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Term Structure Modeling Using Exponential Splines

OLDRICH A. VASICEK and H. GIFFORD FONG*

I. Introduction

TERM STRUCTURE OF interest rates provides a characterization of interest rates as a function of maturity. It facilitates the analysis of rates and yields such as discussed in Dobson, Sutch, and Vanderford [1976], and provides the basis for investigation of portfolio returns as for example in Fisher and Weil [1971]. Term structure can be used in pricing of fixed-income securities (cf., for instance, Houglet [1980]), and for valuation of futures contracts and contigent claims, as in Brennan and Schwartz [1977]. It finds applications in analysis of the effect of taxation on bond yields (cf. McCulloch [1975a] and Schaefer [1981]), estimation of liquidity premia (cf. McCulloch [1975b]), and assessment of the accuracy of market-implicit forecasts (Fama [1976]). Because of its numerous uses, estimation of the term structure has received considerable attention from researchers and practitioners alike.

A number of theoretical equilibrium models has been proposed in the recent past to describe the term structure of interest rates, such as Brennan and Schwartz [1979], Cox, Ingersoll, and Ross [1981], Langetieg [1980], and Vasicek [1977]. These models postulate alternative assumptions about the nature of the stochastic process driving interest rates, and deduct a characterization of the term structure implied by these assumptions in an efficiently operating market. The resulting spot rate curves have a specific functional form dependent only on a few parameters.

Unfortunately, the spot rate curves derived by these models (at least in the instances when it was possible to obtain explicit formulas) do not conform well to the observed data on bond yields and prices. Typically, actual yield curves exhibit more varied shapes than those justified by the equilibrium models. It is undoubtedly a question of time until a sufficiently rich theoretical model is proposed that provides a good fit to the data. For the time being, however, empirical fitting of the term structure is very much an unrelated task to investigations of equilibrium bond markets.

The objective in empirical estimation of the term structure is to fit a spot rate curve (or any other equivalent description of the term structure, such as the discount function) that (1) fits the data sufficiently well, and (2) is a sufficiently smooth function. The second requirement, being less quantifiable than the first,

* Gifford Fong Associates

is less often stated. It is nevertheless at least as important as the first, particularly since it is possible to achieve an arbitrary good (or even perfect) fit if the empirical model is given enough degrees of freedom, with the consequence that the resulting term structure makes little sense. For a discussion of this point, see Langetieg and Smoot [1981].

A simple approach to estimation of the term structure is to postulate that bond payments occur only on a discrete set of specified dates, and assume no relationship among the discount factors corresponding to these dates (such as that they lie on a smooth curve). The discount factors can then be estimated as the coefficients in a regression with the bond payments on the given set of dates as the independent variables, and the bond price as the dependent variable. This approach has been taken by Carleton and Cooper [1976]. They include both U.S. Treasury and Federal Home Loan Bank securities in the estimation, with an adjustment for the default risk in the FHLB bonds. The resulting discount function is discrete rather than continuous, and the forward rates are found not to be smooth.

McCulloch [1971] introduced the methodology of fitting the discount function by polynomial splines. This produces estimates of the discount function as a continuous function of time. For cubic or higher order splines, the forward rates are a smooth function. Since the model is linear in the discount function, ordinary least-squares regression techniques can be used.

In addressing the effect of taxation, McCulloch [1975a] estimates the after-tax term structure of interest rates and the marginal income tax rate. Estimates of the tax rate were achieved by minimizing the standard error of the regression. This estimated tax rate is used to convert the after-tax term structure into a before-tax term structure. This procedure makes the estimated forward rates very sensitive to any estimation errors in the tax rate. Moreover, because the tax effect is estimated by best fitting to minimize large errors, the inclusion of special securities such as flower bonds tends to prejudice the results.

Langetieg and Smoot [1981] discuss extensions of McCulloch's spline methodology. These include fitting cubic splines to the spot rates rather than the discount function, and varying the location of the spline knots. Non-linear estimation procedures are required in these models.

