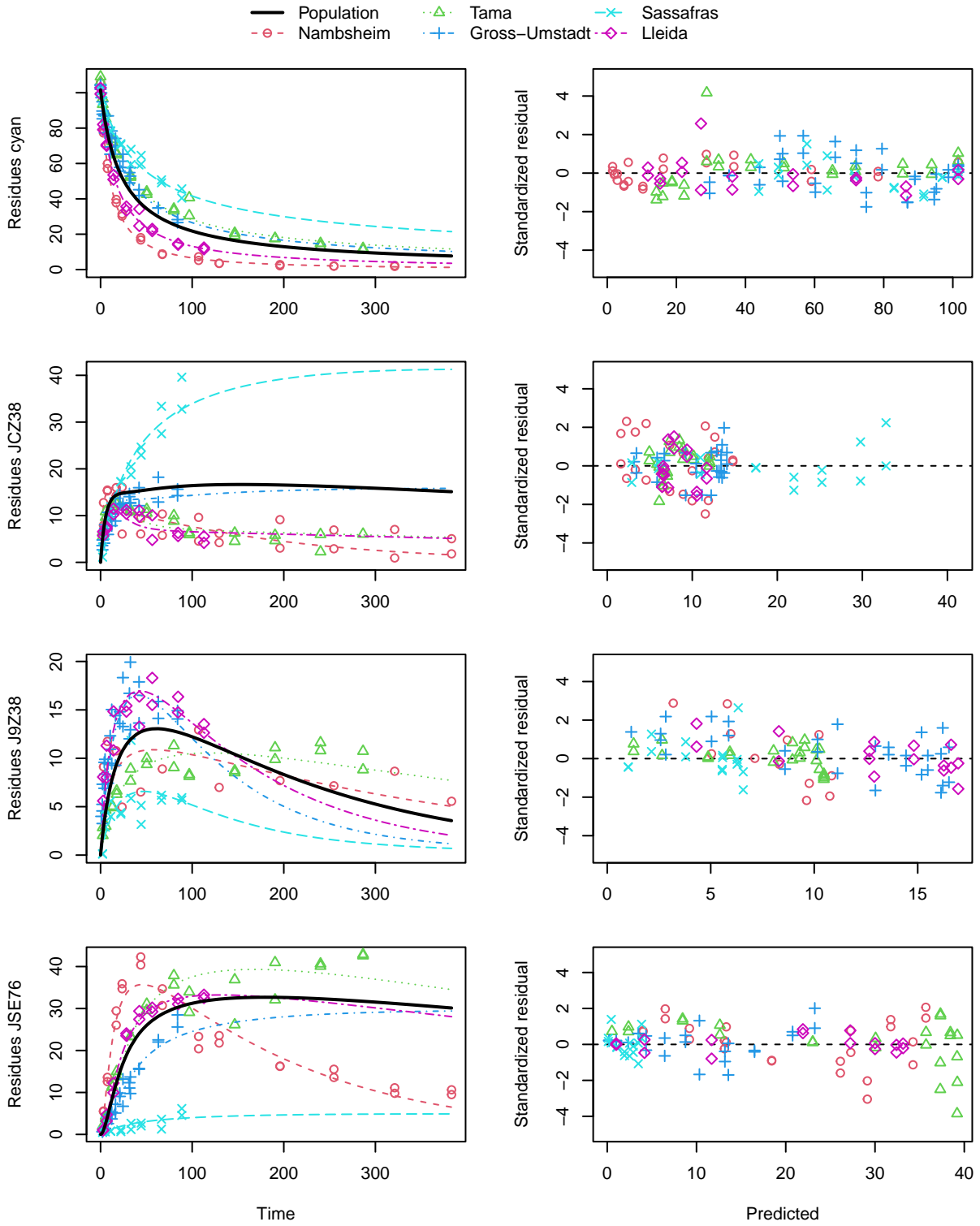

Figure 3: FOMC pathway fit with two-component error, alternative pathway

```
plot(f_saem_2[["dfop_path_2", "tc"]])
```
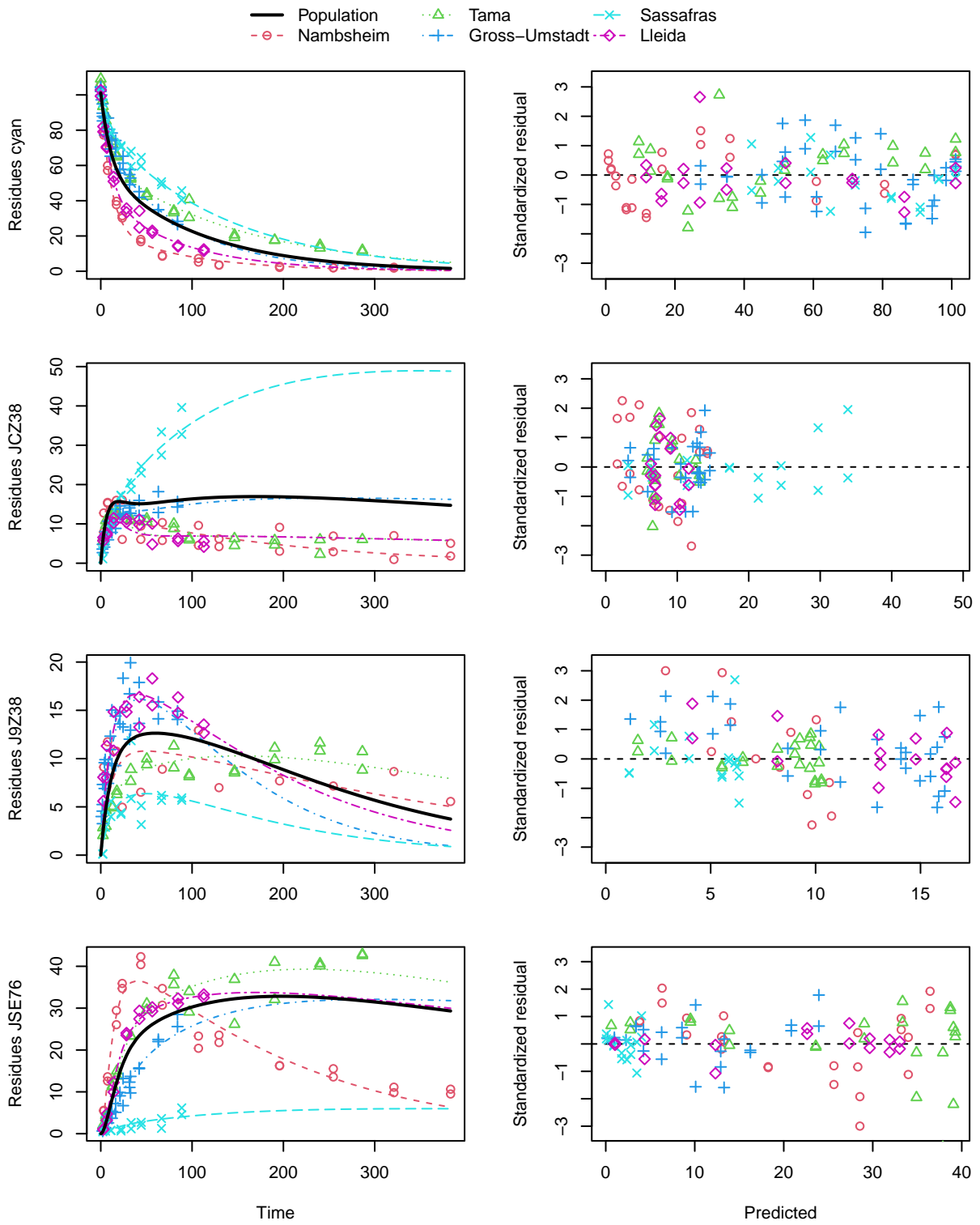

Figure 4: DFOP pathway fit with two-component error, alternative pathway

```
plot(f_saem_2[["sforb_path_2", "tc"]])
```
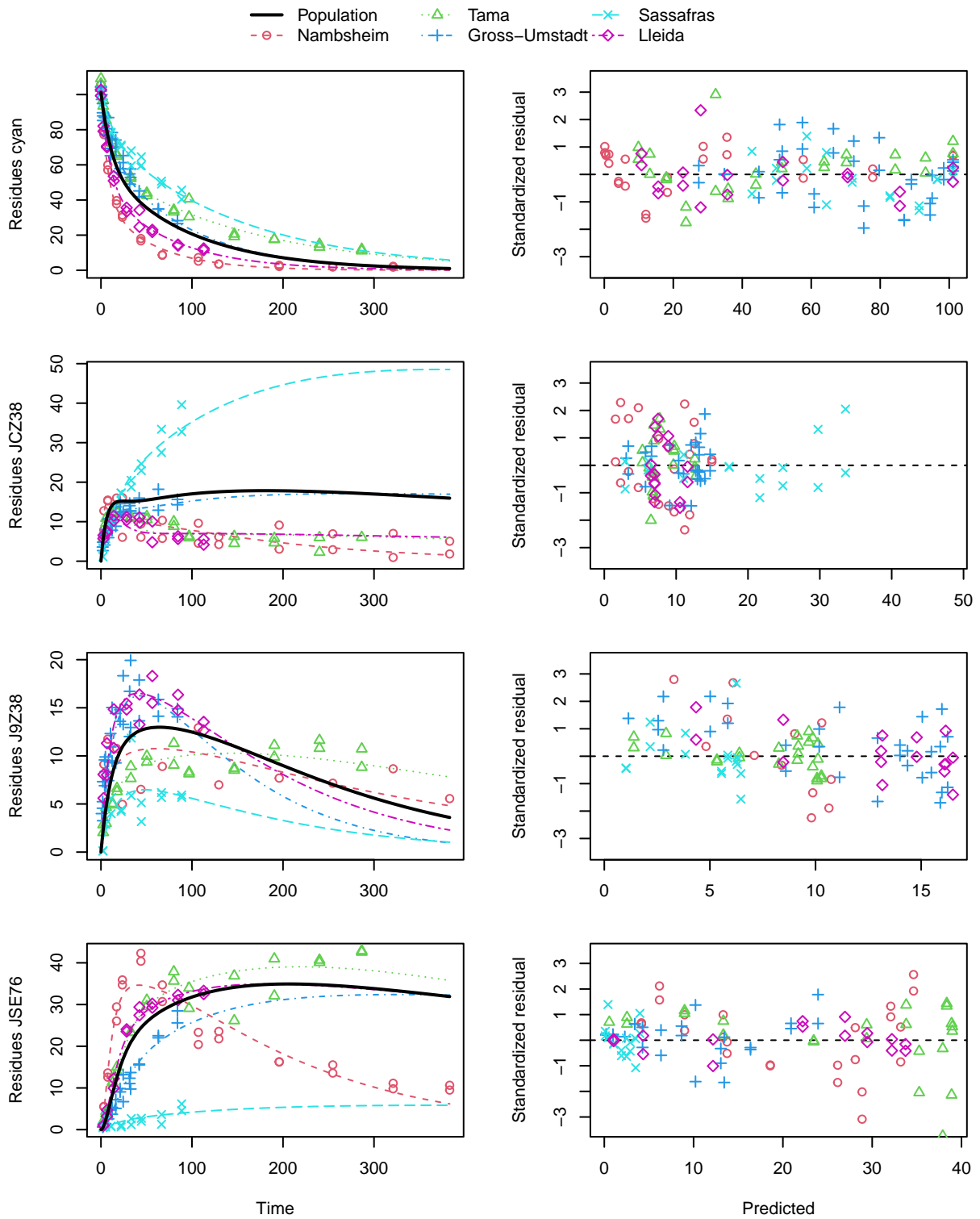

Figure 5: SFORB pathway fit with two-component error, alternative pathway

Refinement of alternative pathway fits

All ill-defined random effects that were identified in the parent only fits and in the above pathway fits, are excluded for the final evaluations below. For this purpose, a list of character vectors is created below that can be indexed by row and column indices, and which contains the degradation parameter names for which random effects should be excluded for each of the hierarchical fits contained in `f_saem_2`.

```
no_ranef <- matrix(list(), nrow = 3, ncol = 2, dimnames = dimnames(f_saem_2))
no_ranef[["fomc_path_2", "const"]] <- c("log_beta", "f_JCZ38_qlogis", "f_JSE76_qlogis")
no_ranef[["fomc_path_2", "tc"]] <- c("cyan_0", "f_JCZ38_qlogis", "f_JSE76_qlogis")
no_ranef[["dfop_path_2", "const"]] <- c("cyan_0", "f_JCZ38_qlogis", "f_JSE76_qlogis")
no_ranef[["dfop_path_2", "tc"]] <- c("cyan_0", "log_k1", "f_JCZ38_qlogis", "f_JSE76_qlogis")
no_ranef[["sforb_path_2", "const"]] <- c("cyan_free_0",
  "f_JCZ38_qlogis", "f_JSE76_qlogis")
no_ranef[["sforb_path_2", "tc"]] <- c("cyan_free_0", "log_k_cyan_free_bound",
  "f_JCZ38_qlogis", "f_JSE76_qlogis")
clusterExport(cl_path_2, "no_ranef")

f_saem_3 <- update(f_saem_2,
  no_random_effect = no_ranef,
  cluster = cl_path_2)
```

```
status(f_saem_3) |> kable()
```

	const	tc
fomc_path_2	E	Fth
dfop_path_2	Fth	Fth
sforb_path_2	Fth	Fth

With the exception of the FOMC pathway fit with constant variance, all updated fits completed successfully. However, the Fisher Information Matrix for the fixed effects (Fth) could not be inverted, so no confidence intervals for the optimised parameters are available.

```
illparms(f_saem_3) |> kable()
```

	const	tc
fomc_path_2	E	
dfop_path_2		
sforb_path_2		

```
anova(f_saem_3) |> kable(digits = 1)
```

	npar	AIC	BIC	Lik
fomc_path_2 tc	19	2250.9	2243.5	-1106.5
dfop_path_2 const	20	2281.7	2273.9	-1120.8
sforb_path_2 const	20	2279.5	2271.7	-1119.7
dfop_path_2 tc	20	2231.5	2223.7	-1095.8
sforb_path_2 tc	20	2235.7	2227.9	-1097.9

While the AIC and BIC values of the best fit (DFOP pathway fit with two-component error) are lower than in the previous fits with the alternative pathway, the practical value of these refined evaluations is limited as no confidence intervals are obtained.

```
stopCluster(cl_path_2)
```

Conclusion

It was demonstrated that a relatively complex transformation pathway with parallel formation of two primary metabolites and one secondary metabolite can be fitted even if the data in the individual datasets are quite different and partly only cover the formation phase.

The run times of the pathway fits were several hours, limiting the practical feasibility of iterative refinements based on ill-defined parameters and of alternative checks of parameter identifiability based on multistart runs.

Acknowledgements

Appendix

Plots of fits that were not refined further
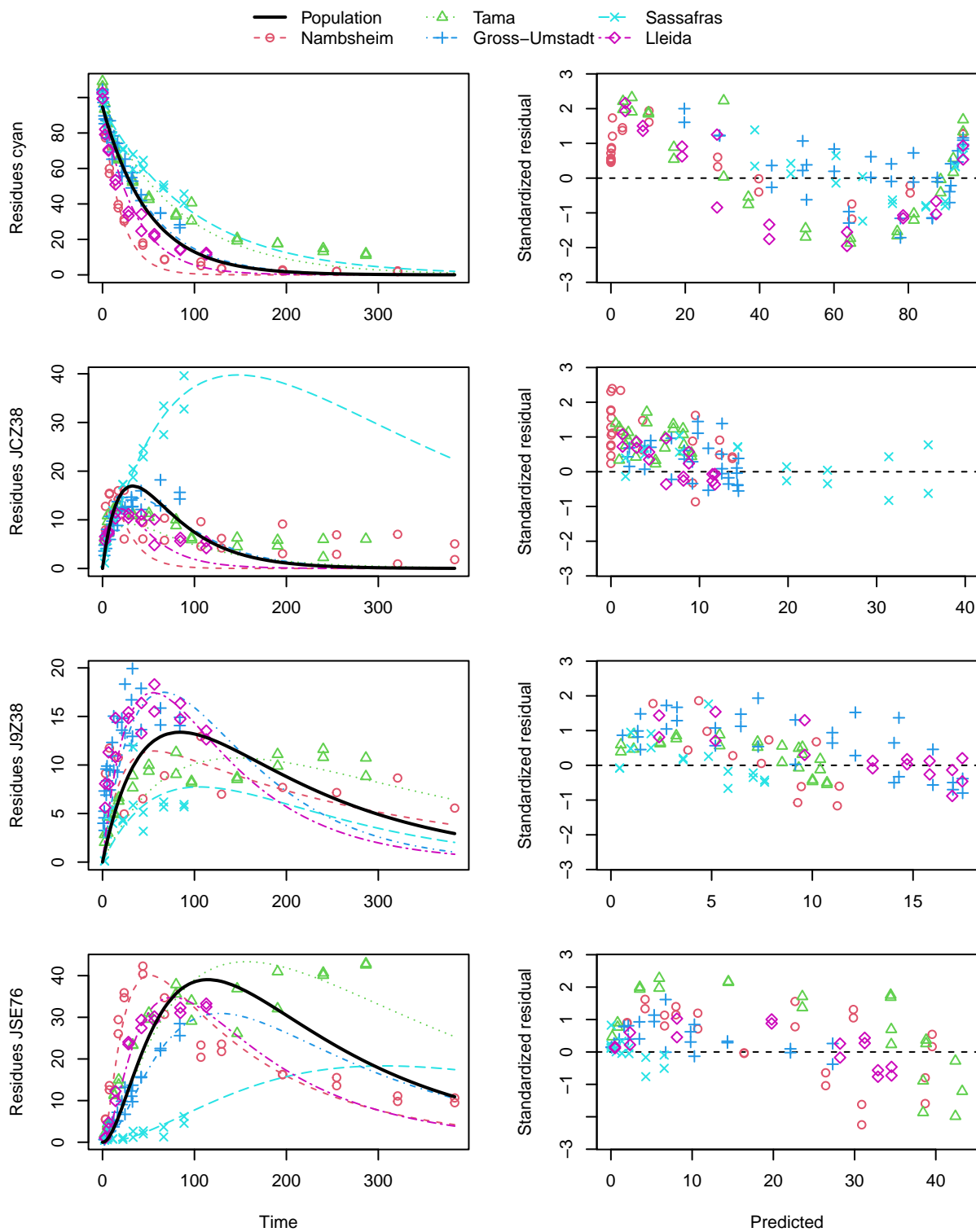
```
plot(f_saem_1[["sfo_path_1", "tc"]])
```


Figure 6: SFO pathway fit with two-component error
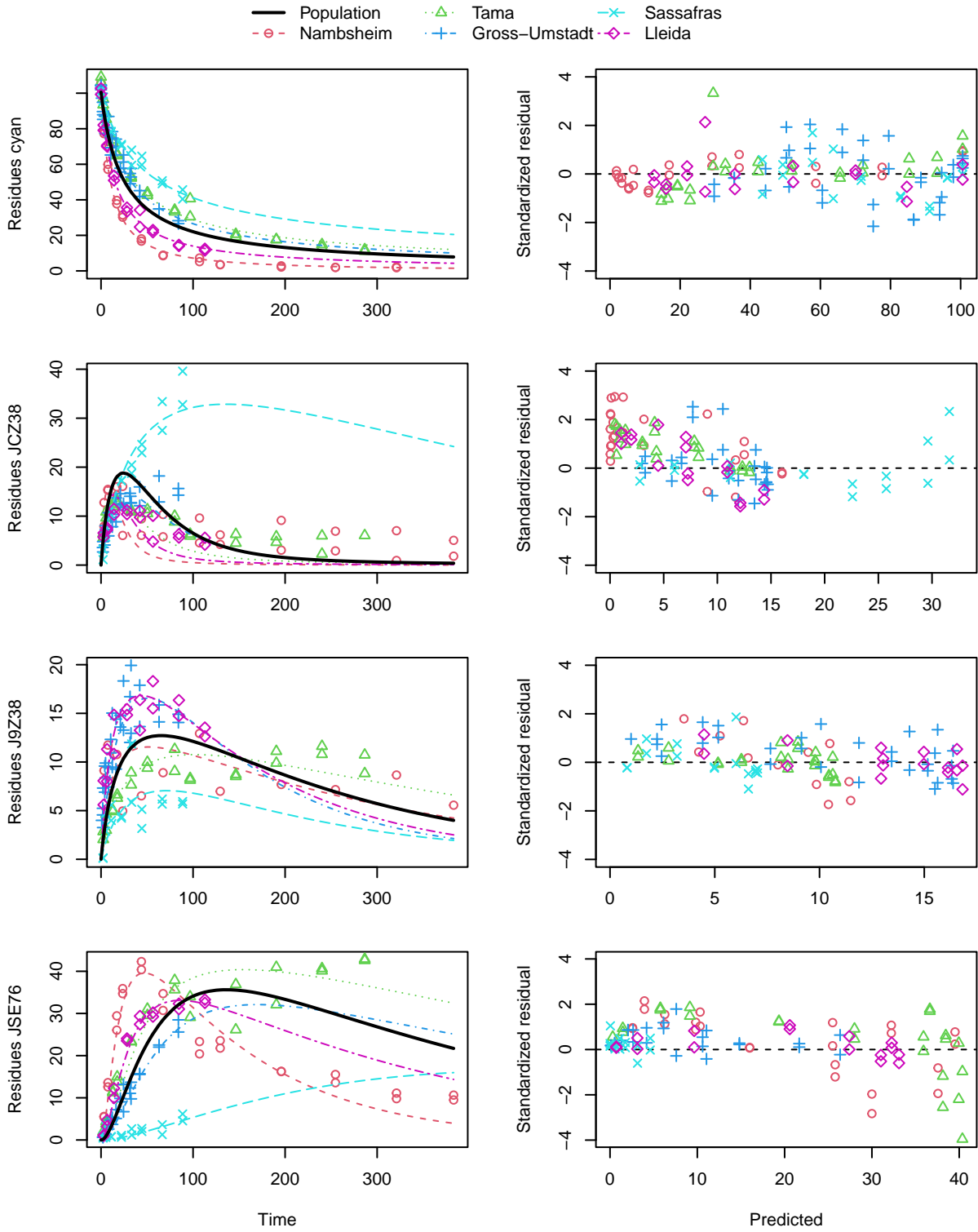
```
plot(f_saem_1[["fomc_path_1", "tc"]])
```


