Parsimonious Estimation of Credit Spreads

RAINER JANKOWITSCH AND STEFAN PICHLER

RAINER JANKOWITSCH
is an assistant professor of finance at Vienna University of Economics and Business Administration in Vienna, Austria.
rainer.jankowitsch@wu-wien.ac.at

STEFAN PICHLER
is a professor of finance at Vienna University of Economics and Business Administration.
stefan.pichler@wu-wien.ac.at

Over the last decade it has become more and more important to understand and model credit risk. Growing markets for corporate bonds and credit derivatives have stimulated development of advanced valuation and risk management models. The new Basel Capital Accord has also fueled the evolution of theoretical and applied research in credit risk.

Many of the new credit risk models require accurate observations of the term structure of interest rates of different credit risk classes as an input. Prominent examples are the Markov chain framework first introduced by Jarrow and Turnbull [1995] and extended by Jarrow, Lando, and Turnbull [1997], and the class of reduced-form models that exogenously model the default intensity of a Poisson process as a function of stochastic state variables. This approach, introduced by Duffie and Singleton [1997] and Lando [1998], enables the use of well-established results and techniques from the world of affine risk-free term structure models.

Both classes of models rely on the availability of reasonable data for term structures of credit spreads, i.e., the term structure of differences between risky term structures and the risk-free one. Apparently, there is a need for accurate and reliable procedures that estimate the term structure of credit spreads from observable coupon bond prices.

The estimation of the discount function or the zero-coupon yield curve from observable prices of coupon bonds is an established field of academic research. The most generally accepted procedures are due to pioneering work by McCulloch [1971, 1975], Schaefer [1981], Vasicek and Fong [1982], Nelson and Siegel [1987], and Langetieg and Smoot [1989]. More advanced estimation methods have been suggested by Steeley [1991], Svensson [1994], Linton et al. [2001], and Subramanian [2001].

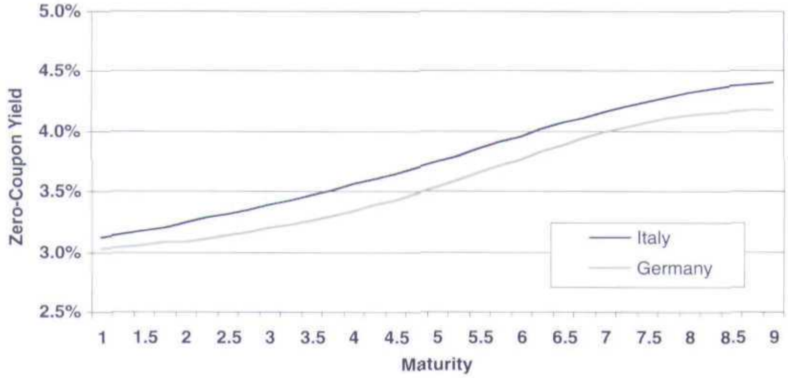
Interestingly, there are only rare contributions to the specific problems of credit spread estimation. Traditionally, credit spreads are calculated by subtracting independently estimated risk-free and risky term structures of interest rates, which in many cases yields unrealistically shaped and often irregular credit spread curves. Düllmann and Windfuhr [2000] and Geyer, Kossmeier, and Pichler [2004] report twisted credit spread curves for European Monetary Union government debt when the German curve is used as the risk-free reference curve.

Exhibit 1 shows a representative example where the yield curves for two government issuers have a similar shape. This suggests a rather well-behaved or even linear shape of the credit spread curve. Exhibit 2, however, shows a strong non-linear twisted pattern of the credit spread curve. Many researchers see this twisted pattern as unrealistic or irregular, in that it contradicts the implications of many economic credit risk models.

Yet before jumping to conclusions, one

EXHIBIT 1

Zero-Coupon Yield Curves Obtained Using Cubic Splines Model for Italy and Germany—March 11, 1999



should carefully check whether this observation is an artifact of the estimation procedure instead of an economic fact. In general, traditional models of calculating the credit spread may produce questionable results that hinder their economic interpretation.

Houweling, Hoek, and Kleibergen [2001] were the first to propose a new framework to overcome this possible drawback. They suggest using a multicurve approach where the risk-free term structure and the credit spread curve are estimated jointly. This method leads to more realistic and smoother credit spread curves in their empirical examples. Again it is hard to decide whether a seemingly irregular shape of the credit spread curve is caused by the data or is merely an artifact of the functional form of the estimation model. There is no natural benchmark to use to assess the quality of an estimation model.

Houweling, Hoek, and Kleibergen tried to formalize the trade-off between the gain in smoothness and the reduced explanatory power of the more parsimonious joint estimation procedure. According to this rather relative and arbitrary benchmark, they find strong evidence in favor of their model.

We develop an alternative joint estimation procedure to serve as an additional benchmark in a peer comparison, and examine the performance of two different traditional models and two different joint estimation models.

Our main objective is to provide empirical evidence that a procedure that jointly estimates the risk-free term structure and the credit spread curve is better than traditional models. We do this by comparing the results of two completely different single-curve and two completely different multicurve models. This peer comparison provides an additional benchmark and allows for deciding whether a twisted shape of an estimated curve is an artifact of the functional form of the model or not. The empirical study makes use of EMU government bond data already used by other studies, but the results should hold for corporate bond markets as well.

I. ESTIMATION MODELS FOR THE CREDIT SPREAD

We start by briefly describing the basic estimation problem arising when prices of coupon bonds can be observed only in an incomplete bond market. We apply two different approaches to this estimation problem, the cubic splines model and the Svensson model.

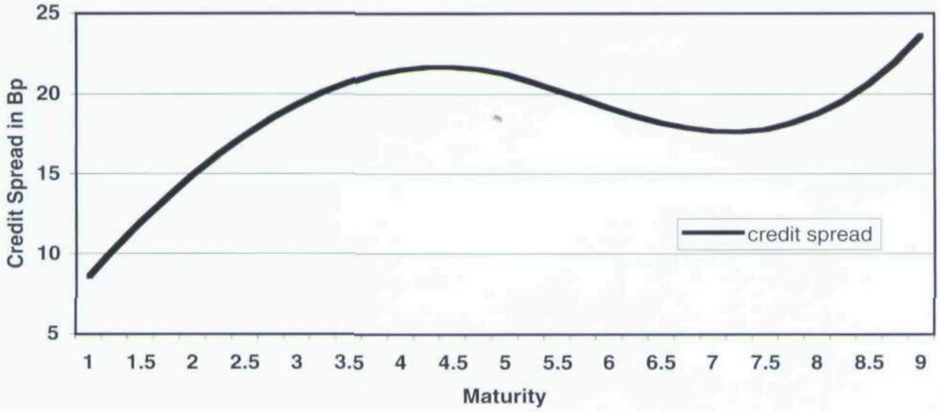
The choice of these models may seem arbitrary, but they cover a broad class of estimation procedures and because of their practical importance may be seen as representative examples. Our main conclusions, however, are very unlikely to be affected by the choice of the specific models. Thus, we do not refer to more advanced spline interpolation methods, like B-splines or exponential splines.

