

THE TERM STRUCTURE OF INTEREST RATES

ROBERT J SHILLER*

Yale University

with an Appendix by

J HUSTON McCULLOCH

Ohio State University

Contents

1.	Introduction	629
2.	Simple analytics of the term structure: Discount bonds	630
3.	Fundamental concepts	633
3.1	Bonds Their definition	633
3.2	Interest rates Their definition	634
3.3	Par bonds	635
3.4	Instantaneous and perpetuity rates	636
3.5	Estimates of the term structure	636
3.6	Duration	637
3.7	Forward rates	639
3.8	Holding period rates	644
4.	Theories of the term structure	644
4.1	Expectations theories of the term structure	644
4.2	Risk preferences and the expectations hypothesis	645
4.3	Definitions of term premia	647
4.4	Early presumptions pertaining to the sign of the term premium	649
4.5	Risk preferences and term premia	651
5.	Empirical studies of the term structure	653
5.1	Empirical expectations hypotheses for the term structure	653
5.2	The rational expectations hypothesis in empirical work	654

*The author is indebted to John Campbell, Benjamin Friedman, Jonathan Ingersoll, Edward Kane, Stephen LeRoy, Jeffrey Miron, and J Huston McCulloch for helpful comments and discussions, and to Jeeman Jung, Sejin Kim, Plutarchos Sakellaris, and James Robinson for research assistance. Research was supported by the National Science Foundation.

The research reported here is part of the NBER's research program in Financial Markets and Monetary Economics. Any opinions expressed are those of the authors and not those of the National Bureau of Economic Research.

5.3. The volatility of long-term interest rates	658
5.4. Encouraging results for the rational expectations hypothesis	660
5.5. Interpreting departures from the expectations theory	664
5.6. Seasonality and interest rates	665
5.7. The sign of term premia	666
5.8. Modelling time-varying term premia	667
5.9. Flow of funds models	668
6. Some concluding observations	670
Appendix A: Mathematical symbols	671
Appendix B: U.S. term structure data, 1946–87 (by J. Huston McCulloch)	672
References	716

1. Introduction

The term of a debt instrument with a fixed maturity date is the time until the maturity date. The term structure of interest rates at any time is the function relating interest rate to term. Figure 13.1 shows the U.S. term structure of nominal interest rates according to one definition for each year since 1948. Usually the term structure is upward sloping; long-term interest rates are higher than short-term interest rates and the interest rate rises with term. Sometimes the term structure is downward sloping. Sometimes it is hump shaped, with intermediate terms having highest interest rates.

The study of the term structure inquires what market forces are responsible for the varying shapes of the term structure. In its purest form, this study considers only bonds for which we can disregard default risk (that interest or principal will not be paid by the issuer of the bond), convertibility provisions (an option to convert the bond to another financial instrument), call provisions (an option of the issuer to pay off the debt before the maturity date), floating rate provisions (provisions that change the interest payments according to some rule) or other special features.¹ Thus, the study of the term structure may be regarded as the study of the market price of time, over various intervals, itself.

What follows is an effort to consolidate and interpret the literature on the term structure as it stands today. The notation adopted is a little more complicated than usual, to allow diverse studies to be treated in a uniform notation. Definitions of rates of return, forward rates and holding returns for all time intervals are treated here in a uniform manner and their interrelations, exact or approximate, delineated. The concept of duration is used throughout to simplify mathematical expressions. Continuous compounding is used where possible to avoid arbitrary distinctions based on compounding assumptions. The relations described here can be applied approximately to conventionally defined interest rates or exactly to the continuously compounded McCulloch data in Appendix B. The McCulloch data, published here for the first time, are the cleanest interest rate data available in that they are based on a broad spectrum of government bond prices and are corrected for coupon and special tax effects.

Section 2 is a brief introduction to some key concepts in the simplest case, namely that of pure discount bonds. Section 3 sets forth the full definitions and concepts and their interrelations. Section 4 sets forth theories of the term

¹The U.S. government bonds used to produce Figure 13.1 are in some dimensions good approximations to such bonds: default risk must be considered very low, the bonds are not convertible, and there are no floating rate provisions. However, many long-term U.S. bonds are callable five years before maturity, and some bonds are given special treatment in estate tax law.

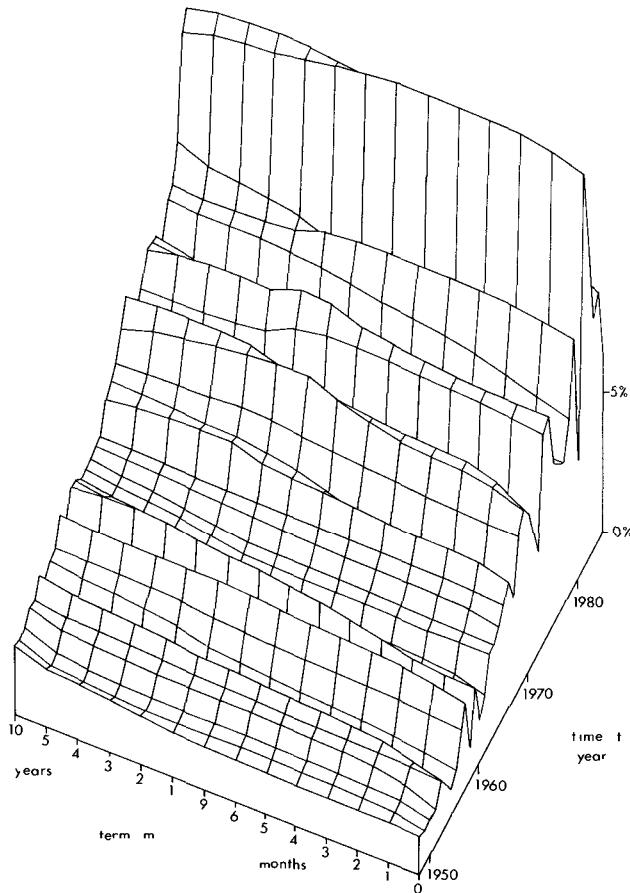


Figure 13.1. The term structure of interest rates. Data plotted are par bond yields to maturity, $r_p(t, t+m)$, against time t and term m , annual data, end of June 1948–85. Curves on the surface parallel to the m -axis show the term structure for various years. Curves on the surface parallel to the t -axis show the path through time of interest rates of various maturities. Maturities shown are 0, 1, 2, 3, 4, 5, 6, and 9 months and 1, 2, 3, 4, 5, and 10 years. Note that longer maturities are at the left, the reverse of the usual plot of term structures, so an “upward sloping” term structure slopes up to the left. Source of data: Table 13.A.3 of Appendix B.

structure, and Section 5 the empirical work on the term structure. Section 6 is an overview and interpretation of the literature.

2. Simple analytics of the term structure: Discount bonds

A discount bond is a promise by the issuer of the bond of a single fixed payment (the “principal”) to the holder of the bond at a given date (the

“maturity”). There are no intervening interest payments; thus the bond sells for less than the principal before the maturity date, i.e. it is expected to sell at a discount. The issuer of the bond has no other obligation than to pay the principal on the maturity date. An investment in a discount bond is not illiquid because the holder can sell it at any time to another investor. Let us denote by $p_d(t, T)$ the market price at time t of a discount bond whose principal is one dollar and whose maturity date is T , $t \leq T$. The subscript d denotes discount bond, to contrast this price from the par bond price to be defined below. The “term” of the bond (which will be represented here by the letter m) is the time to maturity, $m = T - t$. Thus, the term of any given bond steadily shrinks through time: a three-month bond becoming a two-month bond after one month and a one-month bond after two months.

All discount bonds maturing at date T for which there is no risk of default by the issuer ought to be perfectly interchangeable, and to sell at time t for $p_d(t, T)$ times the principal. The price $p_d(t, T)$ is thus determined by the economy-wide supply and demand at time t for credit to be repaid at time T . The determination of $p_d(t, T)$ is thus macroeconomic in nature, and is not at the discretion of any individual issuer or investor.

The price $p_d(t, T)$ of a discount bond may be generally expected to increase gradually with time t until the maturity date T , when it reaches its maximum, equal to one dollar. The increase in price for any holder of the bond over the period of time that he or she holds it is the return to holding it. The actual increase in price, since it is determined by market forces, may not be steady and may vary from time to time. It is useful to have some measure of the prospective increase in price that is implicit in the price $p_d(t, T)$. The yield to maturity (or interest rate) $r_d(t, T)$ at time t on the discount bond maturing at time T can be defined, given $p_d(t, T)$, as the *steady* rate at which the price should increase if the bond is to be worth one dollar at time T . If the growth of price is to be steady, then the price at time t' , $t \leq t' \leq T$, should be given by $p_d(t, T) e^{(t'-t)r_d(t, T)}$. Setting this price equal to one dollar where $t' = T$, and solving for $r_d(t, T)$, we find that the yield to maturity is given by

$$r_d(t, T) = -\log_n(p_d(t, T))/(T - t).$$

The term structure of interest rates, for discount bonds, is the function relating $r_d(t, t + m)$ to m . We may also refer to $r_d(t, t + m)$ as the “ m -period rate”; if m is very small as the “short rate” and if m is very large as the “long rate”.

Note that the term structure at any given date is determined exclusively by bond prices quoted on that day; there is a term structure in every daily newspaper. Those making plans on any day might well consult the term structure on that day. We can all lend (that is, invest) at the rates shown in the paper, and while we cannot all borrow (that is, issue bonds) at these rates, the rates shown are likely to be indicative of the rates at which we can borrow. If

the one-year interest rate is high and the two-year interest rate is low (i.e. if there is a descending term structure in this range), then individual firms, or governments, who plan to borrow for one year may be rather discouraged, and inclined to defer their borrowing plans for another year. Those who plan to lend this year rather than next would be encouraged. The reverse would happen if the term structure were ascending. Most individuals, of course, do not pay close attention to the term structure, but many do, and firms and governments do as well. The term structure on any day is determined by those who enter their preferences in the market on that day. A descending term structure on that day means that if the term structure had been flat there would be an excess supply of one-year bonds or an excess demand for two-year bonds. The descending term structure arises, of course, to choke off this excess supply or demand.

In making plans using the term structure it is helpful to realize that the term structure on any given date has in it implicit future interest rates, called forward rates. In the above example, where the term structure is descending between one and two years, it is implicit in the term structure that the one-year interest rate can be guaranteed to be lower next year than it is this year. To guarantee the forward rate one must be able both to buy and to issue bonds at quoted prices. One achieves this by trading in bonds of different maturities available today. One buys a discount bond at time t maturing at time T at price $p_d(t, T)$ and issues an amount of discount bonds maturing at t' at price $p_d(t, t')$, where $t < t' < T$. If the number of bonds issued equals $p_d(t, T)/p_d(t, t')$, then one will have broken even, at time t . That is, one will not have acquired or lost any cash today in the transaction. However, at time t' one must pay the principal on the bonds issued, equal to $p_d(t, T)/p_d(t, t')$. At time T one will receive the principal on the $(T - t)$ -period bond, equal to 1. Thus, the outcome of the transaction is in effect that one is committing oneself at time t to buy a discount bond at time t' maturing at time T with price $p_d(t, T)/p_d(t, t')$. The forward rate $f_d(t, t', T)$ at time t applying to the time interval t' to T is the yield to maturity on this contract:

$$f_d(t, t', T) = -\log_n(p_d(t, T)/p_d(t, t'))/(T - t') , \quad t < t' < T .$$

This may also be called the $t' - t$ period ahead forward rate of term $T - t'$. One can also guarantee that one can borrow at the forward rate $f_d(t, t', T)$ by buying discount bonds maturing at t' and issuing bonds maturing at T .

One might thus consider, in deciding whether or not to defer borrowing or lending plans, a comparison of the spot rate $r_d(t, T)$ with the forward rate of corresponding maturity k periods in the future, $f_d(t, t+k, T+k)$. There is also another margin to consider. One might hold one's borrowing or lending plans fixed, deciding, let us say, to invest at time $t+k$, but to consider whether to tie

down the interest rate today at $f_d(t, t+k, T+k)$ or to wait and take one's chances with regard to the future spot rate $r_d(t+k, T+k)$.

The subject of the literature surveyed here is how people who are making decisions at the various margins interact to determine the term structure. Before embarking on this, it is important to broaden our definitions and concepts.

3. Fundamental concepts

3.1. Bonds: Their definition

The term "bond" will be used here for any debt instrument, whether technically bond, bill, note, commercial paper, etc. and whether or not payments are defined in nominal (money) terms or in real terms (that is, tied to a commodity price index).

A bond represents a claim on a prespecified sequence of payments. A bond which is issued at time I and matures at time T is defined by a w -element vector of payment dates $(t_1, t_2, \dots, t_{w-1}, T)$, where $I < t_i \leq T$ for all i , and by a w -element vector of corresponding positive payments $(s_1, s_2, s_3, \dots, s_w)$. In theoretical treatments of the term structure, payments may be assumed to be made continually in time, so that the payment stream is represented by a positive function of time $s(t)$, $I < t \leq T$.

Two kinds of payment sequences are common. For the discount bond referred to above the vector of payment dates contains a single element T and the vector of payments contains the single element called the principal. A coupon bond, in contrast, promises a payment at regular intervals of an amount c called the coupon and a payment of the last coupon and principal (the latter normalized here at 1) at the maturity date. Thus, for example, a coupon bond that will mature in an integer number of periods and whose coupons are paid at integer intervals has vector of payment dates $(I+1, I+2, \dots, I+w-1, I+w)$, and vector of payments $(c, c, \dots, c, c+1)$. A perpetuity or consol is a special case of a coupon bond for which T , the maturity date, is infinity.

The purchaser at time t of a bond maturing at time T pays price $p(t, T)$ and is entitled to receive those payments corresponding to the t_i that are greater than t , so long as the purchaser continues to hold the bond.² A coupon bond is

²In the United States coupon bonds are typically traded "and accrued interest" (rather than "flat") which means that the price $p(t, T)$ actually paid for a coupon bond between coupon dates is equal to its quoted price plus accrued interest which is a fraction of the next coupon. The fraction is the time elapsed since the last coupon payment divided by the time interval between coupons.

said to be selling at par at time t if $p(t, T)$ is equal to the value of the principal, by our convention equal to 1.00.

A coupon bond may be regarded as a portfolio of discount bonds. If coupons are paid once per time period, for example, then the portfolio consists of an amount c of discount bonds maturing at time $I + 1$, an amount c discount bonds maturing at time $I + 2$, etc. and an amount $c + 1$ of discount bonds maturing at time T . Should all such discount bonds be traded, we would expect, by the law of one price, that (disregarding discrepancies allowed by taxes, transactions costs and other market imperfections) the price of the portfolio of discount bonds should be the same as the price of the coupon bond.³

There is thus (abstracting from market imperfections) a redundancy in bond prices, and if both discount and coupon bonds existed for all maturities, we could arbitrarily confine our attention to discount bonds only or coupon bonds only. In practice, we do not generally have prices on both kinds of bonds for the same maturities. In the United States, for example, discount bonds were until recently available only for time to maturity of one year or less. There is also redundancy among coupon bonds, in that one can find coupon bonds of differing coupon for the same maturity date.

3.2. Interest rates: Their definition

The *yield to maturity* (or, loosely, interest rate) at time t of a bond maturing at time T is defined implicitly as the rate $r(t, T)$ that discounts its vector of payments s to the price $p(t, T)$:

$$p(t, T) = \sum_{t_i > t} s_i e^{-(t_i - t)r(t, T)}. \quad (1)$$

The right-hand side of this expression is just the present value, discounted at rate $r(t, T)$, of the remaining payments accruing to bond holders. For discount bonds, this expression reduces to the expression given in Section 2 above. The yield to maturity may also be given an interpretation as above. Given the price $p(t, T)$, $r(t, T)$ is that steady rate of appreciation of price between payment dates so that if the price falls by the amount s_i at each t_i before T , the price equals S_7 at time T . In theoretical treatments of the term structure in which the

³Conversely, a discount bond may be considered a portfolio of coupon bonds, though in this case the portfolio involves negative quantities. For example, a two-period discount bond may be regarded as a portfolio of one- and two-period coupon bonds whose coupons are c_1 and c_2 , respectively. The portfolio would consist of $-(c_2)/[(c_1 + 1)(c_2 + 1)]$ of the one-period coupon bonds and $1/(c_2 + 1)$ of the two-period coupon bonds.

payments are assumed to be made continually in time, the summation in (1) is replaced by an integral.

The expression (1) gives the continuously compounded yield to maturity $r(t, T)$. One can define a yield to maturity with any compounding interval h : $r(t, T, h) = (e^{hr(t, T)} - 1)/h$. In the United States, where coupons are traditionally paid semiannually, it is customary to express yields to maturity at annual rates with semiannual compounding.⁴ Continuous compounding will be assumed here for consistency, as we do not wish to allow such things as the interval between coupon dates to dictate the compounding interval.⁵

For coupon bonds it is customary to define the *current yield* as the total coupons paid per year divided by the price. Current yield is not used to represent the interest rate and should not be confused with the yield to maturity.

If coupon payments are made once per period, then equation (1) is a $(T-t)$ -order polynomial equation in $e^{-r(t, T)}$ which therefore has $T-t$ roots. However, given that $s_i \geq 0$ for all i , there is only one real positive root, and this is taken for the purpose of computing the yield to maturity $r(t, T)$.

Roots of polynomials of order n can be given an explicit formula in terms of the coefficients of the polynomial only if n is less than five. Thus, yields to maturity for $T-t$ greater than or equal to five can be determined from price only by iterative or other approximation procedures, or with the use of bond tables.

The term structure of interest rates at time t is the function relating yield to maturity $r(t, t+m)$ to term m . A plot of $r(t, t+m)$ against m is also known as a yield curve at time t . There is a term structure for discount bonds and a term structure for coupon bonds. If we assume the law of one price as described in the preceding section, then, given the coupons, there is a relation between the different term structures.

3.3. Par bonds

Consider a bond that pays coupons continuously at rate c per period until the maturity date T when a lump-sum payment of 1 is made. If we disregard taxes and other market imperfections, the law of one price implies that the price of

⁴Thus, computing yield by solving (1) and converting to semiannual compounding (using $h = 0.5$) gives us exactly the yields in bond value tables, as in Financial Publishing Company (1970), so long as the term m is an integer multiple of $h = 0.5$. Whether or not m is an integer multiple of h , this also gives exactly yields to maturity as presented in Stigum (1981, p. 111) if $p(t, T)$ is represented as price plus accrued interest.

⁵Continuously compounded yield to maturity has also been referred to as “instantaneous compound interest”, “force of interest”, or “nominal rate convertible instantaneously”. See, for example, Skinner (1913).

this bond in terms of $p_d(t, T)$ is given by

$$p_p(t, T) = \int_t^T cp_d(t, s) ds + p_d(t, T). \quad (2)$$

The yield $r_p(t, T)$ of a par bond is found from $p_p(t, T)$ by setting the left-hand side of this expression to 1 and solving for c :⁶

$$r_p(t, T) = \frac{1 - p_d(t, T)}{\int_t^T p_d(t, s) ds}. \quad (3)$$

3.4. Instantaneous and perpetuity rates

The interest rate of term zero is $r_p(t, t)$, defined as the limit of $r_p(t, T)$ as $T \rightarrow t$, or as $r_d(t, t)$ defined as the limit of $r_d(t, T)$ as T approaches t . It is the instantaneous interest rate, which is of course not directly observed in any market. Since $r_p(t, t) = r_d(t, t)$ we can adopt the simpler notation r_i to refer to this instantaneous rate of interest. At the other extreme is $r_p(t, \infty)$, the limit of $r_p(t, T)$ as T approaches ∞ . This is the consol or perpetuity yield, which is just the inverse of the integral of $p_d(t, s)$ from $s = t$ to $s = \infty$.⁷

3.5. Estimates of the term structure

At any point of time t there will be an array of outstanding bonds differing by term, $m = T - t$, and by payment streams. Of course, not all possible times to maturity will be observed on available bonds at any given time t , and for some terms there will be more than one bond available. There has long been interest in estimates of rates of interest on standard bonds in terms of a standard list of times to maturity, interpolated from the rates of interest on bonds of those maturities that are actively traded.

⁶Note that for a par bond the yield to maturity equals the coupon. Note also that in the presence of taxes the law of one price need not imply (2) or (3). McCulloch's (1975b) formula for $r_p(t, T)$ collapses to (3) if the income tax rate is zero.

⁷Corresponding to the consol yield, we may also define the yield of a discount bond of infinite term, $r_d(t, \infty)$, defined as the limit of $r_d(t, T)$ as T goes to infinity. Dybvig, Ingersoll and Ross (1986) have a curious result concerning $r_d(t, \infty)$ in the context of a state price density model. They show that if $r_d(t, \infty)$ exists for all t , then $r_d(t, \infty) \leq r_d(s, \infty)$ with probability one when $t < s$. Otherwise, arbitrage profits would obtain. Thus, the long-term interest rate so defined can never fall. Intuitively, this seemingly strange result follows from the fact that for large enough T the price $p_d(t, T)$ is virtually zero and hence cannot decline, but will rise dramatically if there is any decline in $r_d(t, T)$.

The U.S. Treasury reports constant maturity yields for its own securities, that appear regularly in the *Federal Reserve Bulletin*. Salomon Brothers (1983) provides yield curve data for government bonds of a wide range of maturities. Durand (1942, and updated) provides yield curve data for corporate bonds. These data are interpolated judgmentally.⁸

McCulloch (1971, 1975b) used a spline interpolation method that deals statistically with the redundancy of bonds and deals systematically with some differences among bonds, such as tax provisions pertaining to them. His method produced an estimate of an after-tax discount function and from that the price $p_d(t, T)$ of a taxable discount bond as a continuous function of T . Expression (1) was then used to convert this estimated function into a function $r_d(t, T)$. Values of his estimated continuous function for various values of t and T appear in Table 13.A.1. His method allows for the fact that, in the U.S. personal income tax law, capital gains are not taxable until the bond is sold and that, until the 1986 Tax Act, capital gains on bonds originally issued at par were taxed at a rate which was lower than the income tax rate. He describes his function in Appendix B. Other functional forms for estimation of the term structure have been discussed by Chambers, Carlton and Waldman (1984), Jordan (1984), Nelson and Siegel (1985), Schaefer (1981), Shea (1984, 1985), and Vasicek and Fong (1982).

McCulloch used his estimated $p_d(t, T)$ to produce an estimate of the term structure of par bond yields, using an equation differing from (3) above only for tax effects.

3.6. Duration

The term $m = T - t$ of a bond is the time to the last payment, and is unrelated to the times or magnitudes of intervening payments s_1, s_2, \dots, s_{w-1} . Since bonds can be regarded as portfolios of discount bonds, it may be more useful to describe bonds by a weighted average of the terms of the constituent bonds rather than by the term of the longest bond in the portfolio. The duration of a bond, as defined by Macaulay (1938), is such a weighted average of the terms of the constituent discount bonds, where the weights correspond to the amount of the payments times a corresponding discount factor.⁹ The use of the discount factor in the definition implies that terms of very long-term constituent bonds

⁸Other important sources of historical data may be noted. Homer (1963) and Macaulay (1938) provide long historical time series. Amsler (1984) has provided a series of high quality preferred stock yields that might proxy for a perpetuity yield in the United States, a series which is much longer than that supplied by Salomon Brothers (1983).

⁹Hicks (1946) independently defined “average period”, which is equivalent to Macaulay’s first definition of duration

will tend to have relatively little weight in the duration formula. Thus, 30-year coupon bonds and 40-year coupon bonds have similar durations; indeed, they are similar instruments since the payments beyond 30 years into the future are heavily discounted and not important today relative to the coupons that come much sooner.

Macaulay actually gave two different definitions of duration that differed in the specification of the discount factor. The first definition of the duration of a bond of term m at time t uses the yield to maturity of the bond:¹⁰

$$D(m, t) = \frac{\sum_{t_i > t} (t_i - t) s_i e^{-(t_i - t)r(t, t+m)}}{\sum_{t_i > t} s_i e^{-(t_i - t)r(t, t+m)}}. \quad (4)$$

The second definition of duration of a bond of term m at time t uses prices of discount bonds as discount factors:

$$D'(m, t) = \frac{\sum_{t_i > t} (t_i - t) s_i p_d(t, t_i)}{\sum_{t_i > t} s_i p_d(t, t_i)}. \quad (4')$$

By either definition of duration, if the bond is a discount bond, then the duration equals the term m , that is, we shall write $D_d(m, t) = m$. Otherwise, (since payments s_i are positive) the duration is less than the time to maturity.

If a bond is selling at par and coupons are paid continually, then duration using (4) is:

$$D_p(m, t) = \frac{1 - e^{-mr_p(t, t+m)}}{r_p(t, t+m)}. \quad (5)$$

Thus, the duration of a perpetuity, whose term is infinite, is $1/r_p(t, \infty)$.

The duration using yields to maturity to discount $D(m, t)$ is the derivative of the log of $p(t, T)$, using (1), with respect to the yield to maturity $r(t, T)$.¹¹ Thus, duration may be used as an index of the “risk” of a bond. The concept of duration has thus played a role in the literature on “immunization” from interest rate risk of portfolios of financial intermediaries. A portfolio is fully immunized if there is complete cash-flow matching, that is, if the payments

¹⁰The second argument, t , of duration will be dropped below in contexts where the interest rate $r(t, t+m)$ is replaced by a constant

¹¹This fact was used by Hicks and was rediscovered by Samuelson (1945), Fisher (1966), Hopewell and Kaufman (1973), and others.

received on assets exactly equal payments paid on liabilities. When such cash-flow matching is infeasible, portfolio managers may instead try to match the overall duration of their assets with the duration of their liabilities. As long as the term structure makes only parallel shifts, the yields on bonds of all terms being increased or decreased by the same amount, then duration matching will perfectly immunize the portfolio and there is no uncertainty about net worth. However, the term structure rarely makes a parallel shift, long-term interest rates being more stable than short-term interest rates, and so duration tends to overstate the relative riskiness of long-term bonds. Other methods of immunization have been proposed that take this into account [see Ingersoll, Skelton and Weil (1978)].

3.7. Forward rates¹²

The time t discount bond forward rate applying to the interval from t' to T , $f_d(t, t', T)$, alluded to in Section 2 above, is defined in terms of yields to maturity and duration in Table 13.1, expression (a). Using (1) one verifies that this expression is the same as the expression given in Section 2 above. The forward rate compounded once per h periods is $f_d(t, t', T, h) = (\exp(hf_d(t, t', T)) - 1)/h$.

The limit of expression (a) of Table 13.1 as t' approaches T , denoted $f_d(t, T, T)$ or just $f(t, T)$ is the instantaneous forward rate:¹³

$$f(t, T) = r_d(t, T) + (T - t) dr_d(t, T) / dT \quad (6)$$

¹²The earliest use of the term “forward rate”, and the first indication that it can be thought of as a rate on a forward contract that can be computed from the term structure, appears to be in Hicks (1946) [first published (1939)] Kaldor (1939) speaks of forward rates and their interpretation as rates in forward contracts, but attributes the idea to Hicks Macaulay (1938) speaks of computing “implicit interest rates” without making an analogy to forward contracts (p 30) Of course, the notion that long rates are averages of future short rates has a longer history, the earlier authors appear not to have written of computing forward rates from the long rates, or of showing an analogy of such rates to rates in forward contracts.

I wrote to Sir John Hicks asking if he had coined the term forward rate in the term structure He replied that he only remembers being influenced by a 1930 paper in Swedish by Lindahl, later published in English (1939), which is couched, Hicks writes, “in terms of expected rates rather than forward rates; it is likely that the change from one to the other is my own contribution”

¹³McCulloch, who was concerned with the effects of taxation, writes (1975b, p 823) what appears to be a different expression for the instantaneous forward rate If we adopt some of the notation of this paper this is

$$f(t, T) = -\{\partial\delta(t, T)/\partial T\}/\{(1 - z)\delta(t, T)\},$$

where $\delta(t, T)$ is the price at time t of an *after-tax* dollar at time T , and z is the marginal tax rate However, since $\delta(t, T) = \exp(-(1 - z)(T - t)r_d(t, T))$, his formula is identical to the one shown here, i.e. the tax rate drops out of the formula expressed in terms of $r_d(t, T)$ The tax rate *should* drop out because both the interest rate $r_d(t, T)$ and the forward rate are taxable

Table 13.1
Formulas for computation of forward rates and holding rates

I. Time t forward rate applying to interval from t' to T , $t \leq t' \leq T$:

$$(a) f_i(t, t', T) = \frac{D_i(T-t)r_i(t, T) - D_i(t'-t)r_i(t, t')}{D_i(T-t) - D_i(t'-t)}$$

II Holding period rate or return from t to t' on bond maturing at time T , $t \leq t' \leq T$:

$$(b) h_i(t, t', T) = \frac{D_i(T-t)r_i(t, T) - [D_i(T-t) - D_i(t'-t)]r_i(t', T)}{D_i(t'-t)}.$$

III Holding period rate of return from t to t' rolling over bonds of term $m = T-t$, $t \leq T \leq t'$:

$$(c) h_i(t, t', T) = \left\{ \sum_{k=0}^{s-1} (D_i(km+m) - D_i(km))r_i(t+km, t+km+m) + [D_i(t'-t) - D_i(sm)]h_i(t+sm, t', t+sm+m) \right\} / D_i(t'-t),$$

where $s = \text{largest integer} \leq (t'-t)/m$.

Note In the above formulas, substitute $i = d$ for discount bonds, $i = p$ for par bonds. Par bond formulas give linear approximation to true rates. Duration (from which the second argument, t , has been dropped here) is given by $D_d(m) = m$, $D_p(m) = (1 - e^{-R_i m})/R_i$, where R_i is the point of linearization, which might be taken as $r_p(t, T)$. These formulas may be applied to data in Tables 13.A 1 and 13.A 3.

or

$$f(t, t+m) = r_d(t, t+m) + m dr_d(t, t+m)/dm. \quad (7)$$

It follows that the instantaneous forward rate follows the same relation to the spot rate as does marginal cost to average cost. To see this relation, think of m as output produced and $r_d(t, t+m)$ as price of a unit of output. As with the familiar cost curves, the instantaneous forward rate (marginal) equals the instantaneous spot rate (average) when m equals zero, that is, $f(t, t) = r_i$. The forward rate is less than the spot rate where the slope of the term structure is negative, and is greater than the spot rate where the slope of the term structure is positive. An example showing a term structure and forward rate curve is shown in Figure 13.2.¹⁴

Solving the differential equation (7) we can show:

$$f_d(t, t', T) = (T-t')^{-1} \int_{t'}^T f(t, s) ds. \quad (8)$$

¹⁴ Instantaneous forward rates computed using McCulloch's data for large $T-t$ seem to be very erratic. Vasicek and Fong (1982) have suggested that the problem would be eliminated if McCulloch had used exponential splines instead of the ordinary splines of his procedure; however, McCulloch (1984) has disputed whether this would solve the problem.

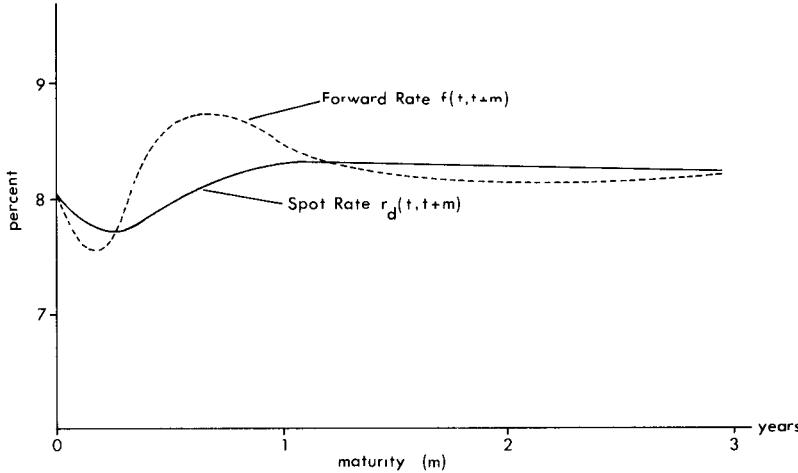


Figure 13.2 The term structure of interest rates $r_d(t, t+n)$ (solid line) and the instantaneous forward rate $f(t, t+n, t+n)$ (dashed line) for the end of August 1978. Source of data: Tables 13.A 1 and 13.A 2 in Appendix B.

Thus, the forward rate $f_d(t, t', T)$ is a simple average of the instantaneous forward rates between t' and T (see Figure 13.3).

Par bond forward rates can also be computed. These are especially useful if one wishes to make comparisons with spot interest rates as commonly quoted, since longer-term bonds usually trade near par. At time t one can guarantee for oneself a par bond issued at time t' and maturing at time T ($t \leq t' \leq T$) by buying at time t one discount bond maturing at time T , buying discount bonds maturing continually between t' and T whose principal accrues at rate c , and selling a discount bond maturing at date t' such that the proceeds of the sale exactly equal the total purchases made. If one then chooses c such that the number of bonds maturing at time t' sold is 1, one will have guaranteed for oneself, in effect, the rate of interest on a par bond at time t' maturing at T . The par forward rate $F_p(t, t', T)$ equals c or:¹⁵

$$F_p(t, t', T) = \frac{p_d(t, t') - p_d(t, T)}{\int_{t'}^T p_d(t, s) ds}. \quad (9)$$

¹⁵This formula for the forward rate differs slightly from that in McCulloch (1975b, p. 825). His formula replaces $p_d(x, y)$ with $\delta(x, y) = p_d(x, y)^{1-z}$ and divides by $(1-z)$, where z is the marginal tax rate. His formula is *not* quite identical to the one shown here if $z > 0$. However, the Volterra-Taylor linearization (like that which follows immediately in the text) of his expression in terms of instantaneous forward rates is identical to equation (10) below, and the tax rate drops out of that.