Figure 7: FOMC pathway fit with two-component error

```
plot(f_saem_1[["sforb_path_1", "tc"]])
```
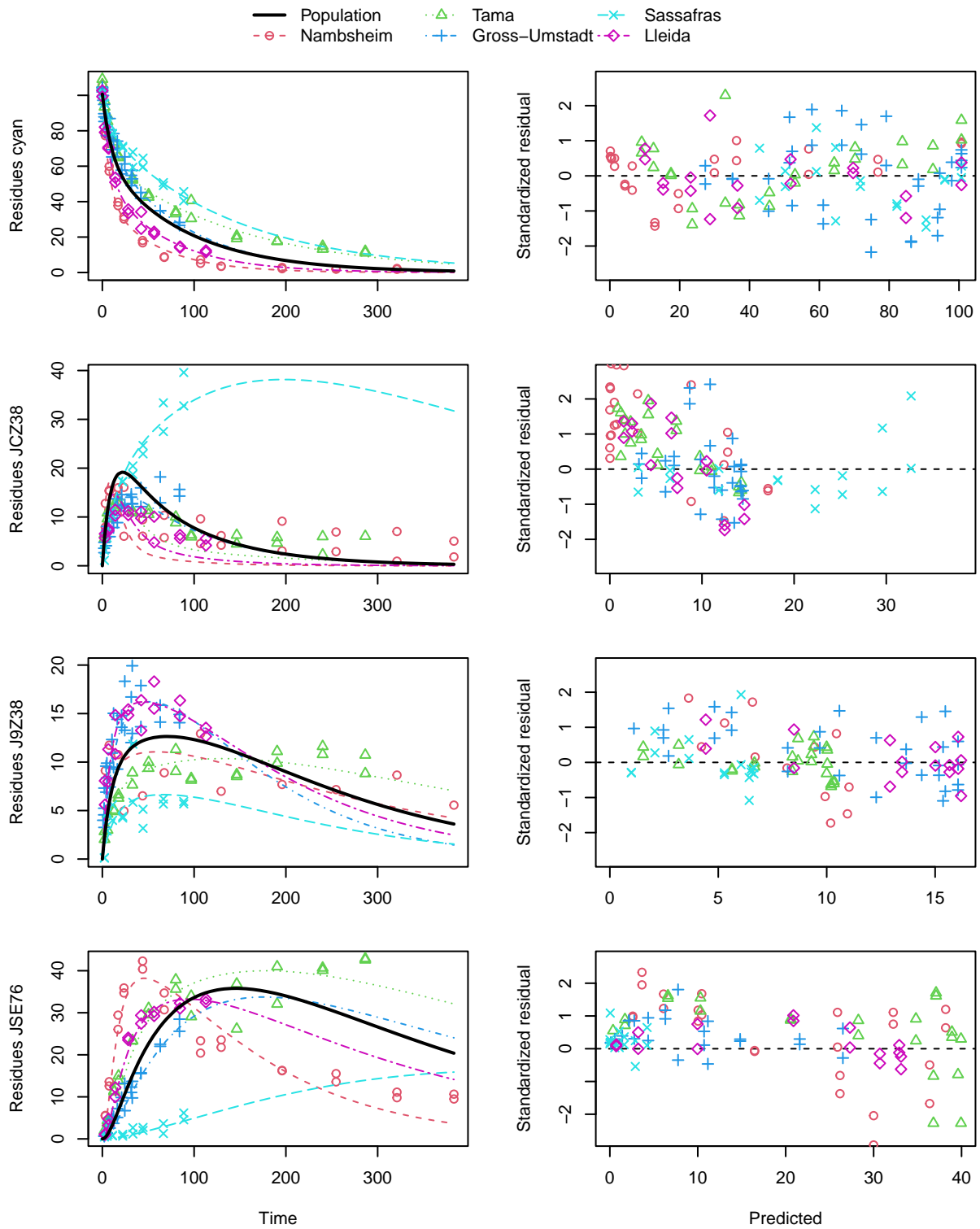

Figure 8: HS pathway fit with two-component error

Hierarchical fit listings

Pathway 1

Listing 1: Hierarchical SFO path 1 fit with constant variance

```
saemix version used for fitting:      3.2
mkin version used for pre-fitting:  1.2.3
R version used for fitting:        4.2.3
Date of fit:     Thu Apr 20 09:54:06 2023
Date of summary: Thu Apr 20 19:09:25 2023


Equations:
d_cyan/dt = - k_cyan * cyan
d_JCZ38/dt = + f_cyan_to_JCZ38 * k_cyan * cyan - k_JCZ38 * JCZ38
d_J9Z38/dt = + f_cyan_to_J9Z38 * k_cyan * cyan - k_J9Z38 * J9Z38
d_JSE76/dt = + f_JCZ38_to_JSE76 * k_JCZ38 * JCZ38 - k_JSE76 * JSE76

Data:
433 observations of 4 variable(s) grouped in 5 datasets

Model predictions using solution type deSolve

Fitted in 431.481 s
Using 300, 100 iterations and 10 chains


Variance model: Constant variance

Starting values for degradation parameters:
       cyan_0     log_k_cyan    log_k_JCZ38    log_k_J9Z38    log_k_JSE76
      95.3304       -3.8459        -3.1305        -5.0678        -5.3196
  f_cyan_ilr_1   f_cyan_ilr_2 f_JCZ38_qlogis
       0.8158       22.5404        10.4289

Fixed degradation parameter values:
None

Starting values for random effects (square root of initial entries in omega):
             cyan_0 log_k_cyan log_k_JCZ38 log_k_J9Z38 log_k_JSE76
cyan_0        4.797     0.0000       0.000       0.000      0.0000
log_k_cyan    0.000     0.9619       0.000       0.000      0.0000
log_k_JCZ38   0.000     0.0000       2.139       0.000      0.0000
log_k_J9Z38   0.000     0.0000       0.000       1.639      0.0000
log_k_JSE76   0.000     0.0000       0.000       0.000      0.7894
f_cyan_ilr_1  0.000     0.0000       0.000       0.000      0.0000
f_cyan_ilr_2  0.000     0.0000       0.000       0.000      0.0000
f_JCZ38_qlogis 0.000    0.0000       0.000       0.000      0.0000
             f_cyan_ilr_1 f_cyan_ilr_2 f_JCZ38_qlogis
cyan_0             0.0000        0.000           0.00
log_k_cyan         0.0000        0.000           0.00
log_k_JCZ38        0.0000        0.000           0.00
log_k_J9Z38        0.0000        0.000           0.00
log_k_JSE76        0.0000        0.000           0.00
f_cyan_ilr_1       0.7714        0.000           0.00
f_cyan_ilr_2       0.0000        8.684           0.00
f_JCZ38_qlogis     0.0000        0.000          13.48

Starting values for error model parameters:
a.1
  1


Results:

Likelihood computed by importance sampling
   AIC  BIC logLik
  2693 2687  -1330

Optimised parameters:
                est. lower upper
cyan_0         95.0946   NA    NA
log_k_cyan     -3.8544   NA    NA
log_k_JCZ38    -3.0402   NA    NA
log_k_J9Z38    -5.0109   NA    NA
log_k_JSE76    -5.2857   NA    NA
f_cyan_ilr_1    0.8069   NA    NA
f_cyan_ilr_2   16.6623   NA    NA
f_JCZ38_qlogis  1.3602   NA    NA
a.1             4.8326   NA    NA
SD.log_k_cyan   0.5842   NA    NA
SD.log_k_JCZ38  1.2680   NA    NA
SD.log_k_J9Z38  0.3626   NA    NA
SD.log_k_JSE76  0.5244   NA    NA
SD.f_cyan_ilr_1 0.2752   NA    NA
```

```
SD.f_cyan_ilr_2   2.3556   NA    NA
SD.f_JCZ38_qlogis 0.2400   NA    NA


Correlation is not available

Random effects:
                  est. lower upper
SD.log_k_cyan     0.5842   NA    NA
SD.log_k_JCZ38    1.2680   NA    NA
SD.log_k_J9Z38    0.3626   NA    NA
SD.log_k_JSE76    0.5244   NA    NA
SD.f_cyan_ilr_1   0.2752   NA    NA
SD.f_cyan_ilr_2   2.3556   NA    NA
SD.f_JCZ38_qlogis 0.2400   NA    NA

Variance model:
    est. lower upper
a.1 4.833   NA    NA

Backtransformed parameters:
                   est. lower upper
cyan_0          95.094581   NA    NA
k_cyan           0.021186   NA    NA
k_JCZ38          0.047825   NA    NA
k_J9Z38          0.006665   NA    NA
k_JSE76          0.005063   NA    NA
f_cyan_to_JCZ38  0.757885   NA    NA
f_cyan_to_J9Z38  0.242115   NA    NA
f_JCZ38_to_JSE76 0.795792   NA    NA

Resulting formation fractions:
                  ff
cyan_JCZ38  7.579e-01
cyan_J9Z38  2.421e-01
cyan_sink   5.877e-10
JCZ38_JSE76 7.958e-01
JCZ38_sink  2.042e-01

Estimated disappearance times:
        DT50    DT90
cyan   32.72  108.68
JCZ38  14.49   48.15
J9Z38 103.99  345.46
JSE76 136.90  454.76
```

Listing 2: Hierarchical SFO path 1 fit with two-component error

```
saemix version used for fitting:      3.2
mkin version used for pre-fitting:  1.2.3
R version used for fitting:        4.2.3
Date of fit:     Thu Apr 20 09:53:58 2023
Date of summary: Thu Apr 20 19:09:25 2023


Equations:
d_cyan/dt = - k_cyan * cyan
d_JCZ38/dt = + f_cyan_to_JCZ38 * k_cyan * cyan - k_JCZ38 * JCZ38
d_J9Z38/dt = + f_cyan_to_J9Z38 * k_cyan * cyan - k_J9Z38 * J9Z38
d_JSE76/dt = + f_JCZ38_to_JSE76 * k_JCZ38 * JCZ38 - k_JSE76 * JSE76


Data:
433 observations of 4 variable(s) grouped in 5 datasets


Model predictions using solution type deSolve


Fitted in 422.681 s
Using 300, 100 iterations and 10 chains


Variance model: Two-component variance function


Starting values for degradation parameters:
        cyan_0     log_k_cyan     log_k_JCZ38     log_k_J9Z38     log_k_JSE76
       96.0039        -3.8907        -3.1276         -5.0069         -4.9367
  f_cyan_ilr_1    f_cyan_ilr_2 f_JCZ38_qlogis
        0.7937        20.0030        15.1336


Fixed degradation parameter values:
None


Starting values for random effects (square root of initial entries in omega):
              cyan_0 log_k_cyan log_k_JCZ38 log_k_J9Z38 log_k_JSE76
cyan_0         4.859      0.000        0.00        0.00       0.0000
log_k_cyan     0.000      0.962        0.00        0.00       0.0000
log_k_JCZ38    0.000      0.000        2.04        0.00       0.0000
log_k_J9Z38    0.000      0.000        0.00        1.72       0.0000
log_k_JSE76    0.000      0.000        0.00        0.00       0.9076
f_cyan_ilr_1   0.000      0.000        0.00        0.00       0.0000
f_cyan_ilr_2   0.000      0.000        0.00        0.00       0.0000
f_JCZ38_qlogis 0.000      0.000        0.00        0.00       0.0000
              f_cyan_ilr_1 f_cyan_ilr_2 f_JCZ38_qlogis
cyan_0              0.0000        0.000           0.00
log_k_cyan          0.0000        0.000           0.00
log_k_JCZ38         0.0000        0.000           0.00
log_k_J9Z38         0.0000        0.000           0.00
log_k_JSE76         0.0000        0.000           0.00
f_cyan_ilr_1        0.7598        0.000           0.00
f_cyan_ilr_2        0.0000        7.334           0.00
f_JCZ38_qlogis      0.0000        0.000          11.78


Starting values for error model parameters:
a.1 b.1
  1   1


Results:


Likelihood computed by importance sampling
   AIC  BIC logLik
  2658 2651  -1312


Optimised parameters:
                   est. lower upper
cyan_0          94.72923   NA    NA
log_k_cyan      -3.91670   NA    NA
log_k_JCZ38     -3.12917   NA    NA
log_k_J9Z38     -5.06070   NA    NA
log_k_JSE76     -5.09254   NA    NA
f_cyan_ilr_1     0.81116   NA    NA
f_cyan_ilr_2    39.97850   NA    NA
f_JCZ38_qlogis   3.09728   NA    NA
a.1              3.95044   NA    NA
b.1              0.07998   NA    NA
SD.log_k_cyan    0.58855   NA    NA
SD.log_k_JCZ38   1.29753   NA    NA
SD.log_k_J9Z38   0.62851   NA    NA
SD.log_k_JSE76   0.37235   NA    NA
SD.f_cyan_ilr_1  0.37346   NA    NA
SD.f_cyan_ilr_2  1.41667   NA    NA
SD.f_JCZ38_qlogis 1.81467  NA    NA


Correlation is not available
```

```
Random effects:
                  est. lower upper
SD.log_k_cyan    0.5886    NA    NA
SD.log_k_JCZ38   1.2975    NA    NA
SD.log_k_J9Z38   0.6285    NA    NA
SD.log_k_JSE76   0.3724    NA    NA
SD.f_cyan_ilr_1  0.3735    NA    NA
SD.f_cyan_ilr_2  1.4167    NA    NA
SD.f_JCZ38_qlogis 1.8147   NA    NA


Variance model:
      est. lower upper
a.1 3.95044    NA    NA
b.1 0.07998    NA    NA


Backtransformed parameters:
                      est. lower upper
cyan_0           94.729229    NA    NA
k_cyan            0.019907    NA    NA
k_JCZ38           0.043754    NA    NA
k_J9Z38           0.006341    NA    NA
k_JSE76           0.006142    NA    NA
f_cyan_to_JCZ38   0.758991    NA    NA
f_cyan_to_J9Z38   0.241009    NA    NA
f_JCZ38_to_JSE76  0.956781    NA    NA


Resulting formation fractions:
                 ff
cyan_JCZ38   0.75899
cyan_J9Z38   0.24101
cyan_sink    0.00000
JCZ38_JSE76  0.95678
JCZ38_sink   0.04322


Estimated disappearance times:
         DT50   DT90
cyan    34.82 115.67
JCZ38   15.84  52.63
J9Z38  109.31 363.12
JSE76  112.85 374.87
```

Listing 3: Hierarchical FOMC path 1 fit with constant variance

```
saemix version used for fitting:      3.2
mkin version used for pre-fitting:  1.2.3
R version used for fitting:        4.2.3
Date of fit:     Thu Apr 20 09:54:48 2023
Date of summary: Thu Apr 20 19:09:25 2023


Equations:
d_cyan/dt = - (alpha/beta) * 1/((time/beta) + 1) * cyan
d_JCZ38/dt = + f_cyan_to_JCZ38 * (alpha/beta) * 1/((time/beta) + 1) *
           cyan - k_JCZ38 * JCZ38
d_J9Z38/dt = + f_cyan_to_J9Z38 * (alpha/beta) * 1/((time/beta) + 1) *
           cyan - k_J9Z38 * J9Z38
d_JSE76/dt = + f_JCZ38_to_JSE76 * k_JCZ38 * JCZ38 - k_JSE76 * JSE76


Data:
433 observations of 4 variable(s) grouped in 5 datasets


Model predictions using solution type deSolve


Fitted in 472.763 s
Using 300, 100 iterations and 10 chains


Variance model: Constant variance


Starting values for degradation parameters:
        cyan_0    log_k_JCZ38    log_k_J9Z38    log_k_JSE76    f_cyan_ilr_1
       101.2314       -3.3680        -5.1108        -5.9416          0.7144
   f_cyan_ilr_2 f_JCZ38_qlogis      log_alpha       log_beta
         7.3870       15.7604        -0.1791         2.9811


Fixed degradation parameter values:
None


Starting values for random effects (square root of initial entries in omega):
              cyan_0 log_k_JCZ38 log_k_J9Z38 log_k_JSE76 f_cyan_ilr_1
cyan_0         5.416       0.000         0.0       0.000       0.0000
log_k_JCZ38    0.000       2.439         0.0       0.000       0.0000
log_k_J9Z38    0.000       0.000         1.7       0.000       0.0000
log_k_JSE76    0.000       0.000         0.0       1.856       0.0000
f_cyan_ilr_1   0.000       0.000         0.0       0.000       0.7164
f_cyan_ilr_2   0.000       0.000         0.0       0.000       0.0000
f_JCZ38_qlogis 0.000       0.000         0.0       0.000       0.0000
log_alpha      0.000       0.000         0.0       0.000       0.0000
log_beta       0.000       0.000         0.0       0.000       0.0000
              f_cyan_ilr_2 f_JCZ38_qlogis log_alpha log_beta
cyan_0                0.00           0.00    0.0000   0.0000
log_k_JCZ38           0.00           0.00    0.0000   0.0000
log_k_J9Z38           0.00           0.00    0.0000   0.0000
log_k_JSE76           0.00           0.00    0.0000   0.0000
f_cyan_ilr_1          0.00           0.00    0.0000   0.0000
f_cyan_ilr_2         12.33           0.00    0.0000   0.0000
f_JCZ38_qlogis        0.00          20.42    0.0000   0.0000
log_alpha             0.00           0.00    0.4144   0.0000
log_beta              0.00           0.00    0.0000   0.5077


Starting values for error model parameters:
a.1
  1


Results:


Likelihood computed by importance sampling
   AIC  BIC logLik
  2428 2421  -1196


Optimised parameters:
                    est.       lower     upper
cyan_0          101.0225  98.306270 103.7387
log_k_JCZ38      -3.3786  -4.770657  -1.9866
log_k_J9Z38      -5.2603  -5.902085  -4.6186
log_k_JSE76      -6.1427  -7.318336  -4.9671
f_cyan_ilr_1      0.7437   0.421215   1.0663
f_cyan_ilr_2      0.9108   0.267977   1.5537
f_JCZ38_qlogis    2.0487   0.524897   3.5724
log_alpha        -0.2268  -0.618049   0.1644
log_beta          2.8986   2.700701   3.0964
a.1               3.4058   3.169913   3.6416
SD.cyan_0         2.5279   0.454190   4.6016
SD.log_k_JCZ38    1.5636   0.572824   2.5543
SD.log_k_J9Z38    0.5316  -0.004405   1.0677
SD.log_k_JSE76    0.9903   0.106325   1.8742
SD.f_cyan_ilr_1   0.3464   0.112066   0.5807
```

```
SD.f_cyan_ilr_2    0.2804 -0.393900  0.9546
SD.f_JCZ38_qlogis  0.9416 -0.152986  2.0362
SD.log_alpha       0.4273  0.161044  0.6936


Correlation:
                cyan_0 l__JCZ3 l__J9Z3 l__JSE7 f_cy__1 f_cy__2 f_JCZ38 log_lph
log_k_JCZ38    -0.0156
log_k_J9Z38    -0.0493  0.0073
log_k_JSE76    -0.0329  0.0018  0.0069
f_cyan_ilr_1   -0.0086  0.0180 -0.1406  0.0012
f_cyan_ilr_2   -0.2629  0.0779  0.2826  0.0274  0.0099
f_JCZ38_qlogis  0.0713 -0.0747 -0.0505  0.1169 -0.1022 -0.4893
log_alpha      -0.0556  0.0120  0.0336  0.0193  0.0036  0.0840 -0.0489
log_beta       -0.2898  0.0460  0.1305  0.0768  0.0190  0.4071 -0.1981  0.2772

Random effects:
                   est.     lower  upper
SD.cyan_0        2.5279  0.454190 4.6016
SD.log_k_JCZ38   1.5636  0.572824 2.5543
SD.log_k_J9Z38   0.5316 -0.004405 1.0677
SD.log_k_JSE76   0.9903  0.106325 1.8742
SD.f_cyan_ilr_1  0.3464  0.112066 0.5807
SD.f_cyan_ilr_2  0.2804 -0.393900 0.9546
SD.f_JCZ38_qlogis 0.9416 -0.152986 2.0362
SD.log_alpha     0.4273  0.161044 0.6936

Variance model:
     est. lower upper
a.1 3.406  3.17 3.642

Backtransformed parameters:
                   est.      lower      upper
cyan_0         1.010e+02 9.831e+01 1.037e+02
k_JCZ38        3.409e-02 8.475e-03 1.372e-01
k_J9Z38        5.194e-03 2.734e-03 9.867e-03
k_JSE76        2.149e-03 6.633e-04 6.963e-03
f_cyan_to_JCZ38 6.481e-01        NA        NA
f_cyan_to_J9Z38 2.264e-01        NA        NA
f_JCZ38_to_JSE76 8.858e-01 6.283e-01 9.727e-01
alpha          7.971e-01 5.390e-01 1.179e+00
beta           1.815e+01 1.489e+01 2.212e+01


Resulting formation fractions:
               ff
cyan_JCZ38  0.6481
cyan_J9Z38  0.2264
cyan_sink   0.1255
JCZ38_JSE76 0.8858
JCZ38_sink  0.1142

Estimated disappearance times:
        DT50    DT90 DT50back
cyan   25.15  308.01    92.72
JCZ38  20.33   67.54       NA
J9Z38 133.46  443.35       NA
JSE76 322.53 1071.42       NA
```

```
saemix version used for fitting:      3.2
mkin version used for pre-fitting:  1.2.3
R version used for fitting:         4.2.3
Date of fit:     Thu Apr 20 09:54:58 2023
Date of summary: Thu Apr 20 19:09:25 2023


Equations:
d_cyan/dt = - (alpha/beta) * 1/((time/beta) + 1) * cyan
d_JCZ38/dt = + f_cyan_to_JCZ38 * (alpha/beta) * 1/((time/beta) + 1) *
             cyan - k_JCZ38 * JCZ38
d_J9Z38/dt = + f_cyan_to_J9Z38 * (alpha/beta) * 1/((time/beta) + 1) *
             cyan - k_J9Z38 * J9Z38
d_JSE76/dt = + f_JCZ38_to_JSE76 * k_JCZ38 * JCZ38 - k_JSE76 * JSE76


Data:
433 observations of 4 variable(s) grouped in 5 datasets


Model predictions using solution type deSolve


Fitted in 482.972 s
Using 300, 100 iterations and 10 chains


Variance model: Two-component variance function


Starting values for degradation parameters:
         cyan_0    log_k_JCZ38    log_k_J9Z38    log_k_JSE76   f_cyan_ilr_1
       101.13827       -3.32493       -5.08921       -5.93478        0.71330
   f_cyan_ilr_2 f_JCZ38_qlogis      log_alpha       log_beta
       10.05989       12.79248       -0.09621        3.10646


Fixed degradation parameter values:
None


Starting values for random effects (square root of initial entries in omega):
               cyan_0 log_k_JCZ38 log_k_J9Z38 log_k_JSE76 f_cyan_ilr_1
cyan_0          5.643       0.000       0.000        0.00       0.0000
log_k_JCZ38     0.000       2.319       0.000        0.00       0.0000
log_k_J9Z38     0.000       0.000       1.731        0.00       0.0000
log_k_JSE76     0.000       0.000       0.000        1.86       0.0000
f_cyan_ilr_1    0.000       0.000       0.000        0.00       0.7186
f_cyan_ilr_2    0.000       0.000       0.000        0.00       0.0000
f_JCZ38_qlogis  0.000       0.000       0.000        0.00       0.0000
log_alpha       0.000       0.000       0.000        0.00       0.0000
log_beta        0.000       0.000       0.000        0.00       0.0000
               f_cyan_ilr_2 f_JCZ38_qlogis log_alpha log_beta
cyan_0                 0.00           0.00    0.0000   0.0000
log_k_JCZ38            0.00           0.00    0.0000   0.0000
log_k_J9Z38            0.00           0.00    0.0000   0.0000
log_k_JSE76            0.00           0.00    0.0000   0.0000
f_cyan_ilr_1           0.00           0.00    0.0000   0.0000
f_cyan_ilr_2          12.49           0.00    0.0000   0.0000
f_JCZ38_qlogis         0.00          20.19    0.0000   0.0000
log_alpha              0.00           0.00    0.3142   0.0000
log_beta               0.00           0.00    0.0000   0.7331


Starting values for error model parameters:
a.1 b.1
  1   1


Results:


Likelihood computed by importance sampling
   AIC  BIC logLik
  2423 2416  -1193


Optimised parameters:
                      est. lower upper
cyan_0           100.57649    NA    NA
log_k_JCZ38       -3.46250    NA    NA
log_k_J9Z38       -5.24442    NA    NA
log_k_JSE76       -5.75229    NA    NA
f_cyan_ilr_1       0.68480    NA    NA
f_cyan_ilr_2       0.61670    NA    NA
f_JCZ38_qlogis    87.97407    NA    NA
log_alpha         -0.15699    NA    NA
log_beta           3.01540    NA    NA
a.1                3.11518    NA    NA
b.1                0.04445    NA    NA
SD.log_k_JCZ38     1.40732    NA    NA
SD.log_k_J9Z38     0.56510    NA    NA
SD.log_k_JSE76     0.72067    NA    NA
SD.f_cyan_ilr_1    0.31199    NA    NA
```

```
SD.f_cyan_ilr_2      0.36894    NA    NA
SD.f_JCZ38_qlogis    6.92892    NA    NA
SD.log_alpha         0.25662    NA    NA
SD.log_beta          0.35845    NA    NA


Correlation is not available

Random effects:
                   est. lower upper
SD.log_k_JCZ38    1.4073    NA    NA
SD.log_k_J9Z38    0.5651    NA    NA
SD.log_k_JSE76    0.7207    NA    NA
SD.f_cyan_ilr_1   0.3120    NA    NA
SD.f_cyan_ilr_2   0.3689    NA    NA
SD.f_JCZ38_qlogis 6.9289    NA    NA
SD.log_alpha      0.2566    NA    NA
SD.log_beta       0.3585    NA    NA

Variance model:
      est. lower upper
a.1 3.11518    NA    NA
b.1 0.04445    NA    NA

Backtransformed parameters:
                    est. lower upper
cyan_0           1.006e+02    NA    NA
k_JCZ38          3.135e-02    NA    NA
k_J9Z38          5.277e-03    NA    NA
k_JSE76          3.175e-03    NA    NA
f_cyan_to_JCZ38  5.991e-01    NA    NA
f_cyan_to_J9Z38  2.275e-01    NA    NA
f_JCZ38_to_JSE76 1.000e+00    NA    NA
alpha            8.547e-01    NA    NA
beta             2.040e+01    NA    NA

Resulting formation fractions:
              ff
cyan_JCZ38  0.5991
cyan_J9Z38  0.2275
cyan_sink   0.1734
JCZ38_JSE76 1.0000
JCZ38_sink  0.0000

Estimated disappearance times:
       DT50   DT90 DT50back
cyan   25.50 281.29    84.68
JCZ38  22.11  73.44       NA
J9Z38 131.36 436.35       NA
JSE76 218.28 725.11       NA
```