Basic Estimation Problem

First we describe the estimation problem in a single market setting, and then turn to the multicurve problem when credit spread curves are to be estimated. Consider a bond market with N coupon bonds. Each bond i is characterized by its market price P_i (quoted price plus accrued interest), its cash flow vector Z_i, and its vector of cash flow dates T_i. A bond market is said to be complete if the total number of distinct cash flow dates in the market is equal to the number of linearly independent cash flow vectors, i.e., if arbitrary cash flow structures can be repli-

EXHIBIT 2

Credit Spread Curve Between Italy and Germany Using Cubic Splines Model—March 11, 1999



cated using portfolios of outstanding bonds. The absence of arbitrage implies a unique set of discount factors $D(t)$, where t denotes the time to any cash flow date in the market, for which

$$P_i = \sum_j Z_{ij} D(T_{ij})$$

holds for all $i = 1, ..., N$. The zero-coupon yields $r(t)$ are related to the discount factors via

$$r(t) = \left(\frac{1}{D(t)}\right)^{\frac{1}{t}} - 1$$

In incomplete markets like the government bond markets in European Monetary Union countries, the set of arbitrage-free discount factors is not unique. Given an arbitrary set of discount factors:

$$P_i = \sum_j Z_{ij} D(T_{ij}) + \varepsilon_i$$

applies, where ε_i denotes the pricing error of bond i.

The basic estimation problem is to find a set of discount factors that have optimal explanatory power, i.e., that minimize the pricing errors with respect to a given norm, and are represented by a continuous function depending on a parsimonious number of free parameters. Let a denote the set of parameters and f denote the specified function. Then we have:

$$D(t) = f(t; a)$$

and

$$P_i = \sum_j Z_{ij} f(T_{ij}; a) + \varepsilon_i$$

There are numerous models suggested to solve this estimation problem. These models differ mainly in their functional specification, the number of free parameters, and the quantity for which the functional form is specified, i.e., for the discount factors, the zero-coupon yields, or the forward rates.

Single-Curve Splines Model

In the *single-curve splines model* introduced by McCulloch [1975], cubic splines are used to model the discount function. In this approach, the maturity spectrum is divided into not necessarily equal intervals.

If the maturity spectrum is divided by $k - 1$ knots, there are k free parameters to describe the entire discount function, which is modeled as a linear combination of k prespecified component functions. Where f_k denotes the component functions, the estimation problem reads

$$P_i = \sum_j Z_{ij} \sum_k a_k f_k(T_{ij}) + \varepsilon_i = \sum_k a_k \sum_j Z_{ij} f_k(T_{ij}) + \varepsilon_i$$

Employing the linear structure of this model, we can easily obtain the optimal parameters by performing an ordinary least squares regression. Alternatively, the log of the discount function could be modeled by a cubic spline, but since we restrict the maximum maturity to ten years we use the original setup as in Houweling, Hoek, and Kleibergen [2001].

Finally, the credit spread curve of a specific country with respect to a reference curve is calculated by subtracting the reference zero-coupon yield curve from the zero-coupon yield curve of the country:

$$s_c(t) = r_c(t) - r_{ref}(t)$$

where

- $s_c(t)$ is the credit spread between country c and the reference country for maturity t,
- $r_c(t)$ is the zero-coupon yield for country c for maturity t, and
- $r_{ref}(t)$ is the zero-coupon yield for the reference country for maturity t.

Single-Curve Svensson Model

In the *single-curve Svensson model*, the estimation procedure suggested by Svensson [1994] is used to specify the zero-coupon yield curve directly using the functional form:

$$r(t) = \beta_0 + \beta_1\left(\frac{1-\exp(-t/\tau_1)}{(t/\tau_1)}\right) + \beta_2\left(\frac{1-\exp(-t/\tau_1)}{(t/\tau_1)} - \exp(-t/\tau_1)\right) + \beta_3\left(\frac{1-\exp(-t/\tau_2)}{(t/\tau_2)} - \exp(-t/\tau_2)\right)$$

where β_0, β_1, β_2, β_3, τ_1, and τ_2 are parameters.

The interpolation function now becomes

$$f(T_{ij}; a) = \left[1 + r(T_{ij})\right]^{-T_{ij}}$$

where the function $r(.)$ is defined as above.

The parameters can be obtained by performing a nonlinear optimization with the parameter restrictions $\beta_0 > 0$, $\tau_1 > 0$, and $\tau_2 > 0$. This procedure, which is an extension of the well-known Nelson and Siegel [1987] approach, is widely used by practitioners as well as central banks and other financial institutions. It is seen as producing smoother or more regular functions than a splines interpolation.

Again the credit spread curves are calculated by subtracting the reference curve from the zero-coupon yield curve of a specific country.

Multicurve Splines Model

In the *multicurve splines model*, the zero-coupon yield curve of the reference country and the credit spread curves of the other countries are estimated jointly. All the curves are estimated with cubic splines as described. For a more parsimonious specification, the number of parameters for the credit spread curve can be reduced compared to the reference curve. The framework for the discount functions is as follows:

$$D_0(t) = \sum_{k=1}^{k_{ref}} f_{ref,k}(t)a_{0,k}$$

is the discount function for the reference country, and

$$D_c(t) = D_0(t) + \sum_{k=1}^{k_{spread}} f_{spread,k}(t)a_{c,k}$$

is the discount function for country $c = 1, ..., C$

where

- k_{ref} is the number of parameters for the zero-coupon yield curve of the reference country;
- $f_{ref,k}(t)$ are the component functions, which use the chosen knots of the zero-coupon yield curve of the reference country;
- $a_{0,k}$ are the parameters of the zero-coupon yield curve of the reference country;
- k_{spread} is the number of parameters for the credit spread curves;
- $f_{spread,k}(t)$ are the component functions, which use the chosen knots of the spread curves; and
- $a_{c,k}$ are the parameters of the credit spread curve of the country c.

