# Deep Generative Models
# Lecture 11

Roman Isachenko

Moscow Institute of Physics and Technology

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# Disentangled representations

## Goal

Learning an interpretable factorised representation of the independent data gen- erative factors of the world without supervision.

## Informal definition

A disentangled representation can be defined as one where single latent units are sensitive to changes in single generative factors, while being relatively invariant to changes in other factors.

## Example

Model trained on a dataset of 3D objects might learn independent latent units sensitive to single independent data generative factors, such as object identity, position, scale, lighting or colour.

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https://openreview.net/references/pdf?id=Sy2fzU9gl

# $\beta$-VAE, 2017

## Generative process

- $p(\mathbf{x}|\mathbf{v},\mathbf{w}) = \mathrm{Sim}(\mathbf{v},\mathbf{w})$ – true world simulator;
- $\mathbf{v}$ – conditionally independent factors: $p(\mathbf{v}|\mathbf{x}) = \prod_{k=1}^{K} p(v_k|\mathbf{x})$;
- $\mathbf{w}$ – conditionally dependent factors.

## Goal

Develop an unsupervised deep generative model

$$p(\mathbf{x}|\mathbf{z}) \approx p(\mathbf{x}|\mathbf{v},\mathbf{w}).$$

- Ensure that the inferred latent factors $q(\mathbf{z}|\mathbf{x})$ capture the factors $\mathbf{v}$ in a disentangled manner.
- The conditionally dependent factors $\mathbf{w}$ can remain entangled in a separate subset of $\mathbf{z}$ that is not used for representing $\mathbf{v}$.

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https://openreview.net/references/pdf?id=Sy2fzU9gl

# $\beta$-VAE, 2017

## Constrained optimization

$$\max_{q,\boldsymbol{\theta}} \mathbb{E}_{q(\mathbf{z}|\mathbf{x})} \log p(\mathbf{x}|\mathbf{z},\boldsymbol{\theta}), \quad \text{subject to } KL(q(\mathbf{z}|\mathbf{x})||p(\mathbf{z})) < \epsilon.$$

## Objective

$$\mathcal{L}(q,\boldsymbol{\theta},\beta) = \mathbb{E}_{q(\mathbf{z}|\mathbf{x})} \log p(\mathbf{x}|\mathbf{z},\boldsymbol{\theta}) - \beta \cdot KL(q(\mathbf{z}|\mathbf{x})||p(\mathbf{z})).$$

What do we get at $\beta = 1$?

## Hypothesis

To learn disentangled representations of the conditionally independent factors $\mathbf{v}$, it is important to set stronger constraint on the latent bottleneck: $\beta > 1$.

**Note:** It could lead to poorer reconstructions due to the loss of high frequency details when passing through a constrained latent bottleneck.

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https://openreview.net/references/pdf?id=Sy2fzU9gl

# $\beta$-VAE, 2017

## Disentangling metric

Accuracy of classifier $p(y|\mathbf{z}_{\text{diff}})$ with a low VC-dimension in order to ensure that it has no capacity to perform nonlinear disentangling itself.

$$\mathbf{x}_{li} \sim \text{Sim}(\mathbf{v}_{li}, \mathbf{w}_{li}); \quad \mathbf{x}_{lj} \sim \text{Sim}(\mathbf{v}_{lj}, \mathbf{w}_{lj}); \quad y \sim U[1, K].$$

$$\mathbf{v}_{li} \sim p(\mathbf{v}); \quad \mathbf{w}_{li} \sim p(\mathbf{w}); \quad \mathbf{v}_{lj} \sim p(\mathbf{v})\,([v_{li}]_y = [v_{lj}]_y); \quad \mathbf{w}_{lj} \sim p(\mathbf{w}).$$

$$q(\mathbf{z}|\mathbf{x}) = \mathcal{N}\left(\mu(\mathbf{x})|\sigma^2(\mathbf{x})\right); \quad \mathbf{z}_{li} = \mu(\mathbf{x}_{li}); \quad \mathbf{z}_{lj} = \mu(\mathbf{x}_{lj}).$$

$$\mathbf{z}_{\text{diff}} = \frac{1}{L}\sum_{l=1}^{L} |\mathbf{z}_{li} - \mathbf{z}_{lj}|.$$

