



Measuring Systematic Risk in EMU Government Yield Spreads^{*}

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Abstract. This paper focuses on the joint dynamics of yield spreads derived from government bonds issued by member states of the European Monetary Union (EMU). A descriptive analysis shows that there are substantial and volatile spreads between zero coupon yields of EMU member countries and German Bund yields. These yield spreads form an important source of additional risk that has to be taken into account by any pricing or risk management model dealing with EMU government bonds. We extract risk factors driving observed yield spreads by employing a multi-issuer version of the model originally proposed by Duffie and Singleton (1999). We adopt a state-space approach to implement the model whereby we can extract factor series and model parameters simultaneously. Our findings indicate that a parsimonious two-factor version of the multi-issuer model sufficiently captures the main features of the data. In this model the first factor turns out to be related to long term yield spreads across different issuers, whereas the second factor is related to short term yield spreads. Our evidence suggests that EMU government bond spreads are related to corporate bond spreads and swap spreads whereas we do not find evidence for a significant impact of macroeconomic or liquidity related variables.

Key words: affine term structure model, credit spreads, EMU, intensity model, Kalman filter, multi-issuer model.

JEL classification codes: C51, E43, G13, G15.

1. Introduction

The formation of the third stage of the European Monetary Union (EMU) in January 1999 based on the Maastricht Treaty changed the structure of the European bond market fundamentally. Exchange rates among member countries were irrevocably fixed, and the Euro was introduced as the new single currency. All EMU member countries have to meet specific economic

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“convergence” criteria, most notably the public finance criteria restricting the public debt below 60% of GDP and the public deficit below 3% of GDP. With the commencement of the EMU, the most important reason for the existence of spreads between interest rates among member states, i.e., exchange rate risk, disappeared. As of January 1999 “riskless” bonds denominated in Euro and traded in a frictionless market can be expected to be perfect substitutes and, thus, to trade on the same yield curve.

However, several reasons that yield spreads still exist are conceivable. First, there could be country-specific default risk perhaps due to differences in the fiscal policy of different governments or due to differing exposures to external shocks. Second, there could be differences in liquidity and taxation among countries and market segments. Finally, there is a small but positive probability of a failure of the EMU that might reintroduce exchange rate risk at some date prior to the redemption of some bonds.

The first three years of EMU experience show that there are substantial and volatile spreads between government bond yields of EMU member countries other than Germany and German Bund yields. These yield spreads form an important source of additional risk that has to be taken into account by any pricing or risk management model. For instance, there are derivatives traded in the markets, e.g., spread options, that explicitly refer to observed yield spreads. In addition, many traders “pick” the high yield bonds and simultaneously hedge their interest rate exposure by shorting German Bund futures leaving the trader exposed to yield spread risk. Hence, there is a need for a model that prices EMU government bonds and related derivatives based on explicitly modelling the risk factors underlying the yield spreads. It is the main objective of this paper to formulate and estimate such a model and to put special emphasis on its empirical properties. Additionally, we examine the economic meaning of the extracted factor time series in regressions with macroeconomic and financial variables.

The reduced form default model proposed by Duffie and Singleton (1999) (DS hereafter) seems to be a natural candidate for modelling zero coupon yield spreads. Under the assumption of a loss rate proportional to the market value of the defaultable bond and modelling the default event as the first jump of a Cox process,¹ the resulting bond pricing model has a rather simple structure. The product of the intensity and the loss rate, and the riskless short rate are assumed to be driven by affine processes.² The pricing model preserves its affine structure when the factors driving the intensity/loss rate process are just treated as additional factors in a familiar affine term structure model like Vasicek (1977) or Cox, Ingersoll and Ross (1985) (CIR hereafter). Besides its analytical tractability, the use of this model benefits from the

¹ See Lando (1998).

² See Duffie and Kan (1996).

experience in implementing and testing affine term structure models in a default-free context. To capture the specific structure of the EMU bond market we formulate a multi-issuer extension of the original model, which allows a differentiation between 'global' factors, i.e., factors which affect spreads of all issuers, and 'local' factors, i.e., factors which affect spreads of a subset of issuers only.

It has to be emphasized that the model is more general than a pure default model. First, the model can also be used to capture liquidity effects. DS discuss this issue with respect to the interpretation of endogenously derived factor series whereas Kempf and Uhrig-Homburg (2000) add an exogenously specified factor representing liquidity effects to a standard CIR model. Second, it is not necessary to assume that the reference term structure is perfectly liquid and default free. Spreads can be interpreted as relative credit and liquidity discounts. This is particularly important in the context of this application, since the choice of reference issuer in the EMU government market is not unique.

To estimate various specifications of the general affine model structure we adopt a state-space approach. This approach was first introduced in a default-free term structure setting by Chen and Scott (1995) and Geyer and Pichler (1999). It enables us to extract latent factor series and model parameters simultaneously. In contrast to other approaches no additional assumptions about the observation errors³ are necessary and the errors-in-variables problem inherent in two-step approaches is avoided.

The state-space approach has been applied by Duffee (1999) to estimate a pricing model for U.S. corporate bonds and Lund (1998) uses a non-linear Kalman filter model to analyze the pre-1999 effects of the EMU. The paper most closely related to our work is Düllmann and Windfuhr (2000), who examine the suitability of one-factor affine models to explain the observed spreads between Italian and German government bond yields. While some empirical work has been conducted on spreads of individual issuers, empirical evidence on factors driving spreads in a multi-issuer context is scarce. A notable exception is Driessen (2002), who examines corporate bond spreads of various rating classes. The author proposes a model, in which spreads of different rating classes depend on three sets of factors: Global factors, which are identical to the factors driving the risk-free term structure, rating class specific factors, and issuer-specific factors. The model is estimated in three consecutive and separate steps, corresponding to each class of factors.

Our paper is the first attempt to apply the state-space approach to estimate a model that can be used to price EMU sovereign debt in a multi-issuer

³ Some approaches are based on the assumptions that a subset of bonds is priced without any error.

context. We present the joint estimation of multi-issuer spread models in a state-space framework that allows for arbitrary, linear cross-issuer restrictions in the factor structure. To uncover the dependence structure of factors driving EMU spreads, we formulate various empirical specifications of the class of affine multi-factor models. The specifications differ in terms of the restrictions imposed on the joint factor structure. We find strong empirical evidence for the existence of common or global factors as opposed to issuer-specific or local factors.

Regressing the time series of global factors against macroeconomic and financial variables, we find that the spread factors are related to Euro corporate bond spreads and swap spreads. We do not find significant effects from macroeconomic variables and proxies for liquidity like size or specialness. This suggests that credit risk is a major driving force of systematic risk in EMU government yield spreads.

Our results have the following implications: (i) there is considerable systematic risk in EMU government yield spreads that limits the possible effects of diversification of investments in bonds of different issuers; (ii) spread structures of EMU government bonds relative to German government bonds are dominated by maturity effects and not by issuer effects; (iii) the systematic risk of exposures in these yield spreads, e.g., with spread options, or long positions in 'high yield' bonds hedged by a short position in Bund futures, etc., can be modelled by the multi-issuer model proposed in this paper that fits the data sufficiently well and is computationally attractive in relation to other models; (iv) contrary to widely held beliefs by market practitioners, there is evidence that spreads between EMU issuers cannot be explained by liquidity differences only.