This multicurve model uses two features to improve the single-curve model. First, it directly estimates the spread curves with a parsimonious splines model and, second, the reference zero-coupon yield curve and all spread curves are estimated jointly.

Using this framework, we obtain the linear regression model for $C + 1$ countries:

$$Y = Xa + \varepsilon \Leftrightarrow \begin{pmatrix} Y_0 \\ Y_1 \\ \vdots \\ Y_C \end{pmatrix} = \begin{pmatrix} X_0 & 0 & \cdots & 0 \\ X_1 & S_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ X_C & 0 & \cdots & S_C \end{pmatrix} \cdot \begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_C \end{pmatrix} + \begin{pmatrix} \varepsilon_0 \\ \varepsilon_1 \\ \vdots \\ \varepsilon_C \end{pmatrix}$$

where Y_c is a vector consisting of elements

$$y_i = P_i - \sum_j Z_{ij}$$

X_c is a matrix with elements

$$x_{ik} = \sum_j Z_{ij} f_{ref,k}(T_{ij})$$

S_c is a matrix with elements

$$s_{ik} = \sum_j Z_{ij} f_{spread,k}(T_{ij})$$

a_c is a vector of parameters for country c, and
ε_c is a vector of residuals for country c assumed to be iid $(0, \sigma_c^2)$.

The residual term is allowed to have different variances for each country because in some countries bond prices are generally more noisy than in other countries. To estimate the parameters, we apply a restricted feasible generalized least squares procedure (see, e.g., Greene [2000]). Numerical examples for applications in the given framework show, however, that relaxation of the variance structure is of minor importance for the results.

Multicurve Svensson Model

In the *multicurve Svensson model*, the zero-coupon yield curve of the reference country and the credit spread curves of the other countries are estimated jointly. For the zero-coupon yield curve of the reference country, we use the original Svensson approach with the single-curve form. To allow for a parsimonious specification, the credit spread curves are modeled using the four-parameter approach suggested by Nelson and Siegel [1987], which has the functional form:

$$s_c(t) = \gamma_{0,c} + \gamma_{1,c}\left(\frac{1-\exp(-t/\kappa_c)}{(t/\kappa_c)}\right) + \gamma_{2,c}\exp(-t/\kappa_c)$$

where $\gamma_{0,c}$, $\gamma_{1,c}$, $\gamma_{2,c}$ and κ_c are parameters.

Note that the four-parameter Nelson-Siegel model is nested in the six-parameter Svensson model with $\beta_0 = \gamma_{0,c}$, $\beta_1 + \beta_2 = \gamma_{1,c}$, $\beta_2 = -\gamma_{2,c}$, $\beta_3 = 0$, $\tau_1 = \kappa_c$, and $\tau_2 = 1$. The Nelson-Siegel model is regarded as a four-parameter version of the Svensson model.

The zero-coupon yield curve for the reference country $r_{ref}(t)$ is again specified by a Svensson model with six parameters. The interpolation function is now defined as:

$$f_c(T_{ij}; a) = \left[1 + r_c(T_{ij})\right]^{-T_{ij}} = \left[1 + r_{ref}(T_{ij}) + s_c(T_{ij})\right]^{-T_{ij}}$$

where $a = \{\beta_0, \beta_1, \beta_2, \beta_3, \tau_1, \tau_2; \gamma_{0,1}, \gamma_{1,1}, \gamma_{2,1}, \kappa_1; \ldots; \gamma_{0,C}, \gamma_{1,C}, \gamma_{2,C}, \kappa_C\}$.

The reference zero-coupon yield curve and all spread curves are estimated jointly by performing a non-linear GLS procedure analogous to the linear procedure described under the multicurve splines model.

II. EMPIRICAL ANALYSIS

We first describe the data and methodology, and then present the results.

Data

The time series data used in this study include daily prices of euro-denominated European Monetary Union (EMU) government bonds from January 1999 through March 2001. At present there are 12 countries in the EMU: Germany (rated AAA by Moody's), France (AAA), Italy (AA2), Austria (AAA), The Netherlands (AAA), Belgium (AA1), Portugal (AA2), Spain (AAA), Finland (AAA), Greece (A1), Ireland (AAA), and Luxembourg (AAA).

We choose this data set for several reasons: 1) It constitutes the largest and most liquid bond market where bonds of different issuers are traded in a homogeneous environment; 2) in comparison to the corporate bond market, the quality and reliability of the price information is much better; and 3) credit spread curves obtained from EMU government bond prices have already been used by other researchers, e.g., Düllmann and Windfuhr [2000] and Geyer, Kossmeier, and Pichler [2004].

We are aware nonetheless that the spreads observed between different EMU countries are potentially caused by other effects (e.g., liquidity) than credit risk. Preliminary results from the EMU government bond market, however, show that liquidity effects cannot fully explain the size and the variation of cross-country spreads (see Jankowitsch, Mösenbacher, and Pichler [2003]). Moreover, our conclusions are based only on technical properties of the estimation procedures and are expected to hold even if the economic reason for the spread is different.

We restrict our analysis to coupon bonds with no

EXHIBIT 3

Number of Government Bonds for Each Country and Amounts Observed Each Day

Number of Bonds	Germany	France	Italy	Austria	Netherlands	Belgium	Portugal	Spain	Finland
Overall	104	106	104	99	45	36	26	39	56
Average	65	37	59	53	30	21	13	24	24
Min	57	18	43	43	15	16	7	22	12
Max	71	45	63	60	37	23	15	26	27

option features and with reliable price information and times to maturity shorter than ten years. The data set includes the basic features of the bonds, e.g., the maturity and the exact cash flow schedule. Daily prices are taken from Bloomberg using the Bloomberg generic price source (BGN). For a detailed description of this data set, see Geyer, Kossmeier, and Pichler [2004]. To provide meaningful results, we make use of data collected from nine countries: Germany, France, Italy, Austria, The Netherlands, Belgium, Portugal, Spain, and Finland. Luxembourg is not included because it has no publicly traded government debt. Greece is excluded because it did not take part in the starting phase of the EMU in 1999. Ireland is also excluded because it only has five liquid bonds outstanding.

Exhibit 3 shows the overall number of government bonds for each country and the average, maximum, and minimum number of outstanding government bonds observed on each day. Exhibit 4 shows the average number of outstanding government bonds observed on each day for different maturity intervals (0-3 years, over 3-5 years, and over 5-10 years).

