Deep Generative Models
Lecture 8

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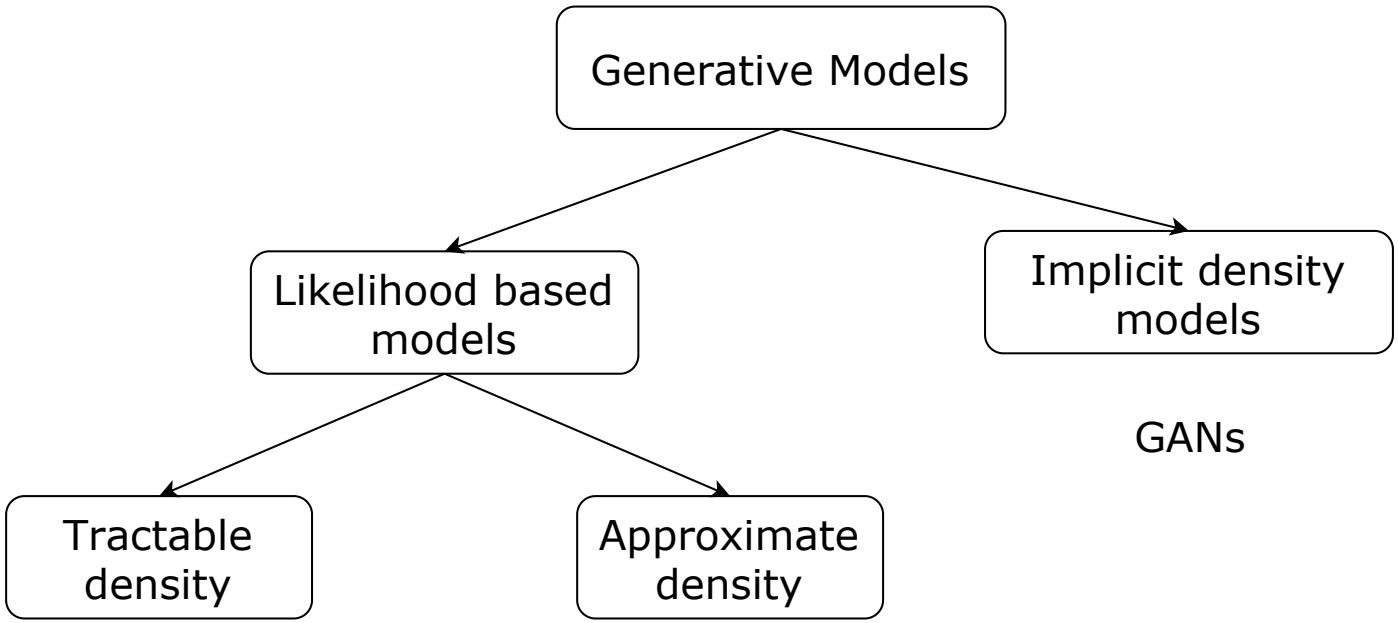
Generative models zoo



Likelihood based models

Is likelihood a good measure of model quality?

Poor likelihood
Great samples

$$p_1(\mathbf{x}) = \frac{1}{n}\sum_{i=1}^{n}\mathcal{N}(\mathbf{x}|\mathbf{x}_i, \epsilon\mathbf{I})$$

For small ϵ this model will generate samples with great quality, but likelihood will be very poor.

Great likelihood
Poor samples

$$p_2(\mathbf{x}) = 0.01p(\mathbf{x}) + 0.99p_{\text{noise}}(\mathbf{x})$$

$$\log\left[0.01p(\mathbf{x}) + 0.99p_{\text{noise}}(\mathbf{x})\right] \geq$$
$$\geq \log\left[0.01p(\mathbf{x})\right] = \log p(\mathbf{x}) - \log 100$$

Noisy irrelevant samples, but for high dimensions $\log p(\mathbf{x})$ becames larger.

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https://arxiv.org/abs/1511.01844

Likelihood-free learning

- Likelihood is not a perfect measure for quality measure for generative model.
- Likelihood could be intractable.