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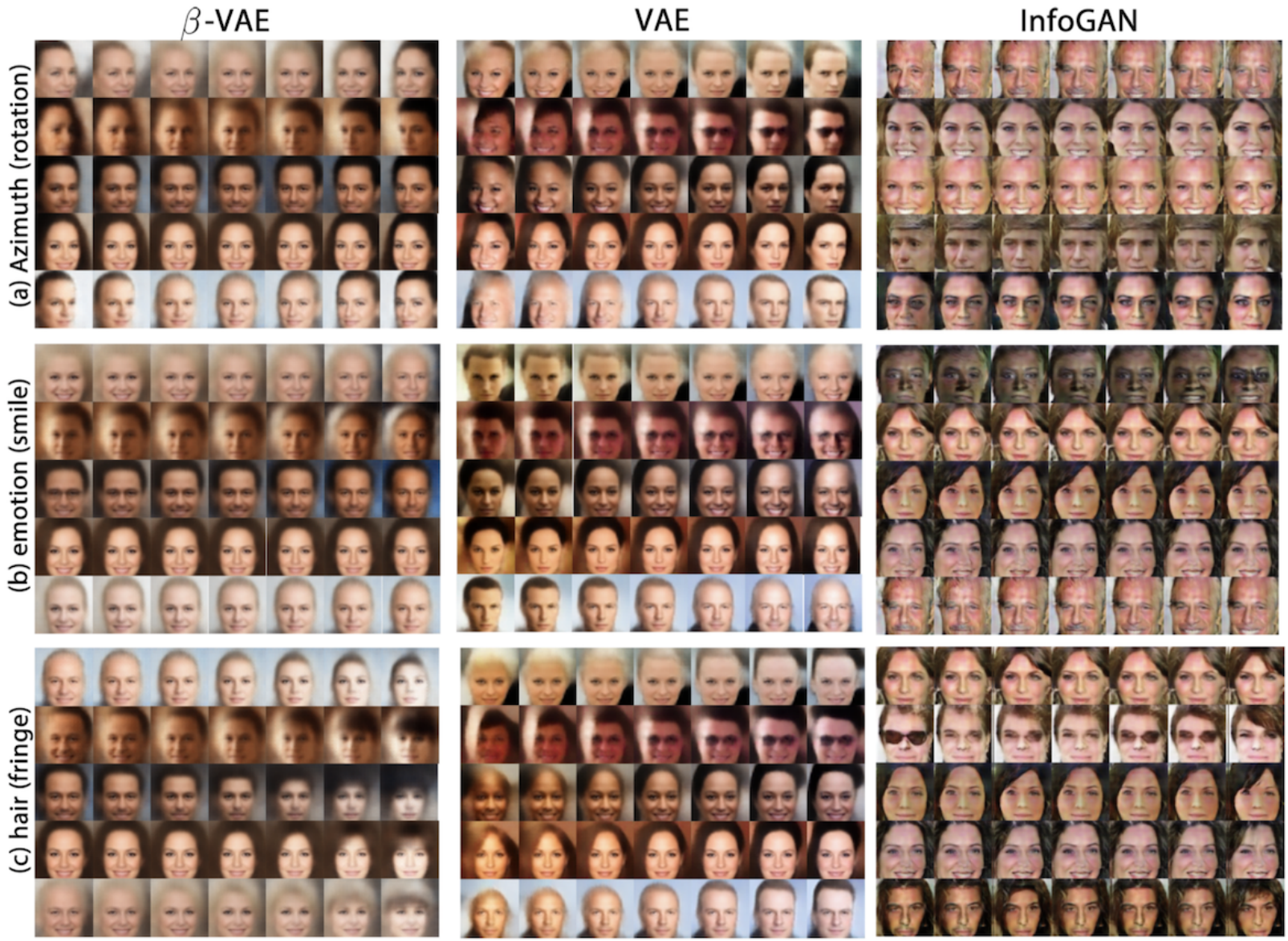
https://openreview.net/references/pdf?id=Sy2fzU9gl