Listing 5: Hierarchical DFOP path 1 fit with constant variance

```
saemix version used for fitting:      3.2
mkin version used for pre-fitting:  1.2.3
R version used for fitting:         4.2.3
Date of fit:      Thu Apr 20 09:55:38 2023
Date of summary: Thu Apr 20 19:09:25 2023


Equations:
d_cyan/dt = - ((k1 * g * exp(-k1 * time) + k2 * (1 - g) * exp(-k2 *
            time)) / (g * exp(-k1 * time) + (1 - g) * exp(-k2 * time)))
            * cyan
d_JCZ38/dt = + f_cyan_to_JCZ38 * ((k1 * g * exp(-k1 * time) + k2 * (1 -
            g) * exp(-k2 * time)) / (g * exp(-k1 * time) + (1 - g) *
            exp(-k2 * time))) * cyan - k_JCZ38 * JCZ38
d_J9Z38/dt = + f_cyan_to_J9Z38 * ((k1 * g * exp(-k1 * time) + k2 * (1 -
            g) * exp(-k2 * time)) / (g * exp(-k1 * time) + (1 - g) *
            exp(-k2 * time))) * cyan - k_J9Z38 * J9Z38
d_JSE76/dt = + f_JCZ38_to_JSE76 * k_JCZ38 * JCZ38 - k_JSE76 * JSE76


Data:
433 observations of 4 variable(s) grouped in 5 datasets


Model predictions using solution type deSolve


Fitted in 522.862 s
Using 300, 100 iterations and 10 chains


Variance model: Constant variance


Starting values for degradation parameters:
        cyan_0    log_k_JCZ38    log_k_J9Z38    log_k_JSE76   f_cyan_ilr_1
       102.0644       -3.4008        -5.0024        -5.8613         0.6855
   f_cyan_ilr_2 f_JCZ38_qlogis         log_k1          log_k2       g_qlogis
         1.2365       13.7245        -1.8641        -4.5063        -0.6468


Fixed degradation parameter values:
None


Starting values for random effects (square root of initial entries in omega):
              cyan_0 log_k_JCZ38 log_k_J9Z38 log_k_JSE76 f_cyan_ilr_1
cyan_0         4.466       0.000       0.000       0.000       0.0000
log_k_JCZ38    0.000       2.382       0.000       0.000       0.0000
log_k_J9Z38    0.000       0.000       1.595       0.000       0.0000
log_k_JSE76    0.000       0.000       0.000       1.245       0.0000
f_cyan_ilr_1   0.000       0.000       0.000       0.000       0.6852
f_cyan_ilr_2   0.000       0.000       0.000       0.000       0.0000
f_JCZ38_qlogis 0.000       0.000       0.000       0.000       0.0000
log_k1         0.000       0.000       0.000       0.000       0.0000
log_k2         0.000       0.000       0.000       0.000       0.0000
g_qlogis       0.000       0.000       0.000       0.000       0.0000
              f_cyan_ilr_2 f_JCZ38_qlogis log_k1 log_k2 g_qlogis
cyan_0                0.00           0.00 0.0000 0.0000    0.000
log_k_JCZ38           0.00           0.00 0.0000 0.0000    0.000
log_k_J9Z38           0.00           0.00 0.0000 0.0000    0.000
log_k_JSE76           0.00           0.00 0.0000 0.0000    0.000
f_cyan_ilr_1          0.00           0.00 0.0000 0.0000    0.000
f_cyan_ilr_2          1.28           0.00 0.0000 0.0000    0.000
f_JCZ38_qlogis        0.00          16.11 0.0000 0.0000    0.000
log_k1                0.00           0.00 0.9866 0.0000    0.000
log_k2                0.00           0.00 0.0000 0.5953    0.000
g_qlogis              0.00           0.00 0.0000 0.0000    1.583


Starting values for error model parameters:
a.1
  1


Results:


Likelihood computed by importance sampling
   AIC  BIC logLik
  2403 2395  -1182


Optimised parameters:
                 est. lower upper
cyan_0         102.6079    NA    NA
log_k_JCZ38     -3.4855    NA    NA
log_k_J9Z38     -5.1686    NA    NA
log_k_JSE76     -5.6697    NA    NA
f_cyan_ilr_1     0.6714    NA    NA
f_cyan_ilr_2     0.4986    NA    NA
f_JCZ38_qlogis  55.4760    NA    NA
log_k1          -1.8409    NA    NA
log_k2          -4.4915    NA    NA
```

```
g_qlogis              -0.6403    NA    NA
a.1                    3.2387    NA    NA
SD.log_k_JCZ38         1.4524    NA    NA
SD.log_k_J9Z38         0.5151    NA    NA
SD.log_k_JSE76         0.6514    NA    NA
SD.f_cyan_ilr_1        0.3023    NA    NA
SD.f_cyan_ilr_2        0.2959    NA    NA
SD.f_JCZ38_qlogis      1.9984    NA    NA
SD.log_k1              0.5188    NA    NA
SD.log_k2              0.3894    NA    NA
SD.g_qlogis            0.8579    NA    NA


Correlation is not available

Random effects:
                  est. lower upper
SD.log_k_JCZ38    1.4524    NA    NA
SD.log_k_J9Z38    0.5151    NA    NA
SD.log_k_JSE76    0.6514    NA    NA
SD.f_cyan_ilr_1   0.3023    NA    NA
SD.f_cyan_ilr_2   0.2959    NA    NA
SD.f_JCZ38_qlogis 1.9984    NA    NA
SD.log_k1         0.5188    NA    NA
SD.log_k2         0.3894    NA    NA
SD.g_qlogis       0.8579    NA    NA


Variance model:
     est. lower upper
a.1 3.239    NA    NA


Backtransformed parameters:
                    est. lower upper
cyan_0          1.026e+02    NA    NA
k_JCZ38         3.064e-02    NA    NA
k_J9Z38         5.692e-03    NA    NA
k_JSE76         3.449e-03    NA    NA
f_cyan_to_JCZ38 5.798e-01    NA    NA
f_cyan_to_J9Z38 2.243e-01    NA    NA
f_JCZ38_to_JSE76 1.000e+00   NA    NA
k1              1.587e-01    NA    NA
k2              1.120e-02    NA    NA
g               3.452e-01    NA    NA


Resulting formation fractions:
                ff
cyan_JCZ38   0.5798
cyan_J9Z38   0.2243
cyan_sink    0.1958
JCZ38_JSE76  1.0000
JCZ38_sink   0.0000


Estimated disappearance times:
        DT50   DT90 DT50back DT50_k1 DT50_k2
cyan   25.21 167.73    50.49   4.368   61.87
JCZ38  22.62  75.15       NA      NA      NA
J9Z38 121.77 404.50       NA      NA      NA
JSE76 200.98 667.64       NA      NA      NA
```

```
saemix version used for fitting:      3.2
mkin version used for pre-fitting:  1.2.3
R version used for fitting:         4.2.3
Date of fit:     Thu Apr 20 09:58:17 2023
Date of summary: Thu Apr 20 19:09:25 2023


Equations:
d_cyan/dt = - ((k1 * g * exp(-k1 * time) + k2 * (1 - g) * exp(-k2 *
            time)) / (g * exp(-k1 * time) + (1 - g) * exp(-k2 * time)))
            * cyan
d_JCZ38/dt = + f_cyan_to_JCZ38 * ((k1 * g * exp(-k1 * time) + k2 * (1 -
            g) * exp(-k2 * time)) / (g * exp(-k1 * time) + (1 - g) *
            exp(-k2 * time))) * cyan - k_JCZ38 * JCZ38
d_J9Z38/dt = + f_cyan_to_J9Z38 * ((k1 * g * exp(-k1 * time) + k2 * (1 -
            g) * exp(-k2 * time)) / (g * exp(-k1 * time) + (1 - g) *
            exp(-k2 * time))) * cyan - k_J9Z38 * J9Z38
d_JSE76/dt = + f_JCZ38_to_JSE76 * k_JCZ38 * JCZ38 - k_JSE76 * JSE76


Data:
433 observations of 4 variable(s) grouped in 5 datasets


Model predictions using solution type deSolve


Fitted in 681.326 s
Using 300, 100 iterations and 10 chains


Variance model: Two-component variance function


Starting values for degradation parameters:
        cyan_0    log_k_JCZ38    log_k_J9Z38    log_k_JSE76    f_cyan_ilr_1
      101.3964        -3.3626        -4.9792        -5.8727          0.6814
  f_cyan_ilr_2 f_JCZ38_qlogis          log_k1          log_k2        g_qlogis
        6.7799        13.7245        -1.9222        -4.5035         -0.7172


Fixed degradation parameter values:
None


Starting values for random effects (square root of initial entries in omega):
               cyan_0 log_k_JCZ38 log_k_J9Z38 log_k_JSE76 f_cyan_ilr_1
cyan_0          5.317       0.000       0.000       0.000       0.0000
log_k_JCZ38     0.000       2.272       0.000       0.000       0.0000
log_k_J9Z38     0.000       0.000       1.633       0.000       0.0000
log_k_JSE76     0.000       0.000       0.000       1.271       0.0000
f_cyan_ilr_1    0.000       0.000       0.000       0.000       0.6838
f_cyan_ilr_2    0.000       0.000       0.000       0.000       0.0000
f_JCZ38_qlogis  0.000       0.000       0.000       0.000       0.0000
log_k1          0.000       0.000       0.000       0.000       0.0000
log_k2          0.000       0.000       0.000       0.000       0.0000
g_qlogis        0.000       0.000       0.000       0.000       0.0000
               f_cyan_ilr_2 f_JCZ38_qlogis log_k1 log_k2 g_qlogis
cyan_0                 0.00           0.00 0.0000 0.0000    0.000
log_k_JCZ38            0.00           0.00 0.0000 0.0000    0.000
log_k_J9Z38            0.00           0.00 0.0000 0.0000    0.000
log_k_JSE76            0.00           0.00 0.0000 0.0000    0.000
f_cyan_ilr_1           0.00           0.00 0.0000 0.0000    0.000
f_cyan_ilr_2          11.77           0.00 0.0000 0.0000    0.000
f_JCZ38_qlogis         0.00          16.11 0.0000 0.0000    0.000
log_k1                 0.00           0.00 0.9496 0.0000    0.000
log_k2                 0.00           0.00 0.0000 0.5846    0.000
g_qlogis               0.00           0.00 0.0000 0.0000    1.719


Starting values for error model parameters:
a.1 b.1
  1   1


Results:


Likelihood computed by importance sampling
   AIC  BIC logLik
  2398 2390  -1179


Optimised parameters:
                   est. lower upper
cyan_0         100.8076    NA    NA
log_k_JCZ38     -3.4684    NA    NA
log_k_J9Z38     -5.0844    NA    NA
log_k_JSE76     -5.5743    NA    NA
f_cyan_ilr_1     0.6669    NA    NA
f_cyan_ilr_2     0.7912    NA    NA
f_JCZ38_qlogis  84.1825    NA    NA
log_k1          -2.1671    NA    NA
log_k2          -4.5447    NA    NA
```

```
g_qlogis            -0.5631    NA    NA
a.1                  2.9627    NA    NA
b.1                  0.0444    NA    NA
SD.log_k_JCZ38       1.4044    NA    NA
SD.log_k_J9Z38       0.6410    NA    NA
SD.log_k_JSE76       0.5391    NA    NA
SD.f_cyan_ilr_1      0.3203    NA    NA
SD.f_cyan_ilr_2      0.5038    NA    NA
SD.f_JCZ38_qlogis    3.5865    NA    NA
SD.log_k2            0.3119    NA    NA
SD.g_qlogis          0.8276    NA    NA


Correlation is not available

Random effects:
                  est. lower upper
SD.log_k_JCZ38    1.4044    NA    NA
SD.log_k_J9Z38    0.6410    NA    NA
SD.log_k_JSE76    0.5391    NA    NA
SD.f_cyan_ilr_1   0.3203    NA    NA
SD.f_cyan_ilr_2   0.5038    NA    NA
SD.f_JCZ38_qlogis 3.5865    NA    NA
SD.log_k2         0.3119    NA    NA
SD.g_qlogis       0.8276    NA    NA


Variance model:
     est. lower upper
a.1 2.9627    NA    NA
b.1 0.0444    NA    NA


Backtransformed parameters:
                  est. lower upper
cyan_0         1.008e+02    NA    NA
k_JCZ38        3.117e-02    NA    NA
k_J9Z38        6.193e-03    NA    NA
k_JSE76        3.794e-03    NA    NA
f_cyan_to_JCZ38 6.149e-01   NA    NA
f_cyan_to_J9Z38 2.395e-01   NA    NA
f_JCZ38_to_JSE76 1.000e+00  NA    NA
k1             1.145e-01    NA    NA
k2             1.062e-02    NA    NA
g              3.628e-01    NA    NA


Resulting formation fractions:
               ff
cyan_JCZ38  0.6149
cyan_J9Z38  0.2395
cyan_sink   0.1456
JCZ38_JSE76 1.0000
JCZ38_sink  0.0000


Estimated disappearance times:
        DT50   DT90 DT50back DT50_k1 DT50_k2
cyan   26.26 174.32    52.47   6.053   65.25
JCZ38  22.24  73.88       NA      NA      NA
J9Z38 111.93 371.82       NA      NA      NA
JSE76 182.69 606.88       NA      NA      NA
```

