A Cellular Automata Approach for Modelling Complex River Systems

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Abstract. Rivers can be treated as transportation networks which supply or collect and remove certain resources from the surrounding environment. A positive feedback between environment and river network both reshapes the configuration of the terrain and produces dynamically stable web of river channels. Anastomosing rivers exemplify such interactions very clearly. In case of this specific type of river, nutrients carried by water disseminate to the surrounding soil and stimulate growth of peat-forming plants. Vertical accumulation of peats changes the shape of terrain and influence river network. We present a model of anastomosing river system based on Cellular Automata paradigm. Principal phenomena that contribute to evolution of such a river system are encoded as rules of local interactions. We discuss extensively the parameters and their influence on simulation results.

1 Introduction

Transportation networks are unique structures occurring only in highly organised dissipative systems, such as biological organisms, geological structures and evolving animal colonies. Functionality of these networks is the principal factor of evolution. Transportation networks can play two different roles. They can supply the necessary nutrients, which are consumed by the environment. Transportation networks can also collect and remove some products from productive environment. These two functions are usually performed simultaneously, as in the vascular tissue (arteries and veins), road and railway networks and river systems.

Rivers and river systems have been investigated for years as an important environmental aspect of human life and as the largest natural transportation network. Dynamics of river system is a good example of both self-organising complex system [1] and the source of natural fractals [2]. As shown in [3], a wide range of natural objects and phenomena possess a fractal–like structure. Examples of these so-called fractal trees include actual trees in gardens, plants, such as cauliflowers, river and cardiovascular systems. Unlike transportation networks in biological organisms, the growth factors influencing river systems can be directly observed and scrutinised. One can distinguish both global environmental factors such as terrain configuration, geology, ecological features, climate and local ones, e.g., erosion, deposition and sedimentation [4].

The existence of diverse topological, biological, geomorphological degrees of freedom, sharp interfaces between interacting components, multifaceted boundary conditions and self-organised criticality phenomena driving the system dynamics, make the river topology intrinsically complex. Consequently, it seems the phenomena cannot be placed in an appropriate integrable function space and therefore the classical approaches involving partial or ordinary differential equations cannot be used in modelling of such the systems. The existing models prefer statistical methods (e.g., Monte-Carlo simulations, diffusion limited aggregation) and the cellular automata (CA) [13], which employs rules instead of equations (see [5], [7], [6]).

"Anastomosing river" term refers to river system that possess extremely complex network of forking and joining channels (see Figure 1). Anastomosing rivers are usually formed by repeated avulsions i.e. sudden change of route by whole or part of the stream. Avulsions are primarily driven by aggradation of the channel belt and/or loss of channel capacity and throughput by in-channel deposition [14]. Both processes are triggered by a low floodplain gradient.

Fig. 1. Part of the Narew River in eastern Poland with clearly visible anastomosing pattern (illustration courtesy Prof. Gradziński [8])

The area of the river valley, with growing layer of peat bog represents a typical consuming environment. The nutrients (nourishing resources as ions of nitrogen, phosphorus and potassium), supplied by river, penetrate the soil surrounding the riverbeds and stimulate the vegetation of peat-forming plants. Products of their partial decay accumulates as a peat, what results in gradual raising the level of terrain. Gradient of nutrients saturation, which appears mainly as a result of suction of root system, decreases the rate of peat accumulation proportionally to the distance from the channel. At the same time, sedimentation of organic and mineral material decreases the throughput of river channels. Water level fluctuations or jams occurring in channels can lead to avulsion, when part of

stream leaves the main channel. The route of new channel is determined by the local terrain topography. The new channels usually merge with the others, creating a complex network composed of splitting and merging water channels and small lakes (see Fig. 1). An example of such the river is Narew (Poland, shown in Fig. 3A). There are many other examples such as the fragments of: upper Columbia River (south eastern British Columbia, Canada), Ob (Siberia), Okawango (Africa) and more. The detailed description of the factors, which govern the evolution of the anastomosing river can be found in [8] and [14].

The hypothesis of "starving environment" [12] is a focal point of transportation networks expansion. In the anastomosing river system the peat bog environment is "starving". It means that the supply of nutrients is insufficient, they are consumed very fast and the peat bog growth is restrained. These factors fuel up the expansion of the river network towards "hungry" areas. Conversely, "starving" plant explores productive environment by the huge network of roots in search for water and minerals. The positive feedback interaction between two factors: the network and environment, results in a mutual growth. While dysfunction in mechanisms stimulating the network expansion causes the death of the entire system.