Although this data set is unique with respect to the countries and variables included, offering a great opportunity for research, it is important to realize its possible weaknesses. We have only quotation prices and not actual trade prices. The quoted price may hold for only a relatively small quantity, while traders demanding higher quantities cannot determine in advance the actual price for the entire quantity they wish to trade. Practitioners do use these quotations in their daily work, and the quotes are regarded by market participants to be good for a certain size, typically €10 million, and most of the transactions take place within the quoted bid-ask spread.

Methodology

As the starting point of our empirical analysis we estimate the zero-coupon yield curves for each country of the EMU government bond market using a standard cubic splines model (*single-curve splines model*) as introduced by McCulloch [1975], with equidistant knots, and the common non-linear procedure (*single-curve Svensson model*) suggested by Svensson [1994]. For the single-curve splines models we use a four-parameter specification, which turns out to be optimal for most countries throughout the analysis.

By taking the German government bonds as risk-free, we can calculate the credit spreads for the other EMU countries by subtracting the German reference curve from the respective zero-coupon yield curves. We perform a visual inspection of these credit spread curves to check whether their shapes are driven by economic reasons or by the functional form of the model specifications.

Given the scarcity of bonds with longer maturities, we restrict our analysis to the maturity spectrum from one to ten years for all countries. The short end of the term structure is eliminated from the analysis mainly because of the lack of liquid price information for very short maturities.

We analyze the pricing errors of government bonds resulting from the single-curve splines model and the single-curve Svensson model for each country. Following Bliss [1997], we calculate the coincidence frequencies of the pricing errors for the two different methodologies. The coincidence frequency measures in what percentage of all cases the two different models result in a pricing error for the same bond that falls in the same error category: highly negative, negative, zero, positive, highly positive. In this way we can test whether the pricing errors are related across models.

If there is a high degree of coincidence, we can conclude that the observed errors, the resulting zero-coupon yield curves, and the credit spread curves are data-driven. If there is a low degree of coincidence, then the pricing errors are method-driven, and the functional forms of the curves are potentially misspecified in an economic sense.

EXHIBIT 4

Average Number of Outstanding Government Bonds Observed Each Day for Different Maturities

maturity	Germany	France	Italy	Austria	Netherlands	Belgium	Portugal	Spain	Finland
0 – 3	36	15	34	29	14	6	4	10	14
3 – 5	14	8	13	12	6	4	4	6	4
5 – 10	15	14	12	12	10	11	5	8	6

Thus, for given estimation results with a low degree of coincidence additional checks concerning the economic meaning are necessary. We then compare the shape, the smoothness, and the volatility of the credit spread curves for the single-curve and multicurve models to find economic misspecifications.

In a first step, we extend the analysis to the use of multicurve models to estimate the credit spread curves. In these models, the zero-coupon yield curve of the risk-free country (Germany) and the credit spread curves of the other countries are estimated jointly by using a parsimonious modeling technique for the credit spreads to filter out possible effects that might produce irregular shapes of the credit spread curves.

We transform the cubic splines model as proposed by Houweling, Hoek, and Kleibergen [2001] into a multicurve model by using a three-parameter credit spread curve, i.e., $k_{ref} = 4$ and $k_{spread} = 3$. The six-parameter Svensson model is transformed by using the nested four-parameter Nelson-Siegel model for the credit spread curve.

To formalize the basis of our conclusions and to incorporate the findings of all countries and of the entire observation period, we calculate several statistics that summarize the main properties of the different models. Along the lines of Houweling, Hoek, and Kleibergen [2001], we calculate the smoothness of the spread curve function to quantify the effect of eliminating the potential of irregular shapes of the credit spread curves.

Following Poirier [1976] and Powell [1981], the smoothness is derived by computing the integral of the square of the second derivative of the credit spread function $s(t)$ over the interval $[t_1, t_2]$:

$$\text{Smoothness} = \int_{t_1}^{t_2} s''(t)^2 \, dt$$

We calculate the smoothness of the credit spread curve of each country for each day and each model. By comparing the average and the volatility of the smoothness in the observation period, we can analyze whether the multicurve models can significantly reduce the S-shape of the credit spread curves better than the single-curve models.

With the four different estimation models, we have four time series of credit spread curves for each country. If we assume that the S-shaped form of the credit spread curves estimated from the single-curve models is caused by the functional form, we would expect these spread curves to be more volatile because a small change in one bond price can have a rather strong effect on the spread curve due to its overparameterization. To test this hypothesis, we compare the volatility of the credit spreads for different maturities.

To examine whether an improvement in the smoothness parameter and in the volatility has a significant negative influence on the explanatory power of the models, we calculate the mean absolute pricing error of each country for each day and each model:

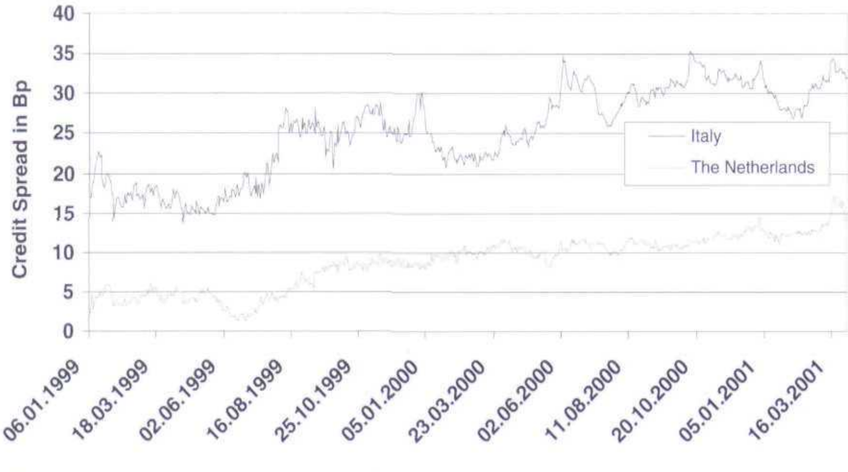
$$\text{Mean absolute pricing error} = \frac{\sum_{i=1}^{N_{c,u}} \left| P_{i,u}^{market} - P_{i,u}^{model} \right|}{N_{c,u}}$$

where

$P_{i,u}^{market}$ market price of bond i on day u,
$P_{i,u}^{model}$ model price of bond i on day u, and
$N_{c,u}$ number of bonds in country c on day u.