The evidence of systematic risk in EMU government yield spreads supports the hypothesis that there is a small but positive probability of a general failure of the EMU that might re-introduce exchange rate risk at some date prior to the redemption of some bonds. The risk factors driving the probability of such an event are expected to be common across different issuers which is consistent with the observed global risk factor structure and the evidence of economic drivers of these global risk factors. The differences in the levels of yield spreads may reflect different market beliefs about the possible effects of a EMU failure to the individual countries.

The paper is structured as follows: Section 2 introduces the market and the data set. Section 3 shortly summarizes the main cornerstones of the DS model and describes the multi-issuer extension developed in this paper. Section 4 reviews related literature and introduces the state-space approach. In Section 5 we present empirical results and interpret extracted factor series. In Section 6 we present results from a regression analysis to investigate the covariation between the factor series and some macroeconomic and financial variables. Section 7 concludes the paper.

2. The Data and Preliminary Analysis

The data set analyzed in this paper consists of time series of weekly yield spreads for selected EMU member states for maturities of two to nine years. The spread curve of a particular issuer is defined as the difference between the zero bond yield curves of that issuer and the reference issuer. Zero bond yield curves are computed from quoted prices of liquid, non-callable government bonds of the respective issuer. As the main source of price data we use averaged daily closing dealer quotes provided by Bloomberg (Bloomberg Generic, BGN). Cash flow data and all other relevant information were also collected from Bloomberg. The resulting price information was cross-checked with data provided by Reuters. In only a few cases we observed inconsistencies and removed this information from our data base. Spread curves in our sample are fitted jointly with the reference yield curve in a multi-curve procedure originally proposed by Houweling, Hoek, and Kleibergen (2001), which ensures relatively stable and well behaved curves. For this estimation procedure we use standard cubic splines for the functional form of the discount function. This allows for a comparably parsimonious parametrization of the model, i.e., we set the number of free parameters equal to four for the reference curve and equal to three for the spread curves, respectively. We obtain parameter estimates by a restricted feasible GLS procedure, see for instance Greene (2000). By comparing alternative functional representations of the discount function (e.g., the model proposed by Svensson 1994) we observe that the yield curves obtained by the multi-curve procedure are rather insensitive to the choice of the cubic splines model at least for this data set (see Jankowitsch and Pichler (2002) for a detailed description of the method and the database). The sample period starts with the inception of EMU in January 1999 and lasts until May 2002 for a total of 178 weekly observations.

Although there are at the moment 12 issuers of government debt in the EMU, we use only a subset of them in our analysis for various reasons: first, Greece and Luxembourg are not part of the sample, the former because it has joined the EMU only in January 2001, the latter because the amount of its outstanding debt is negligible; second, Ireland has consolidated its debt into five large issues, three of which are outside of the maturity spectrum analyzed in this paper. With only two bonds, the zero curve extraction procedure described in the previous paragraph is not feasible; third, for some member countries fitted zero spread curves display large upward or downward spikes for all maturities, potentially due to lack of market depth. Using these data could potentially distort the parameter estimates and the resulting factor time series. This is the case for Portugal and Finland. And, finally, another two issuer countries, i.e., France and The Netherlands have been excluded from the analysis because their spreads are very small (in the case of France at

times even negative) compared to the rest of the countries.⁴ As a consequence we analyze spreads of Austria, Belgium, Italy and Spain relative to the German zero bond curve, which is fixed as the reference curve. We take the German Bund curve as the baseline of our analysis following the standard used by most practitioners as well as academics (see for instance Düllmann Windfuhr 2000). Of course, this choice may seem somewhat arbitrary since French OATs are also regarded as Euro benchmark bonds particularly for shorter maturities. To check whether this choice would have any effect on our results we conducted an exploratory factor analysis using yield spreads relative to the French curve. We obtain a very similar factor structure for the two alternative spreads. This can be explained by the remarkably high correlation between their levels and changes. We presume that the main results of this paper would not be influenced by this choice of the reference yield curve.

Table I shows the means and standard deviations of spreads for all issuers and maturities. Average mean spreads across issuers range from 12 basis points (bp) for short maturities to approximately 28 bp for long maturities. Italy has the highest mean spread curve for all but the shortest maturity in the sample. This is in line with Italy being the lowest rated issuer (AA Standard and Poor's long term issuer rating) in our sample. However, for shorter maturities, Austria has the highest or second highest spread, although it has the best credit rating in the sample (AAA S&P long term issuer rating). For longer maturities, Belgium has the second highest mean spread followed by Spain and Austria. Belgium and Spain are rated AA+ by Standard and Poor's, Spain was upgraded from AA in March 1999. All mean spread curves are monotonically increasing, Austrian mean curves are the flattest, while curves of Belgium are the steepest. Note that despite of the well behaved mean spread curves, at several points in the sample period, all issuers display hump-shaped, downward sloping, or convex spread curves. As shown by the rows labelled "SD" in Table I, term structures of spread volatility of Austria and Belgium show a peculiar hill shaped pattern with volatilities peaking at middle maturities. Also, volatilities for these issuers are highest except for the longest maturity in the sample, where Italy has the highest volatility. Volatility curves for Italy and Spain are upward sloping, with Spanish spread volatilities being the lowest in the sample.

Next, it is interesting to take a look at the time series behaviour of spreads. Figures 1 and 2 show the evolution of the 9 and 2 years to maturity series, i.e., the longest and shortest maturity spreads in the sample, for all issuers.

⁴ Obviously it can be disputed whether this is reason enough to exclude an issuer from the analysis. However, the latent factor models estimated in this paper are computationally quite demanding. Therefore, there is a need to reduce the sample to a smaller number of issuers than would be available.

Table I. Mean spreads (mean) and spread volatilities (SD) per issuer and maturity (ytm) in basis points

	2ytm	3ytm	4ytm	5ytm	6ytm	7ytm	8ytm	9ytm
Austria								
Mean	16.17	17.42	18.63	19.75	20.77	21.64	22.32	22.74
SD	4.18	5.27	6.30	7.06	7.48	7.48	7.04	6.29
Belgium								
Mean	7.26	10.23	13.27	16.39	19.59	22.84	26.10	29.30
SD	3.21	4.19	5.47	6.53	7.18	7.36	7.08	6.60
Italy								
Mean	15.56	18.69	21.67	24.44	26.95	29.10	30.79	31.88
SD	3.95	4.26	4.87	5.53	6.10	6.52	6.77	6.93
Spain								
Mean	8.97	11.14	13.42	15.83	18.38	21.06	23.85	26.71
SD	3.21	3.90	4.60	5.16	5.50	5.63	5.63	5.73

The graphs reveal two characteristic features of spreads: first, there is high short term volatility and, second, there is a clear, slowly changing common trend in spreads of all issuers over the sample period. This common trend is less clear for the short end of the maturity spectrum. The impression of a strong interrelation of spreads of all maturities across issuers is confirmed by a median ‘inter-issuer’ correlation in levels of 0.85 with an upper 5% quantile at 0.94 and a lower 5% quantile at 0.49, respectively. However, because of this long term trend, time series of spreads are only borderline stationary at best. To check for stationarity we use augmented Dickey–Fuller tests. For no time series the unit root hypothesis can be rejected at conventional significance levels.⁵ Correlations in first differences are lower with a median of 0.44 and upper and lower quantiles at 0.66 and 0.12, respectively.

3. A Pricing Model for EMU Government Bonds

This section briefly summarizes the cornerstones of the DS model and discusses the extension of the model to a multi-issuer context. We take as given an arbitrage free setting, where all contingent claims are priced in terms of a numeraire and an equivalent martingale measure. The dynamics of the numeraire asset are described as

$$d\Pi_t = R_t \Pi_t dt, \quad (1)$$

⁵ Numerical results are not reported, but are available from the authors upon request.

