Term Structure of Interest Rates in India: Issues in Estimation and Pricing

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Abstract

The term structure of interest rates – the relationship between interest rates in the economy and the term to maturity – forms the basis for the valuation of all fixed income instruments. While empirical analysis of the term structure of interest rates has a long history in developed countries like the US and UK, work on term structure estimation in India, by comparison, is of more recent origin. The financial sector reforms that have been implemented in India since 1991, in particular those relating to the development of the debt market, have evoked interest in the topic. This exercise attempts to provide a framework for the estimation of the daily term structure taking into account important institutional details related to the Indian debt market. We provide daily estimates of the sovereign zero coupon yield curve using information on secondary market trades in Government of India securities available from the Wholesale Debt Market of the National Stock Exchange (NSE-WDM).

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I. Introduction

The term structure of interest rates – the relationship between interest rates in the economy and the term to maturity – forms the basis for the valuation of all fixed income instruments. Modeled as a series of cashflows due at different points of time in the future, the underlying price of such an instrument can be calculated as the present value of the stream of cashflows. Each cashflow, in such a formulation, has to be discounted using the interest rate for the associated term to maturity. Arriving at the appropriate set of interest rates - the 'term structure' or the 'zero coupon yield curve' (ZCYC) – for the Indian debt market, is the objective of the present exercise.

Empirical analysis of the term structure of interest rates has a long history in developed countries like the US and UK [see McCulloch (1971, 1975), Carleton & Cooper (1976), Robichek & Neibuhr (1970), Schaefer (1981)]; work on term structure estimation in India, by comparison, is of more recent origin [Nag & Ghose (2000), Thomas & Saple (2000), Subramanian (2000)]. The primary reason for the latter can be traced to the dormant debt market in India prior to the phased implementation of financial sector reforms since 1991.

Among the major reforms that have been initiated in the Government securities market, the most significant, for the purpose of the present study, is the gradual shift to market-related rates of interest on Government borrowings. This was a great step forward from the pre-reform period when the return differential between different Government securities was set by statute [see Krishnan (1989) for details]. A paradigm shift in this context has been the virtual elimination of the automatic deficit financing route via the phasing out of ad-hoc T-Bills from 1997-98, and the replacement in its place of a system of Ways and Means Advances (WMA). The Government has thus had to access the market for major part of its borrowings since 1997-98. For the Reserve Bank of India (RBI), this has meant, in its role as debt manager for the Government, a gradual move towards a market-aligned yield curve to enable success of the Government's borrowing program. To the extent the RBI would try and minimise devolvement at primary issuances, an estimate of the sovereign yield curve can be used to arrive at prices (yields) that can be reasonably expected at primary auctions.

A second policy move of major significance has been the gradual reduction in statutory pre-emptions via SLR (Statutory Liquidity Ratio) prescriptions. Compulsory SLR holdings had earlier ensured a captive market for Government securities. As against which, for most part of 1997-98 for instance, banks had invested in Government securities in excess of the prescribed 25 per cent for lack of more profitable alternatives, reflecting conscious portfolio choice in favour of these instruments. In view of the fact that valuation of Government securities portfolio exerts an impact on the balance sheets of financial sector entities, an analysis of the term structure would enable more efficient portfolio decisions.

Once an estimate of the term structure based on default-free government securities is obtained, it can be used to value government securities that do not trade on a given day, to construct a Government bond index, or to provide default-free valuations for corporate bonds. It can be used to price all non-sovereign fixed income instruments after adding an appropriate credit spread. Estimates of the ZCYC at regular intervals over a period of time provides us with a time-series of the interest rate structure in the economy, which can be used to analyse the extent of impact of monetary policy. Time series of ZCYC also form an input for VaR systems for fixed income systems and portfolios.

Considering the usefulness of a model of the term structure and the sparse empirical literature on the subject in the Indian context, the present paper attempts to provide a framework for the estimation of the daily term structure taking into account important aspects related to the Indian debt market. Most existing empirical literature on term structure estimation uses volume-weighted average (VWA) prices (for multiple trades in the same security) as indicative of 'the' price for a particular security on a given day, an approach that conceals the extent of dispersion in prices for multiple trades. We improve upon existing empirical work by highlighting the importance of various institutional details that lead to such intra-security price dispersion. Factors that cause inter-security variation in prices over and above that implied by the term structure, such as liquidity and settlement days, are also analysed. The exercise provides daily estimates of the sovereign ZCYC using information on secondary market trades in Government securities available from the Wholesale Debt Market of the National Stock Exchange (NSE-WDM). Section II provides a brief description of the theoretical framework underlying the econometric estimation of the term structure of interest rates. Section III outlines the econometric methodology and the empirical specification of the model estimated. An account of the data and related estimation issues is presented in Section IV. Results of our exercise are presented in Section V. Section VI concludes.

II. Theoretical Framework

Yield is commonly defined as the return on capital invested in fixed income earning securities. The yield on any instrument has two distinct aspects: (i) a regular income in the form of interest income (coupon payments) and (ii) changes in the market value of fixed interest bearing securities.