Where did we start

We would like to approximate true data distribution $\pi(\mathbf{x})$. Instead of searching true $\pi(\mathbf{x})$ over all probability distributions, learn function approximation $p(\mathbf{x}|\boldsymbol{\theta}) \approx \pi(\mathbf{x})$.

Imagine we have two sets of samples

- $\mathcal{S}_1 = \{\mathbf{x}_i\}_{i=1}^{n_1} \sim \pi(\mathbf{x})$
- $\mathcal{S}_2 = \{\mathbf{x}_i\}_{i=1}^{n_2} \sim p(\mathbf{x}|\boldsymbol{\theta})$

Two sample test

$$H_0 : \pi(\mathbf{x}) = p(\mathbf{x}|\boldsymbol{\theta}), \quad H_1 : \pi(\mathbf{x}) \neq p(\mathbf{x}|\boldsymbol{\theta})$$

Likelihood-free learning

Two sample test

$$H_0 : \pi(\mathbf{x}) = p(\mathbf{x}|\boldsymbol{\theta}), \quad H_1 : \pi(\mathbf{x}) \neq p(\mathbf{x}|\boldsymbol{\theta})$$

Define test statistic $T(\mathcal{S}_1, \mathcal{S}_2)$. Test statictic is likelihood free.
If $T(\mathcal{S}_1, \mathcal{S}_2) < \alpha$, then accept H_0, else reject it.

Desired behaviour

- The generative model $p(\mathbf{x}|\boldsymbol{\theta})$ minimizes the value of test statistic $T(\mathcal{S}_1, \mathcal{S}_2)$.
- Find appropriate test statistic in high dimensions is hard. We could try to learn the appropriate $T(\mathcal{S}_1, \mathcal{S}_2)$.

Vanila GAN

- **Generator:** latent variable model $\mathbf{x} = G(\mathbf{z})$, which minimizes two-sample test objective.
- **Discriminator:** function $D(\mathbf{x})$, which distinguish real samples from model samples and maximizes two-sample test statistic.

Objective

$$V(G, D) = \min_G \max_D \mathbb{E}_{\pi(\mathbf{x})} \log D(\mathbf{x}) + \mathbb{E}_{p(\mathbf{z})} \log(1 - D(G(\mathbf{z})))$$

For fixed generator $G(\mathbf{z})$ discriminator is performing binary classification with cross entropy loss.
This minimax game has global optimum $\pi(\mathbf{x}) = p(\mathbf{x}|\boldsymbol{\theta})$.

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https://arxiv.org/abs/1406.2661

Vanila GAN optimality

Objective (fixed G)

$$\max_D V(D) = \mathbb{E}_{\pi(\mathbf{x})} \log D(\mathbf{x}) + \mathbb{E}_{p(\mathbf{z})} \log(1 - D(G(\mathbf{z})))$$

Optimal discriminator

$$\begin{aligned} V(G, D) &= \mathbb{E}_{\pi(\mathbf{x})} \log D(\mathbf{x}) + \mathbb{E}_{p(\mathbf{x}|\boldsymbol{\theta})} \log(1 - D(\mathbf{x})) \\ &= \int \underbrace{[\pi(\mathbf{x}) \log D(\mathbf{x}) + p(\mathbf{x}|\boldsymbol{\theta}) \log(1 - D(\mathbf{x})]}_{y(D)} \, d\mathbf{x} \end{aligned}$$

$$\frac{dy(D)}{dD} = \frac{\pi(\mathbf{x})}{D(\mathbf{x})} - \frac{p(\mathbf{x}|\boldsymbol{\theta})}{1 - D(\mathbf{x})} = 0 \quad \Rightarrow \quad D^*(\mathbf{x}) = \frac{\pi(\mathbf{x})}{\pi(\mathbf{x}) + p(\mathbf{x}|\boldsymbol{\theta})}$$

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https://arxiv.org/abs/1406.2661

Vanila GAN optimality

Objective (fixed D)

$$\max_G V(G, D^*) = \mathbb{E}_{\pi(\mathbf{x})} \log D^*(\mathbf{x}) + \mathbb{E}_{p(\mathbf{z})} \log(1 - D^*(G(\mathbf{z})))$$