Listing 7: Hierarchical SFORB path 1 fit with constant variance

```
saemix version used for fitting:       3.2
mkin version used for pre-fitting:  1.2.3
R version used for fitting:         4.2.3
Date of fit:     Thu Apr 20 09:55:38 2023
Date of summary: Thu Apr 20 19:09:25 2023


Equations:
d_cyan_free/dt = - k_cyan_free * cyan_free - k_cyan_free_bound *
          cyan_free + k_cyan_bound_free * cyan_bound
d_cyan_bound/dt = + k_cyan_free_bound * cyan_free - k_cyan_bound_free *
          cyan_bound
d_JCZ38/dt = + f_cyan_free_to_JCZ38 * k_cyan_free * cyan_free - k_JCZ38
          * JCZ38
d_J9Z38/dt = + f_cyan_free_to_J9Z38 * k_cyan_free * cyan_free - k_J9Z38
          * J9Z38
d_JSE76/dt = + f_JCZ38_to_JSE76 * k_JCZ38 * JCZ38 - k_JSE76 * JSE76


Data:
433 observations of 4 variable(s) grouped in 5 datasets


Model predictions using solution type deSolve


Fitted in 523.165 s
Using 300, 100 iterations and 10 chains


Variance model: Constant variance


Starting values for degradation parameters:
          cyan_free_0       log_k_cyan_free log_k_cyan_free_bound
             102.0643              -2.8987               -2.7077
log_k_cyan_bound_free           log_k_JCZ38           log_k_J9Z38
              -3.4717               -3.4008               -5.0024
          log_k_JSE76           f_cyan_ilr_1          f_cyan_ilr_2
              -5.8613                0.6855                1.2366
       f_JCZ38_qlogis
              13.7418


Fixed degradation parameter values:
None


Starting values for random effects (square root of initial entries in omega):
                      cyan_free_0 log_k_cyan_free log_k_cyan_free_bound
cyan_free_0                 4.466          0.0000                 0.000
log_k_cyan_free             0.000          0.6158                 0.000
log_k_cyan_free_bound       0.000          0.0000                 1.463
log_k_cyan_bound_free       0.000          0.0000                 0.000
log_k_JCZ38                 0.000          0.0000                 0.000
log_k_J9Z38                 0.000          0.0000                 0.000
log_k_JSE76                 0.000          0.0000                 0.000
f_cyan_ilr_1                0.000          0.0000                 0.000
f_cyan_ilr_2                0.000          0.0000                 0.000
f_JCZ38_qlogis              0.000          0.0000                 0.000
                      log_k_cyan_bound_free log_k_JCZ38 log_k_J9Z38 log_k_JSE76
cyan_free_0                           0.000       0.000       0.000       0.000
log_k_cyan_free                       0.000       0.000       0.000       0.000
log_k_cyan_free_bound                 0.000       0.000       0.000       0.000
log_k_cyan_bound_free                 1.058       0.000       0.000       0.000
log_k_JCZ38                           0.000       2.382       0.000       0.000
log_k_J9Z38                           0.000       0.000       1.595       0.000
log_k_JSE76                           0.000       0.000       0.000       1.245
f_cyan_ilr_1                          0.000       0.000       0.000       0.000
f_cyan_ilr_2                          0.000       0.000       0.000       0.000
f_JCZ38_qlogis                        0.000       0.000       0.000       0.000
                      f_cyan_ilr_1 f_cyan_ilr_2 f_JCZ38_qlogis
cyan_free_0                 0.0000         0.00           0.00
log_k_cyan_free             0.0000         0.00           0.00
log_k_cyan_free_bound       0.0000         0.00           0.00
log_k_cyan_bound_free       0.0000         0.00           0.00
log_k_JCZ38                 0.0000         0.00           0.00
log_k_J9Z38                 0.0000         0.00           0.00
log_k_JSE76                 0.0000         0.00           0.00
f_cyan_ilr_1                0.6852         0.00           0.00
f_cyan_ilr_2                0.0000         1.28           0.00
f_JCZ38_qlogis              0.0000         0.00          16.14


Starting values for error model parameters:
a.1
  1


Results:


Likelihood computed by importance sampling
```

```
   AIC  BIC logLik
  2401 2394  -1181


Optimised parameters:
                          est. lower upper
cyan_free_0             102.7803    NA    NA
log_k_cyan_free         -2.8068    NA    NA
log_k_cyan_free_bound   -2.5714    NA    NA
log_k_cyan_bound_free   -3.4426    NA    NA
log_k_JCZ38             -3.4994    NA    NA
log_k_J9Z38             -5.1148    NA    NA
log_k_JSE76             -5.6335    NA    NA
f_cyan_ilr_1             0.6597    NA    NA
f_cyan_ilr_2             0.5132    NA    NA
f_JCZ38_qlogis          37.2090    NA    NA
a.1                      3.2367    NA    NA
SD.log_k_cyan_free       0.3161    NA    NA
SD.log_k_cyan_free_bound 0.8103    NA    NA
SD.log_k_cyan_bound_free 0.5554    NA    NA
SD.log_k_JCZ38           1.4858    NA    NA
SD.log_k_J9Z38           0.5859    NA    NA
SD.log_k_JSE76           0.6195    NA    NA
SD.f_cyan_ilr_1          0.3118    NA    NA
SD.f_cyan_ilr_2          0.3344    NA    NA
SD.f_JCZ38_qlogis        0.5518    NA    NA


Correlation is not available

Random effects:
                          est. lower upper
SD.log_k_cyan_free       0.3161    NA    NA
SD.log_k_cyan_free_bound 0.8103    NA    NA
SD.log_k_cyan_bound_free 0.5554    NA    NA
SD.log_k_JCZ38           1.4858    NA    NA
SD.log_k_J9Z38           0.5859    NA    NA
SD.log_k_JSE76           0.6195    NA    NA
SD.f_cyan_ilr_1          0.3118    NA    NA
SD.f_cyan_ilr_2          0.3344    NA    NA
SD.f_JCZ38_qlogis        0.5518    NA    NA


Variance model:
     est. lower upper
a.1 3.237    NA    NA


Backtransformed parameters:
                         est. lower upper
cyan_free_0            1.028e+02    NA    NA
k_cyan_free            6.040e-02    NA    NA
k_cyan_free_bound      7.643e-02    NA    NA
k_cyan_bound_free      3.198e-02    NA    NA
k_JCZ38                3.022e-02    NA    NA
k_J9Z38                6.007e-03    NA    NA
k_JSE76                3.576e-03    NA    NA
f_cyan_free_to_JCZ38   5.787e-01    NA    NA
f_cyan_free_to_J9Z38   2.277e-01    NA    NA
f_JCZ38_to_JSE76       1.000e+00    NA    NA


Estimated Eigenvalues of SFORB model(s):
cyan_b1 cyan_b2  cyan_g
0.15646 0.01235 0.33341


Resulting formation fractions:
                 ff
cyan_free_JCZ38 0.5787
cyan_free_J9Z38 0.2277
cyan_free_sink  0.1936
cyan_free       1.0000
JCZ38_JSE76     1.0000
JCZ38_sink      0.0000


Estimated disappearance times:
        DT50   DT90 DT50back DT50_cyan_b1 DT50_cyan_b2
cyan    24.48  153.7   46.26         4.43        56.15
JCZ38   22.94   76.2     NA           NA           NA
J9Z38  115.39  383.3     NA           NA           NA
JSE76  193.84  643.9     NA           NA           NA
```

```
saemix version used for fitting:      3.2
mkin version used for pre-fitting:  1.2.3
R version used for fitting:        4.2.3
Date of fit:     Thu Apr 20 09:58:21 2023
Date of summary: Thu Apr 20 19:09:25 2023


Equations:
d_cyan_free/dt = - k_cyan_free * cyan_free - k_cyan_free_bound *
         cyan_free + k_cyan_bound_free * cyan_bound
d_cyan_bound/dt = + k_cyan_free_bound * cyan_free - k_cyan_bound_free *
         cyan_bound
d_JCZ38/dt = + f_cyan_free_to_JCZ38 * k_cyan_free * cyan_free - k_JCZ38
         * JCZ38
d_J9Z38/dt = + f_cyan_free_to_J9Z38 * k_cyan_free * cyan_free - k_J9Z38
         * J9Z38
d_JSE76/dt = + f_JCZ38_to_JSE76 * k_JCZ38 * JCZ38 - k_JSE76 * JSE76

Data:
433 observations of 4 variable(s) grouped in 5 datasets

Model predictions using solution type deSolve

Fitted in 685.784 s
Using 300, 100 iterations and 10 chains

Variance model: Two-component variance function

Starting values for degradation parameters:
        cyan_free_0      log_k_cyan_free log_k_cyan_free_bound
           101.3964             -2.9881              -2.7949
log_k_cyan_bound_free         log_k_JCZ38          log_k_J9Z38
            -3.4376             -3.3626              -4.9792
        log_k_JSE76          f_cyan_ilr_1         f_cyan_ilr_2
            -5.8727              0.6814               6.8139
      f_JCZ38_qlogis
           13.7419

Fixed degradation parameter values:
None

Starting values for random effects (square root of initial entries in omega):
                    cyan_free_0 log_k_cyan_free log_k_cyan_free_bound
cyan_free_0               5.317          0.0000                 0.000
log_k_cyan_free           0.000          0.7301                 0.000
log_k_cyan_free_bound     0.000          0.0000                 1.384
log_k_cyan_bound_free     0.000          0.0000                 0.000
log_k_JCZ38               0.000          0.0000                 0.000
log_k_J9Z38               0.000          0.0000                 0.000
log_k_JSE76               0.000          0.0000                 0.000
f_cyan_ilr_1              0.000          0.0000                 0.000
f_cyan_ilr_2              0.000          0.0000                 0.000
f_JCZ38_qlogis            0.000          0.0000                 0.000
                    log_k_cyan_bound_free log_k_JCZ38 log_k_J9Z38 log_k_JSE76
cyan_free_0                         0.000       0.000       0.000       0.000
log_k_cyan_free                     0.000       0.000       0.000       0.000
log_k_cyan_free_bound               0.000       0.000       0.000       0.000
log_k_cyan_bound_free               1.109       0.000       0.000       0.000
log_k_JCZ38                         0.000       2.272       0.000       0.000
log_k_J9Z38                         0.000       0.000       1.633       0.000
log_k_JSE76                         0.000       0.000       0.000       1.271
f_cyan_ilr_1                        0.000       0.000       0.000       0.000
f_cyan_ilr_2                        0.000       0.000       0.000       0.000
f_JCZ38_qlogis                      0.000       0.000       0.000       0.000
                    f_cyan_ilr_1 f_cyan_ilr_2 f_JCZ38_qlogis
cyan_free_0               0.0000         0.00           0.00
log_k_cyan_free           0.0000         0.00           0.00
log_k_cyan_free_bound     0.0000         0.00           0.00
log_k_cyan_bound_free     0.0000         0.00           0.00
log_k_JCZ38               0.0000         0.00           0.00
log_k_J9Z38               0.0000         0.00           0.00
log_k_JSE76               0.0000         0.00           0.00
f_cyan_ilr_1              0.6838         0.00           0.00
f_cyan_ilr_2              0.0000        11.84           0.00
f_JCZ38_qlogis            0.0000         0.00          16.14

Starting values for error model parameters:
a.1 b.1
  1   1

Results:

Likelihood computed by importance sampling
```

```
   AIC  BIC logLik
  2400 2392  -1180


Optimised parameters:
                            est.  lower upper
cyan_free_0             100.69983   NA    NA
log_k_cyan_free          -3.11584   NA    NA
log_k_cyan_free_bound    -3.15216   NA    NA
log_k_cyan_bound_free    -3.65986   NA    NA
log_k_JCZ38              -3.47811   NA    NA
log_k_J9Z38             -5.08835   NA    NA
log_k_JSE76             -5.55514   NA    NA
f_cyan_ilr_1             0.66764   NA    NA
f_cyan_ilr_2             0.78329   NA    NA
f_JCZ38_qlogis          25.35245   NA    NA
a.1                      2.99088   NA    NA
b.1                      0.04346   NA    NA
SD.log_k_cyan_free       0.48797   NA    NA
SD.log_k_cyan_bound_free 0.27243   NA    NA
SD.log_k_JCZ38           1.42450   NA    NA
SD.log_k_J9Z38           0.63496   NA    NA
SD.log_k_JSE76           0.55951   NA    NA
SD.f_cyan_ilr_1          0.32687   NA    NA
SD.f_cyan_ilr_2          0.48056   NA    NA
SD.f_JCZ38_qlogis        0.43818   NA    NA


Correlation is not available

Random effects:
                          est. lower upper
SD.log_k_cyan_free       0.4880   NA    NA
SD.log_k_cyan_bound_free 0.2724   NA    NA
SD.log_k_JCZ38           1.4245   NA    NA
SD.log_k_J9Z38           0.6350   NA    NA
SD.log_k_JSE76           0.5595   NA    NA
SD.f_cyan_ilr_1          0.3269   NA    NA
SD.f_cyan_ilr_2          0.4806   NA    NA
SD.f_JCZ38_qlogis        0.4382   NA    NA


Variance model:
      est. lower upper
a.1 2.99088   NA    NA
b.1 0.04346   NA    NA


Backtransformed parameters:
                       est. lower upper
cyan_free_0         1.007e+02   NA    NA
k_cyan_free         4.434e-02   NA    NA
k_cyan_free_bound   4.276e-02   NA    NA
k_cyan_bound_free   2.574e-02   NA    NA
k_JCZ38             3.087e-02   NA    NA
k_J9Z38             6.168e-03   NA    NA
k_JSE76             3.868e-03   NA    NA
f_cyan_free_to_JCZ38 6.143e-01   NA    NA
f_cyan_free_to_J9Z38 2.389e-01   NA    NA
f_JCZ38_to_JSE76    1.000e+00   NA    NA


Estimated Eigenvalues of SFORB model(s):
cyan_b1 cyan_b2  cyan_g
0.10161 0.01123 0.36636


Resulting formation fractions:
                  ff
cyan_free_JCZ38 6.143e-01
cyan_free_J9Z38 2.389e-01
cyan_free_sink  1.468e-01
cyan_free       1.000e+00
JCZ38_JSE76     1.000e+00
JCZ38_sink      9.763e-12


Estimated disappearance times:
        DT50  DT90 DT50back DT50_cyan_b1 DT50_cyan_b2
cyan   25.91 164.4   49.49        6.822        61.72
JCZ38  22.46  74.6     NA           NA           NA
J9Z38 112.37 373.3     NA           NA           NA
JSE76 179.22 595.4     NA           NA           NA
```

Listing 9: Hierarchical HS path 1 fit with constant variance

```
saemix version used for fitting:      3.2
mkin version used for pre-fitting:  1.2.3
R version used for fitting:         4.2.3
Date of fit:     Thu Apr 20 09:55:31 2023
Date of summary: Thu Apr 20 19:09:25 2023


Equations:
d_cyan/dt = - ifelse(time <= tb, k1, k2) * cyan
d_JCZ38/dt = + f_cyan_to_JCZ38 * ifelse(time <= tb, k1, k2) * cyan -
           k_JCZ38 * JCZ38
d_J9Z38/dt = + f_cyan_to_J9Z38 * ifelse(time <= tb, k1, k2) * cyan -
           k_J9Z38 * J9Z38
d_JSE76/dt = + f_JCZ38_to_JSE76 * k_JCZ38 * JCZ38 - k_JSE76 * JSE76


Data:
433 observations of 4 variable(s) grouped in 5 datasets


Model predictions using solution type deSolve


Fitted in 515.987 s
Using 300, 100 iterations and 10 chains


Variance model: Constant variance


Starting values for degradation parameters:
        cyan_0   log_k_JCZ38    log_k_J9Z38    log_k_JSE76   f_cyan_ilr_1
       102.8738       -3.4490        -4.9348        -5.5989         0.6469
   f_cyan_ilr_2 f_JCZ38_qlogis         log_k1         log_k2         log_tb
         1.2854        9.7193        -2.9084        -4.1810         1.7813


Fixed degradation parameter values:
None


Starting values for random effects (square root of initial entries in omega):
              cyan_0 log_k_JCZ38 log_k_J9Z38 log_k_JSE76 f_cyan_ilr_1
cyan_0         5.409        0.00        0.00       0.000       0.0000
log_k_JCZ38    0.000        2.33        0.00       0.000       0.0000
log_k_J9Z38    0.000        0.00        1.59       0.000       0.0000
log_k_JSE76    0.000        0.00        0.00       1.006       0.0000
f_cyan_ilr_1   0.000        0.00        0.00       0.000       0.6371
f_cyan_ilr_2   0.000        0.00        0.00       0.000       0.0000
f_JCZ38_qlogis 0.000        0.00        0.00       0.000       0.0000
log_k1         0.000        0.00        0.00       0.000       0.0000
log_k2         0.000        0.00        0.00       0.000       0.0000
log_tb         0.000        0.00        0.00       0.000       0.0000
              f_cyan_ilr_2 f_JCZ38_qlogis log_k1 log_k2 log_tb
cyan_0               0.000           0.00 0.0000 0.0000 0.0000
log_k_JCZ38          0.000           0.00 0.0000 0.0000 0.0000
log_k_J9Z38          0.000           0.00 0.0000 0.0000 0.0000
log_k_JSE76          0.000           0.00 0.0000 0.0000 0.0000
f_cyan_ilr_1         0.000           0.00 0.0000 0.0000 0.0000
f_cyan_ilr_2         2.167           0.00 0.0000 0.0000 0.0000
f_JCZ38_qlogis       0.000          10.22 0.0000 0.0000 0.0000
log_k1               0.000           0.00 0.7003 0.0000 0.0000
log_k2               0.000           0.00 0.0000 0.8928 0.0000
log_tb               0.000           0.00 0.0000 0.0000 0.6774


Starting values for error model parameters:
a.1
  1


Results:


Likelihood computed by importance sampling
   AIC  BIC logLik
  2427 2420  -1194


Optimised parameters:
                   est. lower upper
cyan_0         101.84849    NA    NA
log_k_JCZ38     -3.47365    NA    NA
log_k_J9Z38     -5.10562    NA    NA
log_k_JSE76     -5.60318    NA    NA
f_cyan_ilr_1     0.66127    NA    NA
f_cyan_ilr_2     0.60283    NA    NA
f_JCZ38_qlogis  45.06408    NA    NA
log_k1          -3.10124    NA    NA
log_k2          -4.39028    NA    NA
log_tb           2.32256    NA    NA
a.1              3.32683    NA    NA
SD.log_k_JCZ38   1.41427    NA    NA
SD.log_k_J9Z38   0.54767    NA    NA
```

```
SD.log_k_JSE76        0.62147      NA    NA
SD.f_cyan_ilr_1       0.30189      NA    NA
SD.f_cyan_ilr_2       0.34960      NA    NA
SD.f_JCZ38_qlogis     0.04644      NA    NA
SD.log_k1             0.39534      NA    NA
SD.log_k2             0.43468      NA    NA
SD.log_tb             0.60781      NA    NA


Correlation is not available

Random effects:
                     est. lower upper
SD.log_k_JCZ38     1.41427    NA    NA
SD.log_k_J9Z38     0.54767    NA    NA
SD.log_k_JSE76     0.62147    NA    NA
SD.f_cyan_ilr_1    0.30189    NA    NA
SD.f_cyan_ilr_2    0.34960    NA    NA
SD.f_JCZ38_qlogis  0.04644    NA    NA
SD.log_k1          0.39534    NA    NA
SD.log_k2          0.43468    NA    NA
SD.log_tb          0.60781    NA    NA


Variance model:
    est. lower upper
a.1 3.327    NA    NA


Backtransformed parameters:
                    est. lower upper
cyan_0           1.018e+02    NA    NA
k_JCZ38          3.100e-02    NA    NA
k_J9Z38          6.063e-03    NA    NA
k_JSE76          3.686e-03    NA    NA
f_cyan_to_JCZ38  5.910e-01    NA    NA
f_cyan_to_J9Z38  2.320e-01    NA    NA
f_JCZ38_to_JSE76 1.000e+00    NA    NA
k1               4.499e-02    NA    NA
k2               1.240e-02    NA    NA
tb               1.020e+01    NA    NA


Resulting formation fractions:
             ff
cyan_JCZ38  0.591
cyan_J9Z38  0.232
cyan_sink   0.177
JCZ38_JSE76 1.000
JCZ38_sink  0.000


Estimated disappearance times:
        DT50   DT90 DT50back DT50_k1 DT50_k2
cyan   29.09 158.91    47.84   15.41   55.91
JCZ38  22.36  74.27       NA      NA      NA
J9Z38 114.33 379.80       NA      NA      NA
JSE76 188.04 624.66       NA      NA      NA
```

```
saemix version used for fitting:       3.2
mkin version used for pre-fitting:  1.2.3
R version used for fitting:           4.2.3
Date of fit:      Thu Apr 20 09:55:40 2023
Date of summary: Thu Apr 20 19:09:25 2023


Equations:
d_cyan/dt = - ifelse(time <= tb, k1, k2) * cyan
d_JCZ38/dt = + f_cyan_to_JCZ38 * ifelse(time <= tb, k1, k2) * cyan -
         k_JCZ38 * JCZ38
d_J9Z38/dt = + f_cyan_to_J9Z38 * ifelse(time <= tb, k1, k2) * cyan -
         k_J9Z38 * J9Z38
d_JSE76/dt = + f_JCZ38_to_JSE76 * k_JCZ38 * JCZ38 - k_JSE76 * JSE76


Data:
433 observations of 4 variable(s) grouped in 5 datasets


Model predictions using solution type deSolve


Fitted in 524.851 s
Using 300, 100 iterations and 10 chains


Variance model: Two-component variance function


Starting values for degradation parameters:
       cyan_0    log_k_JCZ38    log_k_J9Z38    log_k_JSE76   f_cyan_ilr_1
       101.168         -3.358         -4.941         -5.794          0.676
  f_cyan_ilr_2 f_JCZ38_qlogis        log_k1         log_k2         log_tb
         5.740         13.863         -3.147         -4.262          2.173


Fixed degradation parameter values:
None


Starting values for random effects (square root of initial entries in omega):
             cyan_0 log_k_JCZ38 log_k_J9Z38 log_k_JSE76 f_cyan_ilr_1
cyan_0         5.79       0.000       0.000       0.000       0.0000
log_k_JCZ38    0.00       2.271       0.000       0.000       0.0000
log_k_J9Z38    0.00       0.000       1.614       0.000       0.0000
log_k_JSE76    0.00       0.000       0.000       1.264       0.0000
f_cyan_ilr_1   0.00       0.000       0.000       0.000       0.6761
f_cyan_ilr_2   0.00       0.000       0.000       0.000       0.0000
f_JCZ38_qlogis 0.00       0.000       0.000       0.000       0.0000
log_k1         0.00       0.000       0.000       0.000       0.0000
log_k2         0.00       0.000       0.000       0.000       0.0000
log_tb         0.00       0.000       0.000       0.000       0.0000
             f_cyan_ilr_2 f_JCZ38_qlogis log_k1 log_k2 log_tb
cyan_0              0.000           0.00 0.0000 0.0000  0.000
log_k_JCZ38         0.000           0.00 0.0000 0.0000  0.000
log_k_J9Z38         0.000           0.00 0.0000 0.0000  0.000
log_k_JSE76         0.000           0.00 0.0000 0.0000  0.000
f_cyan_ilr_1        0.000           0.00 0.0000 0.0000  0.000
f_cyan_ilr_2        9.572           0.00 0.0000 0.0000  0.000
f_JCZ38_qlogis      0.000          19.19 0.0000 0.0000  0.000
log_k1              0.000           0.00 0.8705 0.0000  0.000
log_k2              0.000           0.00 0.0000 0.9288  0.000
log_tb              0.000           0.00 0.0000 0.0000  1.065


Starting values for error model parameters:
a.1 b.1
  1   1


Results:


Likelihood computed by importance sampling
   AIC  BIC logLik
  2422 2414  -1190


Optimised parameters:
                 est. lower upper
cyan_0        100.9521    NA    NA
log_k_JCZ38    -3.4629    NA    NA
log_k_J9Z38    -5.0346    NA    NA
log_k_JSE76    -5.5722    NA    NA
f_cyan_ilr_1    0.6560    NA    NA
f_cyan_ilr_2    0.7983    NA    NA
f_JCZ38_qlogis 42.7949    NA    NA
log_k1         -3.1721    NA    NA
log_k2         -4.4039    NA    NA
log_tb          2.3994    NA    NA
a.1             3.0586    NA    NA
b.1             0.0380    NA    NA
SD.log_k_JCZ38  1.3754    NA    NA
```

```
SD.log_k_J9Z38        0.6703    NA    NA
SD.log_k_JSE76        0.5876    NA    NA
SD.f_cyan_ilr_1       0.3272    NA    NA
SD.f_cyan_ilr_2       0.5300    NA    NA
SD.f_JCZ38_qlogis     6.4465    NA    NA
SD.log_k1             0.4135    NA    NA
SD.log_k2             0.4182    NA    NA
SD.log_tb             0.6035    NA    NA


Correlation is not available

Random effects:
                   est. lower upper
SD.log_k_JCZ38     1.3754   NA    NA
SD.log_k_J9Z38     0.6703   NA    NA
SD.log_k_JSE76     0.5876   NA    NA
SD.f_cyan_ilr_1    0.3272   NA    NA
SD.f_cyan_ilr_2    0.5300   NA    NA
SD.f_JCZ38_qlogis  6.4465   NA    NA
SD.log_k1          0.4135   NA    NA
SD.log_k2          0.4182   NA    NA
SD.log_tb          0.6035   NA    NA

Variance model:
     est. lower upper
a.1 3.059   NA    NA
b.1 0.038   NA    NA

Backtransformed parameters:
                     est. lower upper
cyan_0          1.010e+02   NA    NA
k_JCZ38         3.134e-02   NA    NA
k_J9Z38         6.509e-03   NA    NA
k_JSE76         3.802e-03   NA    NA
f_cyan_to_JCZ38 6.127e-01   NA    NA
f_cyan_to_J9Z38 2.423e-01   NA    NA
f_JCZ38_to_JSE76 1.000e+00   NA    NA
k1              4.191e-02   NA    NA
k2              1.223e-02   NA    NA
tb              1.102e+01   NA    NA

Resulting formation fractions:
                ff
cyan_JCZ38   0.6127
cyan_J9Z38   0.2423
cyan_sink    0.1449
JCZ38_JSE76  1.0000
JCZ38_sink   0.0000

Estimated disappearance times:
        DT50   DT90 DT50back DT50_k1 DT50_k2
cyan   29.94 161.54    48.63   16.54   56.68
JCZ38  22.12  73.47       NA      NA      NA
J9Z38 106.50 353.77       NA      NA      NA
JSE76 182.30 605.60       NA      NA      NA
```