This paper presents a different approach, which can be termed an *exponential spline fitting*. The methodology described here has been applied to historical price data on U.S. Treasury securities with satisfactory results. The technique produces forward rates that are a smooth continuous function of time. The model has desirable asymptotic properties for long maturities, and exhibits both a sufficient flexibility to fit a wide variety of shapes of the term structure, and a sufficient robustness to produce stable forward rate curves. An adjustment for the effect of taxes and for call features on U.S. Treasury bonds is included in the model.

In the next section, we provide a brief description of the basic concepts of the term structure, such as spot and forward rates, market-implicit forecasts, and the discount function. This provides some background for understanding some of the prior work, and of the model to be proposed in the last section.

II. Concepts and Terms

The spot interest rate of a given maturity is defined as the yield on a pure discount bond of that maturity. The spot rates are the discount rates determining the present value of a unit payment at a given time in the future. Spot rates considered as a function of maturity are referred to as the *term structure of interest rates.*

Spot rates are not directly observable, since there are few pure discount bonds beyond maturities of one year. They have to be estimated from the yields on actual securities by means of a *term structure model.* Each actual coupon bond can be considered a package of discount bonds, namely one for each of the coupon payments and one for the principal payment. The price of such component discount bonds is equal to the amount of the payment discounted by the spot rate of the maturity corresponding to this payment. The price of the coupon bond is then the sum of the prices of these component discount bonds. The *yield to maturity* on a coupon bond is the internal rate of return on the bond payments, or the discount rate that would equate the present value of the payments to the bond price. It is seen that the yield is thus a mixture of spot rates of various maturities. In calculation of yield, each bond payment is discounted by the same rate, rather than by the spot rate corresponding to the maturity of that payment. Decomposing the actual yields on coupon bonds into the spot rates is the principal task of a term structure model.

Spot rates describe the term structure by specifying the current interest rate of any given maturity. The implications of the current spot rates for future rates can be described in terms of the *forward rates.* The forward rates are one-period future reinvestment rates, implied by the current term structure of spot rates.

Mathematically, if R_1, R_2, R_3 $\cdots$ are the current spot rates, the forward rate F_t for period t is given by the equation

$$1 + F_t = \frac{(1 + R_t)^t}{(1 + R_{t-1})}, \qquad t = 1, 2, 3, \cdots. \tag{1}$$

This equation means that the forward rate for a given period in the future is the marginal rate of return from committing an investment in a discount bond for one more period. By definition, the forward rate for the first period is equal to the one period spot rate, $F_1 = R_1$.

The relationship of spot and forward rates described by Equation (1) can be stated in the following equivalent form:

$$(1 + R_t)^t = (1 + F_1)(1 + F_2) \cdots (1 + F_t). \tag{2}$$

This equation shows that spot rates are obtained by compounding the forward rates over the term of the spot rate. Thus, the forward rate F_t can be interpreted as the interest rate over the period from $t - 1$ to t that is implicit in the current structure of spot rates.

Just as the forward rates are determined by the spot rates using Equation (1), the spot rates can be obtained from the forward rates by Equation (2). Thus, either the spot rates or the forward rates can be taken as alternative forms of describing the term structure. The choice depends on which of these two equivalent characterizations is more convenient for the given purpose. Spot rates describe interest rates over periods from the current date to a given future date. Forward rates describe interest rates over one-period intervals in the future.

There is a third way of characterizing the term structure, namely by means of the *discount function.* The discount function specifies the present value of a unit payment in the future. It is thus the price of a pure discount riskless bond of a given maturity. The discount function D_t is related to the spot rates by the equation

$$D_t = \frac{1}{(1 + R_t)^t} \tag{3}$$

and to the forward rates by the equation

$$D_t = \frac{1}{(1 + F_1)(1 + F_2) \cdots (1 + F_t)}. \tag{4}$$

The discount function D_t considered in continuous time t is a smooth curve decreasing from the starting value $D_0 = 1$ for $t = 0$ (since the value of one dollar now is one dollar) to zero for longer and longer maturities. It typically has an exponential shape.