Optimal generator

$$V(G, D^*) = \mathbb{E}_{\pi(\mathbf{x})} \log \frac{\pi(\mathbf{x})}{\pi(\mathbf{x}) + p(\mathbf{x}|\boldsymbol{\theta})} + \mathbb{E}_{p(\mathbf{x}|\boldsymbol{\theta})} \log \frac{p(\mathbf{x}|\boldsymbol{\theta})}{\pi(\mathbf{x}) + p(\mathbf{x}|\boldsymbol{\theta})}$$

$$= KL\left(\pi(x)||\frac{\pi(\mathbf{x}) + p(\mathbf{x}|\boldsymbol{\theta})}{2}\right) + KL\left(p(x|\boldsymbol{\theta})||\frac{\pi(\mathbf{x}) + p(\mathbf{x}|\boldsymbol{\theta})}{2}\right) - 2\log 2$$

$$= 2JSD(\pi(\mathbf{x})||p(\mathbf{x}|\boldsymbol{\theta})) - 2\log 2.$$

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https://arxiv.org/abs/1406.2661

Vanila GAN optimality

Optimal generator

$$V(G, D^*) = 2JSD(\pi(\mathbf{x})||p(\mathbf{x}|\boldsymbol{\theta})) - 2\log 2.$$

Jensen-Shannon divergence

$$JSD(\pi(\mathbf{x})||p(\mathbf{x}|\boldsymbol{\theta})) = \frac{1}{2}\left[KL\left(\pi(x)||\frac{\pi(\mathbf{x}) + p(\mathbf{x}|\boldsymbol{\theta})}{2}\right) + KL\left(p(x|\boldsymbol{\theta})||\frac{\pi(\mathbf{x}) + p(\mathbf{x}|\boldsymbol{\theta})}{2}\right)\right]$$

Also called symmetric KL divergence. Could be used as a distance measure!

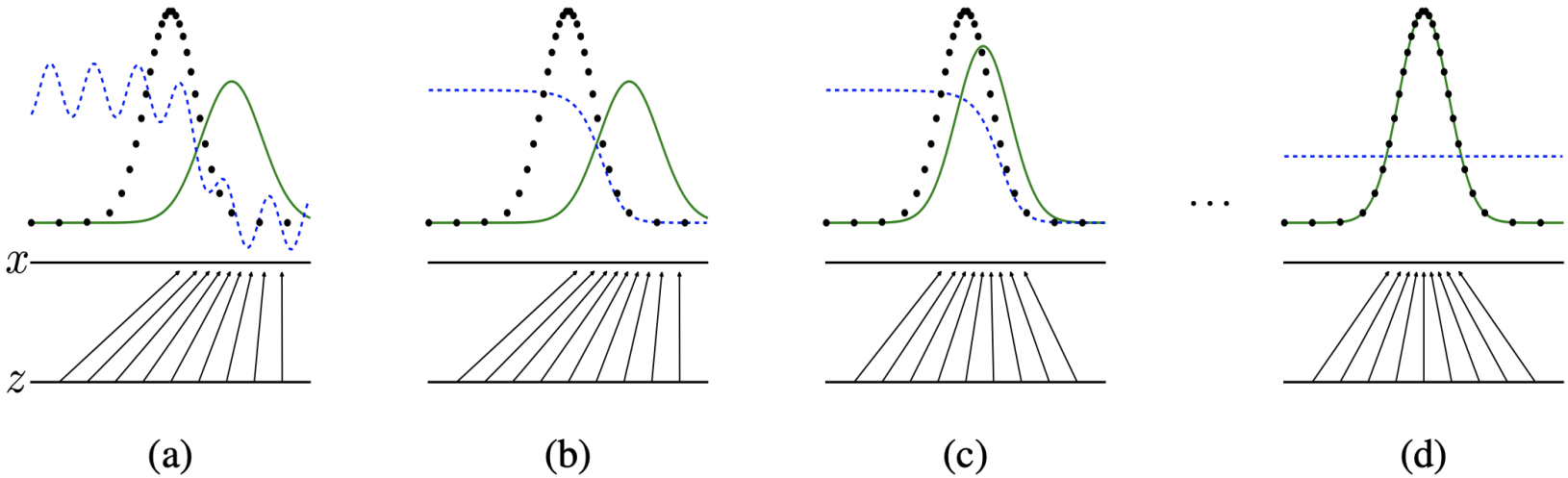
$$V(G^*, D^*) = -2\log 2, \quad \pi(\mathbf{x}) = p(\mathbf{x}|\boldsymbol{\theta}).$$

If the generator updates are made in function space and discriminator is optimal at every step, then the generator is guaranteed to converge to the data distribution.