# $\beta$-VAE, 2017

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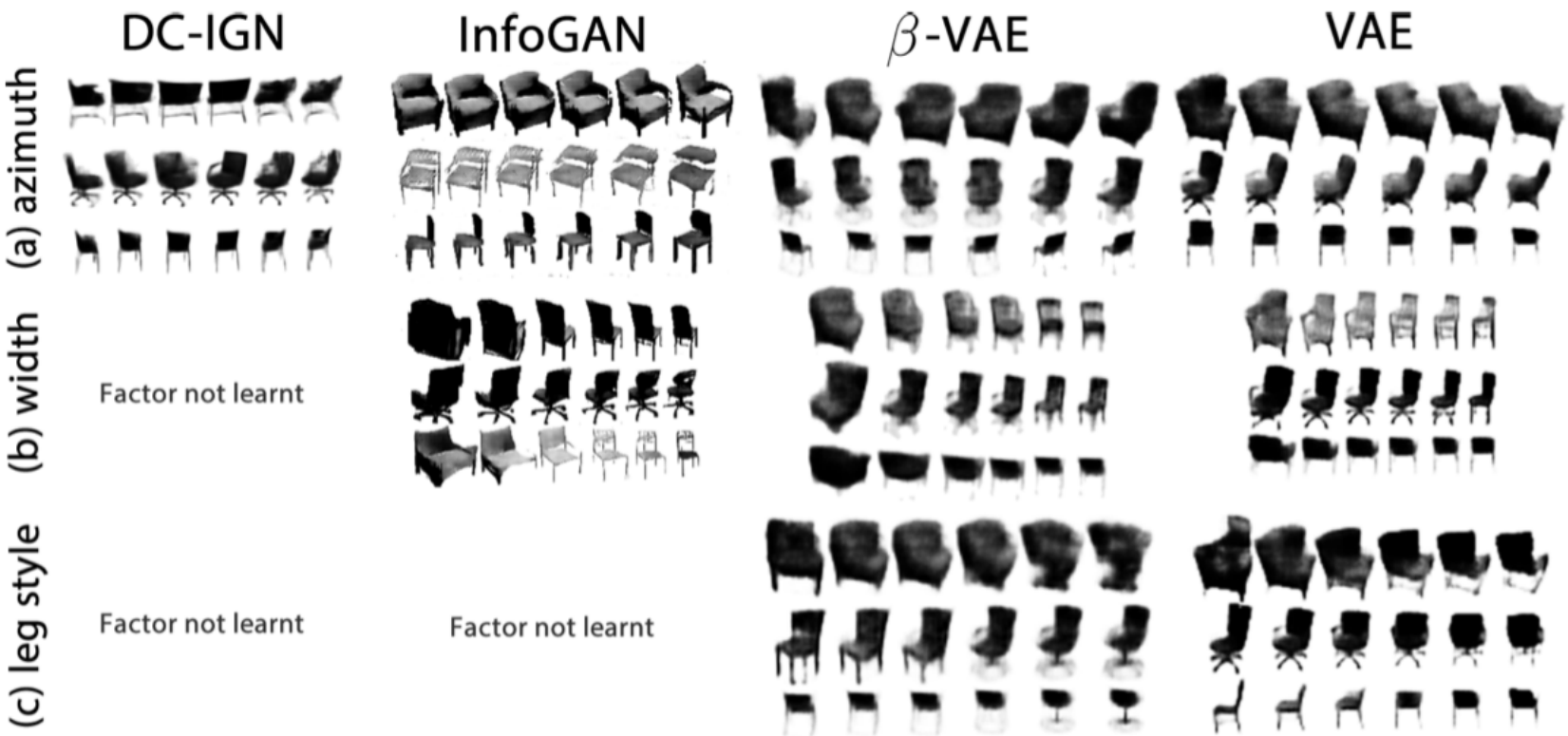
# $\beta$-VAE, 2017