Some of the major factors that lead to yield differentials among fixed interest bearing securities are (i) maturity period, (ii) coupon rates, (iii) tax rates (iv) marketability and (v) risk factor. Government securities, considered the safest form of investment, also carry with them some element of risk in terms of 'purchasing power risk' and 'interest rate risk'. Purchasing power risk arises due to inflationary tendencies in the economy and leads to a consideration of the real rate of return[4]. Interest rate risk arises on account of fluctuating prices of securities which, in turn, requires a re-adjustment of portfolio.

A yield curve depicts the relationship between yield and maturity of a set of 'similar' securities, as on a given date. To derive the 'true' term structure, we need to have a sample of bonds that are identical in every respect except in term to maturity. While Government securities, in practice, differ by coupon rates, nonetheless, these come closest to being identical[5], hence most empirical studies have concentrated on this segment of the securities market.

Valuation of a Bond

The valuation of a bond by an investor can be expressed in terms of three alternative present value formulations.

(i) <u>Spot Interest Rates</u>

Suppose that the *spot rates of interest* (r_t) for every future period are known, then the *present value* of an m-period bond making a series of coupon payments C every period and with redemption value R is:

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[4] Capital indexed bonds are intended to insure investors against this type of risk. The Government of India has till date issued only one bond of this type – the 6% capex bonds issued in December 1996.

[5] As compared to, say, corporate bonds which also differ in terms of creditworthiness of the issuer or default risk.

Equation 1

$$PV = \frac{C}{(1+r_1)} + \frac{C}{(1+r_2)^2} + \ldots\ldots + \frac{C+R}{(1+r_m)^m}$$

The spot interest rate r_t is the interest rate applicable on a cash payment due in t periods. The set of spot rates is the *term structure of interest rates*.

The factor $1/(1+r_t)^t$ used to discount the value of the future payment to the current period is called the *discount factor*. The bond price equation (1) can be written in terms of the discount function as follows:

Equation 2

$$\begin{aligned} p &= d(t_1)C + d(t_2)C + \ldots\ldots\ldots d(t_m)(C+R) \\ &= C\sum_{j=1}^{m} d(t_i) + d(t_m)R \end{aligned}$$

The discount function describes the present value of 1 unit payable at any time in the future. It follows that if an instrument exists that provides a single, unit cashflow t years into the future, its price should correspond to the value of the discount function at that point.

(ii) Yield to maturity

The *yield to maturity* (YTM) is the bond's internal rate of return – it is the single interest rate at which the price of a bond is equal to the present value of the stream of cashflows. It is derived from the bond price equation, constraining all cashflows to be discounted at a single rate. The *yield relationship* is expressed as:

Equation 3

$$PV = \frac{C}{(1+i)} + \frac{C}{(1+i)^2} + \ldots\ldots\ldots + \frac{C+R}{(1+i)^m}$$

(iii) Forward interest rates

The present value relation can also be explained in terms of *forward interest rates*. While the spot rate is the single rate of return applicable over all discrete periods from the present until the end of period j, and is defined as the average of the one-period rates applicable to periods 1,2….j, the forward interest rates describe the marginal return over period j. If γ_j represents the interest rate applicable from the end of period j-1 to end of period j, then

$1/\delta(1) = (1+r_1) = (1+\gamma_1)$

$1/\delta(2) = (1+r_2)^2 = (1+\gamma_1)(1+\gamma_2)$

.

$1/\delta(j) = (1+r_j)^j = (1+\gamma_1)(1+\gamma_2).....(1+\gamma_j)$

In terms of *forward rates*, the present value relationship can be formulated as:

Equation 4

$$PV = \frac{C}{1+g_1} + \frac{C}{(1+g_1)(1+g_2)} + ... + \frac{C+R}{\prod_1^m (1+g_t)}$$

Accrued Interest

There are in practice a number of factors that complicate the specification of equation (1). In the formulation above, the first coupon payment is assumed to be due in exactly one period's time. In reality, while coupon payments are only made at regular intervals during the year, bonds can be traded on any working day. Whenever a bond changes hands on a day that is not a coupon payment date, the valuation of the bond will reflect the proximity of the next coupon payment date. This is effected by the payment of *accrued interest*[6] to compensate the seller for the period since the last coupon payment date during which the seller has held the bond but for which no coupon payment is made. The *total* or *dirty* price paid by the buyer can thus be decomposed into two components: accrued interest and the *clean* or *quoted* price. If market conditions are stable, such that factors underlying the valuation of the bond do not change, then the dirty price of the bond will still increase daily by the amount of accrued interest. A similar adjustment needs to be made to the discounting of coupon payments since, when a bond is traded on a day that is not a coupon payment date, cashflows are no longer an exact number of coupon periods into the future. The time to the earliest coupon payment date is then computed as the proportion of a coupon payment period represented by the time from the trade settlement date to the next coupon payment date[7]. The bond price equation should accordingly be modified to account for accrued interest:

Equation 5

$$p + ai = \frac{C/v}{(1+r(t_1)/v)^{vt_1}} + \frac{C/v}{(1+r(t_2)/v)^{vt_2}} + \ldots\ldots\frac{C/v+R}{(1+r(t_m)/v)^{vt_m}}$$

where:

p = clean price of the bond

ai = accrued interest

C = annual coupon payment

R = redemption payment

t_m = maturity of bond (in years)

$r(t_j)$ = spot rate applicable to payment j due at time t_j

v = frequency of coupon payments

[6] Computed as ai = t_0C, where C is the (annual) coupon on the bond and t_0 the proportion of a period passed since the last coupon payment was made.