Vanila GAN

Objective

$$V(G, D) = \min_G \max_D \mathbb{E}_{\pi(\mathbf{x})} \log D(\mathbf{x}) + \mathbb{E}_{p(\mathbf{z})} \log(1 - D(G(\mathbf{z})))$$



- Generator updates are made in parameter space.
- Discriminator is not optimal at every step.
- Generator and discriminator loss keeps oscillatingduring GAN training.

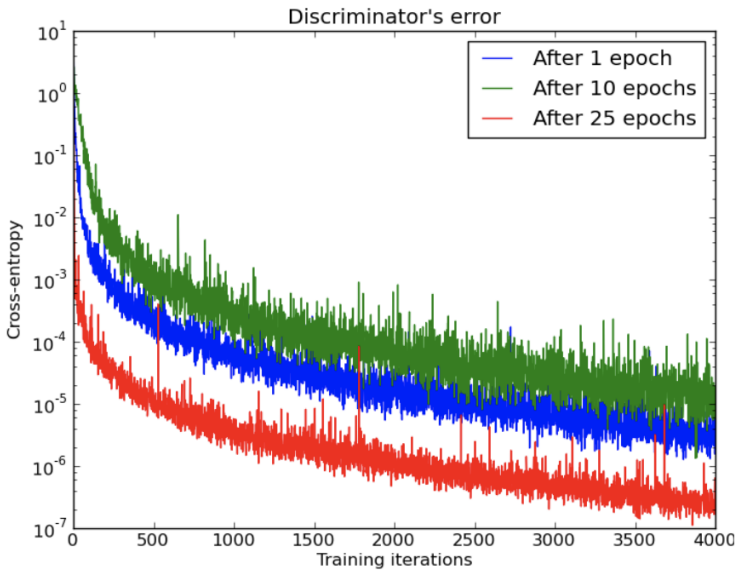
https://arxiv.org/abs/1406.2661

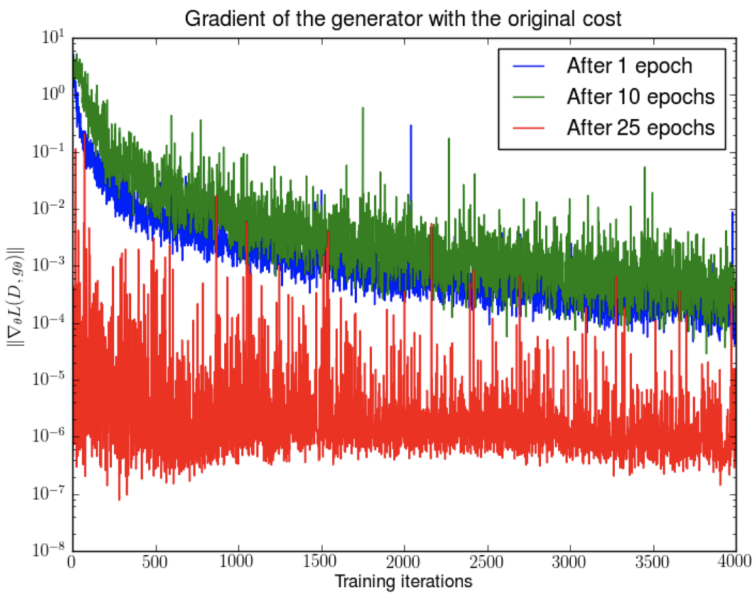
Vanishing gradients

Objective

$$V(G, D) = \min_G \max_D \mathbb{E}_{\pi(\mathbf{x})} \log D(\mathbf{x}) + \mathbb{E}_{p(\mathbf{z})} \log(1 - D(G(\mathbf{z})))$$

Early in learning, G is poor, D can reject samples with high confidence. In this case, $\log(1 - D(G(\mathbf{z})))$ saturates.