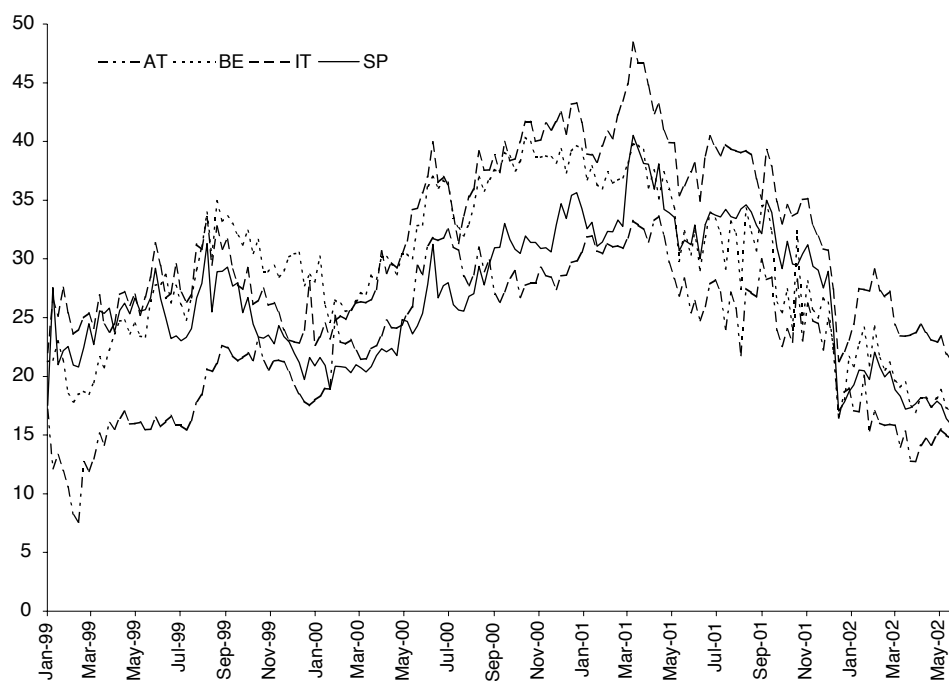


Figure 1. Time series of nine years to maturity spreads.

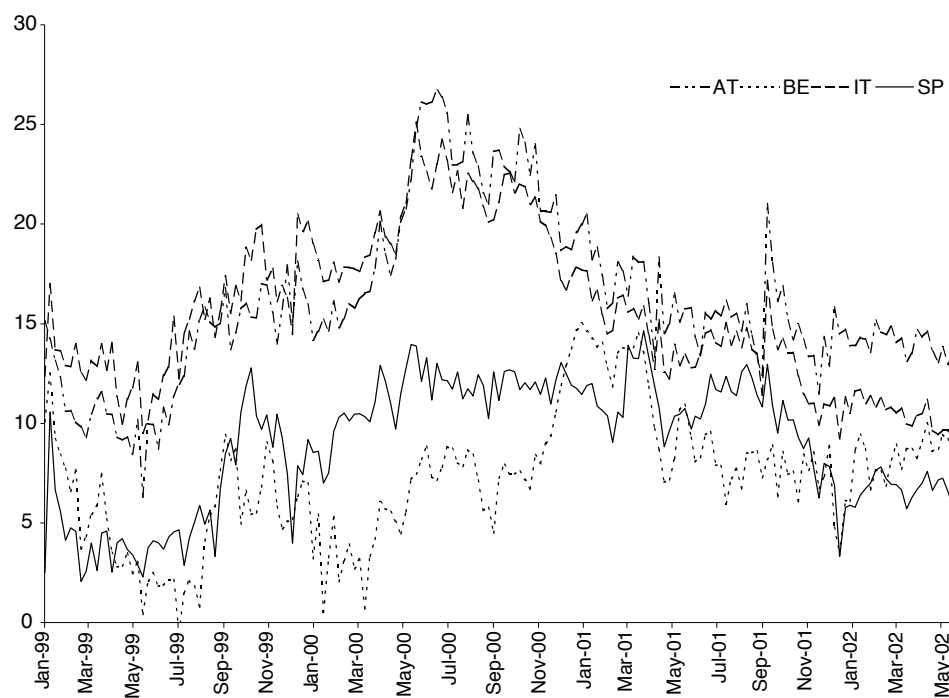


Figure 2. Time series of two years to maturity spreads.

where $R_t = R(x_t)$ denotes the short rate governing the German reference yield curve which in turn depends on a vector, x_t , of state variables or factors driven by Ito processes.

Consider two classes of claims in this economy: first, default-free zero coupon bonds, which pay one unit of currency at time to maturity $t + \tau$, $P(t, \tau)$. The prices of these bonds are given as the expectation of the stochastic discount factor under the equivalent martingale measure:

$$P(t, \tau) = E_t^Q \left[\exp \left(- \int_t^{t+\tau} R_u du \right) \right]. \quad (2)$$

Equation (2) characterizes the default-free term structure in the economy. Throughout this paper, we assume that this default-free term structure is represented by the observed prices of German sovereign bonds. Note, that it is not necessary at all to assume that the German term structure is perfectly free of default risk and perfectly liquid. The reference German short rate R_t can be interpreted so as to include effects of liquidity and credit risk (in the DS setting) as well. In this case the short spread introduced in Equation (3) below contains *relative* credit and liquidity effects and none of the numerical results and the conclusions of this analysis change.

Second, there is a set of defaultable zero bonds, $V_C(t, \tau)$, promising to pay one unit of currency at time to maturity $t + \tau$. The defaultable term structures are defined by the sovereign debt of other EMU countries.⁶ Here $C = \{AT, BE, IT, SP\}$ is an index set containing the issuer countries under analysis. Along the lines of Lando (1998), DS assume that the default event is the first jump of a Cox process with intensity $h_C(y_t)$ (i.e., a Poisson counting process with stochastic parameter), where y_t denotes a $I \times 1$ vector of state variables or factors driven by Ito processes. When one issuer country defaults, the values of all bonds issued by this particular country are reduced by a (constant) fraction L_C of their pre-default market value. This assumption is crucial for the derivation of the following key result. DS show that under the assumption of fractional recovery of market value, defaultable claims can be priced in the same way as their default-free counterparts just with a risk-adjusted short rate. Therefore, the prices of the defaultable bonds are given by

$$V_C(t, \tau) = E_t^Q \left[\exp \left(- \int_t^{t+\tau} (R_u + S_{C,u}) du \right) \right], \quad (3)$$

⁶ Note, that modelling these term structures as defaultable does not necessarily imply that there is default risk in a strict sense. Liquidity effects or the possibility of a general failure of the EMU may also explain the existence of spreads (see Section 1).

where $S_{C,t} = h_{C,t}L_C$ is usually called the short spread and can be interpreted as the mean instantaneous loss rate due to default under the risk neutral measure.⁷ Thus, the risk-adjusted short rate is simply the sum of the risk-free short rate and the short spread for the issuing country.

Equation (3) can be seen as a simple multi-issuer version of the basic DS model. Comparison of Equations (2) and (3) sheds light on the convenience of the DS model. There is a clear analogy between conventional models of the risk-free term structure and models of defaultable term structures. By parameterizing the sum in the integral in Equation (3) or its components, the technical toolbox that has been developed for risk-free interest rate modelling becomes available. However, due to the fact that only the product of the two relevant quantities is considered, default probabilities and recovery rates are not separately identifiable from observed prices of risky bonds.

