

# Term Structure and Credit Spread Estimation with R

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# Basic principles of bond pricing

- coupon bond which matures in  $n$  years
- investor gets at the times  $i = 1, \dots, n$  coupon payments  $C$  and a redemption payment  $R$  at  $t = n$
- **clean price**  $p_c$  is quoted on the market
- seller also receives **accrued interest** for holding the bond over the period since the last coupon payment

$$a = \frac{\text{number of days since last coupon}}{\text{number of days in current coupon period}} C$$

- investor has to pay the **dirty price**  $p_d$
- bond pricing equation with continuous compounding

$$p_c + a = C \sum_{i=1}^n e^{-s_i m_i} + R e^{-s_n m_n}$$

# Basic principles of bond pricing

- **yield to maturity**

$$p_c + a = C \sum_{i=1}^n e^{-ym_i} + Re^{-ym_n}$$

- equivalent formulation of the bond price equation uses the **discount factors**  $d_i = \delta(m_i) = e^{-s_i m_i}$
- continuous **discount function**  $\delta(\cdot)$  is formed by interpolation of the discount factors

$$p_c + a = C \sum_{i=1}^n \delta(m_i) + \delta(m_n)R$$

- implied  $j$ -period **forward rate**

$$f_{t|j} = \frac{js_j - ts_t}{j - t}$$

- **duration** is a weighted average of time to cash flows

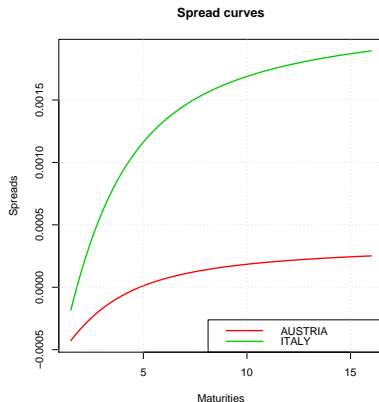
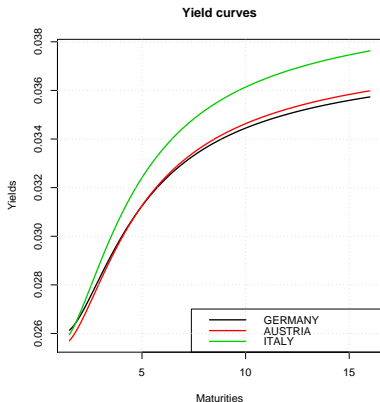
$$D = \frac{1}{p_c + a} \left[ C \sum_{i=1}^n \delta(m_i) m_i + \delta(m_n) R m_n \right]$$

# Term structure estimation

- estimate zero-coupon yield curves and credit spread curves from market data
- usual way for calculation of **credit spread curves**

$$c_i(t) = s_i(t) - s_{ref}(t)$$

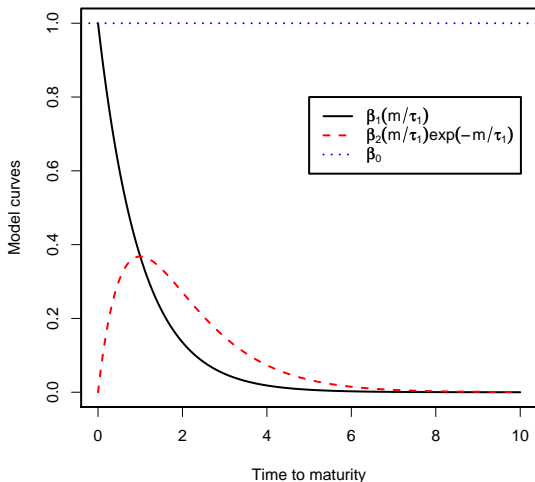
- parsimonious approach widely used by central banks



# Nelson and Siegel (1987) approach

## Instantaneous forward rates

$$f(m, \mathbf{b}) = \beta_0 + \beta_1 \exp\left(-\frac{m}{\tau_1}\right) + \beta_2 \frac{m}{\tau_1} \exp\left(-\frac{m}{\tau_1}\right)$$



# Nelson and Siegel (1987) approach

## Spot rates

$$s(m, \mathbf{b}) = \beta_0 + \beta_1 \frac{1 - \exp(-\frac{m}{\tau_1})}{\frac{m}{\tau_1}} + \beta_2 \left( \frac{1 - \exp(-\frac{m}{\tau_1})}{\frac{m}{\tau_1}} - \exp(-\frac{m}{\tau_1}) \right)$$

## Objective function

$$\mathbf{b}_{opt} = \min_{\mathbf{b}} \sum_{i=1}^n \omega_i (\hat{P}_i - P_i)^2 \quad \text{weighted price errors}$$

$$\mathbf{b}_{opt} = \min_{\mathbf{b}} \sum_{i=1}^n (\hat{y}_i - y_i)^2 \quad \text{yield errors}$$

- Svensson (1994) extended the functional form by two additional parameters which allows for a second hump-shape

## Instantaneous forward rates

$$f(m, \mathbf{b}) = \beta_0 + \beta_1 \exp\left(-\frac{m}{\tau_1}\right) + \beta_2 \frac{m}{\tau_1} \exp\left(-\frac{m}{\tau_1}\right) + \beta_3 \frac{m}{\tau_2} \exp\left(-\frac{m}{\tau_2}\right)$$

- simple calculation method of credit spread curves could lead to twisting curves
- Jankowitsch and Pichler (2004) proposed a **joint estimation method**, which leads to smoother and more realistic credit spread curves



Bank for International Settlements

Zero-coupon yield curves: technical documentation

*BIS Papers*, No. 25, October 2005



David Bolder, David Streliski

Yield Curve Modelling at the Bank of Canada

*Bank of Canada, Technical Report*, No. 84, 1999



Alois Geyer, Richard Mader

Estimation of the Term Structure of Interest Rates - A Parametric Approach

*OeNB, Working Paper*, No. 37, 1999





Rainer Jankowitsch, Stefan Pichler  
Parsimonious Estimation of Credit Spreads  
*The Journal of Fixed Income*, 14(3):49–63, 2004



Charles R. Nelson, Andrew F. Siegel  
Parsimonious Modeling of Yield Curves  
*The Journal of Business*, 60(4):473–489, 1987



Lars E.O. Svensson  
Estimating and Interpreting Forward Interest Rates:  
Sweden 1992 -1994  
*National Bureau of Economic Research,  
Technical Report*, No. 4871, 1994