DELTA Pg No.

MINOR - 1 (COL-100)

Solution Made By : R HARI SHANKAR 2019CS10386

PROBLEM 3 :

To Show That : divide $(x, y) = (q, r)$

where $x = qy + r$

for $x \geq 0$, $y > 0$, $q \geq 0$, $0 \leq r < y$

Proof : By PMI Version 3 on x

Basis : For $x = 0$ divide(x, y) returns $(q, r) = (0, 0)$

IH : For $0 \leq k < x$, divide (k, y) returns (q, r) such that $k = qy + r$ $q \geq 0$ and $0 \leq r < y$

IS :

Case 1 ($x = 2k$) : EVEN

Let x be even i.e. $x = 2k$. Then by IH divide (x div 2, y) returns (q, r) such that $k = qy + r$ $q \geq 0$ and $0 \leq r < y$. Thus by algorithm

$$x = 2k = q_1 y + r_1$$

where $q_1 = 2q$ & $r_1 = 2r$.

If $r_1 < y$ the algo returns (q_1, r_1) as required.

If $r_1 \geq y$ then $r_1 - y = 2r - y < y$ (as $r < y$).

Thus $x = (q_1 + 1)y + (r_1 - y)$ and the algo returns $(q_1 + 1, r_1 - y)$ as required.

Case 2: ODD $(x = 2k+1)$

By IH

divide $(x \text{ div } 2, y)$ returns (q, r)

$k = qy + r$. $q \geq 0$ $0 \leq r < y$

Thus by algo

$x = 2k+1 = q_1 y + r_1 + 1 = q_1 y + r_2$ &

where $q_1 = 2q$ & $r_2 = 2r+1$

If $r_2 \leq y$ algo returns (q_1, r_2) as required.

If $r_2 > y$ then

$r_2 - y = 2r+1 - y < y$ because $r < y$

Thus $x = (q_1+1)y + (r_2 - y)$ and the algo

returns $(q_1+1, r_2 - y)$ as required.

Hence Proved by PMI V3

Finding Time Complexity

$x = 2^m$

$$T(x) = T(x \text{ div } 2) + O(1)$$

$$= \cancel{m\,O(1)}$$

$$= T(2^m \text{ div } 2) + O(1)$$

$$= m\,O(1)$$

$$= k \log_2 x$$

$$\therefore T(x) = O(\log_2 x)$$