There is an alternative interpretation of $S_{C,t}$ in Equation (3) in terms of a fractional carrying cost of the defaultable bond due to illiquidity. If $S_{C,t}$ is negative, it can be interpreted as a fractional convenience yield due to liquidity. Grinblatt (1994) introduces this setup and fits a Vasicek model of liquidity effects to bond prices. Ultimately, it is an empirical question whether yield spreads are predominantly caused by default risk or illiquidity. One of the advantages of latent factor models is that the economic meaning of factors need not be specified in advance. It is very likely that in the market we analyze both effects play a role. In that sense $S_{C,t}$ can be interpreted as the sum of a credit risk component and an illiquidity component.

Assuming that $S_{C,t}$ is a linear combination of “affine factors”,⁸ i.e.,

$$S_{C,t} = \delta_{C,0} + \sum_{i=1}^I \delta_{C,i} Y_{t,i} \quad (4)$$

and that the short spread factors are independent from the factors driving the short rate under the risk-neutral measure, the solution to Equation (3) has the following structure in the multi-issuer setup:

$$\begin{aligned} V_C(t, \tau) = & P(t, \tau) \exp(-\delta_{C,0}\tau) \prod_{i=1}^I A_{C,i}(\delta_{C,i}, \tau) \\ & \times \exp\left\{-\sum_{i=1}^I [B_{C,i}(\delta_{C,i}, \tau) \delta_{C,i} Y_{t,i}]\right\}, \end{aligned} \quad (5)$$

⁷ For a rigorous derivation in a general setup see Bielecki and Rutkowski (2002).

⁸ For a characterization of affine term structure models see Duffie and Kan (1996).

where the expressions $A_{C,i}(\delta_{C,i}, \tau)$ and $B_{C,i}(\delta_{C,i}, \tau)$ denote country and maturity-dependent functions related to the factors driving the yield spreads (i.e., $Y_{t,i}$).

Zero bond yield spreads for non-reference issuers, $Z_C(t, \tau)$, which are the object of our empirical analysis, can be derived from Equation (5) as

$$Z_C(t, \tau) = \delta_{C,0} - \frac{1}{\tau} \sum_{i=1}^I \ln A_{C,i}(\delta_{C,i}, \tau) + \frac{1}{\tau} \sum_{i=1}^I [B_{C,i}(\delta_{C,i}, \tau) \delta_{C,i} Y_{t,i}]. \quad (6)$$

The main question in a multi-issuer context is what dependence structure to impose on the stochastic evolution of the short spreads. The country-specific weights $\delta_{C,i}$ in Equation (5) enable us to incorporate arbitrary linear dependence structures into the model. $\delta_{C,i}$ can be estimated as a free parameter, or can be constrained to zero or one. Depending on this choice some factors can affect the spreads of all issuers, possibly with an issuer-specific factor weight and thus introduce correlation among spreads of different issuers. Other factors can affect only a subset of issuers and still others can be specific to one single issuer and would therefore capture idiosyncratic risks, which are particular to that issuer.

The country-specific constant $\delta_{C,0}$ allows to capture differences in the level of spread structures in the common factor models. Notice, that the components of the risk-free term structure have been omitted in Equation (5) to economize on notation. Of course, the risk-free zero coupon bond can similarly be written as the affine sum of factors driving the risk-free term structure. Linear dependence between the short rate and the short spread, as in Driessen (2002), can be introduced by letting some $Y_{t,i}$ be linear combinations of some factors driving the risk-free term structure.

Different model specifications can be conveniently described in terms of the weight matrix of country-specific factor loadings, i.e.,

$$D = \begin{bmatrix} \delta_{AT,1} & \dots & \delta_{AT,I} \\ \delta_{BE,1} & \dots & \delta_{BE,I} \\ \delta_{IT,1} & \dots & \delta_{IT,I} \\ \delta_{SP,1} & \dots & \delta_{SP,I} \end{bmatrix}. \quad (7)$$

Then the vector of short spreads of all issuers,

$$s_t = (S_{AT,t}, S_{BE,t}, S_{IT,t}, S_{SP,t})'$$

can be written as

$$s_t = d + Dy_t, \quad (8)$$

where d is the vector of country-specific constants, $\delta_{C,0}$, and y_t is the vector of factors (or state variables)

$$y_t = (Y_{t,1}, \dots, Y_{t,I})'.$$

The exact functional forms of A and B in Equation (5) depend on the specification of the factor processes. Given the limited availability of data and also the convenience offered by analytical pricing equations we choose factor specifications with known analytical solutions to Equation (5). The two most widely used specifications that satisfy this condition are the Vasicek model, where the evolution of the factors is governed by Ornstein–Uhlenbeck processes and the CIR model, where the factor dynamics are modelled by square root processes. The factor process specification of the CIR model under the empirical measure is given by

$$dY_{t,i} = \kappa_i(\theta_i - Y_{t,i})dt + \sigma_i\sqrt{Y_{t,i}}dW_{t,i}, \quad i = 1, \dots, I. \quad (9)$$

Here, $W_{t,i}$ denotes the i -th element of a $I \times 1$ vector of independent Brownian motions, κ_i denotes the speed of mean reversion, θ_i is the long term mean of the process, and σ_i is the volatility parameter, respectively. For default-free interest rate modelling, the CIR model is usually considered to be superior to Gaussian specifications. First, under certain parameter restrictions non-negativity of interest rates is assured and second, the volatility structure of the process allows for some degree of heteroscedasticity, which matches empirical evidence for time series of short term interest rates. In the Vasicek model the short rate is normally distributed and homoscedastic.

For credit spread modelling it is not yet clear which model is preferable. From a pure credit spread perspective, non-negativity of spreads seems to be a desirable property. Also, almost all empirical work to date has employed the CIR specification. However, if spreads are not only due to credit risk but also due to liquidity differences, then non-negativity of factors is not necessarily an advantage. It is conceivable that in some cases or in some time periods the market for defaultable instruments is more liquid and that therefore liquidity effects have a negative influence on yields. If the liquidity effect can dominate the default effect then spreads can become negative and a Vasicek-type model should be preferred.

Identification of parameters is another issue that has to be considered in model specification. This is a major disadvantage of the Vasicek model. Dai and Singleton (2000) show that in a multi-factor Vasicek model it is not

possible to separately identify market prices of risk parameters from zero coupon bond data. This is due to the fact that the market prices of risk only appear in the pricing function for the Vasicek model as a linear combination with the mean of the process, θ . Identification is only possible by using time series information. However, the time series of the short rate is usually highly persistent and therefore time series estimates of θ are unstable and highly imprecise. This identification problem in Gaussian multi-factor specifications is also discussed in De Jong (2000), and Ball and Torous (1996) describe the unit root problem in time series estimation of term structure parameters.

Given the considerations above we use CIR-type factor processes. In the CIR model the expressions for the functions A and B for a specific factor and issuer are given by⁹

$$A(\delta, \tau) = \left[\frac{2\phi_1 \exp(\phi_2 \tau / 2)}{\phi_4} \right]^{\phi_3}, \quad (10)$$

$$B(\delta, \tau) = \frac{2[\exp(\phi_1 \tau) - 1]}{\phi_4}, \quad (11)$$

where $\phi_1 = \sqrt{(\kappa + \lambda)^2 + 2\sigma^2\delta}$, $\phi_2 = \kappa + \lambda + \phi_1$, $\phi_3 = 2\kappa\theta/\sigma^2$, $\phi_4 = 2\phi_1 + \phi_2[\exp(\phi_1 \tau) - 1]$, and δ is an element of the weight matrix D in Equation (7). The parameter λ denotes the market price of risk associated with the factor.