Pathway 2

Listing 11: Hierarchical FOMC path 2 fit with constant variance

```
saemix version used for fitting:      3.2
mkin version used for pre-fitting:  1.2.3
R version used for fitting:       4.2.3
Date of fit:     Thu Apr 20 10:06:44 2023
Date of summary: Thu Apr 20 19:09:25 2023


Equations:
d_cyan/dt = - (alpha/beta) * 1/((time/beta) + 1) * cyan
d_JCZ38/dt = + f_cyan_to_JCZ38 * (alpha/beta) * 1/((time/beta) + 1) *
            cyan - k_JCZ38 * JCZ38 + f_JSE76_to_JCZ38 * k_JSE76 * JSE76
d_J9Z38/dt = + f_cyan_to_J9Z38 * (alpha/beta) * 1/((time/beta) + 1) *
            cyan - k_J9Z38 * J9Z38
d_JSE76/dt = + f_JCZ38_to_JSE76 * k_JCZ38 * JCZ38 - k_JSE76 * JSE76


Data:
433 observations of 4 variable(s) grouped in 5 datasets


Model predictions using solution type deSolve


Fitted in 494.416 s
Using 300, 100 iterations and 10 chains


Variance model: Constant variance


Starting values for degradation parameters:
        cyan_0    log_k_JCZ38   log_k_J9Z38    log_k_JSE76    f_cyan_ilr_1
       101.8173       -1.8998      -5.1449        -2.5415          0.6705
  f_cyan_ilr_2 f_JCZ38_qlogis f_JSE76_qlogis    log_alpha        log_beta
        4.4669        16.1281       13.3327       -0.2314          2.8738


Fixed degradation parameter values:
None


Starting values for random effects (square root of initial entries in omega):
            cyan_0 log_k_JCZ38 log_k_J9Z38 log_k_JSE76 f_cyan_ilr_1
cyan_0       5.742       0.000       0.000       0.00       0.0000
log_k_JCZ38  0.000       1.402       0.000       0.00       0.0000
log_k_J9Z38  0.000       0.000       1.718       0.00       0.0000
log_k_JSE76  0.000       0.000       0.000       3.57       0.0000
f_cyan_ilr_1 0.000       0.000       0.000       0.00       0.5926
f_cyan_ilr_2 0.000       0.000       0.000       0.00       0.0000
f_JCZ38_qlogis 0.000     0.000       0.000       0.00       0.0000
f_JSE76_qlogis 0.000     0.000       0.000       0.00       0.0000
log_alpha    0.000       0.000       0.000       0.00       0.0000
log_beta     0.000       0.000       0.000       0.00       0.0000
            f_cyan_ilr_2 f_JCZ38_qlogis f_JSE76_qlogis log_alpha log_beta
cyan_0              0.00           0.00           0.00    0.0000   0.0000
log_k_JCZ38         0.00           0.00           0.00    0.0000   0.0000
log_k_J9Z38         0.00           0.00           0.00    0.0000   0.0000
log_k_JSE76         0.00           0.00           0.00    0.0000   0.0000
f_cyan_ilr_1        0.00           0.00           0.00    0.0000   0.0000
f_cyan_ilr_2       10.56           0.00           0.00    0.0000   0.0000
f_JCZ38_qlogis      0.00          12.04           0.00    0.0000   0.0000
f_JSE76_qlogis      0.00           0.00          15.26    0.0000   0.0000
log_alpha           0.00           0.00           0.00    0.4708   0.0000
log_beta            0.00           0.00           0.00    0.0000   0.4432


Starting values for error model parameters:
a.1
  1


Results:


Likelihood computed by importance sampling
   AIC  BIC logLik
  2308 2301  -1134


Optimised parameters:
                 est.     lower     upper
cyan_0        101.9586 99.22024 104.69700
log_k_JCZ38    -2.4861 -3.17661  -1.79560
log_k_J9Z38    -5.3926 -6.08842  -4.69684
log_k_JSE76    -3.1193 -4.12904  -2.10962
f_cyan_ilr_1    0.7368  0.42085   1.05276
f_cyan_ilr_2    0.6196  0.06052   1.17861
f_JCZ38_qlogis  4.8970 -4.68003  14.47398
f_JSE76_qlogis  4.4066 -1.02087   9.83398
log_alpha      -0.3021 -0.68264   0.07838
log_beta        2.7438  2.57970   2.90786
a.1             2.9008  2.69920   3.10245
```

```
SD.cyan_0         2.7081  0.64216   4.77401
SD.log_k_JCZ38    0.7043  0.19951   1.20907
SD.log_k_J9Z38    0.6248  0.05790   1.19180
SD.log_k_JSE76    1.0750  0.33157   1.81839
SD.f_cyan_ilr_1   0.3429  0.11688   0.56892
SD.f_cyan_ilr_2   0.4774  0.09381   0.86097
SD.f_JCZ38_qlogis 1.5565 -7.83970  10.95279
SD.f_JSE76_qlogis 1.6871 -1.25577   4.63000
SD.log_alpha      0.4216  0.15913   0.68405


Correlation:
              cyan_0  l__JCZ3  l__J9Z3  l__JSE7  f_cy__1  f_cy__2  f_JCZ38  f_JSE76
log_k_JCZ38   -0.0167
log_k_J9Z38   -0.0307  0.0057
log_k_JSE76   -0.0032  0.1358  0.0009
f_cyan_ilr_1  -0.0087  0.0206 -0.1158 -0.0009
f_cyan_ilr_2  -0.1598  0.0690  0.1770  0.0002 -0.0007
f_JCZ38_qlogis 0.0966 -0.1132 -0.0440  0.0182 -0.1385 -0.4583
f_JSE76_qlogis -0.0647 0.1157  0.0333 -0.0026  0.1110  0.3620 -0.8586
log_alpha     -0.0389  0.0113  0.0209  0.0021  0.0041  0.0451 -0.0605  0.0412
log_beta      -0.2508  0.0533  0.0977  0.0098  0.0220  0.2741 -0.2934  0.1999
              log_lph
log_k_JCZ38
log_k_J9Z38
log_k_JSE76
f_cyan_ilr_1
f_cyan_ilr_2
f_JCZ38_qlogis
f_JSE76_qlogis
log_alpha
log_beta        0.2281

Random effects:
                 est.     lower    upper
SD.cyan_0         2.7081  0.64216   4.7740
SD.log_k_JCZ38    0.7043  0.19951   1.2091
SD.log_k_J9Z38    0.6248  0.05790   1.1918
SD.log_k_JSE76    1.0750  0.33157   1.8184
SD.f_cyan_ilr_1   0.3429  0.11688   0.5689
SD.f_cyan_ilr_2   0.4774  0.09381   0.8610
SD.f_JCZ38_qlogis 1.5565 -7.83970  10.9528
SD.f_JSE76_qlogis 1.6871 -1.25577   4.6300
SD.log_alpha      0.4216  0.15913   0.6840

Variance model:
    est. lower upper
a.1 2.901 2.699 3.102

Backtransformed parameters:
                    est.      lower      upper
cyan_0          101.95862 99.220240  1.047e+02
k_JCZ38           0.08323  0.041727  1.660e-01
k_J9Z38           0.00455  0.002269  9.124e-03
k_JSE76           0.04419  0.016098  1.213e-01
f_cyan_to_JCZ38   0.61318        NA         NA
f_cyan_to_J9Z38   0.21630        NA         NA
f_JCZ38_to_JSE76  0.99259  0.009193  1.000e+00
f_JSE76_to_JCZ38  0.98795  0.264857  9.999e-01
alpha             0.73924  0.505281  1.082e+00
beta             15.54568 13.193194  1.832e+01

Resulting formation fractions:
                ff
cyan_JCZ38   0.613182
cyan_J9Z38   0.216298
cyan_sink    0.170519
JCZ38_JSE76  0.992586
JCZ38_sink   0.007414
JSE76_JCZ38  0.987950
JSE76_sink   0.012050

Estimated disappearance times:
         DT50    DT90 DT50back
cyan   24.157  334.68    100.7
JCZ38   8.328   27.66       NA
J9Z38 152.341  506.06       NA
JSE76  15.687   52.11       NA
```

```
saemix version used for fitting:      3.2
mkin version used for pre-fitting:  1.2.3
R version used for fitting:         4.2.3
Date of fit:     Thu Apr 20 10:06:50 2023
Date of summary: Thu Apr 20 19:09:25 2023


Equations:
d_cyan/dt = - (alpha/beta) * 1/((time/beta) + 1) * cyan
d_JCZ38/dt = + f_cyan_to_JCZ38 * (alpha/beta) * 1/((time/beta) + 1) *
             cyan - k_JCZ38 * JCZ38 + f_JSE76_to_JCZ38 * k_JSE76 * JSE76
d_J9Z38/dt = + f_cyan_to_J9Z38 * (alpha/beta) * 1/((time/beta) + 1) *
             cyan - k_J9Z38 * J9Z38
d_JSE76/dt = + f_JCZ38_to_JSE76 * k_JCZ38 * JCZ38 - k_JSE76 * JSE76


Data:
433 observations of 4 variable(s) grouped in 5 datasets


Model predictions using solution type deSolve


Fitted in 499.982 s
Using 300, 100 iterations and 10 chains


Variance model: Two-component variance function


Starting values for degradation parameters:
       cyan_0   log_k_JCZ38   log_k_J9Z38   log_k_JSE76    f_cyan_ilr_1
      101.9028       -1.9055       -5.0249       -2.5646          0.6807
  f_cyan_ilr_2 f_JCZ38_qlogis f_JSE76_qlogis     log_alpha       log_beta
        4.8883        16.0676        9.3923       -0.1346         3.0364


Fixed degradation parameter values:
None


Starting values for random effects (square root of initial entries in omega):
             cyan_0 log_k_JCZ38 log_k_J9Z38 log_k_JSE76 f_cyan_ilr_1
cyan_0        6.321       0.000       0.000       0.000       0.0000
log_k_JCZ38   0.000       1.392       0.000       0.000       0.0000
log_k_J9Z38   0.000       0.000       1.561       0.000       0.0000
log_k_JSE76   0.000       0.000       0.000       3.614       0.0000
f_cyan_ilr_1  0.000       0.000       0.000       0.000       0.6339
f_cyan_ilr_2  0.000       0.000       0.000       0.000       0.0000
f_JCZ38_qlogis 0.000      0.000       0.000       0.000       0.0000
f_JSE76_qlogis 0.000      0.000       0.000       0.000       0.0000
log_alpha     0.000       0.000       0.000       0.000       0.0000
log_beta      0.000       0.000       0.000       0.000       0.0000
             f_cyan_ilr_2 f_JCZ38_qlogis f_JSE76_qlogis log_alpha log_beta
cyan_0               0.00           0.00           0.00    0.0000   0.0000
log_k_JCZ38          0.00           0.00           0.00    0.0000   0.0000
log_k_J9Z38          0.00           0.00           0.00    0.0000   0.0000
log_k_JSE76          0.00           0.00           0.00    0.0000   0.0000
f_cyan_ilr_1         0.00           0.00           0.00    0.0000   0.0000
f_cyan_ilr_2        10.41           0.00           0.00    0.0000   0.0000
f_JCZ38_qlogis       0.00          12.24           0.00    0.0000   0.0000
f_JSE76_qlogis       0.00           0.00          15.13    0.0000   0.0000
log_alpha            0.00           0.00           0.00    0.3701   0.0000
log_beta             0.00           0.00           0.00    0.0000   0.5662


Starting values for error model parameters:
a.1 b.1
  1   1


Results:


Likelihood computed by importance sampling
   AIC  BIC logLik
  2248 2240  -1103


Optimised parameters:
                    est.      lower      upper
cyan_0          101.55545  9.920e+01  1.039e+02
log_k_JCZ38      -2.37354 -2.928e+00 -1.819e+00
log_k_J9Z38      -5.14736 -5.960e+00 -4.335e+00
log_k_JSE76      -3.07802 -4.243e+00 -1.913e+00
f_cyan_ilr_1      0.71263  3.655e-01  1.060e+00
f_cyan_ilr_2      0.95202  2.701e-01  1.634e+00
f_JCZ38_qlogis    3.58473  1.251e+00  5.919e+00
f_JSE76_qlogis   19.03623 -1.037e+07  1.037e+07
log_alpha        -0.15297 -4.490e-01  1.431e-01
log_beta          2.99230  2.706e+00  3.278e+00
a.1               2.04816        NA         NA
b.1               0.06886        NA         NA
SD.log_k_JCZ38    0.56174        NA         NA
```

```
SD.log_k_J9Z38        0.86509        NA        NA
SD.log_k_JSE76        1.28450        NA        NA
SD.f_cyan_ilr_1       0.38705        NA        NA
SD.f_cyan_ilr_2       0.54153        NA        NA
SD.f_JCZ38_qlogis     1.65311        NA        NA
SD.f_JSE76_qlogis     7.51468        NA        NA
SD.log_alpha          0.31586        NA        NA
SD.log_beta           0.24696        NA        NA


Correlation is not available

Random effects:
                   est. lower upper
SD.log_k_JCZ38     0.5617    NA    NA
SD.log_k_J9Z38     0.8651    NA    NA
SD.log_k_JSE76     1.2845    NA    NA
SD.f_cyan_ilr_1    0.3870    NA    NA
SD.f_cyan_ilr_2    0.5415    NA    NA
SD.f_JCZ38_qlogis  1.6531    NA    NA
SD.f_JSE76_qlogis  7.5147    NA    NA
SD.log_alpha       0.3159    NA    NA
SD.log_beta        0.2470    NA    NA

Variance model:
        est. lower upper
a.1 2.04816    NA    NA
b.1 0.06886    NA    NA

Backtransformed parameters:
                       est.     lower     upper
cyan_0             1.016e+02  99.20301 103.9079
k_JCZ38            9.315e-02   0.05349   0.1622
k_J9Z38            5.815e-03   0.00258   0.0131
k_JSE76            4.605e-02   0.01436   0.1477
f_cyan_to_JCZ38    6.438e-01        NA        NA
f_cyan_to_J9Z38    2.350e-01        NA        NA
f_JCZ38_to_JSE76   9.730e-01   0.77745   0.9973
f_JSE76_to_JCZ38   1.000e+00   0.00000   1.0000
alpha              8.582e-01   0.63824   1.1538
beta               1.993e+01  14.97621  26.5262

Resulting formation fractions:
                   ff
cyan_JCZ38   6.438e-01
cyan_J9Z38   2.350e-01
cyan_sink    1.212e-01
JCZ38_JSE76  9.730e-01
JCZ38_sink   2.700e-02
JSE76_JCZ38  1.000e+00
JSE76_sink   5.403e-09

Estimated disappearance times:
          DT50    DT90 DT50back
cyan    24.771  271.70    81.79
JCZ38    7.441   24.72       NA
J9Z38  119.205  395.99       NA
JSE76   15.052   50.00       NA
```

```
saemix version used for fitting:      3.2
mkin version used for pre-fitting:  1.2.3
R version used for fitting:        4.2.3
Date of fit:     Thu Apr 20 10:07:39 2023
Date of summary: Thu Apr 20 19:09:25 2023


Equations:
d_cyan/dt = - ((k1 * g * exp(-k1 * time) + k2 * (1 - g) * exp(-k2 *
            time)) / (g * exp(-k1 * time) + (1 - g) * exp(-k2 * time)))
            * cyan
d_JCZ38/dt = + f_cyan_to_JCZ38 * ((k1 * g * exp(-k1 * time) + k2 * (1 -
            g) * exp(-k2 * time)) / (g * exp(-k1 * time) + (1 - g) *
            exp(-k2 * time))) * cyan - k_JCZ38 * JCZ38 +
            f_JSE76_to_JCZ38 * k_JSE76 * JSE76
d_J9Z38/dt = + f_cyan_to_J9Z38 * ((k1 * g * exp(-k1 * time) + k2 * (1 -
            g) * exp(-k2 * time)) / (g * exp(-k1 * time) + (1 - g) *
            exp(-k2 * time))) * cyan - k_J9Z38 * J9Z38
d_JSE76/dt = + f_JCZ38_to_JSE76 * k_JCZ38 * JCZ38 - k_JSE76 * JSE76


Data:
433 observations of 4 variable(s) grouped in 5 datasets


Model predictions using solution type deSolve


Fitted in 549.349 s
Using 300, 100 iterations and 10 chains


Variance model: Constant variance


Starting values for degradation parameters:
      cyan_0    log_k_JCZ38    log_k_J9Z38    log_k_JSE76    f_cyan_ilr_1
     102.4358        -2.3107        -5.3123        -3.7120          0.6753
  f_cyan_ilr_2 f_JCZ38_qlogis f_JSE76_qlogis          log_k1          log_k2
        1.1462        12.4095        12.3630         -1.9317         -4.4557
      g_qlogis
       -0.5648


Fixed degradation parameter values:
None


Starting values for random effects (square root of initial entries in omega):
               cyan_0 log_k_JCZ38 log_k_J9Z38 log_k_JSE76 f_cyan_ilr_1
cyan_0          4.594      0.0000       0.000         0.0       0.0000
log_k_JCZ38     0.000      0.7966       0.000         0.0       0.0000
log_k_J9Z38     0.000      0.0000       1.561         0.0       0.0000
log_k_JSE76     0.000      0.0000       0.000         0.8       0.0000
f_cyan_ilr_1    0.000      0.0000       0.000         0.0       0.6349
f_cyan_ilr_2    0.000      0.0000       0.000         0.0       0.0000
f_JCZ38_qlogis  0.000      0.0000       0.000         0.0       0.0000
f_JSE76_qlogis  0.000      0.0000       0.000         0.0       0.0000
log_k1          0.000      0.0000       0.000         0.0       0.0000
log_k2          0.000      0.0000       0.000         0.0       0.0000
g_qlogis        0.000      0.0000       0.000         0.0       0.0000
               f_cyan_ilr_2 f_JCZ38_qlogis f_JSE76_qlogis log_k1 log_k2
cyan_0               0.000           0.00            0.0  0.000 0.0000
log_k_JCZ38          0.000           0.00            0.0  0.000 0.0000
log_k_J9Z38          0.000           0.00            0.0  0.000 0.0000
log_k_JSE76          0.000           0.00            0.0  0.000 0.0000
f_cyan_ilr_1         0.000           0.00            0.0  0.000 0.0000
f_cyan_ilr_2         1.797           0.00            0.0  0.000 0.0000
f_JCZ38_qlogis       0.000          13.85            0.0  0.000 0.0000
f_JSE76_qlogis       0.000           0.00           14.1  0.000 0.0000
log_k1               0.000           0.00            0.0  1.106 0.0000
log_k2               0.000           0.00            0.0  0.000 0.6141
g_qlogis             0.000           0.00            0.0  0.000 0.0000
               g_qlogis
cyan_0            0.000
log_k_JCZ38       0.000
log_k_J9Z38       0.000
log_k_JSE76       0.000
f_cyan_ilr_1      0.000
f_cyan_ilr_2      0.000
f_JCZ38_qlogis    0.000
f_JSE76_qlogis    0.000
log_k1            0.000
log_k2            0.000
g_qlogis          1.595


Starting values for error model parameters:
a.1
  1
```

```
Results:

Likelihood computed by importance sampling
   AIC  BIC logLik
  2290 2281  -1123

Optimised parameters:
                       est.      lower     upper
cyan_0              102.6903 101.44420 103.9365
log_k_JCZ38          -2.4018  -2.98058  -1.8230
log_k_J9Z38          -5.1865  -5.92931  -4.4437
log_k_JSE76          -3.0784  -4.25226  -1.9045
f_cyan_ilr_1          0.7157   0.37625   1.0551
f_cyan_ilr_2          0.7073   0.20136   1.2132
f_JCZ38_qlogis        4.6797   0.43240   8.9269
f_JSE76_qlogis        5.0080  -1.01380  11.0299
log_k1               -1.9620  -2.62909  -1.2949
log_k2               -4.4894  -4.94958  -4.0292
g_qlogis             -0.4658  -1.34443   0.4129
a.1                   2.7158   2.52576   2.9059
SD.log_k_JCZ38        0.5818   0.15679   1.0067
SD.log_k_J9Z38        0.7421   0.16751   1.3167
SD.log_k_JSE76        1.2841   0.43247   2.1356
SD.f_cyan_ilr_1       0.3748   0.13040   0.6192
SD.f_cyan_ilr_2       0.4550   0.08396   0.8261
SD.f_JCZ38_qlogis     2.0862  -0.73390   4.9062
SD.f_JSE76_qlogis     1.9585  -3.14773   7.0647
SD.log_k1             0.7389   0.25761   1.2201
SD.log_k2             0.5132   0.18143   0.8450
SD.g_qlogis           0.9870   0.35773   1.6164


Correlation:
                cyan_0 l__JCZ3 l__J9Z3 l__JSE7 f_cy__1 f_cy__2 f_JCZ38 f_JSE76
log_k_JCZ38    -0.0170
log_k_J9Z38    -0.0457  0.0016
log_k_JSE76    -0.0046  0.1183  0.0005
f_cyan_ilr_1    0.0079  0.0072 -0.0909  0.0003
f_cyan_ilr_2   -0.3114  0.0343  0.1542  0.0023 -0.0519
f_JCZ38_qlogis  0.0777 -0.0601 -0.0152  0.0080 -0.0520 -0.2524
f_JSE76_qlogis -0.0356  0.0817  0.0073  0.0051  0.0388  0.1959 -0.6236
log_k1          0.0848 -0.0028  0.0010 -0.0010 -0.0014 -0.0245  0.0121 -0.0177
log_k2          0.0274 -0.0001  0.0075  0.0000 -0.0023 -0.0060  0.0000 -0.0130
g_qlogis        0.0159  0.0002 -0.0095  0.0002  0.0029 -0.0140 -0.0001  0.0149
                log_k1  log_k2
log_k_JCZ38
log_k_J9Z38
log_k_JSE76
f_cyan_ilr_1
f_cyan_ilr_2
f_JCZ38_qlogis
f_JSE76_qlogis
log_k1
log_k2          0.0280
g_qlogis       -0.0278 -0.0310


Random effects:
                   est.   lower  upper
SD.log_k_JCZ38   0.5818 0.15679 1.0067
SD.log_k_J9Z38   0.7421 0.16751 1.3167
SD.log_k_JSE76   1.2841 0.43247 2.1356
SD.f_cyan_ilr_1  0.3748 0.13040 0.6192
SD.f_cyan_ilr_2  0.4550 0.08396 0.8261
SD.f_JCZ38_qlogis 2.0862 -0.73390 4.9062
SD.f_JSE76_qlogis 1.9585 -3.14773 7.0647
SD.log_k1        0.7389 0.25761 1.2201
SD.log_k2        0.5132 0.18143 0.8450
SD.g_qlogis      0.9870 0.35773 1.6164


Variance model:
    est. lower upper
a.1 2.716 2.526 2.906


Backtransformed parameters:
                      est.      lower     upper
cyan_0            1.027e+02 1.014e+02 103.93649
k_JCZ38          9.056e-02 5.076e-02   0.16154
k_J9Z38          5.591e-03 2.660e-03   0.01175
k_JSE76          4.603e-02 1.423e-02   0.14890
f_cyan_to_JCZ38  6.184e-01        NA        NA
f_cyan_to_J9Z38  2.248e-01        NA        NA
f_JCZ38_to_JSE76 9.908e-01 6.064e-01   0.99987
f_JSE76_to_JCZ38 9.934e-01 2.662e-01   0.99998
k1               1.406e-01 7.214e-02   0.27393
k2               1.123e-02 7.086e-03   0.01779
```

