STATISTICAL DATA ANALYSIS: RECURRENCE PLOT

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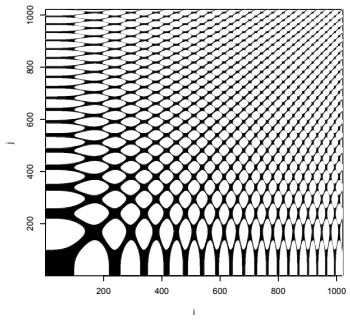

Contents

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1. Background

1.1. **Takens' Recurrence States.** Recurrence plots have been introduced in an attempt to understand near periodic behaviour in hydrodynamics, in particular the transition to turbulence. On the one hand, and extended theory on dynamical systems was available, covering deterministic models. A fundamental concept is that at a certain time a system is in some state, and developing from this. Defining the proper state space is a critical step in modelling.

The other toolkit is that of stochastics processes, in particular Markov models. Classical time series assumes stationarity, and this is obviously not the way to go. A fundamental idea for Markov models is that the system state is seen in a temporal context: you have a Markov process, if you can define a (non-anticipating) state that has sufficient information for prediction: given this state, the future is independent from the past.

Recurrence, coming back to some state, is often a key to understand a near periodic system. A classical field is the movement of celestial bodies.

Hydrodynamics is a challenging problem. Understanding planetary motion is a historical challenge, and may be useful as an illustration.

As a simple illustration, let $x = (x_i)$ be a sequence, maybe near periodic. For now, think of i as a time index.

Recurrence plots have two steps. The first was a bold step by Floris Takens. If you do not know the state space of a system, for a choice of "dimension" d, take the sequence of d tuples taken from your data to define the states.

$$u_i = (x_i, \ldots, x_{i+(d-1)})$$

This is Takens' delay embedding state (re)construction Takens [1981].

As a mere technical refinement: you may know that your data are a flattened representation of m dimensional data. So you take

$$u_i = (x_i, x_{i+m} \ldots, x_{i+(d-1)*m}).$$

ToDo: add support for higher dimensional signals

This may be a relict of FORTRAN times, where it was common to flatten two-dimensional structures by case.

Conceptually, you define states by observed histories. For classical Markov setup, the state is defined by the previous information x_{i-1}, but for more complex situations you may have to step back in the past. Finding the appropriate d is the challenge. So it may be appropriate to view the Takens states as a family, indexed by the time scope d. The rest is structural information how to arrange items.

Of course it is possible to compress information here, sorting states and removing duplicates. Keeping the original definition as the advantage that we have the index i, so that u_i is the state at index position i.

But the states may have an inherent structure, which we may take into account or ignore. Since for this example, we are just in 4-dimensional space, marginal scatterplots may give enough information.

Takens states are vectors in M dimensions. There are standard statistical techniques to visiualize aspects of M dimensional vectors, at least for not too high dimension. One is the draftsman's plot, a scatterplot matrix by marginals. For Takens states, this is

implemented as *statepairs()*. The other is coplots, a variant of scatterplot matrix by marginals, conditionend by one or two additional variables. For Takens states, this is implemented as *statecoplots()*

To display the Takens state space, we us a variant of pairs().

ToDo: the takTakensens state plot may be critically affected by outliers. Find a good rescaling.

By convention, the states are defined using overlapping sliding windows. This imposes considerable dependence between the states: one state is the shifted previous states, with only the end sub-state replaced. As an option, the states can be subsampled, using only non-overlapping ranges.

The Takens states may be stationary, that is asymptotically the states starting at i do not depend on i. In this case, the first row (or column) contains all information, and pairs plot form an inclusion sequence by. In general, we will use state plots in 4 or 8 dimensions, where the limits are suggested by the print area.

ToDo: consider dimension-adjusted radius

1.2. **Recurrence Plots.** The next step, taken in Eckmann *et al.* [1987] was to use a two dimensional display. Take a scatterplot with the Taken's states a marginal. Take a sliding window of your process data, and for each i, find the "distance" of u_i from and to any of the collected states. If the distance is below some chosen threshold, mark the point (i, j) for which u(j) is in the ball of radius r(i) centred at u(i).

The original publication Eckmann *et al.* [1987] actually used a nearest neighbourhood environment to cover about 10 data points.

The construction has considerable arbitrary choices. The critical radius may depend on the point i. In practical applications, using a constant radius is a common first step. Using a dichotomous marking was what presumably was necessary when the idea was introduced. With todays technology, we can allow a markup on a finer scale, as has been seen in Orion-1.

ToDo: support distance instead of 0/1 indicators

We can gain additional freedom by using a correlation view: instead of looking from one axis, we can walk along the diagonal, using two reference axis.

Helpful hints how to interpret recurrence plots are in "Recurrence Plots At A Glance" `<http://www.recurrence-plot.tk/glance.php>`.

1.3. **Recurrence Quantification Analysis.** While visual inspection is the prime way to assess recurrence plots, quantification of some aspects revealed of the plot may be helpful. A collection of indices is provided by a recurrence quantification analysis (RQA) Zbilut and Webber [2006], Webber Jr and Zbilut [2005].

See Table 1 on the following page.

2. R Setup

Input

```
save.RNGseed <- 87149 #.Random.seed
save.RNGkind <- RNGkind()
set.seed(save.RNGseed, save.RNGkind[1])
save.RNGkind
```

Output

TABLE 1. Recurrence Quantification Analysis (RQA)

REC	Recurrence. Percentage of recurrence points in a recurrence Plot.
DET	Determinism. Percentage of recurrence points that form diagonal lines.
LAM	Percentage of recurrent points that form vertical lines.
RATIO	Ratio between *DET* and *RR*.
Lmax	Length of the longest diagonal line.
Lmean	Mean length of the diagonal lines. The main diagonal is not taken into account.
DIV	Inverse of *Lmax*.
Vmax	Longest vertical line.
Vmean	Average length of the vertical lines. This parameter is also referred to as the Trapping time.
ENTR	Shannon entropy of the diagonal line lengths distribution
TREND	Trend of the number of recurrent points depending on the distance to the main diagonal
diagonalHistogram	Histogram of the length of the diagonals.
recurrenceRate	Number of recurrent points depending on the distance to the main diagonal.

```
[1] "Mersenne-Twister" "Inversion"
```

Input

```
laptime <- function(){
return(round(structure(proc.time() - chunk.time.start, class = "proc_time")[3],3))
chunk.time.start <<- proc.time()
}
```

Input

```
# install.packages("sintro",repos="http://r-forge.r-project.org",type="source")
library(sintro)
library(nonlinearTseries)
```

2.0.1. *Takens States.* Takens states are represented as a matrix, one state per row. The number of colums is the imbedding dimension. If present, the *time.lag* attribute is the lag parameter, *id* an identification string for the basic data set.

ToDo: improve choice of alpha

`statepairs` *Show marginal scatterplots of Takens states*

Usage.

```
statepairs(states, main,
range  = NULL,
rank = FALSE, nooverlap = FALSE,
alpha)
```

Arguments.

states	A matrix: Takens states by dimension, one state per row.
main	Optional: the main header.
range	Optional: an interval selecting values to be displayd.
rank	An experimental variant. If *rank*, the values are rank transformed.
nooverlap	An experimental variant. If *nooverlap*, the cases are subsampled by dimension.
alpha	The alpha value used for plotting. The current choice is $\sqrt{\frac{1}{nr\ states}}$ as a default.

ToDo: colour by time

Input

```
statepairs <- function(states, main,
                rank=FALSE, nooverlap= FALSE, range=NULL,
                alpha, ...){
        n <- dim(states)[1]; dim <-  dim(states)[2]
        time.lag <- attr(states,"time.lag"); if (is.null(time.lag)) time.lag <- 1

        if (missing(main)) {
                        stateid <- attr(states, "id")
                        if (is.null(stateid)) stateid <- deparse(substitute(states))
                        main <- paste("Takens states:", stateid, "\n",
                                "n:", n," dim:", dim)
                        if (time.lag != 1) main <- paste(main, " time lag:", time.lag)
                }

        if (nooverlap) {states <- states[ seq(1,n, by=dim),]
        main <- paste(main," no overlap")}

        if (!is.null(range)) { states[states[]< range[1]] <- NA;
                states[states[]> range[2]] <- NA
                main <- paste(main," trimmed")}

        if (missing(alpha)) {alpha <- 1/sqrt(dim(states)[1])}
        #cat("alpha=",alpha)

        if (rank) {states <- apply(states, 2, rank, ties.method="random")
        main <- paste(main," ranked")}

        pairs(states, main=main,
        col=rgb(0,0,0, alpha), pch=19, ...)
        #title(main=main, outer=TRUE, line=-2, cex.main=0.8)
}
```

Assuming the index is time, the *statecoplot()* is layed out to show the highest index as response, i.e. $(x_{t-1}, x_t | x_{t-2}, x_{t-3})$.

ToDo: extend for low dimensions, extend parameters

`statecoplot` *Show conditioning marginal scatterplots of Takens states*

Usage.

```
statecoplot(states, main,
range  = NULL,
rank = FALSE, nooverlap = FALSE,
alpha, number = c(5,5))
```

Arguments.

states	A matrix: Takens states by dimension, one state per row.
main	Optional: the main header.
range	Optional: an interval selecting values to be displayd.
rank	An experimental variant. If *rank*, the values are rank transformed.
nooverlap	An experimental variant. If *nooverlap*, the cases are subsampled by dimension.
alpha	The alpha value used for plotting. The current choice is $\sqrt{\frac{1}{nr\ states}}$ as a default.
number	integer; the number of conditioning intervals, for a and b, possibly of length 2.

Input

```
statecoplot <- function(states, main,
                rank = FALSE, nooverlap = FALSE, range = NULL,
                alpha, number = c(5,5), ...){
        n <- dim(states)[1]; dim <-  dim(states)[2]
        time.lag <- attr(states,"time.lag")
        if (is.null(time.lag)) time.lag <- 1
        if (missing(main)) {
        stateid <- attr(states, "id")
        if (is.null(stateid)) stateid <- deparse(substitute(states))
        main <- paste("Takens states:", stateid, "\n",
                "n=", n," dim=", dim)
        if (time.lag != 1) main <- paste(main, " time lag=", time.lag)
                }

        if (nooverlap) {states <- states[ seq(1,n, by=dim),]
        main <- paste(main," no overlap")}


        if (!is.null(range)) { states[states[]< range[1]] <- NA; states[states[]> range[2]] <- NA
        main <- paste(main," trimmed")}


        if (missing(alpha)) {alpha <- 1/sqrt(dim(states)[1]/32)}
        #cat("alpha=",alpha)

        if (rank) {states <- apply(states, 2, rank, ties.method="random")
        main <- paste(main," ranked")}

        coplot((states[,4]~states[,3]|states[,1]+ states[,2]),
        number=number, main=main,
        col=rgb(0,0,0, alpha), pch=19, ...)
        #title(main=main, outer=TRUE, line=-2, cex.main=0.8)
}
```

2.1. **Local Bottleneck.** To allow experimental implementations, functions from *nonlinearTseries* are aliased here.

Input

```
local.buildTakens <- function (time.series,
        embedding.dim, time.lag=1,
        id=deparse(substitute(time.series)))
{
        takens <- nonlinearTseries:::buildTakens(time.series, embedding.dim, time.lag)
```

```
        attr(takens,"time.lag") <- time.lag
        attr(takens,"embedding.dim") <- embedding.dim
        attr(takens,"id") <- id
        return(takens)
}
```

Input

```
local.findAllNeighbours <- function (takens, radius, number.boxes = NULL)
{
        allneighs <- nonlinearTseries:::findAllNeighbours(takens, radius, number.boxes = NULL)
        mostattributes(allneighs) <- attributes(takens)
        attr(allneighs, "radius") <- radius
        return(allneighs)
}
```

minor cosmetics added to recurrencePlotAux

ToDo: propagate parameters from *buildTakens* and *findAllNeighbours* in a slot of the result, instead of using explicit parameters in recurrencePlotAux.

Input

```
#non-sparse variant
#local.recurrencePlotAux <- nonlinearTseries:::recurrencePlotAux
local.recurrencePlotAux = function(neighs, dim=NULL, lag=NULL, radius=NULL){

        # just for reference. This function is inlined
        neighbourListNeighbourMatrix  = function(){
          #neighs.matrix = Diagonal(ntakens)
          for (i in 1:ntakens){
            if (length(neighs[[i]])>0){
              for (j in neighs[[i]]){
                neighs.matrix[i,j] = 1
              }
            }
          }
          return (neighs.matrix)
        }

  ntakens=length(neighs)
  neighs.matrix <- matrix(nrow=ntakens, ncol=ntakens)
  #neighbourListNeighbourMatrix()
    #neighs.matrix = Diagonal(ntakens)
          for (i in 1:ntakens){
                  neighs.matrix[i,i] = 1 # do we want the diagonal fixed to 1
            if (length(neighs[[i]])>0){
              for (j in neighs[[i]]){
                neighs.matrix[i,j] = 1
              }
            }
          }

#! clean up. only one main id should be presentes
        main <- paste("Recurrence Plot: ",
                      deparse(substitute(neighs))
                      )
        id <-  attr(neighs, "id"); if (!is.null(id)) main <- paste(main," id:",id)

        more <- NULL

        more <- paste(more," n:",length(neighs))

        #use components of neights if available
        embedding.dim <- attr(neighs, "embedding.dim")
```

```
        if (is.null(embedding.dim)) embedding.dim <- dim
        if (!is.null(dim)) {
                if (embedding.dim != dim) warning(paste("Embedding dim:", embedding.dim,
                " does not match dim argument=", dim))
                more <- paste(more," dim:",dim)
        }


        attrradius <- attr(neighs, "radius")
        if (!is.null(lag)) more <- paste(more," lag:",lag)
        if (is.null(radius)) radius <- attrradius
        if (!is.null(radius)) {
                more <- paste(more," radius:",radius)
                if (!is.null(attrradius) && (attrradius != radius)) warning(paste("Radius attribute:", at
                " does not match radius argument=", radius))
                }
        # if (!is.null(attrradius)) more <- paste(more," radius attr:",attrradius)
        if (!is.null(more)) main <- paste(main,"\n",more)

  # need no print because it is not a trellis object!!
  #print(
          image(x=1:ntakens, y=1:ntakens,
                 z=neighs.matrix,xlab="i", ylab="j",
                col="black",
                #xlim=c(1,ntakens), ylim=c(1,ntakens),
                useRaster=TRUE,  #? is this safe??
                main=main
                )
 #        )

 }
```

ToDo: improve feedback for data structures in *nonlinearTseries*

2.2. **Recurrence Plots.** This is a hack to report RQA information. $dim = NULL$ is added to align calling with other functions.

ToDo: improve to a full *show* method for class *rqa*.

Needs improvement.

Input

```
showrqa <- function(takens, dim=NULL, radius,
                digits=3,
                do.hist = TRUE, rm.hist = TRUE, log = TRUE ,...)
{
        xxrqa <- rqa(takens=takens,radius=radius)
        xxrqa$radius <- radius
        id <- attr(takens,"id")
        if (is.null(id)) id <- deparse(substitute(takens))
        xxrqa$id<- id
        xxrqa$time.lag <- attr(takens,"time.lag")
        cat(id, " n:", dim(takens)[1], " Dim:", dim(takens)[2], "\n")
        xxrqa$n <- dim(takens)[1]
        xxrqa$dim <- dim(takens)[2]
        cat(paste("Radius:", radius,
                " Recurrence coverage REC:", round(xxrqa[1]$REC, digits),
                " log(REC)/log(R):", round(log(xxrqa[1]$REC)/log(radius), digits), "\n"))
        xxrqa$logratio <- log(xxrqa[1]$REC)/log(radius)
        cat(paste("Determinism:", round(xxrqa$DET, digits),
                " Laminarity:",round(xxrqa$LAM, digits), "\n"))
        cat(paste("DIV:", round(xxrqa$DIV, digits), "\n"))
```

```
        cat(paste("Trend:", round(xxrqa$TREND, digits),
                " Entropy:",round(xxrqa$ENTR, digits), "\n"))
        cat(paste("Diagonal lines max:", round(xxrqa$Lmax, digits),
                 " Mean:",round(xxrqa$Lmean, digits),
                " Mean off main:",round(xxrqa$LmeanWithoutMain, digits), "\n"))
        cat(paste("Vertical lines max:", round(xxrqa$Vmax, digits),
                " Mean:",round(xxrqa$Vmean, digits), "\n"))
                # str(xxrqa[4:12])

        if (do.hist){
        if (log==TRUE) log<-"y"
        oldpar <- par(mfrow=c(2,1))

                xxrqa$diagonalHistogram[xxrqa$diagonalHistogram==0] <- NA # hack for log zero counts
                dh<- xxrqa$diagonalHistogram
                #if (log=="y")  {dh <- dh+1}

                id <- attr(takens,"id"); if (is.null(id) ) id <- deparse(substitute(takens))
                pars <- paste("\n n=", dim(takens)[1], " Dim:",
                        dim(takens)[2])
                lag<- attr(takens,"time.lag")
                  if (!is.null(lag) && (lag !=1) ) pars <- paste(pars, " Lag:", lag)
                plot(dh, type="h", main=paste( id, " Diagonal:",
                        pars," Radius: ",radius, " REC:", round(xxrqa[1]$REC, digits)),
                        xlab="length of diagonals",
                        log = log, ...)

        xxrqa$recurrenceRate[xxrqa$recurrenceRate==0] <- NA # hack for log zero counts
        drR<- xxrqa$recurrenceRate
                #if (log=="y")  {drR <- drR+1}

                id <- attr(takens,"id"); if (is.null(id) ) id <- deparse(substitute(takens))
                pars <- paste("\n n=", dim(takens)[1], " Dim:",
                        dim(takens)[2])
                lag<- attr(takens,"time.lag")
                  if (!is.null(lag) && (lag !=1) ) pars <- paste(pars, " Lag:", lag)
        plot(drR, type="h",
                 main=paste( id," Recurrence Rate",
                        pars," Radius: ",radius, " REC:", round(xxrqa[1]$REC, digits)),
                        xlab="distance to diagonal",
                        #ylim=c(1,3),
                        log = log, ...)
        par(oldpar)
                }

        if (rm.hist) { xxrqa$recurrenceRate <- NULL; xxrqa$diagonalHistogram <- NULL }
        invisible(xxrqa)
}
```

3. Test Signals

We set up a small series of test signals. Some synthetic test signals are introduced here. More test cases used later are the Geyser data set (Section 6 on page 36) and two examples of heart rate data sets (Section 7 on page 58 and Section 8 on page 74) from *library(rhrv)*.

For convenience, some source code from other libraries is included to make this self-contained.

As a global constant, we set up the length of the series to be used for test signals.

Input

```
#nsignal <- 256
nsignal <- 1024
#nsignal <- 4096
system.time.start <- proc.time()
```

For signal representation, we use a common layout.

Input

```
plotsignal <- function(signal, main, ylab) {
        #! alpha level should depend on expected number of overlaps

        if (missing(ylab)) { ylab <- deparse(substitute(signal)) }

        par(mfrow = c(1,
                2))
        plot(signal,
                main = "", xlab = "index",  ylab = ylab,
                col = rgb(0, 0, 1, 0.3), pch = 20)

        plot(signal, type = "l",
                main = "",  xlab = "index", ylab = ylab,
                col = rgb(0, 0, 0, 0.4))
        points(signal,
                col = rgb(0, 0, 1, 0.3), pch = 20)
        if (missing(main)) { main = deparse(substitute(signal)) }
        title(main = main,
                outer = TRUE, line = -2, cex.main = 1.2)
}
```

3.1. **Sinus.**

Input

```
sin10 <- function(n=nsignal) {sin( (1:n)/n* 2*pi*10)}
plotsignal(sin10())
```

See Figure 1 on the facing page.

3.2. **Uniform Random Numbers.**

Input

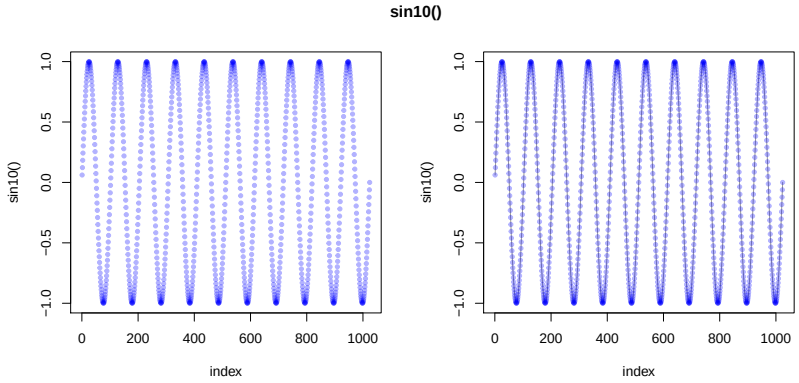

FIGURE 1. Test case: sin10. Signal and linear interpolation.

```
unif <- function(n=nsignal) {runif(n)}
xunif<-unif()
plotsignal(xunif)
```

See Figure 2,

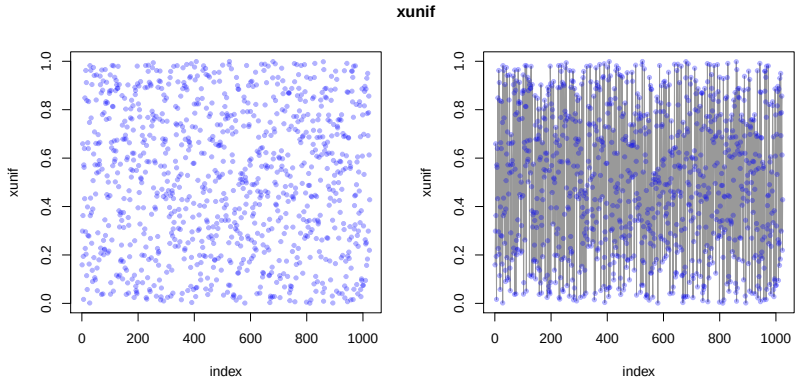

FIGURE 2. Test case: unif - uniform random numbers. Signal and linear interpolation.

3.3. **Chirp Signal.**

Input

```
chirp <- function(n=nsignal)       # this is copied from library(signal)
{signal.chirp <- function(t, f0 = 0, t1 = 1, f1 = 100,
                  form = c("linear", "quadratic", "logarithmic"),
                  phase = 0){

  form <- match.arg(form);  phase <- 2*pi*phase/360

  switch(form,
    "linear" = {
        a <- pi*(f1 - f0)/t1;        b <- 2*pi*f0
        cos(a*t^2 + b*t + phase)
    },
    "quadratic" = {
```

```
        a <- (2/3*pi*(f1-f0)/t1/t1);        b <- 2*pi*f0
        cos(a*t^3 + b*t + phase)
    },
    "logarithmic" = {
        a <- 2*pi * t1 / log(f1 - f0);        b <- 2*pi * f0
        x <- (f1-f0)^(1/t1)
        cos(a*x^t + b*t + phase)
    })
}

signal.chirp(seq(0, 0.6, len=nsignal))
}
plotsignal(chirp())
```

See Figure 3,

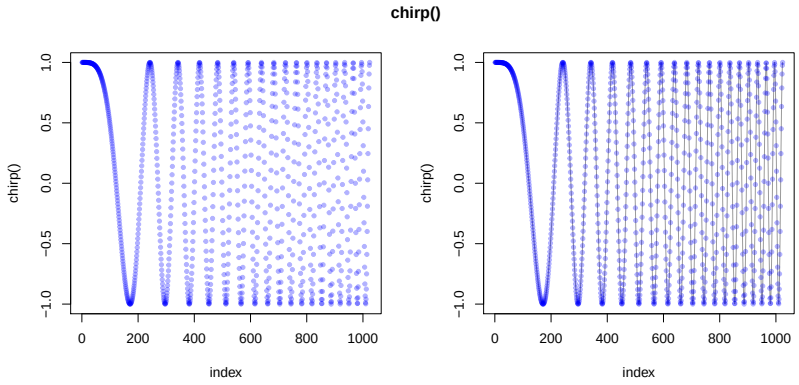

FIGURE 3. Test case: chirp signal. Signal and linear interpolation.

3.4. **Doppler signal.**

Input

```
doppler <- function(n=nsignal) {

dopplersignal  <- function(x) { sqrt(x*(1-x))* sin((2.1*pi)/(x+0.05))}
dopplersignal((1:nsignal)/nsignal)
}
plotsignal(doppler())
```

See Figure 4 on the next page,

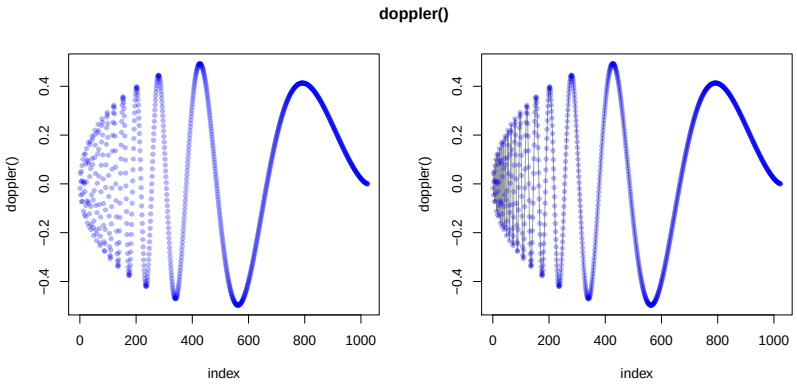

FIGURE 4. Test case: Doppler signal. Signal and linear interpolation.

4. Takens' States for Test Signals

Input
```
sintakens <- local.buildTakens(time.series=sin10(),
        embedding.dim=4, time.lag=1)
statepairs(sintakens) #4
```

See Figure 5.

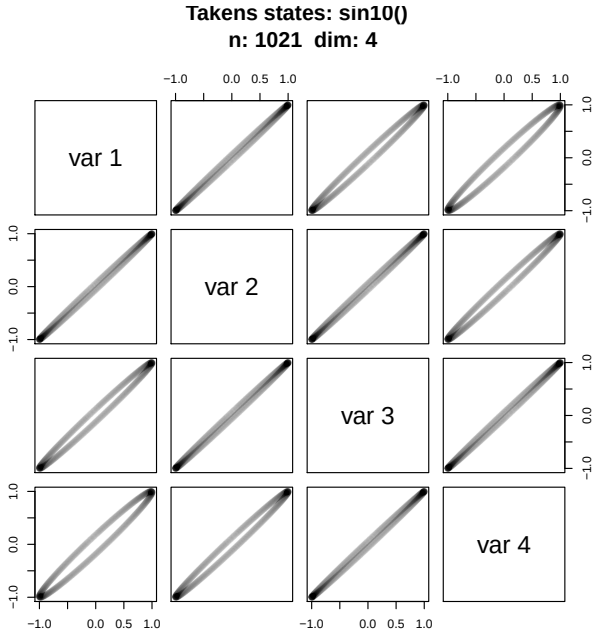

Figure 5. Takens states. Test case: sinus. Note that $2d$ marginal views of 1-dimensional circles in d space generally appear as ellipses. Time used: 0.688 sec.

The states only catch the local behaviour, where "local" depends on the sampling rate and the variation of the signal. For the sinus signal, we get a better picture if we subsample the signal.

Input
```
sintakenslag16 <- local.buildTakens(time.series=sin10(),
        embedding.dim=4, time.lag=16)
statepairs(sintakenslag16) #4
```

See Figure 6 on the next page.

Input
```
statepairs(sintakens, nooverlap=TRUE) #dim=4
```

See Figure 7 on the facing page.