[7] The market conventions to be followed for the computation of accrued interest and time to first coupon payment/maturity vary from one country to another. The commonly used ones are actual/365 and the 30/360 conventions. In the former, the actual number of days between two required dates is used for the computation, with a year comprising 365 days. In the latter, each month comprises 30 days, and a year comprises 360 days.

III. Empirical Specification and Econometric Methodology

Empirical estimation of equation (5) essentially requires specifying a parametric relation between maturity and spot interest rates. The Nelson-Siegel formulation [Nelson & Siegel (1987)] that we adopt in the present exercise provides a framework for the derivation of such a relation. Starting from a parsimonious representation of the forward rate function given by

Equation 6

$$f(m,b) = \beta_0 + \beta_1 * \exp(-m/\tau) + \beta_2[(m/\tau) * \exp(-m/\tau)]$$

where 'm' denotes maturity and b=[β_0, β_1, β_2 and τ] are parameters to be estimated, the relevant spot rate function can be derived as

Equation 7

$$r(m,b) = \beta_0 + (\beta_1 + \beta_2) * [1 - \exp(-m/\tau)]/(m/\tau) - \beta_2 * \exp(-m/\tau)$$

The appeal of the NS functional form lies in its flexibility to cover the entire range of possible shapes that the ZCYC can take, depending on the value of the estimated parameters. In the formulation of the forward rate function *f(m,b)*, the first term represents the long-term component and is a non-zero constant. The second term, which monotonically declines to zero, is the short-term component, and the third term represents the medium-term component. In the spot rate function, the limiting value of *r(m,b)* as maturity gets large is β_0 which therefore depicts the long term component (which is a non-zero constant). The limiting value as maturity tends to zero is $\beta_0 + \beta_1$, which therefore gives the implied short-term rate of interest.

With the spot rate function specified as above, the PV relation is specified using the discount function given by[8]:

[8] This is the continuous form of the discount function specified in equation (2).

Equation 8

$$d(m,b) = \exp\left(-\frac{r(m,b) * m}{100}\right)$$

The present value arrived at is the estimated price (**p_est**) for each bond. It is common to observe market prices (**pmkt**) that deviate from this value. For the purpose of the estimation exercise, we postulate that the observed market price of a bond deviates from its underlying valuation by an error term e_i, which gives us the estimable relation:

Equation 9

$$pmkt_i = p\_est_i + e_i$$

This equation can be estimated by minimising the sum of squared price errors. Earlier empirical studies [Svensson (1994), Bolder & Streliski (1999)] have found that minimising price errors results in fairly large YTM errors for instruments with short maturities. This is on account of the fact that, since the elasticity of price with respect to one plus YTM equals the duration[9] of a bond, prices are relatively insensitive to yields for short maturities. The optimisation technique that seeks to minimse price errors will consequently lead to over-fitting of long–term YTMs at the expense of short-term YTMs. To correct for this in the empirical estimation, Bolder & Streliski (1999) suggest weighting of each price error by the inverse of its duration. The appropriate weighting scheme is given by

$$w_i = \frac{1/D_i}{\sum_{j=1}^{n} 1/D_j}$$

where D_i is the MacCauley duration of the bond given by

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[9] Present-value weighted average maturity of coupon payments and face value.

$$D(T,C)_t = \frac{\sum_{t=1}^{n} \frac{(CF_t * t)}{(1+YTM(T,C)_t)^t}}{\sum_{t=1}^{n} \frac{(CF_t)}{(1+YTM(T,C)_t)^t}}$$

In the empirical exercise, we estimate equation (9) by minimising the weighted sum of squared price errors. Equation (9) could be estimated either by maximising the likelihood function or by minimising some other loss function defined over the errors (e_i). The former approach requires specification of the exact form of the distribution of the errors, while the latter takes a pure optimisation route without prior distributional assumptions about the errors. The parameter estimates, it may be mentioned, are expected to be the same if the errors are normally distributed. Our choice of the second approach is guided by the fact that the data set includes multiple observations on the same security on any given date, making it difficult to specify, *a priori*, a single homogenous error distribution that holds across securities as well as across multiple trades in the same security. As for the form of the loss function, one could use error sum of squares or a quadratic loss function. However, parameters estimated under such loss function are sensitive to 'outliers', a real possibility in the present context. To reduce the impact of outliers on the parameter estimates, we specify a robust loss function [Beaton-Tukey loss function] that downweights large errors (say Re.1 deviation between model and market price) in the objective function (see Seber & Wild (1989) for details). With no prior knowledge of the error distribution, the choice of this approach also implies that standard statistical inference with regard to the parameter vector (b) may be weak.

