The Sceptical *P*-Value

Leonhard Held

University of Zurich[UZH]

Winter School 2022, Les Diablerets

A New Approach to Define Replication Success

J. R. Statist. Soc. A (2020)

A new standard for the analysis and design of replication studies

Leonhard Held

University of Zurich, Switzerland

[*Read before* The Royal Statistical Society *at a meeting on 'Signs and sizes: understanding and replicating statistical findings' at the Society's 2019 annual conference in Belfast on Wednesday, September 4th, 2019, the President*, Professor D. Ashby, *in the Chair*]

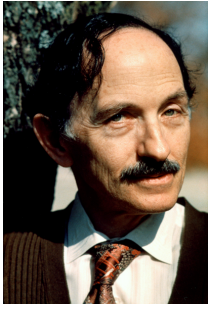
– A Bayes/non-Bayes compromise based on
  1. Reverse-Bayes analysis
  2. Prior criticism

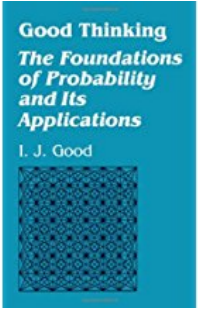
  → The sceptical p-value p_S quantifies degree of replication success

Step 1: Reverse-Bayes Analysis

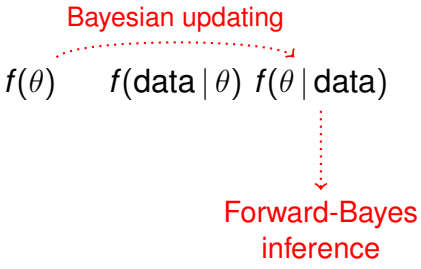
Jack Good (1916-2009)

"We can make judgments of initial probabilities and infer final ones, or we can equally make judgments of final ones and infer initial ones by **Bayes's theorem in reverse**.*"*





Forward- and Reverse-Bayes



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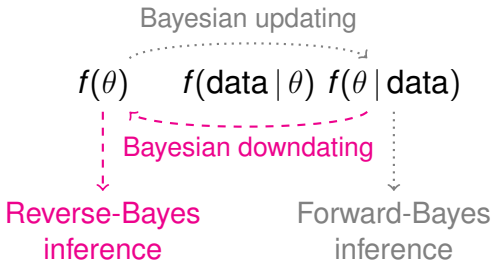

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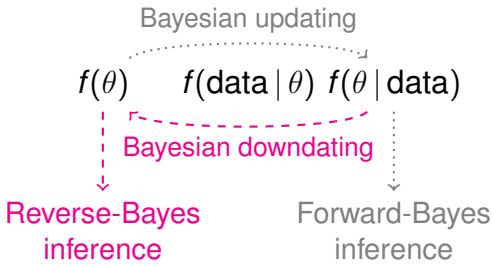


Reverse-Bayes methods for evidence assessment and research synthesis

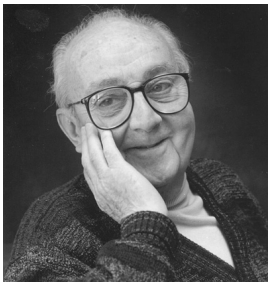
Leonhard Held [*,§], Robert Matthews [†,§], Manuela Ott [*,†], and Samuel Pawel [*,§]

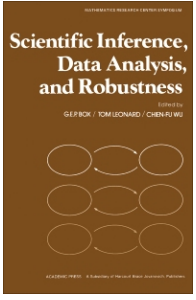
https://arxiv.org/abs/2102.13443

Step 2: Prior-Data Conflict

George Box (1919-2013)

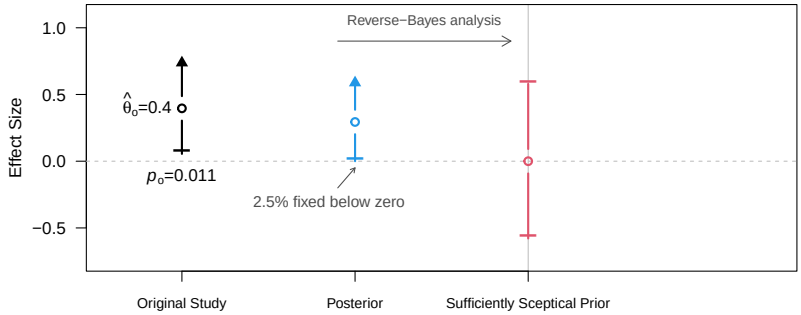
"The process of scientific investigation involves not one but two kinds of inference: estimation and criticism, used iteratively and in alternation."