51

```
g                 3.856e-01 2.068e-01   0.60177

Resulting formation fractions:
                ff
cyan_JCZ38  0.618443
cyan_J9Z38  0.224770
cyan_sink   0.156787
JCZ38_JSE76 0.990803
JCZ38_sink  0.009197
JSE76_JCZ38 0.993360
JSE76_sink  0.006640

Estimated disappearance times:
          DT50    DT90 DT50back DT50_k1 DT50_k2
cyan    21.674 161.70    48.68   4.931   61.74
JCZ38    7.654  25.43       NA      NA      NA
J9Z38  123.966 411.81       NA      NA      NA
JSE76   15.057  50.02       NA      NA      NA
```

```
saemix version used for fitting:      3.2
mkin version used for pre-fitting:  1.2.3
R version used for fitting:         4.2.3
Date of fit:      Thu Apr 20 10:10:25 2023
Date of summary: Thu Apr 20 19:09:25 2023


Equations:
d_cyan/dt = - ((k1 * g * exp(-k1 * time) + k2 * (1 - g) * exp(-k2 *
            time)) / (g * exp(-k1 * time) + (1 - g) * exp(-k2 * time)))
            * cyan
d_JCZ38/dt = + f_cyan_to_JCZ38 * ((k1 * g * exp(-k1 * time) + k2 * (1 -
            g) * exp(-k2 * time)) / (g * exp(-k1 * time) + (1 - g) *
            exp(-k2 * time))) * cyan - k_JCZ38 * JCZ38 +
            f_JSE76_to_JCZ38 * k_JSE76 * JSE76
d_J9Z38/dt = + f_cyan_to_J9Z38 * ((k1 * g * exp(-k1 * time) + k2 * (1 -
            g) * exp(-k2 * time)) / (g * exp(-k1 * time) + (1 - g) *
            exp(-k2 * time))) * cyan - k_J9Z38 * J9Z38
d_JSE76/dt = + f_JCZ38_to_JSE76 * k_JCZ38 * JCZ38 - k_JSE76 * JSE76


Data:
433 observations of 4 variable(s) grouped in 5 datasets


Model predictions using solution type deSolve


Fitted in 715.032 s
Using 300, 100 iterations and 10 chains


Variance model: Two-component variance function


Starting values for degradation parameters:
        cyan_0     log_k_JCZ38     log_k_J9Z38     log_k_JSE76   f_cyan_ilr_1
       101.7523        -1.5948         -5.0119         -2.2723         0.6719
  f_cyan_ilr_2 f_JCZ38_qlogis f_JSE76_qlogis          log_k1         log_k2
        5.1681        12.8238        12.4130         -2.0057         -4.5526
      g_qlogis
       -0.5805


Fixed degradation parameter values:
None


Starting values for random effects (square root of initial entries in omega):
              cyan_0 log_k_JCZ38 log_k_J9Z38 log_k_JSE76 f_cyan_ilr_1
cyan_0         5.627       0.000       0.000       0.000       0.0000
log_k_JCZ38    0.000       2.327       0.000       0.000       0.0000
log_k_J9Z38    0.000       0.000       1.664       0.000       0.0000
log_k_JSE76    0.000       0.000       0.000       4.566       0.0000
f_cyan_ilr_1   0.000       0.000       0.000       0.000       0.6519
f_cyan_ilr_2   0.000       0.000       0.000       0.000       0.0000
f_JCZ38_qlogis 0.000       0.000       0.000       0.000       0.0000
f_JSE76_qlogis 0.000       0.000       0.000       0.000       0.0000
log_k1         0.000       0.000       0.000       0.000       0.0000
log_k2         0.000       0.000       0.000       0.000       0.0000
g_qlogis       0.000       0.000       0.000       0.000       0.0000
              f_cyan_ilr_2 f_JCZ38_qlogis f_JSE76_qlogis log_k1 log_k2
cyan_0                 0.0           0.00           0.00 0.0000 0.0000
log_k_JCZ38            0.0           0.00           0.00 0.0000 0.0000
log_k_J9Z38            0.0           0.00           0.00 0.0000 0.0000
log_k_JSE76            0.0           0.00           0.00 0.0000 0.0000
f_cyan_ilr_1           0.0           0.00           0.00 0.0000 0.0000
f_cyan_ilr_2          10.1           0.00           0.00 0.0000 0.0000
f_JCZ38_qlogis         0.0          13.99           0.00 0.0000 0.0000
f_JSE76_qlogis         0.0           0.00          14.15 0.0000 0.0000
log_k1                 0.0           0.00           0.00 0.8452 0.0000
log_k2                 0.0           0.00           0.00 0.0000 0.5968
g_qlogis               0.0           0.00           0.00 0.0000 0.0000
              g_qlogis
cyan_0           0.000
log_k_JCZ38      0.000
log_k_J9Z38      0.000
log_k_JSE76      0.000
f_cyan_ilr_1     0.000
f_cyan_ilr_2     0.000
f_JCZ38_qlogis   0.000
f_JSE76_qlogis   0.000
log_k1           0.000
log_k2           0.000
g_qlogis         1.691


Starting values for error model parameters:
a.1 b.1
  1   1
```

```
Results:

Likelihood computed by importance sampling
   AIC  BIC logLik
  2234 2226  -1095

Optimised parameters:
                    est.      lower      upper
cyan_0           101.10667  9.903e+01 103.18265
log_k_JCZ38       -2.49437 -3.297e+00  -1.69221
log_k_J9Z38       -5.08171 -5.875e+00  -4.28846
log_k_JSE76       -3.20072 -4.180e+00  -2.22163
f_cyan_ilr_1       0.71059  3.639e-01   1.05727
f_cyan_ilr_2       1.15398  2.981e-01   2.00984
f_JCZ38_qlogis     3.18027  1.056e+00   5.30452
f_JSE76_qlogis     5.61578 -2.505e+01  36.28077
log_k1            -2.38875 -2.517e+00  -2.26045
log_k2            -4.67246 -4.928e+00  -4.41715
g_qlogis          -0.28231 -1.135e+00   0.57058
a.1                2.08190  1.856e+00   2.30785
b.1                0.06114  5.015e-02   0.07214
SD.log_k_JCZ38     0.84622  2.637e-01   1.42873
SD.log_k_J9Z38     0.84564  2.566e-01   1.43464
SD.log_k_JSE76     1.04385  3.242e-01   1.76351
SD.f_cyan_ilr_1    0.38568  1.362e-01   0.63514
SD.f_cyan_ilr_2    0.68046  7.166e-02   1.28925
SD.f_JCZ38_qlogis  1.25244 -4.213e-02   2.54700
SD.f_JSE76_qlogis  0.28202 -1.515e+03 1515.87968
SD.log_k2          0.25749  7.655e-02   0.43843
SD.g_qlogis        0.94535  3.490e-01   1.54174


Correlation:
                cyan_0  l__JCZ3  l__J9Z3  l__JSE7  f_cy__1  f_cy__2  f_JCZ38  f_JSE76
log_k_JCZ38    -0.0086
log_k_J9Z38    -0.0363 -0.0007
log_k_JSE76     0.0015  0.1210 -0.0017
f_cyan_ilr_1   -0.0048  0.0095 -0.0572  0.0030
f_cyan_ilr_2   -0.4788  0.0328  0.1143  0.0027 -0.0316
f_JCZ38_qlogis  0.0736 -0.0664 -0.0137  0.0145 -0.0444 -0.2175
f_JSE76_qlogis -0.0137  0.0971  0.0035  0.0009  0.0293  0.1333 -0.6767
log_k1          0.2345 -0.0350 -0.0099 -0.0113 -0.0126 -0.1652  0.1756 -0.2161
log_k2          0.0440 -0.0133  0.0199 -0.0040 -0.0097 -0.0119  0.0604 -0.1306
g_qlogis        0.0438  0.0078 -0.0123  0.0029  0.0046 -0.0363 -0.0318  0.0736
                log_k1  log_k2
log_k_JCZ38
log_k_J9Z38
log_k_JSE76
f_cyan_ilr_1
f_cyan_ilr_2
f_JCZ38_qlogis
f_JSE76_qlogis
log_k1
log_k2          0.3198
g_qlogis       -0.1666 -0.0954


Random effects:
                   est.      lower      upper
SD.log_k_JCZ38    0.8462  2.637e-01    1.4287
SD.log_k_J9Z38    0.8456  2.566e-01    1.4346
SD.log_k_JSE76    1.0439  3.242e-01    1.7635
SD.f_cyan_ilr_1   0.3857  1.362e-01    0.6351
SD.f_cyan_ilr_2   0.6805  7.166e-02    1.2893
SD.f_JCZ38_qlogis 1.2524 -4.213e-02    2.5470
SD.f_JSE76_qlogis 0.2820 -1.515e+03 1515.8797
SD.log_k2         0.2575  7.655e-02    0.4384
SD.g_qlogis       0.9453  3.490e-01    1.5417


Variance model:
       est.   lower   upper
a.1 2.08190 1.85595 2.30785
b.1 0.06114 0.05015 0.07214

Backtransformed parameters:
                     est.      lower      upper
cyan_0           1.011e+02  9.903e+01 103.18265
k_JCZ38          8.255e-02  3.701e-02   0.18411
k_J9Z38          6.209e-03  2.809e-03   0.01373
k_JSE76          4.073e-02  1.530e-02   0.10843
f_cyan_to_JCZ38  6.608e-01         NA         NA
f_cyan_to_J9Z38  2.419e-01         NA         NA
f_JCZ38_to_JSE76 9.601e-01  7.419e-01   0.99506
f_JSE76_to_JCZ38 9.964e-01  1.322e-11   1.00000
k1               9.174e-02  8.070e-02   0.10430
k2               9.349e-03  7.243e-03   0.01207
```

```
g                       4.299e-01 2.432e-01   0.63890

Resulting formation fractions:
                ff
cyan_JCZ38  0.660808
cyan_J9Z38  0.241904
cyan_sink   0.097288
JCZ38_JSE76 0.960085
JCZ38_sink  0.039915
JSE76_JCZ38 0.996373
JSE76_sink  0.003627

Estimated disappearance times:
          DT50    DT90 DT50back DT50_k1 DT50_k2
cyan    24.359 186.18    56.05   7.555   74.14
JCZ38    8.397  27.89       NA      NA      NA
J9Z38  111.631 370.83       NA      NA      NA
JSE76   17.017  56.53       NA      NA      NA
```

```
saemix version used for fitting:      3.2
mkin version used for pre-fitting:  1.2.3
R version used for fitting:        4.2.3
Date of fit:     Thu Apr 20 10:07:35 2023
Date of summary: Thu Apr 20 19:09:25 2023


Equations:
d_cyan_free/dt = - k_cyan_free * cyan_free - k_cyan_free_bound *
         cyan_free + k_cyan_bound_free * cyan_bound
d_cyan_bound/dt = + k_cyan_free_bound * cyan_free - k_cyan_bound_free *
         cyan_bound
d_JCZ38/dt = + f_cyan_free_to_JCZ38 * k_cyan_free * cyan_free - k_JCZ38
         * JCZ38 + f_JSE76_to_JCZ38 * k_JSE76 * JSE76
d_J9Z38/dt = + f_cyan_free_to_J9Z38 * k_cyan_free * cyan_free - k_J9Z38
         * J9Z38
d_JSE76/dt = + f_JCZ38_to_JSE76 * k_JCZ38 * JCZ38 - k_JSE76 * JSE76


Data:
433 observations of 4 variable(s) grouped in 5 datasets


Model predictions using solution type deSolve


Fitted in 545.815 s
Using 300, 100 iterations and 10 chains


Variance model: Constant variance


Starting values for degradation parameters:
         cyan_free_0      log_k_cyan_free log_k_cyan_free_bound
          102.4394              -2.7673               -2.8942
log_k_cyan_bound_free         log_k_JCZ38           log_k_J9Z38
           -3.6201              -2.3107               -5.3123
         log_k_JSE76          f_cyan_ilr_1          f_cyan_ilr_2
           -3.7120               0.6754                1.1448
      f_JCZ38_qlogis        f_JSE76_qlogis
           13.2672              13.3538


Fixed degradation parameter values:
None


Starting values for random effects (square root of initial entries in omega):
                      cyan_free_0 log_k_cyan_free log_k_cyan_free_bound
cyan_free_0                 4.589          0.0000                  0.00
log_k_cyan_free             0.000          0.4849                  0.00
log_k_cyan_free_bound       0.000          0.0000                  1.62
log_k_cyan_bound_free       0.000          0.0000                  0.00
log_k_JCZ38                 0.000          0.0000                  0.00
log_k_J9Z38                 0.000          0.0000                  0.00
log_k_JSE76                 0.000          0.0000                  0.00
f_cyan_ilr_1                0.000          0.0000                  0.00
f_cyan_ilr_2                0.000          0.0000                  0.00
f_JCZ38_qlogis              0.000          0.0000                  0.00
f_JSE76_qlogis              0.000          0.0000                  0.00
                      log_k_cyan_bound_free log_k_JCZ38 log_k_J9Z38 log_k_JSE76
cyan_free_0                           0.000      0.0000       0.000         0.0
log_k_cyan_free                       0.000      0.0000       0.000         0.0
log_k_cyan_free_bound                 0.000      0.0000       0.000         0.0
log_k_cyan_bound_free                 1.197      0.0000       0.000         0.0
log_k_JCZ38                           0.000      0.7966       0.000         0.0
log_k_J9Z38                           0.000      0.0000       1.561         0.0
log_k_JSE76                           0.000      0.0000       0.000         0.8
f_cyan_ilr_1                          0.000      0.0000       0.000         0.0
f_cyan_ilr_2                          0.000      0.0000       0.000         0.0
f_JCZ38_qlogis                        0.000      0.0000       0.000         0.0
f_JSE76_qlogis                        0.000      0.0000       0.000         0.0
                      f_cyan_ilr_1 f_cyan_ilr_2 f_JCZ38_qlogis f_JSE76_qlogis
cyan_free_0                 0.0000        0.000           0.00           0.00
log_k_cyan_free             0.0000        0.000           0.00           0.00
log_k_cyan_free_bound       0.0000        0.000           0.00           0.00
log_k_cyan_bound_free       0.0000        0.000           0.00           0.00
log_k_JCZ38                 0.0000        0.000           0.00           0.00
log_k_J9Z38                 0.0000        0.000           0.00           0.00
log_k_JSE76                 0.0000        0.000           0.00           0.00
f_cyan_ilr_1                0.6349        0.000           0.00           0.00
f_cyan_ilr_2                0.0000        1.797           0.00           0.00
f_JCZ38_qlogis              0.0000        0.000          13.84           0.00
f_JSE76_qlogis              0.0000        0.000           0.00          14.66


Starting values for error model parameters:
a.1
  1
```

```
Results:

Likelihood computed by importance sampling
   AIC  BIC logLik
  2284 2275  -1120

Optimised parameters:
                            est.      lower      upper
cyan_free_0              102.7730  1.015e+02  1.041e+02
log_k_cyan_free           -2.8530 -3.167e+00 -2.539e+00
log_k_cyan_free_bound     -2.7326 -3.543e+00 -1.922e+00
log_k_cyan_bound_free     -3.5582 -4.126e+00 -2.990e+00
log_k_JCZ38               -2.3810 -2.921e+00 -1.841e+00
log_k_J9Z38               -5.2301 -5.963e+00 -4.497e+00
log_k_JSE76               -3.0286 -4.286e+00 -1.771e+00
f_cyan_ilr_1               0.7081  3.733e-01  1.043e+00
f_cyan_ilr_2               0.5847  7.846e-03  1.162e+00
f_JCZ38_qlogis             9.5676 -1.323e+03  1.342e+03
f_JSE76_qlogis             3.7042  7.254e-02  7.336e+00
a.1                        2.7222  2.532e+00  2.913e+00
SD.log_k_cyan_free         0.3338  1.086e-01  5.589e-01
SD.log_k_cyan_free_bound   0.8888  3.023e-01  1.475e+00
SD.log_k_cyan_bound_free   0.6220  2.063e-01  1.038e+00
SD.log_k_JCZ38             0.5221  1.334e-01  9.108e-01
SD.log_k_J9Z38             0.7104  1.371e-01  1.284e+00
SD.log_k_JSE76             1.3837  4.753e-01  2.292e+00
SD.f_cyan_ilr_1            0.3620  1.248e-01  5.992e-01
SD.f_cyan_ilr_2            0.4259  8.145e-02  7.704e-01
SD.f_JCZ38_qlogis          3.5332 -1.037e+05  1.037e+05
SD.f_JSE76_qlogis          1.6990 -2.771e-01  3.675e+00


Correlation:
                        cyn_f_0 lg_k_c_ lg_k_cyn_f_ lg_k_cyn_b_ l__JCZ3 l__J9Z3
log_k_cyan_free          0.2126
log_k_cyan_free_bound    0.0894  0.0871
log_k_cyan_bound_free    0.0033  0.0410   0.0583
log_k_JCZ38             -0.0708 -0.0280  -0.0147       0.0019
log_k_J9Z38             -0.0535 -0.0138   0.0012       0.0148   0.0085
log_k_JSE76             -0.0066 -0.0030  -0.0021      -0.0005   0.1090  0.0010
f_cyan_ilr_1            -0.0364 -0.0157  -0.0095      -0.0015   0.0458 -0.0960
f_cyan_ilr_2            -0.3814 -0.1104  -0.0423       0.0146   0.1540  0.1526
f_JCZ38_qlogis          0.2507  0.0969   0.0482      -0.0097  -0.2282 -0.0363
f_JSE76_qlogis         -0.1648 -0.0710  -0.0443      -0.0087   0.2002  0.0226
                        l__JSE7 f_cy__1 f_cy__2 f_JCZ38
log_k_cyan_free
log_k_cyan_free_bound
log_k_cyan_bound_free
log_k_JCZ38
log_k_J9Z38
log_k_JSE76
f_cyan_ilr_1             0.0001
f_cyan_ilr_2             0.0031  0.0586
f_JCZ38_qlogis           0.0023 -0.1867 -0.6255
f_JSE76_qlogis           0.0082  0.1356  0.4519 -0.7951


Random effects:
                           est.      lower      upper
SD.log_k_cyan_free        0.3338  1.086e-01 5.589e-01
SD.log_k_cyan_free_bound  0.8888  3.023e-01 1.475e+00
SD.log_k_cyan_bound_free  0.6220  2.063e-01 1.038e+00
SD.log_k_JCZ38            0.5221  1.334e-01 9.108e-01
SD.log_k_J9Z38            0.7104  1.371e-01 1.284e+00
SD.log_k_JSE76            1.3837  4.753e-01 2.292e+00
SD.f_cyan_ilr_1           0.3620  1.248e-01 5.992e-01
SD.f_cyan_ilr_2           0.4259  8.145e-02 7.704e-01
SD.f_JCZ38_qlogis         3.5332 -1.037e+05 1.037e+05
SD.f_JSE76_qlogis         1.6990 -2.771e-01 3.675e+00


Variance model:
     est. lower upper
a.1 2.722 2.532 2.913


Backtransformed parameters:
                          est.      lower      upper
cyan_free_0            1.028e+02  1.015e+02 104.06475
k_cyan_free           5.767e-02  4.213e-02   0.07894
k_cyan_free_bound     6.505e-02  2.892e-02   0.14633
k_cyan_bound_free     2.849e-02  1.614e-02   0.05028
k_JCZ38               9.246e-02  5.390e-02   0.15859
k_J9Z38               5.353e-03  2.572e-03   0.01114
k_JSE76               4.838e-02  1.376e-02   0.17009
f_cyan_free_to_JCZ38  6.011e-01  5.028e-01   0.83792
f_cyan_free_to_J9Z38  2.208e-01  5.028e-01   0.83792
f_JCZ38_to_JSE76      9.999e-01  0.000e+00   1.00000
```

```
f_JSE76_to_JCZ38      9.760e-01 5.181e-01   0.99935

Estimated Eigenvalues of SFORB model(s):
cyan_b1 cyan_b2  cyan_g
0.13942 0.01178 0.35948

Resulting formation fractions:
                    ff
cyan_free_JCZ38 6.011e-01
cyan_free_J9Z38 2.208e-01
cyan_free_sink  1.780e-01
cyan_free       1.000e+00
JCZ38_JSE76     9.999e-01
JCZ38_sink      6.996e-05
JSE76_JCZ38     9.760e-01
JSE76_sink      2.403e-02

Estimated disappearance times:
          DT50    DT90 DT50back DT50_cyan_b1 DT50_cyan_b2
cyan    23.390 157.60    47.44        4.971        58.82
JCZ38    7.497  24.90       NA           NA           NA
J9Z38  129.482 430.13       NA           NA           NA
JSE76   14.326  47.59       NA           NA           NA
```

```
saemix version used for fitting:      3.2
mkin version used for pre-fitting:  1.2.3
R version used for fitting:          4.2.3
Date of fit:     Thu Apr 20 10:10:20 2023
Date of summary: Thu Apr 20 19:09:25 2023


Equations:
d_cyan_free/dt = - k_cyan_free * cyan_free - k_cyan_free_bound *
          cyan_free + k_cyan_bound_free * cyan_bound
d_cyan_bound/dt = + k_cyan_free_bound * cyan_free - k_cyan_bound_free *
          cyan_bound
d_JCZ38/dt = + f_cyan_free_to_JCZ38 * k_cyan_free * cyan_free - k_JCZ38
          * JCZ38 + f_JSE76_to_JCZ38 * k_JSE76 * JSE76
d_J9Z38/dt = + f_cyan_free_to_J9Z38 * k_cyan_free * cyan_free - k_J9Z38
          * J9Z38
d_JSE76/dt = + f_JCZ38_to_JSE76 * k_JCZ38 * JCZ38 - k_JSE76 * JSE76


Data:
433 observations of 4 variable(s) grouped in 5 datasets


Model predictions using solution type deSolve


Fitted in 709.958 s
Using 300, 100 iterations and 10 chains


Variance model: Two-component variance function


Starting values for degradation parameters:
         cyan_free_0      log_k_cyan_free log_k_cyan_free_bound
             101.751               -2.837                -3.016
log_k_cyan_bound_free          log_k_JCZ38          log_k_J9Z38
              -3.660               -2.299                -5.313
          log_k_JSE76          f_cyan_ilr_1          f_cyan_ilr_2
              -3.699                0.672                 5.873
       f_JCZ38_qlogis        f_JSE76_qlogis
              13.216               13.338


Fixed degradation parameter values:
None


Starting values for random effects (square root of initial entries in omega):
                 cyan_free_0 log_k_cyan_free log_k_cyan_free_bound
cyan_free_0             5.629           0.000                0.000
log_k_cyan_free         0.000           0.446                0.000
log_k_cyan_free_bound   0.000           0.000                1.449
log_k_cyan_bound_free   0.000           0.000                0.000
log_k_JCZ38             0.000           0.000                0.000
log_k_J9Z38             0.000           0.000                0.000
log_k_JSE76             0.000           0.000                0.000
f_cyan_ilr_1            0.000           0.000                0.000
f_cyan_ilr_2            0.000           0.000                0.000
f_JCZ38_qlogis          0.000           0.000                0.000
f_JSE76_qlogis          0.000           0.000                0.000
                 log_k_cyan_bound_free log_k_JCZ38 log_k_J9Z38 log_k_JSE76
cyan_free_0                      0.000      0.0000       0.000      0.0000
log_k_cyan_free                  0.000      0.0000       0.000      0.0000
log_k_cyan_free_bound            0.000      0.0000       0.000      0.0000
log_k_cyan_bound_free            1.213      0.0000       0.000      0.0000
log_k_JCZ38                      0.000      0.7801       0.000      0.0000
log_k_J9Z38                      0.000      0.0000       1.575      0.0000
log_k_JSE76                      0.000      0.0000       0.000      0.8078
f_cyan_ilr_1                     0.000      0.0000       0.000      0.0000
f_cyan_ilr_2                     0.000      0.0000       0.000      0.0000
f_JCZ38_qlogis                   0.000      0.0000       0.000      0.0000
f_JSE76_qlogis                   0.000      0.0000       0.000      0.0000
                 f_cyan_ilr_1 f_cyan_ilr_2 f_JCZ38_qlogis f_JSE76_qlogis
cyan_free_0            0.0000         0.00           0.00           0.00
log_k_cyan_free        0.0000         0.00           0.00           0.00
log_k_cyan_free_bound  0.0000         0.00           0.00           0.00
log_k_cyan_bound_free  0.0000         0.00           0.00           0.00
log_k_JCZ38            0.0000         0.00           0.00           0.00
log_k_J9Z38            0.0000         0.00           0.00           0.00
log_k_JSE76            0.0000         0.00           0.00           0.00
f_cyan_ilr_1           0.6519         0.00           0.00           0.00
f_cyan_ilr_2           0.0000        10.78           0.00           0.00
f_JCZ38_qlogis         0.0000         0.00          13.96           0.00
f_JSE76_qlogis         0.0000         0.00           0.00          14.69


Starting values for error model parameters:
a.1 b.1
  1   1
```

```
Results:

Likelihood computed by importance sampling
   AIC  BIC logLik
  2240 2232  -1098

Optimised parameters:
                             est.     lower    upper
cyan_free_0              101.10205  98.99221 103.2119
log_k_cyan_free           -3.16929  -3.61395  -2.7246
log_k_cyan_free_bound     -3.38259  -3.63022  -3.1350
log_k_cyan_bound_free     -3.81075  -4.13888  -3.4826
log_k_JCZ38               -2.42057  -3.00756  -1.8336
log_k_J9Z38               -5.07501  -5.85138  -4.2986
log_k_JSE76               -3.12442  -4.21277  -2.0361
f_cyan_ilr_1               0.70577   0.35788   1.0537
f_cyan_ilr_2               1.14824   0.15810   2.1384
f_JCZ38_qlogis             3.52245   0.43257   6.6123
f_JSE76_qlogis             5.65140 -21.22295  32.5257
a.1                        2.07062   1.84329   2.2980
b.1                        0.06227   0.05124   0.0733
SD.log_k_cyan_free         0.49468   0.18566   0.8037
SD.log_k_cyan_bound_free   0.28972   0.07188   0.5076
SD.log_k_JCZ38             0.58852   0.16800   1.0090
SD.log_k_J9Z38             0.82500   0.24730   1.4027
SD.log_k_JSE76             1.19201   0.40313   1.9809
SD.f_cyan_ilr_1            0.38534   0.13640   0.6343
SD.f_cyan_ilr_2            0.72463   0.10076   1.3485
SD.f_JCZ38_qlogis          1.38223  -0.20997   2.9744
SD.f_JSE76_qlogis          2.07989 -72.53027  76.6901


Correlation:
                      cyn_f_0 lg_k_c_ lg_k_cyn_f_ lg_k_cyn_b_ l__JCZ3 l__J9Z3
log_k_cyan_free        0.1117
log_k_cyan_free_bound  0.1763  0.1828
log_k_cyan_bound_free  0.0120  0.0593  0.5030
log_k_JCZ38           -0.0459 -0.0230 -0.0931      -0.0337
log_k_J9Z38           -0.0381 -0.0123 -0.0139       0.0237     0.0063
log_k_JSE76           -0.0044 -0.0038 -0.0175      -0.0072     0.1120  0.0003
f_cyan_ilr_1          -0.0199 -0.0087 -0.0407      -0.0233     0.0268 -0.0552
f_cyan_ilr_2          -0.4806 -0.1015 -0.2291      -0.0269     0.1156  0.1113
f_JCZ38_qlogis         0.1805  0.0825  0.3085       0.0963    -0.1674 -0.0314
f_JSE76_qlogis        -0.1586 -0.0810 -0.3560      -0.1563     0.2025  0.0278
                      l__JSE7 f_cy__1 f_cy__2 f_JCZ38
log_k_cyan_free
log_k_cyan_free_bound
log_k_cyan_bound_free
log_k_JCZ38
log_k_J9Z38
log_k_JSE76
f_cyan_ilr_1           0.0024
f_cyan_ilr_2           0.0087  0.0172
f_JCZ38_qlogis        -0.0016 -0.1047 -0.4656
f_JSE76_qlogis         0.0119  0.1034  0.4584 -0.8137


Random effects:
                           est.     lower    upper
SD.log_k_cyan_free        0.4947   0.18566   0.8037
SD.log_k_cyan_bound_free  0.2897   0.07188   0.5076
SD.log_k_JCZ38            0.5885   0.16800   1.0090
SD.log_k_J9Z38            0.8250   0.24730   1.4027
SD.log_k_JSE76            1.1920   0.40313   1.9809
SD.f_cyan_ilr_1           0.3853   0.13640   0.6343
SD.f_cyan_ilr_2           0.7246   0.10076   1.3485
SD.f_JCZ38_qlogis         1.3822  -0.20997   2.9744
SD.f_JSE76_qlogis         2.0799 -72.53027  76.6901


Variance model:
       est.   lower  upper
a.1 2.07062 1.84329 2.2980
b.1 0.06227 0.05124 0.0733


Backtransformed parameters:
                         est.      lower     upper
cyan_free_0         1.011e+02  9.899e+01 103.21190
k_cyan_free         4.203e-02  2.695e-02   0.06557
k_cyan_free_bound   3.396e-02  2.651e-02   0.04350
k_cyan_bound_free   2.213e-02  1.594e-02   0.03073
k_JCZ38             8.887e-02  4.941e-02   0.15984
k_J9Z38             6.251e-03  2.876e-03   0.01359
k_JSE76             4.396e-02  1.481e-02   0.13054
f_cyan_free_to_JCZ38 6.590e-01 5.557e-01   0.95365
f_cyan_free_to_J9Z38 2.429e-01 5.557e-01   0.95365
f_JCZ38_to_JSE76    9.713e-01  6.065e-01   0.99866
```

```
f_JSE76_to_JCZ38     9.965e-01 6.067e-10   1.00000

Estimated Eigenvalues of SFORB model(s):
cyan_b1 cyan_b2  cyan_g
0.08749 0.01063 0.40855

Resulting formation fractions:
                   ff
cyan_free_JCZ38 0.65905
cyan_free_J9Z38 0.24291
cyan_free_sink  0.09805
cyan_free       1.00000
JCZ38_JSE76     0.97132
JCZ38_sink      0.02868
JSE76_JCZ38     0.99650
JSE76_sink      0.00350

Estimated disappearance times:
        DT50    DT90 DT50back DT50_cyan_b1 DT50_cyan_b2
cyan   24.91 167.16    50.32        7.922        65.19
JCZ38   7.80  25.91       NA           NA           NA
J9Z38 110.89 368.36       NA           NA           NA
JSE76  15.77  52.38       NA           NA           NA
```