In this paper we propose the Cellular Automata approach for modelling river networks in consuming environment. The CAMAN model ((CAMAN stands for Cellular Automata Model of Anastomosing Networks) is extended and modified version of older SCAMAN model [12]. We present the definition of Cellular Automata and outline of the main algorithm. The results are presented and discussed extensively.

2   Cellular Automata Model of Anastomosing River

For modelling a river, which is undergoing anastomosis we shall construct the algorithm of water distribution in terrain of a predefined topography. We use the modified version [9] of the algorithm described by Di Gregorio and Serra in [10], which was used for modelling lava and mud flow. The model of water spreading can be easily extended on anastomosing rivers. According to the definition of anastomosing river given in the Introduction, we have supplemented the model with the rules of both nutrients distribution and vertical growth of the peat bog.

Let us define this cellular automata model as follows :

$$CA_{CAMAN} = < Z^2, A_{in}, A_{out}, X, S, \delta >$$

where:

- Z^2 — is the $Z \times Z$ square mesh of cellular automata,
- $A_{in} \subset Z^2$ — collection of cells modelling sources - inlets,
- $A_{out} \subset Z^2$ — collection of cells modelling outlets,
- $X(ij)$ — defines the collection of neighbouring cells for an (i, j) cell,
- $S_{ij} = (g_{ij}, w_{ij}, n_{ij}, p_{ij}), i, j = 1, ..., Z$ — the vector describing state of an (i, j) cell:

- g_{ij} — the height of the terrain,
- w_{ij} — the height of water,
- n_{ij} — concentration of nutrients,
- p_{ij} — the peat-bog thickness,

– δ — is a transition function defined as follows:

$$\delta((g_{ij}^t, w_{ij}^t, n_{ij}^t, p_{ij}^t)) = (g_{ij}^{t+1}, w_{ij}^{t+1}, n_{ij}^{t+1}, p_{ij}^{t+1}).$$
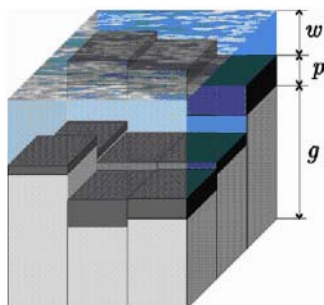
Fig. 2. The heights of columns on the cellular automata lattice represent the elevation of the terrain g and the thickness of both water w and peat-bog layers p

The terrain is modelled by a rectangular mesh $Z \times Z$ of cellular automata. The Moore neighbourhood and the fixed boundary conditions are applied [13]). The borders of the mesh are simulated as extremely high barriers, which prevent horizontal dissipation of water from the simulation domain. The only exceptions are "inlets" (sources) — from which water is added and "outlets" where water is removed from the system.

We defined the following parameters that tune the model:

– γ — gradient of nutrient distribution,
– ρ — peat bog vertical growth rate,
– μ — sedimentation rate.

Main loop of the algorithm consists of three procedures that implement the rules of CAMAN model (see Algorithm 1). The procedure `calculate_flows()` deals with water flow simulation by using method of difference minimising in the neighbouring cells [9]. The rule is homogeneous for the whole CA system and it mimics the process of water distribution due to gravitation.

The procedure `calculate_nutrient_dist()` calculates the concentration of nutrients in the neighbourhood of cells with non-zero water amount ($w_{ij} > 0$). We assume that the cells flooded by water have the maximum concentration of nutrients. This concentration decreases proportionally to the distance from the nutrient source. The value of γ is the nutrients concentration gradient. The procedure for each cell calculates maximum concentration in its neighbourhood and decrease by γ coefficient (see Algorithm 2).

Algorithm 1. The algorithm of CAMAN model

for all step of simulation **do**
  calculate_flows();
  calculate_nutrient_dist();
  calculate_peat_growth();

Procedure 2. Outline of the `calculate_nutrient_dist()` procedure

for all cell in the mesh **do**
  if $w_{ij} > 0$ **then**
    $n_{ij} \Leftarrow 1.0$;
  else
    $n_{ij} \Leftarrow \gamma \max(n_{X(ij)})$;

The nutrients concentration influences the thickness of the peat bog layer in "dry" cells (`calculate_peat_growth()` procedure). We assume that each time-step its growth increment is proportional to the current concentration of nutrients with proportionality coefficient ρ. In comparison to SCAMAN model [12], the rule of growth of peat layer has been extended and now it distinguish between cells with and without water. Areas covered by water elevate with rate described by μ parameter what reflects the process of sedimentation occurring on the bottom of channels. "Dry" areas grow with ρ rate. Difference between μ and ρ parameters has substantial influence on resulting patterns (see Figure 7).