While the discount function is usually more difficult to interpret as a description of the structure of interest rates than either the spot rates or the forward rates, it is useful in the *estimation* of the term structure from bond prices. The reason is that bond prices can be expressed in a very simple way in terms of the discount function, namely the sum of the payments multiplied by their present value. In terms of the spot or forward rates, bond prices are a more complicated (nonlinear) function of the values of the rates to be estimated.

The concept of forward rates is closely related to that of the *market-implicit forecasts.* The market-implicit forecast $M_{t,s}$ of a rate of maturity s as of a given future date t is the rate that would equate the total return from an investment at the spot rate R_t for t periods reinvested at the rate $M_{t,s}$ for additional s periods, with the straight investment for $t + s$ periods at the current spot rate R_{t+s}. Mathematically, this can be written as follows:

$$(1 + R_t)^t(1 + M_{t,s})^s = (1 + R_{t+s})^{t+s}. \tag{5}$$

The market-implicit forecasts can be viewed as a forecast of future spot rates by the aggregate of market participants. Suppose that the current one-year rate is 12%, and that there is a general agreement among investors that the one-year rate a year from now will be 13%. Then the current two-year spot rate will be 12.50%, since

$$(1 + .1250)^2 = (1 + .12)(1 + .13).$$

The two-year rate would be set in such a way that the two-year security has the same return as rolling over a one-year security for two years. There may not be such a general agreement as to the future rate, and in any case the forecast would not be directly observable. Knowing the current one-year and two-year spot rates, however, enables us to determine the future rate for the second year that would make the two-year bond equivalent in terms of total return to a roll-over of one-year bond. This rate is the market-implicit forecast.

The market-implicit forecasts have a number of interesting properties. The first thing to note is that when a *futures contract* is available for a given future period, the rate on the futures contract is equal to the market-implicit forecast (up to a difference attributable to transaction costs). If this were not true, a riskless arbitrage can be set between a portfolio consisting of the futures contract and a security maturing at the execution date on one hand, and a security maturing at the maturity date of the contract on the other hand. Such riskless arbitrage opportunities should not exist in efficient financial markets.

Another feature of market-implicit forecasts is that the *holding period return* calculated using these forecasts is the same for any default-free security, regardless of its maturity. It is equal to the spot rate corresponding to the length of the holding period. Indeed, the total return over a holding period of length h on an issue with maturity s ($s > h$) is equal to

$$\frac{(1 + R_s)^s}{(1 + M_{h,s-h})^{s-h}}.$$

Recalling the definition of the market-implicit forecast in Equation (5), the total return over the holding period is readily calculated as

$$\frac{(1 + R_s)^s}{(1 + M_{h,s-h})^{s-h}} = \frac{(1 + R_s)^s(1 + R_h)^h}{(1 + R_s)^s} = (1 + R_h)^h.$$

Thus, the holding period return is independent of the maturity of the security, and is given by the spot rate for the holding period.

This is a characterization of the market-implicit forecasts that can actually serve as their definition. No other set of forecasts would have the property that the holding period returns over a given period are the same for securities of all maturities (including coupon bonds). In a sense, the market-implicit forecast is the most "neutral" forecast. It is the equilibrium expectation such that no maturities or payment schedules are ex-ante preferred to others.

