termstrc: A Package for Term Structure and Credit Spread Estimation with R

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Basic principles of bond pricing

- coupon bond which matures in n years
- investor gets cashflows c_t at the times $t = 1, \dots n$ (c_n includes the redemption payment)
- **clean price** p_c is quoted on the market
- seller also receives **accrued interest** for holding the bond over the period since the last coupon payment

$$a = \frac{\text{number of days since last coupon}}{\text{number of days in current coupon period}} C$$

- investor has to pay the **dirty price** p_d
- bond pricing equation with continuous compounding

$$p_c + a = \sum_{t=1}^{n} c_t e^{-s_t m_t}$$

Basic principles of bond pricing

- **yield to maturity**

$$p_c + a = \sum_{t=1}^{n} c_t e^{-y m_t}$$

- equivalent formulation of the bond price equation uses the **discount factors** $d_t = \delta(m_t) = e^{-s_t m_t}$
- continuous **discount function** $\delta(\cdot)$ is formed by interpolation of the discount factors

$$p_c + a = \sum_{t=1}^{n} c_t \delta(m_t)$$

- implied j-period **forward rate**

$$f_{t|j} = \frac{j s_j - t s_t}{j - t}$$

Term structure and credit spread estimation

- estimate zero-coupon yield curves and credit spread curves from market data
- usual way for calculation of **credit spread curves**

$$cs_j(\boldsymbol{m}) = s_j(\boldsymbol{m}, \boldsymbol{b}) - s_{ref}(\boldsymbol{m}, \boldsymbol{b})$$

$cs_j(\boldsymbol{m})$	credit-spread between country j and reference country ref
$s_j(\boldsymbol{m}, \boldsymbol{b})$	spot-rate curve of country j with maturity vector $\boldsymbol{m}$
$s_{ref}(\boldsymbol{m}, \boldsymbol{b})$	spot-rate curve of the reference country

Nelson and Siegel (1987) approach

Instantaneous forward rates

$$f(m,\boldsymbol{b}) = \beta_0 + \beta_1 \exp(-\frac{m}{\tau_1}) + \beta_2 \frac{m}{\tau_1} \exp(-\frac{m}{\tau_1})$$

Spot rates

$$s(m,\boldsymbol{b}) = \beta_0 + \beta_1 \frac{1-\exp(-\frac{m}{\tau_1})}{\frac{m}{\tau_1}} + \beta_2 \left(\frac{1-\exp(-\frac{m}{\tau_1})}{\frac{m}{\tau_1}} - \exp(-\frac{m}{\tau_1}) \right)$$

Svensson (1994) Approach

- Svensson (1994) extended the functional form by two additional parameters which allows for a second hump-shape

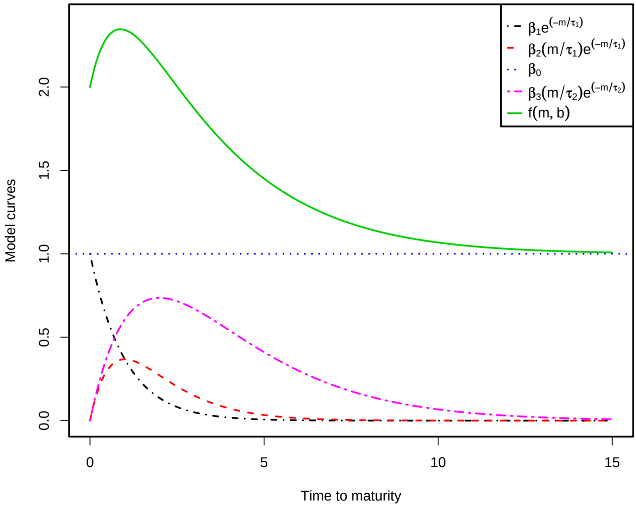
Instantaneous forward rates

$$f(m,\boldsymbol{b}) = \beta_0 + \beta_1 \exp(-\frac{m}{\tau_1}) + \beta_2 \frac{m}{\tau_1} \exp(-\frac{m}{\tau_1}) + \beta_3 \frac{m}{\tau_2} \exp(-\frac{m}{\tau_2})$$