4. Model Estimation

4.1. RELATED LITERATURE

This section discusses some estimation issues and related empirical work. Duffie and Singleton (1997) estimate a two-factor model with independent CIR factors for interest rate swap yields. Duffie (1999) specifies and estimates a three-factor CIR model for corporate bond yields. The short rate is driven by two factors and the short spread by the same two factors plus an idiosyncratic factor. Thus, using exogenous common factors the model can capture some amount of correlation between short spreads of different issuers. Duffie, Pedersen, and Singleton (2003) use a fairly general affine model, which allows for dependence between the short rate and the short spread and stochastic volatility to model Russian sovereign debt. The model looks very promising with common factors for different kinds of sovereign bonds and idiosyncratic factors affecting only certain maturities. However

⁹ For notational convenience we suppress country and factor dependent indices in Equations (10) and (11).

due to data restrictions, the authors have to fix a good part of the model parameters prior to estimation and still hardly any parameter turns out to be significant. Düllmann and Windfuhr (2000) analyze CIR and Vasicek specifications for the spread between German and Italian government bonds in a univariate setting and conclude that no specification performs significantly better than the other in terms of pricing errors.

Econometric estimation of parameters of continuous time processes is a non-trivial issue. One of the advantages of using affine models is that there is a rich and well-developed literature on parameter estimation for such processes. The general problem is that the state variables, in our case the short rate and the short spread, are not observable. Simple approaches usually define proxies for the short rate (e.g., a short term money market rate) and the short spread and use standard time series methods to estimate the parameters under the empirical measure (see for instance Chan et al. 1992). In a second step the model can be calibrated to market prices to obtain the market price of risk parameters. However, there are serious drawbacks associated with this procedure. First, reliable price series of liquid instruments are usually not available for very short maturities. Additionally, even prices of instruments of very short maturity contain risk premia which may lead to substantially biased estimation results. Finally, this approach neglects the information which is available in the cross section of bond prices with different maturities.

There are more advanced methods which use both time series and cross sectional information. The first approach is due to Pearson and Sun (1994) and Chen and Scott (1993). In this setting, the K unobservable factors are measured by assuming that K bonds in the cross section are priced without error and then inverting the pricing relationship of the model. The results, of course, depend strongly on the choice of the 'input' bonds. Like other two-stage estimation procedures this approach suffers from an errors-in-variables problem. A different and technically more involved approach uses a state-space framework and Kalman filtering to jointly estimate the parameters of the process and the factor series. This approach has first been used by Chen and Scott (1995) and Geyer and Pichler (1999). A technical description of the method is provided in the following sub-section. The advantage of this approach is that it is consistent with the underlying theoretical model since no additional restrictions on the structure of observation errors are necessary. The same set of parameters that govern the dynamics of the factor processes also determines the term structure at any given point in time. The factors need not be specified in advance, but are extracted from the data.

4.2. THE STATE-SPACE APPROACH

To estimate parameters and to extract the unobservable state variables from yield spreads observed at discrete time intervals we use a state-space

formulation of the CIR model. The exact state-space formulation for an I -factor yield spread model with state-vector y_t is based on the assumption that y_0, y_1, \dots, y_t is a Markov process with $y_0 \sim p_0(y_0)$ and $y_t|y_{t-1} \sim p(y_t|y_{t-1})$. $p_0(y_0)$ is the density of the initial state and $p(y_t|y_{t-1})$ is the transition density.

It is known that the exact transition density for a I -factor CIR-model is the product of I non-central χ^2 densities (see Cox et al. 1985). Estimation of the unobservable state variables with an approximate Kalman filter in combination with quasi-maximum-likelihood (QML) estimation of the model parameters can be carried out by substituting the exact transition density by a normal density:

$$y_t|y_{t-1} \sim N(\mu_t, Q_t).$$

μ_t and Q_t are determined in such a way that the first two moments of the approximate normal and the exact transition density are equal. The elements of the I -dimensional vector μ_t are defined as

$$\mu_{t,i} = \theta_i[1 - \exp(-\kappa_i\Delta t)] + \exp(-\kappa_i\Delta t)Y_{t-1,i},$$

where Δt is a discrete time interval. Q_t is an $I \times I$ diagonal matrix with non-zero elements

$$Q_{t,i} = \sigma_i^2 \frac{1 - \exp(-\kappa_i\Delta t)}{\kappa_i} \left(\frac{\theta_i}{2} [1 - \exp(-\kappa_i\Delta t)] + \exp(-\kappa_i\Delta t) Y_{t-1,i} \right).$$

QML estimation involves an approximation error. Evidence obtained by Fröhwrth-Schnatter and Geyer (1998) and Brandt and He (2002) suggests that the bias of QML estimates increases as the number of factors increases. For models with one or two factors the deviations are rather small, however. The approximation errors in the present case would probably be similar because their analysis is done in a comparable setup.

Let $z_t = [Z(t, \tau_1), \dots, Z(t, \tau_N)]'$ be the N -dimensional vector of yield spreads of one particular issuer¹⁰ observed at time t , where τ_n is the n -th time to maturity. We assume that z_t are conditionally independent given y_{t-1} and z_t is independent of y_u , $\forall u \neq t$ given y_t with $z_t|y_t \sim p(z_t|y_t)$. The observation density $p(z_t|y_t)$ is based on the linear relation between observed yield spreads and the state variables. The measurement equation for observed yield spreads is

¹⁰ We drop issuer dependent indices for notational convenience.

$$z_t = a + by_t + \epsilon_t \quad \epsilon_t \sim NID(0, H) \quad t = 1, \dots, T,$$

where T is the number of observations. The n -th element of the $N \times 1$ vector a is defined as follows (see Equation (10)):

$$a_n(\tau_n) = \delta_0 - \sum_{i=1}^I \left[\frac{\phi_{i,3}}{\tau_n} \ln \left(\frac{2\phi_{i,1} \exp(\phi_{i,2}\tau_n/2)}{\phi_{i,4}} \right) \right], \quad n = 1, \dots, N. \quad (12)$$

A single element of the $N \times I$ matrix b is defined as (see Equation (11)):

$$b_{n,i}(\tau_n) = \frac{\delta_i 2[\exp(\phi_{i,1}\tau_n) - 1]}{\tau_n \phi_{i,4}}, \quad n = 1, \dots, N. \quad (13)$$

The matrix D in Equation (7) determines whether the weights δ_i in Equations (12) (δ_i is used in the definition of $\phi_{i,1}$) and (13) are estimated, or constrained to 0 or 1.

H is the $N \times N$ variance-covariance matrix of ϵ_t . In our empirical application we assume that H is a diagonal matrix, and the errors for each maturity have the same standard deviation h (i.e., all diagonal elements of H are identical).