Input
```
statecoplot(sintakens) #4
```

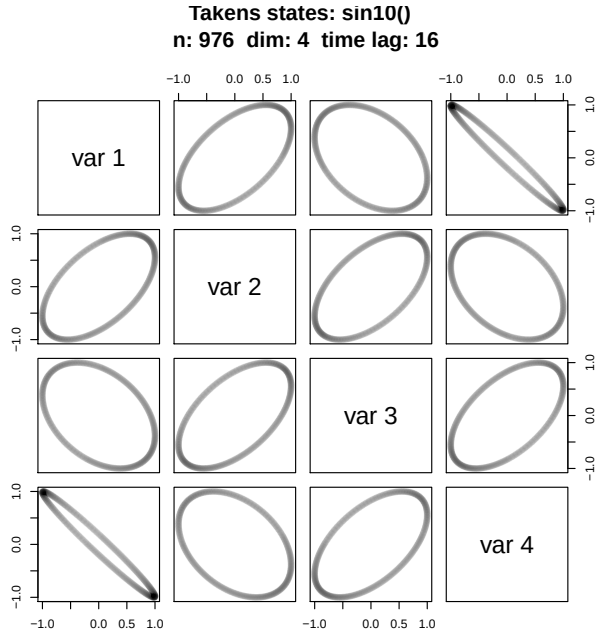

FIGURE 6. Takens states. Test case: sinus at time lag 16. Note that $2d$ marginal views of 1-dimensional circles in d space generally appear as ellipses. Time used: 0.649 sec.

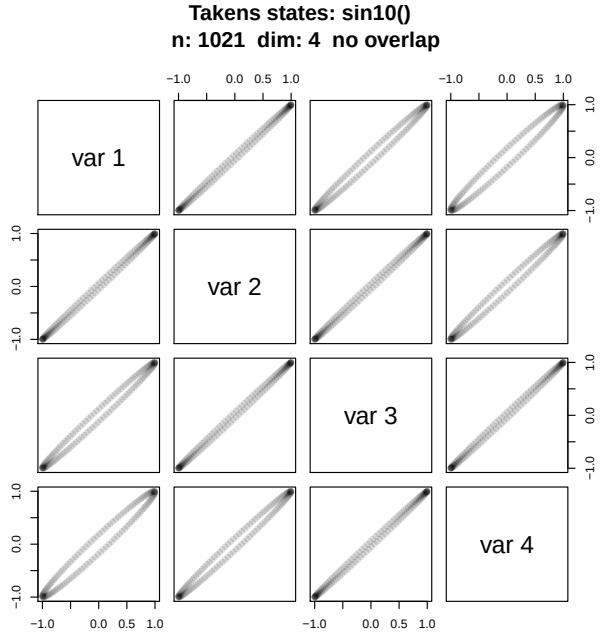

FIGURE 7. Takens states. Test case: sinus, no overlap. Note that $2d$ marginal views of 1-dimensional circles in d space generally appear as ellipses. Time used: 0.953 sec.

See Figure 8.

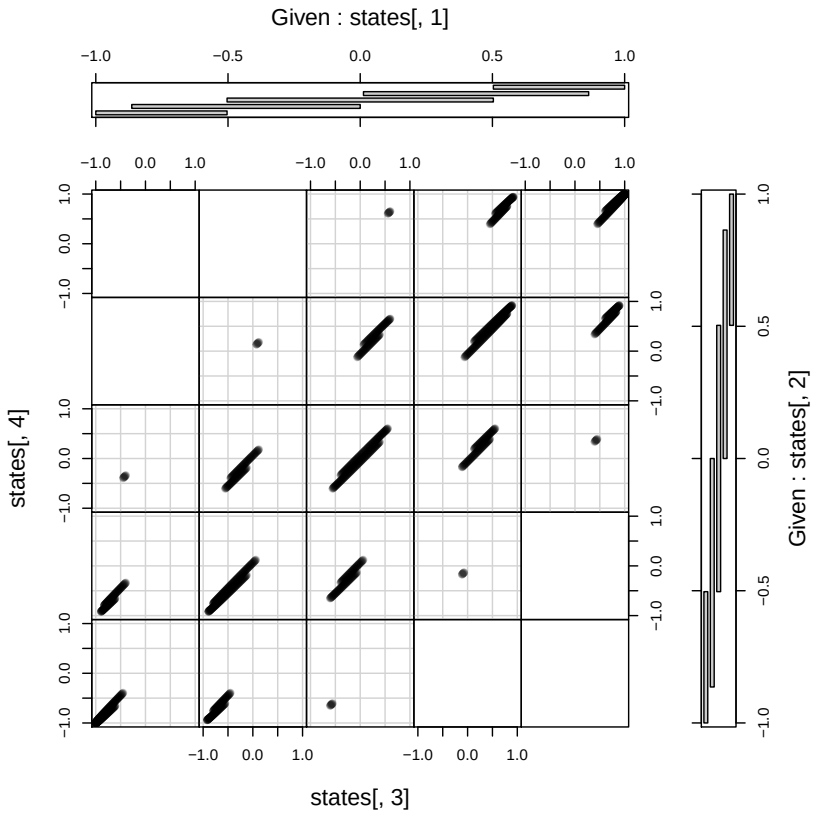

FIGURE 8. State coplot. Test case: sinus. Note that $2d$ marginal views of 1-dimensional circles in d space generally appear as ellipses. Time used: 0.271 sec.

Input

```
statecoplot(sintakenslag16) #4
```

See Figure 9 on the facing page.

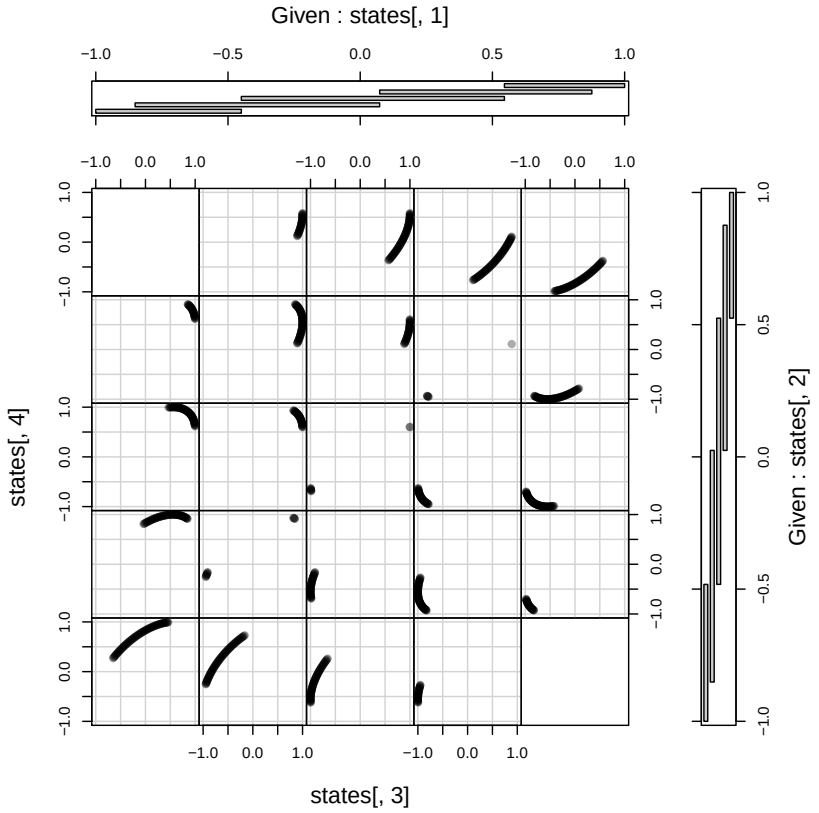

FIGURE 9. State coplot. Test case: sinus, time lag 16. Note that $2d$ marginal views of 1-dimensional circles in d space generally appear as ellipses. Time used: 0.254 sec.

Input

```
uniftakens <- local.buildTakens( time.series=xunif,
        embedding.dim=4, time.lag=1)
statepairs(uniftakens) #dim=4
```

See Figure 10.

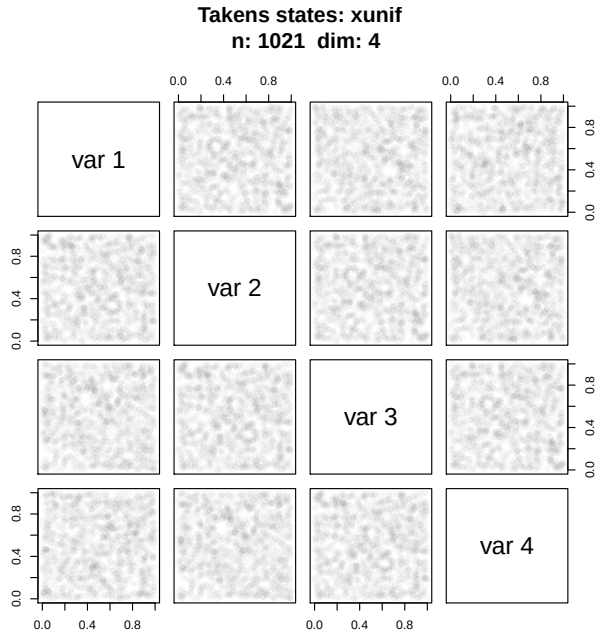

FIGURE 10. Takens states. Test case: uniform random numbers. Time used: 0.697 sec.

Input

```
statecoplot(uniftakens) #dim=4
```

See Figure 11 on the facing page.

Input

```
statepairs(uniftakens, nooverlap=TRUE) #dim=4
```

See Figure 12 on the next page.

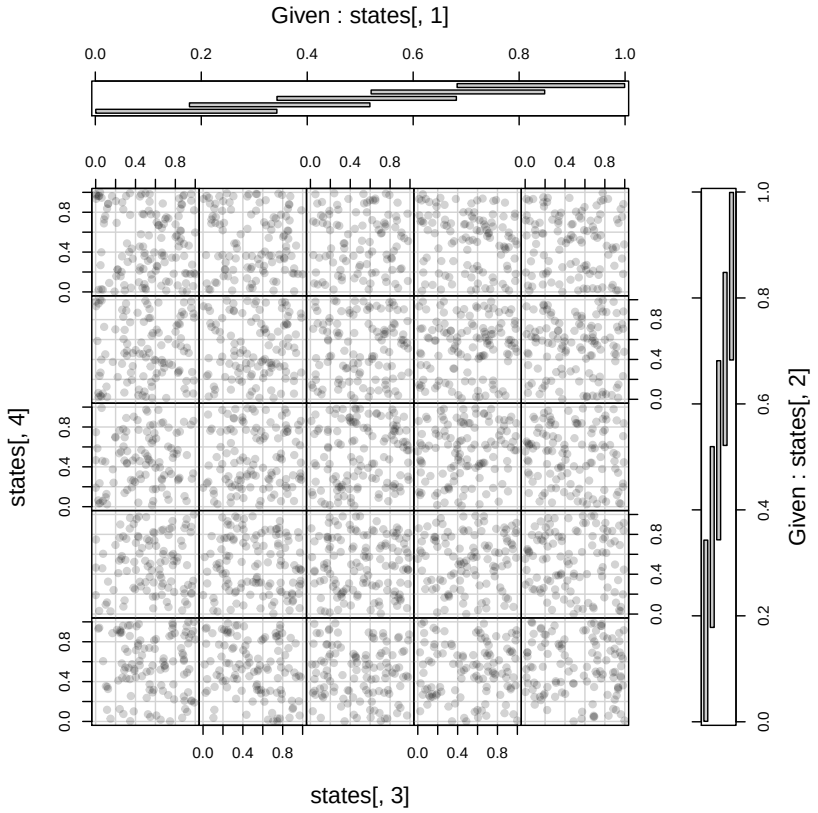

FIGURE 11. State coplot. Test case: uniform random numbers. Time used: 1.007 sec.

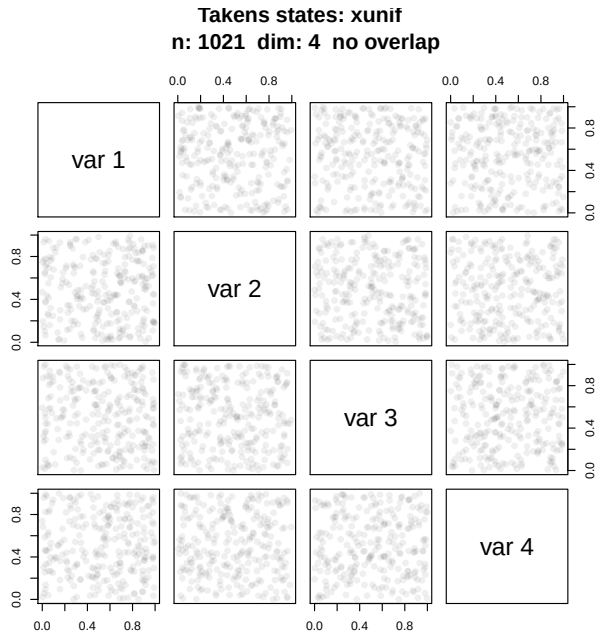

FIGURE 12. Takens states. Test case: uniform random numbers. Time used: 1.294 sec.

Input
```
chirptakens <- local.buildTakens( time.series=chirp(),
        embedding.dim=4,time.lag=1)
statepairs(chirptakens) #dim=4
```

See Figure 13.

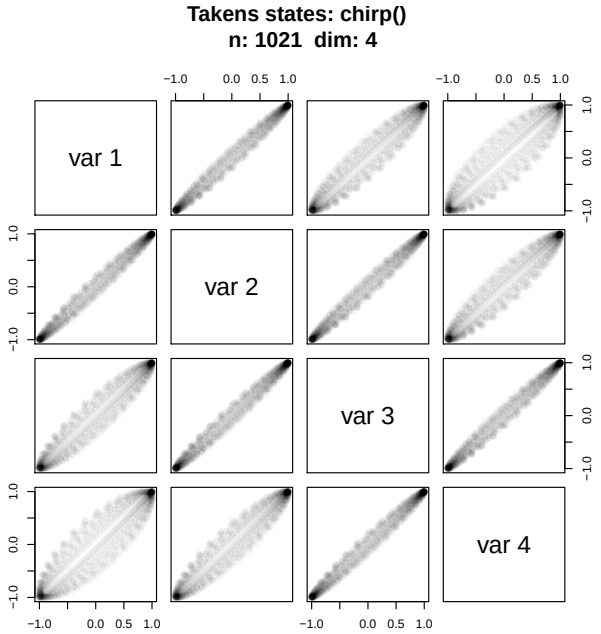

FIGURE 13. Takens states. Test case: chirp signal. Time used: 0.716 sec.

Input
```
statecoplot(chirptakens) #dim=4
```

See Figure 14 on the next page.

Input
```
statepairs(chirptakens, nooverlap=TRUE) #dim=4
```

See Figure 15 on the facing page.

Input
```
dopplertakens <- local.buildTakens(time.series=doppler(),
        embedding.dim=4, time.lag=1)
statepairs(dopplertakens) #4
```

See Figure 16 on page 22.

The states only catch the local behaviour, where “local” depends on the sampling rate and the variation of the signal. For the doppler signal, we get a better picture if we subsample the signal.

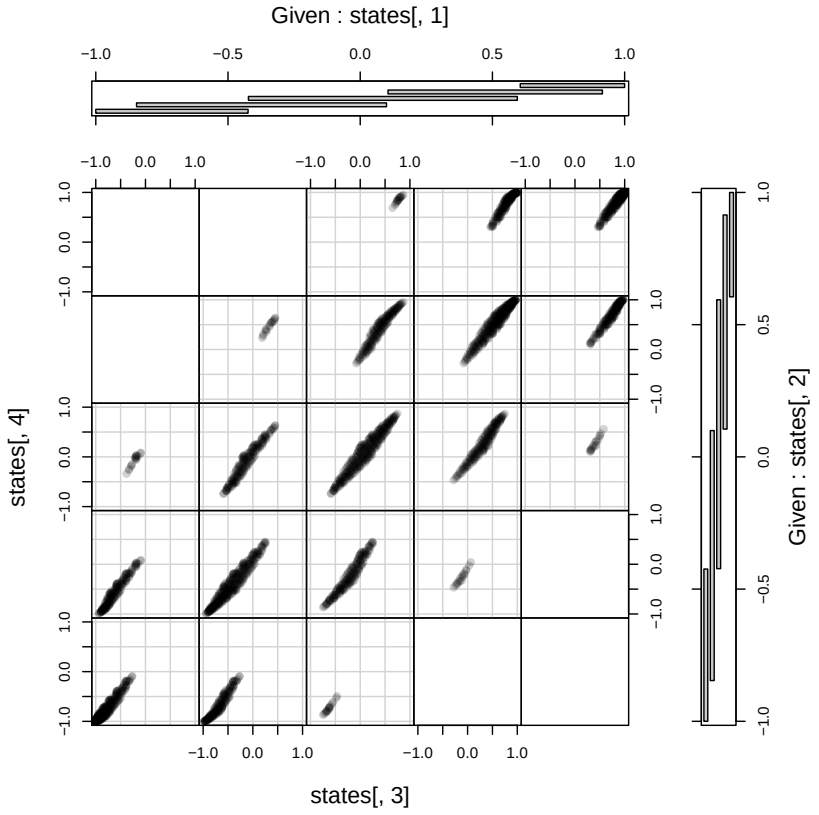

FIGURE 14. State coplot. Test case: chirp signal. Time used: 1.024 sec.

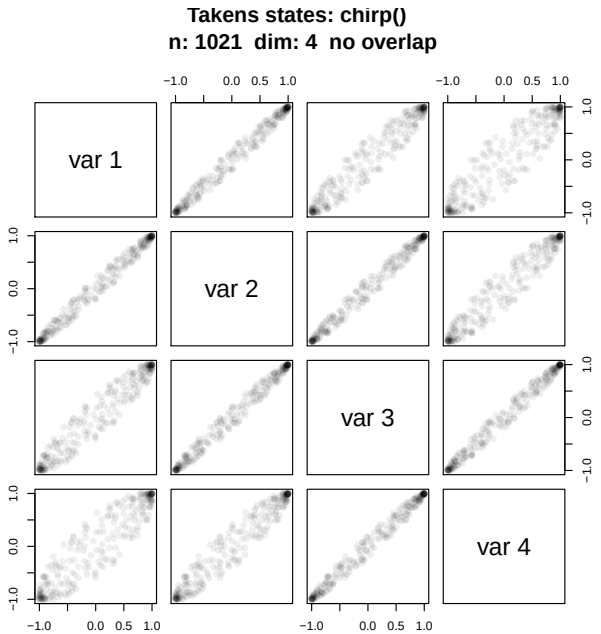

FIGURE 15. Takens states. Test case: chirp signal. Time used: 1.292 sec-

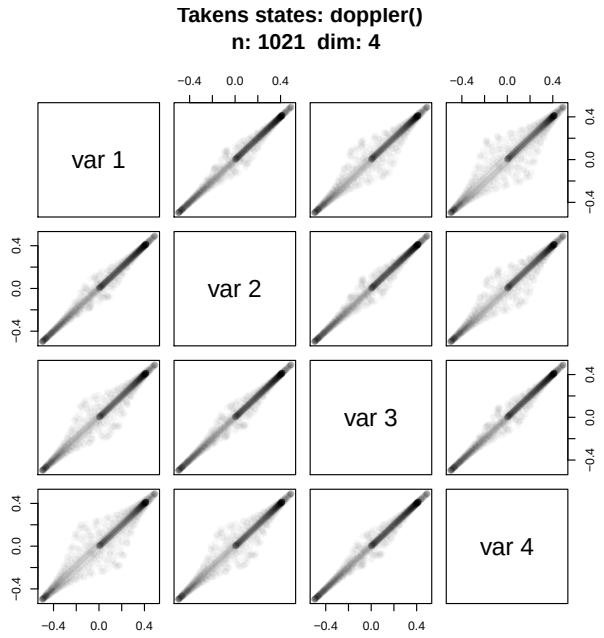

FIGURE 16. Takens states. Test case: Doppler. Note that $2d$ marginal views of 1-dimensional circles in d space generally appear as ellipses. Time used: 0.679 sec.

Input

```
dopplertakenslag16 <- local.buildTakens(time.series=doppler(),
        embedding.dim=4, time.lag=16)
statepairs(dopplertakenslag16) #4
```

See Figure 17 on the facing page.

Input

```
statepairs(dopplertakens, nooverlap=TRUE) #dim=4
```

See Figure 18 on the next page.

Input

```
statecoplot(dopplertakens) #4
```

See Figure 19 on page 24.

Input

```
statecoplot(dopplertakenslag16) #4
```

See Figure 20 on page 25.

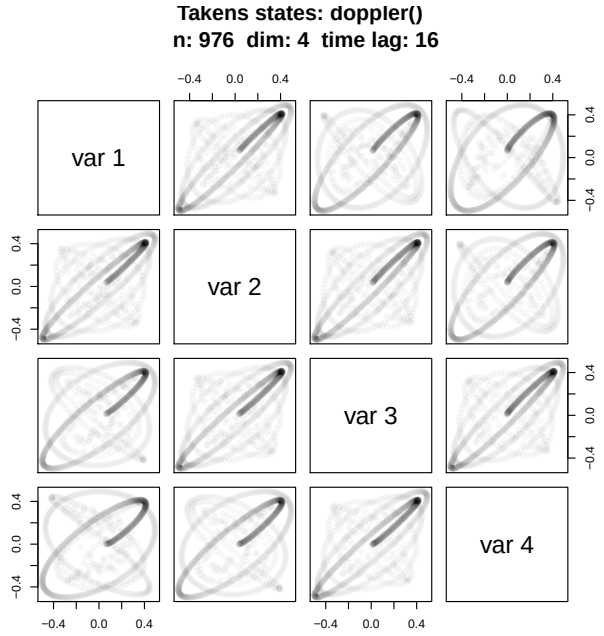

FIGURE 17. Takens states. Test case: doppler at time lag 16. Note that $2d$ marginal views of 1-dimensional circles in d space generally appear as ellipses. Time used: 0.703 sec.

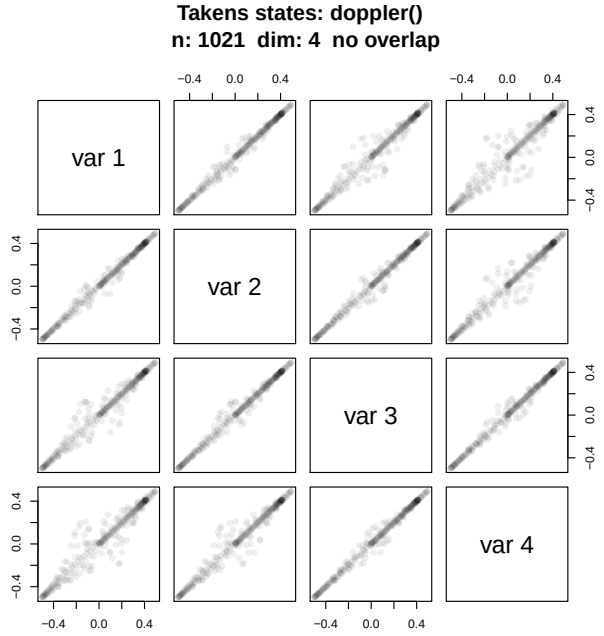

FIGURE 18. Takens states. Test case: doppler, no overlap. Note that $2d$ marginal views of 1-dimensional circles in d space generally appear as ellipses. Time used: 1.067 sec.

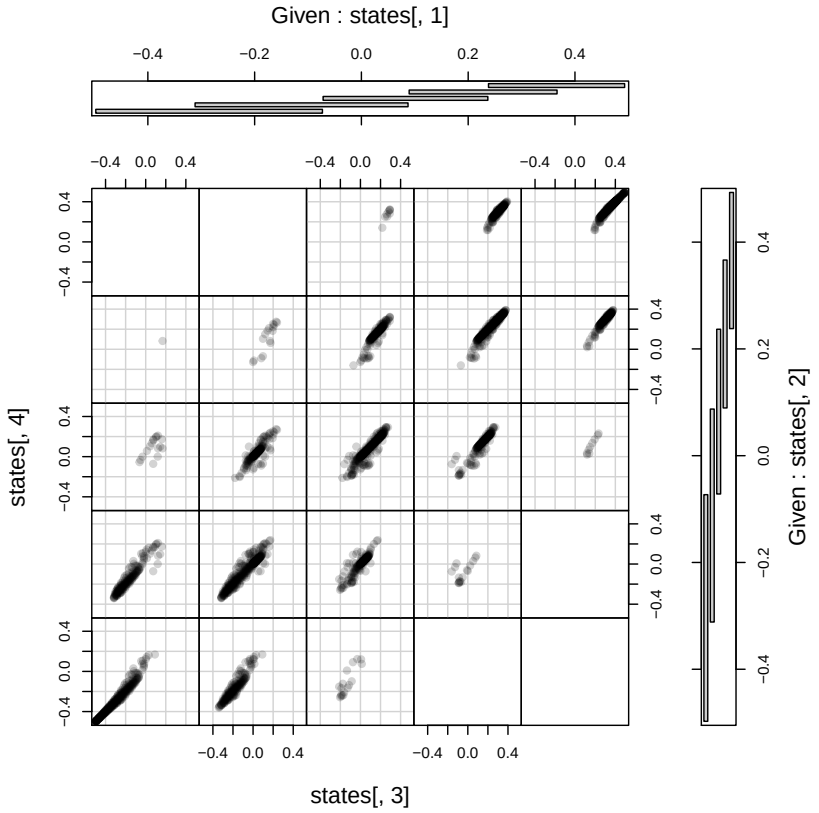

FIGURE 19. State coplot. Test case: Doppler. Note that $2d$ marginal views of 1-dimensional circles in d space generally appear as ellipses. Time used: 0.313 sec.

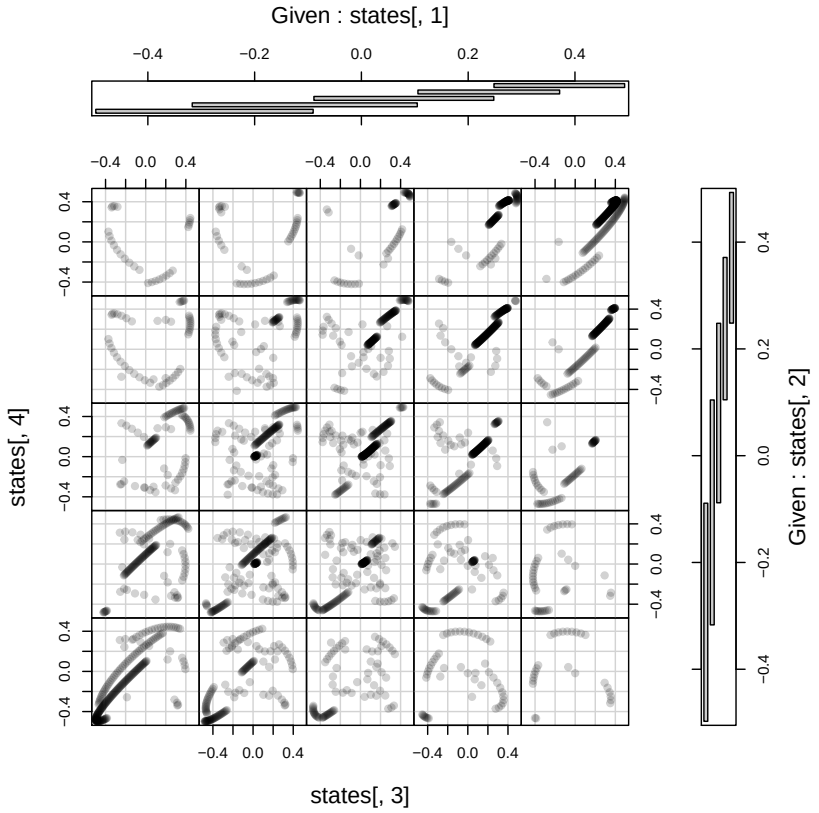

FIGURE 20. State coplot. Test case: Doppler, time lag 16. Note that $2d$ marginal views of 1-dimensional circles in d space generally appear as ellipses. Time used: 0.287 sec.

5. Recurrence Plots for Test Signals

5.0.1. *Sinus Recurrence Plots.*

Input
```
sin10neighs <- local.findAllNeighbours(sintakens, radius=0.8)
save(sin10neighs, file="sin10neighs.Rdata")
# load(file="sin10neighs.RData")
local.recurrencePlotAux(sin10neighs, dim=dim(sintakens)[2], radius=0.8)
```

See Figure 21.