The Proposed Approach: Step 1

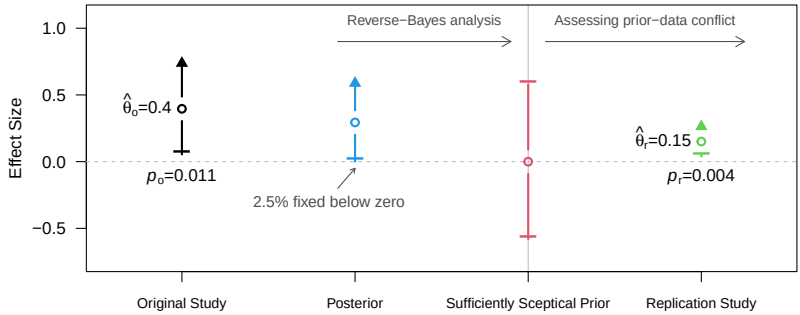
One-sided $\alpha = 2.5\%$



– Determine the variance τ^2 of a sceptical prior $N(0, \tau^2)$ that makes the original result no longer convincing.

The Proposed Approach: Step 2

One-sided $\alpha = 2.5\%$



– Prior-data conflict is quantified based on the prior-predictive distribution:

$$p_{\text{Box}} = \Pr\{\text{N}(0, \tau^2 + \sigma_r^2) \geq \hat{\theta}_r\}.$$

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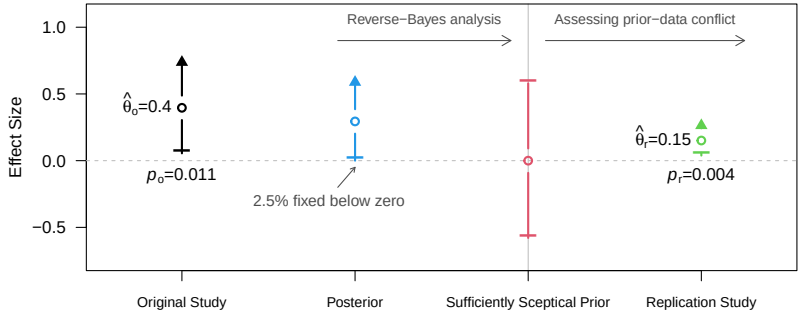

– Prior-data conflict is quantified based on the prior-predictive distribution:

$$p_{\text{Box}} = \Pr\{\text{N}(0, \tau^2 + \sigma_r^2) \geq \hat{\theta}_r\}.$$

– Replication success is achieved if $p_{\text{Box}} \leq \alpha$.

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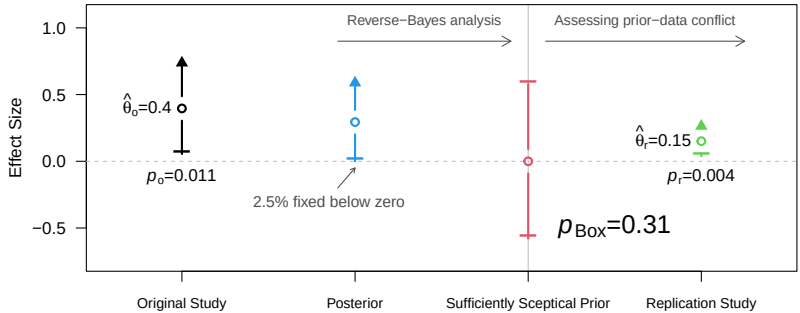
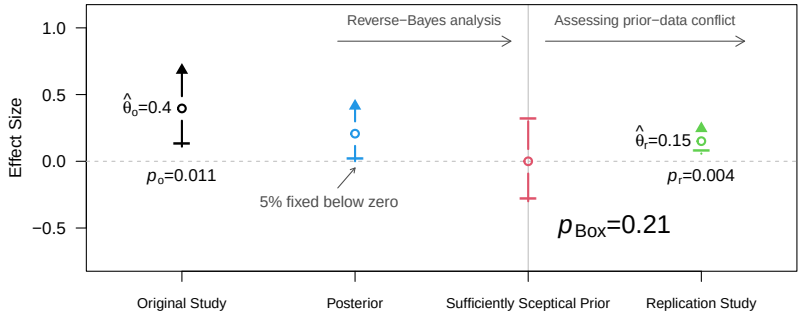