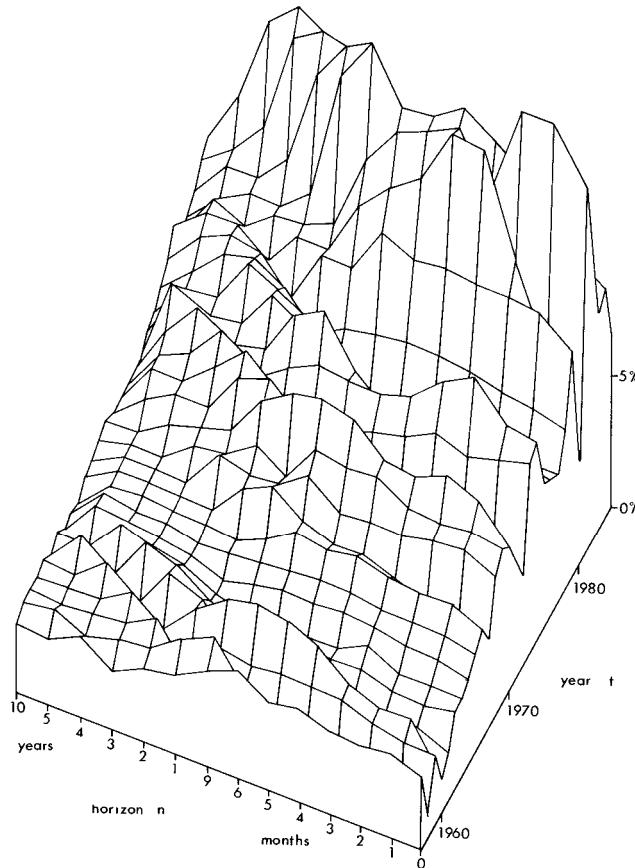


Figure 13.3 Instantaneous forward rates Data plotted are $f(t - n, t, t)$ against time t and horizon n . The annual data series $f(t, t, t) = r_t$, seen at the far right of the surface is the instantaneous interest rate for the end of June of each year, 1957 to 1985. Curves on the surface parallel to the n axis show the path through time of the "forecast" implicit in the term structure of the instantaneous forward rate applying to the date shown on the t axis. If there were perfect foresight, we would expect these curves to be horizontal straight lines. In the expectations theory of the term structure with zero risk premium, they should be random walks. Curves on the surface parallel to the t -axis show the path through time of a forward rate of fixed forecast horizon. Data plotted here are from Table 13 A 2 of Appendix B.

Note the similarity between this expression and expression (3) for the par spot rate, above.

The limit of expression (9) as t' approaches T is the same as the values given by expressions (6) or (7) above for the instantaneous discount forward rate, hence the omission in those expressions of the d or p subscript.

It can be shown that the par bond forward rate is a *weighted* average of

instantaneous forward rates, where the weights are proportional to the prices of the discount bond maturing at the date to which the instantaneous forward rate applies:¹⁶

$$F_p(t, t', T) = \frac{\int_{t'}^T p_d(t, s)f(t, s) ds}{\int_{t'}^T p_d(t, s) ds}. \quad (9')$$

This expression may be compared with the corresponding expression for discount bonds, expression (8) above. This expression gives more weight to instantaneous forward rates in the near future, rather than equal weight to all forward rates as in expression (8).

Using (8) for $t = t'$ and substituting for $p_d(t, s)$ in the above expression makes $F_d(t, t', T)$ a functional of $f(t, s)$ considered as a function of s . Following Campbell (1984, 1986b), this functional can be linearized around $f(t, s) = R$ using a Volterra–Taylor expansion [Volterra (1959)]. We will refer to the linear approximation to the forward rate $F_p(t, t', T)$ as $f_p(t, t', T)$. This is:¹⁷

$$f_p(t, t', T) = \frac{R}{e^{-(t'-t)R} - e^{-(T-t)R}} \int_{t'}^T e^{-R(s-t)} f(t, s) ds. \quad (10)$$

Using expression (10) and that $f_p(t, t, T) = r_p(t, T)$ gives us again expression (a) in Table 13.1 for this forward rate in terms of par interest rates only.¹⁸ The expression for forward rates on par bonds is the same as that on discount bonds except that duration on par bonds, $D_p(s - t)$, replaces the duration on discount bonds, $D_d(s - t) = s - t$, where $s = t', T$. It might also be noted that expression (a) in Table 13.1 gives the true par forward rate $F_p(t, t', T)$ exactly if a slightly different but less convenient, definition of duration is used.¹⁹

¹⁶See McCulloch (1977)

¹⁷The lower case f here denotes a linear approximation to the upper case F above. In contrast, for discount bonds the linear approximation is no different from the true forward rate, so lower case f is used for both. A discrete time version of (10) appears in Shiller (1979).

¹⁸The quality of this approximation to $F_p(t, t', T)$ is discussed in Shiller, Campbell and Schoenholtz (1983). The approximation is good except when both $t' - t$ and $T - t'$ are large.

¹⁹If we use Macaulay's definition of duration using prices of discount bonds as discount factors, (4'), and compute duration at time t of a bond whose continuous coupon is not constant through time but at time $t + \iota$ always equals $f(t, t + \iota)$, then one finds that duration is given by.

$$D'(m) = \int_t^{t+m} p_d(t, s) ds$$

Using expression (9) [and the fact that $F_p(t, t, T) = r_p(t, T)$] one finds that $F_p(t, t', T)$ equals the right-hand side of expression (a) in Table 13.1, where the above $D'(m)$ replaces $D_p(m)$.

3.8. Holding period rates

A holding period rate is a rate of return to buying a bond (or sequence of bonds) and selling at a later date. The simple discount bond holding period rate, $h_d(t, t', T)$, where $t \leq t' \leq T$, is the rate of return from buying at time t a discount bond maturing at date T and selling it at date t' ; see Table 13.1, expression (b). The rollover discount bond holding period rate, $h_d(t, t', T)$, where $t < T < t'$, is the rate of return from buying at time t a discount bond of term $m = T - t$, reinvesting ("rolling over") the proceeds in another m -period discount bond at time $t + m$, and continuing until time t' when the last m -period discount bond is sold [Table 13.1, expression (c)].

The par bond holding period rate, $H_p(t, t', T)$, where $t \leq t' \leq T$, is the yield to maturity on the stream of payments accruing to someone who buys at time t a par bond maturing at T , receives the stream of coupons between t' and T , and sells the bond at time T . This holding period rate can be defined as an implicit function of the coupon on the bond $r_p(t, T)$ and the selling price, which in turn is a function of $r_p(t', T)$ as well as the coupon $r_p(t, T)$. This implicit function may be linearized around $H_p(t, t', T) = r_p(t, T) = r_p(t', T) = R$ to yield the approximate $h_p(t, t', T)$ shown in Table 13.1, expression (b).²⁰ The rollover par bond holding period rate, $H_p(t, t', T)$, $t < T < t'$, is the yield to maturity on the stream of payments accruing to someone who buys at time t a par bond maturing at time $T = t + m$, reinvests proceeds in a par bond maturing at time $t + 2m$, and continues until time t' when the last par bond is sold. The linear approximation $h_p(t, t', T)$ appears in Table 13.1, expression (c).

4. Theories of the term structure

4.1. Expectations theories of the term structure

The expectations hypothesis, in the broadest terms, asserts that the slope of the term structure has something to do with expectations about future interest rates. The hypothesis is certainly very old, although it apparently did not receive an academic discussion until Fisher (1896).²¹ Other important early

²⁰The quality of this linear approximation to $H_p(t, t', T)$ is generally quite good; see Shiller, Campbell and Schoenholz (1983) and Campbell (1986b).

²¹Fisher (1896) appears to say that the market has perfect foresight (p. 91). I have been unable to find any earlier discussion of the expectations theory of the term structure. Bohm-Bawerk (1891) discussed how expectations of future short rates affect today's long rate, but appears to conclude that the term structure is always flat (p. 280). Perhaps there is a hint of the expectations theory in Clark (1895, p. 395). Malkiel (1966) claims (p. 17) that "one can find anticipations of the expectations theory" in Sidgwick (1887) and Say (1853). In reading any of these works, one is led to conclude that the hint of the expectations theory is very slight.

discussions were in Fisher (1930), Williams (1938), Lutz (1940), and Hicks (1946). The expectations hypothesis probably derives from observing the way people commonly discuss choices between long and short debt as investments. They commonly speak of the outlook for future interest rates in deciding whether to purchase a long-term bond rather than a short-term bond as an investment. If interest rates are expected to decline, people may advise “locking in” the high long-term interest rate by buying a long-term bond. If everyone behaves this way, it is plausible that the market yield on long-term interest rates would be depressed in times when the short rate is expected to decline until the high demand for long-term interest rates is eliminated. Thus, relatively downward-sloping term structures are indicative of expectations of a decline in interest rates, and relatively upward-sloping term structures of a rise.

Early term structure theorists apparently could not think of any formal representation of the expectations hypothesis other than that forward rates equalled actual future spot rates (plus possibly a constant).²² Early empirical work finding fault with the expectations hypothesis for the inaccuracy of the forecasts [Macaulay (1938), Hickman (1942), and Culbertson (1957)] were later dismissed by subsequent writers who thought that the issue should instead be whether the forward rates are in accord with a model of expectations [Meiselman (1962), Kessel (1965)]. Since the efficient markets revolution in finance in the 1960s, such a model has generally involved the assumption of rational expectations.

4.2. Risk preferences and the expectations hypothesis

Suppose economic agents can be characterized by a representative individual whose utility function has the simple form:

$$U = \sum_{t=0}^{\infty} u(C_t)/(1 + \mu)^t, \quad (11)$$

where C_t is consumption at time t , μ is the subjective rate of time preference, and $u(C_t)$ is momentary utility or “felicity”. Calling v_t the real value (value in terms of the consumption good rather than money) of any asset or portfolio of assets including reinvested coupons or dividends, a first-order condition for maximization of expected utility is that

$$u'(C_t)v(t) = E_t\{(1 + \mu)^{t-t'}u'(C_{t'})v(t')\}, \quad t < t' < T. \quad (12)$$

²²Conard (1959) wrote: “I assume not only that expectations concerning future rates are held with confidence by all investors, but also that these expectations are realized. Only by adding this last assumption is it possible to build a theory whose predictions can be meaningfully tested empirically” (p. 290).

If there is risk neutrality, then $u(C_t)$ is linear in C_t and $u(C_t) = a + bC_t$. It follows from (12) that

$$\frac{E_t v(t')}{v(t)} = (1 + \mu)^{t' - t}. \quad (13)$$

If the asset is a discount index bond maturing at time T , then $v(t') = p_d(t', T)$ and the left-hand side of this expression is one plus the expected holding return compounded every $t' - t$ periods, i.e. it is one plus $E_t(e^{xh_d(t,t',T)} - 1)/x$, where x equals $t' - t$. This means that under risk neutrality expected holding period returns as computed in the left-hand side of (13) will be equalized, i.e. will not depend on T . This in turn suggests that a particular formal expectations theory of the term structure follows from risk neutrality. Of course, risk neutrality may not seem a very attractive assumption, but approximate risk neutrality might be invoked to justify the intuitive expectations hypothesis described in the preceding section. Invoking risk neutrality to justify an expectations theory of the term structure was done by Meiselman (1962), Bierwag and Grove (1967), Malkiel (1966), Richard (1978), and others.

There are, however, fundamental problems with the expectations hypothesis as derived from risk neutrality. It is not possible for all expected holding period returns as defined in the left-hand side of (13) to be equalized if future interest rates are uncertain. This point was emphasized by Stiglitz (1970), who also attributed it to C.C. von Weizsäcker. If one-period expected holding period returns are equalized for a one-period compounding interval, then $1/p_d(0, 1) = E_0 p_d(1, 2)/p_d(0, 2)$. If two-period expected holding period returns are equalized for a two-period compounding interval, then $1/p_d(0, 2) = E_0(1/p_d(1, 2))/p_d(0, 1)$. It follows that $E_0 p_d(1, 2) = 1/E_0(1/(p_d(1, 2)))$. This is a contradiction, since Jensen's inequality states that for any random variable x that is always greater than zero, unless x is nonstochastic, $E(x) > 1/E(1/x)$.

For index bonds, equation (13) implies that interest rates are *not* random, and that therefore Jensen's inequality does not come into play [Cox, Ingersoll and Ross (1981), LeRoy (1982a)]. This can be easily seen by substituting $p_d(t, t')$ for $v(t)$ in (13). Since t' is the maturity date, and the real value of the index bond at maturity is specified as $v(t') = 1$, it follows that $v(t')$ is not random. Clearly, (13) then implies that $p_d(t, t')$ is not random either. It will be known with certainty at any date before t . Thus, while risk neutrality gives us an expectations hypothesis, it gives us a perfect foresight version that is extreme and uninteresting. It would be possible to alter the utility function (11) to allow the subjective rate of time preference μ to vary through time, and that would give us a time varying yield curve. Still, we would have a perfect

foresight model and a model in which preferences alone determine interest rates.²³

Risk neutrality is of course not a terribly attractive assumption, given various evidence on human behavior. The theoretical literature does not appear to contain any argument for appealing simple restrictions on preferences or technology that singles out for us an attractive version of the expectations hypothesis; see LeRoy (1982a) for a discussion. Cox, Ingersoll and Ross (1981) offered two sets of assumptions other than risk neutrality that can produce an expectations hypothesis for the term structure: one involving locally certain consumption changes, the other involving state-independent logarithmic utility. But by offering such special cases they are not giving any reason to suspect that the expectations hypothesis should be taken seriously in applied work.

Applied workers, actually, have rarely taken seriously the risk neutrality expectations hypothesis as it has been defined in the theoretical literature, and so the theoretical discussion of this expectations hypothesis may be something of a red herring. The applied literature has defined the expectations hypothesis to represent constancy through time of differences in expected holding returns, or constancy through time of the difference between forward rates and expected spot rates, and not that these constants are zero. We shall see in the next subsection that these theories can be described as assuming constancy of the “term premia”. Campbell (1986b) has stressed that some of the important conclusions of this theoretical literature do not carry over to the definitions of the expectations hypothesis in the empirical literature.²⁴

4.3. Definitions of term premia²⁵

There is little agreement in the empirical literature on definitions of term premia, and often term premia are defined only for certain special cases. Here,

²³Cox, Ingersoll and Ross (1981) emphasize that risk neutrality itself does not necessarily imply that interest rates are nonstochastic. Utility is not concave, and investors could be at a corner solution to their maximization problem in which (13) does not hold. However, they argued that in this case the expectations hypothesis will not generally be valid.

LeRoy (1983) showed [correcting errors in his own papers (1982a, 1982b)] a sense in which when there is “near risk neutrality”, that is, when utility functions are nearly linear, the expectations hypothesis is approximately satisfied.

²⁴Cox, Ingersoll and Ross (1981) showed that if there are fewer relevant state variables in an economy than there are bond maturities outstanding and if bond prices follow Itô processes, then only one version of the rational expectations hypothesis, what they called the “local expectations hypothesis”, can obtain in a rational expectations equilibrium. Campbell showed that this conclusion hinges on the assumption of a zero, not just constant, risk premium. He also showed a sense in which the other versions of the expectations hypothesis (which they claimed to reject as inconsistent with rational expectations equilibrium) may not be importantly different from their local expectations hypothesis.

²⁵Much of this subsection follows Campbell and Shiller (1984) and Campbell (1986b).

some definitions will be adopted which are clarifications and generalizations of definitions already commonplace. As suggested in the discussion in the preceding subsection, economic theory does not give us guidance as to how to define term premia, and so choices will be made here to retain essential linearity, which will simplify discussion.

The forward term premium, $\Phi_{f,i}(t, t', T)$, $i = p, d$, will be defined as the difference between the forward rate and the expectation of the corresponding future spot rate. Unless otherwise noted, this expectation will be defined as a rational expectation, i.e. E_t is the mathematical expectation conditional on information available at time t . Thus we have:

$$\Phi_{f,i}(t, t', T) = f_i(t, t', T) - E_t r_i(t', T), \quad t < t' < T, \quad i = p, d. \quad (14)$$

The holding period term premium, $\Phi_{h,i}(t, t', T)$, for $t < t' < T$, will be defined as the difference between the conditional expected holding period yield and the corresponding spot rate:

$$\Phi_{h,i}(t, t', T) = E_t h_i(t, t', T) - r_i(t, t'), \quad t < t' < T, \quad i = p, d. \quad (15)$$

The rollover term premium, $\Phi_{r,i}(t, t', m)$, for $t < t + m < t'$, will be defined as the difference between the yield on a bond maturing at time t' and the conditional expected holding period return from rolling over a sequence of m -period bonds:²⁶

$$\Phi_{r,i}(t, t', m) = r_i(t, t') - E_t h_i(t, t', t + m), \quad t < t + m < t', \quad i = p, d. \quad (16)$$

Although earlier authors did not always clearly intend rational expectations and often used different conventions about compounding, we can loosely identify the above definitions with definitions given by earlier authors. Hicks (1946), who is commonly credited with first defining these in the term structure literature, referred to both $\Phi_f(t, t', T)$ and $\Phi_r(t, t', m)$ as the “risk premium”.²⁷ Because of a subsequent liquidity theory of interest by Lutz (1940), and analogy with the Keynes’ (1936) liquidity preference theory, the risk premium has also become known as the “liquidity premium”. In this survey, the phrase “term premium” will be used throughout as synonymous with risk premium and liquidity premium; it is preferred to these because the phrase does not

²⁶Note that this risk premium has the form interest rate minus expected holding yield, in contrast to expected holding period yield minus interest rate in the preceding expression. This way of defining risk premia seems to be conventional; the rate on the longer asset comes first with a positive sign.

²⁷See Hicks (1946, p. 147)

have an association with a specific theory of the term structure. The holding period term premium $\Phi_{h,i}(t, t', T)$ is referred to as the expected “excess return” in finance textbooks.

From the definitions in Table 13.1, there are simple proportional relations between holding period term premia and forward rate term premia:

$$\Phi_{h,i}(t, t', T) = \{D_i(T-t)/D_i(t'-t) - 1\} \Phi_{f,i}(t, t', T), \quad t < t' < T. \quad (17)$$

We also have the following relations for the rollover term premium, where $t < t + m < t'$, $t' - t = sm$, s integer:

$$\begin{aligned} \Phi_{r,i}(t, t', m) &= (1/D_i(t'-t)) \sum_{k=0}^{s-1} [D_i(km+m) \\ &\quad - D_i(km)] \Phi_{f,i}(t+km, t+km+m), \end{aligned} \quad (18)$$

$$\begin{aligned} \Phi_{r,i}(t, t', m) &= (1/D_i(t'-t)) \sum_{k=0}^{s-1} [D_i(km+m) \\ &\quad - D_i(km)] E_t \Phi_{h,i}(t+km, t+km+m, t+sm), \end{aligned} \quad (19)$$

4.4. Early presumptions pertaining to the sign of the term premium

Hicks (1946) thought that there was a tendency for term premia to be positive. In this context he referred to the forward rate term premium, $\Phi_{f,i}(t, t', T)$, but if this term premium is always positive, then by (17) above so must the holding period term premium $\Phi_{h,i}(t, t', T)$ and, by (18), the rollover term premium $\Phi_{r,i}(t, t', m)$.

Hicks’ reasons to expect that term premia should be positive had their motivation in the theory of “normal backwardation” in commodity forward markets of Keynes (1930). Hicks wrote:²⁸

... the forward market for loans (like the forward market for commodities) may be expected to have a constitutional weakness on one side, a weakness which offers an opportunity for speculation. If no extra return is offered for long lending, most people (and institutions) would prefer to lend short, at least in the sense that they would prefer to hold their money on deposit in some way or other. But this situation would leave a large excess demand to borrow long which would not be met. Borrowers would thus tend to offer better terms in order to persuade lenders to switch over into the long market.

²⁸Hicks (1946, p. 146).

He offered no evidence (other than that on average risk premia themselves) that would support such a “constitutional weakness” on one side of the forward market.

Lutz (1940) offered a “liquidity theory of interest” that also predicted positive term premia:

... The most liquid asset, money, does not bear interest. Securities, being less liquid than money, bear an interest rate which is higher the longer the maturity, since the danger of capital loss due to a change in the interest rate in the market is supposed to be the greater (and therefore liquidity the smaller) the longer the security has to run.²⁹

His theory appears to ascribe term premia to own-variance, contrary to received wisdom in finance theory today.

Such theories were disputed by Modigliani and Sutch (1966) by merely pointing out that it is not clearly rational for individuals to prefer to lend short or to be concerned with short-term capital losses. If one is saving for a child's college education 10 years ahead, it is least risky to put one's savings in the form of a (real) 10-year bond rather than roll over short bonds. They proposed as an alternative to Hicks' theory the “preferred habitat theory”. A trader's habitat is the investment horizon he or she is most concerned about, and that person will prefer to borrow or lend at that term. There is a separate supply and demand for loanable funds in each habitat, which could give rise to any pattern of term premia. Traders may be “tempted out of their natural habitat by the lure of higher expected returns”,³⁰ but because of risk aversion this will not completely level term premia. The idea that individuals have a single habitat must be described as heuristic.³¹ The intertemporal capital asset pricing model typically assumes maximization of an intertemporal utility function that involves the entire future consumption stream, with exponentially declining weights, and thus no single ‘habitat’. However, the Modigliani–Sutch conclusion that term premia might as well, on theoretical grounds, be positive as negative seems now to be generally accepted.³²

²⁹Lutz (1940, p. 62)

³⁰Modigliani and Sutch (1966, p. 184).

³¹Cox, Ingersoll and Ross (1981) consider an economy in which all investors desire to consume at one fixed date. They find that the risk premium, defined as the expected instantaneous return minus the instantaneous interest rate, may not be lowest for bonds maturing at this date. Still, they argue that a preferred habitat theory holds if the habitat is defined in terms of a “stronger or weaker tendency to hedge against changes in the interest rate” (p. 786).

³²LeRoy (1982b) has argued that the risk premia are likely to be positive on theoretical grounds in a model without production, but had no results on the sign of the risk premium when production is introduced.

4.5. Risk preferences and term premia

If the representative agent maximizes the utility function (11), and therefore satisfies the first-order condition (12), then it follows that:³³

$$e^{-(\tau_2 - \tau_1)r_d(\tau_1, \tau_2)} = E_{\tau_1} S(\tau_1, \tau_2), \quad (20)$$

where $S(\tau_1, \tau_2)$ is the marginal rate of substitution between time τ_1 and τ_2 .³⁴ For equation (20), the precise definition of $S(\tau_1, \tau_2)$ will depend on whether we are dealing with index bonds or bonds whose principal is defined in nominal terms, that is, on whether $r_d(\tau_1, \tau_2)$ is a real or nominal rate. With index bonds, $S(\tau_1, \tau_2)$ is defined as $u'(C(\tau_2))/(u'(C(\tau_1))(1 + \mu)^{(\tau_2 - \tau_1)})$. With bonds whose principal is defined in nominal terms, $S(\tau_1, \tau_2)$ is defined as $u'(C(\tau_2))/u'(C(\tau_1)) \times (\pi(\tau_1)/\pi(\tau_2))/(1 + \mu)^{(\tau_2 - \tau_1)}$. Here $\pi(\tau)$ is a commodity price index at time τ , that is, the price of the consumption good in terms of the unit of currency. Thus, $S(\tau_1, \tau_2)$ is the marginal rate of substitution between consumption at time τ_1 and consumption at time τ_2 if the bond is an index bond, and between a nominal dollar at time τ_1 and a nominal dollar at time τ_2 if the bond is a conventional nominal bond.³⁵

It follows from equation (20) for $t < t' < T$ [setting (τ_1, τ_2) in (20) as (t, t') , (t, T) and (t', T)] that:

$$\begin{aligned} e^{-(T-t)r_d(t,T)} &= e^{-(t'-t)r_d(t,t')} E_t e^{-(T-t')r_d(t',T)} \\ &\quad + \text{cov}_t(S(t, t'), S(t', T)). \end{aligned} \quad (21)$$

In order to put this in terms of the above definitions of term premia, we use the linearization $e^x \approx (1 + x)$ for small x to derive from the above:

$$\Phi_{f,d}(t, t', T) \approx -\text{cov}_t(S(t, t'), S(t', T))/(T - t'). \quad (22)$$

The term premium $\Phi_{f,d}(t, t', T)$ depends on the covariance between the marginal rate of substitution between t and t' and the marginal rate of substitution between t' and T .³⁶ If this covariance is negative, then forward

³³To show this, use $v(t') = v(\tau_2) = 1$ and $v(t) = v(\tau_1) = p_d(\tau_1, \tau_2) = \exp(-(\tau_2 - \tau_1)r_d(\tau_1, \tau_2))$ in (12) where $\tau_1 < \tau_2$ so that $r_d(\tau_1, \tau_2)$ as well as $v(\tau_1)$ is known at time τ_1 .

³⁴It follows that increasing the uncertainty at time τ_1 about consumption at τ_2 will, if there is diminishing marginal utility, lower $r_o(r_1, \tau_2)$. This point was made by Fisher (1907, p. 214).

³⁵Benninga and Protopapadakis (1983) describe the relation of risk premia on nominal bonds to risk premia on index bonds.

³⁶LeRoy (1984) gives an expression for the term premium defined as the expected real J -period return on an t -period nominal bond minus the return to maturity of a J -period real bond.

rates tend to be above expected spot rates, as Hicks originally hypothesized, and the risk premium is positive.³⁷ In the case of index bonds, a negative covariance means that if real consumption should increase faster than usual between t and t' , it tends to increase less fast than usual between t' and T . In the case of nominal bonds, the interpretation of the sign of the term premium is less straightforward. But consider the utility function $u(C_t) = \log(C_t)$. Then for nominal bonds $S(t, t')$ equals nominal consumption at time t divided by nominal consumption at time $t + 1$. Then, the nominal term premium $\Phi_{f,d}(t, t', T)$ would tend to be positive if it happens that when nominal consumption increases faster than usual between t and t' it tends to increase less fast than usual between t' and T .

One can also derive [taking unconditional expectations of (20)] an expression like (21) for unconditional expectations:

$$\begin{aligned} E e^{-(T-t)r_d(t,T)} &= E e^{-(t'-t)r_d(t,t')} E e^{-(T-t')r_d(t',T)} \\ &\quad + \text{cov}(S(t, t'), S(t', T)). \end{aligned} \quad (23)$$

From which, by a linearization, we have:

$$E(\Phi_{f,d}(t, t', T)) \approx -\text{cov}(S(t, t'), S(t', T))/(T - t'). \quad (24)$$

Thus, the mean term premium $\Phi_{f,d}(t, t', T)$ is positive if the unconditional covariance between $S(t, t')$ and $S(t', T)$ is negative. Such a negative covariance might be interpreted as saying that marginal utility is “unsmooth” between $\tau = t$ and $\tau = T$. This means that when detrended marginal utility increases between $\tau = t$ and $\tau = t'$ it tends to decrease between t' and T . A positive covariance, and hence a negative term premium, would tend to occur if marginal utility is “smooth” between $\tau = t$ and $\tau = T$.

If the values of bonds of all maturities are assumed to be deterministic functions of a small number of state variables that are continuous diffusion processes, then theoretical restrictions on risk premia beyond those defined here can also be derived [e.g. Brennan and Schwartz (1980), Cox, Ingersoll and Ross (1981), Dothan (1978), Langetieg (1980), Marsh (1980), Richard (1978) and Vasicek (1978)]. When bond values are such deterministic functions of diffusion processes, if the restrictions did not hold there would be riskless

³⁷ Woodward (1983) discusses term premia in terms of the serial correlation of marginal utility of consumption rather than the serial correlation of marginal rates of substitution. Her principal result is that in a case where the correlation conditional on information at t between $u'(c_t)$ and $u'(c_T)$ is negative, then the sign of the term premium may be sensitive to the definition of the premium. She defines as an alternative definition of the term premium, the “solidity premium”, based on forward and actual discounts. The negative correlation she defines is an unlikely special case. Actual aggregate consumption in the United States roughly resembles a random walk [Hall (1978)] for which the correlation she defines is positive.

arbitrage opportunities. The assumption of such a state variable representation has been convenient for theoretical models. It has even led to a complete general equilibrium model of the term structure in a macro economy [Cox, Ingersoll and Ross (1985a, 1985b)], a model subjected to empirical testing by Brown and Dybvig (1986).

5. Empirical studies of the term structure

5.1. Empirical expectations hypotheses for the term structure

One need not assume rational expectations to proceed with studying an expectations theory of the term structure if one has data on expectations or can infer expectations from other data. The first study of the term structure using an expectations model was performed by Meiselman (1962). Meiselman proposed the “error learning hypothesis” that economic agents revise their expectations in proportion to the error just discovered in their last period expectation for today’s one-period rate. This hypothesis then implies that $f_i(t, t + n, t + n + 1) - f_i(t - 1, t + n, t + n + 1) = a_n + b_n(r_i(t, t + 1) - f_i(t - 1, t, t + 1))$. He estimated a_n and b_n by regression analysis using U.S. Durand’s annual data 1901–54 for $n = 1, 2, \dots, 8$. He took as encouraging for the model that the signs of the estimated b_n were all positive and declined with n . However, Buse (1967) criticized his conclusion, saying that “... such results are implied by any set of smoothed yield curves in which the short-term interest rates have shown a greater variability than long-term interest rates”.

It was pointed out later by Diller (1969) and Nelson (1970a) that the error learning principle is a property of optimal linear forecasts. They found that the coefficients b_n that Meiselman estimated compared rather favorably with the coefficients implied by an estimated linear forecasting equation. However, the univariate form of the error learning principle proposed by Meiselman applies only to univariate optimal linear forecasts [Shiller (1978)], and thus the Meiselman theory is unfortunately restrictive.

Other authors have used survey expectations data for market expectations of future interest rates. Survey methods seem particularly attractive since surveys can be focused on the institutional investors who hold most government and corporate bonds, and who are probably not well described in terms of the expected utility of consumption models described in the preceding section.³⁸

Friedman (1979) used data 1969–78 from a quarterly survey of financial market participants by the *Goldsmith–Nagan Bond and Money Market Letter*.

³⁸See Board of Governors (1985, pp 20 and 54). Of course, individuals ultimately have claims on the assets of these institutions, still there is an institutional layer between them and the bonds held on their behalf

He found that the term premium on U.S. Treasury bills, $\Phi_d(t, t+1, t+2)$ and $\Phi_d(t, t+2, t+3)$ (where time is measured in months), was positive on average and depended positively on the level of interest rates. He showed [Friedman (1980c)] that his model differed substantially from a rational expectations model, in that the survey expectations could be improved upon easily. Kane and Malkiel (1967) conducted their own survey of banks, life insurance companies and nonfinancial corporations to learn about the relation of expectations to the term structure of interest rates. They learned that many investors seemed not to formulate specific interest rate expectations (especially for the distant future) and those that did, did not have uniform expectations. Kane (1983) found using additional Kane–Malkiel survey data 1969–72 that term premia appear positively related to the level of interest rates.

5.2. *The rational expectations hypothesis in empirical work*

Although the rational expectations hypothesis regarding the term structure has had many forms, it has its simplest form used in empirical work in terms of the continuously compounded yields discussed here. Often, the other forms of the hypothesis do not differ importantly from that discussed here [see Shiller, Campbell and Schoenholtz (1983) and Campbell (1986b)]. In the definition to be used here, the rational expectations hypothesis is that all term premia, $\Phi_{h,i}(t, t+n, t+m+n)$, $0 < m, 0 < n$, $\Phi_{f,i}(t, t+n, t+m+n)$, $0 < m, 0 < n$, and $\Phi_{r,i}(t, t+n, m)$, $0 < m < n$, do not depend on time t .³⁹ This means that all term premia depend only on maturity and not time, and the changing slope of the term structure can only be interpreted in terms of the changing expectations for future interest rates.⁴⁰

The literature testing forms of the rational expectations hypothesis like that defined here is enormous.⁴¹ It is difficult to summarize what we know about the expectations hypothesis from this literature. We are studying a two-dimensional array of term premia; term premia depend on m (the maturity of the

³⁹When dealing with par bonds, the expectations model defined here relates to the linearized model. The assumption here is that the point of linearization R does not depend on the level of interest rates, otherwise the model will not be linear in interest rates.

⁴⁰Note that in this expectations hypothesis, stated in terms of continuously compounded yields, it is possible for all risk premia to be zero. We do not encounter the Jensen's inequality problem alluded to above in connection with risk neutrality. The problem alluded to by von Weizsaecker and Stiglitz was essentially one of compounding, and is eliminated when we couch the model in terms of continuously compounded interest rates.