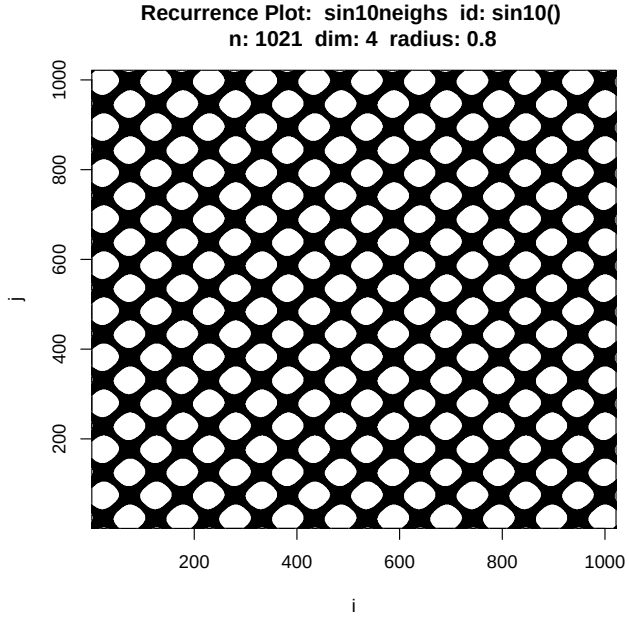

Figure 21. Recurrence plot. Test case: sinus curve. Time used: 3.484 sec.

Input
```
sin10rqa <- showrqa(sintakens, radius=0.8, log=TRUE)
```

Output
```
sin10()  n: 1021  Dim: 4
Radius: 0.8  Recurrence coverage REC: 0.496  log(REC)/log(R): 3.143
Determinism: 1  Laminarity: 1
DIV: 0.001
Trend: 0  Entropy: 3.213
Diagonal lines max: 1020  Mean: 32.712  Mean off main: 32.65
Vertical lines max: 66  Mean: 41.479
```

See Figure 22 on the next page.

Input
```
sin10lag16neighs <- local.findAllNeighbours(sintakenslag16, radius=0.2)
save(sin10lag16neighs, file="sin10lag16neighs.Rdata")
```

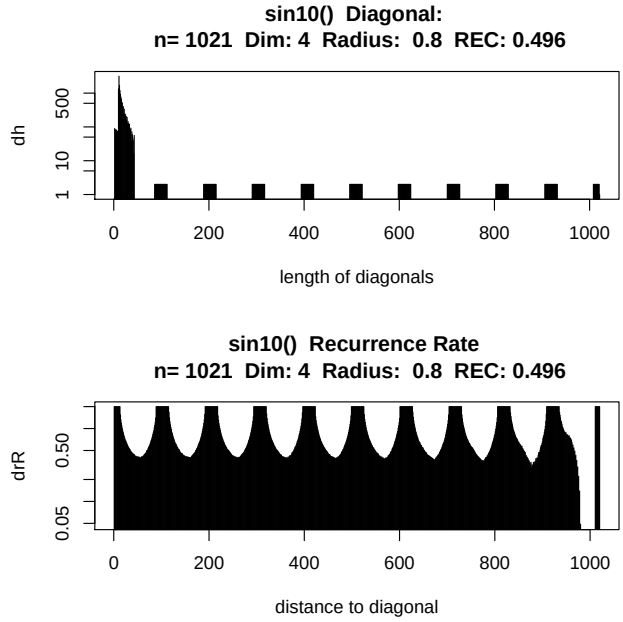

FIGURE 22. RQA. Test case: sinus curve. Time used: 0.835 sec.

```
# load(file="sin10lag16neighs.RData")
local.recurrencePlotAux(sin10lag16neighs, dim=4, radius=0.2)
```

See Figure 23.

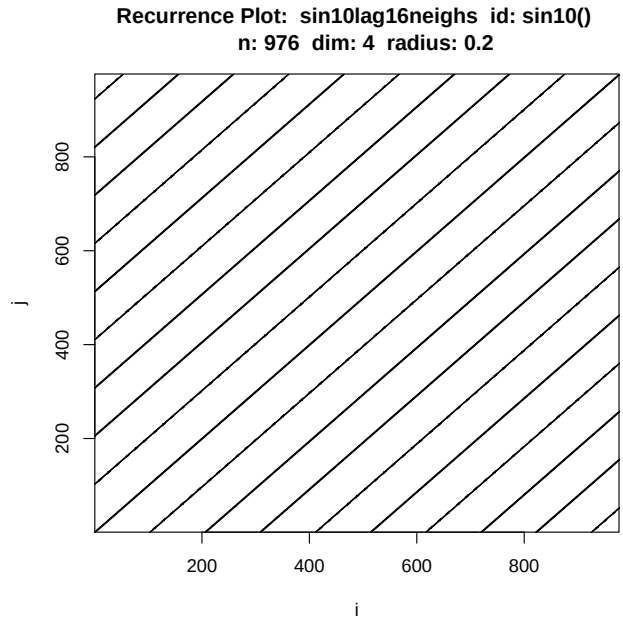

FIGURE 23. Recurrence plot. Recurrence Plot. Test case: sinus curve, time lag 16. Time used: 1.995 sec.

Input

```
sin10lag16rqa <- showrqa(sintakenslag16, radius=0.2)
```

Output

```
sin10()  n: 976  Dim: 4
Radius: 0.2  Recurrence coverage REC: 0.067  log(REC)/log(R): 1.683
Determinism: 0.999  Laminarity: 1
DIV: 0.001
Trend: 0  Entropy: 2.316
Diagonal lines max: 975  Mean: 124.503  Mean off main: 122.827
Vertical lines max: 8  Mean: 6.774
```

See Figure 24.

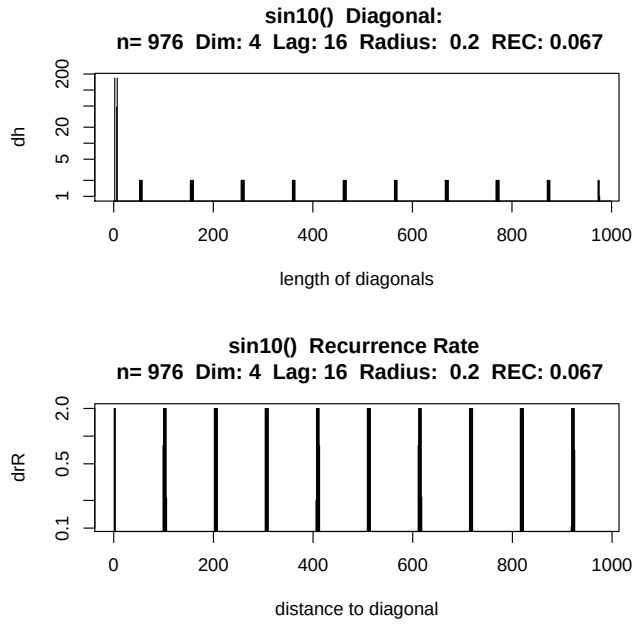

FIGURE 24. RQA. Test case: sinus curve, time lag 16. Time used: 2.444 sec.

5.0.2. *Uniform Random Numbers Recurrence Plots.*

Input

```
unifneighs <- local.findAllNeighbours(uniftakens, radius=0.6)#0.4
save(unifneighs, file="unifneighs.RData")
# load(file="unifneighs.RData")
local.recurrencePlotAux(unifneighs, radius=0.6)
```

See Figure 25.

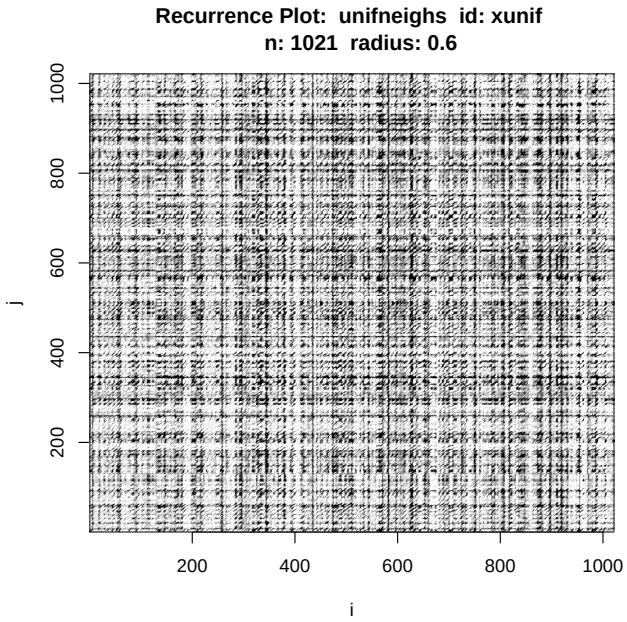

FIGURE 25. Recurrence plot. Test case: uniform random numbers. Time used: 3.734 sec.

Input

```
showrqa(uniftakens, radius=0.6, log=TRUE)
```

Output

```
xunif  n: 1021  Dim: 4
Radius: 0.6  Recurrence coverage REC: 0.491  log(REC)/log(R): 1.392
Determinism: 0.973  Laminarity: 0.785
DIV: 0.017
Trend: 0  Entropy: 2.716
Diagonal lines max: 60  Mean: 7.092  Mean off main: 7.078
Vertical lines max: 1021  Mean: 3.867
```

See Figure 26 on the next page.

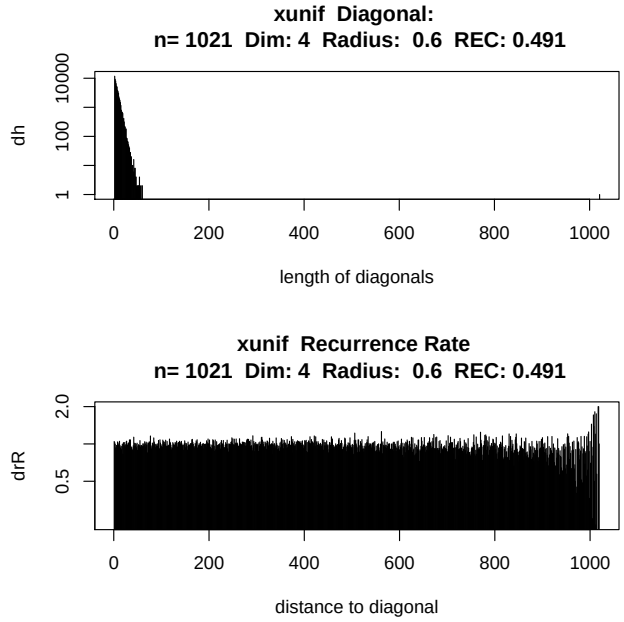

FIGURE 26. RQA. Test case: uniform random numbers, radius=0.6. Time used: 4.606 sec.

5.0.3. *Chirp Signal Recurrence Plots.*

Input

```
chirpneighs <- local.findAllNeighbours(chirptakens, radius=0.6)
save(chirpneighs, file="chirpneighs.RData")
# load(file="chirpneighs.RData")
local.recurrencePlotAux(chirpneighs, radius=0.6)
```

See Figure 27.

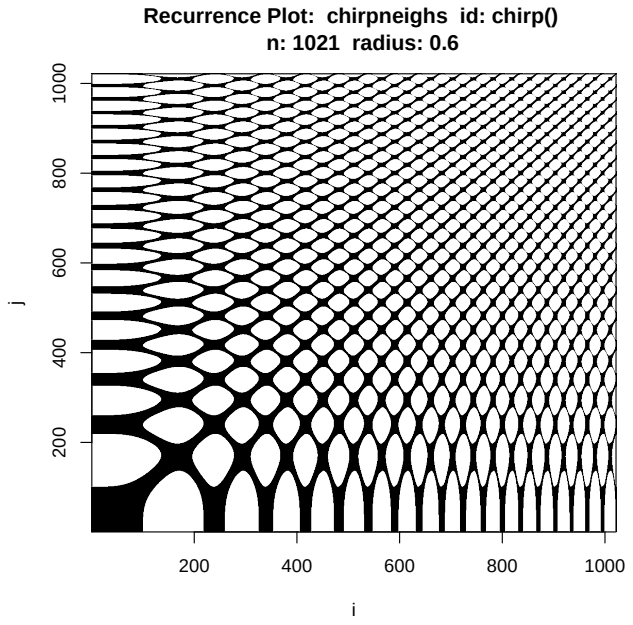

FIGURE 27. Recurrence plot. Test case: chirp signal. Time used: 2.581 sec.

Input

```
showrqa(chirptakens, radius=0.6)
```

Output

```
chirp()  n: 1021  Dim: 4
Radius: 0.6  Recurrence coverage REC: 0.341  log(REC)/log(R): 2.108
Determinism: 0.988  Laminarity: 0.998
DIV: 0.001
Trend: 0  Entropy: 3.254
Diagonal lines max: 1020  Mean: 12.496  Mean off main: 12.46
Vertical lines max: 125  Mean: 14.721
```

See Figure 28 on the next page.

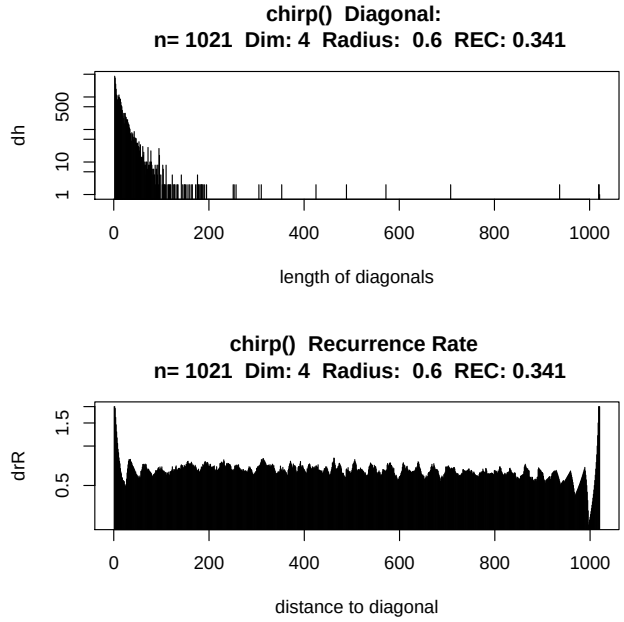

FIGURE 28. RQA. Test case: chirp signal. Time used: 3.242 sec.

5.0.4. *Doppler Recurrence Plots.*

Input

```
dopplerneighs <- local.findAllNeighbours(dopplertakens, radius=0.2)
save(dopplerneighs, file="dopplerneighs.Rdata")
```

Input

```
load(file="dopplerneighs.RData")
local.recurrencePlotAux(dopplerneighs, dim=4, radius=0.2)
```

See Figure 29.

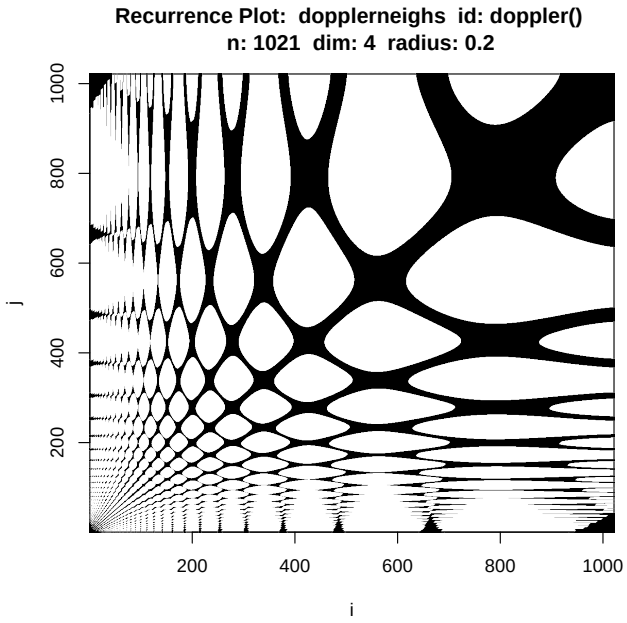

FIGURE 29. Recurrence plot. Test case: Doppler. Time used: 2.056 sec.

Input

```
showrqa(dopplertakens, radius=0.2)
```

Output

```
doppler()  n: 1021  Dim: 4
Radius: 0.2  Recurrence coverage REC: 0.299  log(REC)/log(R): 0.75
Determinism: 0.991  Laminarity: 0.995
DIV: 0.001
Trend: 0  Entropy: 3.445
Diagonal lines max: 1020  Mean: 17.496  Mean off main: 17.439
Vertical lines max: 329  Mean: 24.835
```

See Figure 30 on the next page.

Input

```
dopplerlag16neighs <- local.findAllNeighbours(dopplertakenslag16, radius=0.2)
save(dopplerlag16neighs, file="dopplerlag16neighs.Rdata")
```

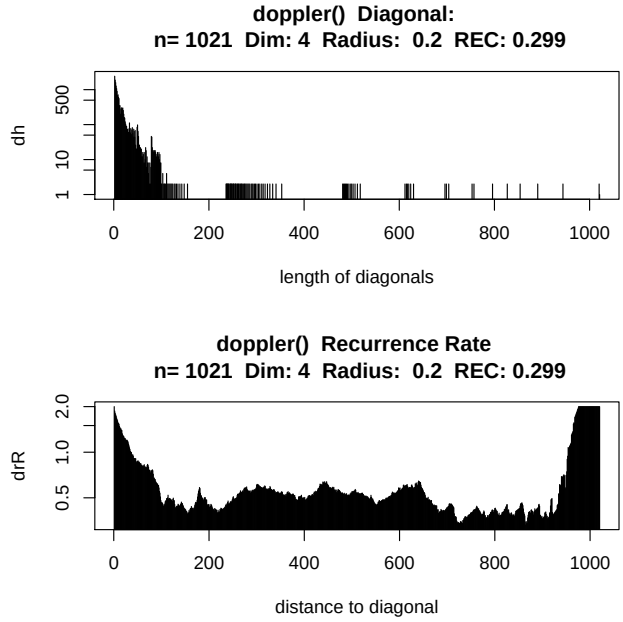

FIGURE 30. RQA. Test case: Doppler. Time used: 0.626 sec.

```
# load(file="dopplerlag16neighs.RData")
local.recurrencePlotAux(dopplerlag16neighs, dim=4, radius=0.2)
```

See Figure 31.

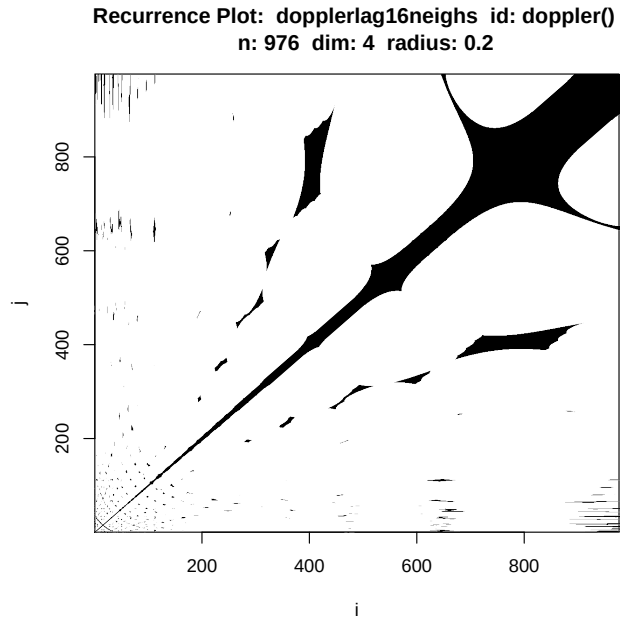

FIGURE 31. Recurrence plot. Test case: Doppler. Time used: 1.12 sec.

Input

```
showrqa(dopplertakenslag16, radius=0.2)
```

Output

```
doppler()  n: 976  Dim: 4
Radius: 0.2  Recurrence coverage REC: 0.098  log(REC)/log(R): 1.443
Determinism: 0.98  Laminarity: 0.989
DIV: 0.001
Trend: 0  Entropy: 2.815
Diagonal lines max: 975  Mean: 28.978  Mean off main: 28.678
Vertical lines max: 256  Mean: 31.539
```

See Figure 32.

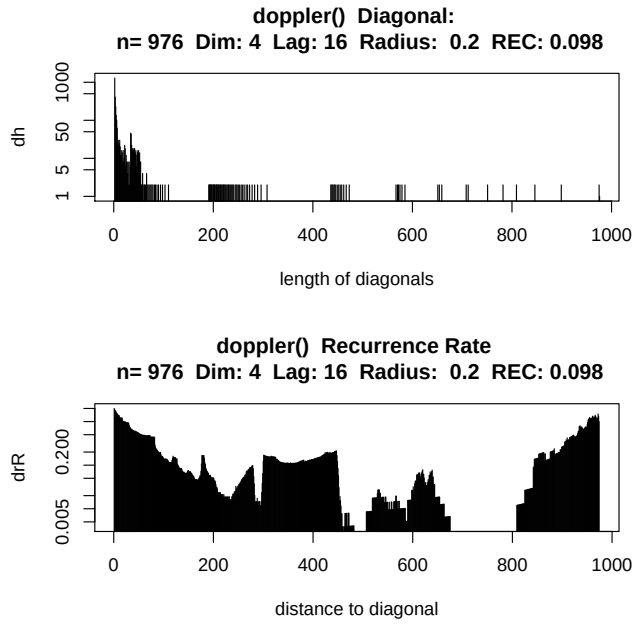

FIGURE 32. RQA. RQA. Test case: Doppler. Time used: 1.61 sec.

6. Case Study: Geyser data

ToDo: double check: *MASS:::geyser* should be used, not *faithful*

ToDo: Geyser: extended to two-dimensional data in *geyserlin*. Experimental only. Check.

This is a classical data set with a two dimensional structure, *duration* and *waiting*.

The data structure asks for a variant of the recurence plot, adapted to a two dimensional series.

Input

```
library(MASS)
data(geyser)
```

6.1. **Geyser Eruption Durations.**

Input

```
plotsignal(geyser$duration)
```

See Figure 33,

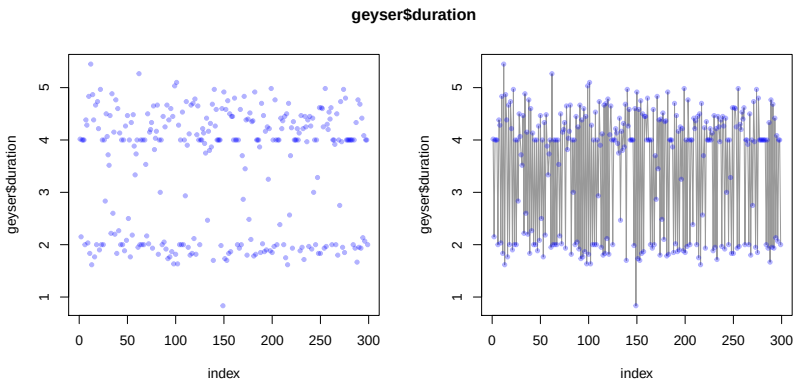

Figure 33. Example case: Old Faithful Geyser eruption durations. Signal and linear interpolation.

Input

```
eruptionstakens4 <-
        local.buildTakens( time.series=geyser$duration,
                embedding.dim=4, time.lag=1)
        statepairs(eruptionstakens4) #dim=4
```

See Figure 34 on the next page.

Input

```
        statecoplot(eruptionstakens4) #dim=4
```

See Figure 35 on the facing page.

Input

```
        statepairs(eruptionstakens4, nooverlap=TRUE) #dim=4
```

See Figure 36 on page 38.

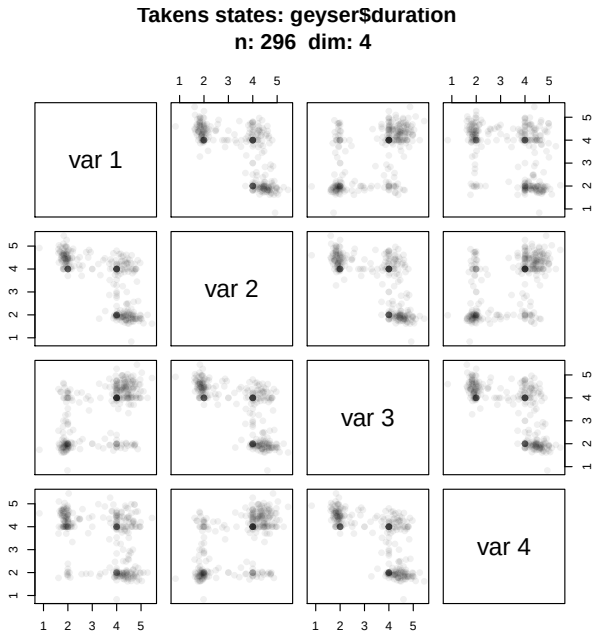

FIGURE 34. Recurrence plot. Example case: Old Faithful Geyser eruption durations. Time used: 0.29 sec.

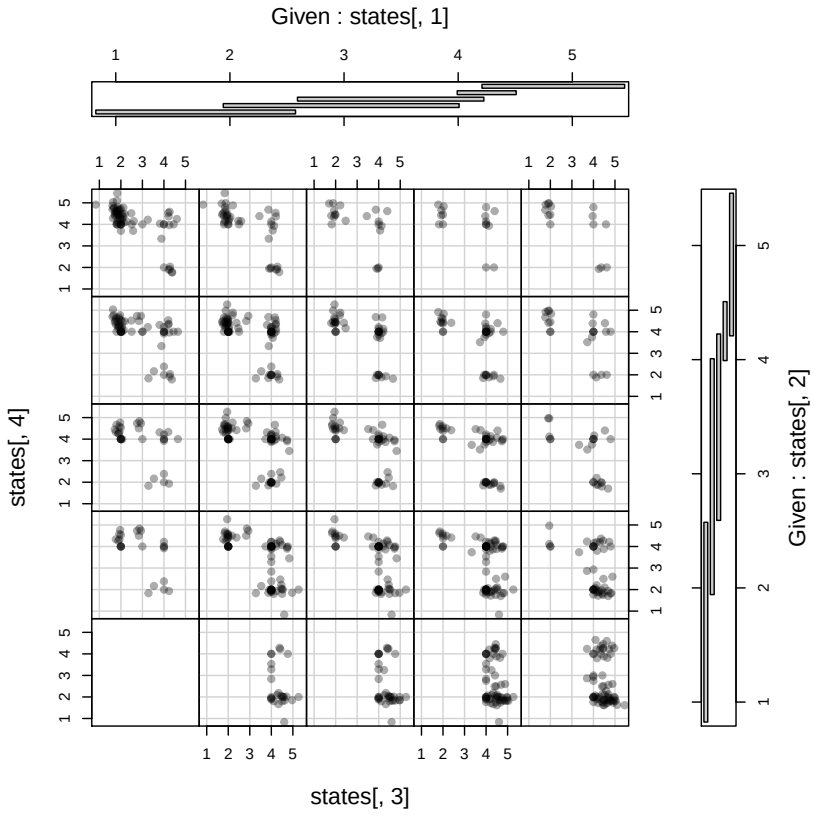

FIGURE 35. State coplot. Example case: Old Faithful Geyser eruption durations. Time used: 0.492 sec.

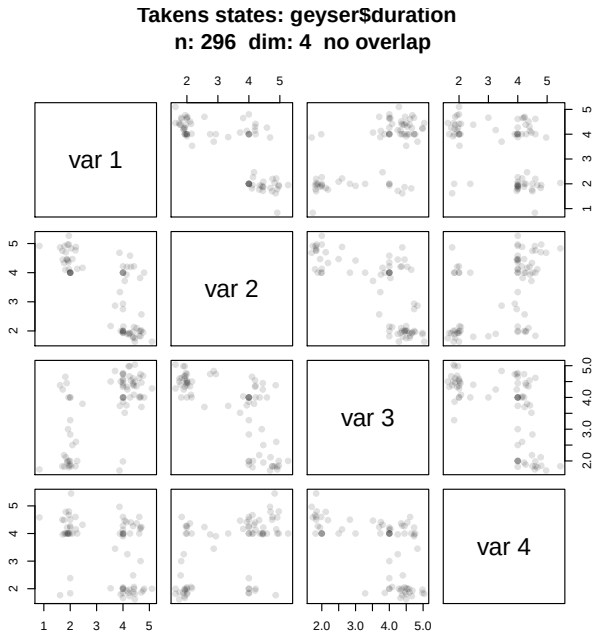

FIGURE 36. Recurrence plot. Example case: Old Faithful Geyser eruption durations. Time used: 0.682 sec.

Input

```
eruptionsneighs4 <- local.findAllNeighbours(eruptionstakens4, radius=3*0.8)
save(eruptionsneighs4, file="eruptionsneighs4.RData")
```

Input

```
#load(file="eruptionsneighs4.RData")
local.recurrencePlotAux(eruptionsneighs4)
```

See Figure 37 on the facing page.