No replication success at level $\alpha = 2.5\%$

The Proposed Approach

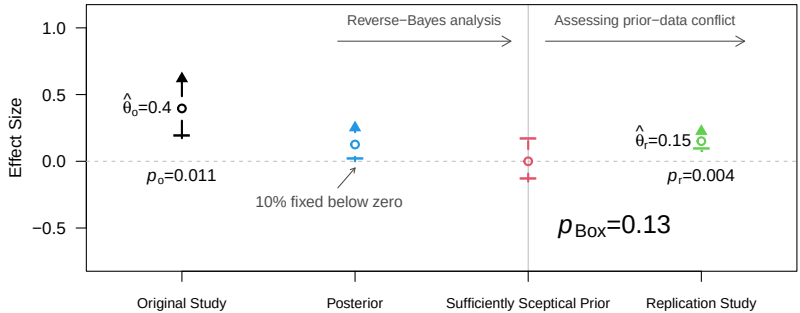
One-sided $\alpha = 5\%$



No replication success at level $\alpha = 5\%$

The Proposed Approach

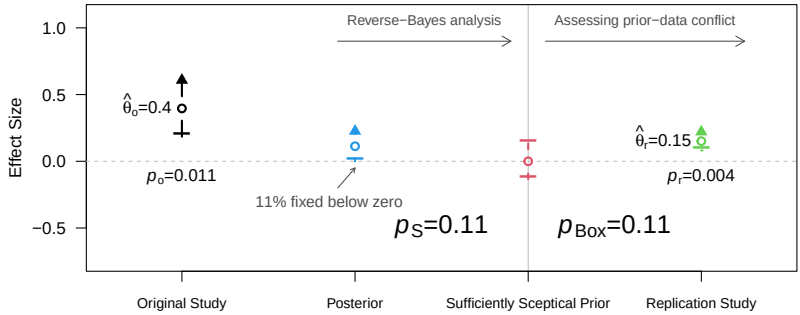
One-sided $\alpha = 10\%$



No replication success at level $\alpha = 10\%$

The Proposed Approach

One-sided $\alpha = 11\%$



Replication success at level $\alpha = 11\%$

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- can be computed analytically under standard normality assumptions
- depends on both p-values p_o and p_r and the relative sample size c
- has a particularly simple form for $c = 1$ with known null distribution

Appl. Statist. (2020)

The harmonic mean χ^2-test to substantiate scientific findings

Leonhard Held
University of Zurich, Switzerland

Replication Success in Terms of Relative Effect Size

Goal: Comparison of

- **sceptical *p*-value**
- **two-trials rule**
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THE ASSESSMENT OF REPLICATION SUCCESS BASED ON RELATIVE EFFECT SIZE

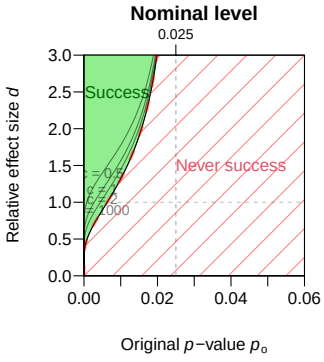
By Leonhard Held, Charlotte Micheloud and Samuel Pawel

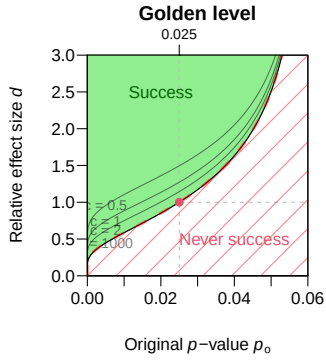
Epidemiology, Biostatistics and Prevention Institute, Center for Reproducible Science, University of Zurich,
leonhard.held@uzh.ch; charlotte.micheloud@uzh.ch; samuel.pawel@uzh.ch

