

Referee's report to Authors on

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Nonparametric goodness-of-fit tests for discrete null distributions

by

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Summary:

This article describes implementation of empirical distribution function tests for fully specified discrete distributions. I found the article clear, well written and pretty comprehensive. I have three comments or suggestions:

1. I think you ought to remind readers that the hypotheses being tested must not have any unknown parameters; it is not ok to estimate the parameters and then use the fitted distribution as the null hypothesis. The relevant large sample theory for Cramér-von Mises statistics is in Lockhart, Spinelli, and Stephens, *CJS*, 2007. Those authors should have written R code to implement their tests but they never did.
2. In the 2007 paper just cited the authors suggest alternative test statistics because they noticed that in non-uniform cases if they reversed the order of the cells they got a different statistic value. The modifications are not great. Suppose you were testing the hypothesis that a sample of X values come from the binomial distribution with k trials and success probability $3/4$. The values $k - X$ would be a sample from the binomial distribution with success probability $1/4$ and you would probably want the test statistics to be the same in both cases. (Switching the meaning of success and failure should not change the conclusion about whether or not the binomial model is appropriate.)
3. The upper bound on the tail of a linear combination of chi-squares can likely be improved by Markov's inequality:

$$P\left(\sum_i \lambda_i Z_i^2 \geq x\right) \leq E\left(\exp\left(t \sum_i \lambda_i Z_i^2\right)\right) \exp(-tx) = \frac{e^{-tx}}{\sqrt{\prod (1 - 2t\lambda_i)}}.$$

Here the Z_i are standard normal and I hope I have the formula for the MGF of a chi-square right. Here the Z_i are standard normal and I hope I have the formula for the mgf of a chi-square right. The final quantity on the right should be minimized over $0 < t < (2\lambda_{\max})^{-1}$ to get a pretty tight bound as in the fashion of extreme value theory.

Overall, well done.