Input

```
showrqa(eruptionstakens4, radius=3*0.8)
```

Output

```
geyser$duration  n: 296  Dim: 4
Radius: 2.4  Recurrence coverage REC: 0.554  log(REC)/log(R): -0.675
Determinism: 0.986  Laminarity: 0.562
DIV: 0.011
Trend: 0  Entropy: 2.994
Diagonal lines max: 88  Mean: 9.147  Mean off main: 9.092
Vertical lines max: 147  Mean: 4.264
```

See Figure 38 on the next page.

6.1.1. *Geyser eruption durations. Dim=2.*

Input

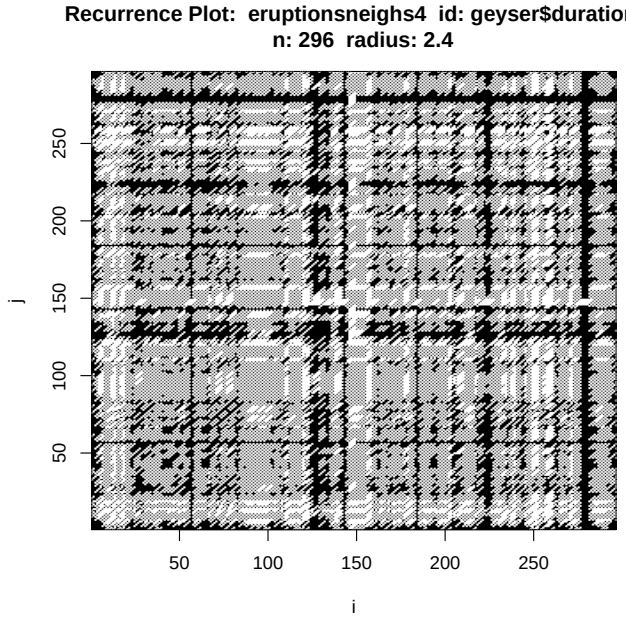

FIGURE 37. Recurrence plot. Example case: Old Faithful Geyser eruption durations. Dim=4. Time used: 0.386 sec.

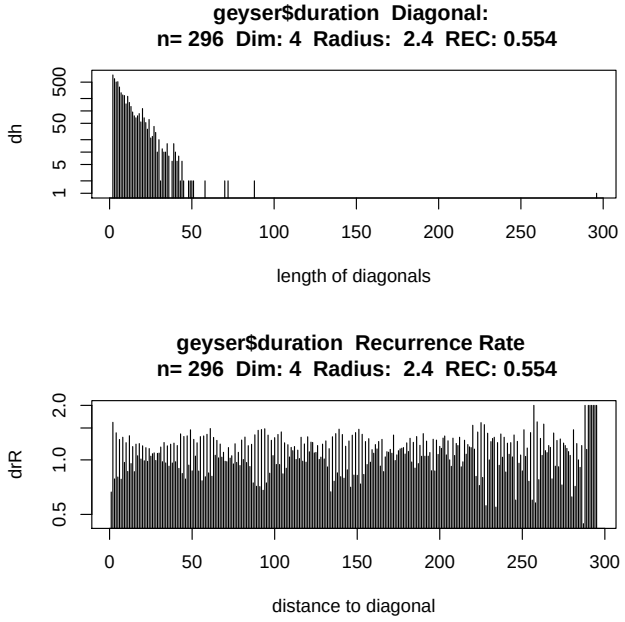

FIGURE 38. RQA. Example case: Old Faithful Geyser eruption durations. Dim=4. Time used: 0.579 sec.

```
eruptionstakens2 <-
        local.buildTakens(time.series=geyser$duration,
                embedding.dim=2, time.lag=1)
statepairs(eruptionstakens2) #dim=2
```

See Figure 39.

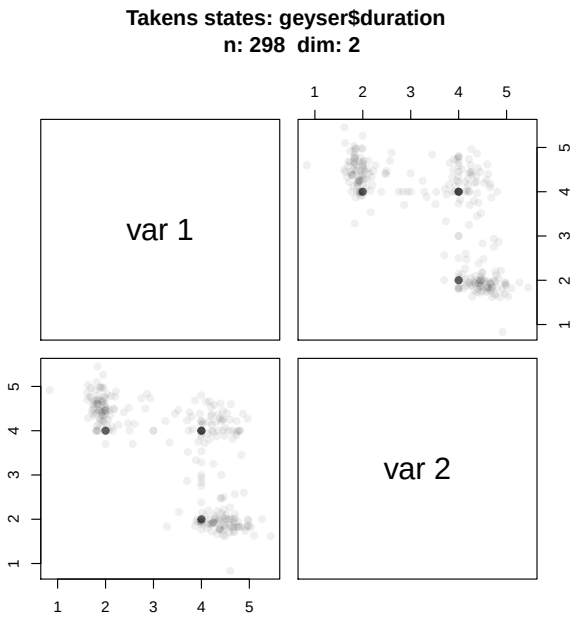

FIGURE 39. Recurrence plot. Example case: Old Faithful Geyser eruption durations. Dim=2. Time used: 0.118 sec.

Input

```
statepairs(eruptionstakens2, nooverlap=TRUE) #dim=2
```

See Figure 40 on the next page

Input

```
eruptionsneighs2 <- local.findAllNeighbours(eruptionstakens2,
        radius=0.8)
save(eruptionsneighs2, file="eruptionsneighs2.RData")
# load(file="eruptionsneighs2.RData")
local.recurrencePlotAux(eruptionsneighs2)
```

See Figure 41 on the facing page.

Input

```
showrqa(eruptionstakens2, radius=0.8)
```

Output

```
geyser$duration  n: 298  Dim: 2
Radius: 0.8  Recurrence coverage REC: 0.274  log(REC)/log(R): 5.806
Determinism: 0.892  Laminarity: 0.205
DIV: 0.045
Trend: 0  Entropy: 1.691
Diagonal lines max: 22  Mean: 3.644  Mean off main: 3.595
Vertical lines max: 7  Mean: 3.457
```

See Figure 42 on page 42.

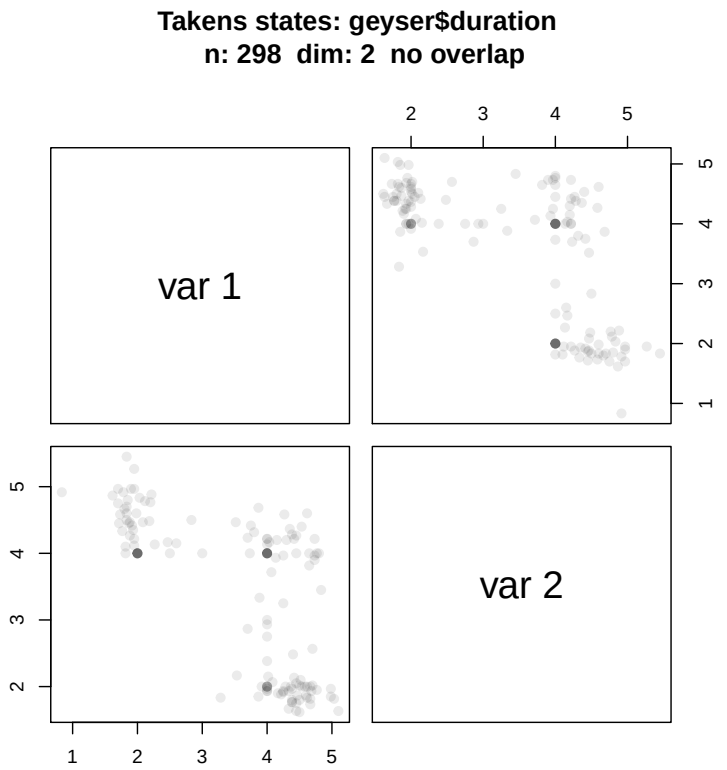

FIGURE 40. Example case: Old Faithful Geyser eruption durations. Dim=2. Time used: 0.225 sec.

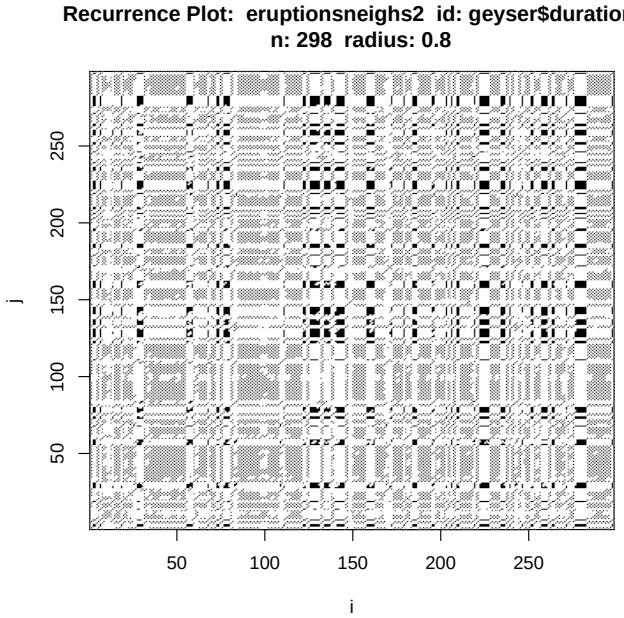

FIGURE 41. Recurrence plot. Example case: Old Faithful Geyser eruption durations. Dim=2. Time used: 0.314 sec.

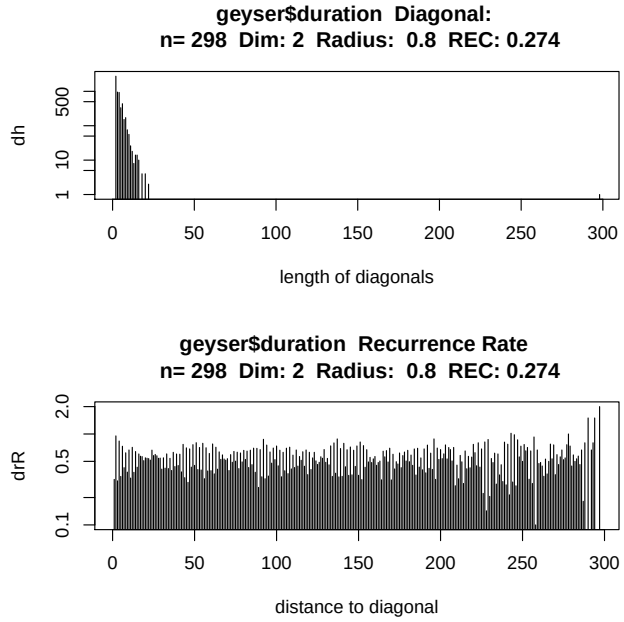

FIGURE 42. RQA. Example case: Old Faithful Geyser eruption durations. Dim=2. Time used: 0.536 sec.

6.1.2. *Geyser eruptions. Dim=4.*

Input

```
eruptionstakens4 <- local.buildTakens( time.series=geyser$duration,
        embedding.dim=4, time.lag=1)
statepairs(eruptionstakens4) #dim=4
```

See Figure 43 on the facing page.

Input

```
statepairs(eruptionstakens4, nooverlap=TRUE) #dim=4
```

See Figure 44 on the next page.

Input

```
eruptionsneighs4 <- local.findAllNeighbours(eruptionstakens4,
        radius=0.8)
save(eruptionsneighs4, file="eruptionsneighs4.RData")
```

Input

```
load(file="eruptionsneighs4.RData")
local.recurrencePlotAux(eruptionsneighs4)
```

See Figure 45 on page 44.

Input

```
showrqa(eruptionstakens4, radius=0.8)
```

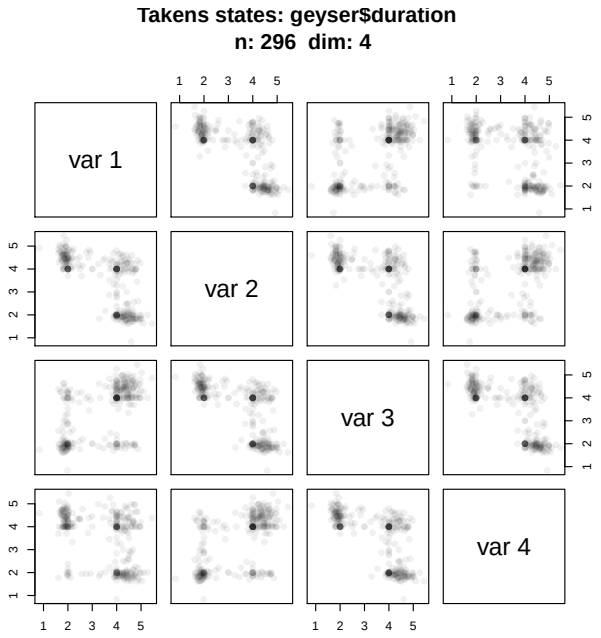

FIGURE 43. Recurrence plot. Example case: Old Faithful Geyser eruption durations. Dim=4. Time used: 0.297 sec.

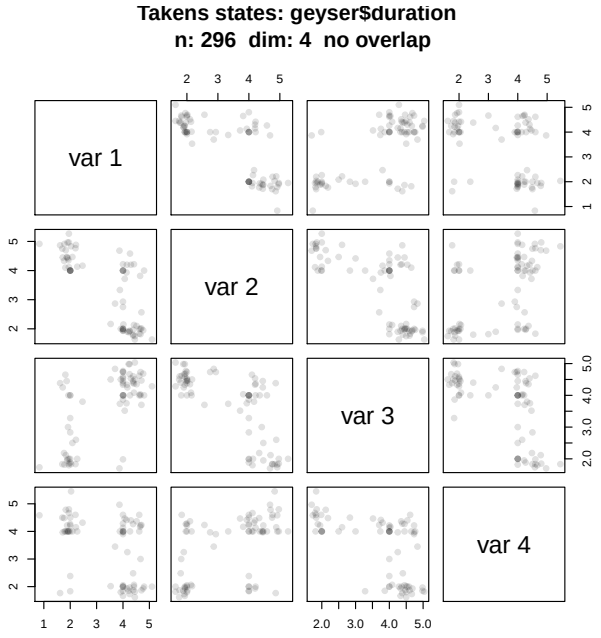

FIGURE 44. Recurrence plot. Example case: Old Faithful Geyser eruption durations. Dim=4. Time used: 0.469 sec.

Output

```
geyser$duration  n: 296  Dim: 4
Radius: 0.8  Recurrence coverage REC: 0.112  log(REC)/log(R): 9.822
Determinism: 0.903  Laminarity: 0.076
DIV: 0.05
```

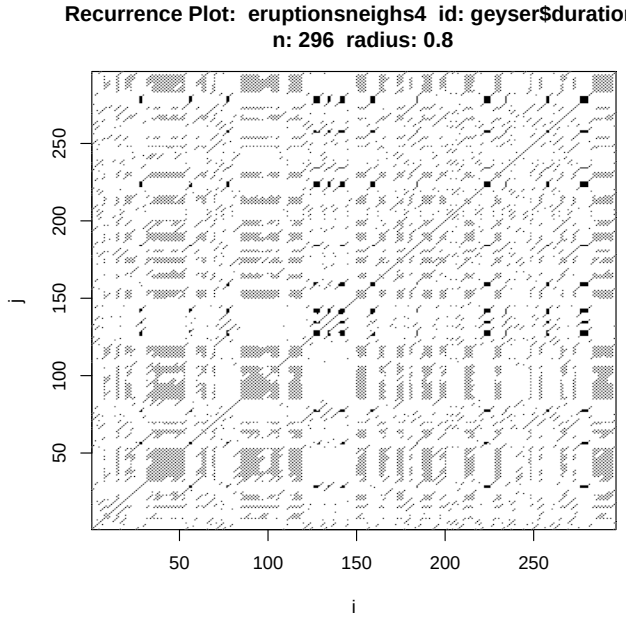

FIGURE 45. Recurrence plot. Example case: Old Faithful Geyser eruption durations. Dim=4. Time used: 0.201 sec.

```
Trend: 0  Entropy: 1.779
Diagonal lines max: 20  Mean: 3.919  Mean off main: 3.79
Vertical lines max: 5  Mean: 3.041
```

See Figure 46.

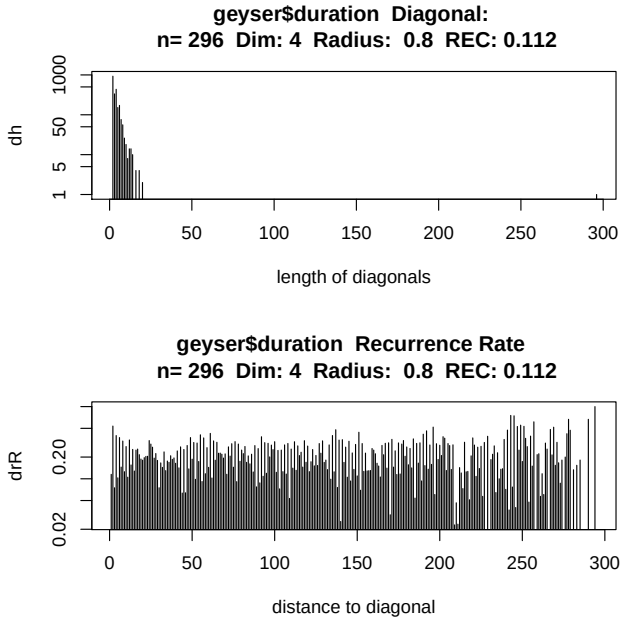

FIGURE 46. Recurrence plot. .

Example case: Old Faithful Geyser eruption durations. Dim=4. Time used: 0.406 sec.

6.1.3. *Geyser eruption durations. Dim=8.*

Input

```
eruptionstakens8 <- local.buildTakens( time.series=geyser$duration,
        embedding.dim=8,time.lag=1)
statepairs(eruptionstakens8) #dim=8
```

See Figure 47.

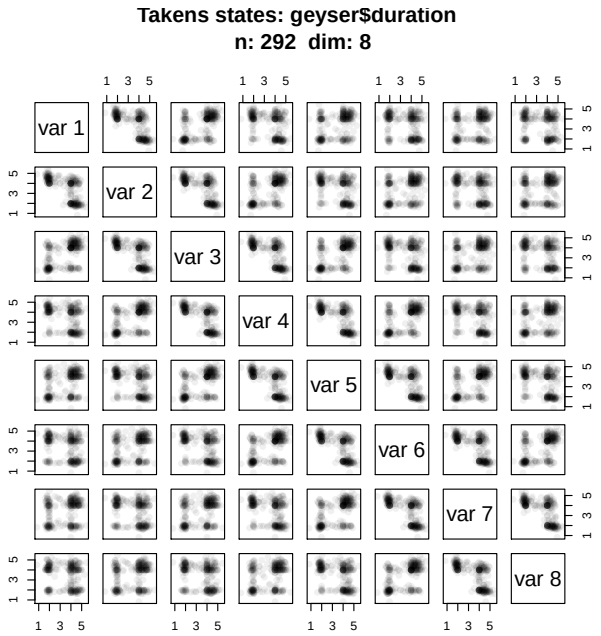

FIGURE 47. Recurrence plot. Example case: Old Faithful Geyser eruption durations. Dim=8. Time used: 0.958 sec.

Input

```
statepairs(eruptionstakens8, nooverlap=TRUE) #dim=8
```

See Figure 48 on the next page.

Input

```
eruptionsneighs8 <- local.findAllNeighbours(eruptionstakens8,
        radius=2.6)
save(eruptionsneighs8, file="eruptionsneighs8.RData")
#load(file="eruptionsneighs8.RData")
local.recurrencePlotAux(eruptionsneighs8)
```

See Figure 49 on page 47.

Input

```
showrqa(eruptionstakens8, radius=2.6)
```

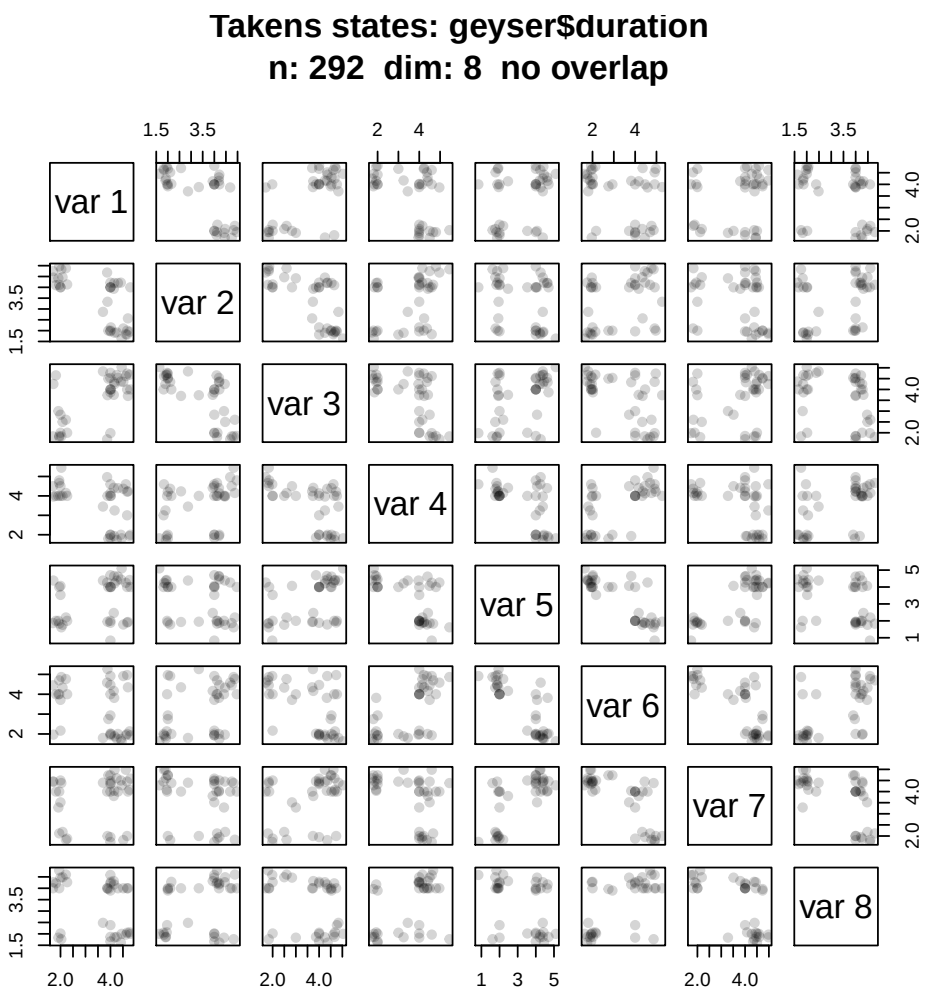

FIGURE 48. Example case: Old Faithful Geyser eruption durations. Dim=8. Time used: 1.26 sec.

Output

```
geyser$duration  n: 292  Dim: 8
Radius: 2.6  Recurrence coverage REC: 0.494  log(REC)/log(R): -0.738
Determinism: 0.991  Laminarity: 0.5
DIV: 0.011
Trend: 0  Entropy: 3.338
Diagonal lines max: 93  Mean: 12.635  Mean off main: 12.55
Vertical lines max: 36  Mean: 4.073
```

See Figure 50 on the next page.

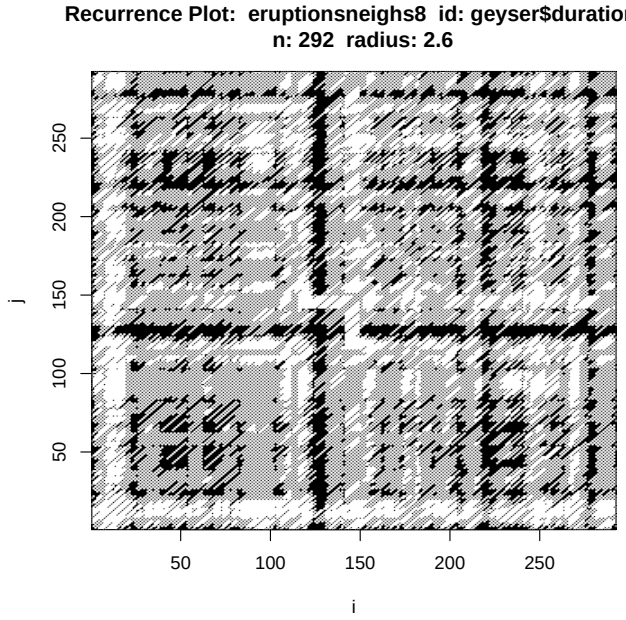

FIGURE 49. Recurrence plot. Example case: Old Faithful Geyser eruption durations. Dim=8. Time used: 0.418 sec.

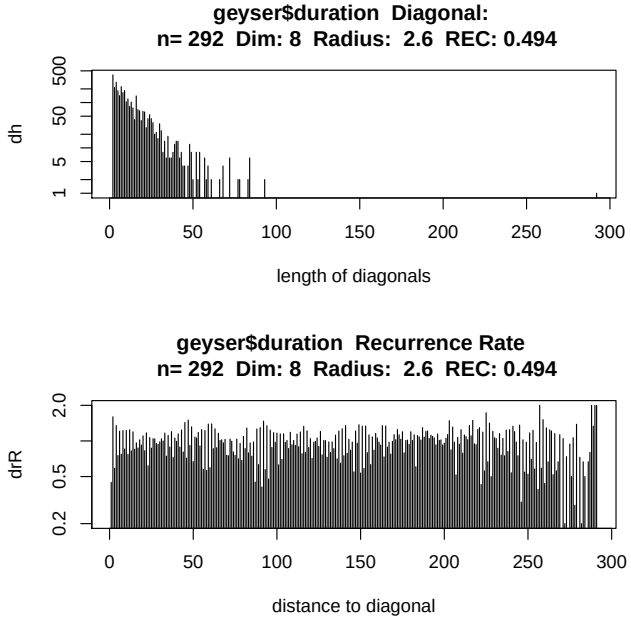

FIGURE 50. RQA. Example case: Old Faithful Geyser eruption durations. Dim=8. Time used: 0.624 sec.

6.1.4. *Geyser eruption durations: Comparison by Dimension.* For comparison, recurrence plots for the Geyser data with varying dimension are in Figure 51

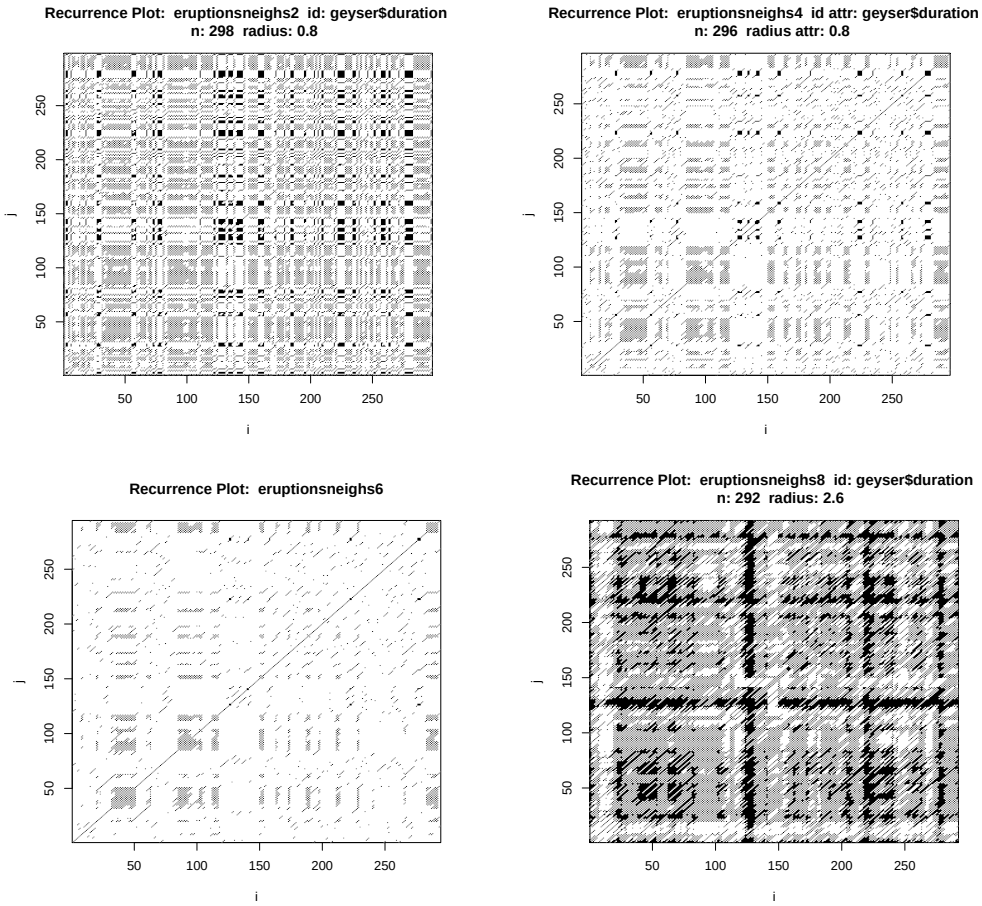

FIGURE 51. Recurrence Plot. Example case: Old Faithful Geyser eruptions. Dim=2, 4, 6, 8.