⁴¹The literature has to do almost entirely with nominal interest rates, as a term structure of index bonds is observed only for brief periods in certain countries. Campbell and Shiller (1988) in effect looked at the real term structure in the postwar U.S. corporate stock price data by correcting the dividend price ratio for predictable changes in real dividends, leaving a long-term real consol component of the dividend price ratio. The expectations hypothesis was not supported by the evidence.

forward instrument) and n (the time into the future that the forward instrument begins). Term premia may be approximately constant for some m and n and not for others; certain functions of term premia may be approximately constant and not others. Term premia may be approximately constant for some time periods and not others, or in some countries and not others.

Testing for the constancy of term premia ultimately means trying to predict the right-hand side of the equations defining term premia [equations (14), (15) or (16) above] from which the conditional expectations operator E , is deleted, in terms of information at time t . This means predicting either excess holding period returns or the difference between forward rates and corresponding spot rates in terms of information at time t . Because of the relations between the definitions of term premia [equations (17), (18), or (19) above] it does not matter whether the regression has excess holding yields or the difference between forward rates and corresponding spot rates as the variable explained; the difference has to do only with a multiplicative constant for the dependent variable. Of course, most studies do not use the exact definitions of term premia defined here, in terms of continuously compounded rates or, in the case of par bonds, linearized holding yields, but the differences in definition are generally not important.

Some studies may report some tests of the rational expectations hypothesis that have the appearance of something very different; for example, Roll (1970) tested (and rejected using 1–13 week U.S. Treasury bill data 1949–64) the martingale property of forward rates by testing whether changes in forward rates $f_d(t, t', T) - f_d(t-1, t', T)$ are serially correlated through time t . But in fact testing the hypothesis that there is no such serial correlation is no different from testing the hypothesis that *changes* in the difference between forward rates and corresponding spot rates cannot be predicted based on information consisting of past changes in forward rates. For another example, some researchers have noted that for large m and small n the holding return $h_i(t, t+n, t+m)$ is approximately equal to $r_i(t, t+m) - r_i(t+n, t+n+m)$, the change in the long rate, divided by $D_i(n)$. If n is very small, $1/D_i(n)$ is a very large number, and the excess holding return is heavily influenced by the change in the long rate. The rational expectations hypothesis thus suggests that $r_i(t, t+m) - r_i(t+n, t+n+m)$ is approximately unforecastable, and hence that long rates are in this sense approximately random walks. The random walk property for long-term interest rates was tested by Phillips and Pippenger (1976, 1979), Pesando (1981, 1983), and Mishkin (1978).⁴²

⁴²The random walk property is an approximation useful only under certain assumptions [see Mishkin (1980) and Begg (1984)]. Phillips and Pippenger (1976, 1979) used the random walk approximation to assert that the Modigliani and Sutch (1966) and Modigliani and Shiller (1973) distributed lag regressions explaining the long rate must be spurious. Looking at the out-of-sample fit of the equation does not suggest that term-structure equations like that in Modigliani–Shiller are completely spurious. see Ando and Kennickell (1983)

Of all the studies of the rational expectations hypothesis for the term structure, of greatest interest are the results in which the explanatory variable is approximately (or approximately proportional to) the spread between a forward rate $f_i(t, m, n)$ and the spot rate of the same maturity as the forward rate, $r_i(t, t + m)$. This spread forecasts the change in $r_i(t, t + m)$ over the next n periods. Regressions in the literature that can be interpreted at least approximately as regressions of the actual change in spot rates $r_i(t + n, t + m + n) - r_i(t, t + m)$ on the predicted change $f_i(t, m, n) - r_i(t, t + m)$ and a constant are shown in Table 13.2.

What is clear from Table 13.2 is that the slope coefficient is quite far below one – and often negative – for low forecast horizon n , regardless of the maturi-

Table 13.2
Regressions of changes in m -period interest rates on changes predicted by the term structure:
 $r_i(t + n, t + m + n) - r_i(t, t + m)$ on $f_i(t, t + n, t + m + n) - r_i(t, t + m)$ and constant

Study	Country	Sample	m (years)	n (years)	Slope coef.	Std. error	R^2
Shiller (1979) ^a	U.S.	1966–77	>20.0	0.25	-5.56	1.67	0.201
	U.S.	1919–58	>20.0	1.00	-0.44	0.75	0.01
	U.K.	1956–77	∞	0.25	-5.88	2.09	0.09
Shiller, Campbell and Schoenholz (1983) ^b	U.S.	1959–74	0.25	0.25	0.27	0.18	0.03
	U.S.	1959–73	30.0	0.50	-1.46	(1.79)	0.02
Mankiw (1986) ^c	Canada	1961–84	0.25	0.25	0.10	(0.07)	0.02
Fama (1984a) ^d	W Germany	1961–84	0.25	0.25	0.14	(0.07)	0.03
	U.S.	1959–82	1/12	1/12	0.46	(0.07)	0.13
			1/12	2/12	0.25	(0.10)	0.02
			1/12	3/12	0.26	(0.12)	0.02
			1/12	4/12	0.17	(0.10)	0.01
Fama and Bliss (1987) ^e	U.S.	1964–84	1.00	1.00	0.09	(0.28)	0.00
			1.00	2.00	0.69	(0.26)	0.08
			1.00	3.00	1.30	(0.10)	0.24
			1.00	4.00	1.61	(0.34)	0.48
Shiller (1986) ^f	U.S.	1953–86	0.25	rollover*	0.61	(0.17)	0.090

Note Expectations theory of the term structure asserts that the slope coefficient should be 1.00. Not all regressions summarized here were in exactly the form shown here, in some cases a linearization was assumed to transform results to the form shown here. Significance level refers to a test of hypothesis that the coefficient is 1.00.

^aDependent variable is approximately $S^*(m, n)$ and the independent variable is $S(m, n)$ as defined in expression (26) in text.

^bPage 1210, table 3, rows 1, 4 and 5. Column 2 coefficient was converted using duration implicit in γ_n given in Table 13.1 rows 1, 4 and 5 column 1.

^cPage 192, table 3, rows 4 and 10, columns 5 and 6.

^dPage 81, table 9, rows 2 and 4, columns 3 and 4.

^ePage 517, table 4, rows 6–10, columns 1–2.

^fPage 686, table 3, rows 1–4, columns 3–4.

*Page 103

ty m of the forward interest rate, but rises closer to one for higher n . This result may at first seem counterintuitive. One might have thought that forecasts into the near future would be more accurate than forecasts into the more distant future; the reverse seems to be true.

When both n and m are small, both less than a year or so, the slope coefficients are positive (the right sign) but substantially lower than one. Thus, for example, when two-month interest rates exceed one-month rates by more than the average term premium, $\Phi_{r,t}(t, 2, 1)$, the one-month rate does tend to increase as predicted, but by substantially less than the predicted amount.⁴³

The results in Table 13.2 look especially bad for the rational expectations hypothesis when the forecast horizon n is small (a year or less in the table) and the maturity of the forward rate m is large (20 or more years in the table). Here, the spread between the forward rate and spot rate predicts the wrong direction of change of interest rates. One might consider it the “essence” of the rational expectations hypothesis that an unusually high spread between the forward rate and current spot rate portends increases in interest rates, not the decreases as observed.

It is helpful in interpreting this result to consider a caricature, the case of a perpetuity (for which $m = \infty$) paying coupon c once per period, and where, for simplicity, the term premium is zero. Then the price of the perpetuity $p_p(t, \infty)$ equals coupon over yield $c/r_p(t, \infty, 1)$, and the spread between the one-period-ahead forward consol yield and the one-period spot rate is proportional to the spread between the consol yield and the one-period rate. When the consol yield is above the one-period interest rate, $r(t, t+1, 1)$, then its current yield $c/p_p(t, \infty)$ is greater than the one-period rate $r(t, t+1, 1)$. This would suggest that consols are then a better investment for the short run than is short debt. Since the rational expectations hypothesis with zero term premium would deny this, it follows that the consol yield $r_p(t, \infty, 1)$ should be expected to increase over the next period, producing a decline in price, a capital loss that offsets the high current yield. But, in fact when the consol yield is high relative to the short rate the consol yields tends to fall subsequently and not rise.⁴⁴ The capital gain tends to augment rather than offset the high current yield. The naive rule that long bonds are a better investment (in an expected value sense) whenever long rates are above short rates is thus confirmed.

⁴³Regressions for large n and small m are not in Table 13.2. Since such forward rates are very sensitive to rounding error or small noise in the long-term interest rates, we cannot accurately measure such forward rates.

Some more favorable results for the expectations theory with small n were reported in Shiller (1981a); however, these results were later found to be related to a couple of anomalous observations [Shiller, Campbell and Schoenholz (1983)].

⁴⁴That long rates tend to move opposite the direction indicated by the expectations theory was first noted by Macaulay (1938, p. 33) “the yields of bonds of the highest grade should *fall* during a period when short rates are higher than the yields of bonds and *rise* during a period in which short rates are the lower. Now experience is more nearly the opposite.”

Froot (1987) attempted a decomposition of the departure from 1.00 of the coefficient in Table 13.2 here into two parts: a part due to expectation error and a part due to time-varying term premium. He used survey data published in the investor newsletter *Reporting on Governments* (continuing the Goldsmith–Nagan data series) to represent expectations. He found that for three-month-ahead forecasts of three-month rates, the departure from 1.00 is due primarily to time-varying term premium. But for forecasts of changes in 30-year mortgage rates, the expectations error bears most of the blame for the departure of the coefficient from 1.00.⁴⁵

5.3. The volatility of long-term interest rates

According to the rational expectations theory of the term structure, n -period interest rates are a weighted moving average of one-period interest rates plus a constant term premium; that is, from (16) and Table 13.1:⁴⁶

$$r_i(t, t+m) = D_i(m)^{-1} \sum_{k=0}^{m-1} (D_i(k+1) - D_i(k)) \times E_i r_i(t+k, t+k+1) + \Phi_m, \quad i = p, d, \quad (25)$$

where $\Phi_m = \Phi_{r,i}(t, t+m, 1)$ is constant through time. Since long moving averages tend to smooth the series averaged, one might expect to see that long rates are a very smooth series. Are long-term rates too “choppy” through time to accord with the expectations theory? It is natural to inquire whether this is so and, if so, whenever it is possibly related to the poor results for the expectations hypothesis that were obtained in the Table 13.2 regressions.

Because of the choppiness of long-term interest rates, short-term holding returns on long-term bonds, which are related to the short-term change in long-term interest rates, are quite variable. Culbertson (1957), in his well-known critique of expectations models of interest rates, thought the volatility of holding yields was evidence against the model. He showed a time-series plot of holding yields on long bonds and, noting their great variability, remarked “what sort of expectations, one might ask, could possibly have produced this result?”⁴⁷

It is possible, using the expectations hypothesis, to put limits on the variability of both long-term interest rates themselves and on short-term

⁴⁵See Froot (1987, table 3). Note that his regressions are run in a slightly different form than in Table 13.2 here, but that our Table 13.2 coefficients can be inferred from his.

⁴⁶For par bonds, it is necessary to evaluate $D_p(k)$ with (5) using a fixed point of linearization r , so that (25) will be linear in interest rates

⁴⁷Culbertson (1957, p. 508)

holding returns on long-term debt. The expectations hypothesis implies that $r_i(t, t+m) = E_i r_i^*(t, t+m) + \Phi_m$, where $r_i^*(t, t+m)$ is the “perfect foresight” or “ex post rational” long-term interest rate defined as:

$$\begin{aligned} r_i^*(t, t+m) &= D_i(m)^{-1} \sum_{k=0}^{m-1} (D_i(k+1) - D_i(k)) \\ &\quad \times E_i r_i(t+k, t+k+1), \quad i = p, d. \end{aligned} \quad (26)$$

It follows that $r_i^*(t, t+m) = r_i(t, t+m) + \Phi_m + u_{mt}$, where u_{mt} is a forecast error made at time t and observed at time $t+m$. Since u_{mt} is a forecast error, if forecasts are rational, u_{mt} cannot be correlated with anything known at time t ; otherwise the forecast could be improved. Hence, u_{mt} must be uncorrelated with $r_i(t, t+m)$. Since the variance of the sum of two independent variables is the sum of their variances it follows that $\text{var}(r_i^*(t, t+m)) = \text{var}(r_i(t, t+m)) + \text{var}(u_{mt})$, and since $\text{var}(u_{mt})$ cannot be negative, the rational expectations model implies [Shiller (1979)]:⁴⁸

$$\text{var}(r_i(t, t+m)) \leq \text{var}(r_i^*(t, t+m)), \quad i = p, d, \quad (27)$$

so there is an upper bound to the variance of m -period rates given by the variance of $r_i^*(t, t+m)$. One can also put an upper bound to the variance of the holding period return in terms of the one-period rate [Shiller (1981a)]:

$$\text{var}(h_i(t, t+1, t+m)) \leq (D(m)/D(1)) \text{var}(r_i(t, t+1)), \quad i = p, d, \quad (28)$$

where in the case $i = p$ of par bonds, $D(m)$ and $D(1)$ are computed from equation (5) above with interest rate $2r$, where r is the point of linearization.

Both of the above inequalities were found to be violated using U.S. data and m of 2 or more years [Shiller (1979, 1981a, 1986) and Singleton (1980b)]. Their rejection could have either of two interpretations: the rational expectations hypothesis could be wrong, in such a way as to make long rates much more volatile than they should be, or the measures of the upper bound in the inequalities could be faulty: the measures of $\text{var}(r_i^*(t, t+m))$ or $\text{var}(r_i(t, t+1))$ could underestimate the true variance.

The latter view of the violation of the inequalities was argued by Flavin (1983) who showed with Monte Carlo experiments that if the one-period interest rate $r_i(t, t+1)$ is a first-order autoregressive process with the autoregressive parameter close to one (see the next subsection), the inequalities are likely to be violated in small samples even if the rational expectations model is true. Such a process shows a great deal of persistence, and $r_i(t, t+1)$ may thus

⁴⁸LeRoy and Porter (1981) also noted this inequality in a different context.

stay on one side of the true mean throughout the sample. Thus, the sample variance around the sample mean of $r_i(t, t+1)$ or of $r_i^*(t, t+m)$ may be a strikingly downward biased measure of their true variance.

Flavin's is apparently a variable interpretation of the excess volatility results. The volatility tests do not allow us to tell whether there is too much variability in long rates or just nonstationarity in short rates. They *do* allow us to reject the idea that movements in long rates can be interpreted in terms of rational expectations of movements in short rates within the range historically observed.

5.4. Encouraging results for the rational expectations hypothesis

It does not follow from the Table 13.2 results with small n that the spread between very long-term interest rates and short-term interest rates is totally wrong from the standpoint of the expectations hypothesis. One way of summarizing the relatively good results [Shiller (1986)] for this spread for larger n is to compute both actual and perfect foresight spreads between very long-term interest rates and short-term interest rates. Defining the spread $S_n(m) = r_i(t, t+m) - r_i(t, t+1)$ (m integer > 1), then the rational expectations hypothesis implies:

$$S_n(m) = E_t S_n^*(m) + \Phi_m, \quad i = p, d, \quad (29)$$

$$S_n^*(m) = r_i^*(t, t+m) - r_i(t, t+1). \quad (30)$$

From the definition (26) of $r_i^*(t, t+m)$, it can be shown that $S_n^*(m)$ is the duration weighted average of expected changes in the n -period rate. Equation (29) thus asserts that when long rates are high relative to short rates the weighted average of increases in short rates should tend to be high. The values of $S_{td}(m)$ and $S_{td}^*(m)$ are plotted for $m = 10$ in Figure 13.4 for those years for which data are available in Appendix B. The correspondence between $S_{td}^*(m)$ and $S_{td}(m)$ is apparent. This might be viewed as a striking confirmation of some element of truth in the expectations hypothesis. Moreover, a variance inequality analogous to (28) above is that $\text{var}(S_n(m)) \leq \text{var}(S_n^*(m))$. This variance inequality is satisfied by the data. This result does not by itself establish whether or not Flavin's view of the variance inequality violation described in the preceding section is correct.⁴⁹

⁴⁹Indeed, even if $S_n(m)$ and $S_n^*(m)$ look good by this criterion, there could be some small noise contaminating $S_n(m)$ which, if the noise is not highly serially uncorrelated, could cause holding period yields to be much more volatile than would be implied by the expectations model. Moreover, the appearance of $S_n(m)$ and $S_n^*(m)$ may also be relatively little affected by a gross overstatement of the variability of $r_i(t, t+m)$, so long as it is substantially less variable than the short rate [see Shiller (1986)].

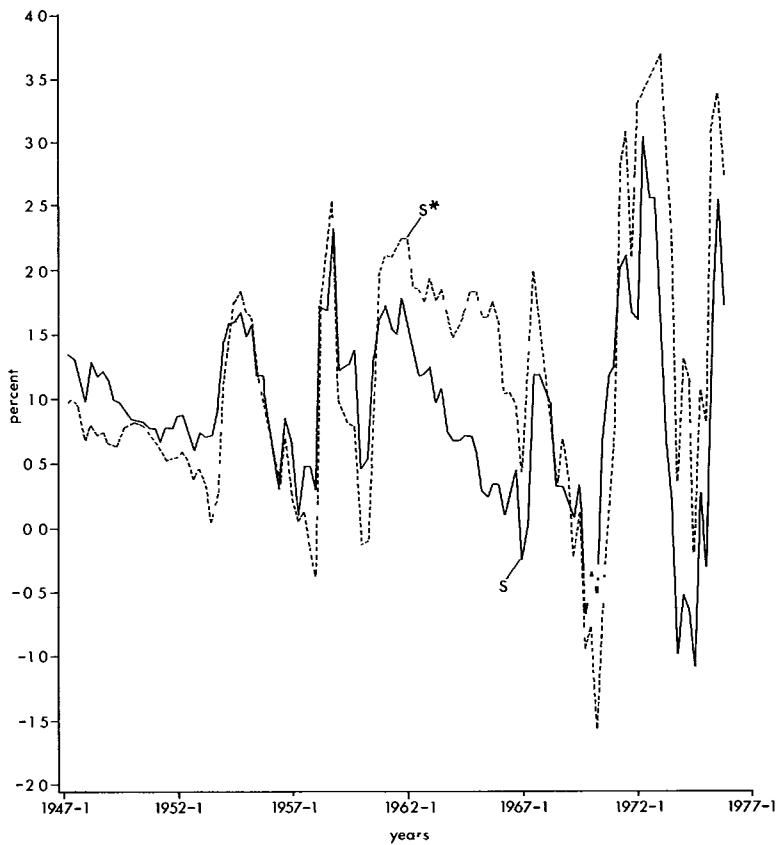


Figure 13.4 The long-short spread $S_{td}(n) = r_d(t, t+n) - r_d(t, t+1)$, solid line, and the perfect-foresight spread $S_{td}^*(n) = r_d^*(t, t+n) - r_d(t, t+1)$, dashed line, where $r_d^*(t, t+n)$ is the perfect foresight n -period rate defined by expression (26), $n = 40$ quarters. Thus, $r_d^*(t, t+n) = (\sum (\tau = t, t+39)r(\tau, \tau+1))/40$. Data plotted are quarterly series for the end of the first month of each quarter using McCulloch's three-month and ten-year discount bond yield series, Table 13 A 1, Appendix B. $S_{td}(n)$ is plotted for 1947 first quarter to 1975 third quarter at annual rates.

One must consider, though, whether this apparent confirmation of the expectations theory for large m could also be described in a less inspiring way: as reflecting largely just that the long rate is much smoother than the short rate. In fact, the correspondence in postwar U.S. data between $S_{td}^*(10, 1)$ and $S_{td}(10, 1)$ would still be apparent if the long rate $r(t, t+10)$ had been a simple trend through the path of short rates. Short-term interest rates have shown an apparent tendency to revert to trend; thus, a duration weighted average of future changes in the short rate is approximately minus the detrended short rate.

It was shown by Modigliani and Shiller (1973) and Shiller (1972) [following

Sutch (1968)] that a regression of a long rate on a distributed lag of short rates produces distributed lag coefficients that crudely resembled the “optimal” distributed lag coefficients implied by an autoregression in first differences for the short rate. Similarly, a regression of the long rate on a distributed lag of short rates and a distributed lag of inflation rates is consistent with a vector autoregression in first differences using the short rate and the inflation rate.⁵⁰ The basic principle of these analyses can be illustrated by assuming for simplicity here (as in Flavin) that the short rate $r_p(t, t+1)$ follows a first-order autoregressive (AR-1) process around a mean μ : $r_p(t+1, t+2) - \mu = \lambda(r_p(t, t+1) - \mu) + \varepsilon_t$, $0 < \lambda < 1$, where ε_t is a realization of a random variable with zero mean independent of ε_{t-k} , $k \neq 0$. The optimal forecast at time t of $r_p(t+k, t+k+1)$ is:

$$E_t r_p(t+k, t+k+1) = \mu + \lambda^k (r_p(t, t+1) - \mu). \quad (31)$$

From (25) and (5) for $m = \infty$ and $i = p$ the consol yield is given by:

$$r_p(t, t+\infty) = (1 - \gamma) \sum_{k=0}^{\infty} \gamma^k E_t r_p(t+k, t+k+1) + \Phi, \quad (32)$$

where $\gamma = e^{-r}$ and r is the point of linearization. Thus, the consol yield is a sort of present value of expected future one period rates. Together, (31) and (32) imply:

$$r_p(t, t+\infty) = \frac{(1-\gamma)}{(1-\gamma\lambda)} r_p(t, t+1) + \Phi. \quad (33)$$

One can therefore evaluate the rational expectations model by first regressing $r_p(t+1, t+2)$ on $r_p(t, t+1)$ and a constant [i.e. estimating λ in (31)], and computing the theoretical coefficient of $r(t)$ using (33). This theoretical coefficient can be compared with the slope coefficient in a regression of $r_p(t, t+\infty)$ onto $r_p(t, t+1)$ and a constant. Now, in fact, our assumption that $r_p(t, t+1)$ was forecast by the market according to (31) would imply that (33) should hold without error. However, it can be shown that whether or not $r_p(t, t+1)$ is an AR-1 process if $E_t r_p(t+k, t+k+1)$ is the optimal forecast of $r_p(t+k, t+k+1)$ conditional on an information set that includes $r_p(t, t+1)$, then a theoretical regression of $r_p(t, t+\infty)$ on $r_p(t, t+1)$ and a constant should produce the coefficient $(1-\gamma)/(1-\gamma\lambda)$, where λ is the slope coefficient in a theoretical regression of $r_p(t+1, t+2)$ on $r_p(t, t+1)$, [Shiller (1972)]. With

⁵⁰Distributed lag regressions explaining the term structure have had different functional forms see, for example, Bierwag and Grove (1967), Cargill and Meyer (1972) or Malkiel (1966). A comparison of eight different distributed lag models of the term structure is in Dobson, Sutch and Vanderford (1976).

this assumption there is an error term in (33) reflecting information held by market participants beyond $r(t, t+1)$. Comparing such estimated coefficients using more complicated autoregressive models was the method used in the aforementioned papers.

Note that if γ and λ are both near one, then $(1 - \gamma)/(1 - \gamma\lambda)$ may be very sensitive to λ . When data are limited, we cannot tell with much accuracy what λ is, and hence cannot pin down what the value of $(1 - \gamma)/(1 - \gamma\lambda)$ is. Thus, we cannot say with much assurance whether the consol yield in fact is or is not too volatile.

Such simple comparisons of estimated coefficients are not formal tests of the rational expectations model. Rather, they are indications of the “fit” of the model. If we are given data on a consol yield $r_p(t, \infty)$ and the one-period rate $r_p(t, 1)$, then a likelihood ratio test of all restrictions of the model [except for a restriction implied by the stationarity of $r_p(t, \infty)$] amounts to nothing more than a regression of the excess return $h_p(t, t+1, \infty) - r_p(t, t+1) = (r_p(t, \infty) - \gamma r_p(t+1, \infty))/(1 - \gamma) - r_p(t, t+1)$ on information at time t . [Shiller (1981a), Campbell and Shiller (1987)].

Note that such tests may not have much power to determine whether long rates are too volatile to accord with market efficiency. Suppose, for example, that the short rate $r_p(t, t+1)$ is a first-order autoregressive process as above, and suppose that the long rate overreacts to the short rate, $r_p(t, \infty) = (\mu + \Phi) + b(r_p(t, 1) - \mu)$, where $b > (1 - \gamma)/(1 - \gamma\lambda)$. Then the excess holding return $h_p(t, t+1, \infty) - r_p(t, t+1)$, defined as $(r_p(t, \infty) - \gamma r_p(t+1, \infty))/(1 - \gamma) - r_p(t, t+1)$, is equal (up to a constant) to $(c - 1)r_p(t, t+1) - \gamma c r_p(t+1, t+2)$, where $c = b/(1 - \gamma)$. If γ is close to one and c large, then this excess return is approximately proportional to b , and changing b would do little more than scale it up or down. If the excess return is not very forecastable for one b , it is likely also to be not very forecastable for another b . Then, a regression of excess holding returns on the short rate may have little power to detect even major departures of b from $(1 - \gamma)/(1 - \gamma\lambda)$.

Tests of the rational expectations model are not so straightforward when using data on a single long rate that is not a consol yield and a short rate. Sargent (1979) showed how, using a companion-form vector autoregression, it is readily possible to test the restrictions implied by the rational expectations model even with such data. He was unable to reject these restrictions on the vector autoregression of long and short rates using a likelihood ratio test. However, it was later discovered that Sargent's paper did not test all restrictions, and when the additional restrictions were incorporated into the analysis, the hypothesis was rejected [Hansen and Sargent (1981), Shiller (1981a)]. These rejections, however, do not deny the *similarity* between actual and optimal distributed lag coefficients. Campbell and Shiller (1987) used a cointegrated vector autoregressive framework, where the vector contained two

elements, the long rate and the short rate, and confirmed both that the rational expectations model is rejected with a Wald test and that the model is of some value in describing how long rates respond to short rates and their own lagged values.

There is also some evidence that the relation of long rates to lagged interest rates changes approximately appropriately when the stochastic properties of interest rates change. It was shown by Shiller (1987) that such a correspondence between the distribution lag coefficients holds up crudely speaking even when one uses nineteenth-century U.S. data, or nineteenth- or twentieth-century British data. In the nineteenth century in Britain, for example, short rates appeared to be sharply mean reverting, so that long rates should have been nearly constant: indeed the distributed lag regressions of the British consol yield on the short rates showed sharply reduced coefficients relative to the twentieth-century coefficients in a distributed lag regression of long rates on short rates. Mankiw, Miron and Weil (1987) found an abrupt, and they interpreted appropriate, given the rational expectations model, change in the distributed lag coefficients, at the time of the founding of the Federal Reserve.

How is it then that the forward-spot spread $f(t, t + m, t + m + n) - r_p(t, t + m)$ seems to predict well only for large n and not small n ? Fama and Bliss (1987) interpreted this finding as reflecting the fact that interest rates are not very forecastable into the near future, but better forecastable into the more distant future. He gave as an example the story of AR-1 Model described in connection with equations (31)–(33) above. The expectation as of time t or the change $r(t + n, t + n + 1) - r(t, t + 1)$ is $(\gamma^n - 1)(r(t, t + 1) - \mu)$. For γ close to, but below, one, the variance of the expected change is quite small for small n , and grows with n . Thus, for small n any noise in the term premium might swamp out the component in the forward-spot spread $F(t, t + n, t + m + n) - r(t, t + m)$ that is due to predictable change in interest rates.

5.5. Interpreting departures from the expectations theory

Of course, as a matter of tautology, the fact that the coefficients in Table 13.2 do not all equal one has something to do with time-varying term premia. But the nature of the time varying term premia has not been given an ample description for all n and m .

One story for the negative coefficients in Table 13.2 for large m (≥ 20 years) and small n (\leq one year) is that there might be noise in term premia on long-term interest rates unrelated to short-term interest rates. The noise might be due to exogenous shift to investor demand, or even to changing fashions and fads in investing. Suppose, for example, that this “noise” is serially uncorrelated, as though it were due to an error in measuring long-term interest

rates.⁵¹ Consider for simplicity consols, $m = \infty$, for which $f_p(t, t+n, \infty) - r_p(t, \infty) = (D_p(n)/(D_p(\infty) - D_p(n)))(r_p(t, \infty) - r_p(t, n))$. If one regresses $r_p(t+n, \infty) - r_p(t, \infty)$ on this, then one has $r_p(t, \infty)$ on both sides of the equation with opposite signs. Thus, any “noise” in $r_p(t, \infty)$ might give a negative slope coefficient in the regression.

This simple story about extraneous noise like measurement error in long rates, while suggestive, is not completely adequate in explaining the wrong signs in the Table 13.2 regressions for large m and small n . If the problem were just exogenous noise in long rates then an instrumental variables approach to the estimation of the above regressions with economic variables as instruments would correct the wrong sign; yet it does not [Mankiw (1986)].

A different story for the wrong sign in the regression is that long rates do not react properly to short rates. The distributed lag regressions noted above of long rates on short rates, while similar to the distributed lag implied by an autoregressive forecasting regression for short rates, are not quite the same. In fact, the distributed lag coefficients of long rates on short rates tend to show too simple a pattern, like a simple exponential decay pattern instead of a relatively choppy pattern seen in the optimal responses of long rates to short rates implied by the forecasting equation [Shiller (1987)]. This result might come about because people who price long bonds tend to blur the past somewhat in their memories, or because people use a simple “conventional” pricing rule for long bonds.⁵²

5.6. Seasonality and interest rates

The above discussion suggests that the expectations hypothesis works best when interest rate movements are well forecastable. With many economic variables seasonal movements are forecastable far into the future. If there is any seasonality in interest rates, one would expect to see a seasonal pattern to the term structure. We would not expect that long rates and short rates should show the same seasonal pattern, that is, reach their highest point in the same month. Instead, the expectations theory would predict a phase shift between

⁵¹ Just as well, the wrong signs in some regressions could be due to measurement error in interest rates, a point considered and rejected as the main explanation for the wrong signs by Shiller (1979) and Mankiw (1986). However, measurement error is taken more seriously by Brown and Dybvig (1986). They needed measurement error to study the one-factor version of the Cox–Ingersoll–Ross model because without it there would be a perfect dependence among the interest rates of different maturity.

⁵² Keynes (1936) said that the long rate is “highly conventional . . . its actual value is largely governed by the prevailing view as to what its value is expected to be.” The idea here is apparently that a simple rule of thumb used to price long-term bonds may become validated when market prices appear to follow the rule.

long and short rates. Macaulay (1938) investigated whether this occurred using data on call money and time rates 1890 to 1913, and concluded that there was “evidence of definite and relatively successful forecasting”,⁵³ for seasonal movements, though not for movements other than seasonals.

Sargent (1971) noted that the maturity on the call rates was not well defined, and in fact the actual maturities of the call loans are likely to have had a seasonality themselves. He thus sought to reproduce Macaulay’s work using more recent data for which maturity can be defined more precisely. Sargent showed that in a perfect foresight model the simple expectations theory for discount bonds implies that the m -period rate $r_d(t, t + m)$ should lead the one-period rate $r_d(t, t + 1)$ by $(m - 1)/2$ periods across all frequencies. He used U.S. Treasury Bill rates on one to thirteen week bills for 1953 to 1960. He found that long rates did tend to lead short rates at the seasonal frequencies, but by much less than the theoretical $(m - 1)/2$.

The post World War II data set that Sargent used, however, contained a much milder seasonal than was evident in the prewar data that Macaulay had used. The Federal Reserve was founded in 1913 to “provide an elastic currency” and this clearly meant that one of their missions was to eliminate seasonals, which they then largely did [see Shiller (1980) and Miron (1984, 1986)].⁵⁴ Mankiw and Miron (1986), using a time series on pre-1913 U.S. interest rates of three and six months maturity, found more encouraging results for the expectations theory.

5.7. The sign of term premia

Kim (1986) investigated whether the observed term premium between nominal three- and six-month treasury bills in the United States 1959–86 could be reconciled with the covariance between $S(t, t + 3)$ and $S(t + 3, t + 6)$ as described by equation (22) or (24) as the theory prescribes. He used a co-integrated vector autoregressive model for the two log interest rates, log consumption and a log price index, and a lognormality assumption for the error term. He transformed the vector with the co-integrating vector so that the transformed vector has as elements the spread between the two log interest rates, the change in one of the interest rates, the change in log real consumption and the change in the log price index. For the model, the covariance in equation (22) is constant through time. He tested the restrictions across the

⁵³ Macaulay (1938, p. 36).

⁵⁴ Clark (1986) questioned whether the decline in seasonality was due to the founding of the Fed. He noted that seasonals disappeared in the United States and other countries at about the same time, and that seasonals disappeared approximately three years before the seasonals in currency and high powered money changed.

mean vector, coefficient vector, and variance matrix of residuals using a Wald test. The test rejected the restrictions; on the other hand, the sign of the term premium is as predicted by the sign of the covariance.

Other studies of consumption and the term structure of interest rates looked at short-term real returns on long and short bonds and their correlation with real consumption changes to see if the difference in mean real returns between long and short bonds could be reconciled with the covariance of real returns with real marginal rates of substitution. Grossman, Melino and Shiller (1987) found that the excess real one-period returns between long-term debt and short-term debt had negligible correlation with real per capita consumption changes with annual U.S. data 1890–1981 and with U.S. quarterly data 1953–83. They rejected at high significance levels the covariance restrictions using a vector autoregression model including real returns on long-term debt, short-term debt, and corporate stocks.⁵⁵

5.8. Modelling time-varying term premia

Since the rational expectations hypothesis can be rejected, as discussed above, it follows that the term premium is time varying. Although the term premium is not observed itself without error, we can study its projection onto any information set by regressing the variables represented on the right-hand sides of the expressions defining term premia, i.e. (14), (15), and (16) above, from which the expectations operators have been deleted, onto information available at time t . The above discussion of the projection onto the forward-spot spread concerns only one possible such regression. There is no theory of the term structure well-developed enough to allow us to predict what variables to use, so the empirical literature here often looks like a “fishing expedition”.