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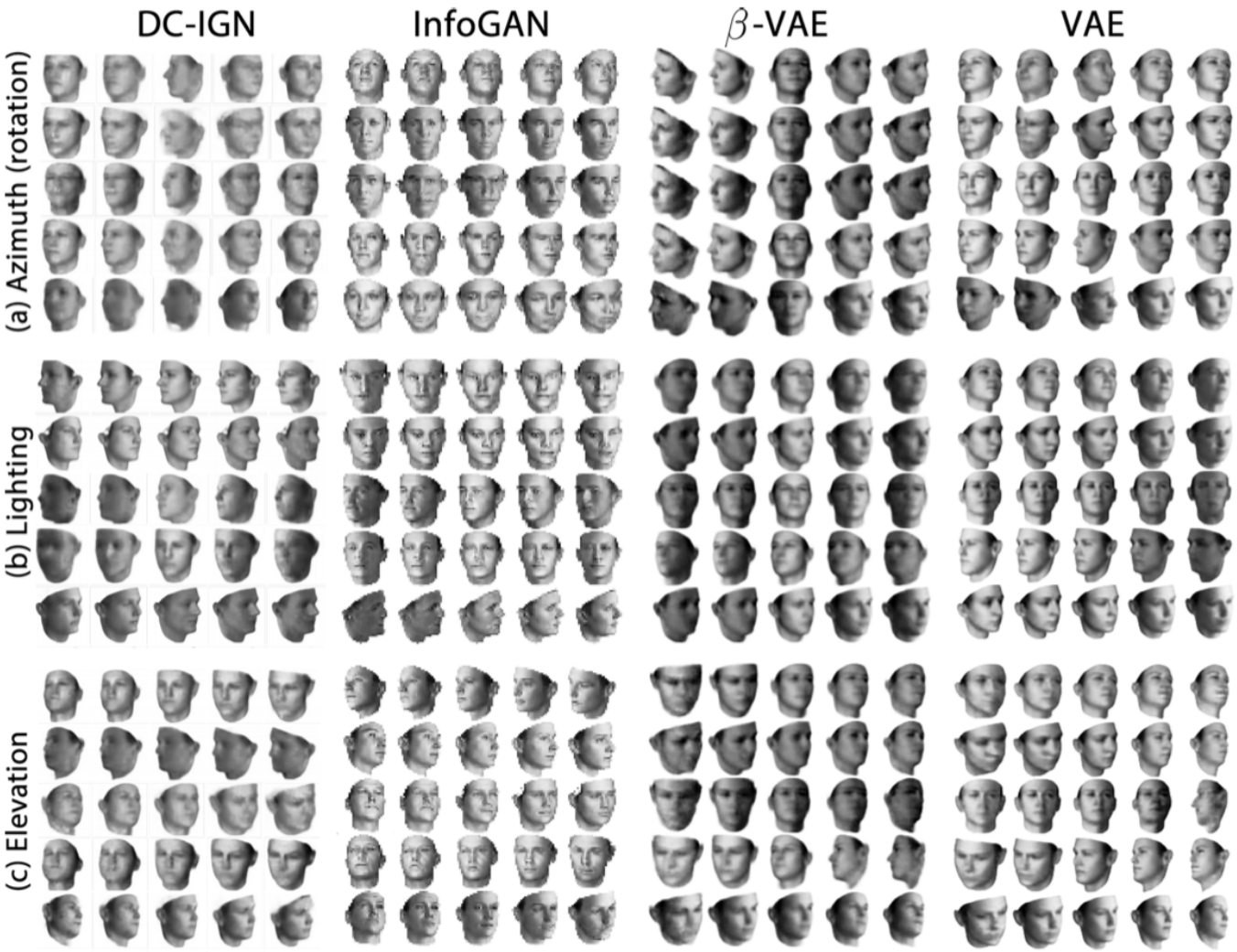
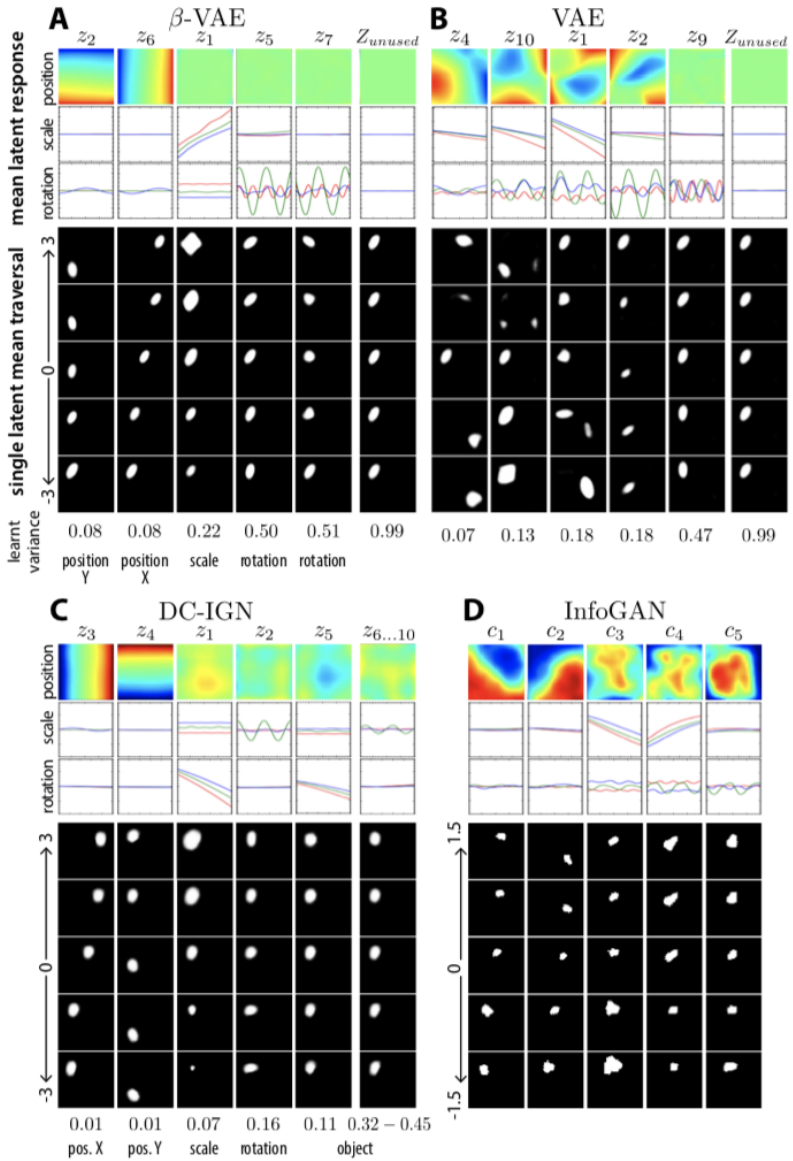
# $\beta$-VAE, 2017