6.2. **Geyser Waiting.**

Input

```
plotsignal(geyser$waiting)
```

See Figure 52 on the next page.

Input

```
waitingtakens <-
        local.buildTakens( time.series=geyser$waiting,
                embedding.dim=4, time.lag=4)
statepairs(waitingtakens) #dim=4
```

See Figure 53 on the facing page.

Input

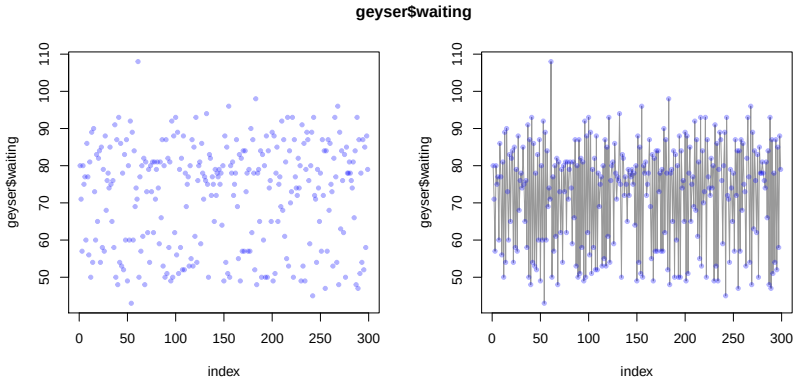

FIGURE 52. Example case: Old Faithful Geyser waiting. Signal and linear interpolation. Time used: 0.851 sec.

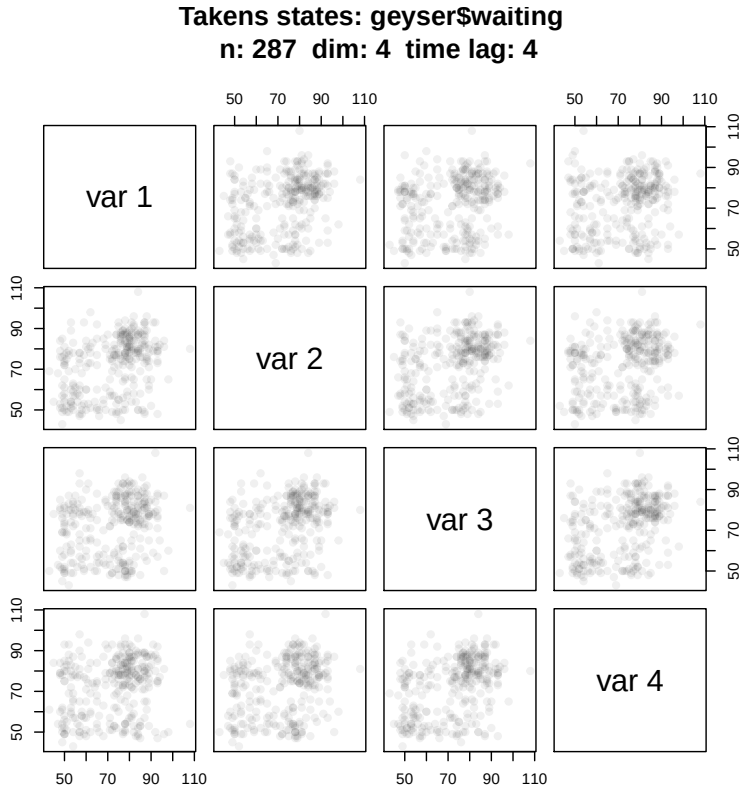

FIGURE 53. Example case: Old Faithful Geyser waiting. Time used: 0.277 sec.

```
waitingneighs <- local.findAllNeighbours(waitingtakens, radius=16)
save(waitingneighs, file="waitingneighs.Rdata")
```

Input

```
showrqa(waitingtakens, radius=16)
```

Output

```
geyser$waiting  n: 287  Dim: 4
Radius: 16  Recurrence coverage REC: 0.137  log(REC)/log(R): -0.718
Determinism: 0.382  Laminarity: 0.053
DIV: 0.053
Trend: 0  Entropy: 1.002
Diagonal lines max: 19  Mean: 2.878  Mean off main: 2.688
Vertical lines max: 3  Mean: 2.315
```

See Figure 54.

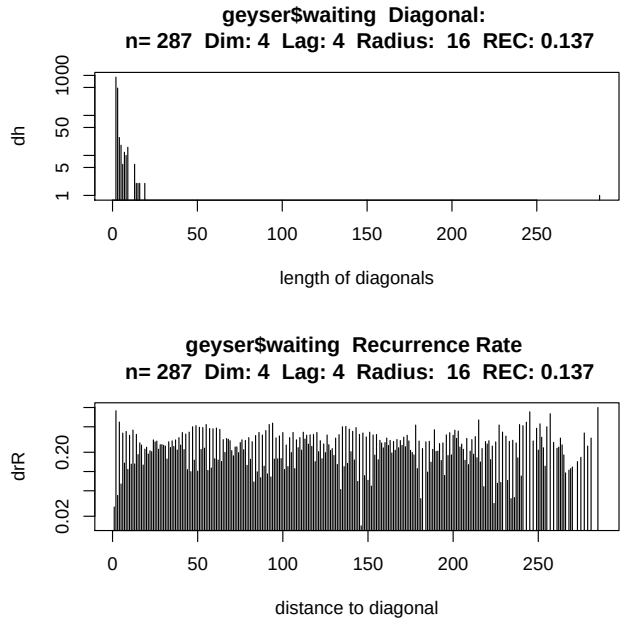

FIGURE 54. RQA. Example case: Old Faithful Geyser waiting. Time used: 0.187 sec.

Input

```
load(file="waitingneighs.RData")
local.recurrencePlotAux(waitingneighs)
```

See Figure 55 on the facing page.

6.3. **Geyser - linearized.** The Geyser data is a bivariate series and asks for bivariate Takens states and recurrence plots. This is a first crude attempt and need improvement.

So far, *nonlinearTseries* only handles multivariate data by FORTRAN conventions, using a lag parameter.

As a hack, we transform the data to FORTRAN conventions.

Input

```
geyserlin <- t(geyser)
dim(geyserlin)<-NULL
dimnames(geyserlin)<-NULL
```

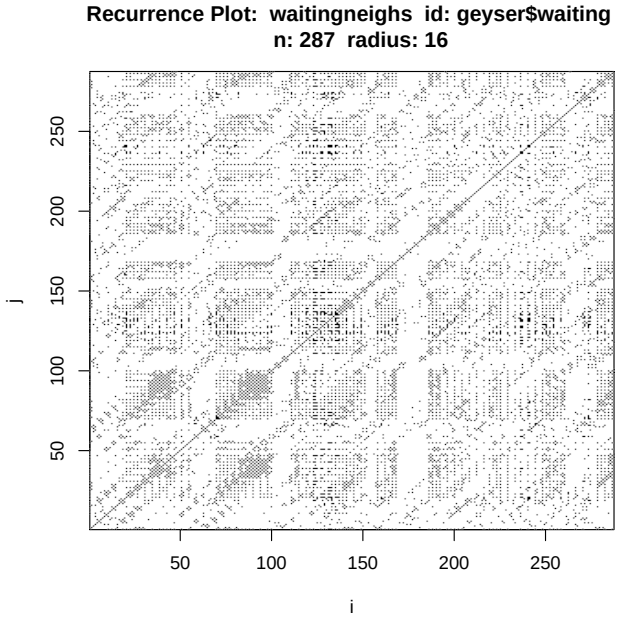

FIGURE 55. Recurrence plot. Recurrence Plot. Example case: Old Faithful Geyser waiting. Time used: 0.379 sec.

Now duration and waiting are mixed. A $lag = 2$ separates the dimension again. The Taken states iterate over the index, giving alternating a duration and waiting state.

Input

```
plotsignal(geyserlin)
```

See Figure 56.

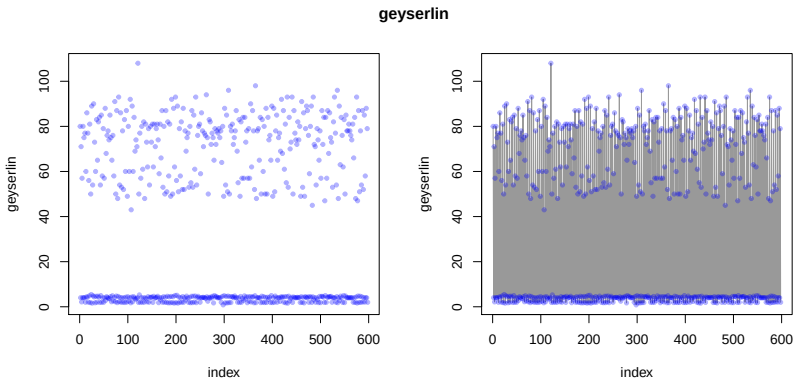

FIGURE 56. Example case: Old Faithful Geyser eruptions. Signal and linear interpolation.

Input

```
glineruptionstakens4 <-
        local.buildTakens( time.series=geyserlin,
```

```
                embedding.dim=4, time.lag=2)
statepairs(glineruptionstakens4) #dim=4
```

See Figure 57.

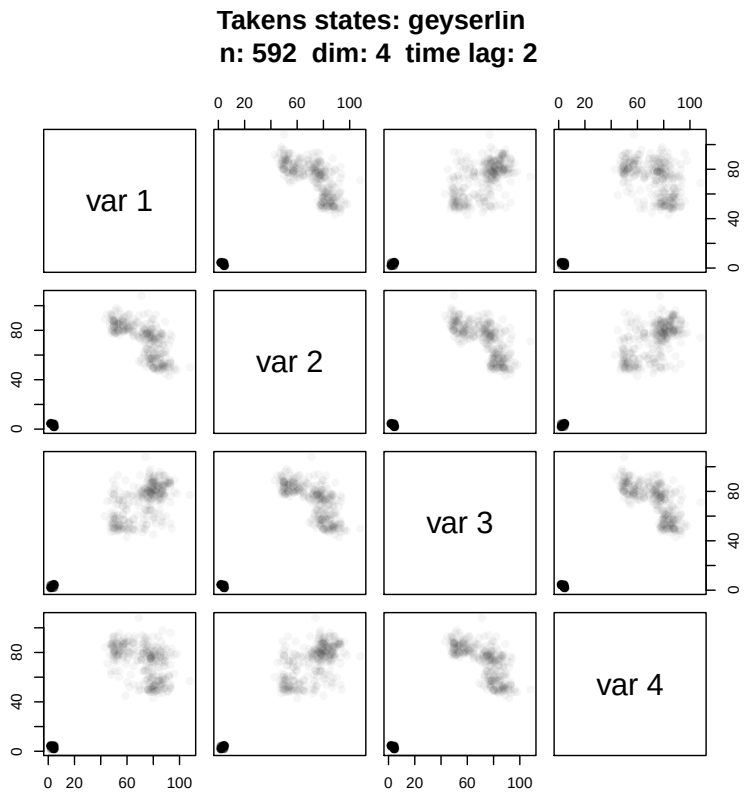

FIGURE 57. Example case: Old Faithful Geyser eruptions. Time used: 0.407 sec.

Input

```
glineruptionsneighs4 <- local.findAllNeighbours(glineruptionstakens4, radius=0.8)
save(glineruptionsneighs4, file="glineruptionsneighs4.RData")
```

Input

```
showrqa(glineruptionstakens4, radius=0.8)
```

Output

```
geyserlin  n: 592  Dim: 4
Radius: 0.8  Recurrence coverage REC: 0.029  log(REC)/log(R): 15.901
Determinism: 0.059  Laminarity: 0
DIV: Inf
Trend: 0  Entropy: 0
Diagonal lines max: 0  Mean: 592  Mean off main: NaN
Vertical lines max: 0  Mean: 0
```

See Figure 58 on the next page.

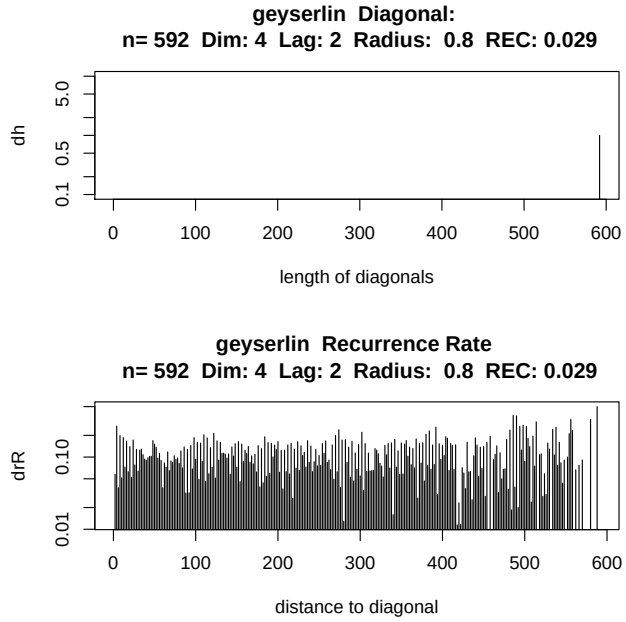

FIGURE 58. Recurrence plot. Old Faithful Geyser - both. Dim=4. Time used: 0.33 sec.

Input

```
load(file="glineruptionsneighs4.RData")
local.recurrencePlotAux(glineruptionsneighs4)
```

See Figure 59.

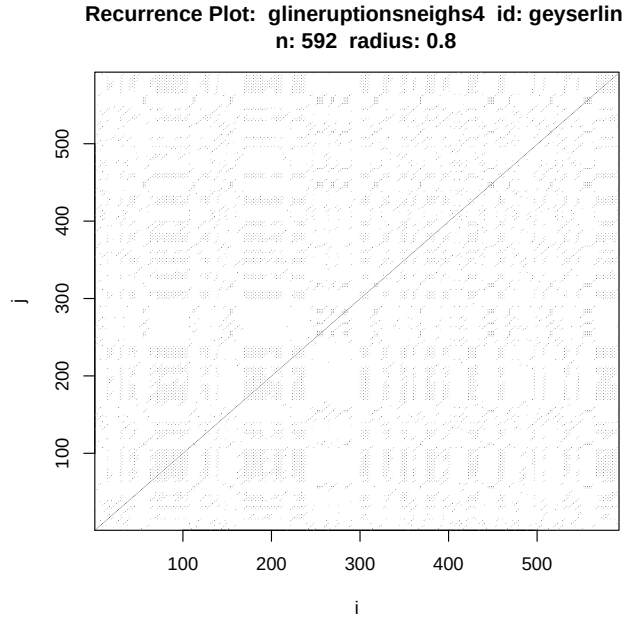

FIGURE 59. Recurrence plot. Example case: Old Faithful Geyser eruptions. Dim=4. Time used: 0.609 sec.

6.3.1. *Geyser eruptions - linearized. Dim=2.*

Input

```
glineruptionstakens2 <-
        local.buildTakens(time.series=geyserlin,
                embedding.dim=2, time.lag=2)
statepairs(glineruptionstakens2) #dim=2
```

See Figure 60.

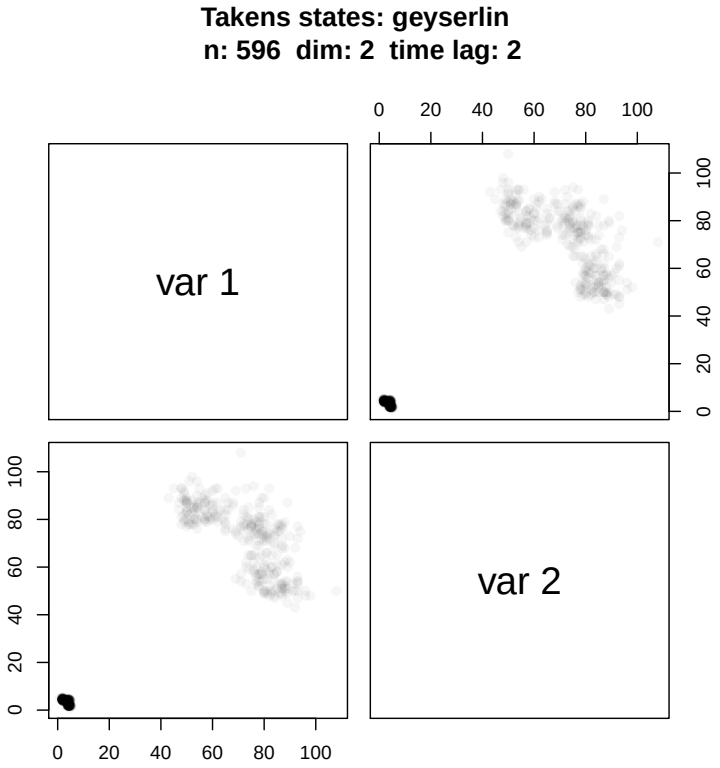

FIGURE 60. Example case: Old Faithful Geyser eruptions. Dim=2. Time used: 0.138 sec.

Input

```
glineruptionsneighs2 <- local.findAllNeighbours(glineruptionstakens2, radius=0.8)
save(glineruptionsneighs2, file="glineruptionsneighs2.RData")
#load(file="glineruptionsneighs2.RData")
local.recurrencePlotAux(glineruptionsneighs2)
```

See Figure 61 on the facing page.

6.3.2. *Geyser eruptions - linearized. Dim=8.*

Input

```
glineruptionstakens8 <- local.buildTakens( time.series=geyserlin,
        embedding.dim=8,time.lag=2)
statepairs(glineruptionstakens8) #dim=8
```

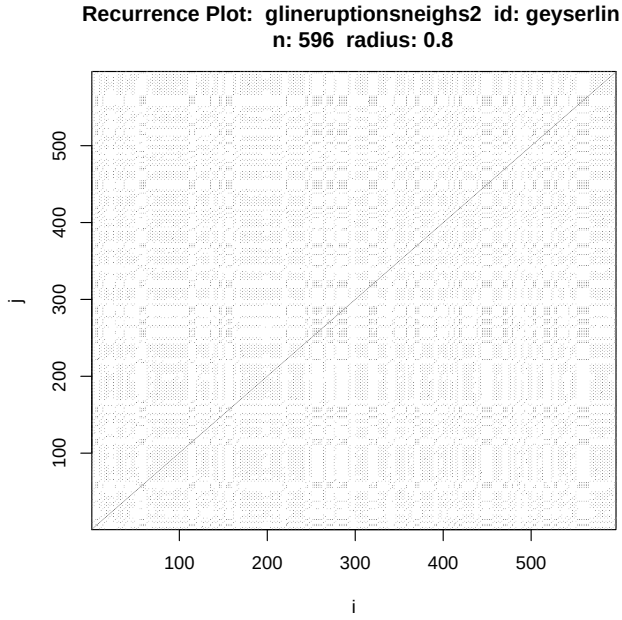

FIGURE 61. Recurrence plot. Example case: Old Faithful Geyser eruptions linearized. Dim=2. Time used: 0.509 sec.

See Figure 62 on the next page

Input

```
glineruptionsneighs8 <- local.findAllNeighbours(glineruptionstakens8,
        radius=0.8)
save(glineruptionsneighs8, file="glineruptionsneighs8.RData")
```

Input

```
load(file="glineruptionsneighs8.RData")
local.recurrencePlotAux(glineruptionsneighs8)
```

See Figure 63 on page 57.

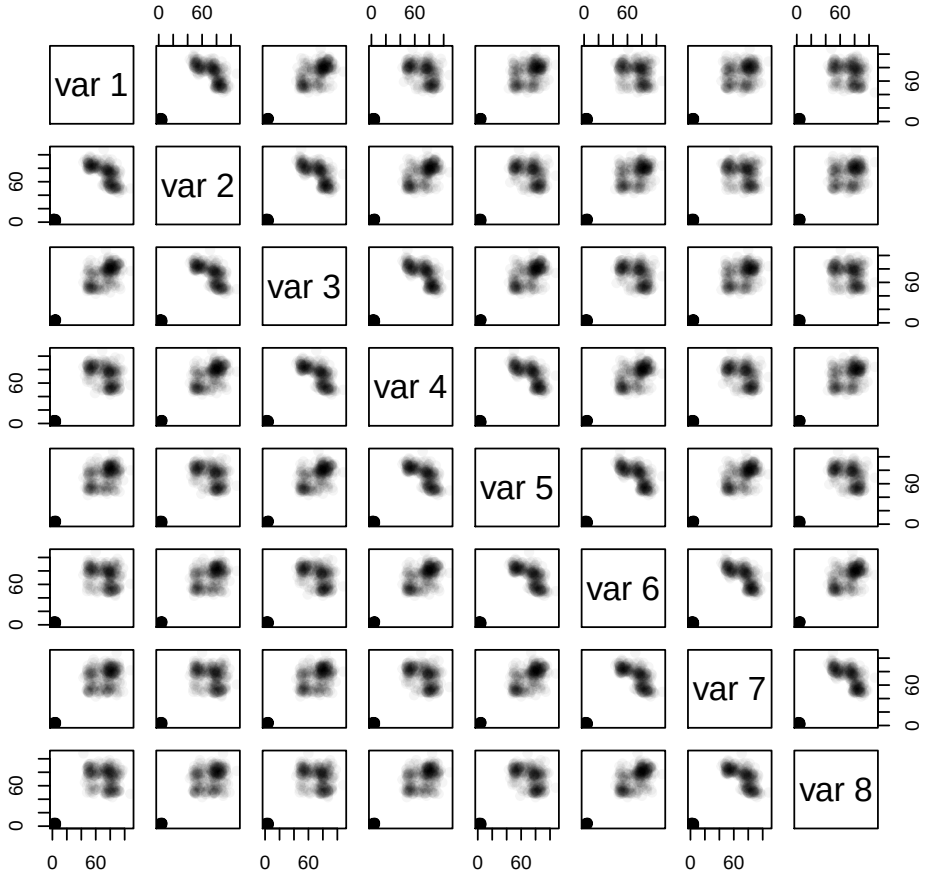

FIGURE 62. Example case: Old Faithful Geyser eruptions. Dim=8. Time used: 1.477 sec.

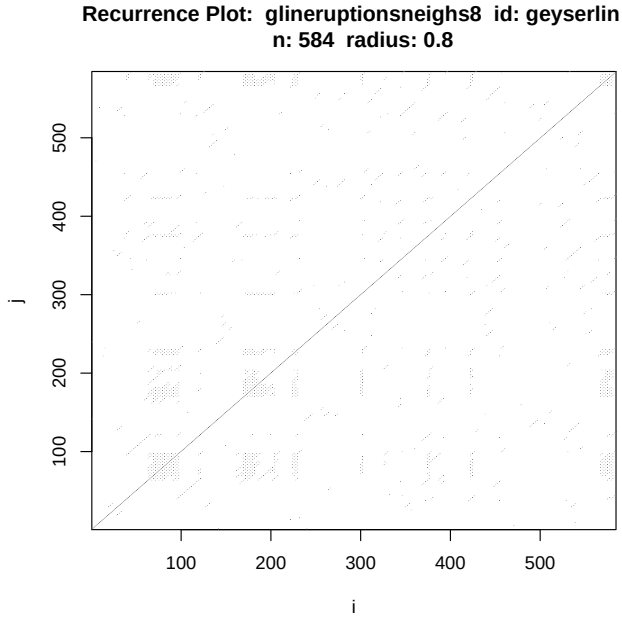

FIGURE 63. Recurrence plot. Recurrence Plot. Example case: Old Faithful Geyser eruptions. Dim=8. Time used: 0.266 sec.

7. Case Study: HRV data example.beats

Only 1024 data points used in recurrence plots in this section

Input

```
library(RHRV)
load("/users/gs/projects/rforge/rhrv/pkg/data/HRVData.rda")
load("/users/gs/projects/rforge/rhrv/pkg/data/HRVProcessedData.rda")
#####################################################
### code chunk number 1: creation
#####################################################
hrv.data  = CreateHRVData()
hrv.data = SetVerbose(hrv.data, TRUE )
#####################################################
### code chunk number 3: loading
#####################################################
hrv.data = LoadBeatAscii(hrv.data, "example.beats",
       RecordPath = "/users/gs/projects/rforge/rhrv/tutorial/beatsFolder")
```

Output

```
** Loading beats positions for record: example.beats **
   Path: /users/gs/projects/rforge/rhrv/tutorial/beatsFolder
   Scale: 1
   Date: 01/01/1900
   Time: 00:00:00
   Number of beats: 17360
```

Input

```
#       RecordPath = "beatsFolder")


#####################################################
### code chunk number 4: derivating
#####################################################
hrv.data = BuildNIHR(hrv.data)
```

Output

```
** Calculating non-interpolated heart rate **
   Number of beats: 17360
```

Input

Input

```
plotsignal(hrv.data$Beat$RR)
```

ToDo: We have outliers at approximately 2*RR. Could this be an artefact of preprocessing, filtering out too many impulses?

See Figure 64 on the next page.

Input

```
hrvRRtakens4 <- local.buildTakens( time.series=hrv.data$Beat$RR[1:nsignal],
        embedding.dim=4,time.lag=1)
statepairs(hrvRRtakens4) #dim=4
```

See Figure 65 on the facing page.

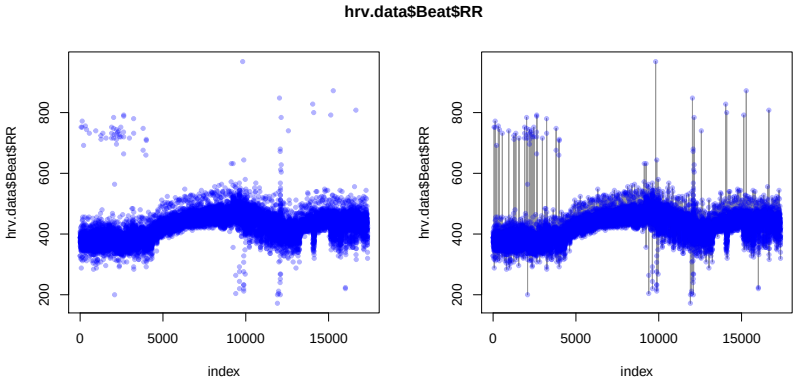

FIGURE 64. RHRV tutorial example.beats. Signal and linear interpolation.

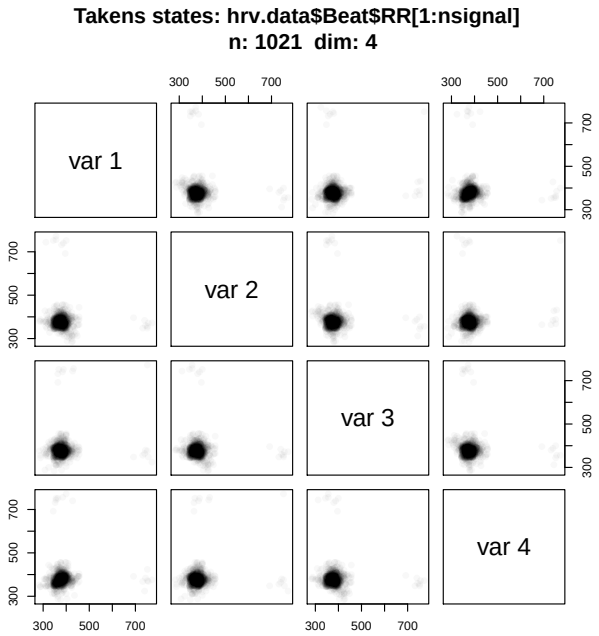

FIGURE 65. Recurrence plot. RHRV tutorial example.beats. Time used: 0.541 sec.