Procedure 3. Outline of the `calculate_peat_growth()` procedure

for all cell in the mesh **do**
  if $w_{ij} > 0$ **then**
    $p_{ij} \Leftarrow \mu \, n_{ij}$;
  else
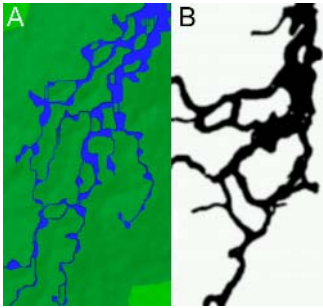 $p_{ij} \Leftarrow \rho \, n_{ij}$;

2.1   Results

To speed up calculations, our model was parallelised and implemented under MPI environment on SGI/Altix cluster. The results has been postprocessed and visualised using Amira package (www.tgs.com). In Table 1 we present the parameters used in our simulations.

In Fig. 3A we illustrate the snapshots from simulations for cellular automata mesh of size 530×530 grid points. The snapshot is compared with a small section of Narew River (Figure 3B). The terrain is slightly inclined (the slope is 0.05%) and rough. However, vertical random amplitude of roughness is assumed to be small and less than 1% of the distance between neighboring cells. Water is supplied to the system by a single source cell. We can observe the creation of small floods in the two pictures and similar backbone structure.

Table 1. Simulation parameters for presented results

| Fig. no.	γ	ρ	μ	slope	mesh
Fig.3A	0.02	0.0002	0.00014	0.05% (0.5m/km)	530×530
Fig.4	0.02	0.0005	0.0004	0.2% (2m/km)	730×730
Fig.5	(A,B) 0.02	0.0005	0.0004	0.05%	330×330
	(C,D) 0.08				
Fig.6	0.04	(A) 0.0005	(A) 0.0004	0.05%	330×330
		(B) 0.001	(B) 0.0009		
Fig.7	0.04	(A) 0.0009	(A) 0.00075	0.05%	330×330
		(B) 0,0009	(B) 0.00085		
Fig.8	0.03	0.00003	0.00002	0.05%	330×330

Fig. 3. Comparison of modelled river network (A) to small fragment of real anastomosing pattern (B, the Narew River from Figure 1)

In comparison to the previous run, the snapshots shown in Fig. 4 represent the terrain of a greater inclination (0.2%) and greater number of source cells. Therefore, despite we used larger system of cellular automata (730×730 grid points), the river system develops faster. The situation from Fig.4A was obtained after 10^3 timesteps. As displayed in Fig.4B, after about 2×10^4 timesteps the environment saturates and the landscape pattern stabilizes in an equilibrium state.

Figs. 5 and 6 illustrate the various influences of the simulation parameters, such as gradient of nutrients distribution γ and peat bog growth factor ρ, respectively, on the evolution of river networks. Large value of γ and small value of peat bog growth factor ρ, cause that smaller area of the terrain is penetrated by the nutrients. Thus the environment is "starving". As a consequence, the complexity of the river system, as measured by the number of channels and bifurcations, increases [14]. Irregular distribution of nutrients stimulates creation of more complex river networks. This confirms the hypothesis that "starving environment" is a driving force developing transportation networks [12].