Kessel (1965) regressed the forward-spot spread $f_d(t, t+1, t+2) - r_d(t+1, t+2)$ on $r_d(t, t+1)$, where the time unit is four weeks, to test whether term premia are related to the level of interest rates. He found, using monthly U.S. Treasury Bill data 1949–61, that there was a positive coefficient on $r_d(t, t+1)$. However, Nelson (1972b), using analogous methodology, found the opposite sign for the coefficient of the interest rate. Both Kessel and Nelson gave theories why risk considerations should imply the sign they got. Shiller (1979) in effect regressed $f_p(t, t+1, t+m+1) - r_p(t+1, t+m+1)$ on $r_p(t, t+m)$ for m very large with quarterly, monthly, and annual time periods for U.S. and U.K. history and found a consistently positive coefficient, which was interpreted as a sign of possible excess volatility of long rates. Campbell and Shiller

⁵⁵Mankiw (1986) inquired whether the time variation in the covariance could be reconciled with time variation in the spread between long and short rates in the United States, Canada, the United Kingdom, and Germany 1961–84. He concluded that it could not

(1984) in effect found a negative slope coefficient in a regression (in effect) of $f_d(t, t+1, t+241) - r_d(t+1, t+241)$ on $r_d(t, t+1)$, where time is measured in months, and interpreted this result as reflecting a possible underreaction of long rates to short rates. It is difficult to produce a useful summary of these conflicting results.

Other researchers have used some indicators of time-varying risk premia in such regressions. Modigliani and Shiller (1973) and Shiller, Campbell and Schoenholtz (1983) used a moving standard deviation of interest rates. Fama (1976), Mishkin (1982), and Jones and Roley (1983) used other measures of the variability in interest rates. Such measures were often statistically significant. Engle, Lilien and Robins (1987) used an ARCH model to model time-varying variance of interest rates, and concluded that the risk premium so modelled helps to explain the failures of the expectations theory.

Still other variables have been used to explain time-varying term premia. Nelson (1972b) used an index of business confidence. Shiller, Campbell and Schoenholtz (1983) used a measure of the volume of trade in bonds. Keim and Stambaugh (1986) used a low-grade yield spread variable (the difference between yields on long-term under-BAA-rated corporate bonds and short-term Treasury bills), and a small-firm variable (the log of the share price, averaged equally across the quintile of smallest market value on the New York Stock Exchange). Campbell (1987) used a latent variable model of the returns on bills, bonds and common stocks to infer time-varying risk premia in all three markets.

5.9. Flow of funds models

Clearly, term premia do vary and are correlated with observable economic variables. But what kind of structural model might clarify why they vary? One might expect that when the federal government issues a large amount of long-term debt, the supply of long-term debt should rise and, other things equal, term premia should rise. One might also expect that in time when funds flow into life insurance companies, major purchasers of long-term bonds, then the demand for long-term debt should rise and, other things equal, term premia should decline. Thus, the term structure might be related to such flows of funds.⁵⁶

⁵⁶Conversely, when the government attempts to peg the term structure, there should be consequential flows of demand across maturities. Walker (1954) noted that when the Federal Reserve attempted to peg an upward-sloping term structure there was a great shift out of short-term securities into long-term securities by the holders of government debt. Such a shift is implied by the expectations hypothesis.

There was a flurry of research on the effects of government debt policy on the term structure following the policy, brought in by the Kennedy Administration in the United States in 1961, known as “Operation Twist”. Operation Twist consisted of Federal Reserve open market operations and Treasury debt management operations directed toward shortening the average term to maturity of outstanding public debt, with the intention of “twisting” the term structure.⁵⁷ Okun (1963) and Scott (1965) correlated the term structure with federal debt measures without accounting for expectations. Modigliani and Sutch (1966, 1967) added dummy or debt composition variables to their distributed lag regressions of long rates on short rates, but found evidence of only a “weak” effect of national debt on the term structure. Indeed, the simple distributed lag on short rates explained long rates so well that there was little room for much improvement of fit using debt policy variables.⁵⁸ The Modigliani–Sutch conclusions were criticized by Wallace (1967) for the assumption that government debt policy is exogenous over the sample period.

There is a substantial literature on models that relate interest rates to such flows of funds; see, for example, Ando and Modigliani (1975), Brainard and Tobin (1968), De Leeuw (1965), Friedman (1977a, 1980a), Hendershott (1971), or Backus, Brainard, Smith and Tobin (1980). But much of this literature makes no explicit use of expectations of future interest rates that ought to play a pivotal role in the term structure of interest rates. Many of the models are not complete, e.g. providing estimates of some demands for funds, and not providing a general equilibrium that might give a theory of the term structure.

Friedman and Roley (1979) and Roley (1982) estimated a flow of funds model [along the lines of Friedman (1977a, 1980b), and Roley (1977)] but incorporating as determinants of the demand functions not yields to maturity but rational expectations of short-run returns.

Flow of funds modelling has offered the promise of estimating consistently general equilibrium models of the determination of interest rates, but such modelling has to date been hampered by the same problems that have prevented any consensus on other macroeconomic models. A lot of subjective judgment goes into specifying the identifying restrictions, exogeneity specifications and other assumptions that lie behind a complicated simultaneous equation model. Hence, there is a lot of uncertainty about the validity of particular models.

⁵⁷ Operation Twist also involved relaxing some interest rate ceilings. The federal debt structure during the early 1960s in fact went in exactly the opposite direction to what was implied by Operation Twist, as the Treasury's debt policy was contradicting the Fed's. See Friedman (1981).

⁵⁸ Friedman (1977a, 1981) did find a significant coefficient in a term structure equation for a variable which was the ratio of outstanding federal long-term securities to outstanding federal short-term securities.

6. Some concluding observations

There has been a lot of progress in our understanding of the term structure in the last twenty years. We now have formal heuristic theoretical models of the term structure in terms of the ultimate objectives of economic agents and the stochastic properties of forcing variables. These models are beginnings that have changed our way of thinking about the term structure. We now have an extensive empirical literature describing in great detail how the term structure is correlated with other economic variables. But we could hope for still more progress.

It is of course very difficult to say where the actual opportunities for productive research lie, but it is possible to say where there are problems to be solved.

Theoretical work on the term structure, while it has offered many insights, still does not allow us to say much about the term structure we observe. Most theorists are currently using a representative individual utility of consumption model, while most corporate and government bonds in the United States are held by institutions. Even if institutions were somehow behaving as if they were representative consumers, we must face the fact that the expected present value of utility of consumption model has not held up well in tests of the returns on assets other than bonds. Probably, the theoretical model is just not a good descriptor of human behavior.

Most of the theoretical work on the expectations hypothesis has worked on the term structure of index bonds, but freely tradable true index bonds of varying maturity are virtually nonexistent. The theoretical literature has tried to find justifications for a zero term premia model, while the assumption of zero term premia has never been an issue for empirical researchers. That term premia are not zero and change through time has not suggested any well-posed problems for theoretical researchers working in the current paradigm that would produce any idea as to how to expect them to change.

Empirical work on the term structure has produced consensus on little more than that the rational expectations model, while perhaps containing an element of truth, can be rejected. There is no consensus on why term premia vary. There does not seem even to be agreement on how to describe the correlation of the term premia with other variables. A lot more research could be done leading to consensus on, for example, the senses in which long rates may be influenced by government fiscal policy, term premia are related to some measures of risk, interest rates overreact or underreact to short rates, or be influenced by or depend on rules of thumb or “satisficing” behavior. Flow of funds models have some interest, but seem to have been largely dropped by researchers in the wake of the rational expectations revolution, just when they should have been integrated with it.

Appendix A: Mathematical symbols

$D_i(m, t)$ = Duration of an m -period bond at time t . Second argument will sometimes be omitted, $i = d$: discount bond, $i = p$: par bond.

$f_d(t, t', T)$ = The forward discount interest rate at time t applying to the interval from t' to T , $t \leq t' \leq T$. The term of the forward instrument is $m = T - t'$.

$F_p(t, t', T)$ = The forward par interest rate at time t applying to the interval from t' to T , $t \leq t' \leq T$. The term of the forward instrument is $m = T - t'$.

$f_p(t, t', T)$ = Linear approximation to $F_p(t, t', T)$.

$h_d(t, t', T)$ = The discount holding period return. If $t \leq t' \leq T$ it is the return from buying a discount bond at time t that matures at time T and selling it at time t' . If $t \leq T \leq t'$, it is the rate of return from rolling over discount bonds of maturity $m = T - t$, until time t' .

$H_p(t, t', T)$ = The par holding period return. If $t \leq t' \leq T$, it is the return from buying a par bond at time t that matures at time T , receiving coupons between t and t' and selling it at time t' . If $t \leq T \leq t'$, it is the rate of return from rolling over par bonds of maturity $m = T - t$ until time t' .

$h_p(t, t', T)$ = Linear approximation to $H_p(t, t', T)$.

m = the term of a bond, equal to the time to maturity $T - t$.

$p(t, T)$ = The price at time t of a bond that matures at time T , whose principal is 1.

$p_i(t, T)$ = The price at time t of a bond that matures at time T , whose principal is 1, $i = d$: discount bond, $i = p$: par bond.

$r(t, T)$ = The interest rate (yield to maturity) at date t on a bond that matures at date T , continuous compounding.

$r(t, T, h)$ = The interest rate (yield to maturity), compounded every h periods, at date t on a bond that matures at date T .

$r_d(t, T)$ = The interest rate (yield to maturity) at date t on a discount bond that matures at date T , continuous compounding.

$r_p(t, T)$ = The interest rate (yield to maturity) at date t on a par bond that matures at date T , continuous compounding.

s_i = The amount of the i th payment made on a bond, made at date t_i on a coupon bond $s_i = c$, $i < T$, $s_T = 1 + c$.

t_i = The date of the i th payment on a bond.

T = The date on which a bond matures.

$\Phi_{f,i}(t, t', T)$ = Forward term premium, equal to $f_i(t, t', T) - E_t r_i(t', T)$, $t < t' < T$, $i = p, d$.

$\Phi_{h,i}(t, t', T)$ = Holding period term premium, equal to $E_t h_i(t, t', T) - r_i(t, t')$, $t < t' < T$, $i = p, d$.

$\Phi_{r,i}(t, t', m)$ = Rollover term premium, equal to $r_i(t, t') - E_t h_i(t, t', t + m)$,
 $t < t + m < t'$, $i = p, d$.

w = The number of payments promised by a bond when it was issued.

Appendix B: U.S. term structure data, 1946–87⁵⁹ (by J. Huston McCulloch)

The three tables that follow summarize the term structure of interest rates on U.S. Treasury securities from December 1946 to February 1987.

Table 13.A.1 shows the zero-coupon yield curve on an annual percentage, continuously compounded basis. This yield curve is inferred from the prices of whole securities, rather than being based on the recently developed (but much less liquid) market for stripped Treasury securities. In Shiller's notation, this is $100r_d(t, t + m)$, as used in his (1).

Table 13.A.2 shows the instantaneous forward rate curve on the same annual percentage, continuous compounding basis. This curve shows the marginal return to lengthening an investment in m -year zeroes by one instant. The zero coupon yield for maturity m is the unweighted average of these forward rates between 0 and m . In Shiller's notation, these forward rates are $100f(t, t + m)$, as used in his (7).

Table 13.A.3 shows the par bond yield curve, again on an annual percentage, continuously compounded basis. This is defined as the (unique) coupon rate that would make a bond of maturity m be quoted at par, and gives a precise meaning to the ambiguous conventional concept of a "yield curve" for coupon bonds. The par bond yield for maturity m is a weighted average, with declining weights, of the forward rates between 0 and m . The tabulated values are essentially $100r_p(t, t + m)$, as used in Shiller's (3).

These values were computed by fitting the discount function that gives the present value of a future dollar to Treasury security prices. This discount function [$p_d(t, T)$ in Shiller's nomenclature] was curve fit with a cubic spline, as described in McCulloch (1975b), and as modified at NBER-West during 1977–78.⁶⁰

Briefly, the data sets include most of the marketable U.S. government bills, notes and bonds. Closing bid and asked quotations for the last working day of

⁵⁹Written by J. Huston McCulloch while he was Visiting Professor at l'Ecole Supérieure des Sciences Économiques et Commerciales (ESSEC), Cergy, France, on professional leave from the Ohio State University Economics Department.

⁶⁰This NBER version fits the actual "flat" price to the sum of the values of the individual payments. This is slightly more accurate than the version I developed at the Treasury in 1973 and described in McCulloch (1975b), which fit the "and interest" price to an idealized continuous coupon flow. McCulloch (1971) contains further background information on this procedure.

In only one instance prior to March 1986, namely the zero-maturity rates for May 1958, did the cubic spline indicate a negative interest rate, of -0.11 percent. This value was not significantly negative, however (its estimated standard error was 0.54 percent), and so it was replaced with a zero in the tables. The zero at $m = 0$ for May 1947 is the actually estimated value. Since March 1986 forward rates in the range 27 to 29 years have often been negative, but these maturities are beyond the range of the tables.

the month indicated, as reported in dealer quote sheets or the next day's *Wall Street Journal*, were averaged. These observations were given weights inversely proportional to the bid-asked spread. Callable bonds were treated as if maturing on their call dates, if currently selling above par, and as if running to final maturity, if currently selling below par. "Flower bonds" (redeemable at par in payment of estate taxes if owned by the decedent at the time of death) could not all be eliminated, as they constituted the bulk of the observations for many maturities during the earlier part of the period. Accordingly, they were selectively eliminated during these years if the estate feature appeared to be active. For further details see McCulloch (1981, pp. 229-230). Since August 1985, callables are not used.

During the early 1970s, a legislative ceiling on the interest rates the Treasury could pay on long-term debt effectively prevented the issue of new bonds. As existing bonds approached maturity, the longest available maturity therefore fell to under 15 years, so that values over 10 years are occasionally missing during this period. The longest available maturity sometimes also fluctuates by five years from month to month if the longest securities are callable and hovering near par. Since the cubic spline does not lend itself to extrapolation, this methodology cannot be used to infer longer term interest rates than those shown.⁶¹

The curve-fitting procedure was adjusted for tax effects, as described in McCulloch (1975b). The capital gains advantage on deep discount bonds could not be ignored during the earlier part of the period, when most of the long-term bonds were heavily discounted. The importance of this adjustment greatly diminished after 1969, however, when the tax laws were changed so that commercial banks were required to treat capital gains and losses symmetrically. After this date, the best fitting apparent marginal tax rate generally was much lower than before, and was often less than 10 percent.⁶²

The par bond yields in Table 13.A.3 are based on hypothetical continuous-coupon bonds, and therefore are on a continuous-compounding basis directly comparable to the rates in Tables 13.A.1 and 13.A.2. "Bond yields" quoted in the press and elsewhere are instead on a semiannual compounding basis. Following Shiller's terminology (Section 2), each continuously compounded value, $r_p(t, t+m)$, in Table 13.A.3 may be converted to its semiannually compounded equivalent value, $r_p(t, t+m, 0.5)$, by means of

$$r_p(t, t+m, 0.5) = 2(e^{0.5r_p(t, t+m)} - 1).$$

This adjustment would make the rates several basis points higher than in the tables.

⁶¹The exponential spline approach proposed by Vasicek and Fong (1982) has the considerable virtue of making such as extrapolation meaningful. Chen (1986) has implemented the VF approach along with a modification proposed by the present author, with mixed preliminary results, the forward curves are better behaved at the long end, but often the restrictions implicit in the VF model and in the modified model can be formally rejected with a likelihood ratio test

⁶²It should be noted that the identity (9') holds for the values in Table 13 A 3, but only using the discount function that applies to *after-tax* payments Cf Shiller's footnote 12.

References

- Amsler, C. (1984), 'A "pure" long-term interest rate and the demand for money', *Journal of Economics and Business*, 36: 359–370
- Ando, A. and A. Kennickell (1983) 'A reappraisal of the Phillips curve and the term structure of interest rates', University of Pennsylvania.
- Ando, A. and F. Modigliani (1975) 'Some reflections on describing structures in financial sectors', in G. Fromm and L. Klein, eds., *The Brookings model: Perspectives and recent developments*. Amsterdam North-Holland
- Backus, D., W.C. Brainard, G. Smith and J. Tobin (1980) 'A model of U.S. financial and nonfinancial economic behavior', *Journal of Money, Credit and Banking*, 12: 259–293
- Begg, D.K.H. (1984) 'Rational expectations and bond pricing: Modelling the term structure with and without certainty equivalence', *Economic Journal*, 94: 45–58.
- Benninga, S. and A. Protopapadakis (1983) 'Real and nominal interest rates under uncertainty: The Fisher theorem and the term structure', *Journal of Political Economy* 91: 856–867
- Bierwag, G.O. and M.A. Grove (1967) 'A model of the term structure of interest rates', *Review of Economics and Statistics*, 49: 50–62.
- Bohm-Bawerk, E.V. (1891) *The positive theory of capital*. G.E. Stechert & Co.
- Board of Governors of the Federal Reserve System (1985) *Flow of funds accounts financial assets and liabilities year-end, 1961–84*. Washington
- Bodie, Z., A. Kane and R. McDonald (1984) 'Why haven't nominal rates declined?', *Financial Analysts Journal*, 40: 16–27
- Brainard, W.C. and J. Tobin (1968) 'Pitfalls in financial model building', *American Economic Review Papers and Proceedings*, 58: 99–122
- Brealey, R. and S. Schaefer (1977) 'Term structure with uncertain inflation', *Journal of Finance*, 32: 277–289.
- Brennan, M.J. and E.S. Schwartz (1980) 'Conditional predictions of bond prices and returns', *Journal of Finance*, 35: 405–417
- Brown, S. and P. Dybvig (1986) 'The empirical implications of the Cox, Ingersoll, Ross theory of the term structure of interest rates', *Journal of Finance*, 41: 616–630
- Buse, A. (1967) 'Interest rates, the Misenman model and random numbers', *Journal of Political Economy*, 75: 49–62
- Campbell, J.Y. (1984) 'Asset duration and time-varying risk premia', unpublished Ph.D. dissertation, Yale University
- Campbell, J.Y. (1986a) 'Bond and stock returns in a simple exchange model', *Quarterly Journal of Economics*, 101: 786–803
- Campbell, J.Y. (1986b) 'A defense of traditional hypotheses about the term structure of interest rates', *Journal of Finance*, 41: 183–193
- Campbell, J.Y. (1987) 'Stock returns and the term structure', *Journal of Financial Economics*, 18: 373–399.
- Campbell, J.Y. and R.J. Shiller (1984) 'A simple account of the behavior of long-term interest rates', *American Economic Review Papers and Proceedings*, 74: 44–48
- Campbell, J.Y. and R.J. Shiller (1987) 'Cointegration and tests of present value models', *Journal of Political Economy*, 95: 1062–1088
- Campbell, J.Y. and R.J. Shiller (1988) 'The dividend price ratio and expectations of future dividends and discount factors', *Review of Financial Studies*, 1: 195–228
- Cargill, T.F. and R.A. Meyer (1972) 'A spectral approach to estimating the distributed lag relationship between long-term and short-term interest rates', *International Economic Review*, 13: 223–238
- Chambers, D.R., W.T. Carlton and D.W. Waldman (1984) 'A new approach to the estimation of the term structure of interest rates', *Journal of Financial and Quantitative Analysis*, 19: 233–252
- Chen, E.T. (1986) 'Estimation of the term structure of interest rates via cubic exponential spline functions', unpublished doctoral dissertation draft, The Ohio State University.
- Clark, J.B. (1895) 'The gold standard of currency in light of recent theory', *Political Science Quarterly*, 10: 389–403

- Clark, T A. (1986) 'Interest rate seasonals and the Federal Reserve', *Journal of Political Economy*, 94 76–125
- Conard, J.W (1959) *An introduction to the theory of interest* Berkeley and Los Angeles: University of California Press.
- Cox, J C , J.E. Ingersoll, Jr and S.A. Ross (1981) 'A reexamination of traditional hypotheses about the term structure of interest rates', *Journal of Finance*, 36. 769–799.
- Cox, J C , J E. Ingersoll, Jr and S.A. Ross (1985a) 'An intertemporal general equilibrium model of asset prices', *Econometrica*, 53. 363–384.
- Cox, J.C , J E Ingersoll, Jr. and S.A. Ross (1985b) 'A theory of the term structure of interest rates', *Econometrica*, 53. 385–408
- Culbertson, J.M (1957) 'The term structure of interest rates', *Quarterly Journal of Economics*, 71: 485–517
- De Leeuw, F (1965) 'A Model of Financial Behavior', in: J Duesenberry et al , eds., *The Brookings quarterly economic model of the United States* Chicago Rand McNally, pp. 465–530
- Diller, S (1969) 'Expectations and the term structure of interest rates', in J. Mincer, ed , *Economic forecasts and expectations* New York. National Bureau of Economic Research.
- Dobson, S W (1978) 'Estimating term structure equations with individual bond data', *Journal of Finance*, 33 75–92
- Dobson, S W , R.C. Sutch and D E Vanderford (1976) 'An evaluation of alternative empirical models of the term structure of interest rates', *Journal of Finance*, 31: 1035–1065
- Dothan, L U (1978) 'On the term structure of interest rates', *Journal of Financial Economics*, 6: 59–69
- Durand, D (1942) *Basic yields of corporate bonds, 1900–1942*, Technical Paper No 3, NBER
- Dybvig, P H , J.E. Ingersoll Jr and S.A. Ross (1986) 'Long forward rates can never fall', Unpublished paper, Yale University
- Dunn, K B and K.J. Singleton (1984) 'Modelling the term structure of interest rates under nonseparable utility and durability of goods', NBER Working Paper 1415.
- Echols, M.E. and J W Elliott (1976) 'A quantitative yield curve model for estimating the term structure of interest rates', *Journal of Financial and Quantitative Analysis*, 11· 87–114
- Engle, R F , D M Lilien and R P Robins (1987) 'Estimating time varying risk premia in the term structure: The ARCH-M model', *Econometrica*, 55. 391–407
- Fama, E F (1976) 'Inflation uncertainty and expected return on Treasury bills', *Journal of Political Economy*, 84 427–448
- Fama, E.F (1984a) 'The information in the term structure', *Journal of Financial Economics*, 13 509–528
- Fama, E F (1984b) 'Term premiums in bond returns', *Journal of Financial Economics*, 13 529–546
- Fama, E F (1986) 'Term premiums and default premiums in money markets', *Journal of Financial Economics*, 17· 175–196
- Fama, E F. and R R Bliss (1987) 'The information in long-maturity forward rates', *American Economic Review*, 77: 680–692
- Financial Publishing Company (1970) *Expanded bond values tables*. London Routledge & Kegan Paul, Ltd
- Fisher, I (1896) 'Appreciation and interest', Publications of the American Economic Association, pp 23–29 and 88–92
- Fisher, I (1907) *The rate of interest, its nature, determination and relation to economic phenomena* New York Macmillan
- Fisher, I (1930) *Theory of interest*. New York: Macmillan
- Fisher, L (1966) 'An algorithm for finding exact rates of return', *Journal of Business*, 39· 111–118
- Flavin, M (1983) 'Excess volatility in the financial markets A reassessment of the empirical evidence', *Journal of Political Economy*, 91 929–956
- Flavin, M (1984a) 'Excess sensitivity of consumption to current income: Liquidity constraints or myopia?', NBER Working Paper 1341.
- Flavin, M (1984b) 'Time series evidence on the expectations hypothesis of the term structure', *Carnegie-Rochester Conference Series on Public Policy*, 20. 211–238

- Friedman, B.M. (1977a) 'Financial flow variables and the short-run determination of long-run interest rates', *Journal of Political Economy*, 85: 661–689.
- Friedman, B.M. (1977b) 'The inefficiency of short-run monetary targets for monetary policy', *Brookings Papers on Economic Activity*, 2: 293–335.
- Friedman, B.M. (1979) 'Interest rate expectations versus forward rates: Evidence from an expectations survey', *Journal of Finance*, 34: 965–973.
- Friedman, B.M. (1980a) 'The determination of long-term interest rates: Implications for fiscal and monetary policies', *Journal of Money, Credit and Banking*, 12(Part 2): 331–352.
- Friedman, B.M. (1980b) 'The effect of shifting wealth ownership on the term structure of interest rates: The case of pensions', *Quarterly Journal of Economics*, 94: 567–590.
- Friedman, B.M. (1980c) 'Survey evidence on the rationality of interest rate expectations', *Journal of Monetary Economics*, 6: 453–465.
- Friedman, B.M. (1981) 'Debt management policy, interest rates and economic activity', NBER Working Paper.
- Friedman, B.M. and V.V. Roley (1979) 'Investors portfolio behavior under alternative models of long-term interest rate expectations: Unitary, rational or autoregressive', *Econometrica*, 47: 1475–1497.
- Froot, K.A. (1987) 'New hope for the expectations hypothesis of the term structure of interest rates', Sloan School of Management.
- Grossman, S.J., A. Melino and R.J. Shiller (1987) 'Estimating the continuous time consumption based asset pricing model', *Journal of Business and Economic Statistics*, 5: 315–327.
- Hall, R.E. (1978) 'Stochastic implications of the life cycle-permanent income hypothesis', *Journal of Political Economy*, 6: 971–988.
- Hamburger, M.J. and E.N. Platt (1975) 'The expectations hypothesis and the efficiency of the Treasury bill market', *Review of Economics and Statistics*, 57: 190–199.
- Hansen, L.P. and T.J. Sargent (1981) 'Exact linear rational expectations models: Specification and estimation', Staff Report, Federal Reserve Bank of Minneapolis.
- Hansen, L.P. and K.J. Singleton (1983) 'Stochastic consumption, risk aversion, and the temporal behavior of asset returns', *Journal of Political Economy*, 91: 249–265.
- Hendershott, P.H. (1971) 'A flow of funds model estimated for the non-bank finance sector', *Journal of Money, Credit and Banking*, 3: 815–832.
- Hickman, W.B. (1942) 'The term structure of interest rates: An exploratory analysis', NBER Results shown in Kessel (1965, Appendix A, pp. 103–105).
- Hicks, J.R. (1946) *Value and capital*, 2nd edn. Oxford: Oxford University Press.
- Homer, S. (1963) *A history of interest rates*. New Brunswick: Rutgers University Press.
- Hopewell, M. and G. Kaufman (1973) 'Bond price volatility and term to maturity: A generalized respecification', *American Economic Review*, 63: 749–753.
- Huzinga, J. and F.S. Mishkin (1984) 'The measurement of ex-ante real interest rates on assets with different risk characteristics', unpublished paper, Graduate School of Business, University of Chicago.
- Ingersoll, J.E., Jr., J. Skelton and R.L. Weil (1978) 'Duration forty years later', *Journal of Financial and Quantitative Analysis*, 13: 627–650.
- Jarrow, R.A. (1981) 'Liquidity premiums and the expectations hypothesis', *Journal of Banking and Finance*, 5: 539–546.
- Jones, D.S. and V.V. Roley (1983) 'Rational expectations and the expectations model of the term structure: A test using weekly data', *Journal of Monetary Economics*, 12: 453–465.
- Jordan, J.V. (1984) 'Tax effects in term structure estimation', *Journal of Finance*, 39: 393–406.
- Kaldor, N. (1939) 'Speculation and instability', *Review of Economic Studies*, 7: 1–27.
- Kane, E.J. (1970) 'The term structure of interest rates: An attempt to reconcile teaching with practice', *The Journal of Finance*, 25: 361–374.
- Kane, E.J. (1980) 'Market incompleteness and divergences between forward and future interest rates', *Journal of Finance*, 35: 221–234.
- Kane, E.J. (1983) 'Nested tests of alternative term structure theories', *Review of Economics and Statistics*, 65: 115–123.
- Kane, E.J. and B.G. Malkiel (1967) 'The term structure of interest rates: An analysis of a survey of interest rate expectations', *Review of Economics and Statistics*, 49: 343–355.

- Keim, D B and R F Stambaugh (1986) 'Predicting returns in the stock and bond markets', *Journal of Financial Economics*, 17: 357–390
- Kessel, R A (1965) *The cyclical behavior of the term structure of interest rates* New York. NBER
- Keynes, J M (1930) *Treatise on money* New York Macmillan.
- Keynes, J M (1936) *The general theory of employment, interest and money* London Macmillan & Co Ltd
- Kim, S.-J (1986) 'Explaining the risk premium Nominal interest rates, inflation and consumption', Yale University
- Langetieg, T.C (1980) 'A multivariate model of the term structure', *Journal of Finance*, 35 71–97
- LeRoy, S F (1982a) 'Expectations models of asset prices A survey of theory', *Journal of Finance*, 37 185–217
- LeRoy, S F (1982b) 'Risk aversion and the term structure of real interest rates', *Economics Letters*, 10: 355–361
- LeRoy, S F (1983) 'Risk aversion and the term structure of real interest rates A correction', *Economics Letters*, 12 339–340
- LeRoy, S F (1984) 'Nominal prices and interest rates in general equilibrium Endowment shocks', *Journal of Business*, 57 197–213
- LeRoy, S F and R D Porter (1981) 'The present value relation: Tests based on implied variance bounds', *Econometrica*, 49 555–574
- Lindahl, E. (1939) *Studies in the theory of money and capital*. New York Rinehart and Company.
- Long, J B (1974) 'Stock prices, inflation and the term structure of interest rates', *Journal of Financial Economics*, 1 131–170
- Lutz, F A (1940) 'The structure of interest rates', *Quarterly Journal of Economics*, 55 36–63
- Macaulay, F.R (1938) *Some theoretical problems suggested by the movements of interest rates, bond yields, and stock prices in the United States since 1856* New York NBER
- Malkiel, B G. (1966) *The term structure of interest rates: Expectations and behavior patterns*. Princeton: Princeton University Press.
- Mankiw, N G (1986) 'The term structure of interest rates revisited', *Brookings Papers on Economic Activity*, 1986, 1 61–96.
- Mankiw, N G. and J.A Miron (1986) 'The changing behavior of the term structure of interest rates', *Quarterly Journal of Economics*, 101 211–228
- Mankiw, N G and L H Summers (1984) 'Do long-term interest rates overreact to short-term interest rates?', *Brookings Papers on Economic Activity*, 00 223–242.
- Mankiw, N G., J A Miron and D N Weil (1987) 'The adjustment of expectations of a change in regime. A study of the founding of the Federal Reserve', *American Economic Review*, 77: 358–374
- Marsh, T A (1980) 'Equilibrium term structure models Test methodology', *Journal of Finance*, 35. 421–435
- Marsh, T A and E R Rosenfeld (1983) 'Stochastic processes for interest rates and equilibrium bond prices', *Journal of Finance*, 38: 635–646.
- McCallum, J.S (1975) 'The expected holding period return, uncertainty and the term structure of interest rates', *Journal of Finance*, 30: 307–323
- McCulloch, J.H (1971) 'Measuring the term structure of interest rates', *Journal of Business*, 44: 19–31
- McCulloch, J H (1975a) 'An estimate of the liquidity premium', *Journal of Political Economy*, 83 95–119.
- McCulloch, J H. (1975b) 'The tax adjusted yield curve', *Journal of Finance*, 30. 811–830
- McCulloch, J H (1977) 'Cumulative unanticipated changes in interest rates', NBER Working Paper 222
- McCulloch, J.H (1981) 'Interest rate risk and capital adequacy for traditional banks and financial intermediaries', in. S J Maisel, ed , *Risk and capital adequacy in commercial banks* Chicago: University of Chicago Press and NBER, pp. 223–248.
- McCulloch, J H (1984) 'Term structure modeling using constrained exponential splines', Ohio State University