Input

```
statepairs(hrvRRtakens4, rank=TRUE) #dim=4
```

See Figure 66 on the next page.

Input

```
statecoplot(hrvRRtakens4) #dim=4
```

See Figure 67 on page 61.

See Figure 68 on page 61.

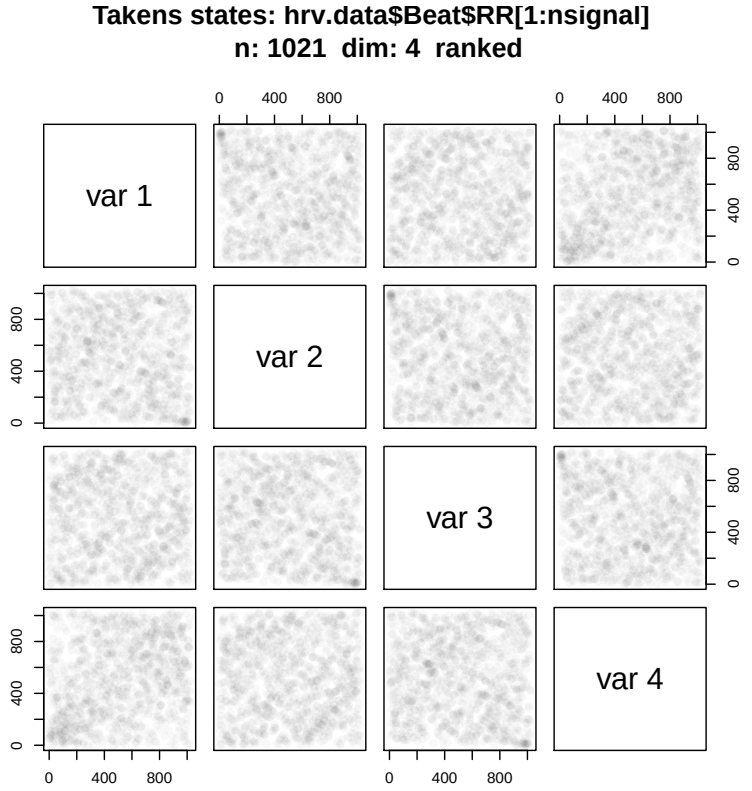

FIGURE 66. RHRV tutorial example.beats. Ranked data. Time used: 1.178 sec.

Input

```
hrvRRneighs4  <- local.findAllNeighbours(hrvRRtakens4, radius=16)
save(hrvRRneighs4, file="hrvRRneighs4.Rdata")
```

Time used: 0.139 sec.

Input

```
load(file="hrvRRneighs4.RData")
local.recurrencePlotAux(hrvRRneighs4)
```

See Figure 69 on page 62.

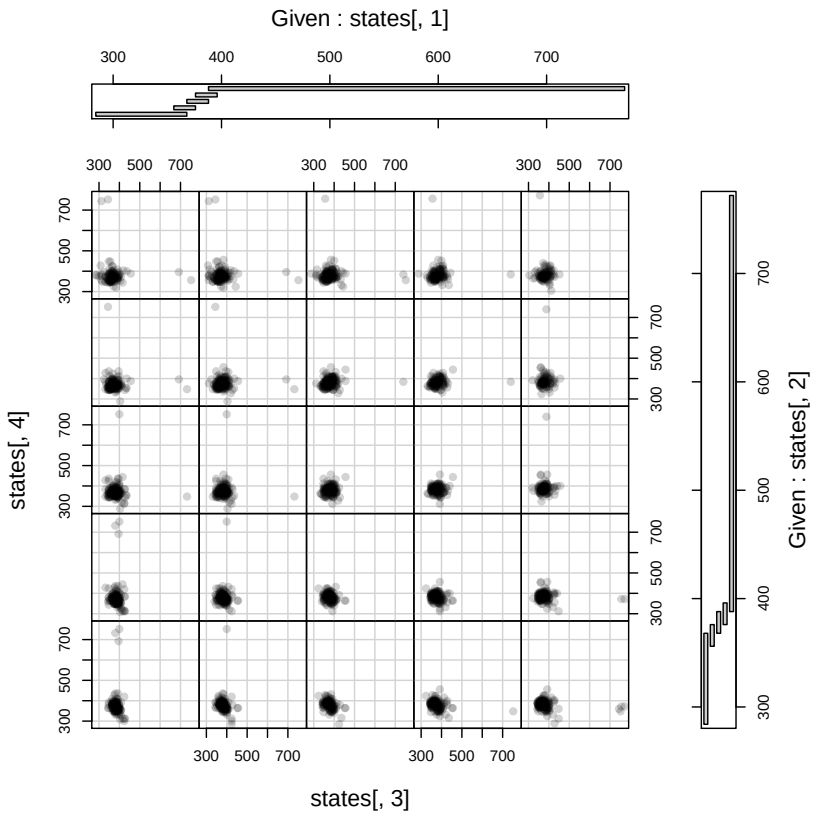

FIGURE 67. State coplot. RHRV tutorial example.beats. Time used: 0.253 sec.

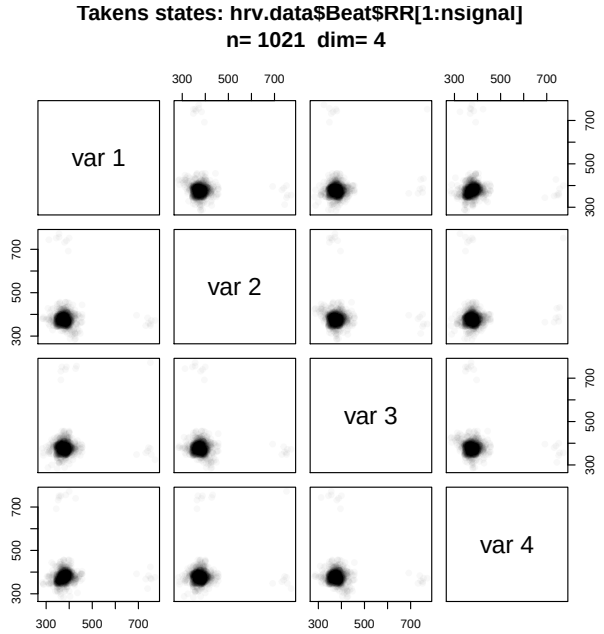

FIGURE 68. Recurrence plot. RHRV tutorial example.beats. Time used: 0.001 sec.

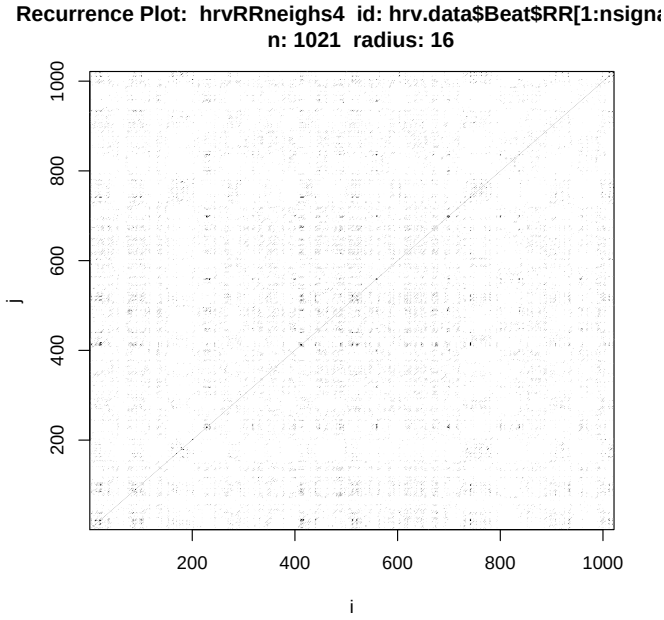

FIGURE 69. Recurrence plot. Recurrence Plot. Example case: RHRV tutorial example.beats. Dim=4. Time used: 0.782 sec.

We should expect the breathing rhythm, so a time lag in the order of 10 is to be expected.

7.0.3. *RHRV: example.beats - Comparison by Dimension.*

Input

```
hrvRRtakens2 <- local.buildTakens( time.series=hrv.data$Beat$RR[1:nsignal],
        embedding.dim=2,time.lag=1)
hrvRRneighs2  <- local.findAllNeighbours(hrvRRtakens2, radius=16)
save(hrvRRneighs2, file="hrvRRneighs2.Rdata")
# load(file="hrvRRneighs2.RData")
local.recurrencePlotAux(hrvRRneighs2)
showrqa(hrvRRtakens2, do.hist=FALSE, radius=16)
```

Output

```
hrv.data$Beat$RR[1:nsignal]  n: 1023  Dim: 2
Radius: 16  Recurrence coverage REC: 0.165  log(REC)/log(R): -0.65
Determinism: 0.651  Laminarity: 0.376
DIV: 0.056
Trend: 0  Entropy: 1.248
Diagonal lines max: 18  Mean: 2.841  Mean off main: 2.816
Vertical lines max: 14  Mean: 2.463
```

See Figure 70.

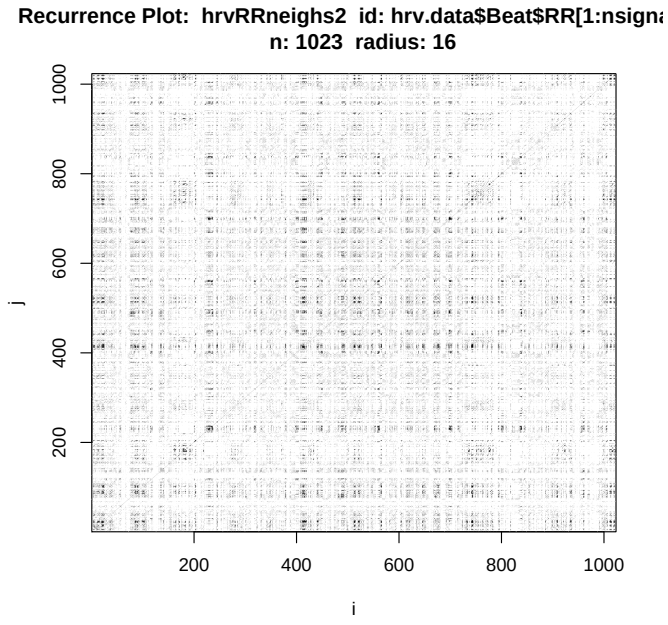

FIGURE 70. Recurrence plot. Dim=2. Time used: 2.394 sec.

Input

```
hrvRRtakens6 <- local.buildTakens( time.series=hrv.data$Beat$RR[1:nsignal],
        embedding.dim=6,time.lag=1)
hrvRRneighs6  <- local.findAllNeighbours(hrvRRtakens6, radius=16)
save(hrvRRneighs6, file="hrvRRneighs6.Rdata")
# load(file="hrvRRneighs6.RData")
local.recurrencePlotAux(hrvRRneighs6)
```

Dim=6. Time used: 0.846 sec.

Input

```
hrvRRtakens8 <- local.buildTakens( time.series=hrv.data$Beat$RR[1:nsignal],
        embedding.dim=8,time.lag=1)
hrvRRneighs8  <- local.findAllNeighbours(hrvRRtakens8, radius=32)
save(hrvRRneighs8, file="hrvRRneighs8.Rdata")
# load(file="hrvRRneighs8.RData")
local.recurrencePlotAux(hrvRRneighs8)
```

Dim=8. Time used: 1.319 sec.

Input

```
hrvRRtakens12 <- local.buildTakens( time.series=hrv.data$Beat$RR[1:nsignal],
        embedding.dim=2,time.lag=1)
hrvRRneighs12  <- local.findAllNeighbours(hrvRRtakens12, radius=16)
save(hrvRRneighs12, file="hrvRRneighs12.Rdata")
# load(file="hrvRRneighs12.RData")
local.recurrencePlotAux(hrvRRneighs12)
```

Dim=12. Time used: 3.318 sec.

Input

```
hrvRRtakens16 <- local.buildTakens(
        time.series=hrv.data$Beat$RR[1:nsignal],
        embedding.dim=16,time.lag=1)
hrvRRneighs16  <- local.findAllNeighbours(hrvRRtakens16, radius=32)
save(hrvRRneighs16, file="hrvRRneighs16.Rdata")
# load(file="hrvRRneighs16.RData")
local.recurrencePlotAux(hrvRRneighs16)
```

Dim=16. Time used: 0.986 sec.

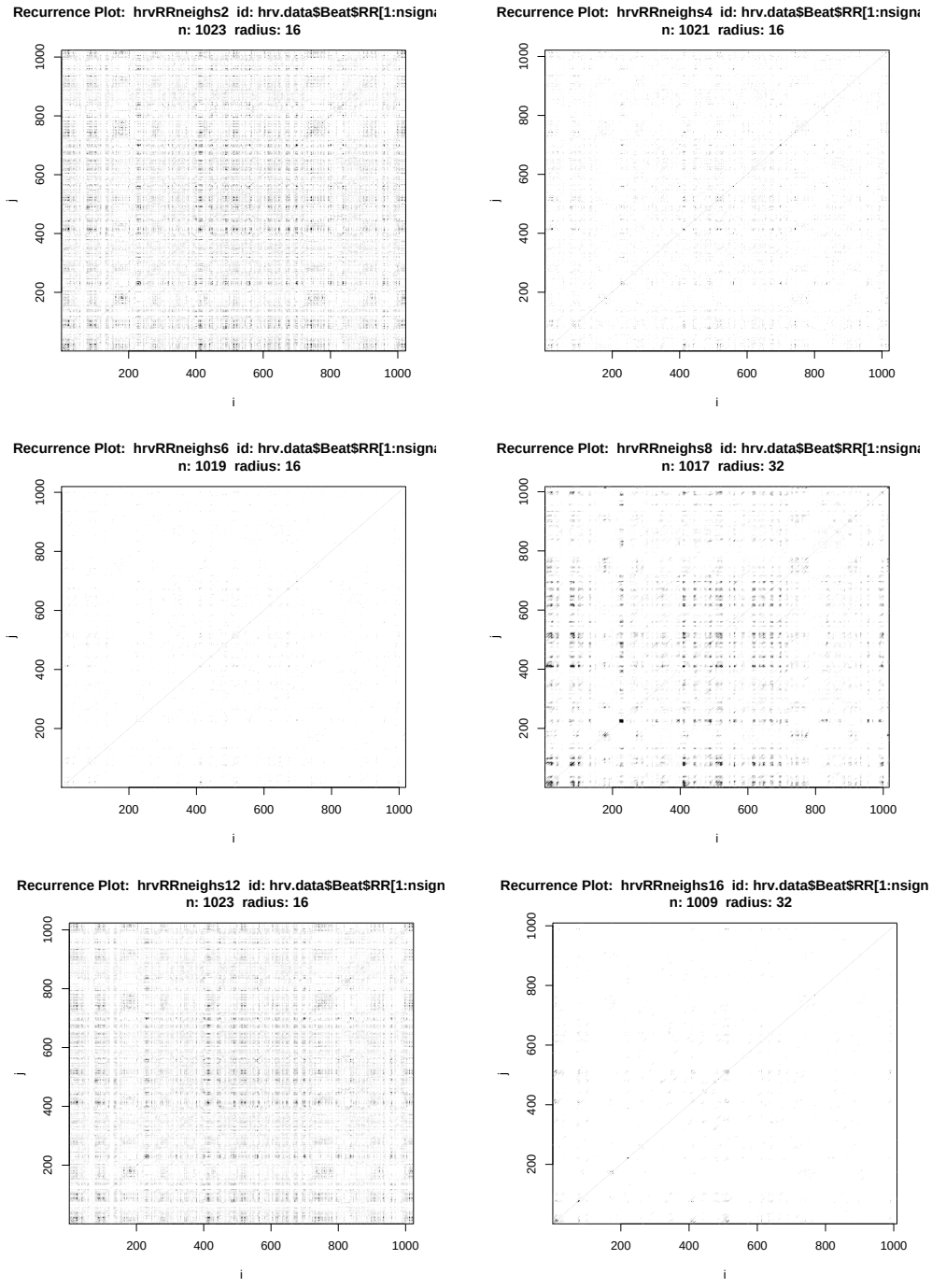

FIGURE 71. Recurrence Plot. Example case: RHRV tutorial example.beats. Dim=2, 4, 6, 8, 12, 16. Time used: 0.986 sec.

ToDo: This is an experimental proposal

7.1. **RHRV: example.beats - Hart Rate Variation.** Since we are not interested in heart rate (or pulse), but in heart rate variation, a proposal is to use scaled differences.

Input

```
# source('/users/gs/projects/rforge/rhrv/pkg/R/BuildNIHR2.R', chdir = TRUE)
BuildNIDHR <-
function(HRVData, verbose=NULL) {
#------------------------------------------------------
# Obtains instantaneous heart rate variation from beats positions
# D for difference. The scaled difference is recorded as variation HRRV
#------------------------------------------------------
        if (!is.null(verbose)) {
                cat("  --- Warning: deprecated argument, using SetVerbose() instead ---\n ",
                        "   --- See help for more information!! ---\n")
                SetVerbose(HRVData,verbose)
        }

        if (HRVData$Verbose) {
                cat("** Calculating non-interpolated heart rate differences **\n")
        }

        if (is.null(HRVData$Beat$Time)) {
                cat("   --- ERROR: Beats positions not present...",
                " Impossible to calculate Heart Rate!! ---\n")
                return(HRVData)
        }

        NBeats=length(HRVData$Beat$Time)
        if (HRVData$Verbose) {
                cat("   Number of beats:",NBeats,"\n");
        }

  # addition gs
   #using NA, not constant extrapolation as else in RHRV
   #drr=c(NA,NA,1000.0*        diff(HRVData$Beat$Time, lag=1 , differences=2))
   HRVData$Beat$dRR=c(NA, NA,
           1000.0*diff(HRVData$Beat$Time, lag=1, differences=2))

   HRVData$Beat$avRR=(c(NA,HRVData$Beat$RR[-1])+HRVData$Beat$RR)/2

   HRVData$Beat$HRRV <- HRVData$Beat$dRR/HRVData$Beat$avRR
        # end addition gs
        return(HRVData)
}
```

differences for HRV

Input

```
hrv.data <- BuildNIDHR(hrv.data)
```

Output

```
** Calculating non-interpolated heart rate differences **
   Number of beats: 17360
```

Input

```
HRRV <- hrv.data$Beat$HRRV
```

These are the displays of the Takens state space we used before, now for HRRV:

Input
```
plotsignal(HRRV)
```

See Figure 72,

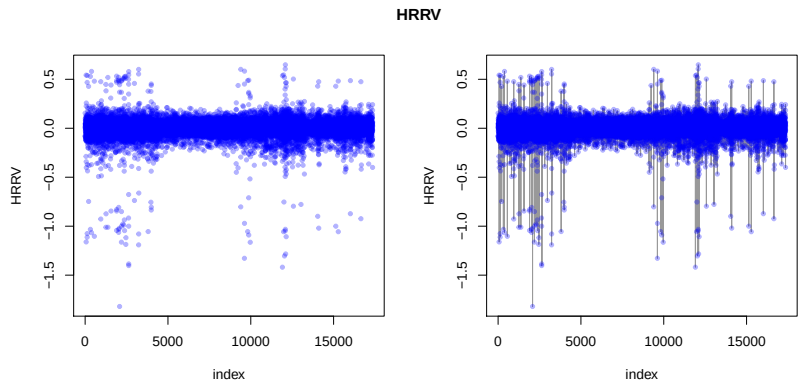

FIGURE 72. RHRV tutorial example.beats. HRRV Signal and linear interpolation.

Only 1024 data points used in this section

Input
```
hrvRRVtakens4 <-
        local.buildTakens( time.series=HRRV[1:nsignal],
                embedding.dim=4,time.lag=1)
statepairs(hrvRRVtakens4) #dim=4
```

See Figure 73 on the next page

Input
```
statepairs(hrvRRVtakens4, rank=TRUE) #dim=4
```

See Figure 74 on page 69

ToDo: findAllNeighbours does not handle NAs

Input
```
#use hack: findAllNeighbours does not handle NAs
hrvRRVneighs4  <- local.findAllNeighbours(hrvRRVtakens4[-(1:2),], radius=0.125)
save(hrvRRVneighs4, file="hrvRRVneighs4.Rdata")
```

Time used: 0.248 sec.

Input
```
load(file="hrvRRVneighs4.RData")
local.recurrencePlotAux(hrvRRVneighs4,dim=4, radius=0.125)
```

ToDo: check. There seem to be strange artefacts.

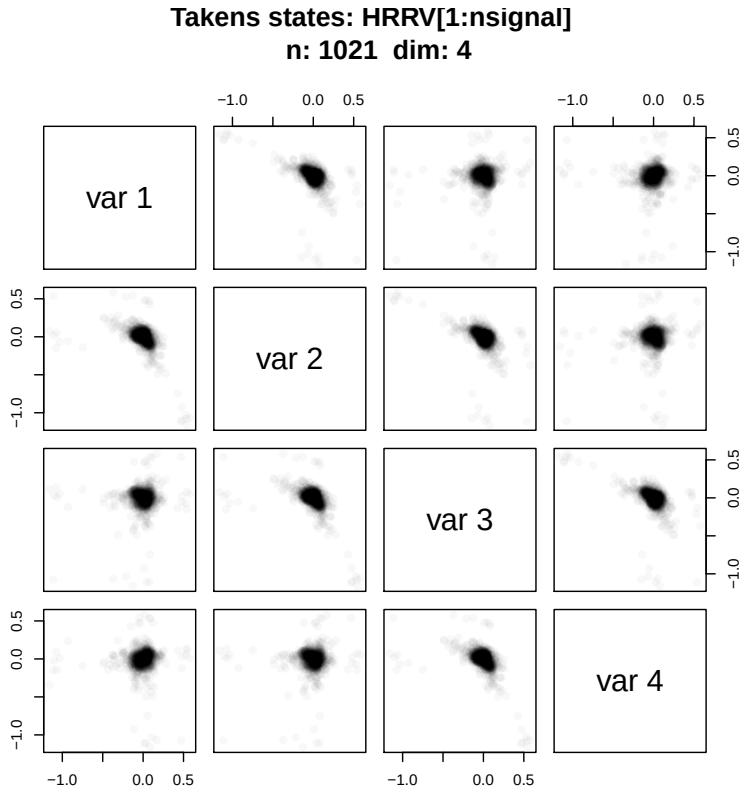

FIGURE 73. RHRV tutorial example.beats. HRRV Time used: 0.736 sec.

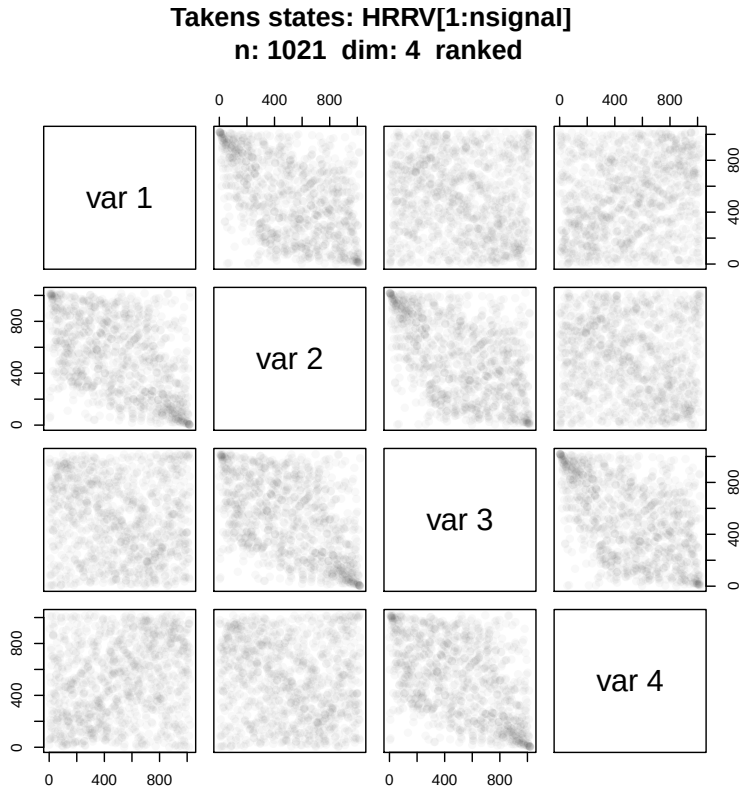

FIGURE 74. RHRV tutorial example.beats. Ranked HRRV data. Time used: 1.461 sec.

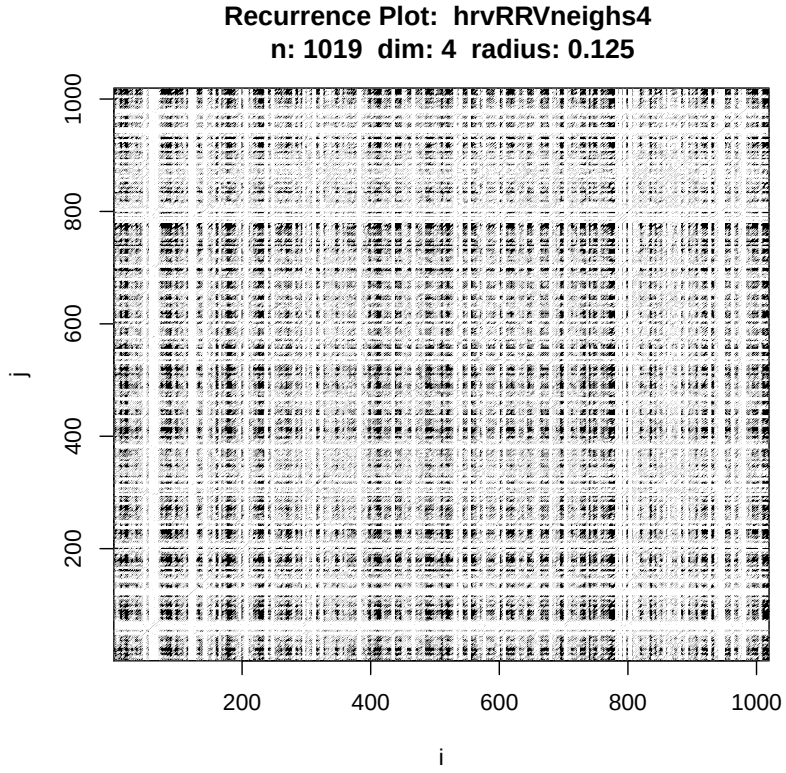

FIGURE 75. Recurrence Plot. Example case: RHRV tutorial example.beats. HRRV Dim=4. Time used: 2.362 sec.

We should expect the breathing rhythm, so a time lag in the order of 10 is to be expected.

ToDo: fix default setting for radius. Eckmann uses nearest neighbours with NN=10

7.1.1. *RHRV: example.beats - RR Variation: Comparison by Dimension.*

Input

```
hrvRRVtakens2 <- local.buildTakens( time.series=HRRV[1:nsignal],
        embedding.dim=2, time.lag=1)
hrvRRVneighs2  <- local.findAllNeighbours(hrvRRVtakens2[-(1:2),],
        radius=0.125)
save(hrvRRVneighs2, file="hrvRRVneighs2.Rdata")
# load(file="hrvRRVneighs2.RData")
local.recurrencePlotAux(hrvRRVneighs2, dim=2,  radius=0.125)
showrqa(hrvRRVtakens2[-(1:2),], radius=0.125, do.hist=FALSE)
```

Output

```
hrvRRVtakens2[-(1:2), ]  n: 1021  Dim: 2
Radius: 0.125  Recurrence coverage REC: 0.515  log(REC)/log(R): 0.319
Determinism: 0.939  Laminarity: 0.81
DIV: 0.027
Trend: 0  Entropy: 2.346
Diagonal lines max: 37  Mean: 5.373  Mean off main: 5.362
Vertical lines max: 48  Mean: 3.998
```

Dim=2. Time used: 4.518 sec.

