Non-positive definite of  matrix for turnover and proportional cost constraints:

Causes:

In the case of turnover and proportional cost constraints, the variables  for the object function  to maximize is the extended weight vector consisted of three components: initial weight, buy weight, sell weight, e.g., , and  is the 2 times the covariance matrix of extended/tripled return matrix: replicates the return matrix two times in the number of columns that suitable to . Hence  is rank deficient, a.k.a. non-positive definite, since it is ablock replication of the covariance matrix of the return matrix and has identical rows. The non-positive definite property of renders errors in solve.QP.

Solution:

In order to use solve.QP, one way is to convert  to its nearest positive definite matrix by using make.positive.definite function in R package corpor.

The distance of the matrix  and its nearest positive definite matrix ,  is defined as:

 ,

where,  is the eigenvector of and  relates to some small perturbation which shifts all negative eigenvalues of polar decomposition of  to the origin. For details of the proof, see Theorem 3.1 and its proof in attached NJ Higham’s reprint paper.

From the empirical observation, the last few eigenvalues are very small negative numbers. Hence, the resulting nearest (Frobenius norm) positive definite matrix  is very similar to .

For instance, suppose we are looking at the first five stocks in mid cap data, the corresponding eigenvalues of the covariance matrix of the extended return matrix is

$values

[1] 2.380478e-01 1.340207e-01 4.777733e-02

[4] 3.438278e-02 1.375631e-02 1.125256e-17

[7] 4.022201e-18 8.334207e-19 -2.269527e-34

[10] -1.855057e-19 -2.261371e-19 -6.674203e-19

[13] -1.425105e-18 -2.356031e-18 -1.831675e-17

As we can see, the last few negative eigenvalues are close to zero, hence the nearest positive definite matrix is very similar to its original.