

# termstrc: A Package for Term Structure and Credit Spread Estimation with R

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# Basic principles of bond pricing

- coupon bond which matures in  $n$  years
- investor gets cashflows  $c_t$  at the times  $t = 1, \dots, n$  ( $c_n$  includes the redemption payment)
- **clean price**  $p_c$  is quoted on the market
- seller also receives **accrued interest** for holding the bond over the period since the last coupon payment

$$a = \frac{\text{number of days since last coupon}}{\text{number of days in current coupon period}} C$$

- investor has to pay the **dirty price**  $p_d$
- bond pricing equation with continuous compounding

$$p_c + a = \sum_{t=1}^n c_t e^{-s_t m_t}$$

# Basic principles of bond pricing

- **yield to maturity**

$$p_c + a = \sum_{t=1}^n c_t e^{-ym_t}$$

- equivalent formulation of the bond price equation uses the **discount factors**  $d_t = \delta(m_t) = e^{-s_t m_t}$
- continuous **discount function**  $\delta(\cdot)$  is formed by interpolation of the discount factors

$$p_c + a = \sum_{t=1}^n c_t \delta(m_t)$$

# Term structure and credit spread estimation

- estimate zero-coupon yield curves and credit spread curves from market data
- usual way for calculation of **credit spread curves**

$$cs_j(\mathbf{m}) = s_j(\mathbf{m}, \mathbf{b}) - s_{ref}(\mathbf{m}, \mathbf{b})$$

$cs_j(\mathbf{m})$	credit-spread between country $j$ and reference country $ref$
$s_j(\mathbf{m}, \mathbf{b})$	spot-rate curve of country $j$ with maturity vector $\mathbf{m}$
$s_{ref}(\mathbf{m}, \mathbf{b})$	spot-rate curve of the reference country

# Nelson and Siegel (1987) approach

## Instantaneous forward rates

$$f(m, \mathbf{b}) = \beta_0 + \beta_1 \exp\left(-\frac{m}{\tau_1}\right) + \beta_2 \frac{m}{\tau_1} \exp\left(-\frac{m}{\tau_1}\right)$$

## Spot rates

$$s(m, \mathbf{b}) = \beta_0 + \beta_1 \frac{1 - \exp\left(-\frac{m}{\tau_1}\right)}{\frac{m}{\tau_1}} + \beta_2 \left( \frac{1 - \exp\left(-\frac{m}{\tau_1}\right)}{\frac{m}{\tau_1}} - \exp\left(-\frac{m}{\tau_1}\right) \right)$$

# Svensson (1994) approach

- Svensson (1994) extended the functional form by two additional parameters which allows for a second hump-shape

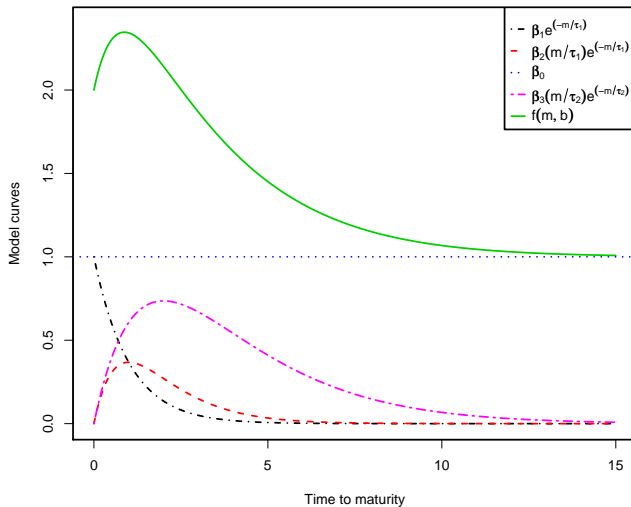
## Instantaneous forward rates

$$f(m, \mathbf{b}) = \beta_0 + \beta_1 \exp\left(-\frac{m}{\tau_1}\right) + \beta_2 \frac{m}{\tau_1} \exp\left(-\frac{m}{\tau_1}\right) + \beta_3 \frac{m}{\tau_2} \exp\left(-\frac{m}{\tau_2}\right)$$

## Spot rates

$$s(m, \mathbf{b}) = \beta_0 + \beta_1 \frac{1 - \exp\left(-\frac{m}{\tau_1}\right)}{\frac{m}{\tau_1}} + \beta_2 \left( \frac{1 - \exp\left(-\frac{m}{\tau_1}\right)}{\frac{m}{\tau_1}} - \exp\left(-\frac{m}{\tau_1}\right) \right) \\ + \beta_3 \left( \frac{1 - \exp\left(-\frac{m}{\tau_2}\right)}{\frac{m}{\tau_2}} - \exp\left(-\frac{m}{\tau_2}\right) \right)$$

# Decomposition of the Svensson forward rate function



# Term structure estimation procedure

## Notation I

Maturity matrix  $M$

$$M_{[n \times m]} = \{m_{ij}\}$$

Cashflow matrix  $C$

$$C_{[n \times m]} = \{c_{ij}\}$$

Discount factor matrix  $D$

$$D_{[n \times m]} = \{d_{ij}\}; \quad d_{ij} = e^{-m_{ij}s(m_{ij}, \mathbf{b})}$$

Clean price vector  $p^c$

$$p^c_{[1 \times m]} = \{p_j^c\}$$



# Term structure estimation procedure

## Notation II

Accrued interest vector  $\mathbf{a}$

$$\mathbf{a}_{[1 \times m]} = \{a_j\}$$

Dirty price vector  $\mathbf{p}^d$

$$\mathbf{p}_{[1 \times m]}^d = \{p_j^d\}$$

$$\mathbf{p}^d = \mathbf{p}^c + \mathbf{a}$$

Weights vector  $\mathbf{w}$

$$\mathbf{w}_{[1 \times m]} = \{w_j\}; \quad w_j = \frac{\frac{1}{D_j}}{\sum_{i=1}^m \frac{1}{D_i}}$$

# Term structure estimation procedure

## Objective function

- Minimization of the weighted pricing or yield errors

### Objective function

$$\mathbf{b}_{opt} = \min_b \left( (\boldsymbol{\iota}_{[1 \times n]} [\mathbf{C} \cdot \mathbf{D}] - \mathbf{p}^d)^2 \mathbf{w} \boldsymbol{\iota}_{[m \times 1]} \right)$$

- The parameter vector is subject to constraints  
( $\beta_0 > 0, \tau_1 > 0, \tau_2 > 0$ )

# Examples



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Zero-coupon yield curves: technical documentation

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