With their reduced flexibility, the more parsimonious multicurve specifications are expected to produce higher pricing errors. The reduction in explanatory power should be minimal if the flexible shape of the single-curve models is caused exclusively by the functional form of the specification. If the flexible shape of the credit spread curve estimates is caused at least partly by economic reasons, the pricing errors implied by the parsimonious multicurve models are expected to be much higher than their single-curve counterparts. By comparing the average and

EXHIBIT 5
Time Series of Five-Year Credit Spread for Italy and The Netherlands



Multicurve splines model—Germany as the reference curve.

EXHIBIT 6
Zero-Coupon Yield Curve for Italy Using Single-Curve and Multicurve Splines Models—March 11, 1999

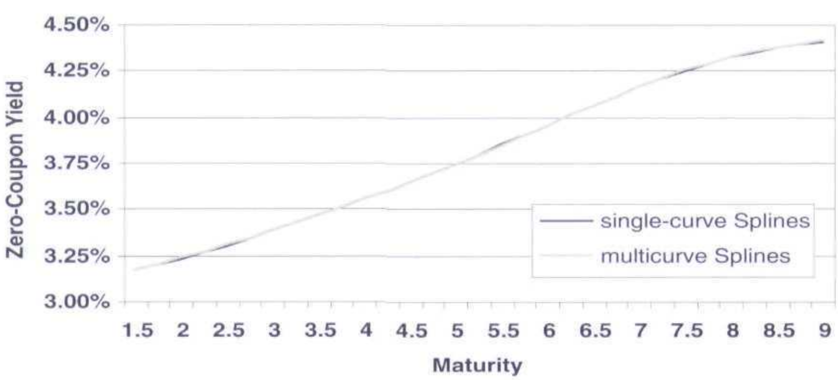
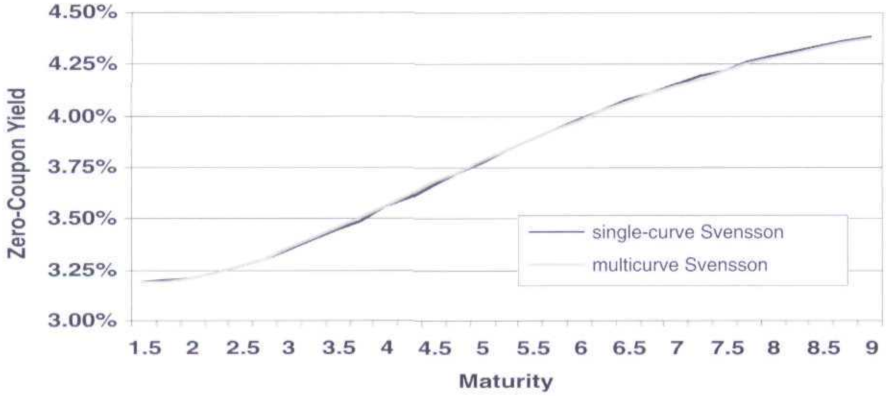


EXHIBIT 7
Zero-Coupon Yield Curve for Italy Using Single-Curve and Multicurve Svensson Models—March 11, 1999



volatility of the mean absolute pricing error in the observation period we analyze this effect.

First we use all bonds, and in a next step we concentrate on bonds of certain maturity intervals.

Results

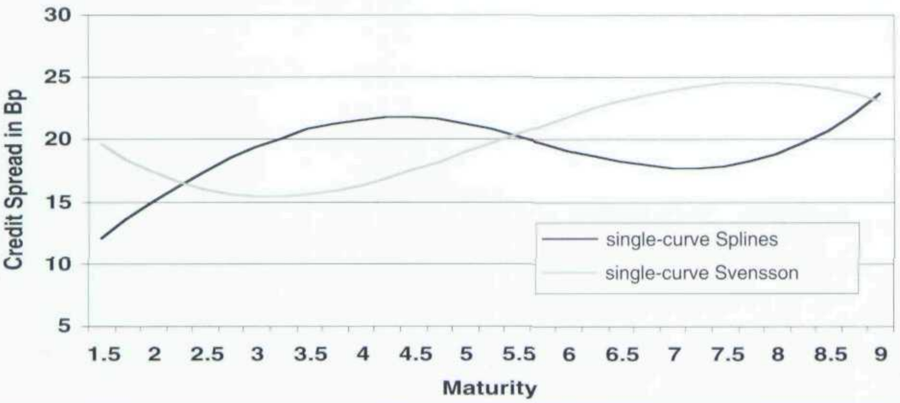
In the first step of the procedure, we estimate zero-coupon yield curves for each country on a daily basis using four different estimation procedures: 1) the single-curve splines model; 2) the multicurve splines model; 3) the single-curve Svensson model; and 4) the multicurve Svensson model. The resulting yield and spread curves for Italy and The Netherlands appear in Exhibit 5. The results in Exhibits 6 and 7 show that all models produce very similar zero-coupon yield curves, but the credit spread curves are quite different.

In most cases, both single-curve models produce twisted curves for the credit spread, but the shapes of these curves are very different, as we see in Exhibit 8 for Italy. In general, the minima and maxima of the resulting curves are in different maturity segments, and in many cases the Svensson model implies an S-shape while the splines model implies a reverse S-shape. Note that the comparisons are based on identical input data drawn from a particular country for one particular day.

In the next step, we compare *the coincidence frequency* of the pricing errors for the single-curve splines and the single-curve Svensson model. We define five error categories: highly negative (pricing error < -30 bp), negative (-30 bp $\leq$ pricing error < -5 bp), zero (-5 bp $\leq$ pricing error < 5 bp), positive (5 bp $\leq$ pricing error < 30 bp), and highly positive (30 bp $\leq$ pricing error). For each bond

EXHIBIT 8

Credit Spread Curve for Italy Using Single-Curve Splines and Single-Curve Svensson Models—March 11, 1999



we examine whether the pricing error from the two models falls into the same error category.

Exhibit 9 shows the percentage of the bonds falling into the same category for each country in the observation period. For the different countries, only around 50% of the bonds fall into the same error category, and categories are defined to be rather broad. This low degree of coincidence indicates that the pricing errors are method-driven, and the functional forms of the curves are potentially misspecified in an economic sense. We conclude that the specific form of the resulting credit spread curves is an artifact of the functional form of the estimation model rather than implied by the data.