https://openreview.net/references/pdf?id=Sy2fzU9gl

# $\beta$-VAE, 2017

| Model | Disentanglement metric score |
|---|---|
| *Ground truth* | *100%* |
| Raw pixels | $45.75 \pm 0.8\%$ |
| PCA | $84.9 \pm 0.4\%$ |
| ICA | $42.03 \pm 10.6\%$ |
| DC-IGN | $\mathbf{99.3 \pm 0.1\%}$ |
| InfoGAN | $73.5 \pm 0.9\%$ |
| VAE untrained | $44.14 \pm 2.5\%$ |
| VAE | $61.58 \pm 0.5\%$ |
| **$\beta$-VAE** | $\mathbf{99.23 \pm 0.1\%}$ |

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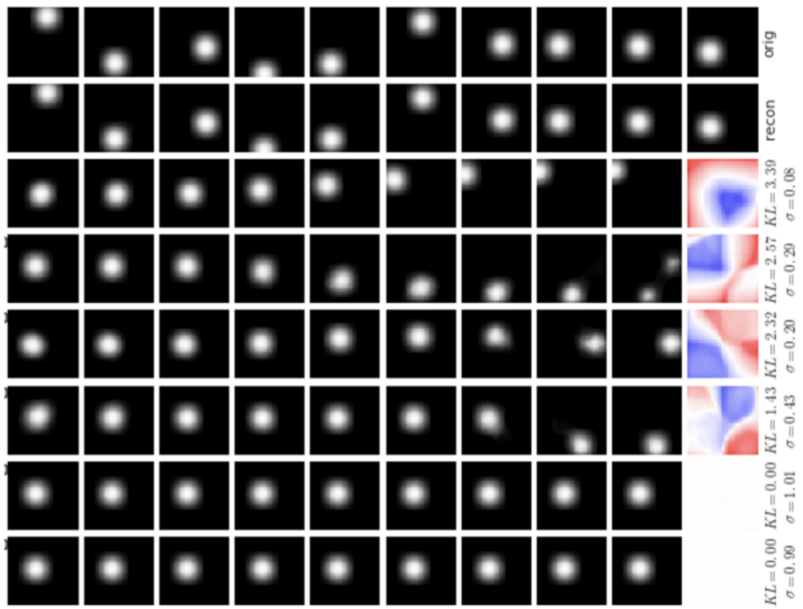
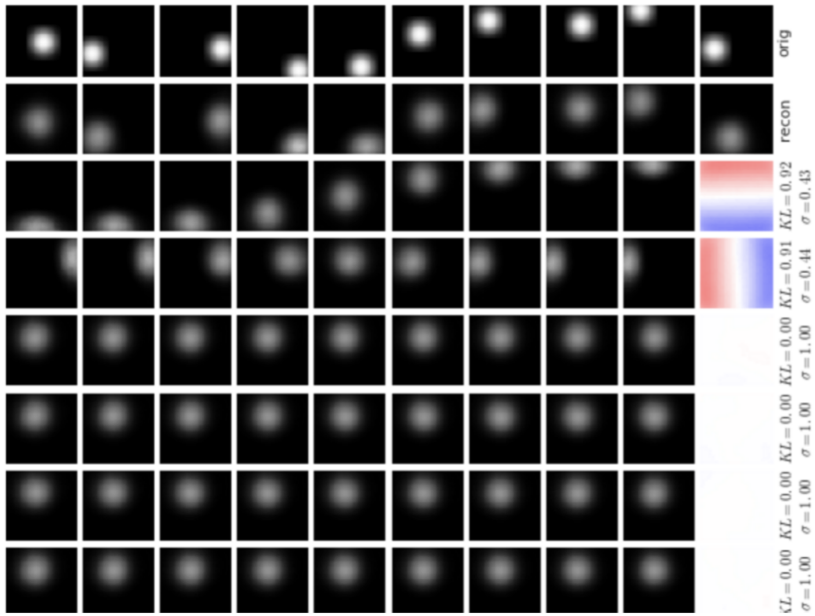
# $\beta$-VAE, 2018