- Meiselman, D (1962) *The term structure of interest rates* Englewood Cliffs: Prentice-Hall.
- Melino, A (1983) 'Estimation of a rational expectations model of the term structure', in. *Essays on estimation and inference in linear rational expectations models*, unpublished Ph D. Dissertation, Harvard University
- Melino, A. (1986) 'The term structure of interest rates: Evidence and theory', NBER Working Paper 1828.
- Michaelsen, J B (1965) 'The term structure of interest rates and holding period yields on government securities', *Journal of Finance*, 20 444-463
- Miron, J A (1984) 'The economics of seasonal time series', Ph D dissertation, M I T
- Miron, J A. (1986) 'Financial panics, the seasonality of the nominal interest rate, and the founding of the fed', *American Economic Review*, 76: 125-140
- Mishkin, F S (1978) 'Efficient markets theory Implications for monetary policy', *Brookings Papers on Economic Activity*, 1978, 2 707-752
- Mishkin, F S (1980) 'Is the preferred habitat model of the term structure inconsistent with financial market efficiency?', *Journal of Political Economy*, 88 406-411.
- Mishkin, F S (1982) 'Monetary policy and short-term interest rates An efficient markets-rational expectations approach', *Journal of Monetary Economics*, 37: 63-72
- Modigliani, F and R.J. Shiller (1973) 'Inflation, rational expectations and the term structure of interest rates', *Economica*, 40: 12-43.
- Modigliani, F.R. and R. Sutch (1966) 'Innovations in interest rate policy', *American Economic Review*, 56. 178-197
- Modigliani, F and R Sutch (1967) 'Debt management and the term structure of interest rates. An analysis of recent experience', *Journal of Political Economy*, 75: 569-589
- Nelson, C.R (1970a) 'A critique of some recent empirical research in the explanation of the term structure of interest rates', *Journal of Political Economy*, 78 764-767
- Nelson, C.R. (1970b) 'Testing a model of the term structure of interest rates by simulation of market forecasts', *Journal of the American Statistical Association*, 65: 1163-1190
- Nelson, C R (1972a) 'Estimation of term premiums from average yield differentials in the term structure of interest rates', *Econometrica*, 40: 277-287
- Nelson, C R (1972b) *The term structure of interest rates*. New York Basic Books
- Nelson, C R and A F Siegel (1985) 'Parsimonious modelling of yield curves for U.S Treasury bills', NBER Working Paper 1594.
- Okun, A M (1963) 'Monetary policy, debt management, and interest rates: A quantitative appraisal', in. Commission on Money and Credit, *Stabilization Policies* Englewood Cliffs Prentice-Hall, pp. 331-380
- Pesando, J E (1975) 'Determinants of term premiums in the market for United States Treasury bills', *Journal of Finance*, 30: 1317-1327
- Pesando, J.E (1978) 'On the efficiency of the bond market: Some Canadian evidence', *Journal of Political Economy*, 86 1057-1076
- Pesando, J E (1981) 'On forecasting interest rates An efficient markets perspective', *Journal of Monetary Economics*, 8: 305-318
- Pesando, J E (1983) 'On expectations, term premiums and the volatility of long-term interest rates', *Journal of Monetary Economics*, 12: 467-474
- Phillips, L and J. Pippenger (1976) "Preferred habitat vs. efficient market: A test of alternative hypotheses", *Federal Reserve Bank of St Louis Review*, 58. 151-164
- Phillips, L. and J Pippenger (1979) 'The term structure of interest rates in the MPS model Reality or illusion?", *Journal of Money, Credit and Banking*, 11. 151-164
- Richard, S F (1978) 'An arbitrage model of the term structure of interest rates', *Journal of Financial Economics*, 6 33-57
- Roley, VV (1977) 'A structural model of the U S government securities market', unpublished Ph D dissertation, Harvard University.
- Roley, VV. (1981) 'The determinants of the Treasury security yield curve', *Journal of Finance*, 36 1103-1126
- Roley, VV (1982) 'The effect of federal debt management policy on corporate bond and equity yields', *Quarterly Journal of Economics*, 97 645-668.
- Roll, R (1970) *The behavior of interest rates* New York Basic Books

- Roll, R. (1971) 'Investment diversification and bond maturity', *Journal of Finance*, 26: 51–66
- Salomon Brothers, Inc (1983) *An analytical record of yields and yield spreads: From 1945* New York.
- Samuelson, P.A. (1945) 'The effect of interest rate increases on the banking system', *American Economic Review*, 35: 16–27.
- Sargent, T.J. (1971) 'Expectations at the short end of the yield curve. An application of Macaulay's test', in: J M Guttentag, ed., *Essays on interest rates*, Vol II New York NBER, pp. 391–412
- Sargent, T.J. (1979) 'A note on the estimation of the rational expectations model of the term structure', *Journal of Monetary Economics*, 5: 133–143.
- Say, J.B. (1853) *A treatise on political economy*. Philadelphia: Lippincott Grambo & Co
- Schaefer, S.M. (1981) 'Measuring a tax-specific term structure of interest rates in the market for British Government securities', *Economic Journal*, 91: 415–438.
- Scott, R.H. (1965) 'Liquidity and the term structure of interest rates', *Quarterly Journal of Economics*, 79: 135–145.
- Shea, G.S. (1984) 'Pitfalls in smoothing interest rate term structure data. Equilibrium models and spline approximations', *Journal of Financial and Quantitative Analysis*, 19: 253–269
- Shea, G.S. (1985) 'Interest rate term structure equations with exponential splines. A note', *Journal of Finance*, 40: 319–325.
- Shiller, R.J. (1972) 'Rational expectations and the structure of interest rates', unpublished Ph D dissertation, M.I.T.
- Shiller, R.J. (1978) 'Rational expectations and the dynamic structure of macroeconomic models: A critical review', *Journal of Monetary Economics*, 4: 1–44
- Shiller, R.J. (1979) 'The volatility of long-term interest rates and expectations models of the term structure', *Journal of Political Economy*, 87: 1190–1219
- Shiller, R.J. (1980) 'Can the federal reserve control real interest rates?', in: S. Fischer, ed., *Rational expectations and economic policy*, Chicago: NBER and University of Chicago Press
- Shiller, R.J. (1981a) 'Alternative tests of rational expectations Models: The case of the term structure', *Journal of Econometrics*, 16: 71–87
- Shiller, R.J. (1981b) 'Do stock prices move too much to be justified by subsequent changes in dividends?', *American Economic Review*, 71: 421–436
- Shiller, R.J. (1986) 'Comments and discussion', *Brookings Papers on Economic Activity*, 1986, 1: 100–107.
- Shiller, R.J. (1987) 'Conventional valuation and the term structure of interest rates', in: R. Dornbusch, S. Fischer and J. Bossoms, eds., *Macroeconomics and finance: Essays in honour of Franco Modigliani* Cambridge, Mass.: MIT Press.
- Shiller, R.J., J.Y. Campbell and K.L. Schoenholtz (1983) 'Forward rates and future policy: Interpreting the term structure of interest rates', *Brookings Papers on Economic Activity*, 1983, 1: 173–217
- Sidgwick, H. (1887) *The principles of political economy*. London: Macmillan
- Singleton, K.J. (1980a) 'A latent time series model of the cyclical behavior of interest rates', *International Economic Review*, 21: 559–575
- Singleton, K.J. (1980b) 'Expectations models of the term structure and implied variance bounds', *Journal of Political Economy*, 88: 1159–1176
- Singleton, K.J. (1980c) 'Maturity-specific disturbances and the term structure of interest rates', *Journal of Money, Credit and Banking*, 12 (Part I): 603–614
- Skinner, E.B. (1913) *The mathematical theory of investment*. Boston: Ginn and Co
- Startz, R. (1982) 'Do forecast errors or term premia really make the difference between long and short rates?', *Journal of Financial Economics*, 10: 323–329
- Stiglitz, J. (1970) 'A consumption-oriented theory of the demand for financial assets and the term structure of interest rates', *Review of Economic Studies*, 37: 321–351
- Stigum, M. (1978) *The money market: Myth, reality and practice*. Homewood: Dow Jones-Irwin
- Stigum, M. (1981) *Money market calculations: Yields, break-evens and arbitrage*. Homewood: Dow Jones-Irwin
- Summers, L.H. (1982) 'Do we really know that markets are efficient?', NBER Working Paper.

- Sutch, R. (1968) 'Expectations, risk and the term structure of interest rates,' unpublished Ph D dissertation, M I T
- Telser, L G (1967) 'A critique of some recent empirical research on the explanation of the term structure of interest rates', *Journal of Political Economy*, 75: 546-561
- Vasicek, O A (1978) 'An equilibrium characterization of the term structure', *Journal of Financial Economics*, 6: 33-57
- Vasicek, O A and H G Fong (1982) 'Term structure modelling using exponential splines', *Journal of Finance*, 37: 339-348
- Volterra, V (1959) *Theory of functionals and of integral and integrodifferential equations* New York: Dover
- Walker, C E (1954) 'Federal reserve policy and the structure of interest rates in government securities', *Quarterly Journal of Economics*, 68: 19-42
- Wallace, N (1967) 'Comment', *Journal of Political Economy*, 75: 590-592
- Walsh, C E (1985) 'A rational expectations model of term premia with some implications for empirical asset demand functions', *Journal of Finance*, 40: 63-83.
- Williams, J B (1938) *The theory of investment value*. Cambridge, Mass: Harvard University Press
- Wood, J.H. (1963) 'Expectations, errors and the term structure of interest rates', *Journal of Political Economy*, 71: 160-171
- Woodward, S (1983) 'The liquidity premium and the solidity premium', *American Economic Review*, 73: 348-361

Table 13 A 1
McCulloch zero coupon yield curve series, continuous compounding, end of month data. 12/46-2/87

Table 13 A.1 (cont.)

		0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr
1952		0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr
10	1.32	1.59	1.75	1.82	1.86	1.89	1.92	1.97	2.01	2.13	2.22	2.28	2.34	2.54	2.69	2.75		
11	1.55	1.78	1.92	1.99	2.02	2.05	2.07	2.11	2.14	2.16	2.27	2.31	2.35	2.52	2.69	2.77		
12	1.82	1.94	2.02	2.06	2.08	2.09	2.11	2.14	2.16	2.16	2.30	2.34	2.38	2.56	2.76			
1953		0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr
1	1.63	1.82	1.93	1.97	2.00	2.02	2.03	2.06	2.09	2.18	2.26	2.33	2.40	2.66	2.80			
2	1.86	2.02	2.11	2.15	2.16	2.18	2.19	2.20	2.22	2.24	2.31	2.37	2.44	2.73	2.90			
3	1.95	1.99	2.01	2.03	2.05	2.06	2.07	2.10	2.13	2.14	2.33	2.46	2.48	2.76	2.92			
4	1.78	2.06	2.23	2.23	2.35	2.42	2.47	2.50	2.58	2.63	2.75	2.83	2.91	3.01	3.09	3.31	3.29	3.19
5	1.43	1.94	2.23	2.35	2.42	2.47	2.50	2.58	2.63	2.75	2.88	2.91	2.99	3.06	3.11	3.16	3.12	3.12
6	1.18	1.61	1.69	2.03	2.11	2.17	2.20	2.28	2.33	2.48	2.59	2.66	2.74	2.99	3.05	3.11	3.15	
7	1.72	1.94	2.06	2.12	2.16	2.19	2.21	2.26	2.31	2.45	2.57	2.64	2.77	2.86	3.09	3.13		
8	1.62	1.79	1.89	1.95	1.99	2.03	2.06	2.14	2.16	2.21	2.45	2.57	2.64	2.77	2.86	3.05	3.11	
9	0.86	1.22	1.46	1.59	1.67	1.72	1.77	1.87	1.95	2.18	2.34	2.43	2.49	2.65	2.80	2.98		
10	0.65	0.98	1.19	1.31	1.39	1.44	1.49	1.54	1.60	1.69	2.00	2.34	2.44	2.65	2.80	2.98		
11	1.36	1.44	1.49	1.53	1.56	1.62	1.69	1.76	1.88	1.98	2.16	2.30	2.40	2.75	2.95	3.04		
12	1.09	1.25	1.35	1.42	1.46	1.49	1.52	1.56	1.60	1.66	1.88	2.03	2.13	2.21	2.46	2.71	2.88	
1954		0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr
1	1.05	0.96	0.92	0.93	0.95	0.97	1.00	1.10	1.19	1.53	1.78	1.95	2.09	2.49	2.69	2.82		
2	0.84	0.89	0.92	0.94	0.97	0.99	1.01	1.06	1.12	1.34	1.56	1.78	1.98	2.47	2.53	2.61		
3	0.99	0.99	0.99	1.00	1.00	1.01	1.02	1.04	1.09	1.14	1.34	1.55	1.75	2.03	2.42	2.54	2.63	
4	0.62	0.67	0.71	0.75	0.78	0.81	0.83	0.91	0.99	1.23	1.42	1.60	1.77	2.34	2.51	2.60		
5	0.51	0.60	0.66	0.69	0.73	0.76	0.79	0.87	0.96	1.28	1.58	1.84	2.03	2.51	2.61	2.67		
6	0.67	0.62	0.60	0.63	0.66	0.69	0.70	0.74	0.85	1.15	1.41	1.65	1.85	2.43	2.51	2.56		
7	0.64	0.68	0.71	0.73	0.75	0.77	0.79	0.84	0.90	1.16	1.44	1.69	1.89	2.39	2.48	2.53		
8	1.30	1.11	1.00	0.96	0.95	0.97	1.00	1.03	1.12	1.20	1.45	1.49	1.71	1.91	2.46	2.51	2.54	
9	0.90	0.90	0.92	0.94	0.97	1.00	1.06	1.09	1.18	1.26	1.55	1.64	1.81	1.98	2.12	2.44	2.52	2.59
10	0.72	0.85	0.93	0.98	1.02	1.06	1.09	1.18	1.25	1.53	1.79	1.98	2.12	2.44	2.63	2.78	2.88	
11	0.86	0.93	0.98	1.02	1.04	1.07	1.10	1.20	1.29	1.61	1.88	2.08	2.21	2.51	2.64	2.79		
12	0.92	0.95	0.98	1.01	1.04	1.07	1.10	1.12	1.20	1.53	1.79	1.99	2.14	2.49	2.63	2.78		
1955		0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr
1	1.19	1.10	1.07	1.09	1.13	1.17	1.22	1.35	1.47	1.85	2.05	2.19	2.29	2.65	2.79	2.82	2.82	
2	0.91	1.14	1.29	1.38	1.45	1.51	1.56	1.70	1.82	2.16	2.32	2.40	2.47	2.71	2.83	2.88	2.90	
3	1.33	1.33	1.35	1.39	1.43	1.48	1.52	1.65	1.78	2.13	2.30	2.40	2.47	2.70	2.80	2.84	2.85	
4	1.44	1.50	1.55	1.60	1.65	1.69	1.73	1.85	1.95	2.25	2.38	2.46	2.53	2.76	2.86	2.88	2.85	
5	0.91	1.15	1.32	1.43	1.51	1.57	1.62	1.76	1.87	2.19	2.35	2.46	2.53	2.73	2.82	2.86	2.88	
6	1.04	1.27	1.43	1.53	1.60	1.66	1.71	1.85	1.96	2.29	2.47	2.58	2.66	2.86	2.90	2.88	2.85	

Table 13A 1 (cont.)

	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr
1958	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr
8	1.35	1.87	2.20	2.38	2.49	2.58	2.65	2.82	2.96	3.36	3.55	3.65	3.70	3.76	3.74	3.73	3.72
9	1.00	1.95	2.57	2.89	3.04	3.14	3.20	3.31	3.36	3.50	3.62	3.71	3.75	3.72	3.72	3.72	3.72
10	1.08	1.82	2.30	2.53	2.67	2.76	2.83	2.98	3.09	3.38	3.54	3.64	3.68	3.71	3.69	3.68	3.71
11	0.83	1.90	2.58	2.91	3.07	3.16	3.23	3.33	3.38	3.48	3.55	3.60	3.63	3.64	3.65	3.68	3.71
12	2.13	2.42	2.62	2.74	2.84	2.91	2.97	3.10	3.19	3.46	3.64	3.75	3.82	3.90	3.85	3.78	3.77
1959	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr
1	2.32	2.47	2.61	2.76	2.90	3.01	3.09	3.27	3.40	3.74	3.90	3.97	4.00	3.99	4.00	4.02	4.02
2	2.04	2.38	2.64	2.83	2.97	3.07	3.15	3.29	3.40	3.60	3.79	3.79	3.90	3.90	4.02	4.07	4.07
3	2.44	2.57	2.72	2.89	3.06	3.19	3.29	3.48	3.59	3.83	3.92	3.97	3.98	3.97	3.98	3.99	3.99
4	2.78	2.74	2.80	2.94	3.10	3.23	3.23	3.32	3.49	3.61	3.88	4.04	4.13	4.18	4.12	4.12	3.97
5	2.80	2.75	2.82	2.97	3.10	3.27	3.47	3.60	3.79	3.86	3.98	4.10	4.18	4.21	4.11	4.05	4.02
6	2.70	2.77	2.93	3.16	3.42	3.62	3.75	3.91	3.97	4.14	4.33	4.41	4.41	4.41	4.14	4.10	4.07
7	1.97	2.40	2.75	3.04	3.31	3.56	3.79	4.26	4.41	4.34	4.40	4.53	4.58	4.41	4.20	4.05	3.98
8	3.14	3.74	3.95	4.13	4.27	4.39	4.55	4.74	4.93	4.56	4.63	4.67	4.71	4.69	4.49	4.25	4.00
9	3.43	3.64	3.90	4.22	4.53	4.77	4.90	4.99	4.69	4.70	4.71	4.69	4.61	4.69	4.47	4.26	4.09
10	2.14	3.17	3.79	4.06	4.20	4.32	4.42	4.56	4.55	4.36	4.44	4.44	4.44	4.48	4.23	4.14	3.98
11	2.31	3.53	4.23	4.52	4.68	4.77	4.84	4.93	4.95	4.87	4.81	4.77	4.73	4.52	4.31	4.14	4.06
12	4.02	4.08	4.28	4.56	4.77	4.88	4.94	5.01	5.02	4.95	4.88	4.83	4.81	4.70	4.51	4.30	4.20
1960	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr
1	3.28	3.54	3.82	4.11	4.33	4.47	4.56	4.68	4.71	4.69	4.72	4.74	4.76	4.63	4.36	4.34	4.16
2	3.58	3.81	4.05	4.25	4.35	4.38	4.38	4.38	4.40	4.49	4.57	4.61	4.60	4.40	4.25	4.16	4.07
3	2.40	2.68	2.91	3.10	3.23	3.34	3.41	3.55	3.63	3.79	3.91	3.99	4.03	4.07	4.05	4.05	4.23
4	2.89	3.02	3.02	3.18	3.34	3.34	3.45	3.53	3.98	4.16	4.25	4.36	4.46	4.34	4.27	4.27	4.23
5	1.82	2.60	3.02	3.16	3.24	3.34	3.45	3.76	3.93	4.14	4.35	4.31	4.31	4.15	4.16	4.20	4.18
6	1.25	1.65	1.97	2.20	2.38	2.52	2.65	2.79	2.91	3.18	3.70	3.94	3.94	3.99	4.20	3.93	3.84
7	1.65	1.83	2.02	2.18	2.38	2.50	2.60	2.79	2.91	3.17	3.28	3.47	3.47	3.63	3.70	3.70	3.70
8	1.45	1.97	2.36	2.61	2.77	2.85	2.86	2.88	2.89	3.05	3.30	3.48	3.54	3.59	3.86	3.91	3.86
9	2.69	3.34	2.31	2.53	2.75	2.86	2.88	2.89	2.89	2.91	3.14	3.30	3.54	3.66	3.80	3.93	3.88
10	1.22	1.61	1.93	2.19	2.38	2.52	2.61	2.76	2.81	3.05	3.14	3.30	3.48	3.66	3.81	3.94	3.94
11	1.24	2.19	2.19	2.44	2.59	2.69	2.77	2.91	2.91	3.05	3.31	3.53	3.70	3.81	4.00	4.02	3.98
12	1.87	2.03	2.16	2.27	2.36	2.41	2.44	2.44	2.44	2.51	2.58	2.87	3.12	3.44	3.75	3.83	3.85
1961	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr
1	1.88	2.04	2.18	2.30	2.40	2.47	2.52	2.64	2.76	3.15	3.40	3.55	3.65	3.85	3.91	3.93	3.93
2	2.47	2.48	2.54	2.62	2.69	2.75	2.80	2.90	2.97	3.19	3.34	3.53	3.73	3.80	3.82	3.84	3.84
3	2.02	2.19	2.32	2.42	2.50	2.57	2.64	2.76	2.86	3.05	3.25	3.43	3.56	3.78	3.87	3.85	3.82
4	1.42	1.93	2.20	2.27	2.30	2.38	2.43	2.57	2.73	2.85	3.04	3.20	3.35	3.47	3.76	3.82	3.77

Table 13 A.1 (cont.)

		0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr	
1964		0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr	
6	3.28	3.46	3.54	3.55	3.59	3.63	3.72	3.75	3.83	3.92	4.00	4.06	4.16	4.14	4.13				
7	3.02	3.22	3.46	3.51	3.54	3.60	3.65	3.73	3.74	3.75	3.86	4.00	4.08	4.21	4.19	4.18			
8	3.21	3.40	3.51	3.56	3.60	3.65	3.69	3.77	3.79	3.84	3.93	4.02	4.08	4.23	4.21	4.18			
9	3.50	3.53	3.57	3.63	3.69	3.74	3.78	3.85	3.89	3.95	3.92	3.97	4.01	4.19	4.20	4.18			
10	3.39	3.45	3.53	3.62	3.70	3.79	3.83	3.93	4.03	4.14	4.15	4.13	4.14	4.15	4.17	4.19	4.16		
11	3.37	3.63	3.81	3.90	3.97	4.03	4.08	4.14	4.15	4.13	4.13	4.14	4.15	4.20	4.20	4.18			
12	3.13	3.56	3.80	3.89	3.94	3.98	4.00	4.02	4.00	4.00	3.99	4.04	4.08	4.12	4.23	4.22	4.19		
1965		0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr	
1	3.81	3.85	3.89	3.93	3.97	4.02	4.06	4.03	4.06	4.02	4.06	4.10	4.19	4.21	4.20	4.19			
2	3.61	3.87	4.00	4.04	4.07	4.11	4.13	4.15	4.15	4.14	4.15	4.10	4.13	4.23	4.23	4.20	4.18		
3	3.60	3.82	3.94	3.98	4.01	4.05	4.07	4.10	4.10	4.09	4.07	4.09	4.12	4.14	4.20	4.21	4.19		
4	3.82	3.89	3.94	3.99	4.02	4.05	4.07	4.08	4.07	4.07	4.07	4.10	4.12	4.14	4.20	4.21	4.19		
5	3.74	3.86	3.93	3.95	3.98	4.00	4.03	4.06	4.06	4.07	4.07	4.08	4.11	4.14	4.23	4.23	4.19		
6	3.71	3.79	3.84	3.87	3.89	3.92	3.93	3.94	3.94	3.93	3.95	4.02	4.07	4.11	4.20	4.22	4.19		
7	3.81	3.81	3.84	3.89	3.93	3.96	3.98	3.99	3.99	4.00	4.00	4.11	4.15	4.22	4.23	4.22	4.21		
8	3.70	3.81	3.88	3.93	3.98	4.04	4.09	4.12	4.13	4.17	4.20	4.23	4.28	4.27	4.25	4.24			
9	3.93	3.95	4.01	4.11	4.19	4.24	4.28	4.33	4.35	4.34	4.33	4.34	4.36	4.41	4.37	4.31	4.28		
10	3.85	3.92	4.02	4.13	4.22	4.26	4.29	4.33	4.35	4.38	4.40	4.43	4.44	4.40	4.40	4.33	4.29		
11	3.67	3.93	4.09	4.18	4.25	4.32	4.37	4.42	4.44	4.45	4.50	4.54	4.55	4.54	4.54	4.49	4.44	4.37	
12	4.31	4.44	4.52	4.57	4.66	4.75	4.82	4.92	4.96	4.96	5.08	5.10	5.04	4.97	4.70	4.59	4.51		
1966		0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr	
1	4.50	4.52	4.61	4.72	4.79	4.82	4.84	4.87	4.90	5.04	5.10	5.08	5.02	5.08	5.25	5.28	5.20	5.14	
2	4.24	4.50	4.65	4.74	4.82	4.90	4.97	5.08	5.13	5.18	5.21	5.25	5.28	5.32	5.35	5.38	5.40	5.37	
3	4.19	4.40	4.53	4.61	4.70	4.79	4.85	4.94	4.97	5.02	5.05	5.04	5.04	5.08	5.12	5.16	5.17	5.15	
4	4.44	4.63	4.69	4.70	4.74	4.82	4.87	4.93	4.95	5.02	5.06	5.25	5.26	5.30	5.34	5.39	5.41	5.39	
5	4.21	4.49	4.65	4.69	4.75	4.86	4.95	4.95	4.95	5.02	5.06	5.36	5.39	5.31	5.46	5.51	5.47	5.43	
6	4.22	4.49	4.60	4.62	4.69	4.80	4.87	4.95	4.95	5.01	5.02	5.40	5.48	5.48	5.46	5.26	5.01	4.71	
7	4.09	4.58	4.78	4.81	4.90	5.01	5.09	5.17	5.17	5.21	5.40	5.48	5.48	5.48	5.48	5.46	5.36	5.23	
8	4.21	4.79	5.06	5.12	5.28	5.56	5.76	5.89	5.94	6.30	6.28	6.14	6.01	5.57	5.29	5.08	4.97		
9	5.00	5.20	5.35	5.48	5.61	5.74	5.83	5.92	5.91	5.79	5.68	5.56	5.42	5.07	4.96	4.89	4.77		
10	4.24	4.85	5.19	5.36	5.50	5.62	5.70	5.74	5.72	5.61	5.53	5.46	5.38	5.08	4.86	4.71	4.65		
11	3.66	4.62	5.15	5.28	5.32	5.37	5.41	5.43	5.48	5.68	5.55	5.41	5.37	5.27	5.05	4.86	4.76		
12	4.60	4.70	4.82	4.94	5.02	5.08	5.07	5.05	5.05	5.03	5.02	4.98	4.93	4.87	4.72	4.67	4.63		
1967		0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr	
1	4.37	4.54	4.61	4.61	4.63	4.64	4.63	4.64	4.63	4.63	4.68	4.70	4.70	4.69	4.62	4.56	4.51	4.48	
2	3.77	4.37	4.59	4.59	4.60	4.61	4.64	4.66	4.64	4.68	4.91	4.86	4.79	4.80	4.87	4.83	4.75	4.68	
3	3.91	4.01	4.10	4.16	4.18	4.17	4.16	4.16	4.14	4.14	4.24	4.30	4.35	4.42	4.60	4.64	4.63	4.60	

Table 13 A 1 (cont.)

		0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr	
1970																			
	6	5.46	6.12	6.45	6.52	6.58	6.68	6.80	7.06	7.22	7.52	7.63	7.63	7.50					
	7	4.32	5.92	6.50	6.43	6.38	6.50	6.66	6.86	6.89	7.29	7.46	7.48	7.60					
	8	6.07	6.24	6.33	6.38	6.56	6.56	6.77	6.79	7.15	7.31	7.32	7.35	7.57					
	9	5.61	5.56	5.71	6.03	6.32	6.48	6.54	6.59	6.60	6.68	6.82	6.97	7.08	7.30				
	10	5.05	5.45	5.76	5.96	6.09	6.17	6.23	6.30	6.34	6.52	6.71	6.86	6.97	7.20				
	11	4.40	4.75	4.97	5.07	5.08	5.06	5.05	5.10	5.16	5.39	5.62	5.88	6.08	6.49	6.52	6.32		
	12	3.98	4.57	4.84	4.90	4.92	4.93	4.91	4.89	4.99	5.47	5.74	5.89	6.00	6.34	6.51	6.39		
1971																			
	1	3.99	4.11	4.15	4.17	4.22	4.25	4.25	4.19	4.24	4.73	5.23	5.62	5.87	6.17	6.04	5.38		
	2	3.13	3.25	3.35	3.43	3.50	3.56	3.61	3.69	3.75	4.23	4.78	5.20	5.49	6.12	6.32	6.32		
	3	3.46	3.46	3.54	3.64	3.69	3.69	3.69	3.74	3.81	4.15	4.52	4.86	5.15	5.72	5.85	5.91		
	4	3.49	3.81	3.97	4.04	4.14	4.24	4.33	4.51	4.64	5.20	5.63	5.90	6.05	6.15				
	5	3.92	4.20	4.35	4.38	4.40	4.44	4.51	4.75	4.97	5.49	5.78	5.97	6.12	6.53				
	6	4.52	4.95	5.13	5.12	5.15	5.29	5.47	5.83	6.06	6.49	6.66	6.74	6.80	6.83				
	7	4.78	5.21	5.33	5.31	5.42	5.64	5.82	6.02	6.06	6.50	6.77	6.86	6.91	6.95				
	8	4.49	4.46	4.42	4.40	4.47	4.64	4.82	5.12	5.27	5.55	5.70	5.82	5.93	6.30				
	9	4.55	4.47	4.52	4.65	4.78	4.89	4.99	5.17	5.27	5.51	5.70	5.87	5.95	5.97				
	10	3.56	4.06	4.29	4.37	4.44	4.47	4.48	4.49	4.60	5.06	5.19	5.49	5.76	5.96	6.12			
	11	3.67	4.06	4.28	4.35	4.39	4.45	4.50	4.63	4.75	5.07	5.27	5.60	5.84	5.85				
	12	3.24	3.33	3.51	3.70	3.87	3.99	4.03	4.11	4.29	4.84	5.04	5.29	5.49	5.97				
1972																			
	1	2.41	3.02	3.28	3.38	3.53	3.68	3.78	3.99	4.18	4.84	5.29	5.56	5.76	6.40				
	2	3.36	3.25	3.29	3.44	3.60	3.72	3.82	4.05	4.24	4.83	5.23	5.49	5.68	6.26				
	3	3.32	3.42	3.64	3.93	4.16	4.33	4.47	4.78	5.00	5.52	5.78	5.94	6.04	6.22				
	4	3.30	3.28	3.44	3.67	3.86	4.01	4.12	4.34	4.50	5.15	5.72	5.83	6.21					
	5	3.31	3.50	3.67	3.81	3.95	4.09	4.21	4.43	4.56	5.00	5.34	5.56	5.73	6.14				
	6	3.39	3.68	3.91	4.11	4.31	4.51	4.69	5.02	5.17	5.48	5.68	5.83	5.94	6.24				
	7	3.41	3.56	3.70	3.86	4.04	4.25	4.43	4.74	4.88	5.36	5.72	5.86	5.97	6.40				
	8	4.27	4.35	4.42	4.51	4.65	4.85	5.02	5.28	5.37	5.67	5.89	6.02	6.13	6.51				
	9	4.65	4.48	4.52	4.74	4.98	5.17	5.31	5.53	5.63	5.86	5.97	6.01	6.09	6.62				
	10	4.49	4.54	4.65	4.82	4.97	5.10	5.19	5.36	5.48	5.81	5.98	6.04	6.10	6.45				
	11	4.82	4.79	4.83	4.92	5.04	5.16	5.26	5.36	5.36	5.67	5.88	5.93	5.99	6.34				
	12	4.77	4.93	5.08	5.22	5.34	5.43	5.48	5.56	5.59	5.90	6.04	6.07	6.12	6.41				
1973																			
	1	5.22	5.44	5.62	5.77	5.87	5.93	5.97	6.02	6.06	6.26	6.35	6.34	6.45	6.65				
	2	5.49	5.59	5.73	5.90	6.03	6.11	6.16	6.29	6.39	6.61	6.66	6.62	6.55	6.60				
	3	5.77	6.08	6.34	6.53	6.70	6.82	6.91	6.99	6.96	6.79	6.71	6.68	6.65	6.58	6.60			

Table 13 A 1 (cont.)