Exhibit 10 summarizes the coincidence statistics for the multicurve models. We do not expect these more parsimonious models to have a higher degree of coincidence, but to produce credit spread curves that are more reasonable from an economic perspective. The levels are comparable, which indicates that there is no loss of economic information when we use the multicurve methods. Given these results, we perform additional checks to see whether multicurve models outperform single-curve models in an economic sense. We compare the shape, the smoothness, and the volatility of the credit spread curves for the single-curve and multicurve models to find economic misspecifications.

Compared to the single-curve models, the two multicurve models produce smoother credit spread curves in all observed cases. Moreover, usually the resulting curves are similar to each other, although they are not identical (*see Exhibit 11*). Typically, the twists and S-shapes are not observed in the multicurve results. Since the multicurve models lead to coinciding results and look more plausible from an intuitive perspective, we conclude that the shapes of the spread curves resulting from these models are implied only by the data and are not adversely affected by the functional form of a specific model.

The difference between the results of the multicurve models can be explained. Both produce smoother curves than the single-curve models by estimating the zero-coupon yield curve of the reference country jointly with the credit spread curves and by reducing the flexibility of the credit spread curves due to their more parsimonious specification. The main difference between the multicurve models is how much they reduce the flexibility of the spread curves. Given the specifications chosen in our study, the multicurve splines model reduces the flexibility more

EXHIBIT 9

Coincidence Frequency of Each Country—Single-Curve Models

	Germany	France	Italy	Austria	Netherlands	Belgium	Portugal	Spain	Finland
Coincidence	49.02%	43.91%	50.35%	50.14%	52.28%	41.67%	46.17%	53.90%	60.71%

Coincidence frequency measures percentage of the pricing errors falling into the same error category for the observation period (01/1999-03/2001).

EXHIBIT 10

Coincidence Frequency of Each Country—Multicurve Models

	Germany	France	Italy	Austria	Netherlands	Belgium	Portugal	Spain	Finland
Coincidence	49.32%	44.67%	51.35%	50.00%	52.77%	42.28%	45.29%	53.75%	61.18%

Coincidence frequency measures percentage of the pricing errors falling into the same error category for the observation period (01/1999-03/2001).

EXHIBIT 11

Credit Spread Curve for Italy Using Multicurve Splines and Multicurve Svensson Models—March 11, 1999

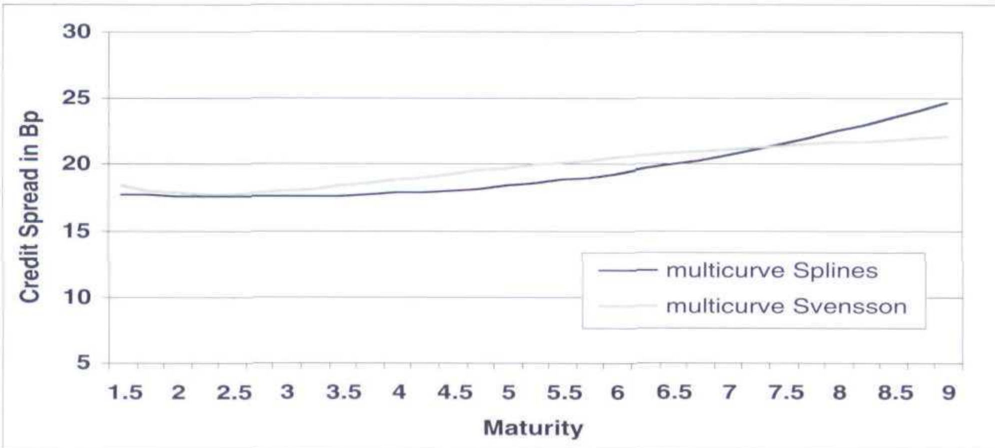


EXHIBIT 12

Average and Volatility (in parentheses) of Smoothness of Credit Spread Curves

	Germany	France	Italy	Austria	Netherlands	Belgium	Portugal	Spain	Finland
SC-Splines	N/A	6.04	0.95	5.77	1.81	1.23	4.55	1.27	2.54
		(5.15)	(0.87)	(4.02)	(2.20)	(2.42)	(4.74)	(1.81)	(2.27)
SC-Svensson	N/A	6.38	1.37	2.13	0.42	1.77	6.44	1.92	1.63
		(5.67)	(1.18)	(1.70)	(0.41)	(2.67)	(8.23)	(2.36)	(2.66)
MC-Splines	N/A	3.49	0.31	0.42	0.52	0.45	0.56	0.26	0.23
		(3.14)	(0.37)	(0.49)	(0.66)	(0.53)	(0.62)	(0.28)	(0.43)
MC-Svensson	N/A	3.32	0.76	1.21	0.40	0.78	2.51	1.45	1.25
		(2.78)	(0.70)	(1.32)	(0.53)	(0.83)	(3.55)	(1.75)	(1.31)

Average and volatility are calculated from the time series of credit spread curves for each country and each model.

than the multicurve Svensson model.

Which of the models performs better depends on the quality of the data. If nearly all government bond prices are exactly explained by the zero-coupon yield curve, the flexibility need not be reduced as much as if there are some larger pricing errors. So, in the first case, the multicurve Svensson model is preferred over the multicurve splines model, and vice versa.

Note, however, that these conclusions are based only on an informal and subjective visual inspection of the resulting yield and spread curves. In order to formalize the basis of our conclusions and to incorporate the findings of all countries and of the entire observation period, we calculate several statistics to summarize the main properties of the different models.

We examine the improvement of the smoothness of the resulting curves by calculating the smoothness parameter between the maturities of two and ten years ($t_1 = 2$, $t_2 = 10$) for all estimated curves. Exhibit 12 shows the average and the volatility of the smoothness of the credit spread curves (multiplied by 10^7). Average and volatility are calculated from the time series of credit spread curves for each country and each model.