β = 1




β = 150

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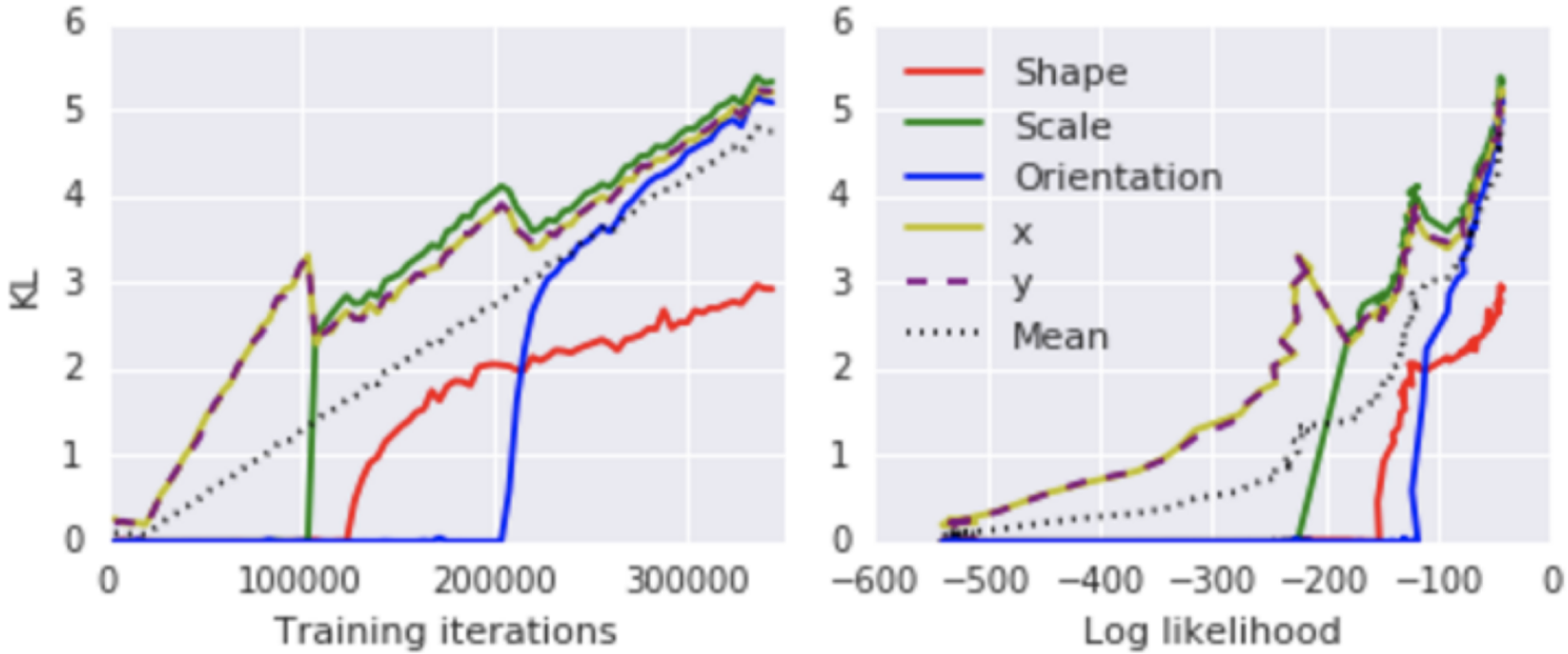
https://arxiv.org/pdf/1804.03599.pdf