Pathway 2, refined fits

Listing 17: Hierarchical FOMC path 2 fit with reduced random effects, two-component error

```
saemix version used for fitting:      3.2
mkin version used for pre-fitting:  1.2.3
R version used for fitting:         4.2.3
Date of fit:    Thu Apr 20 10:23:48 2023
Date of summary: Thu Apr 20 19:09:25 2023


Equations:
d_cyan/dt = - (alpha/beta) * 1/((time/beta) + 1) * cyan
d_JCZ38/dt = + f_cyan_to_JCZ38 * (alpha/beta) * 1/((time/beta) + 1) *
             cyan - k_JCZ38 * JCZ38 + f_JSE76_to_JCZ38 * k_JSE76 * JSE76
d_J9Z38/dt = + f_cyan_to_J9Z38 * (alpha/beta) * 1/((time/beta) + 1) *
             cyan - k_J9Z38 * J9Z38
d_JSE76/dt = + f_JCZ38_to_JSE76 * k_JCZ38 * JCZ38 - k_JSE76 * JSE76


Data:
433 observations of 4 variable(s) grouped in 5 datasets


Model predictions using solution type deSolve


Fitted in 801.809 s
Using 300, 100 iterations and 10 chains


Variance model: Two-component variance function


Starting values for degradation parameters:
        cyan_0    log_k_JCZ38    log_k_J9Z38    log_k_JSE76    f_cyan_ilr_1
       101.9028        -1.9055        -5.0249        -2.5646          0.6807
  f_cyan_ilr_2 f_JCZ38_qlogis f_JSE76_qlogis      log_alpha        log_beta
        4.8883        16.0676         9.3923        -0.1346          3.0364


Fixed degradation parameter values:
None


Starting values for random effects (square root of initial entries in omega):
              cyan_0 log_k_JCZ38 log_k_J9Z38 log_k_JSE76 f_cyan_ilr_1
cyan_0         6.321       0.000       0.000       0.000       0.0000
log_k_JCZ38    0.000       1.392       0.000       0.000       0.0000
log_k_J9Z38    0.000       0.000       1.561       0.000       0.0000
log_k_JSE76    0.000       0.000       0.000       3.614       0.0000
f_cyan_ilr_1   0.000       0.000       0.000       0.000       0.6339
f_cyan_ilr_2   0.000       0.000       0.000       0.000       0.0000
f_JCZ38_qlogis 0.000       0.000       0.000       0.000       0.0000
f_JSE76_qlogis 0.000       0.000       0.000       0.000       0.0000
log_alpha      0.000       0.000       0.000       0.000       0.0000
log_beta       0.000       0.000       0.000       0.000       0.0000
              f_cyan_ilr_2 f_JCZ38_qlogis f_JSE76_qlogis log_alpha log_beta
cyan_0              0.00          0.00          0.00      0.0000   0.0000
log_k_JCZ38         0.00          0.00          0.00      0.0000   0.0000
log_k_J9Z38         0.00          0.00          0.00      0.0000   0.0000
log_k_JSE76         0.00          0.00          0.00      0.0000   0.0000
f_cyan_ilr_1        0.00          0.00          0.00      0.0000   0.0000
f_cyan_ilr_2       10.41          0.00          0.00      0.0000   0.0000
f_JCZ38_qlogis      0.00         12.24          0.00      0.0000   0.0000
f_JSE76_qlogis      0.00          0.00         15.13      0.0000   0.0000
log_alpha           0.00          0.00          0.00      0.3701   0.0000
log_beta            0.00          0.00          0.00      0.0000   0.5662


Starting values for error model parameters:
a.1 b.1
  1   1


Results:


Likelihood computed by importance sampling
   AIC  BIC logLik
  2251 2244  -1106


Optimised parameters:
                    est.    lower    upper
cyan_0         101.05768      NA      NA
log_k_JCZ38     -2.73252      NA      NA
log_k_J9Z38     -5.07399      NA      NA
log_k_JSE76     -3.52863      NA      NA
f_cyan_ilr_1     0.72176      NA      NA
f_cyan_ilr_2     1.34610      NA      NA
f_JCZ38_qlogis   2.08337      NA      NA
f_JSE76_qlogis 1590.31880      NA      NA
log_alpha       -0.09336      NA      NA
log_beta         3.10191      NA      NA
a.1              2.08557 1.85439 2.31675
```

```
b.1                0.06998 0.05800 0.08197
SD.log_k_JCZ38     1.20053 0.43329 1.96777
SD.log_k_J9Z38     0.85854 0.26708 1.45000
SD.log_k_JSE76     0.62528 0.16061 1.08995
SD.f_cyan_ilr_1    0.35190 0.12340 0.58039
SD.f_cyan_ilr_2    0.85385 0.15391 1.55378
SD.log_alpha       0.28971 0.08718 0.49225
SD.log_beta        0.31614 0.05938 0.57290


Correlation is not available

Random effects:
                  est.   lower  upper
SD.log_k_JCZ38  1.2005 0.43329 1.9678
SD.log_k_J9Z38  0.8585 0.26708 1.4500
SD.log_k_JSE76  0.6253 0.16061 1.0900
SD.f_cyan_ilr_1 0.3519 0.12340 0.5804
SD.f_cyan_ilr_2 0.8538 0.15391 1.5538
SD.log_alpha    0.2897 0.08718 0.4923
SD.log_beta     0.3161 0.05938 0.5729

Variance model:
      est. lower   upper
a.1 2.08557 1.854 2.31675
b.1 0.06998 0.058 0.08197

Backtransformed parameters:
                     est. lower upper
cyan_0            1.011e+02   NA   NA
k_JCZ38           6.506e-02   NA   NA
k_J9Z38           6.257e-03   NA   NA
k_JSE76           2.935e-02   NA   NA
f_cyan_to_JCZ38   6.776e-01   NA   NA
f_cyan_to_J9Z38   2.442e-01   NA   NA
f_JCZ38_to_JSE76  8.893e-01   NA   NA
f_JSE76_to_JCZ38  1.000e+00   NA   NA
alpha             9.109e-01   NA   NA
beta              2.224e+01   NA   NA


Resulting formation fractions:
                ff
cyan_JCZ38  0.67761
cyan_J9Z38  0.24417
cyan_sink   0.07822
JCZ38_JSE76 0.88928
JCZ38_sink  0.11072
JSE76_JCZ38 1.00000
JSE76_sink  0.00000

Estimated disappearance times:
        DT50   DT90 DT50back
cyan   25.36 256.37    77.18
JCZ38  10.65  35.39       NA
J9Z38 110.77 367.98       NA
JSE76  23.62  78.47       NA
```

Listing 18: Hierarchical DFOP path 2 fit with reduced random effects, constant variance

```
saemix version used for fitting:      3.2
mkin version used for pre-fitting:  1.2.3
R version used for fitting:        4.2.3
Date of fit:     Thu Apr 20 10:25:03 2023
Date of summary: Thu Apr 20 19:09:25 2023


Equations:
d_cyan/dt = - ((k1 * g * exp(-k1 * time) + k2 * (1 - g) * exp(-k2 *
            time)) / (g * exp(-k1 * time) + (1 - g) * exp(-k2 * time)))
            * cyan
d_JCZ38/dt = + f_cyan_to_JCZ38 * ((k1 * g * exp(-k1 * time) + k2 * (1 -
            g) * exp(-k2 * time)) / (g * exp(-k1 * time) + (1 - g) *
            exp(-k2 * time))) * cyan - k_JCZ38 * JCZ38 +
            f_JSE76_to_JCZ38 * k_JSE76 * JSE76
d_J9Z38/dt = + f_cyan_to_J9Z38 * ((k1 * g * exp(-k1 * time) + k2 * (1 -
            g) * exp(-k2 * time)) / (g * exp(-k1 * time) + (1 - g) *
            exp(-k2 * time))) * cyan - k_J9Z38 * J9Z38
d_JSE76/dt = + f_JCZ38_to_JSE76 * k_JCZ38 * JCZ38 - k_JSE76 * JSE76


Data:
433 observations of 4 variable(s) grouped in 5 datasets


Model predictions using solution type deSolve


Fitted in 876.291 s
Using 300, 100 iterations and 10 chains


Variance model: Constant variance


Starting values for degradation parameters:
      cyan_0    log_k_JCZ38    log_k_J9Z38    log_k_JSE76    f_cyan_ilr_1
    102.4358        -2.3107        -5.3123        -3.7120          0.6753
  f_cyan_ilr_2 f_JCZ38_qlogis f_JSE76_qlogis         log_k1         log_k2
        1.1462        12.4095        12.3630        -1.9317        -4.4557
      g_qlogis
       -0.5648


Fixed degradation parameter values:
None


Starting values for random effects (square root of initial entries in omega):
               cyan_0 log_k_JCZ38 log_k_J9Z38 log_k_JSE76 f_cyan_ilr_1
cyan_0          4.594      0.0000       0.000         0.0       0.0000
log_k_JCZ38     0.000      0.7966       0.000         0.0       0.0000
log_k_J9Z38     0.000      0.0000       1.561         0.0       0.0000
log_k_JSE76     0.000      0.0000       0.000         0.8       0.0000
f_cyan_ilr_1    0.000      0.0000       0.000         0.0       0.6349
f_cyan_ilr_2    0.000      0.0000       0.000         0.0       0.0000
f_JCZ38_qlogis  0.000      0.0000       0.000         0.0       0.0000
f_JSE76_qlogis  0.000      0.0000       0.000         0.0       0.0000
log_k1          0.000      0.0000       0.000         0.0       0.0000
log_k2          0.000      0.0000       0.000         0.0       0.0000
g_qlogis        0.000      0.0000       0.000         0.0       0.0000
               f_cyan_ilr_2 f_JCZ38_qlogis f_JSE76_qlogis log_k1 log_k2
cyan_0                0.000           0.00            0.0  0.000 0.0000
log_k_JCZ38           0.000           0.00            0.0  0.000 0.0000
log_k_J9Z38           0.000           0.00            0.0  0.000 0.0000
log_k_JSE76           0.000           0.00            0.0  0.000 0.0000
f_cyan_ilr_1          0.000           0.00            0.0  0.000 0.0000
f_cyan_ilr_2          1.797           0.00            0.0  0.000 0.0000
f_JCZ38_qlogis        0.000          13.85            0.0  0.000 0.0000
f_JSE76_qlogis        0.000           0.00           14.1  0.000 0.0000
log_k1                0.000           0.00            0.0  1.106 0.0000
log_k2                0.000           0.00            0.0  0.000 0.6141
g_qlogis              0.000           0.00            0.0  0.000 0.0000
               g_qlogis
cyan_0            0.000
log_k_JCZ38       0.000
log_k_J9Z38       0.000
log_k_JSE76       0.000
f_cyan_ilr_1      0.000
f_cyan_ilr_2      0.000
f_JCZ38_qlogis    0.000
f_JSE76_qlogis    0.000
log_k1            0.000
log_k2            0.000
g_qlogis          1.595


Starting values for error model parameters:
a.1
  1
```

```
Results:

Likelihood computed by importance sampling
   AIC  BIC logLik
  2282 2274  -1121

Optimised parameters:
                   est.   lower  upper
cyan_0          102.5254     NA     NA
log_k_JCZ38      -2.9358     NA     NA
log_k_J9Z38      -5.1424     NA     NA
log_k_JSE76      -3.6458     NA     NA
f_cyan_ilr_1      0.6957     NA     NA
f_cyan_ilr_2      0.6635     NA     NA
f_JCZ38_qlogis 4984.8163     NA     NA
f_JSE76_qlogis    1.9415     NA     NA
log_k1           -1.9456     NA     NA
log_k2           -4.4705     NA     NA
g_qlogis         -0.5117     NA     NA
a.1               2.7455 2.55392 2.9370
SD.log_k_JCZ38    1.3163 0.47635 2.1563
SD.log_k_J9Z38    0.7162 0.16133 1.2711
SD.log_k_JSE76    0.6457 0.15249 1.1390
SD.f_cyan_ilr_1   0.3424 0.11714 0.5677
SD.f_cyan_ilr_2   0.4524 0.09709 0.8077
SD.log_k1         0.7353 0.25445 1.2161
SD.log_k2         0.5137 0.18206 0.8453
SD.g_qlogis       0.9857 0.35651 1.6148


Correlation is not available

Random effects:
                  est.   lower  upper
SD.log_k_JCZ38  1.3163 0.47635 2.1563
SD.log_k_J9Z38  0.7162 0.16133 1.2711
SD.log_k_JSE76  0.6457 0.15249 1.1390
SD.f_cyan_ilr_1 0.3424 0.11714 0.5677
SD.f_cyan_ilr_2 0.4524 0.09709 0.8077
SD.log_k1       0.7353 0.25445 1.2161
SD.log_k2       0.5137 0.18206 0.8453
SD.g_qlogis     0.9857 0.35651 1.6148


Variance model:
     est. lower upper
a.1 2.745 2.554 2.937


Backtransformed parameters:
                   est. lower upper
cyan_0          1.025e+02    NA    NA
k_JCZ38         5.309e-02    NA    NA
k_J9Z38         5.844e-03    NA    NA
k_JSE76         2.610e-02    NA    NA
f_cyan_to_JCZ38 6.079e-01    NA    NA
f_cyan_to_J9Z38 2.272e-01    NA    NA
f_JCZ38_to_JSE76 1.000e+00   NA    NA
f_JSE76_to_JCZ38 8.745e-01   NA    NA
k1              1.429e-01    NA    NA
k2              1.144e-02    NA    NA
g               3.748e-01    NA    NA


Resulting formation fractions:
              ff
cyan_JCZ38  0.6079
cyan_J9Z38  0.2272
cyan_sink   0.1649
JCZ38_JSE76 1.0000
JCZ38_sink  0.0000
JSE76_JCZ38 0.8745
JSE76_sink  0.1255


Estimated disappearance times:
        DT50   DT90 DT50back DT50_k1 DT50_k2
cyan   22.29 160.20    48.22    4.85   60.58
JCZ38  13.06  43.37       NA      NA      NA
J9Z38 118.61 394.02       NA      NA      NA
JSE76  26.56  88.22       NA      NA      NA
```