The estimation is carried out using the constrained optimisation (CO) module in GAUSS [Schoenberg (1998)]. Constraints imposed relate to the non-negativity of the long-run rate (β_0), the short-rate ($\beta_0+\beta_1$), and the parameter τ. To the extent these constraints restrict the search procedure of the optimisation algorithm within a meaningful parameter space, they would reduce the overall search time. If the function also happens to be smooth in the relevant region, the constraints would increase overall speed of computation as well. Our experience has been that, when estimation is carried out without imposing these constraints, not only does the computational time increase substantially, but also the resulting parameter estimates are very often outside the meaningful range.

The steps followed in the estimation procedure are as follows:

i. A vector of starting parameters (β_0, β_1, β_2 and τ) is selected,

ii. The discount factor function is determined using these starting parameters,

iii. This is used to determine the present value of the bond cash flows and thereby to determine a vector of *starting* 'model' bond prices,

iv. Numerical optimisation procedures are used to estimate a set of parameters (under a given set of constraints *viz*. non-negativity of long run and short run interest rates) that minimise the specified loss function,

v. The estimated set of parameters are used to determine the spot rate function and therefrom the 'model' prices,

vi. These 'model' prices are used to compute associated 'model' YTMs for each bond.

IV. Data Details and Related Estimation Issues

As on August 18, 2000, there were 110 Government of India (GoI) dated securities outstanding, with maturity dates ranging from September 29, 2000 to April 22, 2020. Daily data on secondary market trades in these securities (and in Treasury Bills) are available from two sources – trades reported on NSE-WDM and those reported on the Subsidiary General Ledger account of the Reserve Bank of India (RBI-SGL). All trades are conducted as negotiated deals and subsequently reported to NSE / RBI; - the price information accordingly relates to 'traded prices' rather than 'quotes', and is not time-stamped.

The present exercise uses data from the NSE-WDM[10], which constitutes about 70 per cent of the secondary market volume. These relate to those trades that are conducted through brokers

[10] Information on trades reported on RBI-SGL is publicly disseminated only on the day of settlement, which renders it difficult to use it for the exercise at hand.

registered with the exchange. On a given trade date, the data for each individual trade include information on traded price, traded volume, settlement date[11], days to settlement[12], issue date, maturity date and details of cashflows[13] for the bond. Government dated securities in India make semi-annual coupon payments. Market conventions require computation of accrued interest on a 30/360 basis for instruments with residual maturity exceeding a year and on actual/365 basis otherwise (this includes Treasury Bills), and these are adhered to in the computation of coupon accrual and time to cashflows. Accrued interest, time to coupon payments and term to maturity are calculated with reference to the settlement date.

Most of the existing empirical literature on term structure estimation uses volume-weighted average (VWA) prices (for multiple trades in the same security) as indicative of 'the' price for a particular security on a particular day. While this method would provide reasonably accurate 'indicative' prices in deep, liquid and transparent markets, the loss of information on this count would be substantial in the Indian case. First, while the number of securities that are actively traded on a given day vary in the range of 13-25, it is not uncommon to observe 45-60 trades in securities considered attractive at that particular point of time. For trades of the latter type, the data reveal a significant dispersion in prices across trades in the same bond [Trades in the 12.5% 2004 security on August 9, 2000 provide a case in point. There were 67 trades in this security on this day, with prices ranging from Rs.108.9669 to Rs.109.5264]. Use of VWA prices would conceal this important aspect of secondary market trades. To highlight the institutional details of the Indian bond market that lead to such intra-security price variations, the present exercise deviates from the existing norm in the empirical literature and uses prices for all trades for all bonds[14].

[11] Which lies within a range of 5 days from trade date in the T+0 to T+5 system being followed at present.

[12] Which equals calendar days to settlement excluding holidays.

[13] Coupon rate and interest payment dates.

[14] Thomas and Saple (2000) also use all trades data; the authors, however, do not analyse the possible sources of intra-security price variation.

There are various factors to which intra-security variation in prices can be attributed. First, the scope for price discovery in negotiated deals is limited, and this may contribute to the observed dispersion in prices for different trades in the same security. Further, within the T+5 system in vogue, trades negotiated on a given day can have settlement dates varying from current date to 5 days hence[15]. There are two mechanisms through which this exerts an impact on the price. First, for trades that do not settle on the trade date, the futures price (price for a trade that is settled T days in the future) differs from the spot price (price negotiated for a trade that settles on the trade date itself) by the net cost of carry. From the point of view of the seller, the opportunity cost involved in settling a deal T days into the future is approximated by the foregone return in the call money market, while the return is given by the coupon that accrues for these days. If the net cost of carry is positive (negative), the negotiated futures price will be higher (lower) than the spot price. To compute the net cost of carry for the purpose of the empirical exercise, we proxy the call rate by the short-term rate ($\beta_0 + \beta_1$) derived from the estimated term structure. This factor is added to the estimated spot price to arrive at the estimated futures price.