This model is directly applicable to the Kalman filter recursion which can be briefly described as follows. For a given set of parameter values a prediction of the factors y_t is made at the beginning of time t based on the factor estimate from time $t-1$:

$$Y_{(t|t-1),i} = \theta_i[1 - \exp(-\kappa_i\Delta t)] + \exp(-\kappa_i\Delta t)Y_{(t-1|t-1),i}.$$

The vector of expected yield spreads conditional on time $t-1$ information is given by

$$\hat{z}_t = a + by_{t|t-1}. \quad (14)$$

When z_t is observed, an updated state estimate $y_{t|t}$ is computed such that the fit to observed yield spreads for the current date is optimal in the mean squared error sense. The prediction and updating sequence is repeated for each $t = 1, \dots, T$.

The Kalman filter provides all the necessary information to calculate the quasi log-likelihood (see Harvey (1989) p. 126)

$$\ln L = -0.5 \ln[2\pi(T-I)N] - 0.5 \sum_{t=I+1}^T \ln |F_t| - 0.5 \sum_{t=I+1}^T v_t' F_t^{-1} v_t,$$

where v_t are $N \times 1$ vectors of errors $z_t - \hat{z}_t$, \hat{z}_t is the vector of predicted yield spreads based on Equation (14) using the factor estimate before updating ($y_{t|t-1}$), and $F_t = b\Sigma_{t|t-1}b' + H$. $\Sigma_{t|t-1}$ is the variance-covariance matrix of the factor estimate conditional on $t-1$ (i.e., before updating) (for details see Harvey (1989) p. 106).

The CIR model differs from standard Kalman filter applications because of the non-negativity restriction on state variables. We modify the standard Kalman filter by simply replacing any negative element of the state estimate $y_{t|t-1}$ with zero. Therefore, in general, the Kalman filter is not strictly a linear estimator for the state variables, but it is linear for $y_t > 0$.

5. Empirical Analysis

We have estimated various specifications of the general model.¹¹ As a starting point we specify models with two independent factors, estimated separately for each country. It turns out that for each issuer there is one factor, termed the ‘global’ factor, which is very similar across issuers. The average correlation among the factor levels is 0.80. These global factors track the common trend in observed long term spreads quite accurately.

This finding of global factors justifies a model where spreads of all issuers are driven by one common factor and one country-specific factor. This global–local specification (GL) is best described in terms of the weight matrix

$$D_{GL} = \begin{pmatrix} \delta_{AT,1} & 1 & 0 & 0 & 0 \\ \delta_{BE,1} & 0 & 1 & 0 & 0 \\ \delta_{IT,1} & 0 & 0 & 1 & 0 \\ \delta_{SP,1} & 0 & 0 & 0 & 1 \end{pmatrix}.$$

Each issuer has a country-specific weight on the global factor Y_1 . This model has to be estimated jointly for all four countries. We are aware of only one paper in which common latent variables for multiple spread curves are estimated. Driessen (2002) applies a two-step procedure which allows the estimation of the common factor to be separated from the estimation of the residual factors. However, a comparison of the two-step procedure and the joint estimation strategy shows that the latter is by far superior in terms of residual pricing errors in the present sample. We impose the constraint $\delta_{AT,1} = 1$ for identifiability, which is a frequently used approach in latent factor models (see Bollen 1989).

¹¹ Detailed descriptions of results for alternative specifications and model comparisons are available from the authors.

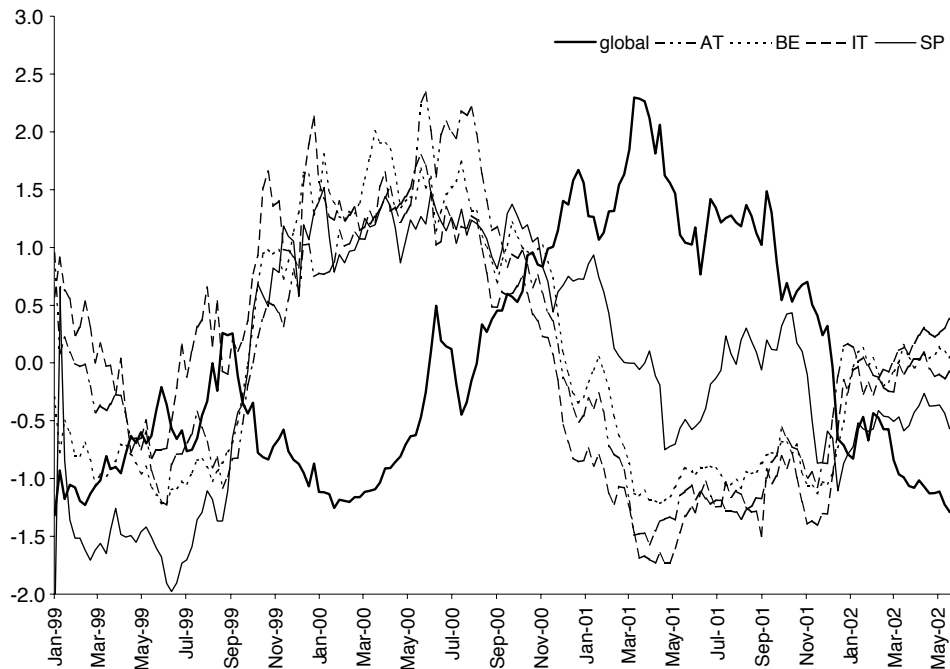


Figure 3. Specification global–local, time series of standardized factors.

Figure 3 shows time series plots of the resulting standardized factor time series. The global factor tracks the overall trend in spreads (see Figure 4). The striking feature of the graph is that the four local factors are rather similar, although they are supposed to capture idiosyncratic risks in terms of country-specific movements in spread structures. The average correlation among these country-specific factors is 0.76, which is nearly as high as the correlation among global factors from individual country models. In other words, fixing one factor to be common to all issuers does not prevent the second, country-specific factors from being again highly correlated. Clearly, the intention of separating systematic from idiosyncratic risk factors has not met with success. A tentative conclusion drawn from Figure 3 is that there is only systematic risk in EMU government bond spreads of the selected issuers, and country-specific variation is best captured by the Gaussian error terms in the model.

To recheck this conclusion we finally estimate a model with two global factors for all issuers. In this global–global specification (GG) spreads of all four issuers are driven by two common factors albeit with country-specific weights and country-specific constants. In this case the weight matrix is written as

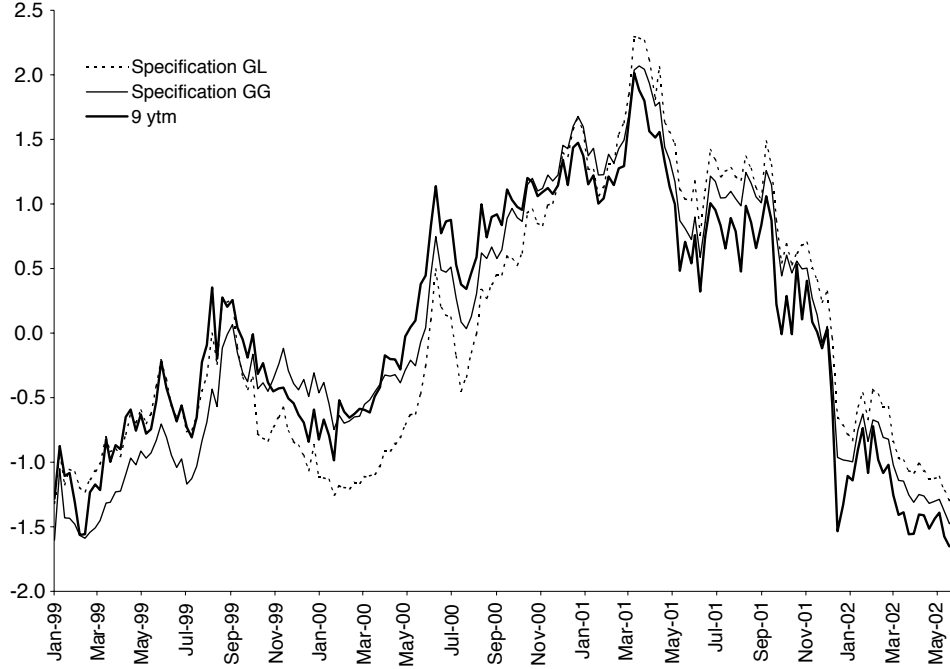


Figure 4. Time series of average nine years to maturity (ytm) spreads and standardized global factors from specifications GL and GG.