	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr
1976	6	5.26	5.30	5.36	5.48	5.63	5.80	5.93	6.15	6.36	6.91	7.17	7.36	7.49	7.81	7.94	8.02
7	5.09	5.10	5.16	5.27	5.41	5.55	5.65	5.85	6.08	6.69	6.97	7.20	7.39	7.85	7.97	8.02	
8	4.80	4.98	5.09	5.17	5.28	5.41	5.50	5.63	5.83	6.43	6.67	6.90	7.11	7.66	7.82	7.98	
9	5.13	5.07	5.09	5.18	5.38	5.47	5.59	5.73	6.27	6.54	6.80	7.01	7.56	7.74	7.82		
10	4.69	4.74	4.85	4.98	5.08	5.15	5.20	5.30	5.46	6.02	6.27	6.50	6.73	7.46	7.72	7.82	
11	4.37	4.41	4.46	4.51	4.56	4.63	4.68	4.77	4.88	5.39	5.66	5.79	6.01	7.09	7.64	7.80	
12	4.22	4.26	4.34	4.45	4.53	4.59	4.63	4.72	4.84	5.35	5.66	5.90	6.11	6.83	7.15	7.32	
1977	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr
1	4.58	4.57	4.66	4.81	4.95	5.06	5.15	5.34	5.55	6.17	6.46	6.72	6.91	7.38	7.62	7.73	
2	4.41	4.52	4.65	4.78	4.90	4.99	5.07	5.30	5.52	6.11	6.46	6.76	6.98	7.44	7.69	7.86	
3	4.54	4.51	4.55	4.65	4.75	4.83	4.91	5.17	5.41	5.97	6.38	6.69	6.91	7.42	7.68	7.85	
4	4.33	4.44	4.58	4.72	4.85	4.95	5.05	5.30	5.51	6.07	6.30	6.70	6.92	7.45	7.67	7.77	
5	4.80	4.90	5.00	5.11	5.21	5.30	5.37	5.55	5.70	6.13	6.38	6.64	6.84	7.39	7.64	7.76	
6	4.98	4.96	5.01	5.12	5.23	5.32	5.39	5.57	5.64	6.06	6.29	6.53	6.73	7.27	7.56		
7	5.16	5.25	5.36	5.48	5.60	5.72	5.81	5.97	6.09	6.41	6.63	6.84	6.99	7.38	7.55	7.63	
8	5.35	5.38	5.49	5.65	5.82	5.94	6.03	6.18	6.26	6.47	6.65	6.78	6.86	7.25	7.47	7.61	
9	5.75	5.74	5.85	6.04	6.19	6.28	6.34	6.47	6.57	6.75	6.95	7.06	7.33	7.53	7.66	7.73	
10	6.27	6.11	6.16	6.37	6.55	6.65	6.71	6.88	7.01	7.20	7.29	7.38	7.55	7.72	7.86	7.90	
11	5.31	5.67	5.97	6.19	6.36	6.48	6.58	6.76	6.87	7.07	7.18	7.24	7.28	7.51	7.71	7.84	
12	5.38	5.62	5.82	6.13	6.32	6.46	6.55	6.63	6.81	6.92	7.10	7.25	7.37	7.45	7.68	7.86	
1978	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr
1	5.69	6.07	6.35	6.53	6.67	6.77	6.86	7.02	7.12	7.28	7.42	7.53	7.61	7.81	8.01	8.16	8.20
2	5.98	6.23	6.41	6.56	6.68	6.81	6.92	7.08	7.16	7.44	7.58	7.69	7.76	7.92	8.09	8.22	8.27
3	6.84	6.56	6.51	6.62	6.76	6.87	6.97	7.18	7.32	7.58	7.69	7.78	7.85	8.02	8.19	8.29	8.32
4	5.86	6.03	6.25	6.51	6.74	6.93	7.09	7.37	7.52	7.78	7.83	7.88	7.93	8.10	8.27	8.36	8.39
5	6.33	6.44	6.60	6.81	7.02	7.21	7.36	7.64	7.78	7.96	8.01	8.08	8.13	8.27	8.44	8.50	
6	6.24	6.61	6.95	7.25	7.48	7.63	7.75	8.02	8.20	8.29	8.34	8.32	8.39	8.45	8.56	8.53	
7	6.40	6.55	6.76	7.01	7.24	7.44	7.62	8.00	8.20	8.26	8.29	8.28	8.29	8.39	8.47	8.46	
8	8.05	7.88	7.76	7.70	7.73	7.84	7.95	8.20	8.30	8.26	8.23	8.21	8.20	8.23	8.29	8.31	
9	7.89	8.00	8.15	8.32	8.48	8.59	8.66	8.86	8.96	8.45	8.30	8.27	8.29	8.43	8.51	8.48	
10	8.35	8.65	8.84	8.94	9.09	9.31	9.50	9.74	9.66	9.25	9.07	8.93	8.83	8.66	8.65	8.69	
11	8.95	8.99	9.06	9.17	9.31	9.45	9.58	9.79	9.76	9.25	8.86	8.68	8.63	8.62	8.60	8.51	8.41
12	7.63	8.63	9.25	9.55	9.70	9.77	9.86	10.22	10.33	9.79	9.33	9.09	9.01	8.86	8.75	8.65	
1979	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr
1	9.64	9.51	9.49	9.56	9.63	9.68	9.73	9.86	9.80	9.34	8.87	8.68	8.67	8.72	8.67	8.63	8.55
2	9.58	9.63	9.65	9.67	9.73	9.82	9.90	10.00	9.97	9.58	9.20	9.02	8.98	8.97	8.90	8.78	8.65

	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr	
1980	11.03	11.87	12.28	12.36	12.39	12.43	12.43	12.05	11.73	11.31	10.90	10.74	10.72	10.90	10.89	10.76	10.52	
2	13.29	13.92	14.37	14.26	14.32	14.53	14.53	14.92	14.90	14.64	13.65	13.25	13.25	12.71	12.38	12.02	11.93	11.47
3	13.89	15.07	15.44	15.24	15.19	15.53	15.92	15.85	15.28	14.15	13.25	13.19	13.19	12.71	12.38	12.02	11.70	10.87
4	10.28	10.39	10.52	10.68	10.82	10.95	11.04	11.07	10.95	10.50	10.38	10.40	10.44	10.71	10.85	10.73	10.40	
5	8.22	7.85	7.76	7.91	8.10	8.25	8.36	8.69	9.02	9.14	9.49	9.70	10.1	10.48	10.71	10.26	10.04	9.65
6	4.09	6.66	7.94	8.16	8.11	8.13	8.20	8.34	8.43	8.95	9.09	9.26	9.42	9.42	9.42	9.42	9.42	
7	6.47	7.84	8.57	8.80	8.87	8.90	8.93	9.01	9.14	9.65	9.77	9.95	10.11	10.70	11.00	10.89	10.40	
8	8.27	9.20	9.86	10.26	10.50	10.64	10.71	10.90	11.14	11.26	11.43	11.48	11.50	11.28	11.28	11.06	10.48	
9	10.98	11.20	11.42	11.63	11.79	11.89	11.95	11.96	11.89	11.84	11.62	11.59	11.59	11.78	11.55	11.55	10.98	
10	10.23	11.66	12.58	12.99	13.14	13.21	13.27	13.32	13.31	12.69	12.56	12.46	12.36	11.99	12.13	11.87	11.24	
11	13.63	14.39	14.80	14.86	14.77	14.75	14.83	14.92	14.45	14.64	13.16	12.87	12.66	12.06	12.19	11.84	11.02	
12	10.88	12.90	14.20	14.76	14.75	14.58	14.40	13.94	13.20	12.52	12.27	12.16	12.10	11.94	11.92	11.41	10.65	
1981	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr	
1	13.97	14.72	15.06	15.02	14.86	14.69	14.54	14.10	13.65	12.80	12.49	12.40	12.36	12.19	12.17	11.74	10.96	
2	13.60	14.20	14.56	14.70	14.68	14.62	14.56	14.31	14.00	13.52	13.46	13.38	13.29	12.21	12.78	12.22	11.43	
3	13.10	13.03	12.94	12.82	12.72	12.66	12.65	12.74	12.86	12.81	12.93	13.00	13.01	12.69	12.65	12.16	11.14	
4	12.57	14.11	14.99	15.00	15.10	15.00	14.96	14.87	14.67	14.35	14.12	14.00	13.90	13.50	13.39	12.68	11.65	
5	16.27	16.21	16.01	15.67	15.31	15.06	14.95	14.66	14.69	14.15	13.82	13.54	13.31	12.99	12.80	12.31	11.41	
6	14.03	14.41	14.61	14.65	14.65	14.65	14.66	14.56	14.56	14.42	14.19	13.93	13.87	13.74	13.29	13.11	12.60	11.57
7	14.50	14.98	15.27	15.41	15.52	15.67	15.80	15.88	15.75	15.04	14.90	14.66	14.66	13.89	13.67	12.93	11.62	
8	15.40	15.64	15.84	16.00	16.15	16.33	16.51	16.63	16.35	16.15	15.60	15.60	15.66	14.91	14.39	13.96	12.61	
9	12.64	13.68	14.38	14.76	15.04	15.35	15.65	16.02	15.91	16.11	15.82	15.70	15.06	15.14	14.41	12.43		
10	12.75	12.97	13.09	13.13	13.21	13.42	13.69	14.12	14.10	14.25	14.35	14.36	14.21	14.21	13.61	12.21		
11	10.17	10.24	10.37	10.57	10.78	10.94	11.06	11.31	11.56	12.23	12.53	12.59	12.72	12.95	12.79	12.59	11.86	
12	8.26	9.71	10.79	11.51	11.97	12.30	12.55	13.05	13.33	13.51	13.58	13.72	13.67	13.70	13.27	11.88		
1982	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr	
1	11.73	12.14	12.51	12.84	13.11	13.29	13.41	13.64	13.81	13.85	13.94	13.90	13.80	13.87	13.26	12.10		
2	11.85	11.98	12.27	12.72	13.19	13.47	13.56	13.55	13.75	13.88	13.82	13.80	13.71	13.85	13.42	12.35		
3	14.57	13.96	13.66	13.77	13.84	13.86	13.88	13.95	14.11	14.06	13.98	13.94	13.64	13.51	13.19	12.00		
4	12.31	12.32	12.46	12.71	12.94	13.09	13.16	13.29	13.46	13.64	13.61	13.46	13.31	13.04	12.76	12.16		

Table 13.A.1 (cont.)

	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr	
1982	5	11.56	11.62	11.72	11.85	12.01	12.14	12.24	12.55	12.87	13.23	13.37	13.49	13.30	13.22	12.96	12.15	
	6	10.65	11.74	12.57	13.12	13.46	13.68	13.81	14.03	14.21	14.32	14.43	14.27	13.84	13.65	13.25	12.07	
	7	8.49	9.15	9.82	10.50	11.08	11.50	11.77	12.26	12.65	12.93	13.31	13.34	13.28	13.27	13.05	12.18	
	8	7.54	7.51	7.78	8.33	8.96	9.48	9.84	10.56	11.17	11.89	12.26	12.34	12.47	12.44	12.25	11.79	
	9	6.88	6.94	7.24	7.77	8.39	8.90	9.23	9.82	10.37	11.21	11.56	11.61	11.65	11.74	11.59	11.11	
	10	7.81	7.72	7.83	8.11	8.41	8.63	8.73	8.97	9.45	10.01	10.52	10.60	10.75	10.87	10.95	10.41	
	11	7.67	7.87	8.11	8.39	8.65	8.81	8.87	8.95	9.32	9.84	10.33	10.52	10.71	10.96	11.19	10.65	
	12	8.05	8.07	8.11	8.17	8.25	8.31	8.34	8.45	8.79	9.45	9.92	10.17	10.33	10.59	10.67	10.37	
1983	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr	
	1	8.01	8.07	8.15	8.26	8.38	8.46	8.52	8.67	8.96	9.60	10.09	10.32	10.55	11.04	11.18	11.52	11.13
	2	7.90	7.91	7.96	8.06	8.17	8.23	8.24	8.28	8.61	9.20	9.70	9.76	9.86	10.36	10.63	11.08	10.35
	3	8.36	8.56	8.72	8.86	8.95	9.01	9.02	9.06	9.25	9.73	10.01	10.18	10.31	10.64	10.83	11.00	10.45
	4	8.01	8.10	8.19	8.28	8.35	8.39	8.41	8.49	8.72	9.23	9.63	9.81	9.97	10.32	10.52	10.69	10.15
	5	8.38	8.56	8.71	8.83	8.93	9.00	9.05	9.16	9.32	9.89	10.16	10.36	10.52	10.86	11.11	11.29	10.85
	6	8.25	8.57	8.81	8.98	9.09	9.18	9.24	9.39	9.58	10.13	10.40	10.57	10.69	11.00	11.21	11.22	10.72
	7	8.63	8.95	9.22	9.43	9.60	9.72	9.71	9.81	10.00	10.35	10.98	11.16	11.53	11.78	11.84	11.82	11.59
	8	8.74	9.01	9.25	9.46	9.65	9.80	9.91	10.13	10.36	11.01	11.35	11.56	11.69	11.79	11.89	12.08	11.44
	9	8.77	8.77	8.83	8.92	9.04	9.15	9.24	9.48	9.70	10.36	10.66	10.92	11.13	11.38	11.43	11.53	10.94
	10	8.46	8.49	8.58	8.72	8.87	9.01	9.12	9.38	9.62	10.38	10.83	11.10	11.29	11.64	11.77	11.81	11.37
	11	8.17	8.58	8.86	9.07	9.19	9.28	9.37	9.58	9.71	10.44	10.77	11.09	11.26	11.47	11.55	11.62	11.14
	12	8.18	8.63	8.97	9.20	9.33	9.42	9.50	9.71	9.89	10.63	10.93	11.25	11.43	11.66	11.74	11.85	11.36
1984	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr	
	1	8.87	8.94	9.02	9.10	9.18	9.25	9.32	9.49	9.63	10.39	10.69	11.03	11.20	11.53	11.58	11.69	11.45
	2	8.45	8.86	9.17	9.37	9.51	9.60	9.68	9.85	9.98	10.77	11.05	11.43	11.61	11.85	11.91	12.06	11.85
	3	9.22	9.50	9.76	9.97	10.12	10.22	10.39	10.59	10.81	11.61	11.30	11.58	11.91	12.07	12.31	12.21	12.29
	4	9.26	9.49	9.73	9.96	10.15	10.27	10.34	10.54	10.81	11.56	11.89	12.17	12.34	12.60	12.82	12.64	12.44
	5	9.65	9.56	9.69	10.04	10.46	10.79	11.01	11.43	11.93	12.69	13.29	13.44	13.43	13.43	13.26	13.25	13.21
	6	8.83	9.39	9.84	10.18	10.47	10.73	10.98	11.60	12.00	12.99	13.05	13.09	13.40	13.51	13.52	13.23	13.21
	7	9.67	10.03	10.33	10.57	10.76	10.91	11.03	11.32	11.63	12.29	12.34	12.44	12.52	12.65	12.56	12.59	12.24
	8	10.78	10.74	10.76	10.82	10.92	11.02	11.13	11.41	11.67	12.21	12.49	12.47	12.35	12.14	12.29	12.14	12.29
	9	10.52	10.44	10.44	10.50	10.60	10.69	10.76	10.94	11.16	11.77	12.14	12.21	12.15	12.00	11.88	11.74	11.59
	10	8.07	8.57	8.96	9.15	9.44	9.57	9.65	9.83	10.15	10.99	11.13	11.28	11.39	11.55	11.48	11.61	11.21
	11	7.47	8.00	8.38	8.62	8.77	8.88	8.99	9.29	9.53	10.31	10.69	11.23	11.43	11.49	11.60	11.20	11.20
	12	6.30	7.23	7.84	8.13	8.19	8.25	8.37	8.61	9.13	9.93	10.44	10.76	11.11	11.46	11.55	11.59	11.20
1985	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr	
	1	7.29	7.73	8.03	8.19	8.27	8.35	8.44	8.73	8.98	9.76	10.22	10.53	10.78	11.12	11.31	11.48	11.93
	2	6.77	7.67	8.32	8.71	8.91	9.04	9.13	9.36	9.65	10.46	10.93	11.24	11.50	11.86	12.03	11.96	11.87

Table 13 A 2
McCulloch instantaneous forward rate data, continuous compounding, end of month data, 12/46-2/87

	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr
1946	0.16	0.45	0.58	0.62	0.66	0.70	0.74	0.86	0.96	1.34	1.61	1.79	1.93	2.57	3.01	2.49	
12	0.18	0.45	0.57	0.61	0.65	0.70	0.74	0.87	0.98	1.38	1.66	1.83	1.94	2.53	3.05	2.26	
1947	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr
1	0.16	0.44	0.56	0.60	0.65	0.70	0.74	0.86	0.96	1.34	1.61	1.79	1.93	2.57	3.01	2.49	
2	0.19	0.44	0.56	0.60	0.65	0.70	0.74	0.86	0.96	1.37	1.65	1.81	1.93	2.52	3.00	2.49	
3	0.13	0.47	0.62	0.66	0.69	0.73	0.76	0.87	0.96	1.30	1.57	1.76	1.91	2.55	2.93	2.60	
4	0.08	0.49	0.65	0.69	0.73	0.77	0.81	0.92	1.03	1.39	1.64	1.78	1.91	2.52	2.98	2.49	
5	0.00	0.52	0.74	0.78	0.81	0.84	0.87	0.96	1.05	1.35	1.58	1.75	1.88	2.53	3.00	2.45	
6	0.08	0.53	0.71	0.75	0.78	0.82	0.85	0.96	1.05	1.39	1.64	1.80	1.94	2.59	3.07	2.15	
7	0.06	0.64	0.85	0.88	0.91	0.94	0.97	1.05	1.13	1.40	1.59	1.73	1.87	2.60	3.17	1.80	
8	0.16	0.82	1.07	1.08	1.09	1.09	1.10	1.11	1.14	1.25	1.40	1.60	1.81	2.68	3.09	1.79	
9	0.52	0.92	1.08	1.09	1.09	1.10	1.11	1.13	1.16	1.29	1.45	1.66	1.86	2.69	3.01	2.03	
10	0.45	0.96	1.17	1.18	1.19	1.21	1.22	1.26	1.30	1.46	1.63	1.80	1.97	2.73	3.08	1.70	
11	0.60	0.99	1.15	1.17	1.19	1.21	1.23	1.28	1.33	1.54	1.74	1.93	2.10	2.81	2.99	2.06	
12	0.93	0.89	0.91	0.96	1.01	1.06	1.11	1.24	1.36	1.76	2.00	2.16	2.31	2.80	2.85		
1948	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr
1	1.04	0.90	0.88	0.93	0.97	1.02	1.06	1.10	1.23	1.34	1.76	2.02	2.18	2.32	2.80	2.86	
2	1.02	0.93	0.93	0.98	1.02	1.06	1.10	1.14	1.24	1.35	1.73	1.98	2.14	2.29	2.82	2.88	
3	1.00	0.96	0.96	0.98	1.02	1.06	1.10	1.14	1.24	1.35	1.69	1.93	2.11	2.27	2.86	2.94	
4	0.98	0.99	1.01	1.01	1.05	1.09	1.12	1.15	1.26	1.35	1.66	1.88	2.07	2.25	2.91	2.97	
5	0.97	1.01	1.03	1.03	1.06	1.08	1.10	1.12	1.19	1.25	1.49	1.72	1.94	2.14	2.91	3.01	
6	0.95	1.00	1.04	1.07	1.10	1.13	1.16	1.24	1.33	1.62	1.85	2.07	2.27	2.95	2.93		
7	1.02	0.97	1.03	1.07	1.10	1.12	1.16	1.28	1.39	1.73	1.94	2.13	2.29	2.85	2.86		
8	1.13	1.02	1.01	1.06	1.11	1.16	1.21	1.34	1.46	1.80	1.99	2.15	2.30	2.81	2.84		
9	1.09	1.08	1.11	1.15	1.20	1.24	1.28	1.40	1.51	1.81	1.98	2.13	2.28	2.81	2.86		
10	1.11	1.07	1.09	1.14	1.19	1.24	1.29	1.43	1.54	1.86	2.03	2.18	2.31	2.78	2.81		
11	1.10	1.10	1.13	1.17	1.21	1.25	1.28	1.39	1.48	1.76	1.94	2.12	2.28	2.78	2.90		
12	1.06	1.15	1.19	1.22	1.24	1.26	1.28	1.35	1.41	1.65	1.87	2.07	2.25	2.90	2.88		
1949	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr
1	1.09	1.16	1.19	1.21	1.22	1.24	1.25	1.30	1.35	1.57	1.61	1.80	2.01	2.21	2.92	2.89	
2	1.16	1.12	1.13	1.15	1.18	1.20	1.23	1.30	1.37	1.61	1.82	2.01	2.21	2.86	2.87		
3	1.03	1.18	1.23	1.24	1.25	1.26	1.27	1.30	1.33	1.51	1.73	1.94	2.14	2.92	2.90		
4	1.10	1.14	1.17	1.18	1.20	1.21	1.23	1.28	1.33	1.53	1.75	1.95	2.14	2.93	2.94		
5	1.00	1.17	1.23	1.24	1.25	1.25	1.25	1.27	1.30	1.47	1.69	1.92	2.13	2.95	2.94		
6	0.81	1.18	1.27	1.25	1.23	1.21	1.20	1.16	1.14	1.23	1.52	1.79	2.04	2.87	2.81		
7	0.80	1.09	1.16	1.14	1.13	1.13	1.11	1.08	1.07	1.21	1.51	1.79	2.04	2.89	2.81		

Table 13 A 2 (cont.)

	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr
1952	10.32	1.80	1.96	1.98	2.00	2.02	2.04	2.10	2.15	2.33	2.45	2.52	2.59	2.88	3.02	2.81	2.81
11	1.55	1.97	2.11	2.13	2.14	2.15	2.16	2.20	2.23	2.35	2.42	2.47	2.53	2.88	3.11	2.81	2.81
12	1.82	2.04	2.12	2.14	2.15	2.17	2.18	2.22	2.26	2.38	2.45	2.49	2.55	2.97	3.28	2.97	2.97
1953	1.63	1.97	2.15	2.06	2.07	2.08	2.10	2.11	2.15	2.35	2.48	2.61	2.72	3.05	3.11		
2	1.86	2.15	2.22	2.22	2.23	2.23	2.24	2.26	2.26	2.44	2.59	2.64	2.77	3.18	3.23		
3	1.95	2.02	2.06	2.08	2.10	2.12	2.14	2.19	2.25	2.49	2.57	2.71	2.82	3.19	3.28		
4	1.78	2.29	2.44	2.45	2.47	2.48	2.49	2.53	2.53	2.71	2.84	2.95	3.33	3.26	3.29	3.53	
5	1.43	2.33	2.59	2.62	2.64	2.66	2.67	2.69	2.76	2.83	2.96	3.06	3.24	3.44	3.48	3.07	3.33
6	1.18	1.96	2.31	2.34	2.36	2.38	2.40	2.47	2.52	2.73	2.90	2.90	3.02	3.12	3.27	3.05	3.14
7	1.72	2.11	2.23	2.26	2.28	2.31	2.33	2.40	2.47	2.71	2.89	3.01	3.11	3.26	3.15	3.52	3.76
8	1.62	1.93	2.04	2.09	2.14	2.19	2.23	2.36	2.49	2.88	3.11	3.19	3.26	3.31	3.17	3.34	
9	0.86	1.52	1.62	1.88	1.92	1.97	2.01	2.13	2.23	2.56	2.71	2.72	2.73	2.92	3.29	3.77	
10	0.65	1.25	1.52	1.58	1.64	1.69	1.75	1.90	2.05	2.49	2.72	2.79	2.84	3.12	3.31	3.33	
11	1.36	1.50	1.59	1.64	1.68	1.73	1.77	1.89	2.01	2.39	2.64	2.77	2.89	3.26	3.37	3.20	
12	1.09	1.38	1.52	1.56	1.61	1.65	1.69	1.81	1.91	2.24	2.40	2.47	2.55	2.97	3.30	3.41	
1954	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr
1	1.05	0.89	0.90	0.97	1.05	1.12	1.19	1.39	1.57	2.12	2.40	2.55	2.68	3.03	3.17	3.24	
2	0.84	0.92	0.98	1.01	1.05	1.09	1.12	1.23	1.34	1.78	2.23	2.62	2.89	3.17	3.64	3.13	
3	0.99	0.98	1.00	1.03	1.06	1.10	1.13	1.23	1.34	1.75	2.17	2.53	2.79	3.22	3.77	3.08	
4	0.62	0.72	0.78	0.84	0.89	0.95	1.00	1.14	1.27	1.64	1.96	2.30	2.62	2.91	2.81	3.01	
5	0.51	0.67	0.74	0.80	0.85	0.91	0.96	1.13	1.29	1.92	2.42	2.73	2.90	2.90	2.78	2.99	
6	0.67	0.59	0.61	0.67	0.73	0.79	0.84	1.01	1.17	1.71	2.15	2.53	2.83	2.84	2.57	2.94	
7	0.64	0.72	0.76	0.79	0.82	0.86	0.91	1.01	1.13	1.73	2.25	2.59	2.77	2.77	2.60	2.83	
8	1.30	0.96	0.87	0.90	0.94	0.97	1.01	1.12	1.23	1.72	2.17	2.57	2.88	2.79	2.52	2.87	
9	0.90	0.91	0.96	1.03	1.09	1.15	1.20	1.36	1.50	1.87	2.15	2.50	2.83	2.73	2.53	2.96	
10	0.72	0.95	1.06	1.12	1.17	1.23	1.28	1.43	1.58	2.07	2.43	2.63	2.73	2.70	2.70	2.93	
11	0.86	0.99	1.07	1.12	1.17	1.21	1.23	1.29	1.47	1.64	2.22	2.59	2.72	2.74	2.86	2.91	2.80
12	0.92	0.98	1.04	1.10	1.17	1.23	1.29	1.47	1.64	2.22	2.59	2.72	2.74	2.86	2.91	2.69	
1955	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr
1	1.19	1.04	1.08	1.19	1.29	1.39	1.48	1.73	1.95	2.38	2.53	2.66	2.79	3.14	2.98	2.82	2.63
2	0.91	1.33	1.51	1.60	1.70	1.78	1.87	2.09	2.27	2.63	2.83	2.70	2.79	3.07	3.04	2.98	2.98
3	1.33	1.34	1.41	1.51	1.61	1.71	1.80	2.04	2.24	2.61	2.80	2.80	2.80	3.02	2.97	2.91	2.94
4	1.44	1.55	1.65	1.74	1.82	1.90	1.98	2.18	2.35	2.64	2.74	2.74	2.84	3.09	2.95	2.81	2.81
5	0.91	1.36	1.60	1.69	1.77	1.85	1.92	2.13	2.29	2.62	2.74	2.81	2.85	2.97	2.90	2.86	2.90
6	1.04	1.47	1.68	1.77	1.85	1.93	1.93	2.01	2.22	2.39	2.75	2.75	2.89	3.00	3.06	2.89	2.77

Table 13 A 2 (cont.)

	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr
1958	0 mo	2.30	2.68	2.78	2.87	2.96	3.05	3.28	3.48	3.92	3.95	3.94	3.90	3.75	3.68	3.67	3.79
8	1.35	2.73	3.48	3.52	3.51	3.52	3.55	3.55	3.76	3.96	3.96	3.96	3.88	3.66	3.66	3.79	4.16
9	1.00	2.43	2.98	3.04	3.10	3.15	3.20	3.35	3.48	3.80	3.91	3.91	3.85	3.66	3.63	3.71	4.04
10	1.08	2.78	3.55	3.56	3.54	3.54	3.53	3.53	3.64	3.75	3.74	3.71	3.65	3.70	3.80	3.96	3.93
11	0.83	2.13	2.66	2.93	3.06	3.16	3.24	3.29	3.41	3.53	3.89	4.07	4.11	4.08	3.87	3.64	3.58
12	2.13	2.61	2.92	3.19	3.38	3.48	3.55	3.72	3.87	4.21	4.22	4.16	4.09	3.94	4.07	4.08	3.88
1959	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr
1	2.32	2.61	2.92	3.19	3.38	3.48	3.55	3.72	3.87	4.21	4.22	4.16	4.09	3.94	4.07	4.08	3.88
2	2.04	2.68	3.09	3.42	3.45	3.49	3.53	3.62	3.70	3.91	3.97	3.93	3.90	3.83	3.83	4.27	4.42
3	2.44	2.70	3.04	3.43	3.67	3.77	3.80	3.90	3.98	4.10	4.12	4.08	4.02	3.99	4.02	4.06	
4	2.78	2.75	3.02	3.43	3.68	3.76	3.80	3.90	3.99	4.27	4.40	4.40	4.35	4.09	3.81	3.68	3.92
5	2.80	2.76	3.10	3.78	4.20	4.20	4.23	4.23	4.23	4.24	4.43	4.43	4.38	4.03	3.91	3.85	4.02
6	2.70	2.89	3.32	3.96	4.37	4.43	4.34	4.16	4.10	4.57	4.75	4.53	4.32	4.30	4.30	4.05	3.96
7	1.97	2.79	3.39	3.85	4.34	4.79	5.10	5.07	4.68	4.17	4.82	4.90	4.68	4.68	3.98	3.97	3.96
8	3.14	3.78	4.21	4.53	4.77	4.92	4.97	4.73	4.50	4.36	4.89	4.93	4.73	4.52	3.62	3.62	3.90
9	3.43	3.87	4.48	5.22	5.67	5.68	5.46	4.92	4.55	4.60	4.79	4.68	4.55	4.01	3.67	3.53	3.99
10	2.14	3.99	4.62	4.60	4.70	4.88	4.95	4.66	4.39	4.19	4.98	5.15	4.86	3.88	3.60	3.53	3.91
11	2.31	4.49	5.10	5.12	5.15	5.18	5.17	5.05	4.94	4.70	4.69	4.63	4.53	4.08	3.73	3.59	3.93
12	4.02	4.21	4.80	5.36	5.38	5.27	5.21	5.11	5.02	4.78	4.70	4.69	4.70	4.39	3.85	3.61	4.11
1960	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr
1	3.28	3.81	4.42	4.91	5.04	5.03	4.99	4.85	4.75	4.70	4.81	4.84	4.81	4.07	3.77	3.18	
2	3.58	4.04	4.53	4.56	4.40	4.36	4.42	4.47	4.68	4.68	4.64	4.64	4.68	4.03	3.91	3.95	4.40
3	2.40	2.94	3.32	3.57	3.71	3.78	3.82	3.84	3.86	4.07	4.23	4.22	4.18	4.07	4.02	3.97	3.96
4	2.89	3.04	3.20	3.52	3.41	3.56	3.89	4.16	4.84	4.36	4.74	4.71	4.49	4.10	4.18		
5	1.82	2.02	2.51	2.79	3.02	3.20	3.36	3.74	4.40	4.39	4.53	4.41	4.21	4.01	4.31	4.31	4.05
6	1.25	2.02	2.40	2.76	3.04	3.10	3.23	3.23	3.32	3.32	4.14	4.14	4.12	4.29	4.26	3.59	3.71
7	1.65	2.02	2.99	3.23	3.23	3.04	2.90	2.86	3.07	3.61	3.77	3.72	3.95	4.11	3.42	3.42	3.73
8	1.45	2.42	2.99	3.23	3.23	3.04	2.90	2.86	3.07	3.61	3.77	3.72	3.92	4.06	4.09	3.73	3.62
9	2.69	2.15	2.58	3.27	3.44	3.18	2.92	2.61	2.85	3.64	3.94	3.94	4.05	4.05	4.00	3.91	3.85
10	1.22	1.96	2.51	2.86	3.04	3.05	3.06	3.20	3.20	3.63	3.96	4.13	4.15	4.09	3.91	3.78	
11	1.24	2.27	2.84	3.01	3.09	3.11	3.14	3.24	3.38	3.83	4.11	4.23	4.23	4.14	3.89	3.74	
12	1.87	2.17	2.41	2.57	2.63	2.61	2.61	2.72	2.88	3.41	3.77	3.93	3.93	4.06	3.95	3.85	3.81
1961	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr
1	1.88	2.19	2.44	2.64	2.74	2.77	2.80	3.00	3.22	3.78	3.97	4.03	4.05	4.06	4.01	3.96	3.93
2	2.47	2.52	2.69	2.87	2.97	3.01	3.05	3.15	3.24	3.55	3.75	3.83	3.87	3.95	3.94	3.90	3.83
3	2.02	2.35	2.54	2.67	2.81	2.94	3.03	3.07	3.11	3.42	3.85	4.04	4.07	4.05	3.89	3.72	3.62
4	1.42	2.32	2.49	2.35	2.52	2.87	3.11	3.21	3.19	3.35	3.70	3.88	3.97	4.02	3.85	3.68	3.56

Table 13.A.2 (cont.)

		0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr
1964	6	3.28	3.59	3.60	3.54	3.63	3.78	3.87	3.88	3.99	4.19	4.28	4.21	4.11	4.07	4.16		
	7	3.02	3.54	3.65	3.60	3.73	3.86	3.93	3.95	3.94	4.29	4.39	4.28	4.17	4.11	4.16		
	8	3.21	3.55	3.65	3.68	3.77	3.89	3.95	3.89	3.84	4.00	4.22	4.32	4.36	4.31	4.18	4.09	4.11
	9	3.50	3.56	3.68	3.81	3.91	3.98	4.00	3.95	3.90	3.99	4.12	4.22	4.29	4.30	4.20	4.13	4.15
	10	3.39	3.52	3.70	3.88	3.97	3.99	3.99	3.96	3.94	4.02	4.12	4.26	4.17	4.10	4.12		
	11	3.37	3.85	4.07	4.13	4.22	4.30	4.31	4.21	4.14	4.14	4.17	4.20	4.23	4.21	4.16	4.10	
	12	3.13	3.90	4.08	4.07	4.12	4.14	4.11	4.11	3.99	4.07	4.19	4.25	4.29	4.30	4.22	4.13	4.07
1965	1	3.81	3.89	3.95	4.05	4.19	4.25	4.20	3.97	3.87	4.02	4.15	4.22	4.27	4.28	4.21	4.14	4.10
	2	3.61	4.06	4.14	4.13	4.22	4.25	4.21	4.14	4.07	4.03	4.09	4.20	4.29	4.31	4.18	4.10	4.13
	3	3.60	3.99	4.08	4.06	4.14	4.21	4.09	4.07	4.02	4.09	4.10	4.18	4.22	4.25	4.20	4.15	4.15
	4	3.82	3.94	4.04	4.11	4.15	4.16	4.14	4.14	4.06	4.02	4.12	4.19	4.22	4.23	4.24	4.21	4.17
	5	3.74	3.96	4.00	4.01	4.08	4.13	4.14	4.11	4.09	4.08	4.14	4.23	4.23	4.29	4.18	4.11	4.14
	6	3.71	3.85	3.91	3.94	3.99	4.03	4.03	4.01	3.90	3.88	4.08	4.20	4.25	4.29	4.27	4.20	4.14
	7	3.81	3.83	3.92	4.03	4.07	4.06	4.07	4.06	4.03	4.18	4.26	4.27	4.28	4.26	4.21	4.17	4.16
	8	3.70	3.90	4.00	4.08	4.22	4.31	4.28	4.28	4.21	4.21	4.29	4.32	4.34	4.37	4.29	4.18	4.24
	9	3.93	3.99	4.19	4.37	4.45	4.47	4.47	4.47	4.42	4.39	4.30	4.33	4.42	4.48	4.37	4.20	4.25
	10	3.85	4.00	4.26	4.44	4.46	4.44	4.43	4.43	4.41	4.41	4.42	4.47	4.51	4.53	4.37	4.18	4.20
	11	3.67	4.14	4.33	4.40	4.54	4.62	4.59	4.49	4.51	4.60	4.61	4.59	4.58	4.47	4.33	4.19	4.09
	12	4.31	4.55	4.62	4.77	5.06	5.20	5.16	5.08	5.13	5.21	5.00	4.76	4.58	4.39	4.30	4.23	4.22
1966	1	4.50	4.58	4.85	4.99	4.98	4.95	4.93	4.94	5.04	5.27	5.11	4.90	4.74	4.44	4.26	4.23	4.51
	2	4.24	4.70	4.86	4.98	5.16	5.27	5.29	5.31	5.28	5.22	5.32	5.40	5.41	4.75	4.16	4.09	5.01
	3	4.19	4.57	4.71	4.85	5.07	5.18	5.15	5.09	5.08	5.03	4.96	4.89	4.82	4.57	4.37	4.33	4.59
	4	4.44	4.75	4.71	4.76	5.02	5.16	5.12	5.10	5.04	5.14	4.97	5.01	4.70	4.20	4.13	4.47	
	5	4.21	4.72	4.82	4.79	5.14	5.40	5.31	5.23	5.19	5.04	4.88	4.84	4.67	4.51	4.43	4.54	
	6	4.22	4.68	4.65	4.72	5.11	5.28	5.17	5.10	5.39	5.16	5.05	5.06	4.83	4.54	4.33	4.34	
	7	4.09	4.92	4.91	4.94	5.36	5.53	5.43	5.28	5.41	5.68	5.55	5.44	5.34	4.77	4.29	4.14	4.66
	8	4.21	5.22	5.29	5.34	6.31	6.87	6.55	6.31	6.65	5.91	5.56	5.38	4.92	4.56	4.42	4.71	
	9	5.00	5.37	5.61	5.87	6.16	6.31	6.23	5.98	5.85	5.54	5.34	5.03	4.77	4.75	4.53	4.08	
	10	4.24	5.32	5.63	5.77	6.06	6.14	5.99	5.69	5.62	5.43	5.32	5.15	4.98	4.60	4.31	4.24	4.62
	11	3.66	5.37	5.75	5.42	5.50	5.64	5.56	5.48	5.74	5.71	5.00	5.07	5.33	4.89	4.40	4.21	4.64
	12	4.60	4.81	5.07	5.25	5.27	5.21	5.09	4.96	4.99	4.98	4.85	4.70	4.60	4.57	4.53	4.48	4.43
1967	1	4.37	4.65	4.65	4.60	4.67	4.69	4.65	4.62	4.67	4.75	4.72	4.66	4.61	4.49	4.39	4.35	4.44
	2	3.77	4.79	4.70	4.56	4.74	4.78	4.67	4.64	4.94	5.05	4.56	4.69	4.96	4.85	4.64	4.44	4.34
	3	3.91	4.11	4.26	4.28	4.18	4.12	4.10	4.11	4.20	4.41	4.44	4.61	4.77	4.76	4.67	4.54	4.36

Table 13.A.2 (cont.)