Exhibit 13 shows the difference in the average smoothness of the credit spread curve between single-curve and multicurve splines models. The difference is measured as the average smoothness of the multicurve splines model minus the average smoothness of the single-curve splines model (divided by the average smoothness of the single-curve splines model for the percentage statistic). Exhibit 14 shows the difference in the average smoothness of the credit spread curve between the single-

EXHIBIT 13

Difference in Average Smoothness Between Single-Curve and Multicurve Splines Models

	Germany	France	Italy	Austria	Netherlands	Belgium	Portugal	Spain	Finland
Absolute	N/A	-2.55	-0.64	-5.35	-1.29	-0.78	-3.99	-1.01	-2.31
Percent	N/A	-42.26	-67.32	-92.69	-71.27	-63.55	-87.72	-79.44	-90.86

EXHIBIT 14

Difference in Average Smoothness Between Single-Curve and Multicurve Svensson Models

	Germany	France	Italy	Austria	Netherlands	Belgium	Portugal	Spain	Finland
Absolute	N/A	-3.06	-0.61	-0.93	-0.02	-0.99	-3.94	-0.47	-0.39
Percent	N/A	-47.96	-44.71	-43.46	-4.71	-56.06	-61.09	-24.64	-23.84

EXHIBIT 15

Volatility of Five-Year Credit Spread

	France	Italy	Austria	Netherlands	Belgium	Portugal	Spain	Finland
SC-Splines	2.78	3.76	3.83	1.34	3.52	5.31	3.49	4.25
SC-Svensson	4.81	5.72	5.23	2.10	4.25	7.86	4.98	4.35
MC-Splines	2.69	3.63	3.43	1.33	3.47	4.78	3.25	4.14
MC-Svensson	4.64	5.43	5.06	2.08	4.15	7.71	4.77	4.21

EXHIBIT 16

Improvement in Volatility of Five-Year Credit Spread With Multicurve Model

	France	Italy	Austria	Netherlands	Belgium	Portugal	Spain	Finland
Splines	-3.10%	-3.48%	-10.45%	-1.03%	-1.44%	-9.88%	-6.85%	-2.54%
Svensson	-3.46%	-5.05%	-3.31%	-0.93%	-2.39%	-1.92%	-4.24%	-3.34%

curve and multicurve Svensson models measured similarly.

The smoothness results clearly support the finding that the artificial twisted form of the credit spread curves in the single-curve models is corrected by the multicurve models in the overall sample. The improvement (–74% on average) is greater when changing from the single-curve to the multicurve splines model because of the greater reduction in the flexibility of the credit spread as noted earlier, but the improvement (–38% on average) for the nonlinear Svensson case is also impressive. The improvement in the volatility of the smoothness strengthens the argument. Note that (in contrast to the spread curves) the yield curves resulting from a multicurve model (not presented here) are virtually as smooth as the single-curve results.

Analysis of the volatility of the credit spread curves indicates virtually the same results for maturities from one to ten years. As an example, Exhibit 15 presents the volatility of the five-year spread for each country and each model, and Exhibit 16 shows the improvement when using multicurve instead of single-curve models. Interestingly, the multicurve credit spreads are about 5% less volatile with the splines model and about 3% less with the Svensson model than with the single-curve model.

Further examination not presented in detail shows that logarithmic interest rate changes over time are virtually equally volatile when moving from a single-curve to a multicurve model. This finding supports our conclusion that a multicurve model explains the variations of the underlying term structures equally well but mitigates the effect of artificial volatility caused by the adverse influence of the functional specifications in the single-curve models.

Exhibit 17

Average and Volatility (in parentheses) of Mean Absolute Pricing Error

	Germany	France	Italy	Austria	Netherlands	Belgium	Portugal	Spain	Finland
SC-Splines	9.91	11.14	8.80	7.88	9.41	9.04	14.97	10.04	6.06
	(2.40)	(5.92)	(1.82)	(1.81)	(1.76)	(3.86)	(9.18)	(3.13)	(2.21)
SC-Svensson	10.52	10.54	9.21	7.41	9.29	9.00	15.79	10.48	6.27
	(1.75)	(4.70)	(2.93)	(1.77)	(1.56)	(3.36)	(9.63)	(3.91)	(2.01)
MC-Splines	10.06	11.55	9.06	9.52	9.63	9.40	16.58	10.92	6.66
	(2.30)	(5.86)	(1.78)	(1.96)	(1.80)	(4.01)	(9.79)	(3.25)	(2.44)
MC-Svensson	10.33	12.41	9.22	7.72	9.35	10.05	16.58	10.68	6.51
	(1.69)	(5.75)	(2.74)	(1.82)	(1.58)	(4.08)	(9.39)	(3.88)	(2.26)

Average and volatility are calculated from the time series of the mean absolute pricing errors for each country and each model.

We have shown that the use of a multicurve model diminishes the flexibility of the credit spread function, and the unrealistic twisted shapes that are artifacts of the estimation procedure can be avoided. The important question is whether this improvement in smoothness and volatility has a significant negative influence on other important aspects of model quality, such as pricing errors.

Obviously, the more parsimonious multicurve specification is expected to produce higher pricing errors, i.e., is expected to have less explanatory power, due to its lower degree of flexibility. But the reduction in explanatory power should be negligible if the flexible shape of the single-curve models is caused exclusively by the functional form of the specification. Yet if the twisted shape of most of the credit spread curve estimates is caused at least partly by economic reasons, the pricing errors implied by the parsimonious multicurve models would be expected to be much greater than the single-curve errors.

To see whether there are significant differences in explanatory power across the models, we calculate the absolute pricing errors (absolute value of the difference between market price and model price). Exhibit 17 shows the average and the volatility of the mean absolute pricing error measured in basis points for bonds of all maturities. Average and volatility are calculated from the time series of the mean absolute pricing errors for each country and each model. Exhibit 18 shows the average and the volatility of the mean absolute pricing error measured in basis points for bonds of specific maturity intervals (0-3 years, over 3-5 years, and over 5-10 years).

Exhibit 19 shows the difference in the average of the mean absolute pricing error for bonds of all maturities between single-curve and multicurve splines models. The difference is measured as the average of the multicurve splines model minus the average of the single-curve splines model (divided by the average of the single-curve splines model for the percentage statistics). Exhibit 20 shows the same difference between single-curve and multicurve Svensson models.

The results clearly indicate a negligible reduction in explanatory power of the multicurve models at least in absolute terms. For most countries, the increase in pricing errors is modest even measured in percent (on average 7.0% for the splines models and 4.7% for the Svensson models). The only outliers are Austria and Portugal for the splines models and France and Belgium for the Svensson models, where the pricing error percentage increase exceeds 10%, although even these differences are not economically significant, given the usual bid-ask spreads of 3-10 basis points in these markets (see Jankowitsch, Mösenbacher, and Pichler [2003]). These results remain unchanged if we look at specific maturity intervals (*see Exhibit 18*).

Compared to the reductions in smoothness and volatility, the other important property of the models does not change significantly, which supports our hypothesis that the flexible shape of the single-curve models is caused exclusively by the functional form of the specification. We conclude that multicurve models are better than single-curve models for the estimation of credit spread curves.