Secondly, expectations about the likely directionality of interest rates would be built into the contract if the term structure is expected to undergo a significant change by the time the deal is settled. To discount the cashflows for deals that do not settle on the current day, therefore, the appropriate rates to be used are those that are expected to prevail on the settlement date. Implied forward rates being the best predictors of expected future spot rates, we use the former to discount these cashflows. The forward rates are derived from the estimated term structure using the relation

$$r_{t1}^{t2} = \left(\frac{(1+r_0^{t2})^{t2}}{(1+r_0^{t1})^{t1}} \right)^{1/(t2-t1)} - 1$$

where r_0^{t2} denotes the spot rate for maturity date t2 as on date 0, r_0^{t1} the spot rate for maturity date t1 as on date 0 and r_{t1}^{t2} denotes the spot rate for maturity date t2 expected to prevail at date t1.

[15] Settlement days beyond 1 day are infrequent; however, it is not uncommon to have 20-30 per cent of trades settling the next day.

The price equation is estimated with the above factors incorporated to account for intra-security price variation across trades that settle on different days. If this model specification adequately captures expectations, intra-security price variation arising on account of this factor would be fully controlled for. However, risk premia and/or investors' preferences for liquidity drive a wedge between implied forwards and expected spots[16]. The price error would therefore be higher the further into the future a deal settles, probably reflecting the higher uncertainty about the extent of shift in the term structure. This, in turn, could result from high volatility in interest rates during these periods.

With reference to dispersion in prices for multiple trades settling on the same day, among other variables that can, in principle, exert an impact on prices, transaction costs (brokerage, for instance) that vary with the size of trade are important. The cap on brokerage charges for different trade volumes are set by notification, the actual brokerage charged could, however, be lower. Incorporating the 'size of trade' variable in the estimation, we find that the impact of brokerage costs on market prices is statistically insignificant. This could be on account of either/both of the following. The data do not reveal significant variation in volume of individual trades, plausibly on account of the fact that most transactions take place in market lots (Rs. 5 crore and multiples thereof). [There is evidence, however, that odd lots trade at prices significantly different than that of market lots.] More importantly, comparison of NSE-WDM trades with those reported on RBI-SGL reveal no systematic pattern in the impact of brokerage costs on prices.

If the term structure is the only factor that determines the price of a bond, pricing errors (inter-security as well as across days for the same security) should be minimal and entirely random. In practice, however, while the term structure provides the basis for the valuation of fixed income securities, there are other factors that also influence the price of a bond. Attributes of particular securities play an important role in explaining inter-security price variation. Illiquid bonds, which sell at a premium compared to otherwise similar liquid securities, are a case in point. The buyer of an illiquid bond has to be

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[16] See Hall, Anderson and Granger (1992).

compensated for this undesirable attribute of the security in terms of a higher yield. This consequently results in transacted prices that are lower than that derived from the term structure.

In empirical work, the bid-ask spread is one of the most commonly used proxies for liquidity. Dealers require greater compensation for maintaining inventories of illiquid assets, which manifests in larger bid-ask spreads for such assets. Elton & Green (1998), examining tax and liquidity effects in pricing of US Government bonds, find that the bid-ask spread bears a negative relationship with time to maturity (positive) and with volume traded (negative), indicating that these variables can be used as proxies for liquidity. The authors include the natural log of 'volume traded' as a measure of liquidity in the present value equation and find that it improves the explanatory power of the model.

As mentioned earlier, the data that we use relate to traded prices, and bid-ask spreads / quotes are not available. The liquidity impact on negotiated prices works in the following manner. The price discovery process for liquid securities is expected to lead to prices that closely mirror the underlying valuation of the bond given by the term structure. Observed prices for illiquid bonds, on the other hand, would be 'off' the term structure on account of the price discovery process being weaker for such securities. With 'absolute' error as the dependent variable and a measure of liquidity as the independent variable, the sign on the liquidity coefficient is expected to be negative. The data set that we use contain two indicators of liquidity, the 'number of trades' in a bond and the 'total volume of transactions' in a bond, on a given day. We include the natural log of 'total volume' as a proxy variable for liquidity[17].

[17] This approach differs from that of Subramanian (2000) who uses both the available measures of liquidity in a liquidity-weighted error function.

V. Results

a. Analysis of the Term Structure

Summary statistics of residuals for the period January – November 2000 are presented in Table 1 below[18]. The month-wise mean is the average over all trading days in the month. The mean absolute price error ranges between 12 to 18 paise, comparable to those obtained by earlier studies in the Indian context [refer Thomas & Saple (2000) and Subramanian (2000)]. The standard deviation statistic reported is a measure of the variation in the absolute error across trading days within a month. High values of standard deviation relative to the mean indicate that there is significant intra-month variation in the fit of the model.