Third parameter μ, which represents growth of sedimentation layer, has to be slightly smaller than the gradient of nutrients spreading. Greater differences between them result in formation of river networks with deep and narrow channels
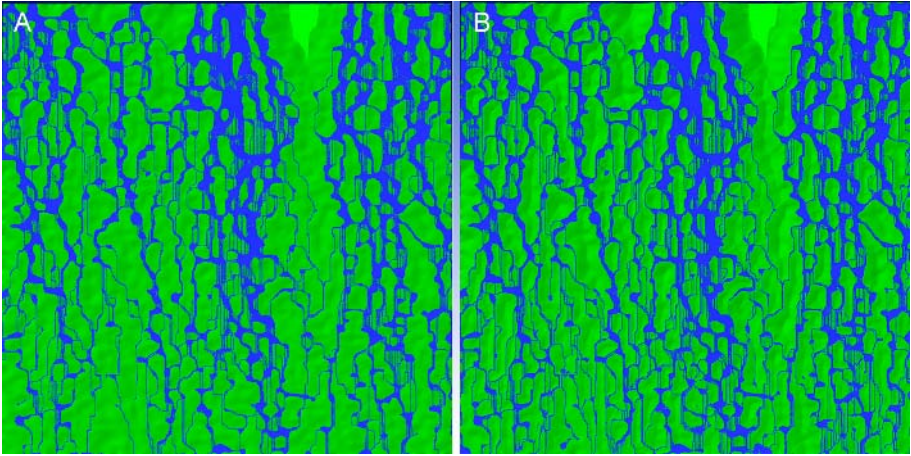
Fig. 4. The river system after A) 10^3 and B) 2×10^4 time-steps. The saturation of the environment can be observed.
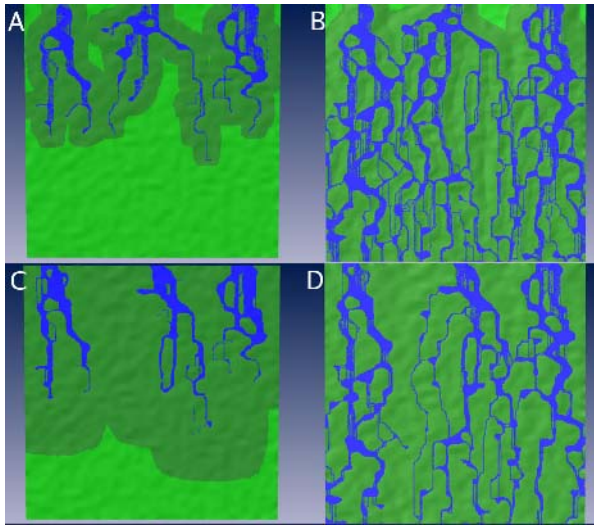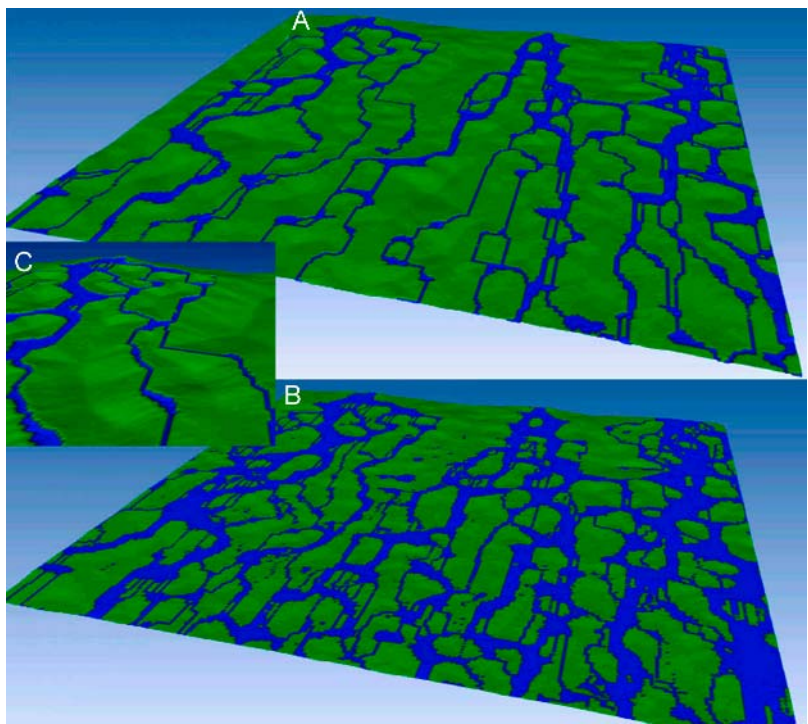

Fig. 5. The snapshots from simulations of river networks performed on 330×330 point mesh. After 500 (A,C) and 4×10^3 (B,D) time-steps, illustrate influence of nutrient gradient parameter γ: 0.02 (A,B) and 0.08 (C,D). Areas covered by peat-bogs are depicted by using dark green colour.

