termstrc: A Package for Term Structure and Credit Spread Estimation with R

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January 2007

Basic principles of bond pricing

- coupon bond which matures in n years
- investor gets cashflows c_t at the times $t = 1, \ldots n$ (c_n includes the redemption payment)
- **clean price** p_c is quoted on the market
- seller also receives **accrued interest** for holding the bond over the period since the last coupon payment

$$a = \frac{\text{number of days since last coupon}}{\text{number of days in current coupon period}} C$$

- investor has to pay the **dirty price** p_d
- bond pricing equation with continuous compounding

$$p_c + a = \sum_{t=1}^{n} c_t e^{-s_t m_t}$$

Basic principles of bond pricing

- **yield to maturity**

$$p_c + a = \sum_{t=1}^{n} c_t e^{-y m_t}$$

- equivalent formulation of the bond price equation uses the **discount factors** $d_t = \delta(m_t) = e^{-s_t m_t}$
- continuous **discount function** $\delta(\cdot)$ is formed by interpolation of the discount factors

$$p_c + a = \sum_{t=1}^{n} c_t \delta(m_t)$$

Term structure and credit spread estimation

- estimate zero-coupon yield curves and credit spread curves from market data
- usual way for calculation of **credit spread curves**

$$cs_j(\boldsymbol{m}) = s_j(\boldsymbol{m}, \boldsymbol{b}) - s_{ref}(\boldsymbol{m}, \boldsymbol{b})$$

$cs_j(\boldsymbol{m})$	credit-spread between country j and reference country ref
$s_j(\boldsymbol{m}, \boldsymbol{b})$	spot-rate curve of country j with maturity vector $\boldsymbol{m}$
$s_{ref}(\boldsymbol{m}, \boldsymbol{b})$	spot-rate curve of the reference country

Nelson and Siegel (1987) approach

Instantaneous forward rates

$$f(m,\boldsymbol{b}) = \beta_0 + \beta_1 \exp(-\frac{m}{\tau_1}) + \beta_2 \frac{m}{\tau_1} \exp(-\frac{m}{\tau_1})$$

Spot rates

$$s(m,\boldsymbol{b}) = \beta_0 + \beta_1 \frac{1-\exp(-\frac{m}{\tau_1})}{\frac{m}{\tau_1}} + \beta_2 \left(\frac{1-\exp(-\frac{m}{\tau_1})}{\frac{m}{\tau_1}} - \exp(-\frac{m}{\tau_1}) \right)$$

Svensson (1994) approach

- Svensson (1994) extended the functional form by two additional parameters which allows for a second hump-shape

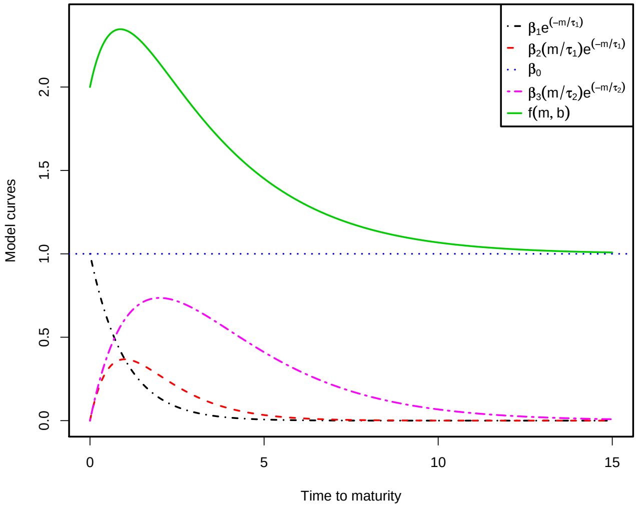
Instantaneous forward rates

$$f(m,\boldsymbol{b}) = \beta_0 + \beta_1 \exp(-\frac{m}{\tau_1}) + \beta_2 \frac{m}{\tau_1} \exp(-\frac{m}{\tau_1}) + \beta_3 \frac{m}{\tau_2} \exp(-\frac{m}{\tau_2})$$

Spot rates

$$\begin{aligned} s(m,\boldsymbol{b}) = \beta_0 + \beta_1 \frac{1-\exp(-\frac{m}{\tau_1})}{\frac{m}{\tau_1}} + \beta_2 \left(\frac{1-\exp(-\frac{m}{\tau_1})}{\frac{m}{\tau_1}} - \exp(-\frac{m}{\tau_1}) \right) \\ + \beta_3 \left(\frac{1-\exp(-\frac{m}{\tau_2})}{\frac{m}{\tau_2}} - \exp(-\frac{m}{\tau_2}) \right) \end{aligned}$$

Decomposition of the Svensson forward rate function



Term structure estimation procedure
Notation I

Maturity matrix $\boldsymbol{M}$

$$\boldsymbol{M}_{[n \times m]} = \{m_{ij}\}$$

Cashflow matrix $\boldsymbol{C}$

$$\boldsymbol{C}_{[n \times m]} = \{c_{ij}\}$$

Discount factor matrix $\boldsymbol{D}$

$$\boldsymbol{D}_{[n \times m]} = \{d_{ij}\}; \qquad d_{ij} = e^{-m_{ij} s(m_{ij}, \boldsymbol{b})}$$

Clean price vector $\boldsymbol{p}^c$

$$\boldsymbol{p}^c_{[1 \times m]} = \{p^c_j\}$$

Term structure estimation procedure
Notation II

Accrued interest vector $\boldsymbol{a}$

$$\boldsymbol{a}_{[1\times m]} = \{a_j\}$$

Dirty price vector $\boldsymbol{p}^d$

$$\boldsymbol{p}^d_{[1\times m]} = \{p^d_j\}$$

$$\boldsymbol{p}^d = \boldsymbol{p}^c + \boldsymbol{a}$$

Weights vector $\boldsymbol{w}$

$$\boldsymbol{w}_{[1\times m]} = \{w_j\}; \qquad w_j = \frac{\frac{1}{D_j}}{\sum_{i=1}^{m} \frac{1}{D_i}}$$

Objective function

- Minimization of the weighted pricing or yield errors

Objective function

$$\boldsymbol{b}_{opt} = \min_{b} \left(\left(\boldsymbol{\iota}_{[1\times n]} \left[\boldsymbol{C} \cdot \boldsymbol{D} \right] - \boldsymbol{p}^{d} \right)^{2} \boldsymbol{w} \boldsymbol{\iota}_{[m\times 1]} \right)$$

- The parameter vector is subject to constraints ($\beta_0 > 0, \tau_1 > 0, \tau_2 > 0$)

Examples

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