# PHY407 Lab-09 Report

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[1] *Wrote and solved Q1,Q2*

## QUESTION 1 - TIME-DEPENDENT SCHRODINGER EQUATION

Consider a particle in an infinite square well of length $L = 10^{-8}$m with mass $m_e = 9.109 \times 10^{-31}$kg. We will construct initial condition

$$\psi(x, t = 0) = \psi_0 e^{-\frac{(x-x_0)^2}{4\sigma^2} + ikx} \tag{1}$$

With $\sigma = L/25$, $\kappa = 500/L$, $x_0 = L/5$. Note that this initial condition follows a Gaussian, and we should expect such shape when plotting.

### *PART (A)*

The Crank-Nicholson method follows an intermediate point of the FTCS method and implicit equation. Combining the two equations through an average, the Crank-Nicholson method for the Schrodinger equation is,

$$\psi(x,t+\tau)-\tau\frac{i\hbar}{4ma^2}[\psi(x+a,t+\tau)+\psi(x-a,t+\tau)-2\psi(x,t+\tau)] = \psi(x,t)+\tau\frac{i\hbar}{4ma^2}[\psi(x+a,t)+\psi(x-a,t)-2\psi(x,t)] \tag{2}$$

This can be written in vector notation as,

$$\vec{A}\psi(x, t + \tau) = \vec{B}\psi(t) \tag{3}$$

The discretized Hamiltonian can be broken into two matrices (following Ex.9 in the textbook) $A$ and $B$.

$$\vec{A} = \begin{pmatrix} a_1 & a_2 & 0 & 0 & ... \\ a_2 & a_1 & a_2 & 0 & ... \\ 0 & a_2 & a_2 & a_1 & ... \\ ... & ... & ... & ... & ... \end{pmatrix} \qquad \vec{B} = \begin{pmatrix} b_1 & b_2 & 0 & 0 & ... \\ b_2 & b_1 & b_2 & 0 & ... \\ 0 & b_2 & b_2 & b_1 & ... \\ ... & ... & ... & ... & ... \end{pmatrix} \tag{4}$$

Where $\vec{A}, \vec{B}$ are tri-diagonal matrices with

$$a_1 = 1 + \tau\frac{i\hbar}{2ma^2} \quad a_2 = -\tau\frac{i\hbar}{4ma^2} \quad b_1 = 1 - \tau\frac{i\hbar}{2ma^2} \quad b_2 = \tau\frac{i\hbar}{4ma^2} \tag{5}$$

We solve the equation

$$\mathbf{\vec{v}} = \vec{B}\psi \tag{6}$$

In Equation 3, then solve the linear system $\vec{A}\vec{x} = \mathbf{\vec{v}}$. We can then loop over this method in time for the specified interval.

### *PART (B)*

Looping over the time interval from our initial condition of the wave-function, we get the Figure 1 for various time intervals in the well.

### *PART (C)*

We can see at a glance these wave-functions do not appear to be normalized, as the combinations of the heights and widths do not appear to integrate to one. I was not able to get the normalization factor (my own fault), but a segment in the textbook argues that the linearity of the wave-function causes the normalization constant to be canceled out on both sides of the equation and plays no part in the solution (pg. 451).

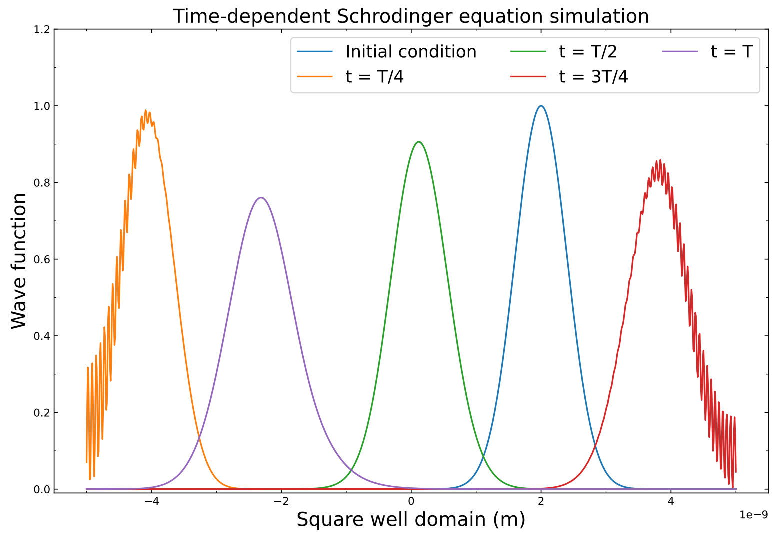


**Figure 1**: The initial condition on our wave-function has a Gaussian shape, and "bounces" off the right wall of the well and travels back towards the right.

## QUESTION 2 - RESONANT EM CAVITY

We have a two-dimensional resonance cavity with conducting walls of size $L_x \times L_y$. We excite the cavity with a $z$

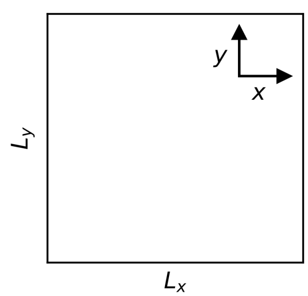


**Figure 2**: Two-dimensional resonance cavity.

directed alternating current of frequency $\omega$. Our goal is to calculate the electric and magnetic field patterns excited within the cavity by the current pattern.