$$D_{GG} = \begin{pmatrix} \delta_{AT,1} & \delta_{AT,2} \\ \delta_{BE,1} & \delta_{BE,2} \\ \delta_{IT,1} & \delta_{IT,2} \\ \delta_{SP,1} & \delta_{SP,2} \end{pmatrix}.$$

Specification GG only captures systematic risk. Issuer-specific risk together with maturity specific risk is captured in the Gaussian error terms. This specification is clearly the most parsimonious one. However, in forward simulations the factors which drive model spreads of all issuers would be perfectly correlated. This can be disadvantageous in some applications. Obviously, this specification has to be estimated jointly for all issuers. For identifiability we impose the constraints $\delta_{AT,1} = 1$ and $\delta_{AT,2} = 1$.

Table II contains the parameter estimates and robust standard errors according to White (1982). Country-specific constants are insignificant, while country-specific factor weights are highly significant for all issuers. This suggests that multiplicative constants are sufficient to capture levels of short spreads. Further, it turns out that the first factor has an insignificant κ , which means that this factor is non-stationary under the empirical measure. As pointed out by Ball and Torous (1996), near unit root factor

dynamics do also influence the stability of the long term mean (θ_1) and the market price of risk of the factor process (λ_1). All other parameters are highly significant. Standard errors of residuals are around 1.5 *bp*. This is rather small compared to the average spread across issuers and maturities which is at about 20 *bp*.

The first factor of the GL specification is very similar to one of the factors in the GG specification, whereas the second factor of specification GG matches the local factors of specification GL. This can be interpreted as evidence that there is no local, or issuer-specific structure in EMU government bond spreads. To be more specific, there is no structure that can be captured by an affine factor, and country-specific variation in spreads is best modelled by country-specific Gaussian errors.

Table II. Specification GG, parameters and robust standard errors in parenthesis

κ_1	0.1156 (0.1821)
κ_2	0.1700** (0.0415)
σ_1	0.1896** (0.0704)
σ_2	0.2156** (0.0081)
λ_1	0.0000 (—)
λ_2	−0.5316** (0.0544)
θ_1	0.1513 (1.1741)
θ_2	0.2055** (0.0778)
$\delta_{AT,0}$	−0.5543 (0.5746)
$\delta_{BE,0}$	−0.1473 (0.1242)
$\delta_{IT,0}$	−0.2248 (0.2994)
$\delta_{SP,0}$	−0.1082 (0.1248)
$\delta_{AT,1}$	1 (—)
$\delta_{BE,1}$	0.2208** (0.0269)
$\delta_{IT,1}$	0.5036** (0.0215)
$\delta_{SP,1}$	0.2186** (0.0152)
$\delta_{AT,2}$	1 (—)
$\delta_{BE,2}$	0.6399** (0.0473)
$\delta_{IT,2}$	0.6942** (0.0323)
$\delta_{SP,2}$	0.4673** (0.0317)
h_{AT}	0.0135** (0.0005)
h_{BE}	0.0227** (0.0007)
h_{IT}	0.0218** (0.0009)
h_{SP}	0.0175** (0.0006)
$\ln L$	14143.88

* significance 10%.

** significance 5%.

To provide an interpretation of the factor time series Figure 4 shows the global factors of specifications GL and GG, as well as the average of the nine year spread across issuers. It becomes evident that the global factors track the overall level of long term spreads in the market. The correlations between global factors and average long term spreads are 0.91 (GL) and 0.96 (GG).

The situation is less clear with local factors which are shown in Figure 5. To provide an interpretation of the second factors we find that they are best associated with the shortest maturity spread in the sample. The average correlation between local factors and the average of the two year spreads is at 0.41. This suggests that the factors are maturity-specific rather than country-specific.

6. What Drives EMU Government Yield Spreads?

In this section, we examine the covariation between the global behaviour of EMU government yield spreads and some macroeconomic and financial variables that may be related with their causes. Since the objective of the paper is to measure systematic risk, we focus on the global factors resulting from the GG specification i.e., we use the long maturities factor ('long') and the short maturities factor ('short').

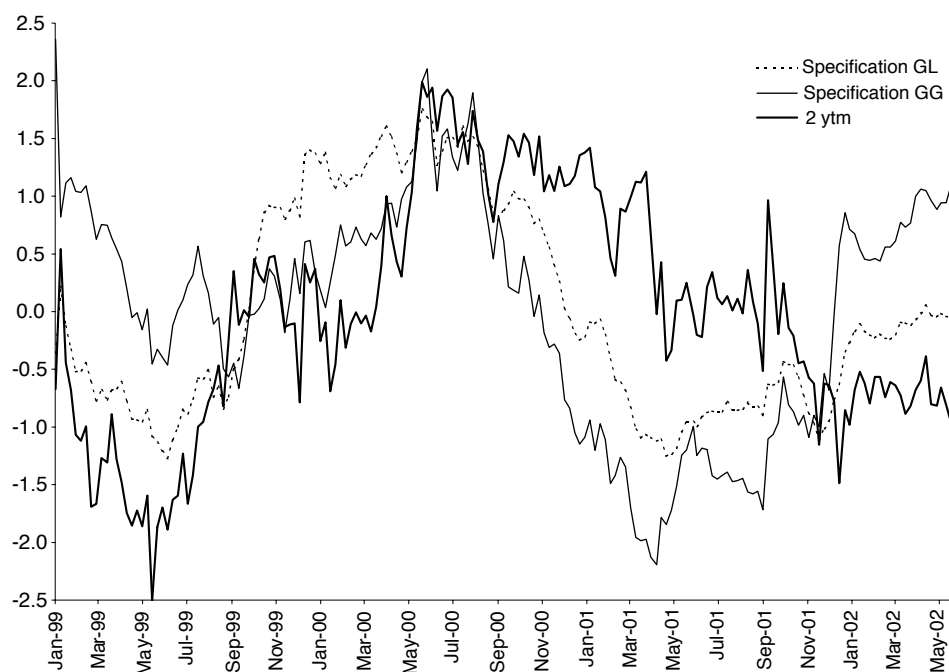


Figure 5. Time series of average two years to maturity (ytm) spreads, average standardized local factors from specification GL and the local factor from specification GG.

There is hardly any empirical or theoretical work on determinants of EMU bond spreads. One exception is the paper by Codogno, Favero, and Missale (2003) who use US corporate bond and swap spreads and debt-to-GDP ratios (as a measure of default risk) to explain the ten year spreads of all EMU issuers. Based on an interaction model they present weak evidence that the effects of shocks in the international bond markets (proxied by U.S. bond data) differ with default risk. Statistically significant results can be established for Austria, Italy and Spain, three of the four issuers in our sample. Codogno et al. (2003) conclude that default risk plays a small but important role in explaining EMU government bond spreads.