Table 1
Summary Statistics of Residuals

	Mean	Stdev
Jan-00	0.152	0.202
Feb-00	0.184	0.244
Mar-00	0.161	0.239
Apr-00	0.118	0.165
May-00	0.149	0.216
Jun-00	0.166	0.244
Jul-00	0.162	0.221
Aug-00	0.127	0.194
Sep-00	0.126	0.175
Oct-00	0.128	0.166
Nov-00	0.127	0.208
Overall	**0.145**	**0.207**

It is useful to mention at this point that the mean absolute error, as a statistic, masks a significant amount variation across securities and/or across multiple trades in the same security. An analysis of the residuals reveals, in fact, the presence of infrequent outliers on certain days as also systematic patterns in residuals for certain securities. The former phenomenon is attributable to factors such as reporting day error or 'odd-lot' trades. The impact of outliers on the estimation has been minimised through an appropriate choice of loss function in the estimation. The latter phenomenon could be on account of security –specific attributes such as liquidity. We address these issues in subsection (b) below.

The analysis above focuses on in-sample error as a measure of performance of the model. Earlier empirical studies have noted that alternate functional forms for the discount function (such as cubic splines), while being less parsimonious in terms of parameters, may perform better with respect to in-sample errors. These models, however, may not perform well when examined on an out-of-sample basis. While any measure of model performance should be related to both in and out-of-sample errors, designing a performance metric for the latter approach is rendered difficult in the present context where an individual trade is the unit of observation and the data include multiple trades for the same security. We propose to analyse this issue further in a future study.

Taking that the statistics presented above indicate reasonably good fit of the estimated model, we now turn to an analysis of the estimated parameters. Table 2 below presents month-wise mean and standard deviation of the estimated parameters. As mentioned earlier, β_0 provides a measure of the implied long-term rate. It should be noted here that this is the long-term rate only in a mathematical sense (as maturity $\rightarrow \propto$). For any economically relevant maturity horizon (say 15 years), the appropriate rates can be obtained by plugging in the estimated values into the spot rate function (7).

Table 2
Estimated parameters for January-November 2000

		Jan-00	Feb-00	Mar-00	Apr-00	May-00	Jun-00	Jul-00	Aug-00	Sep-00	Oct-00	Nov-00
β_0	Mean	12.39	12.28	11.79	11.96	12.10	12.12	12.09	11.95	12.13	12.65	12.67
	Std. Dev	0.15	0.17	0.16	0.13	0.29	0.30	0.35	0.19	0.18	0.14	0.14
β_1	Mean	-2.42	-2.73	-1.46	-2.65	-2.64	-1.83	-2.24	-1.13	-0.99	-1.93	-2.14
	Std. Dev	0.26	0.23	2.17	0.25	0.29	0.62	0.41	0.59	0.56	0.48	0.29
β_2	Mean	-3.30	-3.28	-3.27	-2.86	-4.62	-5.73	-3.42	-2.60	-4.48	-4.80	-4.70
	Std. Dev	0.69	1.06	2.93	0.36	0.58	0.55	2.36	1.44	0.66	0.85	0.55
τ	Mean	2.47	3.57	2.66	3.37	2.12	1.49	2.23	2.66	1.28	1.61	2.01
	Std. Dev	0.42	0.48	1.03	0.36	0.19	0.35	0.93	2.83	0.25	0.14	0.25

[18] Throughout this section, estimation results for November 2000 are upto November 20, 2000.

The implied long and short term rates from the model are presented in Table 3 below. The high volatility in short term interest rates in the current fiscal is clearly evident in the higher standard deviations of the short term rate compared to the long term rate. This, coupled with changes in the estimated β_2 and τ parameters, indicates the sensitivity of market expectations (as reflected in the trade prices) with respect to changes in monetary policy during this period.

Table 3
Estimates of long term and short term rates

	Long term *				Short term **			
	mean	min	max	st.dev	mean	min	max	st.dev
Jan-00	12.39	12.06	12.61	0.15	9.97	9.46	10.32	0.26
Feb-00	12.28	12.02	12.53	0.17	9.55	9.08	10.22	0.33
Mar-00	11.79	11.58	12.06	0.15	10.33	9.08	11.43	0.45
Apr-00	11.96	11.62	12.13	0.13	9.30	9.05	9.96	0.22
May-00	12.10	11.67	12.69	0.29	9.46	9.10	10.16	0.28
Jun-00	12.12	11.73	13.04	0.30	10.29	9.56	11.20	0.39
Jul-00	12.09	11.02	12.51	0.35	9.85	8.74	10.85	0.41
Aug-00	11.95	11.30	12.20	0.19	10.82	9.90	12.13	0.53
Sep-00	12.13	11.78	12.39	0.18	11.15	10.65	12.03	0.42
Oct-00	12.65	12.41	12.95	0.14	10.72	10.26	12.39	0.44
Nov-00	12.67	12.46	12.90	0.15	10.53	10.15	10.95	0.22

* β_0; ** $\beta_0 + \beta_1$

The short term rate is indicative of the risk-free overnight rate in the economy and is expected to have a high correlation with the indicative actual overnight rates [eg. NSE MIBID/MIBOR] and the RBI repo rate. The former, bearing as it does some default risk, would have a credit spread over & above the overnight rates estimated from the model. Analysis of the credit spread for different maturity periods[19] is an issue for further research.

