# Term Structure and Credit Spread Estimation

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# Basic principles of bond pricing

- coupon bond which matures in $n$ years
- investor gets at the times $i = 1, \ldots n$ coupon payments $C$ and a redemption payment $R$ at $t = n$
- **clean price** $p_c$ is quoted on the market
- seller also receives **accrued interest** for holding the bond over the period since the last coupon payment

$$a = \frac{\text{number of days since last coupon}}{\text{number of days in current coupon period}} C$$

- investor has to pay the **dirty price** $p_d$
- bond pricing equation with continuous compounding

$$p_c + a = C \sum_{i=1}^{n} e^{-s_i m_i} + R e^{-s_n m_n}$$

# Basic principles of bond pricing

- **yield to maturity**

$$p_c + a = C \sum_{i=1}^{n} e^{-ym_i} + Re^{-ym_n}$$

- equivalent formulation of the bond price equation uses the **discount factors** $d_i = \delta(m_i) = e^{-s_i m_i}$
- continuous **discount function** $\delta(\cdot)$ is formed by interpolation of the discount factors

$$p_c + a = C \sum_{i=1}^{n} \delta(m_i) + \delta(m_n) R$$

- implied $j$-period **forward rate**

$$f_{t|j} = \frac{js_j - ts_t}{j - t}$$

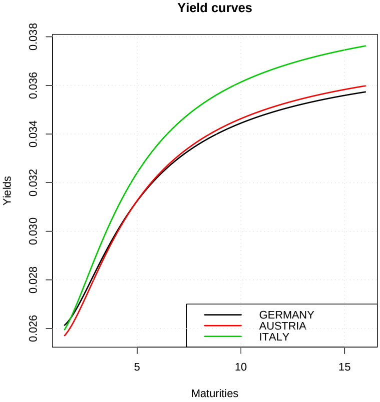
- **duration** is a weighted average of time to cash flows

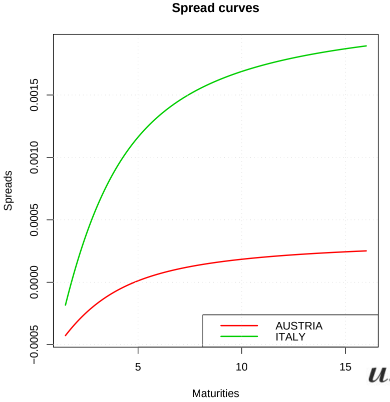
$$D = \frac{1}{p_c + a} \left[ C \sum_{i=1}^{n} \delta(m_i) m_i + \delta(m_n) R m_n \right]$$

- estimate zero-coupon yield curves and credit spread curves from market data
- usual way for calculation of **credit spread curves**

$$c_i(t) = s_i(t) - s_{ref}(t)$$

- parsimonious approach widely used by central banks

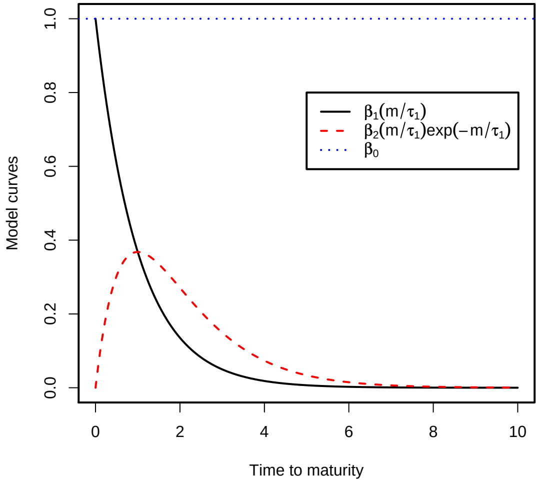


# Nelson and Siegel (1987) approach

## Instantaneous forward rates

$$f(m, \boldsymbol{b}) = \beta_0 + \beta_1 \exp(-\frac{m}{\tau_1}) + \beta_2 \frac{m}{\tau_1} \exp(-\frac{m}{\tau_1})$$



# Nelson and Siegel (1987) approach

## Spot rates

$$s(m, \boldsymbol{b}) = \beta_0 + \beta_1 \frac{1 - \exp(-\frac{m}{\tau_1})}{\frac{m}{\tau_1}} + \beta_2 \left( \frac{1 - \exp(-\frac{m}{\tau_1})}{\frac{m}{\tau_1}} - \exp(-\frac{m}{\tau_1}) \right)$$

## Objective function

$$\boldsymbol{b}_{opt} = \min_{\boldsymbol{b}} \sum_{i=1}^{n} \omega_i \left( \hat{P}_i - P_i \right)^2 \qquad \text{weighted price errors}$$

$$\boldsymbol{b}_{opt} = \min_{\boldsymbol{b}} \sum_{i=1}^{n} \left( \hat{y}_i - y_i \right)^2 \qquad \text{yield errors}$$

# Extensions

- Svensson (1994) extended the functional form by two additional parameters which allows for a second hump-shape

**Instantaneous forward rates**

$$f(m,\boldsymbol{b}) = \beta_0 + \beta_1 \exp(-\frac{m}{\tau_1}) + \beta_2 \frac{m}{\tau_1} \exp(-\frac{m}{\tau_1}) + \beta_3 \frac{m}{\tau_2} \exp(-\frac{m}{\tau_2})$$

- simple calculation method of credit spread curves could lead to twisting curves
- Jankowitsch and Pichler (2004) proposed a **joint estimation method**, which leads to smoother and more realistic credit spread curves

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