The definition of the market-implicit forecasts as given by Equation (5) is perhaps more intuitive if stated in terms of the forward rates. It is given by the following equation:

$$(1 + M_{t,s})^s = (1 + F_{t+1})(1 + F_{t+2}) \cdots (1 + F_{t+s}). \tag{6}$$

Specifically, the market-implicit forecast of one-period rate is equal to the forward rate for that period,

$$M_{t,1} = F_t.$$

It is seen from Equation (6) that the market-implicit forecast is obtained by compounding the forward rates over the period starting at the date of the forecasting horizon and extending for an interval corresponding to the term of the forecasted rate. In other words, the market-implicit forecast corresponds to the scenario of *no change in the forward rates*. The current spot rates then change by rolling along the forward rate series.

One last thing to mention about the market-implicit forecasts is that since it is a forecast of the future spot rates, we can also infer from it the corresponding forecast of yields, discount functions, and all other characterizations of the *future term structure.* The current and future term structures have the forward rates as the one common denominator, which makes the forward rates the basic building blocks of the structure of interest rates.

III. The Model

In specification of the model proposed for estimation of the term structure, we will use the following notation:

t time to payment (measured in half years)

$D(t)$ the discount function, that is, the present value of a unit payment due in time t

$R(t)$ spot rate of maturity t, expressed as the continuously compounded semi-annual rate. The spot rates are related to the discount function by the equation

$$D(t) = e^{-tR(t)}$$

$F(t)$ continuously compounded instantaneous forward rate at time t. The forward rates are related to the spot rate by the equation

$$R(t) = \frac{-d}{dt} \log D(t).$$

n number of bonds used in estimation of the term structure

T_k time to maturity of the k-th bond, measured in half years

C_k the semi-annual coupon rate of the k-th bond, expressed as a fraction of the par value

P_k price of the k-th bond, expressed as a fraction of the par value.

The basic model can be written in the following form:

$$P_k + A_k = D(T_k) + \sum_{j=1}^{L_k} C_k D(T_k - j + 1) - Q_k - W_k + \epsilon_k \qquad k = 1, 2, \cdots, n \tag{7}$$

where

$$A_k = C_k(L_k - T_k)$$

is the accrued interest portion of the market value of the k-th bond,

$$L_k = [T_k] + 1$$

is the number of coupon payments to be received, Q_k is the price discount attributed to the effect of taxes, W_k is the price discount due to call features, and ϵ_k is a residual error with $E\epsilon_k = 0$.

The model specified by Equation (7) is expressed in terms of the discount function, rather than the spot or forward rates. The reason for this specification is that the price of a given bond is linear in the discount function, while it is nonlinear in either the spot or forward rates. Once the discount function is estimated, the spot and forward rates can easily be calculated.

An integral part of the model specification is a characterization of the structure of the residuals. We will postulate that the model be *homoscedastic in yields*, rather than in prices. This means that the variance of the residual error on yields is the same for all bonds. The reason for this requirement is that a given price increment, say \$1 per \$100 face value, has a very different effect on a short bond than on a long bond. Obviously, an error term in price on a three-month Treasury bill cannot have the same magnitude as that in price of a twenty-year bond. It is, however, reasonable to assume that the magnitude of the error term would be the same for yields.

With this assumption, the residual variance in Equation (7) is given as

$$E\epsilon_k^2 = \sigma^2\omega_k, \qquad k = 1, 2, \cdots, n \tag{8}$$

where

$$\omega_k = \left(\frac{dP}{dY}\right)_k^2 \tag{9}$$

is the squared derivative of price with respect to yield for the k-th bond, taken at the current value of yield. The derivative dP/dY can easily be evaluated from time to maturity, the coupon rate, and the present yield. In addition, we will assume that the residuals for different bonds are uncorrelated,

$$E\epsilon_k\epsilon_\ell = 0, \qquad \text{for } k \neq \ell.$$

In specification of the effect of taxes, we will assume that the term Q_k is proportional to the current yield C_k/P_k on the bond,

$$Q_k = q\,\frac{C_k}{P_k}\left(\frac{dP}{dY}\right)_k, \qquad k = 1, 2, \cdots, n. \tag{10}$$

For the call effect, the simplest specification is to introduce a dummy variable I_k, equal to 1 for callable bonds and to 0 for noncallable bonds, and put

$$W_k = wI_k, \qquad k = 1, 2, \cdots, n. \tag{11}$$

Although more complicated specifications (such as those based on option pricing) are possible, the form (11) seems to work well with Treasury bonds, which invariably have the same structure of calls five years prior to maturity at par.