Spot rates

$$s(m,\boldsymbol{b}) = \beta_0 + \beta_1 \frac{1-\exp(-\frac{m}{\tau_1})}{\frac{m}{\tau_1}} + \beta_2 \left(\frac{1-\exp(-\frac{m}{\tau_1})}{\frac{m}{\tau_1}} - \exp(-\frac{m}{\tau_1})\right) + \beta_3 \left(\frac{1-\exp(-\frac{m}{\tau_2})}{\frac{m}{\tau_2}} - \exp(-\frac{m}{\tau_2})\right)$$

Decomposition of the Svensson forward rate function



Term structure estimation procedure
Notation I

Maturity matrix $\boldsymbol{M}$

$$\boldsymbol{M}_{[n \times m]} = \{m_{ij}\}$$

Cashflow matrix $\boldsymbol{C}$

$$\boldsymbol{C}_{[n \times m]} = \{c_{ij}\}$$

Discount factor matrix $\boldsymbol{D}$

$$\boldsymbol{D}_{[n \times m]} = \{d_{ij}\}; \qquad d_{ij} = e^{-m_{ij} s(m_{ij}, \boldsymbol{b})}$$

Clean price vector $\boldsymbol{p}^c$

$$\boldsymbol{p}^c_{[1 \times m]} = \{p^c_j\}$$

Term structure estimation procedure
Notation II

Accrued interest vector $\boldsymbol{a}$

$$\boldsymbol{a}_{[1\times m]} = \{a_j\}$$

Cashflow matrix $\boldsymbol{C}$

$$\boldsymbol{C}_{[n\times m]} = \{c_{ij}\}$$

Dirty price vector $\boldsymbol{p}^d$

$$\boldsymbol{p}^d_{[1\times m]} = \{p^d_j\}$$

$$\boldsymbol{p}^d = \boldsymbol{p}^c + \boldsymbol{a}$$

Weights vector $\boldsymbol{w}$

$$\boldsymbol{w}_{[1\times m]} = \{w_j\}; \qquad w_j = \frac{\frac{1}{D_j}}{\sum_{i=1}^{m} \frac{1}{D_i}}$$

Term structure estimation procedure
Objective function

- Minimization of the weighted pricing or yield errors

Objective function

$$\boldsymbol{b}_{opt} = \min_{b} \left(\left(\boldsymbol{\iota}_{[1 \times n]} \left[\boldsymbol{C} \cdot \boldsymbol{D} \right] - \boldsymbol{p}^{d} \right)^{2} \boldsymbol{w} \boldsymbol{\iota}_{[m \times 1]} \right)$$

- The parameter vector is subject to constraints $(\beta_0 > 0, \tau_1 > 0, \tau_2 > 0)$

Examples

References I

Bank for International Settlements
Zero-coupon yield curves: technical documentation
BIS Papers, No. 25, October 2005

David Bolder, David Streliski
Yield Curve Modelling at the Bank of Canada
Bank of Canada, Technical Report, No. 84, 1999

Alois Geyer, Richard Mader
Estimation of the Term Structure of Interest Rates - A Parametric Approach
OeNB, Working Paper, No. 37, 1999

References II

Rainer Jankowitsch, Stefan Pichler
Parsimonious Estimation of Credit Spreads
The Journal of Fixed Income, 14(3):49–63, 2004

Charles R. Nelson, Andrew F. Siegel
Parsimonious Modeling of Yield Curves
The Journal of Business, 60(4):473–489, 1987

Lars E.O. Svensson
Estimating and Interpreting Forward Interest Rates: Sweden 1992 -1994
National Bureau of Economic Research, Technical Report, No. 4871, 1994