Input

```
hrvRRVtakens6 <- local.buildTakens( time.series=HRRV[1:nsignal],
        embedding.dim=6,time.lag=1)
hrvRRVneighs6  <- local.findAllNeighbours(hrvRRVtakens6[-(1:2),], radius=0.125)
save(hrvRRVneighs6, file="hrvRRVneighs6.Rdata")
# load(file="hrvRRVneighs6.RData")
local.recurrencePlotAux(hrvRRVneighs6, dim=6, radius=0.125)
showrqa(hrvRRVtakens6[-(1:2),], radius=0.125, do.hist=FALSE)
```

Output

```
hrvRRVtakens6[-(1:2), ]  n: 1017  Dim: 6
Radius: 0.125  Recurrence coverage REC: 0.179  log(REC)/log(R): 0.827
Determinism: 0.943  Laminarity: 0.451
DIV: 0.03
Trend: 0  Entropy: 2.305
Diagonal lines max: 33  Mean: 5.243  Mean off main: 5.213
Vertical lines max: 22  Mean: 2.887
```

Dim=6. Time used: 2.538 sec.

Input

```
hrvRRVtakens8 <- local.buildTakens( time.series=HRRV[1:nsignal],
        embedding.dim=8,time.lag=1)
hrvRRVneighs8  <- local.findAllNeighbours(hrvRRVtakens8[-(1:2),], radius=0.125)
save(hrvRRVneighs8, file="hrvRRVneighs8.Rdata")
# load(file="hrvRRVneighs8.RData")
local.recurrencePlotAux(hrvRRVneighs8, dim=8, radius=0.125)
showrqa(hrvRRVtakens8[-(1:2),], radius=0.125, do.hist=FALSE)
```

Output

```
hrvRRVtakens8[-(1:2), ]  n: 1015  Dim: 8
Radius: 0.125  Recurrence coverage REC: 0.105  log(REC)/log(R): 1.084
Determinism: 0.943  Laminarity: 0.337
DIV: 0.032
Trend: 0  Entropy: 2.329
Diagonal lines max: 31  Mean: 5.361  Mean off main: 5.308
Vertical lines max: 17  Mean: 2.715
```

Dim=8. Time used: 1.925 sec.

Input

```
hrvRRVtakens12 <-
        local.buildTakens( time.series=HRRV[1:nsignal],
                embedding.dim=12,time.lag=1)
hrvRRVneighs12 <-
        local.findAllNeighbours(hrvRRVtakens12[-(1:2),], radius=3/16)
save(hrvRRVneighs12, file="hrvRRVneighs12.Rdata")
# load(file="hrvRRVneighs12.RData")
local.recurrencePlotAux(hrvRRVneighs12, dim=12,  radius=3/16)
showrqa(hrvRRVtakens12[-(1:2),], radius=3/16, do.hist=FALSE)
```

Output

```
hrvRRVtakens12[-(1:2), ]  n: 1011  Dim: 12
Radius: 0.1875  Recurrence coverage REC: 0.235  log(REC)/log(R): 0.865
Determinism: 0.988  Laminarity: 0.635
DIV: 0.015
Trend: 0  Entropy: 3.075
Diagonal lines max: 68  Mean: 9.775  Mean off main: 9.733
Vertical lines max: 52  Mean: 3.656
```

Dim=12: Time used: 2.967 sec.

Input

```
hrvRRVtakens16 <- local.buildTakens( time.series=HRRV[1:nsignal],
        embedding.dim=16,time.lag=1)
hrvRRVneighs16  <- local.findAllNeighbours(hrvRRVtakens16[-(1:2),], radius=3/16)
save(hrvRRVneighs16, file="hrvRRVneighs16.Rdata")
# load(file="hrvRRVneighs16.RData")
local.recurrencePlotAux(hrvRRVneighs16, dim=16, radius=3/16)
showrqa(hrvRRVtakens16[-(1:2),], radius=3/16, do.hist=FALSE)
```

Output

```
hrvRRVtakens16[-(1:2), ]  n: 1007  Dim: 16
Radius: 0.1875  Recurrence coverage REC: 0.147  log(REC)/log(R): 1.144
Determinism: 0.99  Laminarity: 0.527
DIV: 0.016
Trend: 0  Entropy: 3.163
Diagonal lines max: 64  Mean: 10.594  Mean off main: 10.523
Vertical lines max: 40  Mean: 3.114
```

Dim=16. Time used: 2.404 sec.

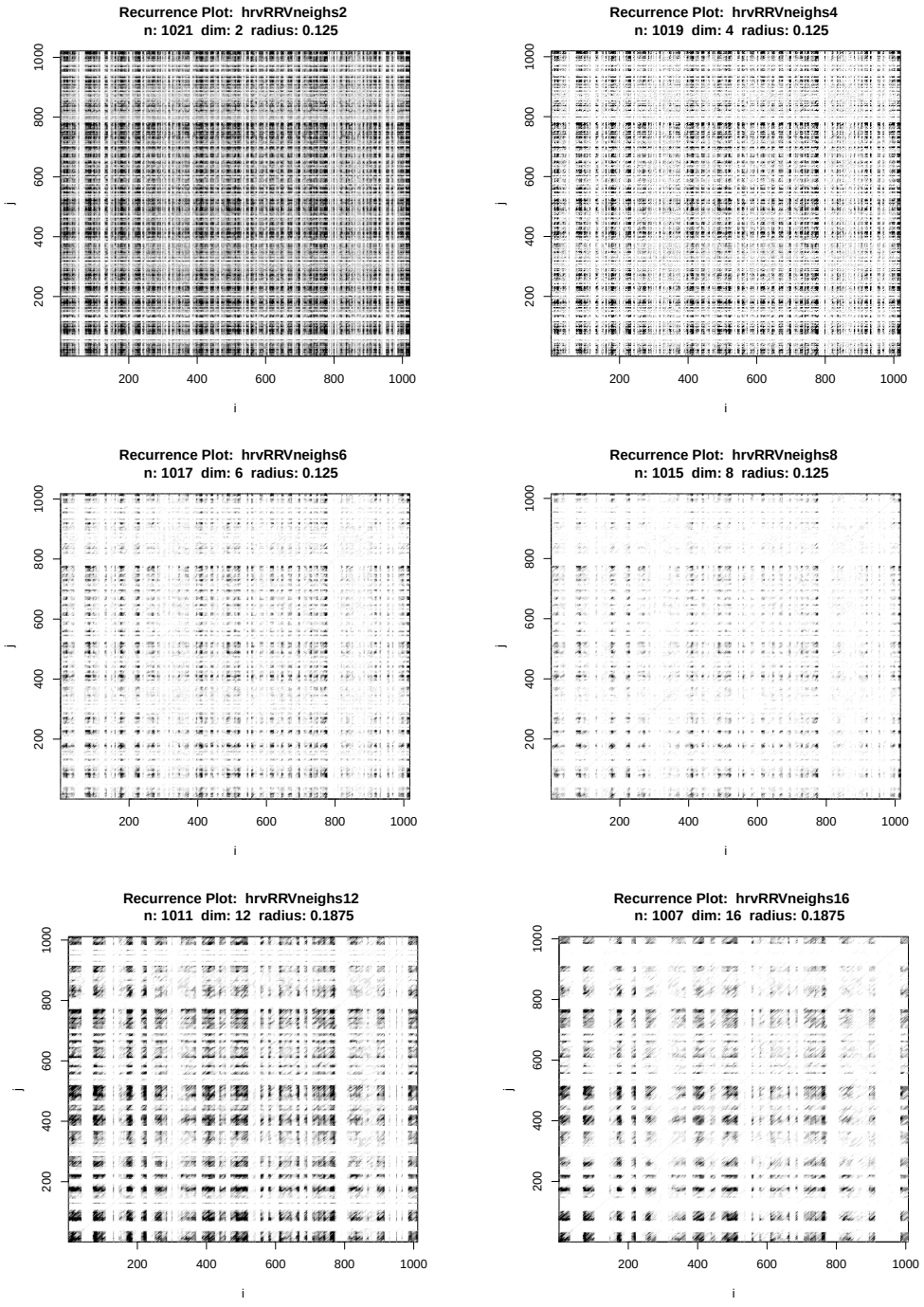

FIGURE 76. Recurrence Plot. Example case: RHRV tutorial example.beats. Dim=2, 4, 6, 8, 12, 16. Time used: 2.405 sec.

8. Case Study: HRV data example2.beats

This is a copy of the previous section, now applied to HRV data example2.beats.

Input

```
library(RHRV)
load("/users/gs/projects/rforge/rhrv/pkg/data/HRVData.rda")
load("/users/gs/projects/rforge/rhrv/pkg/data/HRVProcessedData.rda")
####################################################
### code chunk number 1: creation
####################################################
hrv2.data  = CreateHRVData()
hrv2.data = SetVerbose(hrv2.data, TRUE )
####################################################
### code chunk number 3: loading
####################################################
hrv2.data = LoadBeatAscii(hrv2.data, "example2.beats",
       RecordPath = "/users/gs/projects/rforge/rhrv/tutorial/beatsFolder")
```

Output

```
** Loading beats positions for record: example2.beats **
   Path: /users/gs/projects/rforge/rhrv/tutorial/beatsFolder
   Scale: 1
   Removed 2437 duplicated beats
   Date: 01/01/1900
   Time: 00:00:00
   Number of beats: 2437
```

Input

```
#       RecordPath = "beatsFolder")


####################################################
### code chunk number 4: derivating
####################################################
hrv2.data = BuildNIHR(hrv2.data)
```

Output

```
** Calculating non-interpolated heart rate **
   Number of beats: 2437
```

Input

Input

```
plotsignal(hrv2.data$Beat$RR)
```

ToDo: We have outliers at approximately 2*RR. Could this be an artefact of preprocessing, filtering out too many impulses?

See Figure 77 on the next page.

Input

```
hrv2RRtakens4 <- local.buildTakens( time.series=hrv2.data$Beat$RR[1:nsignal],
        embedding.dim=4, time.lag=1)
statepairs(hrv2RRtakens4) #dim=4
```

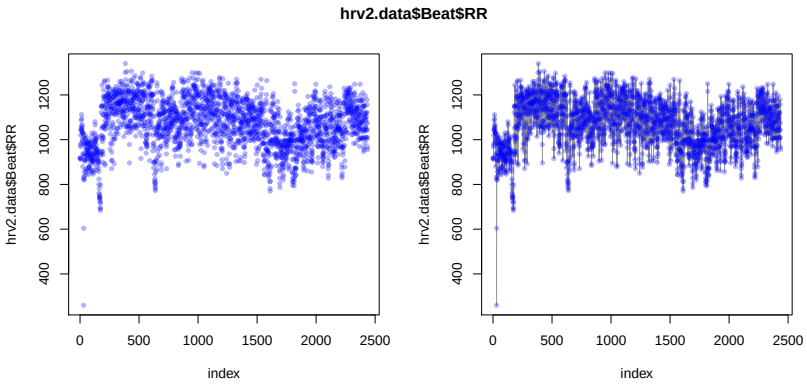

FIGURE 77. RHRV tutorial example2.beats. Signal and linear interpolation.

See Figure 78.

Only 1024 data points used in this plot

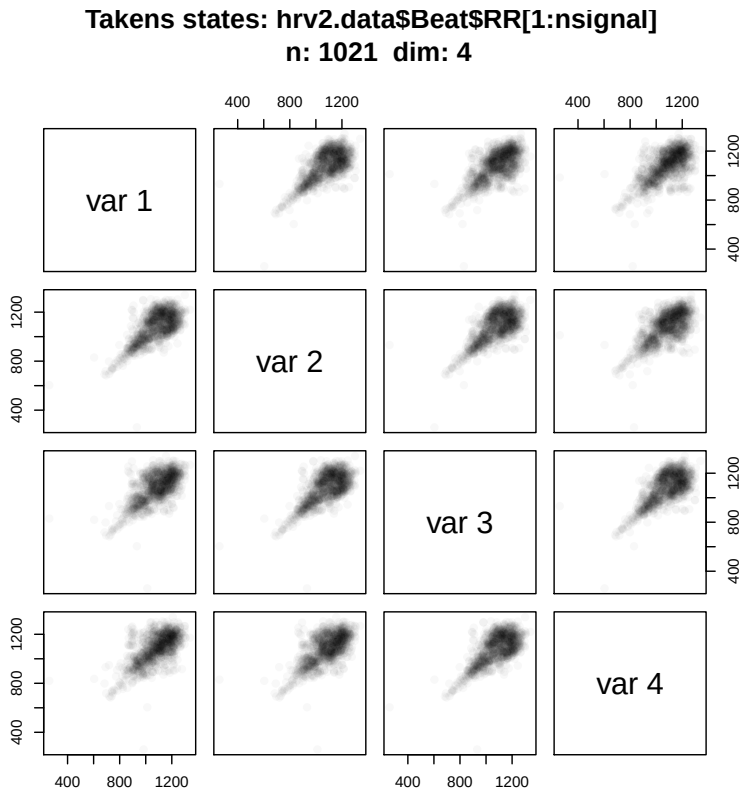

FIGURE 78. RHRV tutorial example2.beats. Time used: 0.68 sec.

Input

```
statepairs(hrv2RRtakens4, rank=TRUE) #dim=4
```

See Figure 79 on the next page.

Input

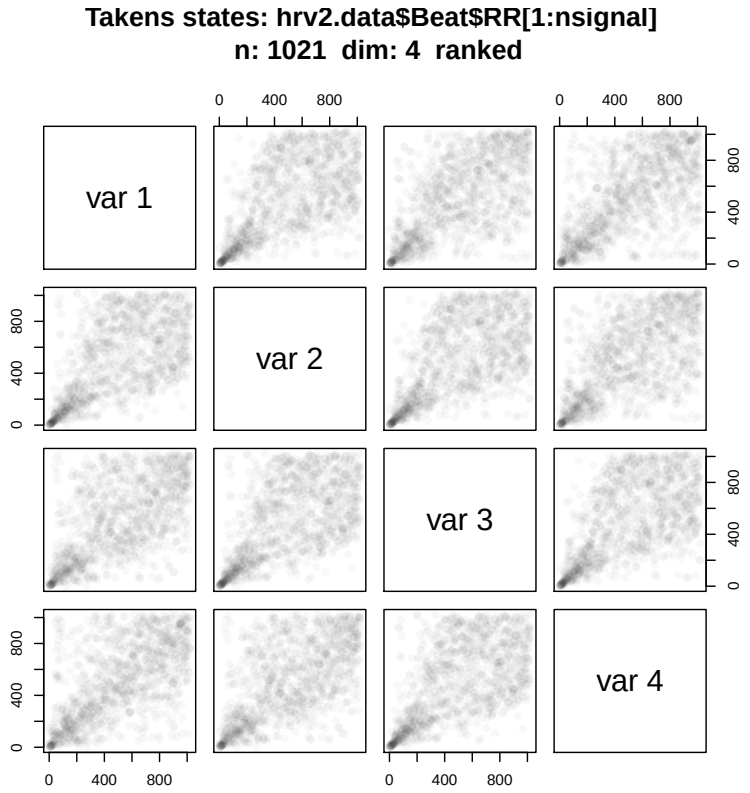

FIGURE 79. RHRV tutorial example2.beats. Ranked data. Time used: 1.376 sec.

```
statecoplot(hrv2RRtakens4) #dim=4
```

See Figure 80 on the facing page.

Input

```
hrv2RRneighs4  <- local.findAllNeighbours(hrv2RRtakens4, radius=12*16)
save(hrv2RRneighs4, file="hrv2RRneighs4.Rdata")
# load(file="hrv2RRneighs4.RData")
local.recurrencePlotAux(hrv2RRneighs4, radius=12*16)
showrqa(hrv2RRtakens4[-(1:2),], radius=12*16, do.hist=FALSE)
```

Output

```
hrv2RRtakens4[-(1:2), ]  n: 1019  Dim: 4
Radius: 192  Recurrence coverage REC: 0.493  log(REC)/log(R): -0.135
Determinism: 0.982  Laminarity: 0.906
DIV: 0.005
Trend: 0  Entropy: 3.158
Diagonal lines max: 191  Mean: 10.396  Mean off main: 10.375
Vertical lines max: 136  Mean: 7.145
```

Dim=4. Time used: 4.27 sec.

See Figure 81 on page 78.

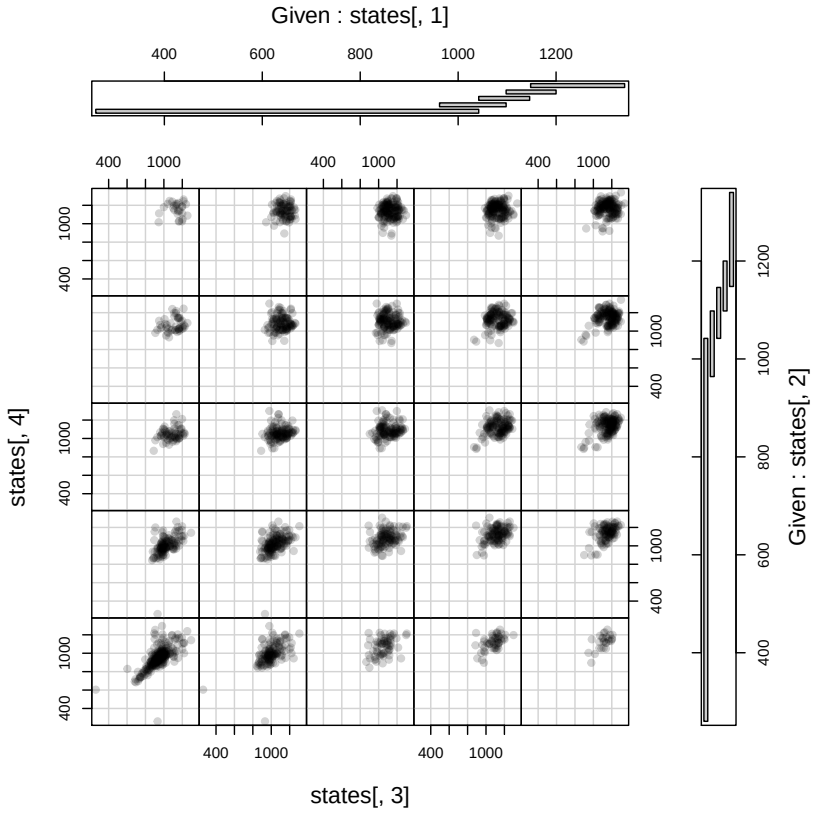

FIGURE 80. State coplot. RHRV tutorial example2.beats. Time used: 0.259 sec.

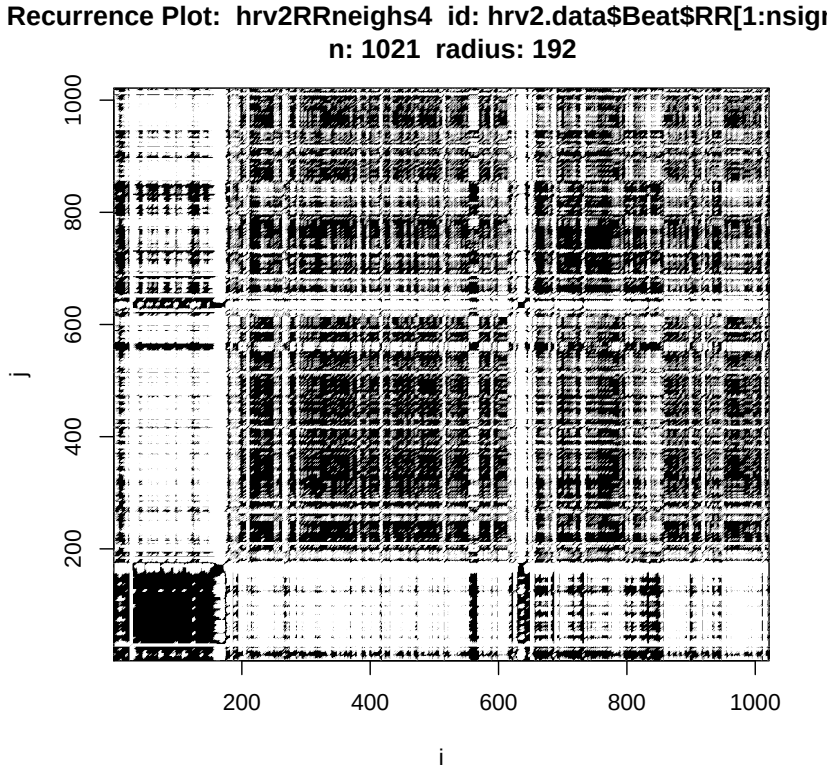

FIGURE 81. Recurrence Plot. Example case: RHRV tutorial example2.beats. Dim=4. Time used: 4.27 sec.

We should expect the breathing rhythm, so a time lag in the order of 10 is to be expected.

8.0.2. *RHRV: example2.beats, RR-intervals. Comparison by Dimension.*

Input

```
hrv2RRtakens2 <- local.buildTakens( time.series=hrv2.data$Beat$RR[1:nsignal],
        embedding.dim=2,time.lag=1)
hrv2RRneighs2  <- local.findAllNeighbours(hrv2RRtakens2, radius=10*16)
save(hrv2RRneighs2, file="hrv2RRneighs2.Rdata")
# load(file="hrv2RRneighs2.RData")
local.recurrencePlotAux(hrv2RRneighs2, radius=10*16)
showrqa(hrv2RRtakens2[-(1:2),], radius=10*16, do.hist=FALSE)
```

Output

```
hrv2RRtakens2[-(1:2), ]  n: 1021  Dim: 2
Radius: 160  Recurrence coverage REC: 0.52  log(REC)/log(R): -0.129
Determinism: 0.948  Laminarity: 0.91
DIV: 0.007
Trend: 0  Entropy: 2.691
Diagonal lines max: 152  Mean: 7.057  Mean off main: 7.043
Vertical lines max: 130  Mean: 6.607
```

Dim=2. Time used: 4.262 sec.

See Figure 82 on page 81.

Input

```
hrv2RRtakens6 <- local.buildTakens( time.series=hrv2.data$Beat$RR[1:nsignal],
        embedding.dim=6,time.lag=1)
hrv2RRneighs6  <- local.findAllNeighbours(hrv2RRtakens6, radius=14*16)
save(hrv2RRneighs6, file="hrv2RRneighs6.Rdata")
# load(file="hrv2RRneighs6.RData")
local.recurrencePlotAux(hrv2RRneighs6, radius=14*16)
showrqa(hrv2RRtakens6[-(1:2),], radius=14*16, do.hist=FALSE)
```

Output

```
hrv2RRtakens6[-(1:2), ]  n: 1017  Dim: 6
Radius: 224  Recurrence coverage REC: 0.528  log(REC)/log(R): -0.118
Determinism: 0.993  Laminarity: 0.925
DIV: 0.004
Trend: 0  Entropy: 3.584
Diagonal lines max: 225  Mean: 15.258  Mean off main: 15.23
Vertical lines max: 167  Mean: 9.007
```

Dim=6. Time used: 4.586 sec.

See Figure 82 on page 81.

Input

```
hrv2RRtakens8 <- local.buildTakens( time.series=hrv2.data$Beat$RR[1:nsignal],
        embedding.dim=8,time.lag=1)
hrv2RRneighs8  <- local.findAllNeighbours(hrv2RRtakens8, radius=16*16)
save(hrv2RRneighs8, file="hrv2RRneighs8.Rdata")
# load(file="hrv2RRneighs8.RData")
local.recurrencePlotAux(hrv2RRneighs8, radius=16*16)
showrqa(hrv2RRtakens8[-(1:2),], radius=16*16, do.hist=FALSE)
```

Output

```
hrv2RRtakens8[-(1:2), ]  n: 1015  Dim: 8
Radius: 256  Recurrence coverage REC: 0.584  log(REC)/log(R): -0.097
Determinism: 0.997  Laminarity: 0.942
DIV: 0.004
Trend: 0  Entropy: 3.986
Diagonal lines max: 241  Mean: 22.506  Mean off main: 22.469
Vertical lines max: 310  Mean: 11.256
```

Dim=8. Time used: 4.922 sec.

See Figure 82 on the facing page.

Input

```
hrv2RRtakens12 <- local.buildTakens( time.series=hrv2.data$Beat$RR[1:nsignal],
        embedding.dim=12,time.lag=1)
hrv2RRneighs12  <- local.findAllNeighbours(hrv2RRtakens12, radius=16*16)
save(hrv2RRneighs12, file="hrv2RRneighs12.Rdata")
# load(file="hrv2RRneighs12.RData")
local.recurrencePlotAux(hrv2RRneighs12, radius=16*16)
showrqa(hrv2RRtakens12[-(1:2),], radius=16*16, do.hist=FALSE)
```

Output

```
hrv2RRtakens12[-(1:2), ]  n: 1011  Dim: 12
Radius: 256  Recurrence coverage REC: 0.488  log(REC)/log(R): -0.129
Determinism: 0.997  Laminarity: 0.914
DIV: 0.004
Trend: 0  Entropy: 4.062
Diagonal lines max: 237  Mean: 24.127  Mean off main: 24.079
Vertical lines max: 299  Mean: 8.972
```

Dim=12. Time used: 4.523 sec.

Input

```
hrv2RRtakens16 <- local.buildTakens(
        time.series=hrv2.data$Beat$RR[1:nsignal],
        embedding.dim=16,time.lag=1)
hrv2RRneighs16  <- local.findAllNeighbours(hrv2RRtakens16, radius=18*16)
save(hrv2RRneighs16, file="hrv2RRneighs16.Rdata")
# load(file="hrv2RRneighs16.RData")
local.recurrencePlotAux(hrv2RRneighs16, radius=18*16)
showrqa(hrv2RRtakens16[-(1:2),], radius=18*16, do.hist=FALSE)
```

Output

```
hrv2RRtakens16[-(1:2), ]  n: 1007  Dim: 16
Radius: 288  Recurrence coverage REC: 0.544  log(REC)/log(R): -0.107
Determinism: 0.999  Laminarity: 0.932
DIV: 0.003
Trend: 0  Entropy: 4.372
Diagonal lines max: 346  Mean: 34.916  Mean off main: 34.854
Vertical lines max: 297  Mean: 9.929
```

Dim=16. Time used: 4.925 sec.

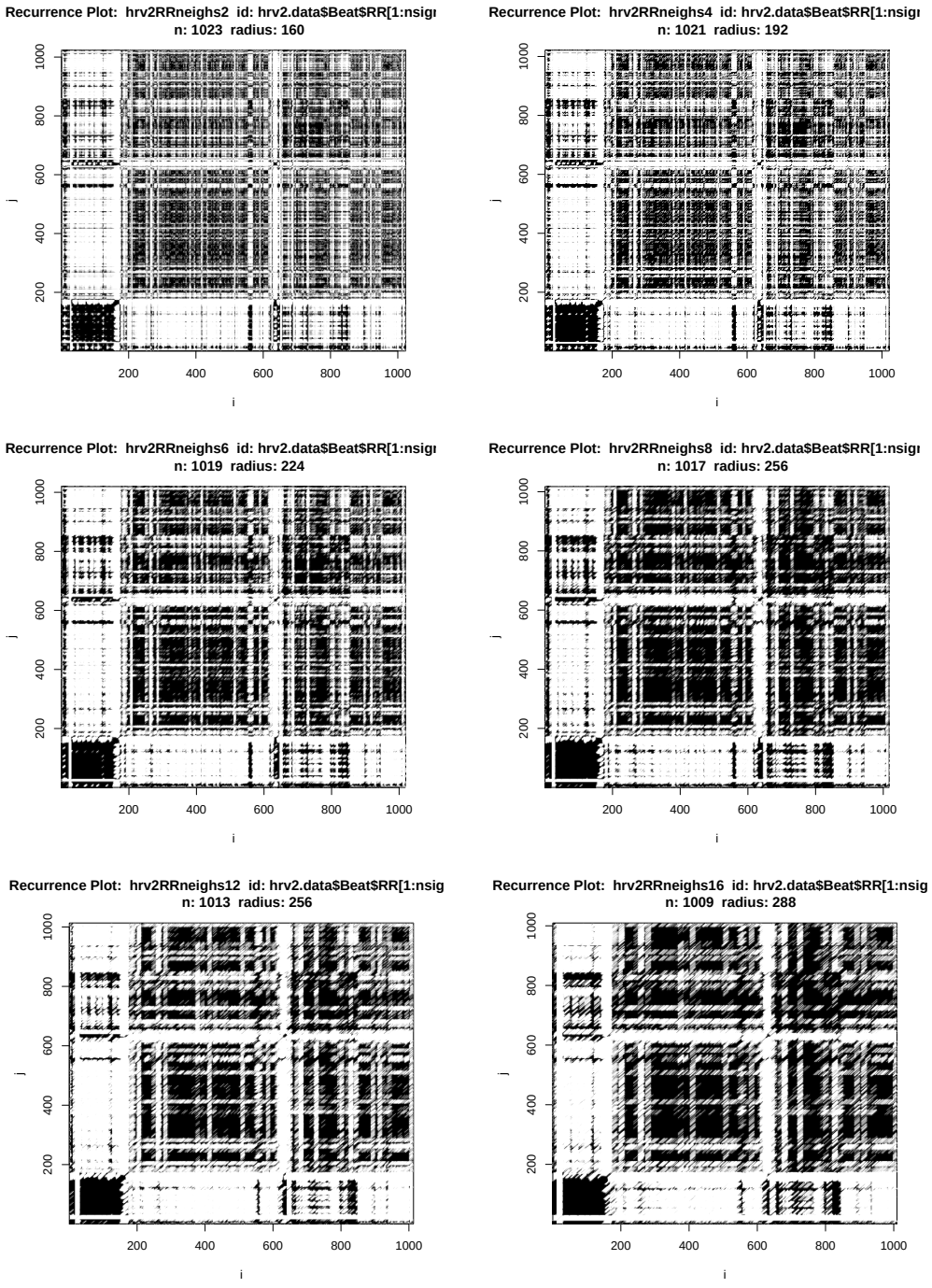

FIGURE 82. Recurrence Plot. Example case: RHRV tutorial example2.beats. Dim=2, 4, 6, 8, 12, 16. Time used: 4.926 sec.