Listing 19: Hierarchical DFOP path 2 fit with reduced random effects, two-component error

```
saemix version used for fitting:       3.2
mkin version used for pre-fitting:  1.2.3
R version used for fitting:         4.2.3
Date of fit:     Thu Apr 20 10:25:36 2023
Date of summary: Thu Apr 20 19:09:25 2023


Equations:
d_cyan/dt = - ((k1 * g * exp(-k1 * time) + k2 * (1 - g) * exp(-k2 *
            time)) / (g * exp(-k1 * time) + (1 - g) * exp(-k2 * time)))
            * cyan
d_JCZ38/dt = + f_cyan_to_JCZ38 * ((k1 * g * exp(-k1 * time) + k2 * (1 -
            g) * exp(-k2 * time)) / (g * exp(-k1 * time) + (1 - g) *
            exp(-k2 * time))) * cyan - k_JCZ38 * JCZ38 +
            f_JSE76_to_JCZ38 * k_JSE76 * JSE76
d_J9Z38/dt = + f_cyan_to_J9Z38 * ((k1 * g * exp(-k1 * time) + k2 * (1 -
            g) * exp(-k2 * time)) / (g * exp(-k1 * time) + (1 - g) *
            exp(-k2 * time))) * cyan - k_J9Z38 * J9Z38
d_JSE76/dt = + f_JCZ38_to_JSE76 * k_JCZ38 * JCZ38 - k_JSE76 * JSE76


Data:
433 observations of 4 variable(s) grouped in 5 datasets


Model predictions using solution type deSolve


Fitted in 909.075 s
Using 300, 100 iterations and 10 chains


Variance model: Two-component variance function


Starting values for degradation parameters:
       cyan_0    log_k_JCZ38    log_k_J9Z38    log_k_JSE76    f_cyan_ilr_1
      101.7523        -1.5948        -5.0119        -2.2723          0.6719
  f_cyan_ilr_2 f_JCZ38_qlogis f_JSE76_qlogis          log_k1          log_k2
        5.1681        12.8238        12.4130         -2.0057         -4.5526
      g_qlogis
       -0.5805


Fixed degradation parameter values:
None


Starting values for random effects (square root of initial entries in omega):
             cyan_0 log_k_JCZ38 log_k_J9Z38 log_k_JSE76 f_cyan_ilr_1
cyan_0        5.627       0.000       0.000       0.000       0.0000
log_k_JCZ38   0.000       2.327       0.000       0.000       0.0000
log_k_J9Z38   0.000       0.000       1.664       0.000       0.0000
log_k_JSE76   0.000       0.000       0.000       4.566       0.0000
f_cyan_ilr_1  0.000       0.000       0.000       0.000       0.6519
f_cyan_ilr_2  0.000       0.000       0.000       0.000       0.0000
f_JCZ38_qlogis 0.000      0.000       0.000       0.000       0.0000
f_JSE76_qlogis 0.000      0.000       0.000       0.000       0.0000
log_k1        0.000       0.000       0.000       0.000       0.0000
log_k2        0.000       0.000       0.000       0.000       0.0000
g_qlogis      0.000       0.000       0.000       0.000       0.0000
             f_cyan_ilr_2 f_JCZ38_qlogis f_JSE76_qlogis log_k1 log_k2
cyan_0               0.0           0.00           0.00 0.0000 0.0000
log_k_JCZ38          0.0           0.00           0.00 0.0000 0.0000
log_k_J9Z38          0.0           0.00           0.00 0.0000 0.0000
log_k_JSE76          0.0           0.00           0.00 0.0000 0.0000
f_cyan_ilr_1         0.0           0.00           0.00 0.0000 0.0000
f_cyan_ilr_2        10.1           0.00           0.00 0.0000 0.0000
f_JCZ38_qlogis       0.0          13.99           0.00 0.0000 0.0000
f_JSE76_qlogis       0.0           0.00          14.15 0.0000 0.0000
log_k1               0.0           0.00           0.00 0.8452 0.0000
log_k2               0.0           0.00           0.00 0.0000 0.5968
g_qlogis             0.0           0.00           0.00 0.0000 0.0000
             g_qlogis
cyan_0          0.000
log_k_JCZ38     0.000
log_k_J9Z38     0.000
log_k_JSE76     0.000
f_cyan_ilr_1    0.000
f_cyan_ilr_2    0.000
f_JCZ38_qlogis  0.000
f_JSE76_qlogis  0.000
log_k1          0.000
log_k2          0.000
g_qlogis        1.691


Starting values for error model parameters:
a.1 b.1
  1   1
```

```
Results:

Likelihood computed by importance sampling
   AIC  BIC logLik
  2232 2224  -1096

Optimised parameters:
                     est.    lower    upper
cyan_0          101.20051       NA       NA
log_k_JCZ38      -2.93542       NA       NA
log_k_J9Z38      -5.03151       NA       NA
log_k_JSE76      -3.67679       NA       NA
f_cyan_ilr_1      0.67290       NA       NA
f_cyan_ilr_2      0.99787       NA       NA
f_JCZ38_qlogis  348.32484       NA       NA
f_JSE76_qlogis    1.87846       NA       NA
log_k1           -2.32738       NA       NA
log_k2           -4.61295       NA       NA
g_qlogis         -0.38342       NA       NA
a.1               2.06184  1.83746  2.28622
b.1               0.06329  0.05211  0.07447
SD.log_k_JCZ38    1.29042  0.47468  2.10617
SD.log_k_J9Z38    0.84235  0.25903  1.42566
SD.log_k_JSE76    0.56930  0.13934  0.99926
SD.f_cyan_ilr_1   0.35183  0.12298  0.58068
SD.f_cyan_ilr_2   0.77269  0.17908  1.36631
SD.log_k2         0.28549  0.09210  0.47888
SD.g_qlogis       0.93830  0.34568  1.53093


Correlation is not available

Random effects:
                   est.  lower   upper
SD.log_k_JCZ38   1.2904 0.4747  2.1062
SD.log_k_J9Z38   0.8423 0.2590  1.4257
SD.log_k_JSE76   0.5693 0.1393  0.9993
SD.f_cyan_ilr_1  0.3518 0.1230  0.5807
SD.f_cyan_ilr_2  0.7727 0.1791  1.3663
SD.log_k2        0.2855 0.0921  0.4789
SD.g_qlogis      0.9383 0.3457  1.5309

Variance model:
       est.   lower   upper
a.1 2.06184 1.83746 2.28622
b.1 0.06329 0.05211 0.07447

Backtransformed parameters:
                    est. lower upper
cyan_0         1.012e+02    NA    NA
k_JCZ38        5.311e-02    NA    NA
k_J9Z38        6.529e-03    NA    NA
k_JSE76        2.530e-02    NA    NA
f_cyan_to_JCZ38 6.373e-01   NA    NA
f_cyan_to_J9Z38 2.461e-01   NA    NA
f_JCZ38_to_JSE76 1.000e+00  NA    NA
f_JSE76_to_JCZ38 8.674e-01  NA    NA
k1             9.755e-02    NA    NA
k2             9.922e-03    NA    NA
g              4.053e-01    NA    NA

Resulting formation fractions:
               ff
cyan_JCZ38  0.6373
cyan_J9Z38  0.2461
cyan_sink   0.1167
JCZ38_JSE76 1.0000
JCZ38_sink  0.0000
JSE76_JCZ38 0.8674
JSE76_sink  0.1326

Estimated disappearance times:
        DT50   DT90 DT50back DT50_k1 DT50_k2
cyan   24.93 179.68    54.09   7.105   69.86
JCZ38  13.05  43.36       NA      NA      NA
J9Z38 106.16 352.67       NA      NA      NA
JSE76  27.39  91.00       NA      NA      NA
```

```
saemix version used for fitting:      3.2
mkin version used for pre-fitting:  1.2.3
R version used for fitting:         4.2.3
Date of fit:     Thu Apr 20 10:24:51 2023
Date of summary: Thu Apr 20 19:09:25 2023


Equations:
d_cyan_free/dt = - k_cyan_free * cyan_free - k_cyan_free_bound *
          cyan_free + k_cyan_bound_free * cyan_bound
d_cyan_bound/dt = + k_cyan_free_bound * cyan_free - k_cyan_bound_free *
          cyan_bound
d_JCZ38/dt = + f_cyan_free_to_JCZ38 * k_cyan_free * cyan_free - k_JCZ38
          * JCZ38 + f_JSE76_to_JCZ38 * k_JSE76 * JSE76
d_J9Z38/dt = + f_cyan_free_to_J9Z38 * k_cyan_free * cyan_free - k_J9Z38
          * J9Z38
d_JSE76/dt = + f_JCZ38_to_JSE76 * k_JCZ38 * JCZ38 - k_JSE76 * JSE76


Data:
433 observations of 4 variable(s) grouped in 5 datasets


Model predictions using solution type deSolve


Fitted in 864.48 s
Using 300, 100 iterations and 10 chains


Variance model: Constant variance


Starting values for degradation parameters:
        cyan_free_0      log_k_cyan_free log_k_cyan_free_bound
           102.4394              -2.7673               -2.8942
log_k_cyan_bound_free          log_k_JCZ38           log_k_J9Z38
            -3.6201              -2.3107               -5.3123
       log_k_JSE76           f_cyan_ilr_1          f_cyan_ilr_2
            -3.7120               0.6754                1.1448
      f_JCZ38_qlogis         f_JSE76_qlogis
            13.2672               13.3538


Fixed degradation parameter values:
None


Starting values for random effects (square root of initial entries in omega):
                  cyan_free_0 log_k_cyan_free log_k_cyan_free_bound
cyan_free_0             4.589          0.0000                  0.00
log_k_cyan_free         0.000          0.4849                  0.00
log_k_cyan_free_bound   0.000          0.0000                  1.62
log_k_cyan_bound_free   0.000          0.0000                  0.00
log_k_JCZ38             0.000          0.0000                  0.00
log_k_J9Z38             0.000          0.0000                  0.00
log_k_JSE76             0.000          0.0000                  0.00
f_cyan_ilr_1            0.000          0.0000                  0.00
f_cyan_ilr_2            0.000          0.0000                  0.00
f_JCZ38_qlogis          0.000          0.0000                  0.00
f_JSE76_qlogis          0.000          0.0000                  0.00
                  log_k_cyan_bound_free log_k_JCZ38 log_k_J9Z38 log_k_JSE76
cyan_free_0                       0.000      0.0000       0.000         0.0
log_k_cyan_free                   0.000      0.0000       0.000         0.0
log_k_cyan_free_bound             0.000      0.0000       0.000         0.0
log_k_cyan_bound_free             1.197      0.0000       0.000         0.0
log_k_JCZ38                       0.000      0.7966       0.000         0.0
log_k_J9Z38                       0.000      0.0000       1.561         0.0
log_k_JSE76                       0.000      0.0000       0.000         0.8
f_cyan_ilr_1                      0.000      0.0000       0.000         0.0
f_cyan_ilr_2                      0.000      0.0000       0.000         0.0
f_JCZ38_qlogis                    0.000      0.0000       0.000         0.0
f_JSE76_qlogis                    0.000      0.0000       0.000         0.0
                  f_cyan_ilr_1 f_cyan_ilr_2 f_JCZ38_qlogis f_JSE76_qlogis
cyan_free_0             0.0000        0.000           0.00           0.00
log_k_cyan_free         0.0000        0.000           0.00           0.00
log_k_cyan_free_bound   0.0000        0.000           0.00           0.00
log_k_cyan_bound_free   0.0000        0.000           0.00           0.00
log_k_JCZ38             0.0000        0.000           0.00           0.00
log_k_J9Z38             0.0000        0.000           0.00           0.00
log_k_JSE76             0.0000        0.000           0.00           0.00
f_cyan_ilr_1            0.6349        0.000           0.00           0.00
f_cyan_ilr_2            0.0000        1.797           0.00           0.00
f_JCZ38_qlogis          0.0000        0.000          13.84           0.00
f_JSE76_qlogis          0.0000        0.000           0.00          14.66


Starting values for error model parameters:
a.1
  1
```

```
Results:

Likelihood computed by importance sampling
   AIC  BIC logLik
  2279 2272  -1120

Optimised parameters:
                            est.    lower   upper
cyan_free_0             102.5621      NA      NA
log_k_cyan_free          -2.8531      NA      NA
log_k_cyan_free_bound    -2.6916      NA      NA
log_k_cyan_bound_free    -3.5032      NA      NA
log_k_JCZ38              -2.9436      NA      NA
log_k_J9Z38             -5.1140      NA      NA
log_k_JSE76              -3.6472      NA      NA
f_cyan_ilr_1              0.6887      NA      NA
f_cyan_ilr_2             0.6874      NA      NA
f_JCZ38_qlogis        4063.6389      NA      NA
f_JSE76_qlogis           1.9556      NA      NA
a.1                      2.7460 2.55451 2.9376
SD.log_k_cyan_free       0.3131 0.09841 0.5277
SD.log_k_cyan_free_bound 0.8850 0.29909 1.4710
SD.log_k_cyan_bound_free 0.6167 0.20391 1.0295
SD.log_k_JCZ38           1.3555 0.49101 2.2200
SD.log_k_J9Z38          0.7200 0.16166 1.2783
SD.log_k_JSE76           0.6252 0.14619 1.1042
SD.f_cyan_ilr_1          0.3386 0.11447 0.5627
SD.f_cyan_ilr_2          0.4699 0.09810 0.8417


Correlation is not available

Random effects:
                            est.    lower   upper
SD.log_k_cyan_free       0.3131 0.09841 0.5277
SD.log_k_cyan_free_bound 0.8850 0.29909 1.4710
SD.log_k_cyan_bound_free 0.6167 0.20391 1.0295
SD.log_k_JCZ38           1.3555 0.49101 2.2200
SD.log_k_J9Z38          0.7200 0.16166 1.2783
SD.log_k_JSE76           0.6252 0.14619 1.1042
SD.f_cyan_ilr_1          0.3386 0.11447 0.5627
SD.f_cyan_ilr_2          0.4699 0.09810 0.8417


Variance model:
     est. lower upper
a.1 2.746 2.555 2.938

Backtransformed parameters:
                         est. lower upper
cyan_free_0         1.026e+02    NA    NA
k_cyan_free         5.767e-02    NA    NA
k_cyan_free_bound   6.777e-02    NA    NA
k_cyan_bound_free   3.010e-02    NA    NA
k_JCZ38             5.267e-02    NA    NA
k_J9Z38             6.012e-03    NA    NA
k_JSE76             2.606e-02    NA    NA
f_cyan_free_to_JCZ38 6.089e-01   NA    NA
f_cyan_free_to_J9Z38 2.299e-01   NA    NA
f_JCZ38_to_JSE76    1.000e+00    NA    NA
f_JSE76_to_JCZ38    8.761e-01    NA    NA

Estimated Eigenvalues of SFORB model(s):
cyan_b1 cyan_b2  cyan_g
 0.1434  0.0121  0.3469

Resulting formation fractions:
                    ff
cyan_free_JCZ38 0.6089
cyan_free_J9Z38 0.2299
cyan_free_sink  0.1612
cyan_free       1.0000
JCZ38_JSE76     1.0000
JCZ38_sink      0.0000
JSE76_JCZ38     0.8761
JSE76_sink      0.1239

Estimated disappearance times:
        DT50   DT90 DT50back DT50_cyan_b1 DT50_cyan_b2
cyan   23.94 155.06    46.68        4.832        57.28
JCZ38  13.16  43.71       NA           NA           NA
J9Z38 115.30 383.02       NA           NA           NA
JSE76  26.59  88.35       NA           NA           NA
```

Listing 21: Hierarchical SFORB path 2 fit with reduced random effects, two-component error

```
saemix version used for fitting:      3.2
mkin version used for pre-fitting:  1.2.3
R version used for fitting:         4.2.3
Date of fit:     Thu Apr 20 10:25:19 2023
Date of summary: Thu Apr 20 19:09:25 2023


Equations:
d_cyan_free/dt = - k_cyan_free * cyan_free - k_cyan_free_bound *
         cyan_free + k_cyan_bound_free * cyan_bound
d_cyan_bound/dt = + k_cyan_free_bound * cyan_free - k_cyan_bound_free *
         cyan_bound
d_JCZ38/dt = + f_cyan_free_to_JCZ38 * k_cyan_free * cyan_free - k_JCZ38
         * JCZ38 + f_JSE76_to_JCZ38 * k_JSE76 * JSE76
d_J9Z38/dt = + f_cyan_free_to_J9Z38 * k_cyan_free * cyan_free - k_J9Z38
         * J9Z38
d_JSE76/dt = + f_JCZ38_to_JSE76 * k_JCZ38 * JCZ38 - k_JSE76 * JSE76


Data:
433 observations of 4 variable(s) grouped in 5 datasets


Model predictions using solution type deSolve


Fitted in 892.076 s
Using 300, 100 iterations and 10 chains


Variance model: Two-component variance function


Starting values for degradation parameters:
        cyan_free_0      log_k_cyan_free log_k_cyan_free_bound
            101.751              -2.837                -3.016
log_k_cyan_bound_free        log_k_JCZ38          log_k_J9Z38
             -3.660              -2.299                -5.313
        log_k_JSE76          f_cyan_ilr_1         f_cyan_ilr_2
             -3.699               0.672                 5.873
      f_JCZ38_qlogis       f_JSE76_qlogis
             13.216               13.338


Fixed degradation parameter values:
None


Starting values for random effects (square root of initial entries in omega):
                      cyan_free_0 log_k_cyan_free log_k_cyan_free_bound
cyan_free_0                 5.629           0.000                 0.000
log_k_cyan_free             0.000           0.446                 0.000
log_k_cyan_free_bound       0.000           0.000                 1.449
log_k_cyan_bound_free       0.000           0.000                 0.000
log_k_JCZ38                 0.000           0.000                 0.000
log_k_J9Z38                 0.000           0.000                 0.000
log_k_JSE76                 0.000           0.000                 0.000
f_cyan_ilr_1                0.000           0.000                 0.000
f_cyan_ilr_2                0.000           0.000                 0.000
f_JCZ38_qlogis              0.000           0.000                 0.000
f_JSE76_qlogis              0.000           0.000                 0.000
                      log_k_cyan_bound_free log_k_JCZ38 log_k_J9Z38 log_k_JSE76
cyan_free_0                           0.000      0.0000       0.000      0.0000
log_k_cyan_free                       0.000      0.0000       0.000      0.0000
log_k_cyan_free_bound                 0.000      0.0000       0.000      0.0000
log_k_cyan_bound_free                 1.213      0.0000       0.000      0.0000
log_k_JCZ38                           0.000      0.7801       0.000      0.0000
log_k_J9Z38                           0.000      0.0000       1.575      0.0000
log_k_JSE76                           0.000      0.0000       0.000      0.8078
f_cyan_ilr_1                          0.000      0.0000       0.000      0.0000
f_cyan_ilr_2                          0.000      0.0000       0.000      0.0000
f_JCZ38_qlogis                        0.000      0.0000       0.000      0.0000
f_JSE76_qlogis                        0.000      0.0000       0.000      0.0000
                      f_cyan_ilr_1 f_cyan_ilr_2 f_JCZ38_qlogis f_JSE76_qlogis
cyan_free_0                 0.0000         0.00           0.00           0.00
log_k_cyan_free             0.0000         0.00           0.00           0.00
log_k_cyan_free_bound       0.0000         0.00           0.00           0.00
log_k_cyan_bound_free       0.0000         0.00           0.00           0.00
log_k_JCZ38                 0.0000         0.00           0.00           0.00
log_k_J9Z38                 0.0000         0.00           0.00           0.00
log_k_JSE76                 0.0000         0.00           0.00           0.00
f_cyan_ilr_1                0.6519         0.00           0.00           0.00
f_cyan_ilr_2                0.0000        10.78           0.00           0.00
f_JCZ38_qlogis              0.0000         0.00          13.96           0.00
f_JSE76_qlogis              0.0000         0.00           0.00          14.69


Starting values for error model parameters:
a.1 b.1
  1   1
```

```
Results:

Likelihood computed by importance sampling
   AIC  BIC logLik
  2236 2228  -1098

Optimised parameters:
                           est.    lower    upper
cyan_free_0            100.72760      NA      NA
log_k_cyan_free         -3.18281      NA      NA
log_k_cyan_free_bound   -3.37924      NA      NA
log_k_cyan_bound_free   -3.77107      NA      NA
log_k_JCZ38             -2.92811      NA      NA
log_k_J9Z38             -5.02759      NA      NA
log_k_JSE76             -3.65835      NA      NA
f_cyan_ilr_1             0.67390      NA      NA
f_cyan_ilr_2             1.15106      NA      NA
f_JCZ38_qlogis         827.82299      NA      NA
f_JSE76_qlogis           1.83064      NA      NA
a.1                      2.06921 1.84443 2.29399
b.1                      0.06391 0.05267 0.07515
SD.log_k_cyan_free       0.50518 0.18962 0.82075
SD.log_k_cyan_bound_free 0.30991 0.08170 0.53813
SD.log_k_JCZ38           1.26661 0.46578 2.06744
SD.log_k_J9Z38           0.88272 0.27813 1.48730
SD.log_k_JSE76           0.53050 0.12561 0.93538
SD.f_cyan_ilr_1          0.35547 0.12461 0.58633
SD.f_cyan_ilr_2          0.91446 0.20131 1.62761


Correlation is not available

Random effects:
                          est.   lower  upper
SD.log_k_cyan_free       0.5052 0.1896 0.8207
SD.log_k_cyan_bound_free 0.3099 0.0817 0.5381
SD.log_k_JCZ38           1.2666 0.4658 2.0674
SD.log_k_J9Z38           0.8827 0.2781 1.4873
SD.log_k_JSE76           0.5305 0.1256 0.9354
SD.f_cyan_ilr_1          0.3555 0.1246 0.5863
SD.f_cyan_ilr_2          0.9145 0.2013 1.6276


Variance model:
      est.   lower   upper
a.1 2.06921 1.84443 2.29399
b.1 0.06391 0.05267 0.07515


Backtransformed parameters:
                      est. lower upper
cyan_free_0        1.007e+02   NA   NA
k_cyan_free        4.147e-02   NA   NA
k_cyan_free_bound  3.407e-02   NA   NA
k_cyan_bound_free  2.303e-02   NA   NA
k_JCZ38            5.350e-02   NA   NA
k_J9Z38            6.555e-03   NA   NA
k_JSE76            2.578e-02   NA   NA
f_cyan_free_to_JCZ38 6.505e-01   NA   NA
f_cyan_free_to_J9Z38 2.508e-01   NA   NA
f_JCZ38_to_JSE76   1.000e+00   NA   NA
f_JSE76_to_JCZ38   8.618e-01   NA   NA


Estimated Eigenvalues of SFORB model(s):
cyan_b1 cyan_b2  cyan_g
0.08768 0.01089 0.39821


Resulting formation fractions:
                   ff
cyan_free_JCZ38 0.65053
cyan_free_J9Z38 0.25082
cyan_free_sink  0.09864
cyan_free       1.00000
JCZ38_JSE76     1.00000
JCZ38_sink      0.00000
JSE76_JCZ38     0.86184
JSE76_sink      0.13816


Estimated disappearance times:
        DT50   DT90 DT50back DT50_cyan_b1 DT50_cyan_b2
cyan   25.32 164.79    49.61        7.906        63.64
JCZ38  12.96  43.04       NA           NA           NA
J9Z38 105.75 351.29       NA           NA           NA
JSE76  26.89  89.33       NA           NA           NA
```

Session info

```
R version 4.2.3 (2023-03-15)
Platform: x86_64-pc-linux-gnu (64-bit)
Running under: Debian GNU/Linux 12 (bookworm)

Matrix products: default
BLAS:   /usr/lib/x86_64-linux-gnu/openblas-serial/libblas.so.3
LAPACK: /usr/lib/x86_64-linux-gnu/openblas-serial/libopenblas-r0.3.21.so

locale:
 [1] LC_CTYPE=de_DE.UTF-8       LC_NUMERIC=C
 [3] LC_TIME=de_DE.UTF-8        LC_COLLATE=de_DE.UTF-8
 [5] LC_MONETARY=de_DE.UTF-8    LC_MESSAGES=de_DE.UTF-8
 [7] LC_PAPER=de_DE.UTF-8       LC_NAME=C
 [9] LC_ADDRESS=C               LC_TELEPHONE=C
[11] LC_MEASUREMENT=de_DE.UTF-8 LC_IDENTIFICATION=C

attached base packages:
[1] parallel  stats     graphics  grDevices utils     datasets  methods
[8] base

other attached packages:
[1] saemix_3.2       npde_3.3            knitr_1.42        mkin_1.2.3
[5] rmarkdown_2.21   nvimcom_0.9-133.1

loaded via a namespace (and not attached):
 [1] compiler_4.2.3   pillar_1.9.0    tools_4.2.3      digest_0.6.31
 [5] mclust_6.0.0     evaluate_0.20   lifecycle_1.0.3  tibble_3.2.1
 [9] gtable_0.3.3     nlme_3.1-162    lattice_0.21-8   pkgconfig_2.0.3
[13] rlang_1.1.0      cli_3.6.1       DBI_1.1.3        yaml_2.3.7
[17] xfun_0.38        fastmap_1.1.1   gridExtra_2.3    dplyr_1.1.1
[21] generics_0.1.3   vctrs_0.6.1     lmtest_0.9-40    grid_4.2.3
[25] tidyselect_1.2.0 inline_0.3.19   glue_1.6.2       R6_2.5.1
[29] fansi_1.0.4      ggplot2_3.4.2   magrittr_2.0.3   scales_1.2.1
[33] htmltools_0.5.5  colorspace_2.1-0 utf8_1.2.3      munsell_0.5.0
[37] zoo_1.8-12
```

Hardware info

```
CPU model: AMD Ryzen 9 7950X 16-Core Processor

MemTotal:       64936316 kB
```