AOAS, to appear

https://arxiv.org/abs/2009.07782

Recalibration of the Sceptical *p*-Value





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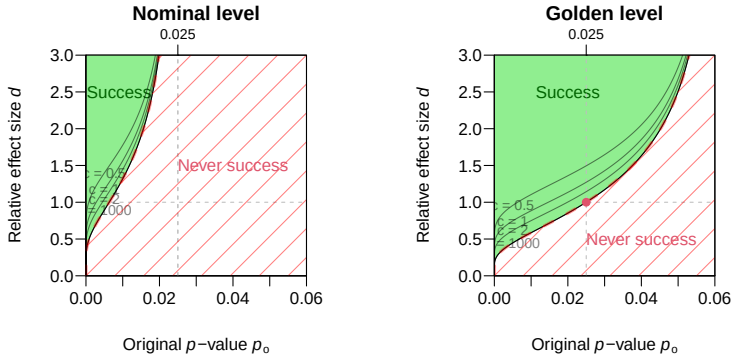
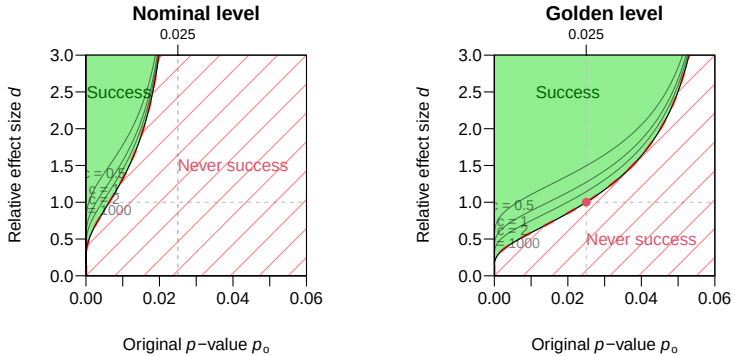


For a borderline convincing original result ($p_o = 0.025$), replication success

– is impossible at the nominal level.

Recalibration of the Sceptical p-Value



For a borderline convincing original result ($p_o = 0.025$), replication success

- is impossible at the nominal level.
- is possible at the golden level if the relative effect size is larger than one.

The Golden Level

Replication success:

$$
\begin{aligned}
p_S &< \alpha_S \\
\alpha_S &= 1 - \Phi(z_\alpha/\sqrt{\varphi}) \\
\text{with golden ratio } \varphi &= (\sqrt{5}+1)/2 \approx 1.62
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(default in the `R`-package `ReplicationSuccess`)

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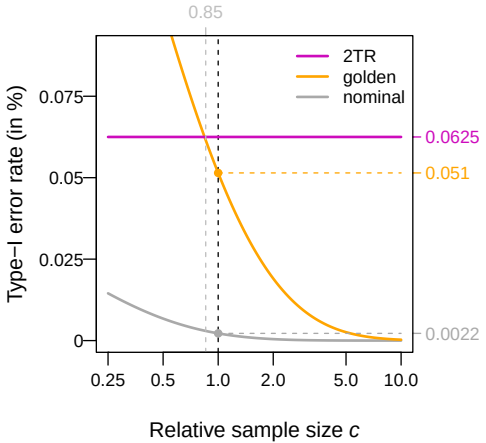
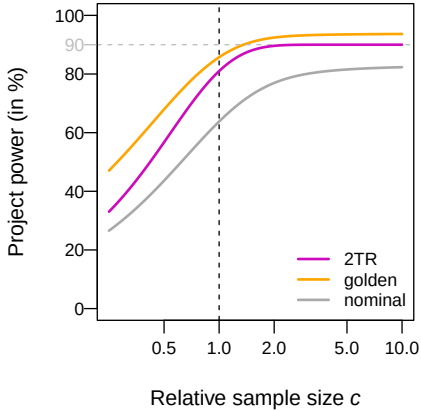
  (default in the `R`-package `ReplicationSuccess`)
- For example, for $p_S = 0.11$ we obtain $\tilde{p}_S = 0.061$

Type-I Error Rate and Project Power





How best to quantify replication success?

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Research

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How best to quantify replication success? A simulation study on the comparison of replication success metrics

Jasmine Muradchanian, Rink Hoekstra, Henk Kiers and Don van Ravenzwaaij

Behavioural and Social Sciences, University of Groningen, The Netherlands

"The sceptical p-value performed particularly well under scenarios of high publication bias."