There are minor differences between the multicurve splines model and the multicurve Svensson model. We have noted that the multicurve splines model is preferable if there are some larger pricing errors, so this model is a better choice except when very small pricing errors are given (e.g., when only market data of benchmark bonds

EXHIBIT 18

Average and Volatility (in parentheses) of Mean Absolute Pricing Error for Bonds of Specific Maturities

	Germany	France	Italy	Austria	Netherlands	Belgium	Portugal	Spain	Finland
maturities: 0 - 3 years									
SC Splines	7.34	8.41	6.82	6.09	5.33	8.43	11.86	6.56	4.24
	(2.20)	(4.21)	(1.46)	(1.78)	(1.81)	(4.42)	(7.24)	(1.99)	(2.01)
MC Splines	7.11	8.30	7.02	6.88	5.66	8.20	11.63	6.79	4.81
	(1.86)	(4.35)	(1.39)	(2.28)	(1.87)	(4.06)	(6.64)	(2.18)	(2.23)
SC Svensson	6.64	6.38	6.29	4.79	4.09	5.70	9.73	6.73	3.99
	(1.21)	(3.44)	(2.02)	(1.25)	(0.92)	(1.87)	(6.71)	(2.87)	(1.56)
MC Svensson	6.32	9.03	6.07	5.10	4.01	7.17	10.81	5.98	4.26
	(1.18)	(4.46)	(1.66)	(1.41)	(0.92)	(2.98)	(6.64)	(2.64)	(2.28)
maturities: 3 - 5 years									
SC Splines	17.98	12.30	13.58	10.17	8.25	8.34	18.72	12.17	8.44
	(4.75)	(7.35)	(3.78)	(2.62)	(2.81)	(5.20)	(14.13)	(3.10)	(3.75)
MC Splines	18.09	12.42	13.66	12.70	8.79	8.37	17.19	11.63	8.98
	(4.93)	(8.73)	(3.70)	(3.08)	(2.90)	(5.10)	(13.43)	(3.39)	(4.90)
SC Svensson	18.59	10.66	13.64	10.45	8.14	7.24	17.48	11.59	8.50
	(4.06)	(6.88)	(3.78)	(2.61)	(2.58)	(4.10)	(12.08)	(2.97)	(3.44)
MC Svensson	18.71	12.85	13.61	10.82	8.78	8.38	17.74	11.50	8.25
	(4.27)	(8.24)	(3.64)	(2.64)	(2.83)	(5.44)	(12.78)	(3.01)	(3.55)
maturities: 5 - 10 years									
SC Splines	8.54	13.79	8.36	9.28	15.62	9.96	15.60	12.48	10.11
	(3.47)	(8.70)	(2.14)	(3.29)	(2.67)	(4.27)	(9.72)	(5.69)	(5.15)
MC Splines	9.74	14.77	9.14	12.03	15.57	10.87	18.19	15.13	10.68
	(3.73)	(8.27)	(2.34)	(4.06)	(2.40)	(4.82)	(10.85)	(6.23)	(4.95)
SC Svensson	12.36	14.92	11.59	9.97	17.04	12.32	18.92	13.97	12.57
	(5.34)	(6.93)	(5.07)	(3.69)	(4.03)	(5.63)	(13.73)	(7.04)	(5.57)
MC Svensson	12.25	15.43	12.28	10.45	17.09	12.88	20.01	15.38	12.86
	(4.82)	(7.21)	(4.65)	(3.49)	(3.82)	(5.63)	(13.59)	(7.23)	(5.71)

Average and volatility are calculated from the time series of the mean absolute pricing error for each country and each model.

are used). Given the obvious numerical advantages of the linear splines model, we suggest using a multicurve splines model to estimate credit spread curves.

III. SUMMARY

We have provided empirical evidence that a parsimonious procedure that jointly estimates the risk-free term structure and the credit spread curve is better than traditional models that obtain the credit spread curve by subtracting the risk-free reference curve from the risky term structure. Credit spread curves from traditional models are often S-shaped; such a twisted pattern is regarded as unrealistic or irregular, as it is seen to contradict the implications of many economic credit risk models.

We extend the framework of a multicurve cubic splines model originally suggested by Houweling, Hoek, and Kleibergen [2001] to cover a multicurve version of the Svensson-Nelson-Siegel approach. This peer comparison provides an additional benchmark and helps us determine whether the twisted shape of an estimated curve is an artifact of the functional form of the traditional models or not. The study uses the European Monetary Union government bond data used in other studies, but the results should hold for corporate bond markets as well.

Results regarding the smoothness parameter show

EXHIBIT 19

Difference in Average of Mean Absolute Pricing Error Between Single-Curve and Multicurve Splines Models

	Germany	France	Italy	Austria	Netherlands	Belgium	Portugal	Spain	Finland
Basis points	0.15	0.41	0.26	1.64	0.22	0.36	1.61	0.89	0.60
Percent	1.49	3.70	2.94	20.79	2.31	3.96	10.74	8.84	9.87

EXHIBIT 20

Difference in Average of Mean Absolute Pricing Error Between Single-Curve and Multicurve Svensson Models

	Germany	France	Italy	Austria	Netherlands	Belgium	Portugal	Spain	Finland
Basis points	-0.19	1.84	0.01	0.32	0.05	1.05	0.78	0.20	0.22
Percent	-1.79	17.52	0.11	4.29	0.58	11.68	4.94	1.87	3.55

impressive evidence that the artificial twisted form of the credit spread curves in the single-curve models is corrected by the multicurve models in the overall sample. This finding is supported by an examination of the volatility of logarithmic interest rate changes, which remains at virtually the same level moving from a single-curve to a multicurve model, while the multicurve credit spreads are remarkably less volatile than under the single-curve framework.

There is a negligible reduction in explanatory power of the multicurve models, at least measured in absolute terms. The observed pricing errors of the more parsimonious multicurve models are about the same as the pricing errors of single-curve models. Given the improvements in smoothness and volatility, this finding supports the hypothesis that the flexible shape of the single-curve models is caused exclusively by the functional form of the specifications. We conclude that multicurve models are clearly better than single-curve models for the estimation of credit spread curves.

There are inconsequential differences between the multicurve splines model and the multicurve Svensson model. Generally, the multicurve splines model is preferable if pricing errors are greater, so it is a better choice except in a setting with very small pricing errors. Given this observation and the obvious numerical advantages of the linear splines model, we suggest using a multicurve splines model to estimate credit spread curves.

ENDNOTE

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