### *PART (A)*

We can test the 2D Fourier transforms and their inverses by transforming a two dimensional array and then recovering it. For this we choose a simple even function, $x^2 + y^2$.

### *PART (B)*

With our Fourier functions working, we can now implement our oscillating current in the $z$ direction in the cavity. We begin with initial conditions $J_z = H_x = H_y = E_z = 0$, where $J_z$ is the current density in the $z$ direction, $H$ are scaled magnetic fields, and $E_z$ is the electric field in the $z$ direction. These electric and magnetic fields follow Maxwell's

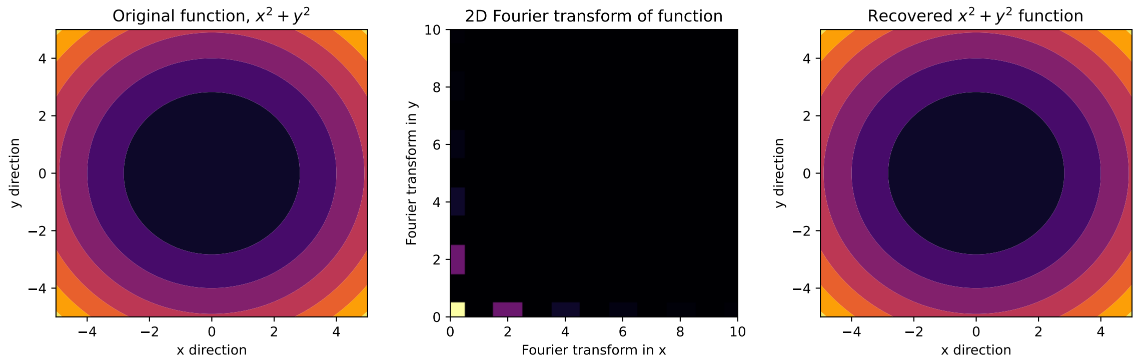


**Figure 3**: The even function $x^2 + y^2$ function was passed through the Fourier and inverse Fourier transforms to test the validity of our functions.

equations,

$$\begin{aligned} \frac{\partial H_x}{\partial t} + c\frac{\partial E_z}{\partial y} &= 0 \\ \frac{\partial H_y}{\partial t} + c\frac{\partial E_z}{\partial x} &= 0 \\ \frac{\partial E_z}{\partial t} + c\frac{\partial H_y}{\partial x} + c\frac{\partial H_x}{\partial y} &= J_z \end{aligned} \tag{7}$$

Where $c$ is the speed of light, and $\mu_0$ is the vacuum permeability. We use a Fourier transform to simplify the original PDE's for the Crank-Nicholson method.

$$(Ez)p, q^n = \sum q' = 0^P \sum p' = 0^P \hat{E}^n_{p',q'} \sin\left(\frac{pp'\pi}{P}\right) \sin\left(\frac{q'q\pi}{P}\right) \tag{8}$$

$$(Hx)p, q^n = \sum q' = 0^P \sum p' = 0^P \hat{X}^n_{p',q'} \sin\left(\frac{pp'\pi}{P}\right) \cos\left(\frac{q'q\pi}{P}\right) \tag{9}$$

$$(Hy)p, q^n = \sum q' = 0^P \sum p' = 0^P \hat{Y}^n_{p',q'} \cos\left(\frac{pp'\pi}{P}\right) \sin\left(\frac{q'q\pi}{P}\right) \tag{10}$$

$$(Jz)p, q^n = \sum q' = 0^P \sum p' = 0^P \hat{J}^n_{p',q'} \sin\left(\frac{pp'\pi}{P}\right) \sin\left(\frac{q'q\pi}{P}\right) \tag{11}$$

In doing so, we get the following plots for the scaled magnetic field in the $x$ and $y$ direction, as well as the electric field in the $z$ direction.

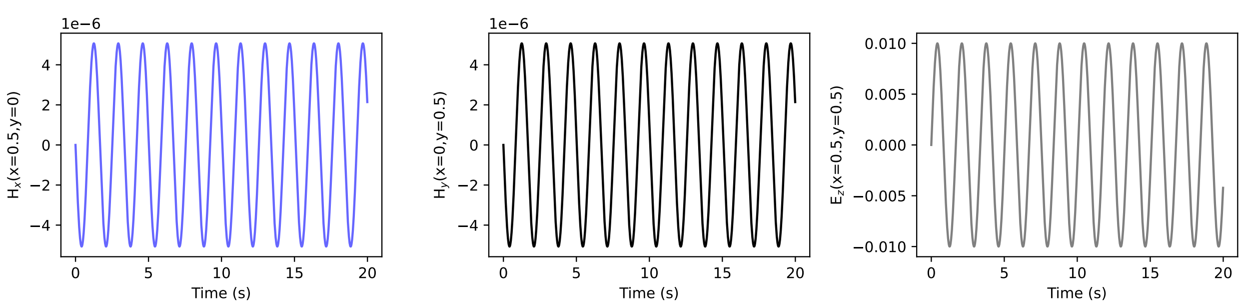


**Figure 4**: We see each field oscillates over time. The magnetic fields in the $x$ and $y$ direction oscillate in response to the alternating electric field. We see that the two fields are out of phase with one another, which makes physical sense and affirms our plots.

*PART (C)*

Although it is not obvious how the electric field should behave given the Fourier transformed function, Equations **??**, we do get a response that makes physical sense for our magnetic field, which is the response to the changing electric field, for it to be out of phase with the changing E-field.