# $\beta$-VAE, 2018

## Controlled encoding capacity

$$\mathcal{L}(q, \boldsymbol{\theta}, \beta) = \mathbb{E}_{q(\mathbf{z}|\mathbf{x})} \log p(\mathbf{x}|\mathbf{z}, \boldsymbol{\theta}) - |KL(q(\mathbf{z}|\mathbf{x})||p(\mathbf{z})) - C|.$$

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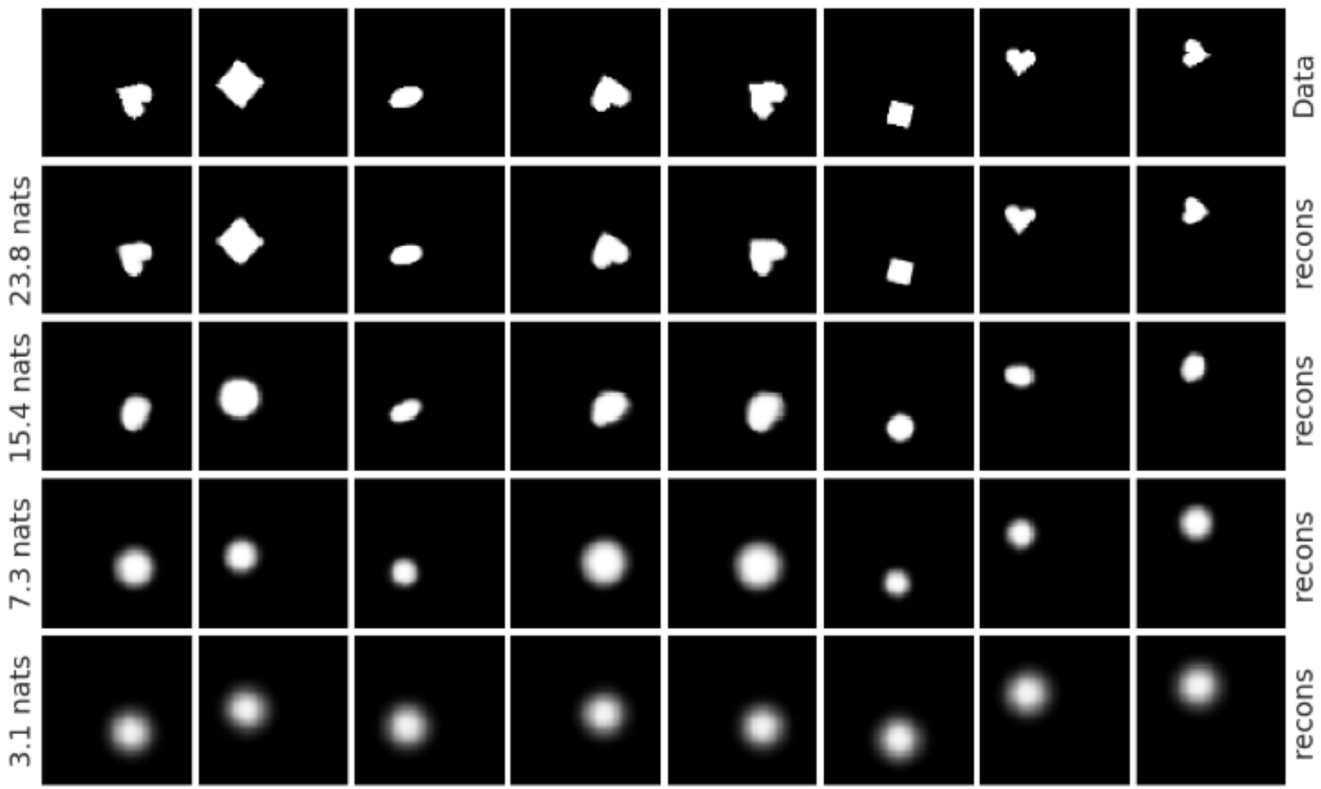