ToDo: Consider using differences

differences for HRV

8.1. **RHRV: example2.beats - Hart Rate Variation.** Since we are not interested in heart rate (or pulse), but in heart rate variation, a proposal is to use scaled differences.

Input

```
hrv2.data <- BuildNIDHR(hrv2.data)
```

Output

```
** Calculating non-interpolated heart rate differences **
   Number of beats: 2437
```

Input

```
HRRV <- hrv2.data$Beat$HRRV
```

These are the displays of the Takens state space we used before, now for HRRV:

Input

```
plotsignal(HRRV)
```

See Figure 83,

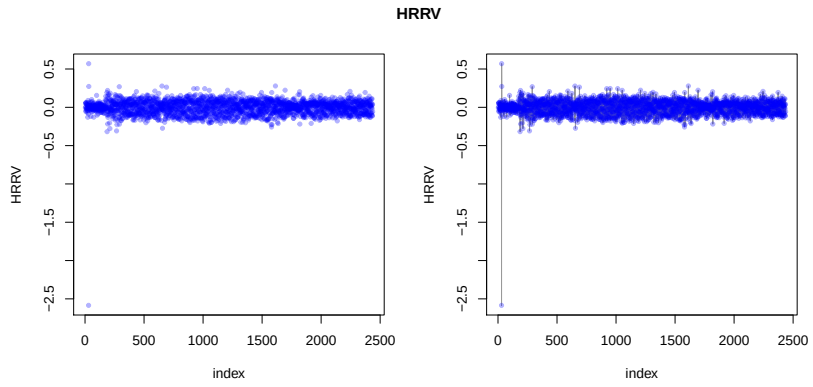

FIGURE 83. RHRV tutorial example2.beats. HRRV Signal and linear interpolation.

Only 1024 data points used in these plots

Input

```
hrv2RRVtakens4 <-
        local.buildTakens( time.series=HRRV[1:nsignal],
                embedding.dim=4,time.lag=1)
statepairs(hrv2RRVtakens4) #dim=4
```

See Figure 84 on the facing page.

Input

```
        statecoplot(hrv2RRVtakens4) #dim=4
```

Output

```
Missing rows: 1, 2
```

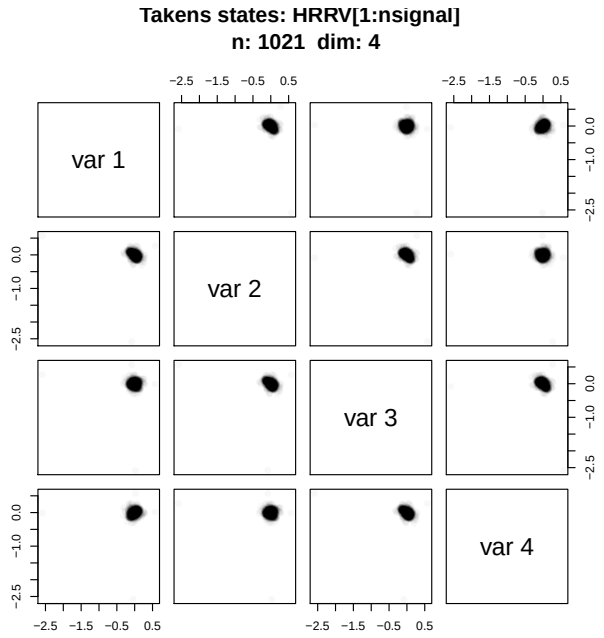

FIGURE 84. Recurrence plot. RHRV tutorial example2.beats. HRRV Time used: 0.7 sec.

See Figure 85 on the next page.

Input

```
statepairs(hrv2RRVtakens4, rank=TRUE) #dim=4
```

See Figure 86 on page 85

Input

```
hrv2RRVtakens4l <- hrv2RRVtakens4
hrv2RRVtakens4l[hrv2RRVtakens4l < -1.5] <- NA
hrv2RRVtakens4l[hrv2RRVtakens4l > 0.45] <- NA
statepairs(hrv2RRVtakens4l) #dim=4
```

See Figure 87 on page 85.

Input

```
statecoplot(hrv2RRVtakens4l) #dim=4
```

Output

```
Missing rows: 1, 2, 27, 28, 29, 30, 31
```

See Figure 88 on page 86.

ToDo: findAllNeighbours does not handle NAs

Input

```
#use hack: findAllNeighbours does not handle NAs
hrv2RRVneighs4  <- local.findAllNeighbours(hrv2RRVtakens4[-(1:2),], radius=0.125)
save(hrv2RRVneighs4, file="hrv2RRVneighs4.Rdata")
```

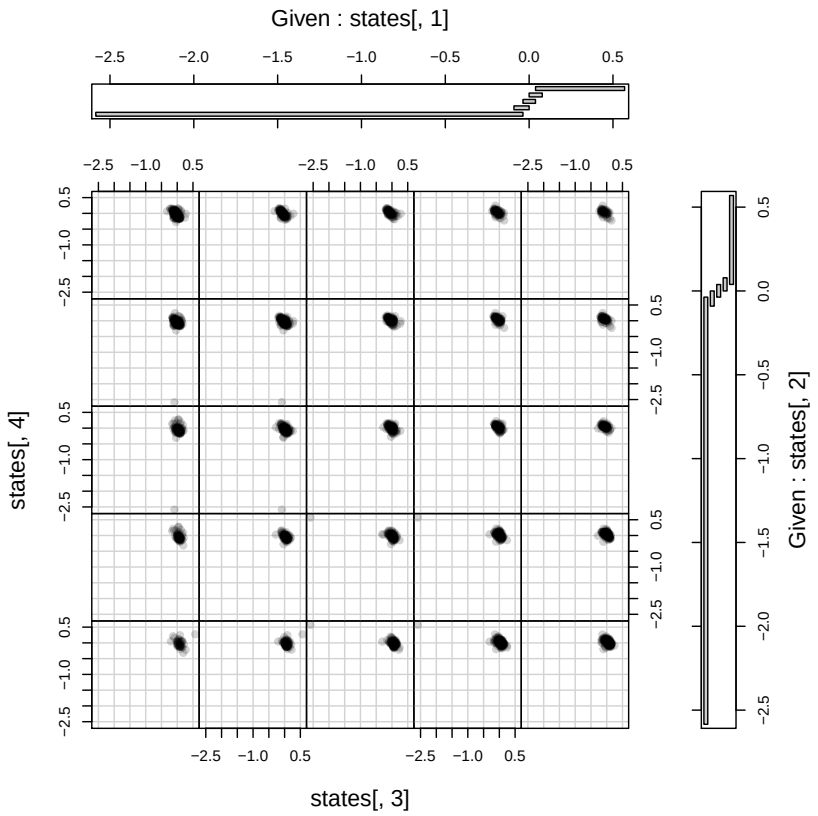

FIGURE 85. State coplot. RHRV tutorial example2.beats. HRRV. Time used: 1.018 sec.

Time used: 0.222 sec.

Input

```
load(file="hrv2RRVneighs4.RData")
local.recurrencePlotAux(hrv2RRVneighs4,dim=4, radius=0.125)
```

ToDo: check. There seem to be strange artefacts.

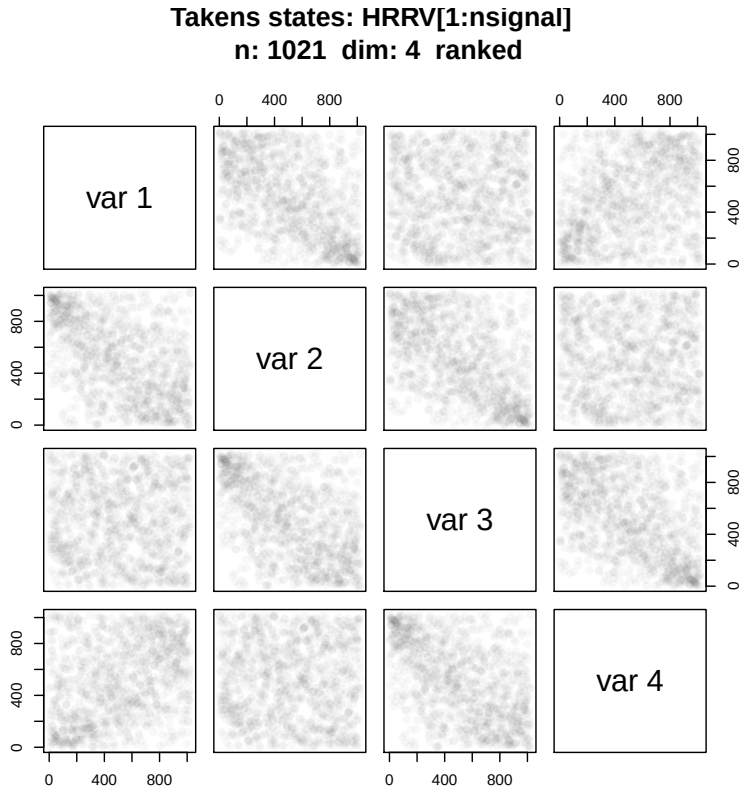

FIGURE 86. RHRV tutorial example2.beats. Ranked HRRV data. Time used: 1.684 sec.

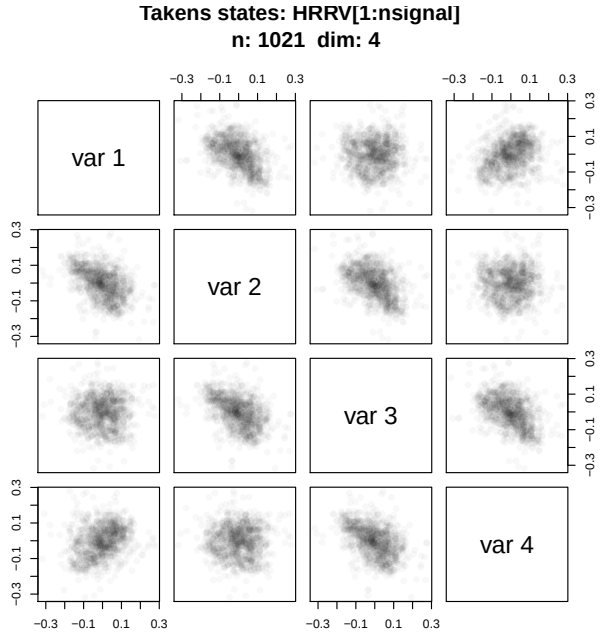

FIGURE 87. Recurrence plot. RHRV tutorial example2.beats. HRRV Time used: 2.391 sec.

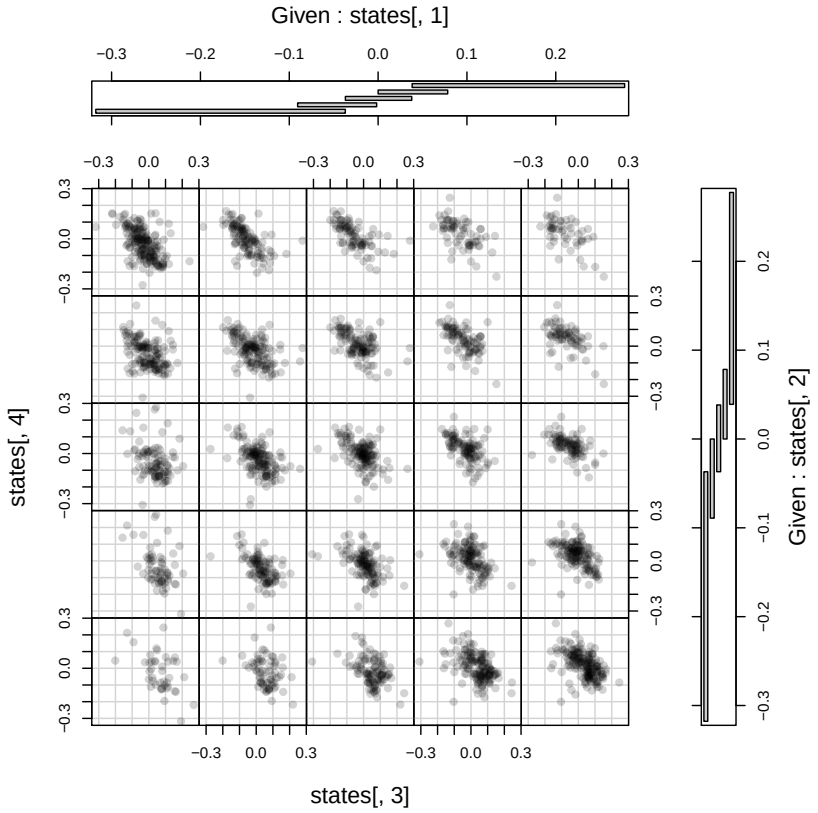

FIGURE 88. State coplot. RHRV tutorial example2.beats. HRRV. Time used: 2.706 sec.

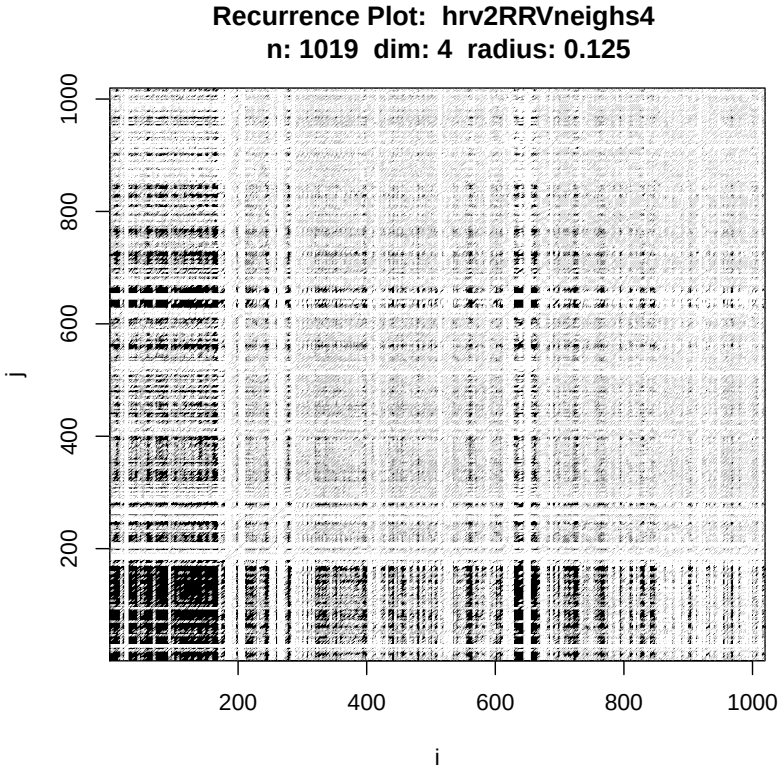

FIGURE 89. Recurrence Plot. Example case: RHRV tutorial example2.beats. HRRV Dim=4. Time used: 2.445 sec.

We should expect the breathing rhythm, so a time lag in the order of 10 beats is to be expected for the base signal.

ToDo: fix default setting for radius. Eckmann uses nearest neighbours with NN=10

8.1.1. *RHRV: example2.beats - RR Variation: Comparison by Dimension.*

Input

```
hrv2RRVtakens2 <- local.buildTakens( time.series=HRRV[1:nsignal],
        embedding.dim=2, time.lag=1)
hrv2RRVneighs2  <- local.findAllNeighbours(hrv2RRVtakens2[-(1:2),],
        radius=0.125)
save(hrv2RRVneighs2, file="hrv2RRVneighs2.Rdata")
# load(file="hrv2RRVneighs2.RData")
local.recurrencePlotAux(hrv2RRVneighs2, dim=2,  radius=0.125)
hrv2RRVrqa2 <- showrqa(hrv2RRVtakens2[-(1:2),], radius=0.125, do.hist=FALSE)
```

Output

```
hrv2RRVtakens2[-(1:2), ]  n: 1021  Dim: 2
Radius: 0.125  Recurrence coverage REC: 0.508  log(REC)/log(R): 0.326
Determinism: 0.913  Laminarity: 0.712
DIV: 0.009
Trend: 0  Entropy: 2.289
Diagonal lines max: 117  Mean: 5.305  Mean off main: 5.294
Vertical lines max: 86  Mean: 3.923
```

Time used: 4.865 sec.

Input

```
hrv2RRVtakens6 <- local.buildTakens( time.series=HRRV[1:nsignal],
        embedding.dim=6,time.lag=1)
hrv2RRVneighs6  <- local.findAllNeighbours(hrv2RRVtakens6[-(1:2),], radius=0.125*1.5)
save(hrv2RRVneighs6, file="hrv2RRVneighs6.Rdata")
# load(file="hrv2RRVneighs6.RData")
local.recurrencePlotAux(hrv2RRVneighs6, dim=6, radius=0.125*1.5)
hrv2RRVrqa6 <- showrqa(hrv2RRVtakens6[-(1:2),], radius=0.125*1.5, do.hist=FALSE)
```

Output

```
hrv2RRVtakens6[-(1:2), ]  n: 1017  Dim: 6
Radius: 0.1875  Recurrence coverage REC: 0.516  log(REC)/log(R): 0.395
Determinism: 0.992  Laminarity: 0.736
DIV: 0.007
Trend: 0  Entropy: 3.355
Diagonal lines max: 147  Mean: 12.475  Mean off main: 12.452
Vertical lines max: 193  Mean: 4.902
```

Dim=6. Time used: 4.489 sec.

Input

```
hrv2RRVtakens8 <- local.buildTakens( time.series=HRRV[1:nsignal],
        embedding.dim=8,time.lag=1)
hrv2RRVneighs8  <- local.findAllNeighbours(hrv2RRVtakens8[-(1:2),], radius=3/16)
save(hrv2RRVneighs8, file="hrv2RRVneighs8.Rdata")
# load(file="hrv2RRVneighs8.RData")
local.recurrencePlotAux(hrv2RRVneighs8, dim=8, radius=3/16)
hrv2RRVrqa8 <- showrqa(hrv2RRVtakens8[-(1:2),], radius=3/16, do.hist=FALSE)
```

Output

```
hrv2RRVtakens8[-(1:2), ]  n: 1015  Dim: 8
Radius: 0.1875  Recurrence coverage REC: 0.432  log(REC)/log(R): 0.501
Determinism: 0.992  Laminarity: 0.679
DIV: 0.007
Trend: 0  Entropy: 3.369
Diagonal lines max: 145  Mean: 12.714  Mean off main: 12.685
Vertical lines max: 147  Mean: 4.628
```

Dim=8. Time used: 4.307 sec.

Input

```
hrv2RRVtakens12 <-
        local.buildTakens( time.series=HRRV[1:nsignal],
                embedding.dim=12,time.lag=1)
hrv2RRVneighs12 <-
        local.findAllNeighbours(hrv2RRVtakens12[-(1:2),], radius=3.5/16)
save(hrv2RRVneighs12, file="hrv2RRVneighs12.Rdata")
# load(file="hrv2RRVneighs12.RData")
local.recurrencePlotAux(hrv2RRVneighs12, dim=12,  radius=3.5/16)
hrv2RRVrqa12 <- showrqa(hrv2RRVtakens12[-(1:2),], radius=3.5/16, do.hist=FALSE)
```

Output

```
hrv2RRVtakens12[-(1:2), ]  n: 1011  Dim: 12
Radius: 0.21875  Recurrence coverage REC: 0.479  log(REC)/log(R): 0.484
Determinism: 0.997  Laminarity: 0.746
DIV: 0.006
Trend: 0  Entropy: 3.803
Diagonal lines max: 156  Mean: 19.62  Mean off main: 19.58
Vertical lines max: 192  Mean: 5.298
```

Dim=12. Time used: 4.455 sec.

Input

```
hrv2RRVtakens16 <- local.buildTakens( time.series=HRRV[1:nsignal],
        embedding.dim=16,time.lag=1)
hrv2RRVneighs16  <- local.findAllNeighbours(hrv2RRVtakens16[-(1:2),], radius=4/16)
save(hrv2RRVneighs16, file="hrv2RRVneighs16.Rdata")
# load(file="hrv2RRVneighs16.RData")
local.recurrencePlotAux(hrv2RRVneighs16, dim=16, radius=4/16)
hrv2RRVrqa16 <- showrqa(hrv2RRVtakens16[-(1:2),], radius=4/16, do.hist=FALSE)
```

Output

```
hrv2RRVtakens16[-(1:2), ]  n: 1007  Dim: 16
Radius: 0.25  Recurrence coverage REC: 0.568  log(REC)/log(R): 0.408
Determinism: 0.999  Laminarity: 0.816
DIV: 0.005
Trend: 0  Entropy: 4.186
Diagonal lines max: 221  Mean: 30.105  Mean off main: 30.054
Vertical lines max: 220  Mean: 6.145
```

Dim=16. Time used: 5.219 sec.

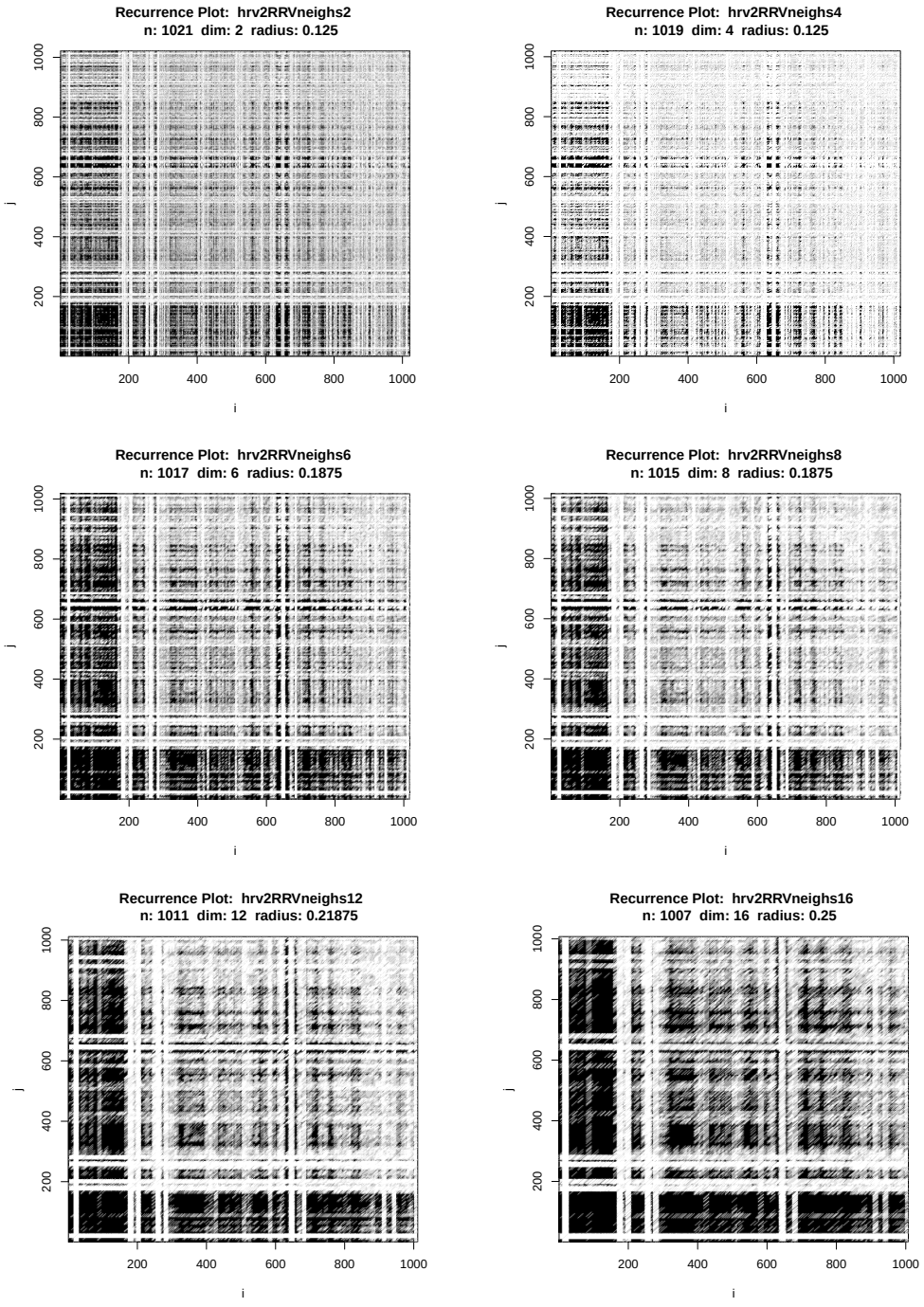

FIGURE 90. Recurrence Plot. Example case: RHRV tutorial example2.beats. Dim=2, 4, 6, 8, 12, 16. Time used: 5.219 sec.

References

Eckmann JP, Kamphorst SO, Ruelle D (1987). "Recurrence plots of dynamical systems." *Europhys. Lett*, **4**(9), 973–977.

Takens F (1981). "Detecting strange attractors in turbulence." In *Dynamical systems and turbulence, Warwick 1980*, pp. 366–381. Springer.

Webber Jr CL, Zbilut JP (2005). "Recurrence quantification analysis of nonlinear dynamical systems." *Tutorials in contemporary nonlinear methods for the behavioral sciences*, pp. 26–94.

Zbilut JP, Webber CL (2006). "Recurrence quantification analysis." *Wiley encyclopedia of biomedical engineering*. URL `http://onlinelibrary.wiley.com/doi/10.1002/9780471740360.ebs1355/full`.

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R session info:

Total Sweave time used: 128.874 sec. at Thu Aug 21 18:14:15 2014.

- R version 3.1.1 (2014-07-10), `x86_64-apple-darwin13.1.0`
- Locale: `en_GB.UTF-8/en_GB.UTF-8/en_GB.UTF-8/C/en_GB.UTF-8/en_GB.UTF-8`
- Base packages: base, datasets, graphics, grDevices, methods, stats, tcltk, utils
- Other packages: leaps 2.9, locfit 1.5-9.1, MASS 7.3-33, Matrix 1.1-4, mgcv 1.8-1, nlme 3.1-117, nonlinearTseries 0.2.1, rgl 0.93.1098, RHRV 4.0, sintro 0.1-3, tkrplot 0.0-23, TSA 1.01, tseries 0.10-32, waveslim 1.7.3
- Loaded via a namespace (and not attached): grid 3.1.1, lattice 0.20-29, quadprog 1.5-5, tools 3.1.1, zoo 1.7-11

LaTeX information:

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Bibliography style: jss

CVS/Svn repository information:

`$Source: /u/math/j40/cvsroot/lectures/src/dataanalysis/Rnw/recurrence.Rnw,v $`

copied to r-forge

`$HeadURL: svn+ssh://gsawitzki@scm.r-forge.r-project.org/svnroot/rhrv/gs/Rnw/recurrence.Rnw $`

`$Revision: 163 $`

`$Date: 2014-08-21 18:10:31 +0200 (Thu, 21 Aug 2014) $`

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