We present below charts depicting the behaviour of interest rates, at different maturity horizons, over the period January – November 2000. Estimated short-term rates corresponding to different maturities for which risk-free instruments (T-Bills) are available are plotted in Figure 1. The plot reveals, first, higher interest rates, in general, in the current financial year. Further, volatility in interest rates is

[19] NSE MIBID/MIBOR are available for maturity periods of 1 day (overnight), 14 days, 1 month and 3 months.

found to be high in June 2000 and during August-September 2000, along with a marked increase in spreads between interest rates for different maturity horizons. This could probably be attributed to the uncertainty regarding interest rate policy measures generated by the uncertainties on the exchange rate front.

[Insert Figure 1 here]

Rates at maturities beyond 1 year have also witnessed significant volatility during this period, as evident in Figure 2. However, in contrast to the behaviour in short-term rates, the term premium between rates at different maturity horizons is in this case found to be near-uniform over most of the sample period.

[Insert Figure 2 here]

b. Analysis of Residuals

As mentioned earlier in Section IV, while the term structure is the primary factor determining the valuation/pricing of a bond, there may be other factors that also influence pricing. If the expectational elements have been adequately captured via the 'cost of carry' and the use of forward rates, the term structure should provide the 'fair' value of the bond and the additional 'settlement day' effect is expected to be statistically insignificant. If this has not been adequately captured, however, the extent of pricing error is expected to be larger the larger the gap between trade and maturity dates, and the sign on the estimated coefficient on this variable is expected to be positive. Table 4 below provides a snap-shot view of the statistical significance of the estimated coefficient. That the coefficient is statistically insignificant for more than 75 per cent of the sample days indicates that our model specification, ie. incorporation of 'cost of carry' and use of forward rates for discounting of 'futures' trades, adequately captures the expectational effects on prices associated with settlement days. The average value of the estimated coefficient (for statistically significant coefficients) is 0.28 (Table 5).

Table 4
‘Settlement day’ effect

	Trd_days	Number of times	
		Significant	Insignificant
Jan-00	25	3	22
Feb-00	24	5	19
Mar-00	24	4	20
Apr-00	20	8	12
May-00	25	5	20
Jun-00	25	5	20
Jul-00	26	8	18
Aug-00	25	4	21
Sep-00	25	7	18
Oct-00	22	5	17
Nov-00	20	5	15
Overall	**261**	**59**	**202**

Observations in Nov’00 are upto Nov 20, 2000

Table 5
Settlement day and Liquidity Effects
Estimated Coefficients
(average of significant coefficients)

	sdays	ln(vol)
Jan-00	0.21	-0.19
Feb-00	0.33	-0.07
Mar-00	1.46	-0.32
Apr-00	-1.02	-0.39
May-00	1.24	-0.34
Jun-00	1.34	-0.38
Jul-00	0.69	-0.26
Aug-00	-1.45	-0.34
Sep-00	-0.44	-0.43
Oct-00	1.01	-0.42
Nov-00	-0.30	-0.33
Overall	**0.28**	**-0.32**

Among the attributes of a bond that cause price differentials over and above that implied by the term structure is its relative liquidity. As explained earlier, the more illiquid a security, the larger is the pricing error, and hence the sign of the coefficient on ‘log volume’ is expected to be negative. The estimated coefficient is found to be statistically significant at the 95% confidence level for almost 50 per cent of the sample days (**Table 6**). It has the expected negative sign for 41 per cent of the sample (107 days of 261 in the sample). The average value of the coefficient is –0.32 (**refer Table 5**). To sum up, the supplementary exercise on analysis of residuals provides fairly strong evidence in support of the 'liquidity effect'. Weak evidence in favour of the additional 'settlement day' effect is indicative the adequacy of our model specification in tracking market expectations.

Table 6
Liquidity effect

	Trd_days	Number of times significant	Number of times insignificant
Jan-00	25	9	16
Feb-00	24	14	10
Mar-00	24	9	15
Apr-00	20	9	11
May-00	25	10	15
Jun-00	25	17	8
Jul-00	26	12	14
Aug-00	25	15	10
Sep-00	25	14	11
Oct-00	22	16	6
Nov-00	20	10	10
Overall	**261**	**135**	**126**

VI. Conclusion

The importance being accorded to term structure estimation as a consequence of the phased implementation of financial sector reforms has evoked widespread interest in the subject. The present exercise contributes to, and attempts to improve on, the sparse empirical literature that exists in the Indian context. Using the parsimonious Nelson-Siegel functional form, we present daily estimates of the term structure. The estimated model is found to perform satisfactorily in terms of the mean absolute price errors. Liquidity emerges as an important factor explaining inter-security variation in prices over and above that implied by the term structure. The impact of the T+5 settlement system, an important institutional aspect that leads to intra-security price variation for multiple trades, is captured via the incorporation of 'cost of carry' and use of forward rates for discounting of 'futures' trades. We find that this model specification adequately captures the expectational effects on interest rates (and therefrom on prices) associated with settlement days.

References

Anderson, N., F. Breedon, M. Deacon, A. Derry & G. Murphy (1996); '*Estimating and Interpreting the Yield Curve*'; Wiley Series in Financial Economics and Quantitative Analysis.