We will now turn to the specification of the discount function $D(t)$. Earlier approaches (cf. McCulloch [1971], [1975b]) fit the discount function by means of polynomial splines of the second or third order. While splines constitute a very flexible family of curves, there are several drawbacks to their use in fitting discount functions. The discount function is principally of an exponential shape,

$$D(t) \sim e^{-\gamma t}, \qquad 0 \leq t < \infty.$$

Splines, being piecewise polynomials, are inherently ill suited to fit an exponential type curve. Polynomials have a different curvature from exponentials, and although a polynomial spline can be forced to be arbitrarily close to an exponential curve by choosing a sufficiently large number of knot points, the local fit is not good. A practical manifestation of this phenomenon is that a polynomial spline tends to "weave" around the exponential, resulting in highly unstable forward rates (which are the derivatives of the logarithm of the discount function). Another problem with polynomial splines is their undesirable asymptotic properties. Polynomial splines cannot be forced to tail off in an exponential form with increasing maturities.

It would be convenient if we can work with the logarithm $\log D(t)$ of the discount function, which is essentially a straight line and can be fitted very well with splines. Unfortunately, the model given by Equation (7) would then be nonlinear in the transformed function, which necessitates the use of complicated nonlinear estimation techniques (cf. Langetieg and Smoot [1981]).

A way out of this dilemma is provided by the following approach, which is used in our model. Instead of using a transform of the function $D(t)$, we can apply a transform to the *argument* of the function. Let α be some constant and put

$$t = -\frac{1}{\alpha}\log(1-x), \qquad 0 \leq x < 1. \tag{12}$$

Then $G(x)$ defined by

$$D(t) = D\left(-\frac{1}{\alpha}\log(1-x)\right) \equiv G(x) \tag{13}$$

is a new function with the following properties: (a) $G(x)$ is a decreasing function defined on the finite interval $0 \leq x \leq 1$ with $G(0) = 1$, $G(1) = 0$; (b) to the extent that $D(t)$ is approximately exponential,

$$D(t) \sim e^{-\gamma t}, \qquad 0 \leq t < \infty$$

the function $G(x)$ is approximately a power function,

$$G(x) \sim (1-x)^{\gamma/\alpha} \qquad 0 \leq x \leq 1;$$

(c) the model specified by Equation (7) is linear in G. Thus, we have replaced the function $D(t)$ to be estimated by the approximately power function $G(x)$ which can be very well fitted by polynomial splines, while preserving the linearity of the model. Moreover, desired asymptotic properties can easily be enforced.

If $G(x)$ is polynomial with $G'(1) \neq 0$, then the parameter α constitutes the *limiting value of the forward rates*,

$$\lim_{t\to\infty} F(t) = \alpha.$$

Indeed, in that case

$$G(x) = -G'(1)(1-x) + o(1-x)$$

and consequently

$$D(t) = -G'(1)\, e^{-\alpha t} + o(e^{-\alpha t})$$

as $t \to \infty$. Using polynomial splines to fit the function $G(x)$ will thus assure the desired convergence of the forward rates. The limiting value α can be fitted to the data together with the other estimation parameters.