(see Fig.7A). This increases considerably the total volume of the river system, decreasing simultaneously the probability of new channels formation and branching. Conversely, by diminishing the difference between γ and μ, frequent floods produce deeper river networks.
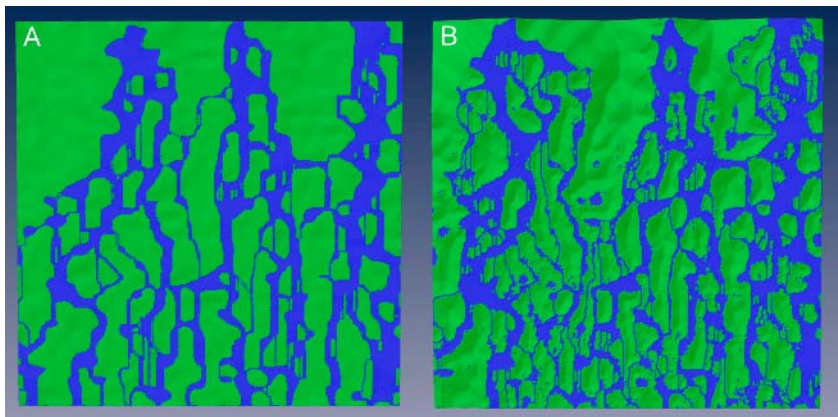
Fig. 6. The snapshots from simulations of river networks after 2×10^4 (mesh 330×330 points) for two different values of the peat bog growth rate ρ: 0.0005 (A) and 0.001 (B)

Fig. 7. The snapshots from simulations illustrating diversity between river networks obtained for large (A) and small (C) differences between peat bog growth rate ρ and sedimentation rate μ. In B) a fragment from Fig A) is presented under different angle to show better the terrain configuration.
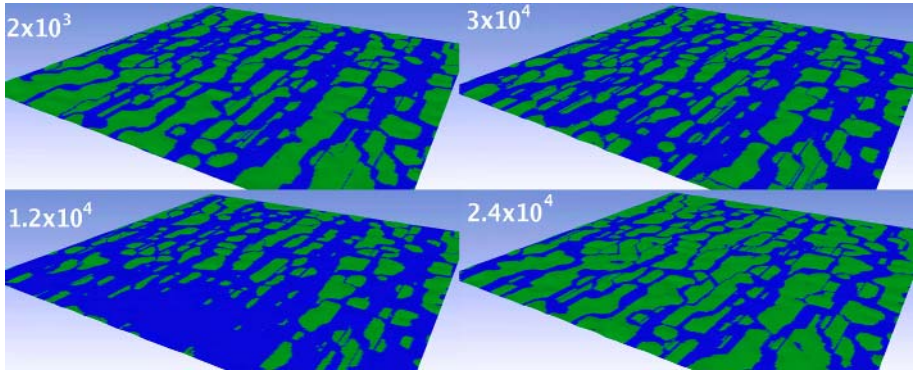
Fig. 8. The landscape changes with time (number of time-steps is given for each picture) produced for variable outputs of water sources. This figure illustrate periodic floods in anastomosing river basin.

As shown in Fig. 8, the configuration of anastomosing river basin is also influenced by periodical changes of water level e.g., spring floods and summer droughts — modelled by high and low water output from the source cells.

3   Conclusions

The classical synchronous cellular automata paradigm is a perfect tool for modelling drainage systems and dendric rivers [15] such as those created by erosion [http://fd.alife.co.uk]. We have shown that CA can be used as a fine grained model for simulating more complex transportation networks. Their role is very different from drainage itself. Erosion and flow can be simulated as two concurrent phenomena driven by local mutual forces. The transportation network distributes nutrients to the environment. The slow changes in the environment (e.g. peat bog growth and changes in terrain configuration) feedbacks the growth of the network.

The main disadvantage of our CA model is its low computational speed for simulating more disparate spatio-temporal scales. This is mainly due to the high degree of spatial and temporal disparity between the processes modelling the evolution of anastomosing rivers. The flow speed of the river is orders of magnitude greater than environmental changes, such as the peat bog growth and sedimentation. This results in the configuration of the terrain not changing too much. The channels are too shallow which prompts wide floods. Therefore, the simulated systems from Figs.4–8 and the real anastomosing network from Fig.1 represent different scales. Modelling realistic anastomosing networks with our CA model would involve $10^6 - 10^8$ cells and a similar number of time-steps. This would be very demanding in terms of CPU time.

Acknowledgements

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