Bolder, D. & D. Streliski (1999); "Yield Curve Modelling at the Bank of Canada", Bank of Canada Working Paper.

Carleton, Willard T. and Ian A. Cooper (1976); "Estimation and Uses of the Term Structure of Interest Rates", *Journal of Finance* XXXI (4), 1067-1083.

Elton, Edwin J. & T. Clifton Green (1998); "Tax and Liquidity Effects in Pricing Government Bonds", *Journal of Finance*, LIII(5), 1532-62.

Hall, Anthony D., Heather M. Anderson and Clive W.J. Granger (1992); "A Cointegration Analysis of Treasury Bill Yields, *Review of Economics and Statistics*, 74(1), 116-26.

Krishnan, R. (1989) '*An Econometric Analysis of the Term Structure of Indian Interest Rates*', PhD thesis, University of Bombay.

McCulloch (1971); "Measuring the Term Structure of Interest Rates", *Journal of Business* XLIV (January); 19-31.

------------- (1975); "The Tax Adjusted Yield Curve", *Journal of Finance* XXX (3), 811- 830.

Nelson, Charles R. and Andrew F. Siegel (1987); "Parsimonious Modeling of Yield Curves", *Journal of Business* 60(4), 473-89.

Nag, Ashok K. and Sudip K. Ghose (2000); "Yield Curve Analysis for Government Securities in India", *Economic and Political Weekly*, January 29, 339-48.

Robichek, Alexander A. & W. David Neibuhr (1970); "Tax-induced Bias in Reported Treasury Yields", *Journal of Finance* XLIII (); 1081-1090.

Schaefer, Stephen M. (1981); "Measuring a Tax-specific Term Structure of Interest Rates in the Market for British Government Securities", *The Economic Journal*, 91 (June), 415-438.

Schoenberg, R. (1998); '*Constrained Optimisation – Application Module*', GAUSS Applications, Aptech Systems Inc.

Seber, G.A.F and C.J. Wild (1989); '*Nonlinear Regression*', Wiley Series in Probability and Mathematical Statistics, John Wiley & Sons Inc.

Subramanian, K.V (2000); "Term Structure Estimation in Illiquid Government Bond Markets: An Empirical Analysis for India", unpublished paper, ICICI Ltd.

Svensson, Lars E.O (1994); "Estimating and Interpreting Forward Interest Rates: Sweden 1992-1994", NBER WP 4871.

Thomas, S. & V. Saple (2000); "Estimating the Term Structure of Interest Rates in India", unpublished paper, IGIDR.

Figure 1.
Short-term spot rates: January-November 2000

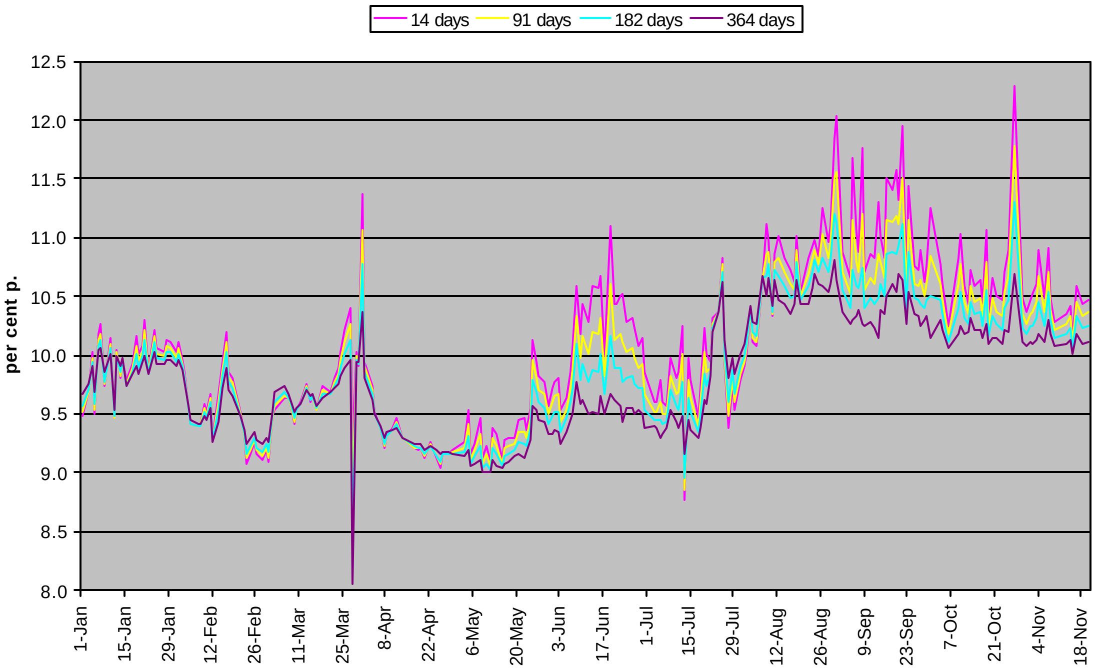

Figure 2.
Long-term spot rates: January-November 2000