	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr
1970	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr
6	5.46	6.61	6.77	6.64	6.91	7.26	7.48	7.64	7.73	7.87	7.72	7.63	7.59	7.06			
7	4.32	7.01	6.69	6.07	6.61	7.28	7.52	7.03	7.07	7.99	7.56	7.46	7.67	7.20			
8	6.07	6.37	6.44	6.53	6.81	7.17	7.27	6.80	6.82	7.86	7.43	7.35	7.58	7.54			
9	5.61	5.60	6.23	7.03	6.95	6.77	6.66	6.63	6.91	7.28	7.49	7.55	7.27				
10	5.05	5.81	6.26	6.44	6.51	6.48	6.44	6.49	6.90	7.24	7.40	7.42	7.37				
11	4.40	5.04	5.28	5.02	4.95	5.06	5.30	5.42	5.81	6.41	6.85	6.90	6.83	6.26	5.09		
12	3.98	5.00	5.08	4.99	4.99	4.90	4.80	5.03	5.54	6.18	6.34	6.36	6.46	6.68	5.14		
1971	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr
1	3.99	4.19	4.18	4.27	4.41	4.31	4.14	4.17	4.57	5.80	6.58	6.91	6.83	6.09	5.55	6.50	
2	3.13	3.36	3.51	3.66	3.78	3.83	3.84	3.86	4.00	5.40	6.26	6.62	6.68	6.79	6.59		
3	3.46	3.50	3.76	3.88	3.76	3.69	3.74	3.92	4.11	4.88	5.60	6.15	6.36	6.20	6.06	6.24	
4	3.49	4.05	4.15	4.28	4.55	4.75	4.83	4.90	5.16	6.25	6.66	6.71	6.60	5.82			
5	3.92	4.42	4.49	4.42	4.53	4.73	4.95	5.48	5.74	6.23	6.44	6.65	6.84	6.55			
6	4.52	5.26	5.22	5.09	5.50	6.15	6.43	6.67	6.80	7.00	7.00	7.00	7.05	6.36			
7	4.78	5.48	5.31	5.38	6.19	6.72	6.71	6.19	6.32	7.34	7.19	7.10	7.13	6.59			
8	4.49	4.42	4.35	4.44	5.01	5.59	5.74	5.72	5.74	5.91	6.10	6.28	6.44	6.66			
9	4.55	4.46	4.73	5.05	5.28	5.42	5.48	5.57	5.62	5.89	6.29	6.38	6.20	6.07			
10	3.56	4.42	4.54	4.57	4.65	4.58	4.48	4.65	5.22	5.36	5.83	6.79	6.75	5.82	7.01		
11	3.67	4.37	4.53	4.45	4.60	4.76	4.81	4.99	5.21	5.43	6.13	6.86	6.61	5.63			
12	3.24	3.47	3.91	4.27	4.47	4.38	4.19	4.53	5.11	5.33	5.73	6.24	6.34	6.53			
1972	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr
1	2.41	3.46	3.52	3.72	4.20	4.31	4.30	4.57	4.94	5.96	6.32	6.44	6.68	7.07			
2	3.36	3.22	3.51	3.94	4.15	4.27	4.37	4.66	4.95	5.82	6.15	6.36	6.56	6.86			
3	3.32	3.58	4.20	4.74	4.96	5.07	5.20	5.56	5.79	6.22	6.38	6.43	6.43	6.32			
4	3.30	3.36	3.89	4.33	4.53	4.64	4.70	4.85	5.13	6.29	6.28	6.21	6.35	6.64			
5	3.31	3.68	3.97	4.23	4.52	4.75	4.85	4.90	5.02	5.81	6.15	6.32	6.42	6.55			
6	3.39	3.94	4.33	4.69	5.13	5.48	5.67	5.64	5.64	5.94	6.19	6.36	6.46	6.43			
7	3.41	3.70	4.00	4.37	4.84	5.25	5.42	5.29	5.33	6.32	6.36	6.29	6.29	6.80			
8	4.27	4.42	4.56	4.85	5.37	5.81	5.92	5.69	5.65	6.25	6.35	6.47	6.68	6.76			
9	4.65	4.42	4.81	5.49	5.86	5.98	6.01	5.94	5.94	6.22	6.10	6.22	6.60	7.22			
10	4.49	4.62	4.94	5.32	5.53	5.62	5.67	5.77	5.89	6.33	6.23	6.26	6.49	6.76			
11	4.82	4.80	4.96	5.25	5.55	5.75	5.73	5.41	5.40	6.39	6.14	6.10	6.35	6.66			
12	4.77	5.08	5.38	5.61	5.75	5.79	5.76	5.67	5.76	6.47	6.17	6.20	6.44	6.57			
1973	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr
1	5.22	5.64	5.96	6.14	6.18	6.15	6.14	6.14	6.21	6.63	6.39	6.29	6.35	6.77	7.36		
2	5.49	5.71	6.06	6.37	6.44	6.41	6.43	6.63	6.77	6.82	6.67	6.55	6.52	6.50	7.05		
3	5.77	6.37	6.78	7.07	7.27	7.37	7.32	6.99	6.76	6.57	6.57	6.56	6.56	6.48	6.92		

Table 13 A.2 (cont.)

	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr
1976	6.26	5.35	5.54	5.88	6.34	6.58	6.55	6.70	7.26	7.49	7.85	7.99	8.03	8.19	8.22	8.28	
7	5.09	5.14	5.33	5.64	6.00	6.17	6.44	7.00	7.37	7.73	8.23	8.28	8.37	8.45	8.51	8.54	
8	4.80	5.13	5.25	5.43	5.79	5.99	5.96	6.06	6.65	7.02	7.39	7.80	8.04	8.21	8.48	8.68	8.04
9	5.13	5.05	5.23	5.48	5.70	5.86	5.86	5.92	6.39	6.91	7.35	7.75	7.96	8.13	8.09	8.06	
10	4.69	4.82	5.11	5.34	5.42	5.44	5.44	5.65	6.19	6.66	6.94	7.43	7.89	8.13	8.19	8.03	
11	4.37	4.45	4.55	4.66	4.82	4.94	4.94	5.03	5.40	6.19	6.48	7.28	8.57	8.76	7.49		
12	4.22	4.33	4.54	4.75	4.82	4.80	4.82	5.02	5.40	6.14	6.42	6.78	7.19	7.71	7.85	7.77	
1977	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr
1	4.58	4.61	4.94	5.28	5.44	5.53	5.60	5.91	6.43	6.85	7.31	7.64	7.68	8.01	8.15	7.85	
2	4.41	4.65	4.92	5.17	5.31	5.40	5.54	5.98	6.32	6.91	7.43	7.86	8.05	8.30	8.45	8.10	
3	4.54	4.51	4.73	4.97	5.10	5.22	5.40	5.95	6.22	6.90	7.45	7.74	7.81	8.09	8.27	8.16	7.66
4	4.33	4.56	4.86	5.13	5.31	5.46	5.60	5.98	6.29	6.82	7.36	7.75	7.84	8.08	8.13	7.98	7.76
5	4.80	5.00	5.21	5.42	5.58	5.68	5.81	6.02	6.27	6.71	7.12	7.58	7.72	8.08	8.16	7.99	7.58
6	4.98	4.97	5.20	5.46	5.63	5.73	5.75	5.84	6.18	6.53	7.04	7.42	7.61	7.84	7.80	8.05	
7	5.16	5.35	5.60	5.86	6.09	6.24	6.30	6.33	6.53	6.82	7.30	7.56	7.65	7.85	7.87	8.07	
8	5.35	5.45	5.77	6.18	6.39	6.45	6.48	6.50	6.53	6.88	7.11	7.25	7.36	7.79	8.01	8.01	
9	5.75	5.79	6.19	6.57	6.66	6.63	6.65	6.81	6.89	6.94	7.22	7.37	7.43	7.78	8.03	8.08	7.83
10	6.27	6.06	6.48	6.99	7.10	7.04	7.06	7.33	7.42	7.36	7.59	7.64	7.56	7.88	8.21	8.26	7.76
11	5.31	6.00	6.49	6.78	6.94	7.00	7.06	7.18	7.21	7.38	7.40	7.41	7.41	7.68	7.78	7.94	7.65
12	5.38	6.20	6.61	6.80	6.91	6.98	7.06	7.25	7.23	7.41	7.68	7.78	7.80	8.07	8.32	8.32	8.09
1978	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr
1	5.69	6.40	6.80	7.00	7.14	7.24	7.31	7.39	7.56	7.82	7.91	7.92	8.21	8.56	8.55	8.07	
2	5.98	6.44	6.74	7.19	7.42	7.49	7.36	7.46	7.79	8.06	8.06	8.03	8.24	8.58	8.60	8.21	
3	6.84	6.40	6.62	7.03	7.27	7.40	7.50	7.70	7.79	8.06	8.01	8.09	8.10	8.36	8.62	8.51	8.09
4	5.86	6.22	6.75	7.26	7.60	7.80	7.91	7.96	8.02	7.94	7.99	8.08	8.13	8.47	8.69	8.46	7.90
5	6.33	6.58	6.98	7.47	7.83	8.05	8.18	8.20	8.19	8.06	8.21	8.33	8.33	8.59	8.87	8.32	6.47
6	6.24	6.96	7.60	8.05	8.24	8.28	8.36	8.73	8.67	8.45	8.32	8.24	8.36	8.76	8.69	8.12	7.87
7	6.40	6.74	7.23	7.73	8.11	8.39	8.60	8.85	8.68	8.34	8.29	8.27	8.34	8.61	8.60	8.17	7.94
8	6.05	7.73	7.57	7.65	8.05	8.43	8.61	8.72	8.46	8.14	8.18	8.17	8.21	8.48	8.48	8.29	
9	7.89	8.13	8.50	8.83	9.02	9.05	8.93	8.60	8.58	7.98	8.07	8.31	8.62	8.67	8.60	8.17	7.85
10	8.35	8.90	9.07	9.28	9.89	10.40	10.48	9.87	8.98	8.88	8.59	8.43	8.44	8.74	8.82	8.20	
11	8.95	9.05	9.25	9.54	9.89	10.16	10.26	10.03	9.31	8.44	8.94	8.53	8.63	8.44	8.04	8.04	
12	7.63	9.44	10.11	10.19	10.09	10.10	10.49	11.11	10.07	8.92	8.16	8.56	8.74	8.64	8.45	8.18	8.07
1979	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr
1	9.64	9.43	9.57	9.80	9.87	9.91	10.05	9.99	9.27	8.53	7.75	8.43	8.78	8.68	8.56	8.39	8.08
2	9.58	9.66	9.69	9.77	10.05	10.30	10.32	10.06	9.69	8.77	8.30	8.68	8.93	8.63	8.23	8.25	

	0mo	1mo	2mo	3mo	4mo	5mo	6mo	9mo	1yr	2yr	3yr	4yr	5yr	10yr	15yr	20yr	25yr
1980	11.03	12.50	12.67	12.44	12.53	12.58	12.17	10.71	11.06	10.46	10.80	11.07	10.68	9.91	9.56		
1	13.29	14.41	14.63	14.38	14.78	15.54	15.77	14.90	14.95	12.36	11.79	11.73	11.56	10.78	8.32	8.62	
2	13.89	15.85	15.37	14.61	15.82	17.71	17.71	14.10	13.56	11.82	11.18	11.07	11.11	12.33	11.95	8.45	8.06
3	10.28	10.51	10.82	11.13	11.38	11.51	11.44	10.86	10.43	9.95	10.32	10.53	10.23	11.17	10.91	9.72	8.61
4	8.22	7.62	7.89	8.13	8.48	8.79	8.91	8.93	8.85	8.65	8.47	8.23	10.59	10.98	10.86	8.65	8.32
5	4.09	8.60	9.19	8.13	7.99	8.00	8.64	8.54	8.97	9.25	9.58	9.92	10.19	10.89	10.27	8.46	8.38
6	6.47	8.89	9.41	9.14	9.06	9.14	9.04	9.10	9.28	9.80	9.80	10.29	10.65	10.86	11.35	9.52	7.72
7	8.27	9.29	10.90	11.16	11.24	11.14	11.04	11.06	11.63	11.95	11.01	11.73	11.91	11.44	11.32	9.23	7.66
8	10.58	11.43	11.85	12.19	12.33	12.27	12.16	11.79	11.68	11.47	11.41	11.48	11.40	11.99	12.02	8.19	8.19
9	10.23	12.83	13.90	13.69	13.51	13.53	12.91	12.39	12.32	12.32	12.06	11.82	12.00	12.29	9.65	8.89	
10	13.63	14.98	15.26	14.64	14.47	14.98	14.97	14.45	12.45	12.53	12.08	11.92	11.69	11.86	12.34	8.94	7.63
11	10.88	14.56	16.07	15.34	14.22	13.67	13.37	11.90	10.97	11.96	11.78	11.87	11.91	11.34	8.32	8.03	
1981	0mo	1mo	2mo	3mo	4mo	5mo	6mo	9mo	1yr	2yr	3yr	4yr	5yr	10yr	15yr	20yr	25yr
1	13.97	15.26	15.32	14.62	14.14	13.91	13.70	12.68	12.07	11.82	12.00	12.22	12.18	12.11	11.76	8.87	7.78
2	13.60	14.68	15.06	14.79	14.48	14.30	14.16	13.39	12.83	13.31	13.27	13.01	12.84	12.54	12.07	8.82	9.18
3	13.10	12.95	12.73	12.46	12.39	12.52	12.20	12.35	12.88	13.20	13.45	12.90	12.35	12.40	8.34	7.34	
4	12.57	15.33	16.06	15.14	14.58	14.66	14.83	14.35	13.91	13.87	13.69	13.58	13.42	13.15	12.64	9.07	9.40
5	16.27	16.08	15.47	14.53	14.05	14.19	14.56	14.8	13.92	13.30	12.91	12.50	12.39	12.64	12.25	8.95	8.03
6	14.03	14.70	14.82	14.66	14.63	14.71	14.6	14.68	14.04	14.11	13.23	13.72	14.67	13.99	12.50	8.85	7.97
7	14.50	15.37	15.65	15.74	16.04	16.41	16.48	15.56	15.30	14.28	14.67	14.13	13.32	13.26	12.79	7.71	7.85
8	15.40	15.86	16.20	16.44	16.81	17.28	17.47	15.9	15.46	16.61	15.31	15.40	14.43	13.58	14.19	9.68	7.17
9	12.64	14.55	15.42	15.66	16.19	16.95	17.31	15.90	15.66	15.40	15.78	15.62	14.83	15.20	7.26	5.77	
10	12.75	13.14	13.22	13.26	13.77	14.72	15.30	14.38	13.88	14.30	14.54	14.24	14.06	14.06	9.07		
11	10.17	10.34	10.71	11.22	11.54	11.62	11.64	12.09	12.52	13.21	12.95	13.52	12.38	12.68	10.53	8.37	
12	8.26	10.98	12.59	13.18	13.51	13.72	13.88	14.15	14.11	13.54	14.00	14.10	13.84	13.45	13.88	8.43	7.72
1982	0mo	1mo	2mo	3mo	4mo	5mo	6mo	9mo	1yr	2yr	3yr	4yr	5yr	10yr	15yr	20yr	25yr
1	11.73	12.53	13.22	13.75	14.01	14.00	14.06	14.26	13.94	14.09	13.91	13.61	14.03	13.49	8.54	9.42	
2	11.85	12.19	13.01	14.23	14.78	14.32	13.66	13.92	14.65	14.65	13.68	13.76	13.59	13.12	14.27	9.56	8.81
3	14.57	13.51	13.39	13.93	14.17	14.06	13.92	14.09	14.22	14.22	14.15	13.76	13.78	12.83	13.59	9.41	7.71
4	12.31	12.39	12.89	13.49	13.71	13.57	13.46	13.79	14.06	13.74	13.25	13.13	13.55	12.39	12.63	10.60	10.40

Table 13 A 2 (cont.)

	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr	
1982	5	11.56	11.70	11.96	12.31	12.58	12.73	12.85	13.52	14.02	13.48	13.72	13.67	13.61	12.65	13.31	9.99	10.38
6	10.65	12.70	13.94	14.40	14.56	14.52	14.44	14.59	14.84	14.48	14.45	13.89	14.01	12.68	13.71	8.69	9.82	
7	8.49	9.81	11.19	12.44	13.17	13.17	13.09	13.54	13.92	13.66	13.21	13.65	13.21	13.81	13.60	11.17	10.26	
8	7.54	7.63	8.61	9.26	11.33	11.62	11.66	12.54	13.27	12.70	12.79	12.63	13.34	11.04	12.62	11.01	10.26	
9	6.88	7.12	8.08	9.65	10.71	10.95	10.83	11.46	12.40	12.07	12.07	11.61	12.05	11.20	11.34	10.03	10.88	
10	7.81	7.73	8.24	9.07	9.48	9.41	9.18	10.12	11.39	11.04	11.42	10.80	11.87	9.68	11.69	10.06	6.60	
11	7.67	8.09	8.64	9.24	9.51	9.36	9.01	9.65	10.99	10.41	11.42	11.00	12.05	9.74	12.13	10.45	6.96	
12	8.05	8.10	8.22	8.40	8.53	8.52	8.46	9.16	10.26	10.20	11.10	10.84	11.15	10.25	11.41	10.21	7.21	
1983	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr	
1	8.01	8.14	8.35	8.63	8.79	8.80	8.78	9.35	10.15	10.61	11.13	11.07	11.98	10.40	12.56	11.55	7.54	
2	7.90	7.94	8.12	8.40	8.52	8.41	8.22	8.84	10.14	9.88	10.73	9.69	10.98	9.39	12.86	10.15	5.94	
3	8.36	8.74	9.02	9.20	9.25	9.17	9.06	9.40	10.10	10.27	10.72	10.68	11.13	10.38	11.96	10.04	7.10	
4	8.01	8.19	8.37	8.53	8.59	8.52	8.47	9.02	9.69	10.01	10.52	10.34	10.96	9.97	11.74	9.65	6.98	
5	8.38	8.73	8.98	9.16	9.26	9.29	9.29	9.54	10.02	10.69	10.82	11.34	10.82	10.88	12.21	10.65	7.85	
6	8.25	8.85	9.23	9.38	9.47	9.52	9.56	9.88	10.45	10.73	11.06	11.12	11.25	11.29	11.83	10.06	7.86	
7	8.63	9.25	9.70	9.99	10.17	10.23	10.27	10.83	11.64	11.41	11.68	12.15	12.26	11.82	11.98	11.42	9.76	
8	8.74	9.26	9.70	10.07	10.33	10.45	10.47	10.76	11.34	11.79	12.17	12.30	11.24	12.98	11.06	7.84	7.80	
9	8.77	8.90	8.98	9.27	9.51	9.65	9.76	10.15	10.57	11.14	11.49	11.88	12.01	11.09	12.16	10.48	7.80	
10	8.46	8.55	8.81	9.19	9.48	9.63	9.71	10.10	10.59	11.50	11.89	12.11	11.64	12.34	10.89	9.11	8.93	
11	8.17	8.93	9.37	9.51	9.61	9.74	9.88	10.02	10.27	11.22	11.82	12.07	11.33	12.19	10.64	8.93	8.93	
12	8.18	9.03	9.54	9.69	9.75	9.85	9.97	10.28	10.64	11.44	11.95	12.26	12.05	11.40	12.52	10.94	9.08	
1984	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr	
1	8.87	9.02	9.18	9.34	9.49	9.61	9.70	9.92	10.29	11.15	11.80	12.02	11.84	11.35	12.13	11.40	10.14	
2	8.45	9.22	9.68	9.85	9.95	10.04	10.12	10.24	10.61	11.36	12.31	12.49	12.23	11.62	12.54	11.91	10.66	
3	9.22	9.77	10.22	10.53	10.63	10.54	10.49	11.91	11.61	12.65	12.87	12.87	12.23	11.41	12.46	12.23	11.41	
4	9.26	9.73	10.21	10.61	10.70	10.61	10.59	11.91	12.29	12.85	13.03	13.06	11.94	12.51	11.39	12.51	11.39	
5	9.65	9.58	10.18	10.31	10.61	10.62	10.16	12.03	12.82	13.80	14.16	13.83	13.47	14.27	13.15	12.94	13.72	
6	8.83	9.89	10.63	11.10	11.55	12.01	12.43	13.07	13.28	13.64	13.82	13.95	13.60	12.79	12.96	13.51	13.03	
7	9.67	10.36	10.86	11.21	11.45	11.57	11.67	12.23	12.81	12.61	12.57	12.84	12.61	12.44	12.71	12.12	9.94	
8	10.78	10.33	10.84	11.09	11.34	11.55	11.72	12.21	12.67	12.51	12.55	12.91	12.73	12.36	11.71	11.88	12.57	
9	10.52	10.40	10.50	10.79	11.00	11.09	11.14	11.54	12.07	12.31	12.55	12.66	12.20	11.84	11.68	11.23	11.74	
10	8.07	9.02	9.64	9.95	10.08	10.07	10.02	10.58	11.51	11.57	11.66	11.79	11.89	11.48	11.36	10.78	10.70	
11	7.47	8.46	9.01	9.15	9.26	9.42	9.63	10.09	10.43	11.44	11.55	12.03	12.44	11.35	12.02	10.93	9.97	
12	6.30	8.00	8.74	8.52	8.35	8.68	9.22	10.00	10.18	11.23	11.52	11.47	11.99	10.76	9.80			
1985	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr	
1	7.29	8.10	8.50	8.51	8.55	8.75	9.02	9.56	9.91	10.94	11.30	11.65	11.83	11.58	11.46	10.08	10.55	
2	6.77	8.45	9.35	9.53	9.55	9.57	9.60	10.12	10.89	11.55	12.05	12.32	12.73	12.23	12.20	11.52	10.76	

Table 13 A.3
McCulloch par bond yield curve series, continuous compounding, end of month data, 12/46-2/87

		0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr	
1946	0 mo	0.16	0.32	0.43	0.48	0.52	0.55	0.58	0.65	0.72	0.95	1.14	1.29	1.41	1.80	2.10	2.25		
12	0.18	0.32	0.42	0.48	0.52	0.55	0.58	0.65	0.72	0.95	1.14	1.29	1.41	1.80	2.10	2.25			
1947	0 mo	0.16	0.32	0.43	0.48	0.52	0.56	0.58	0.65	0.72	0.94	1.12	1.26	1.38	1.80	2.10	2.26		
1	0.16	0.32	0.43	0.48	0.52	0.56	0.58	0.65	0.72	0.94	1.12	1.26	1.38	1.80	2.10	2.26			
2	0.19	0.33	0.42	0.47	0.51	0.55	0.58	0.61	0.68	0.74	0.94	1.14	1.28	1.40	1.79	2.09	2.25		
3	0.13	0.32	0.44	0.49	0.51	0.55	0.58	0.61	0.68	0.78	1.00	1.17	1.24	1.36	1.78	2.07	2.23		
4	0.08	0.30	0.45	0.52	0.57	0.61	0.64	0.71	0.78	0.93	1.00	1.17	1.29	1.39	1.79	2.08	2.24		
5	0.00	0.29	0.47	0.57	0.63	0.67	0.70	0.77	0.83	1.02	1.02	1.19	1.32	1.43	1.82	2.13	2.27		
6	0.08	0.33	0.49	0.57	0.62	0.65	0.68	0.76	0.82	0.92	1.10	1.23	1.34	1.43	1.80	2.15	2.36		
7	0.06	0.38	0.58	0.67	0.73	0.77	0.80	0.87	0.97	1.02	1.05	1.12	1.18	1.26	1.35	1.78	2.13	2.33	
8	0.16	0.53	0.76	0.86	0.92	0.95	0.98	1.01	1.02	1.06	1.08	1.15	1.22	1.30	1.39	1.82	2.14	2.25	
9	0.52	0.74	0.88	0.95	0.98	1.00	1.02	1.05	1.08	1.10	1.14	1.18	1.28	1.36	1.45	1.53	1.92	2.24	
10	0.45	0.73	0.91	1.00	1.05	1.08	1.10	1.12	1.16	1.20	1.24	1.32	1.42	1.52	1.62	2.03	2.30		
11	0.59	0.81	0.95	1.02	1.06	1.09	1.11	1.16	1.20	1.24	1.32	1.42	1.52	1.62	2.03	2.30			
12	0.93	0.90	0.90	0.91	0.93	0.95	0.97	1.04	1.11	1.16	1.20	1.24	1.32	1.42	1.52	1.77	2.17	2.40	
1948	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr		
1	1.04	0.96	0.92	0.91	0.92	0.94	0.96	0.96	1.02	1.08	1.32	1.51	1.66	1.78	2.18	2.40			
2	1.02	0.97	0.94	0.95	0.96	0.97	0.99	1.00	1.05	1.11	1.33	1.50	1.64	1.76	2.16	2.40			
3	1.00	0.98	0.97	0.98	0.98	1.00	1.01	1.03	1.08	1.14	1.33	1.49	1.62	1.73	2.14	2.38			
4	0.98	0.98	0.99	0.99	1.00	1.02	1.04	1.05	1.10	1.15	1.33	1.48	1.60	1.71	2.13	2.39			
5	0.97	0.99	1.00	1.02	1.03	1.04	1.05	1.06	1.09	1.12	1.25	1.37	1.48	1.59	2.05	2.34			
6	0.95	0.98	1.00	1.02	1.03	1.05	1.06	1.11	1.15	1.31	1.45	1.58	1.69	2.14	2.39				
7	1.02	0.99	0.98	0.99	1.01	1.02	1.04	1.06	1.08	1.16	1.37	1.53	1.65	1.76	2.18	2.41			
8	1.13	1.06	1.03	1.04	1.05	1.07	1.09	1.15	1.21	1.33	1.58	1.71	1.81	2.19					
9	1.09	1.08	1.09	1.10	1.12	1.14	1.16	1.22	1.28	1.48	1.61	1.72	1.82	2.19	2.41				
10	1.11	1.11	1.08	1.08	1.11	1.13	1.16	1.22	1.29	1.51	1.65	1.76	1.86	2.21	2.41				
11	1.11	1.10	1.11	1.12	1.14	1.16	1.17	1.23	1.28	1.46	1.59	1.69	1.79	2.17	2.30				
12	1.06	1.11	1.14	1.16	1.18	1.19	1.21	1.24	1.28	1.40	1.52	1.63	1.73	2.15	2.38				
1949	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr		
1	1.09	1.13	1.15	1.17	1.18	1.19	1.20	1.22	1.25	1.35	1.46	1.57	1.67	1.78	2.11	2.37			
2	1.16	1.14	1.13	1.13	1.14	1.15	1.16	1.20	1.23	1.36	1.48	1.58	1.68	1.79	2.10	2.34			
3	1.03	1.11	1.16	1.19	1.20	1.21	1.22	1.24	1.26	1.33	1.43	1.53	1.63	1.73	2.07	2.35			
4	1.10	1.12	1.14	1.15	1.16	1.17	1.18	1.20	1.23	1.33	1.43	1.53	1.63	1.73	2.07	2.35			
5	1.00	1.09	1.15	1.18	1.19	1.20	1.21	1.23	1.24	1.31	1.40	1.50	1.60	1.70	2.06	2.35			
6	0.81	1.02	1.13	1.18	1.19	1.20	1.21	1.23	1.24	1.31	1.41	1.51	1.61	1.71	2.04	2.31			
7	0.79	0.96	1.05	1.08	1.10	1.10	1.10	1.10	1.10	1.10	1.10	1.10	1.10	1.10	1.30	1.42	1.94		

Table 13 A.3 (cont.)

	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr
1952	0	1.32	1.59	1.75	1.82	1.86	1.89	1.92	1.97	2.00	2.12	2.21	2.28	2.33	2.53	2.66	2.72
10	1.35	1.55	1.78	1.92	1.99	2.02	2.05	2.07	2.11	2.13	2.21	2.27	2.31	2.35	2.51	2.66	2.74
11	1.55	1.78	1.94	2.02	2.05	2.08	2.09	2.11	2.14	2.16	2.24	2.30	2.34	2.38	2.55	2.73	
12	1.82	2.02	2.11	2.14	2.16	2.18	2.20	2.23	2.26	2.29	2.37	2.41	2.44	2.49	2.71	2.86	
1953	0	1.63	1.86	1.82	1.92	1.97	2.00	2.02	2.07	2.09	2.18	2.26	2.33	2.39	2.64	2.77	
1	1.95	1.99	2.01	2.03	2.06	2.07	2.09	2.13	2.16	2.24	2.33	2.41	2.48	2.74	2.88		
2	1.95	2.06	2.09	2.11	2.14	2.16	2.18	2.21	2.24	2.27	2.33	2.41	2.48	2.74	2.88		
3	1.95	2.06	2.09	2.11	2.14	2.16	2.18	2.21	2.24	2.27	2.33	2.41	2.48	2.74	2.88		
4	1.78	2.06	2.23	2.30	2.34	2.37	2.41	2.46	2.55	2.62	2.69	2.79	2.91	3.08	3.11	3.16	
5	1.43	1.93	2.22	2.35	2.42	2.47	2.50	2.58	2.63	2.79	2.91	3.00	3.08	3.29	3.22	3.20	
6	1.18	1.61	1.89	2.03	2.11	2.17	2.20	2.28	2.33	2.48	2.59	2.68	2.75	2.99	3.03	3.04	
7	1.72	1.94	2.06	2.12	2.16	2.19	2.21	2.26	2.31	2.45	2.57	2.66	2.74	2.97	3.03	3.09	
8	1.62	1.79	1.89	1.95	1.99	2.03	2.06	2.14	2.21	2.45	2.63	2.76	2.85	3.07	3.12	3.14	
9	0.86	1.22	1.46	1.59	1.67	1.72	1.77	1.87	1.94	2.18	2.33	2.42	2.48	2.64	2.77	2.93	
10	0.65	0.97	1.19	1.31	1.39	1.44	1.49	1.56	1.60	1.99	2.16	2.34	2.43	2.60	2.74	2.96	
11	1.36	1.44	1.49	1.53	1.56	1.59	1.62	1.69	1.75	1.98	2.16	2.29	2.40	2.74	2.92	3.01	
12	1.09	1.25	1.35	1.42	1.46	1.49	1.52	1.60	1.66	1.88	2.03	2.13	2.20	2.47	2.68	2.83	
1954	0	1.05	0.96	0.92	0.92	0.95	0.97	1.00	1.10	1.19	1.53	1.78	1.95	2.08	2.48	2.68	
1	0.84	0.89	0.92	0.94	0.97	0.99	1.00	1.01	1.02	1.04	1.12	1.34	1.56	1.78	1.97	2.05	
2	0.99	0.99	1.00	1.00	1.01	1.01	1.02	1.02	1.03	1.09	1.14	1.34	1.54	1.75	1.93	2.01	
3	0.62	0.67	0.71	0.75	0.78	0.81	0.83	0.87	0.91	0.99	1.23	1.42	1.59	1.77	1.97	2.09	
4	0.51	0.60	0.66	0.69	0.73	0.76	0.79	0.82	0.87	0.96	1.28	1.48	1.63	1.83	2.03	2.20	
5	0.67	0.62	0.60	0.61	0.63	0.66	0.67	0.70	0.77	0.85	1.15	1.34	1.54	1.75	1.93	2.11	
6	0.67	0.62	0.60	0.61	0.63	0.66	0.67	0.70	0.77	0.84	1.16	1.36	1.54	1.75	1.93	2.11	
7	0.64	0.68	0.71	0.73	0.75	0.77	0.79	0.96	0.99	1.04	1.25	1.44	1.64	1.89	2.08	2.27	
8	1.30	1.11	1.00	0.96	0.95	0.95	0.95	0.95	0.95	0.99	1.04	1.25	1.45	1.71	1.91	2.51	
9	0.90	0.90	0.92	0.94	0.97	1.00	1.00	1.03	1.12	1.20	1.45	1.64	1.80	1.98	2.14	2.50	
10	0.72	0.85	0.93	0.98	1.02	1.05	1.08	1.10	1.18	1.25	1.55	1.78	1.97	2.11	2.43	2.58	
11	0.86	0.93	0.98	1.02	1.05	1.08	1.10	1.10	1.20	1.29	1.61	1.88	2.08	2.20	2.49	2.67	
12	0.92	0.95	0.98	1.01	1.04	1.07	1.10	1.10	1.20	1.29	1.61	1.88	2.08	2.20	2.49	2.67	
1955	0	1.00	1.00	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr
1	1.19	1.10	1.07	1.09	1.13	1.17	1.22	1.22	1.35	1.47	1.84	2.05	2.18	2.29	2.63	2.77	2.80
2	1.19	1.14	1.29	1.38	1.45	1.51	1.56	1.70	1.82	2.13	2.31	2.40	2.46	2.69	2.81	2.85	
3	1.33	1.35	1.39	1.43	1.47	1.52	1.65	1.78	1.85	2.13	2.29	2.39	2.46	2.69	2.78	2.83	
4	1.44	1.50	1.55	1.60	1.65	1.69	1.73	1.85	1.95	2.25	2.35	2.46	2.52	2.75	2.84	2.84	
5	1.15	1.32	1.43	1.50	1.57	1.62	1.75	1.87	1.98	2.18	2.35	2.45	2.52	2.75	2.79	2.82	
6	1.04	1.27	1.43	1.53	1.60	1.66	1.71	1.84	1.96	2.28	2.38	2.46	2.58	2.65	2.74	2.87	

Table 13 A 3 (cont.)