Let $g_i(x)$, $0 \leq x \leq 1$, $i = 1, 2, \cdots, m$ be a base of a polynomial spline space. Any spline in this space can be expressed as a linear combination of the base. If $G(x)$

is fitted by a function from this space,

$$G(x) = \textstyle\sum_{i=1}^{m} \beta_i g_i(x), \qquad 0 \leq x \leq 1, \tag{14}$$

the model of Equation (7) can be written as

$$P_k + A_k = \sum_{i=1}^{m} \beta_i \Big(g_i(X_{k1}) + \sum_{j=1}^{L_k} C_k g_i(X_{kj})\Big) - q\,\frac{C_k}{P_k}\left(\frac{dP}{dY}\right)_k - wI_k + \epsilon_k, \tag{15}$$

$$E\epsilon_k = 0, \qquad E\epsilon_k^2 = \sigma^2\omega_k, \qquad E\epsilon_k\epsilon_\ell = 0 \text{ for } k \neq \ell$$

where

$$X_{kj} = 1 - e^{-\alpha(T_k - j + 1)}, \qquad j = 1, 2, \cdots, L_k.$$

The model described by Equation (15) is used in the estimation of the term structure. It is linear in the parameters $\beta_1, \beta_2, \cdots, \beta_m, q, w$, with residual covariance matrix proportional to

$$\Omega = \begin{vmatrix} \omega_1 & & & & & \\ & \omega_2 & & & & \\ & & \cdot & & & \\ & & & \cdot & & \\ & & & & \cdot & \\ & & & & & \cdot \\ & & & & & & \omega_n \end{vmatrix}.$$

If we write

$$U_k = P_k + A_k$$

$$Z_{ki} = g_i(X_{k1}) + \textstyle\sum_{j=1}^{L_k} C_k g_i(X_{kj}), \qquad i = 1, 2, \cdots, m$$

$$Z_{k,m+1} = -\frac{C_k}{P_k}\left(\frac{dP}{dY}\right)_k$$

$$Z_{k,m+2} = -I_k$$

for $k = 1, 2, \cdots, n$, then the least-squares estimate of $\beta = (\beta_1, \beta_2, \cdots, \beta_m, q, w)'$ conditional on the value of α can be directly calculated by the generalized least-squares regression equation

$$\hat{\beta} = (Z'\Omega^{-1}Z)^{-1}Z'\Omega^{-1}U$$

where $U = (U_k)$, $Z = (Z_{ki})$. The sum of squares

$$S(\alpha) = U'\Omega^{-1}U - \hat{\beta}'Z'\Omega^{-1}U$$

is then a function of α only. We can then find the value of α that minimizes $S(\alpha)$ by use of numerical procedures, such as the three-point Newton minimization method.

Once the least-squares values of the regression coefficients $\beta_1, \beta_2, \cdots, \beta_m, q, w$ and the parameter α are determined, the fitted discount function is given by

$$\hat{D}(t) = \textstyle\sum_{i=1}^{m} \hat{\beta}_i g_i(1 - e^{-\hat{\alpha}t}), \qquad t \geq 0. \tag{16}$$

As for the spline space, we choose cubic splines as the lowest odd order with continuous derivatives. The boundary conditions are $G(0) = 1$, $G(1) = 0$. The base $(g_i(x))$ should be chosen to be reasonably close to orthogonal, in order that the regression matrix

$$Z'\Omega^{-1}Z$$

can be inverted with sufficient precision.

Although the model is fitted in its transformed version given by Equation (15), it may be illustrative to rewrite it in the original parameter t. In any interval between consecutive knot points, $G(x)$ is a cubic polynomial, and therefore $D(t)$ takes the form

$$D(t) = a_0 + a_1 e^{-\alpha t} + a_2 e^{-2\alpha t} + a_3 e^{-3\alpha t}$$

on each interval between knots. The function $D(t)$ and its first and second derivatives are continuous at the knot points. This family of curves, used to fit the discount function, can be described as the *third order exponential splines.*

Since least-squares methods are highly sensitive to wrong data, we use a screening procedure to identify and exclude outliers. Observations with residuals larger than four standard deviations are excluded and the model is fitted again. This procedure is repeated until no more outliers are present.

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