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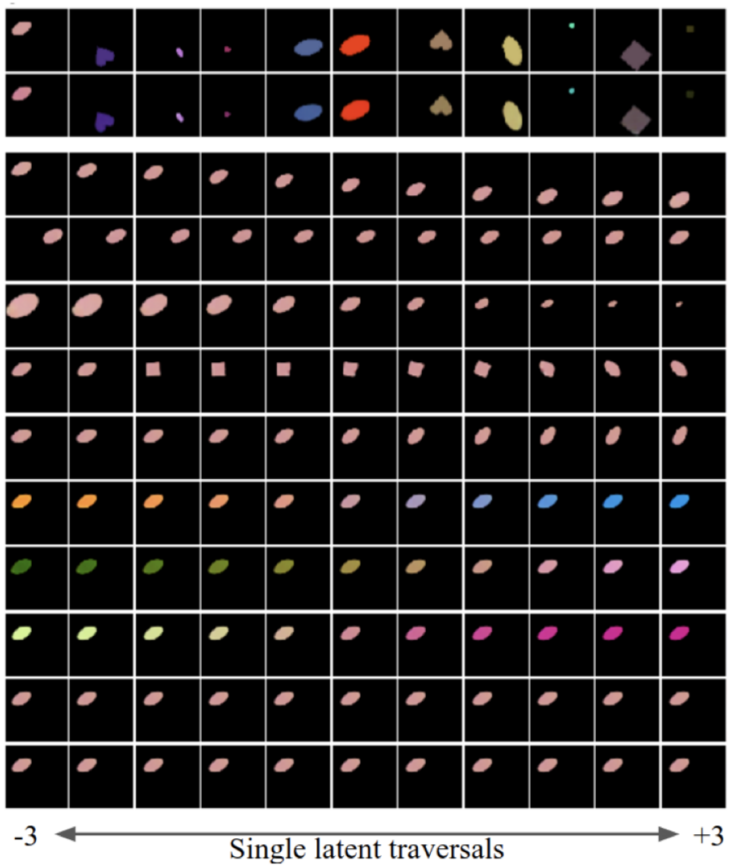
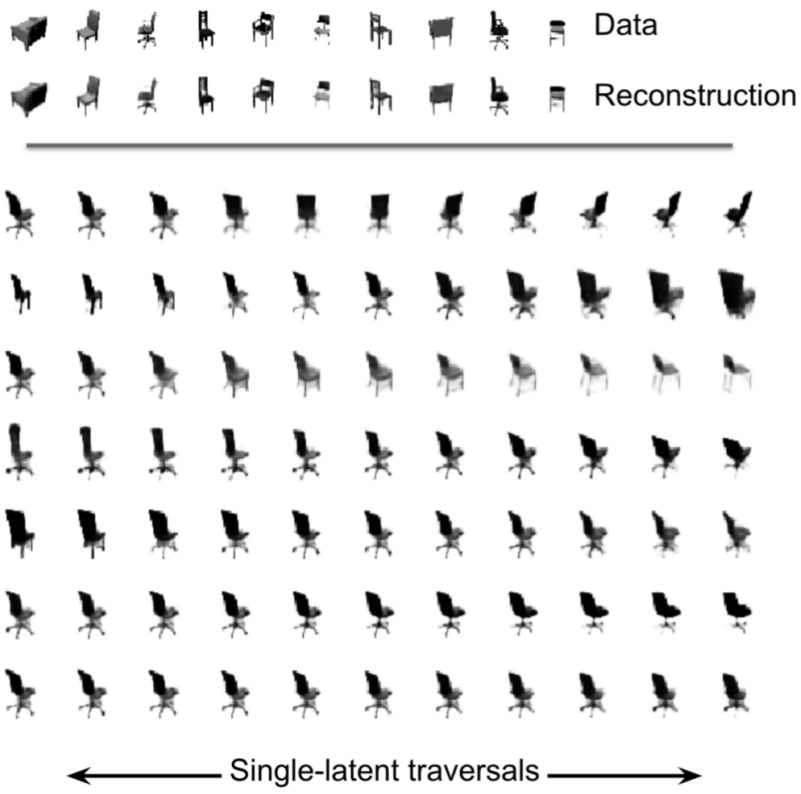
# $\beta$-VAE, 2018

(a) Coloured dSprites




(b) 3D Chairs




https://arxiv.org/pdf/1804.03599.pdf

# References


- **beta-VAE:** Learning Basic Visual Concepts with a Constrained Variational Framework
https://openreview.net/references/pdf?id=Sy2fzU9gl
**Summary:** Modifications of VAE objective. The task is represented as constrained optimization. Increasing the weight of KL divergence term in ELBO allows to disentangle latent space factors and makes model more interpretable. The assessment of disentanglement is provided by constructing the classifier.
- Understanding disentangling in $\beta$-**VAE**
https://arxiv.org/pdf/1804.03599.pdf
**Summary:** Consider beta-VAE from the position of the rate-distortion theory (information bottleneck). Propose the modified ELBO with controlled latent capacity.