We use multiple regressions to analyze the covariation between spread factors and three types of explanatory variables: first, variables directly related to the probability of default such as business cycle variables and current account variables; second, two variables which measure credit risk premia in the corporate bond market; third, two measures of liquidity – size and specialness, both related to the German market.

The first set of explanatory variables is directly related to credit risk. The current account balance measures the difference of funds flowing in and out of an economy. A negative current account balance leads to additional financing needs and should therefore tend to widen spreads. Growth rates in industrial production are a measure of the state of the business cycle. Higher production growth is expected to reduce spreads. Another measure of the business climate and its prospects are economic sentiment indicators published by Eurostat. Again, the expectation of an improving business climate should lead to a tightening of spreads. For these variables, we construct differences between the average¹² of the four issuers and the reference issuer. This seems to be the appropriate approach since we have studied the spreads relative to German yields.

Next we consider proxies for credit risk. The difference between the implied yield to maturity of the Merrill Lynch EMU corporate bond index and the yield to maturity of a corresponding German Bund is used as a measure of corporate bond spreads. This variable can be interpreted as a proxy of the credit cycle or, more importantly, as a proxy for time-varying credit risk premia in the bond market. A second, related variable is the swap versus Bund spread, which is defined as the difference between quoted Euro swap rates (against six months Euribor) and Bund yields of corresponding maturity. Again, this variable is interpreted as a proxy for credit risk. However, both the corporate bond spread and the swap spread might incorporate liquidity effects as well.

Third, we include a measure of specialness in the reference term structure. There is well-established empirical evidence that benchmark and on-the-run

¹² We use the sum of the four issuers in case of the current account balance.

bonds are frequently traded ‘on special’ in the repo market, i.e., they trade at lower refinancing rates which in turn depresses their market yields (see for instance Duffie 1999). If repo specialness affects cash prices in the German Bund market, spreads are expected to increase and they could be correlated with Bund repo specials. In contrast to the U.S. markets only little evidence about specialness effects in the German Bund market is available. The only evidence we are aware of is Buraschi and Menini (2002) who report the existence of comparably large repo spreads for a variety of German Bunds and also of considerable specialness premia observed in the cash market. Time series data of German special repo rates are not publicly available. Given the results obtained by Buraschi and Menini (2002) and the evidence from the U.S. bond market we assume that on-the-run as well as benchmark issues are more likely to trade on special than other bonds. Therefore, we define a specialness measure as the difference between a zero coupon yield for a certain maturity estimated from on-the-run and benchmark bonds only, and the corresponding zero coupon yield estimated from all other available bonds. We presume that using the spread between the curves derived from these market segments might serve as a good proxy for the specialness premium contained in German bond prices.

Significant effects of the specialness variable could indicate that spread movements are due to specialness effects in the German term structure rather than the term structures of the other issuers. Alternatively, the specialness variable can also be interpreted as a liquidity measure, because it captures the premium of a “standard” bond over an on-the-run or benchmark bond. The spread structures of our specialness measure have a relatively stable shape over the whole sample period. They show a peak in level and volatility at medium maturities around four years and descend to around zero for longer maturities. We therefore use the four years to maturity specialness spread.

Since specialness is sufficient as an indicator of liquidity, but not necessary, we include size as another proxy for liquidity. We use a similar procedure as in the case of specialness. We separate German bonds into two groups based on issue size and fit yield curves to each group. The size proxy is defined as the difference between the curves at a maturity of four years as in the case of the specialness variable. We failed to obtain reliable data on other liquidity proxies like the bid-ask spread or trading volume.

All variables have been obtained from Datastream, except for the liquidity proxies which are constructed from our weekly data set described in Section 2. Macroeconomic variables are only available at a monthly frequency. Therefore they are kept constant during the weeks of the same month. To avoid spurious results we use first differences¹³ of all variables except industrial production growth.

¹³ Non-stationarity cannot be rejected at conventional significance levels.

Table III. The table reports the p -value for the estimated coefficients from a multiple regression analysis of the long and short spread factors resulting from specification GG on macroeconomic and financial variables. Signs refer to the signs of the estimated coefficients

Explanatory variables	Long	Short
Current account balance	0.9342	-0.5479
Industrial production growth	0.3109	-0.9231
Economic sentiment indicator	0.0933	-0.0577
EMU corporate bond spread	0.0001	0.0313
Euribor swap spread	0.0044	-0.0115
Specialness indicator	-0.4419	0.7357
Size indicator	-0.6095	-0.6856
R^2	0.17	0.07

Table III shows the results from multiple regressions.¹⁴ Both the corporate bond spread and the swap spread are highly significant, while the economic sentiment indicator is marginally significant. The long spread model explains roughly 17% of spread variation, the short spread model explains only 7%. The liquidity proxies and the macroeconomic variables are not found to be important determinants of the spread factors. We might need much longer time series to be able to measure the (potential) effects of these variables with sufficient accuracy.

Three main conclusions can be drawn from the regression analysis: first, only a small fraction of the variation of aggregate spreads can be explained. A similar result is obtained by Collin-Dufresne, Goldstein and Martin (2001) for U.S. corporate bond spreads and Boss and Scheicher (2002) for EMU corporate bond spreads. Collin-Dufresne et al. (2001) find that a common (latent) factor explains a large part of the variation of residuals from regressions of individual bond spreads on several explanatory variables, but they are unable to find financial or macroeconomic variables which explain the common factor; second, contrary to widely held beliefs, we find no evidence that spreads between EMU issuers can be explained by liquidity differences. Neither specialness nor size are related to the credit spread factors; third, our results provide firm evidence that corporate bond spreads and swap spreads are related to EMU government bond spreads. This can be

¹⁴ We have also run regressions using monthly data where monthly averages of financial variables are used to match the frequency of the macroeconomic variables. We find that the corporate bond spread is the only significant variable of the long spread factor, and no variable has a significant effect on the short spread factor. Given the relatively large number of explanatory variables compared to the small sample size these results have to be interpreted with caution.

interpreted as a sign of bond market integration, i.e., systematic risk in government and corporate bond spreads is driven by a common set of factors and their market risk premia are closely related. Assuming that corporate bond spreads are mainly driven by credit risk factors, these findings support the hypothesis that credit risk explains a substantial part of EMU government bond spreads.

7. Summary and Conclusions

We have analyzed the joint dynamics of yield spreads derived from government bonds issued by four member states of the EMU using the German Bund curve as the baseline. To uncover the dependence structure of factors driving EMU spreads, we jointly estimate multi-issuer spread models with linear restrictions across issuers in the factor structure. We use a state-space approach to estimate various specifications of affine multi-factor models.

For multi-issuer models with one or two global factors we obtain a surprisingly high degree of explanatory power even in a parsimonious model with two global factors but no local factors. We find strong support for the presence of a global factor that captures the level of long term yield spreads. A second global factor is related to short term yield spreads. This leads to the conclusion that the joint variation of EMU spreads is driven by a set of common factors, rather than country-specific ones.

To obtain some understanding of the economic determinants of the extracted spread factors we analyze their covariation with some macroeconomic and financial variables. We find no significant effects from issue size, German repo specials, and credit related macroeconomic variables (current account balance, industrial production growth and economic sentiment). However, the spread factors are related to Euro corporate bond spreads and swap spreads. Assuming that corporate bond spreads are mainly driven by credit risk factors, our findings support the hypothesis that credit risk explains a substantial part of EMU government bond spreads.

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