		0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr
1958	8	1.35	1.87	2.20	2.38	2.49	2.65	2.82	2.96	3.35	3.54	3.64	3.69	3.75	3.74	3.73	3.72	3.72
9	1.00	1.95	2.88	3.04	3.14	3.20	3.30	3.36	3.50	3.62	3.70	3.74	3.74	3.72	3.72	3.72	3.72	3.75
10	1.08	1.82	2.30	2.53	2.67	2.76	2.83	2.98	3.09	3.37	3.53	3.62	3.67	3.70	3.68	3.68	3.68	3.70
11	0.83	1.90	2.58	2.91	3.07	3.16	3.23	3.33	3.38	3.47	3.55	3.60	3.64	3.65	3.67	3.67	3.70	3.70
12	2.13	2.42	2.62	2.75	2.84	2.91	2.97	3.09	3.19	3.45	3.63	3.74	3.80	3.86	3.86	3.86	3.86	3.79
1959	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr	
1	2.32	2.47	2.61	2.76	2.89	3.00	3.09	3.27	3.40	3.73	3.89	3.96	3.99	3.99	3.99	3.99	3.99	4.01
2	2.04	2.38	2.64	2.83	2.97	3.07	3.14	3.29	3.38	3.60	3.71	3.76	3.79	3.79	3.87	3.87	3.87	4.01
3	2.44	2.57	2.72	2.89	3.06	3.19	3.29	3.48	3.59	3.82	3.92	3.96	3.98	3.98	3.98	3.98	3.98	3.99
4	2.78	2.74	2.80	2.94	3.10	3.22	3.32	3.49	3.61	3.87	4.03	4.12	4.16	4.19	4.13	4.05	4.05	4.01
5	2.80	2.82	2.75	3.03	3.47	3.60	3.79	3.96	3.96	4.10	4.17	4.20	4.20	4.18	4.12	4.08	4.08	4.05
6	2.70	2.77	2.93	3.16	3.42	3.62	3.75	3.91	3.96	4.13	4.31	4.39	4.40	4.24	4.17	4.14	4.14	4.11
7	1.97	2.40	2.75	3.04	3.30	3.56	3.79	4.25	4.40	4.34	4.39	4.52	4.57	4.42	4.24	4.12	4.06	4.06
8	3.14	3.48	3.74	3.95	4.27	4.38	4.54	4.56	4.51	4.62	4.66	4.51	4.51	4.52	4.17	4.11	4.11	4.11
9	3.43	3.64	3.90	4.21	4.53	4.76	4.90	4.99	4.92	4.69	4.70	4.71	4.70	4.50	4.32	4.19	4.12	4.12
10	2.14	3.17	3.79	4.06	4.20	4.32	4.42	4.55	4.55	4.36	4.44	4.60	4.67	4.50	4.29	4.15	4.08	4.08
11	2.31	3.53	4.23	4.52	4.67	4.77	4.84	4.93	4.94	4.87	4.88	4.78	4.74	4.55	4.37	4.23	4.23	4.16
12	4.02	4.08	4.27	4.56	4.77	4.88	4.94	5.01	5.02	4.95	4.88	4.84	4.84	4.81	4.72	4.56	4.41	4.33
1960	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr	
1	3.28	3.54	3.82	4.11	4.33	4.47	4.56	4.68	4.71	4.69	4.71	4.74	4.76	4.65	4.43	4.39	4.39	4.21
2	3.58	3.81	4.05	4.25	4.35	4.37	4.37	4.38	4.40	4.48	4.57	4.60	4.59	4.42	4.30	4.23	4.23	4.21
3	2.40	2.68	2.91	3.09	3.23	3.33	3.41	3.55	3.62	3.78	3.90	3.98	4.02	4.06	4.06	4.05	4.04	4.04
4	2.89	2.97	3.02	3.08	3.18	3.34	3.53	3.97	4.15	4.24	4.45	4.48	4.35	4.29	4.27	4.25	4.25	4.25
5	1.82	2.60	3.02	3.16	3.24	3.33	3.45	3.76	3.92	4.13	4.24	4.30	4.30	4.16	4.16	4.20	4.18	4.18
6	1.25	1.66	1.97	2.20	2.37	2.52	2.65	2.95	3.18	3.69	3.88	3.93	3.98	4.18	4.10	3.97	3.90	3.90
7	1.65	1.83	2.02	2.21	2.37	2.50	2.60	2.79	2.91	3.16	3.28	3.36	3.45	3.79	3.79	3.75	3.75	3.71
8	1.45	1.97	2.36	2.61	2.77	2.85	2.87	2.88	2.88	3.13	3.31	3.44	3.55	3.84	3.86	3.81	3.81	3.81
9	2.68	2.34	2.31	2.53	2.75	2.86	2.89	2.83	2.80	3.04	3.29	3.46	3.57	3.79	3.83	3.84	3.84	3.84
10	1.22	1.61	1.93	2.18	2.38	2.51	2.60	2.76	2.85	3.13	3.35	3.52	3.64	3.87	3.91	3.89	3.89	3.89
11	1.24	1.79	2.19	2.44	2.59	2.69	2.77	2.90	3.00	3.30	3.52	3.68	3.78	3.97	3.97	3.97	3.94	3.94
12	1.87	2.03	2.16	2.27	2.36	2.41	2.44	2.51	2.58	2.87	3.11	3.29	3.41	3.71	3.79	3.81	3.82	3.82
1961	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr	
1	1.88	2.04	2.18	2.30	2.40	2.47	2.52	2.64	2.76	3.14	3.38	3.53	3.63	3.82	3.88	3.90	3.91	3.91
2	2.47	2.48	2.54	2.62	2.69	2.75	2.80	2.90	2.97	3.18	3.34	3.45	3.52	3.71	3.77	3.80	3.81	3.81
3	2.02	2.19	2.32	2.42	2.50	2.57	2.64	2.78	2.86	3.05	3.24	3.42	3.54	3.79	3.84	3.83	3.83	3.81
4	1.42	1.93	2.20	2.27	2.30	2.38	2.48	2.72	2.84	3.03	3.20	3.34	3.45	3.78	3.78	3.78	3.78	3.76

Table 13 A 3 (cont.)

Table 13.A.3 (cont.)

	1970	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr
6	5.46	6.12	6.44	6.52	6.57	6.68	6.79	7.05	7.21	7.51	7.60	7.61	7.61	7.52				
7	4.32	5.92	6.50	6.43	6.38	6.66	6.89	7.27	7.43	7.44	7.46	7.46	7.46	7.57				
8	6.07	6.24	6.33	6.38	6.44	6.56	6.67	6.79	6.89	7.13	7.29	7.30	7.33	7.52				
9	5.62	5.56	5.71	6.03	6.32	6.47	6.53	6.59	6.60	6.67	6.81	6.94	7.04	7.24				
10	5.05	5.45	5.76	5.96	6.09	6.17	6.22	6.30	6.34	6.51	6.69	6.83	6.93	7.13				
11	4.40	4.75	4.97	5.07	5.08	5.06	5.05	5.09	5.16	5.38	5.60	5.84	6.02	6.39	6.44	6.34		
12	3.98	4.57	4.84	4.90	4.92	4.93	4.91	4.99	4.99	5.46	5.72	5.86	5.96	6.26	6.39	6.35		
	1971	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr
1	3.99	4.11	4.15	4.17	4.22	4.25	4.25	4.19	4.24	4.72	5.19	5.56	5.79	6.09	6.01	5.98		
2	3.12	3.25	3.34	3.42	3.50	3.56	3.61	3.69	3.74	4.21	4.74	5.14	5.41	5.97	6.15	6.17		
3	3.46	3.46	3.54	3.64	3.69	3.69	3.69	3.74	3.81	4.15	4.50	4.83	5.09	5.62	5.74	5.80		
4	3.49	3.81	3.97	4.04	4.13	4.24	4.33	4.50	4.63	5.18	5.60	5.85	6.00	6.11				
5	3.92	4.20	4.35	4.38	4.40	4.44	4.51	4.75	4.96	5.47	5.75	5.93	6.07	6.43				
6	4.52	4.95	5.12	5.12	5.15	5.29	5.46	5.82	6.05	6.47	6.64	6.72	6.77	6.81				
7	4.79	5.21	5.33	5.31	5.42	5.63	5.82	6.01	6.06	6.48	6.74	6.83	6.88	6.93				
8	4.49	4.46	4.42	4.40	4.47	4.64	4.82	5.11	5.26	5.34	5.69	5.80	5.90	6.22				
9	4.55	4.47	4.52	4.65	4.78	4.89	4.99	5.16	5.27	5.50	5.68	5.84	5.92	5.95				
10	3.56	4.06	4.29	4.37	4.33	4.47	4.48	4.49	4.60	5.04	5.17	5.46	5.71	5.91	6.03			
11	3.67	4.06	4.28	4.35	4.39	4.45	4.50	4.63	4.75	5.06	5.16	5.56	5.78	5.82				
12	3.24	3.33	3.50	3.70	3.87	3.98	4.03	4.11	4.29	4.82	5.02	5.25	5.44	5.87				
	1972	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr
1	2.41	3.02	3.28	3.38	3.53	3.68	3.78	3.99	4.18	4.83	5.26	5.52	5.70	6.28				
2	3.36	3.25	3.44	3.60	3.72	3.82	4.05	4.23	4.82	5.20	5.45	5.63						
3	3.32	3.42	3.64	3.92	4.16	4.33	4.46	4.75	5.00	5.50	5.76	5.91	6.00	6.17				
4	3.30	3.28	3.44	3.67	3.86	4.01	4.12	4.33	4.49	5.12	5.52	5.68	5.79	6.13				
5	3.32	3.50	3.67	3.81	3.95	4.09	4.21	4.43	4.56	4.98	5.31	5.53	5.68	6.06				
6	3.39	3.67	3.91	4.11	4.31	4.51	4.69	5.01	5.16	5.47	5.66	5.80	5.91	6.19				
7	3.41	3.56	3.70	3.86	4.04	4.25	4.43	4.73	4.87	5.35	5.69	5.83	5.93	6.32				
8	4.27	4.35	4.42	4.51	4.65	4.84	5.02	5.27	5.56	5.87	6.00	6.10	6.44					
9	4.64	4.48	4.52	4.74	4.98	5.17	5.31	5.53	5.63	5.95	6.00	6.07	6.53					
10	4.49	4.54	4.65	4.82	4.97	5.09	5.18	5.36	5.47	5.80	5.96	6.02	6.08	6.38				
11	4.82	4.79	4.83	4.92	5.04	5.16	5.26	5.36	5.36	5.65	5.86	5.97	6.27					
12	4.77	4.93	5.08	5.22	5.34	5.48	5.55	5.59	5.89	6.03	6.06	6.10	6.36					
	1973	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr
1	5.22	5.44	5.62	5.77	5.87	5.93	5.96	6.02	6.06	6.26	6.34	6.34	6.42	6.57	6.75			
2	5.49	5.59	5.73	5.90	6.03	6.11	6.16	6.28	6.39	6.50	6.64	6.62	6.62	6.59				
3	5.77	6.08	6.33	6.53	6.69	6.82	6.91	6.99	6.99	6.79	6.72	6.69	6.67	6.60				

Table 13.A 3 (cont.)

		0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr
1976		0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr
6	5.26	5.30	5.36	5.47	5.63	5.80	5.93	6.14	6.35	6.90	7.14	7.32	7.44	7.73	7.84	7.90	7.90	7.90
7	5.09	5.10	5.16	5.27	5.41	5.55	5.65	5.85	6.07	6.67	6.94	7.16	7.33	7.55	7.75	7.86	7.86	7.90
8	4.80	4.98	5.09	5.17	5.28	5.40	5.50	5.65	5.82	6.41	6.65	6.87	7.06	7.56	7.70	7.76	7.76	7.76
9	5.13	5.07	5.09	5.18	5.28	5.36	5.46	5.59	5.73	6.25	6.52	6.77	6.96	7.34	7.62	7.69	7.69	7.69
10	4.69	4.75	4.85	4.98	5.08	5.15	5.20	5.30	5.45	6.01	6.25	6.47	6.69	7.34	7.56	7.65	7.65	7.65
11	4.37	4.41	4.46	4.50	4.56	4.63	4.68	4.77	4.88	5.38	5.63	5.76	5.96	6.85	7.25	7.38	7.38	7.38
12	4.22	4.26	4.34	4.45	4.53	4.59	4.63	4.72	4.84	5.34	5.64	5.87	6.07	6.72	7.00	7.14	7.14	7.14
1977		0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr
1	4.58	4.57	4.66	4.81	4.95	5.06	5.14	5.34	5.54	6.15	6.43	6.68	6.85	7.27	7.46	7.55	7.55	7.55
2	4.41	4.52	4.65	4.79	4.90	4.99	5.07	5.30	5.51	6.09	6.42	6.71	6.92	7.34	7.54	7.66	7.66	7.66
3	4.54	4.51	4.55	4.65	4.75	4.83	4.91	5.17	5.40	5.95	6.35	6.64	6.84	7.31	7.52	7.63	7.63	7.63
4	4.33	4.44	4.58	4.72	4.84	4.95	5.05	5.30	5.50	6.05	6.36	6.65	6.86	7.33	7.52	7.60	7.60	7.60
5	4.80	4.90	5.00	5.11	5.21	5.29	5.37	5.55	5.70	6.12	6.36	6.60	6.79	7.28	7.49	7.58	7.58	7.58
6	4.98	4.96	5.01	5.12	5.23	5.32	5.39	5.52	5.63	6.04	6.27	6.50	6.69	7.17	7.33	7.42	7.42	7.42
7	5.16	5.25	5.36	5.48	5.60	5.72	5.81	5.97	6.08	6.40	6.46	6.61	6.80	6.95	7.30	7.44	7.51	7.55
8	5.35	5.38	5.49	5.65	5.82	5.94	6.02	6.18	6.26	6.64	6.86	6.93	7.17	7.35	7.45	7.53	7.53	7.53
9	5.75	5.74	5.85	6.03	6.18	6.28	6.33	6.47	6.56	6.74	6.84	6.95	7.03	7.27	7.43	7.53	7.53	7.53
10	6.28	6.11	6.16	6.37	6.54	6.65	6.71	6.87	7.00	7.20	7.28	7.36	7.40	7.51	7.64	7.74	7.74	7.74
11	5.30	5.67	5.97	6.19	6.36	6.48	6.57	6.75	6.86	7.06	7.22	7.27	7.47	7.62	7.72	7.75	7.75	7.75
12	5.38	5.82	6.13	6.32	6.45	6.55	6.63	6.80	6.91	7.08	7.23	7.35	7.42	7.62	7.75	7.83	7.83	7.83
1978		0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr
1	5.69	6.07	6.35	6.53	6.67	6.77	6.85	7.02	7.11	7.27	7.40	7.51	7.58	7.75	7.89	7.98	8.01	8.01
2	5.98	6.22	6.41	6.55	6.68	6.81	6.91	7.08	7.15	7.42	7.56	7.67	7.73	7.87	7.99	8.07	8.10	8.10
3	6.84	6.56	6.62	6.75	6.87	6.96	7.08	7.18	7.32	7.56	7.68	7.76	7.82	8.07	8.15	8.15	8.15	8.15
4	5.86	6.03	6.25	6.50	6.74	6.93	7.08	7.36	7.51	7.77	7.82	7.87	7.91	8.05	8.17	8.22	8.22	8.22
5	6.33	6.44	6.60	6.81	7.02	7.20	7.35	7.63	7.77	7.95	7.99	8.06	8.10	8.23	8.38	8.38	8.38	8.38
6	6.24	6.61	6.94	7.24	7.47	7.63	7.74	8.01	8.19	8.48	8.53	8.61	8.71	8.84	8.99	9.12	9.12	9.12
7	6.40	6.55	6.76	7.00	7.23	7.43	7.61	7.98	8.18	8.25	8.28	8.36	8.42	8.56	8.72	8.86	8.86	8.86
8	8.04	7.87	7.76	7.70	7.73	7.84	7.95	8.19	8.29	8.26	8.21	8.21	8.20	8.22	8.25	8.26	8.26	8.26
9	7.89	8.00	8.15	8.32	8.47	8.59	8.65	8.88	8.66	8.32	8.29	8.30	8.41	8.46	8.46	8.46	8.46	8.46
10	8.34	8.65	8.84	8.94	9.09	9.30	9.49	9.73	9.65	9.26	9.10	8.87	8.72	8.70	8.71	8.70	8.70	8.70
11	8.95	9.99	9.06	9.17	9.30	9.45	9.58	9.78	9.75	9.27	8.91	8.74	8.69	8.66	8.64	8.59	8.55	8.55
12	7.63	8.63	9.24	9.55	9.70	9.77	9.85	10.21	10.31	9.82	9.99	9.98	9.98	9.98	9.98	9.98	9.98	9.98
1979		0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr
1	9.64	9.51	9.49	9.56	9.63	9.68	9.73	9.85	9.80	9.36	8.93	8.75	8.73	8.74	8.71	8.69	8.69	8.69
2	9.58	9.63	9.65	9.67	9.73	9.82	9.90	10.00	9.97	9.60	9.25	9.08	9.04	9.00	9.04	8.95	8.89	8.89

	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr
1980	11.03	11.87	12.28	12.36	12.42	12.42	12.42	12.42	12.42	12.42	12.42	12.42	12.42	10.92	10.90	10.89	10.84
1	13.29	13.92	14.25	14.32	14.53	14.72	14.91	14.90	14.90	14.90	14.90	14.90	14.90	13.36	13.11	12.33	12.10
2	13.89	15.07	15.44	15.24	15.19	15.53	15.91	15.85	15.31	14.25	13.43	12.94	12.65	12.26	12.30	12.13	11.89
3	10.28	10.39	10.52	10.67	10.82	10.95	11.04	11.07	10.96	10.52	10.41	10.41	10.45	10.65	10.74	10.71	10.59
4	8.42	7.85	7.76	8.10	8.25	8.36	8.51	8.68	9.01	9.12	9.43	9.62	10.05	10.25	10.26	10.18	
5	6.09	6.66	7.94	8.15	8.11	8.13	8.19	8.33	8.42	8.93	9.06	9.22	9.36	9.85	10.03	9.96	9.82
6	6.47	7.83	8.57	8.80	8.87	8.90	8.93	9.01	9.13	9.63	9.74	9.91	10.05	10.52	10.73	10.72	10.57
7	8.27	9.20	9.45	10.49	10.33	10.43	10.49	10.49	10.49	11.19	11.19	11.26	11.26	11.29	11.29	11.21	
8	10.98	11.20	11.42	11.62	11.78	11.89	11.94	11.95	11.89	11.84	11.70	11.65	11.61	11.57	11.68	11.62	11.44
9	10.23	11.65	12.57	12.99	13.13	13.20	13.26	13.27	13.11	12.72	12.60	12.51	12.42	12.12	12.17	12.08	11.91
10	13.63	14.39	14.80	14.77	14.75	14.83	14.92	14.92	14.47	13.70	13.26	13.00	12.81	12.33	12.19	11.93	
11	10.88	12.90	14.19	14.74	14.74	14.58	14.41	13.86	13.25	12.57	12.34	12.23	12.18	12.03	12.01	11.82	11.56
1981	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr
1	13.97	14.72	15.05	15.02	14.86	14.69	14.55	14.12	13.69	12.87	12.57	12.48	12.43	12.29	12.26	12.10	11.86
2	13.60	14.20	14.56	14.69	14.68	14.62	14.56	14.32	14.02	13.56	13.50	13.42	13.34	13.06	12.97	12.75	12.49
3	13.10	13.03	12.94	12.82	12.72	12.67	12.65	12.86	12.81	12.91	12.98	12.77	12.74	12.56	12.24		
4	12.57	14.11	14.98	15.20	15.10	15.00	14.96	14.87	14.69	14.33	14.17	14.06	13.98	13.67	13.59	13.34	
5	16.27	16.21	16.01	15.68	15.32	15.08	14.97	14.87	14.70	14.20	13.89	13.65	13.45	13.09	13.02	12.83	12.55
6	14.03	14.41	14.61	14.65	14.64	14.65	14.66	14.57	14.43	14.21	13.97	13.92	13.81	13.46	13.34	13.15	12.82
7	14.50	14.98	15.27	15.40	15.15	15.52	15.66	15.79	15.87	15.75	15.36	15.10	14.98	14.78	14.19	14.04	13.36
8	15.39	15.64	15.84	16.00	16.15	16.32	16.50	16.62	16.35	16.17	15.68	15.67	15.56	14.95	14.79	14.64	14.29
9	12.63	13.68	14.37	14.75	15.03	15.33	15.63	15.99	15.90	16.09	15.85	15.75	15.30	15.29	15.10	14.60	
10	12.75	12.97	13.09	13.12	13.21	13.41	13.63	14.16	14.08	14.14	14.23	14.32	14.24	14.23	14.23	14.23	
11	10.17	10.24	10.37	10.57	10.77	10.94	11.05	11.30	11.55	12.19	12.47	12.53	12.65	12.79	12.72	12.48	
12	8.26	9.71	10.79	11.50	11.96	12.28	12.53	13.02	13.29	13.48	13.55	13.67	13.71	13.68	13.56	13.19	
1982	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr
1	11.73	12.14	12.51	12.84	13.10	13.28	13.40	13.63	13.79	13.83	13.90	13.92	13.90	13.82	13.85	13.67	13.34
2	11.85	11.98	12.27	12.71	13.17	13.36	13.54	13.54	13.73	13.87	13.82	13.82	13.73	13.61	13.74	13.62	13.35
3	14.57	13.96	13.67	13.66	13.77	13.84	13.86	13.88	13.94	14.10	14.06	13.99	13.96	13.75	13.66	13.55	13.21
4	12.31	12.32	12.46	12.71	12.94	13.08	13.15	13.28	13.44	13.62	13.60	13.50	13.48	13.38	13.23	13.12	12.93

Table 13.A.3 (cont.)

	0 mo	1 mo	2 mo	3 mo	4 mo	5 mo	6 mo	9 mo	1 yr	2 yr	3 yr	4 yr	5 yr	10 yr	15 yr	20 yr	25 yr	
1982																		
5	11.56	11.62	11.72	11.85	12.00	12.13	12.24	12.53	12.84	13.20	13.34	13.45	13.54	13.28	13.19	12.95		
6	10.65	11.74	12.56	13.11	13.45	13.67	13.80	14.01	14.19	14.30	14.40	14.35	14.28	14.00	13.87	13.72	13.36	
7	8.49	9.15	9.82	10.49	11.07	11.48	12.07	12.51	12.90	12.75	12.33	12.23	12.28	12.30	12.29	12.27	12.31	12.94
8	7.54	7.51	7.78	8.33	8.96	9.46	9.82	10.53	11.12	11.84	12.18	12.27	12.39	12.44	12.30	12.31	12.31	12.16
9	6.88	6.94	7.24	7.77	8.39	8.88	9.21	9.80	10.34	11.16	11.50	11.55	11.60	11.70	11.62	11.53	11.41	
10	7.80	7.72	7.83	8.11	8.41	8.62	8.73	9.66	9.44	9.99	10.47	10.56	10.69	10.81	10.90	10.64		
11	7.67	7.87	8.11	8.39	8.64	8.81	8.87	8.95	9.31	9.82	10.28	10.64	10.69	10.93	10.91	11.05	10.82	
12	8.05	8.07	8.11	8.17	8.25	8.31	8.33	8.45	8.78	9.42	9.87	10.11	10.25	10.51	10.58	10.66	10.49	
1983																		
1	8.01	8.07	8.15	8.26	8.38	8.46	8.51	8.67	8.95	9.57	10.04	10.26	10.47	10.93	11.03	11.23	11.10	
2	7.90	7.91	7.96	8.06	8.17	8.23	8.24	8.28	8.60	9.18	9.65	9.74	9.81	10.28	10.45	10.71	10.45	
3	8.36	8.56	8.72	8.86	8.95	9.00	9.02	9.06	9.24	9.71	9.98	10.13	10.25	10.56	10.68	10.79	10.60	
4	8.01	8.10	8.19	8.28	8.35	8.39	8.41	8.49	8.71	9.21	9.59	9.76	9.91	10.24	10.38	10.49	10.29	
5	8.38	8.56	8.71	8.81	8.93	9.00	9.05	9.16	9.31	9.87	10.14	10.31	10.45	10.76	10.93	11.04	10.90	
6	8.25	8.57	8.81	8.98	9.09	9.17	9.23	9.38	9.57	10.11	10.36	10.52	10.63	10.90	11.05	11.08	10.91	
7	8.63	8.95	9.22	9.43	9.60	9.72	9.80	10.03	10.33	10.95	11.12	11.29	11.45	11.68	11.73	11.73	11.67	
8	8.74	9.01	9.25	9.46	9.65	9.80	9.91	10.12	10.35	10.97	11.29	11.48	11.61	11.78	11.88	11.71		
9	8.77	8.78	8.83	8.92	8.94	9.15	9.24	9.47	9.69	10.32	10.60	10.85	11.03	11.27	11.38	11.20		
10	8.46	8.49	8.58	8.72	8.87	9.01	9.12	9.37	9.61	10.34	10.76	11.01	11.18	11.50	11.59	11.63	11.51	
11	8.17	8.58	8.87	9.07	9.19	9.28	9.37	9.57	9.70	10.40	10.70	10.99	11.15	11.36	11.42	11.46	11.33	
12	8.18	8.63	8.97	9.19	9.32	9.42	9.50	9.70	9.78	10.58	10.87	11.16	11.31	11.54	11.66	11.54		
1984																		
1	8.87	8.94	9.02	9.10	9.18	9.25	9.32	9.48	9.62	10.35	10.63	10.94	11.09	11.39	11.44	11.50	11.44	
2	8.45	8.86	9.17	9.37	9.50	9.60	9.68	9.84	9.98	10.73	10.98	11.32	11.48	11.77	11.84	11.80		
3	9.22	9.50	9.75	9.96	10.12	10.21	10.26	10.38	10.60	11.25	11.51	11.82	12.01	12.18	12.15			
4	9.25	9.49	9.73	9.96	10.10	10.26	10.33	10.52	10.79	11.41	11.82	12.02	12.44	12.49	12.45			
5	9.65	9.56	9.69	10.04	10.45	10.78	10.99	11.41	11.89	12.63	13.18	13.22	13.32	13.44	13.39	13.38		
6	8.83	9.39	9.83	10.18	10.46	10.72	10.96	11.57	11.95	12.78	13.00	13.20	13.29	13.41	13.29	13.31		
7	9.67	10.03	10.32	10.56	10.76	10.92	11.02	11.12	11.39	11.65	12.24	12.30	12.54	12.55	12.55	12.48		
8	10.78	10.74	10.76	10.82	10.91	11.02	11.12	11.39	11.65	12.17	12.25	12.36	12.43	12.45	12.39	12.34		
9	10.52	10.44	10.44	10.50	10.60	10.69	10.76	10.94	11.15	11.73	11.91	12.07	12.14	12.13	12.06	11.97		
10	8.07	8.57	8.96	9.24	9.44	9.57	9.64	9.83	10.13	10.85	11.06	11.21	11.47	11.44	11.42	11.37		
11	7.47	8.00	8.38	8.62	8.76	8.88	8.98	9.28	9.51	10.26	10.62	10.87	11.10	11.32	11.43	11.32		
12	6.30	7.23	7.84	8.12	8.19	8.25	8.36	8.80	9.11	9.88	10.36	10.65	10.95	11.30	11.41	11.31		
1985																		
1	7.29	7.72	8.03	8.19	8.27	8.34	8.43	8.72	8.97	9.72	10.15	10.43	10.65	10.96	11.10	11.07	11.00	
2	6.77	7.67	8.32	8.70	8.91	9.04	9.13	9.35	9.63	10.41	10.86	11.13	11.36	11.70	11.82	11.81	11.80	

	0 MO	1 MO	2 MO	3 MO	4 MO	5 MO	6 MO	9 MO	1 YR	2 YR	3 YR	4 YR	5 YR	10YR	15YR	20YR	25YR
1986	0	0.04	0.32	0.51	0.65	0.77	0.86	0.93	0.94	0.95	0.97	0.98	0.99	1.14	1.43	1.54	1.60
1	6.38	6.80	7.06	7.16	7.19	7.21	7.27	7.40	7.48	7.89	8.15	8.39	8.57	9.07	9.38	9.35	9.15
2	6.51	6.89	7.11	7.20	7.20	7.22	7.22	7.29	7.33	7.63	7.74	7.87	7.97	8.07	8.29	8.34	8.26
3	6.73	6.58	6.49	6.46	6.47	6.47	6.56	6.72	6.91	7.04	7.21	7.28	7.38	7.51	7.65	7.68	
4	5.82	5.98	6.10	6.18	6.23	6.27	6.30	6.39	6.49	6.89	7.05	7.19	7.27	7.41	7.58	7.78	7.81
5	6.11	6.23	6.34	6.43	6.50	6.55	6.59	6.71	6.86	7.37	7.94	7.98	8.16	8.36	8.69	8.70	
6	5.96	6.01	6.06	6.11	6.15	6.17	6.23	6.40	6.84	7.11	7.25	7.33	7.44	7.70	8.07	8.12	
7	5.08	5.50	5.76	5.88	5.92	5.93	5.94	6.03	6.17	6.60	6.86	7.05	7.16	7.51	7.88	8.14	8.07
8	4.37	4.86	5.15	5.24	5.23	5.23	5.26	5.39	5.48	5.97	6.22	6.46	6.59	7.17	7.48	7.67	7.63
9	5.14	5.20	5.26	5.32	5.38	5.45	5.53	5.69	5.81	6.42	6.76	7.03	7.19	7.74	8.15	8.40	8.31
10	4.65	5.03	5.24	5.30	5.32	5.36	5.41	5.57	5.69	6.18	6.47	6.70	6.86	7.44	7.84	7.94	7.80
11	5.07	5.24	5.37	5.45	5.51	5.54	5.56	5.61	5.70	6.12	6.37	6.55	6.68	7.16	7.62	7.91	7.84
12	4.08	5.00	5.57	5.80	5.81	5.76	5.82	5.94	6.35	6.66	6.68	6.77	7.22	7.67	7.98	7.94	
1987	0	0.04	0.32	0.51	0.65	0.77	0.86	0.93	0.94	0.95	0.97	0.98	0.99	1.14	1.43	1.54	1.60
1	5.23	5.51	5.66	5.70	5.71	5.74	5.84	5.88	5.98	6.24	6.44	6.60	6.73	7.20	7.60	7.82	7.77
2	5.44	5.49	5.53	5.55	5.58	5.61	5.64	5.77	5.88	6.27	6.45	6.61	6.71	7.14	7.51	7.73	7.69