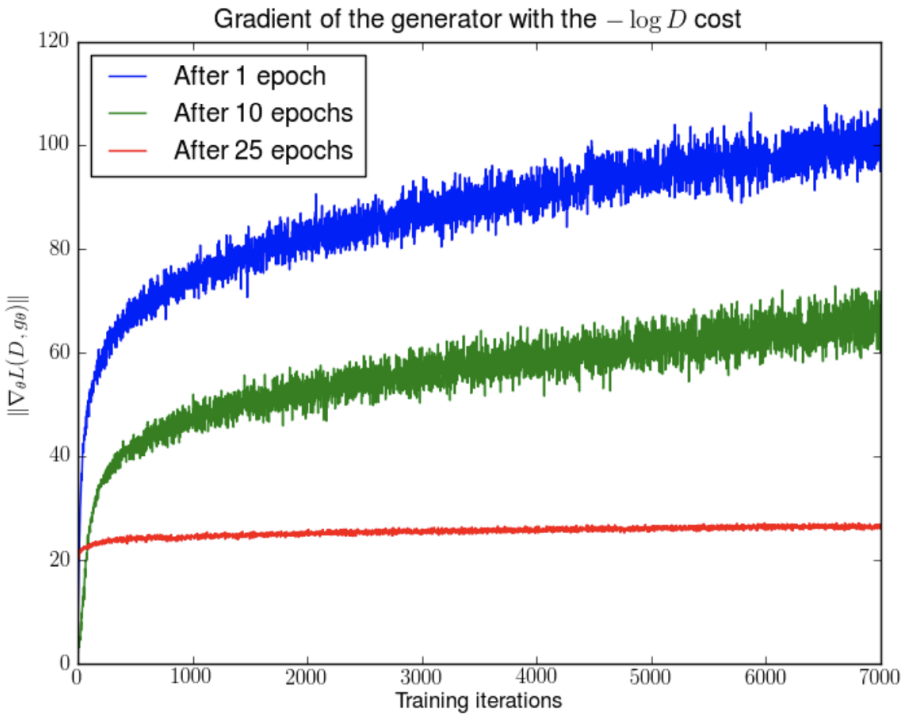
https://arxiv.org/abs/1701.04862

Vanishing gradients

Objective

$$V(G, D) = \min_G \max_D \mathbb{E}_{\pi(\mathbf{x})} \log D(\mathbf{x}) + \mathbb{E}_{p(\mathbf{z})} \log(1 - D(G(\mathbf{z})))$$

Maximize $\log D(G(z))$ instead of $\log(1 - D(G(\mathbf{z})))$.
Gradients are getting much stronger, but the training is instable (with increasing mean and variance).



https://arxiv.org/abs/1701.04862

Likelihood-based vs GAN

GAN objective (for optimal discriminator)

$$V(G, D^*) = 2JSD(\pi(\mathbf{x})||p(\mathbf{x}|\boldsymbol{\theta})) - \log 4.$$

Likelihood model objective

$$\max_{\boldsymbol{\theta}} \log p(\mathbf{X}|\boldsymbol{\theta}) \approx \max_{\boldsymbol{\theta}} \mathbb{E}_{\pi(\mathbf{x})} \log p(\mathbf{x}|\boldsymbol{\theta}) =$$

$$= \max_{\boldsymbol{\theta}} \mathbb{E}_{\pi(\mathbf{x})} \log p(\mathbf{x}|\boldsymbol{\theta}) + \mathbb{E}_{\pi(\mathbf{x})} \log \pi(\mathbf{x}) =$$

$$= \max_{\boldsymbol{\theta}} \mathbb{E}_{\pi(\mathbf{x})} \log \frac{p(\mathbf{x}|\boldsymbol{\theta})}{\pi(\mathbf{x})} = \min_{\boldsymbol{\theta}} KL(\pi(\mathbf{x})||p(\mathbf{x}|\boldsymbol{\theta}))$$

What is the difference between JS and KL divergences?

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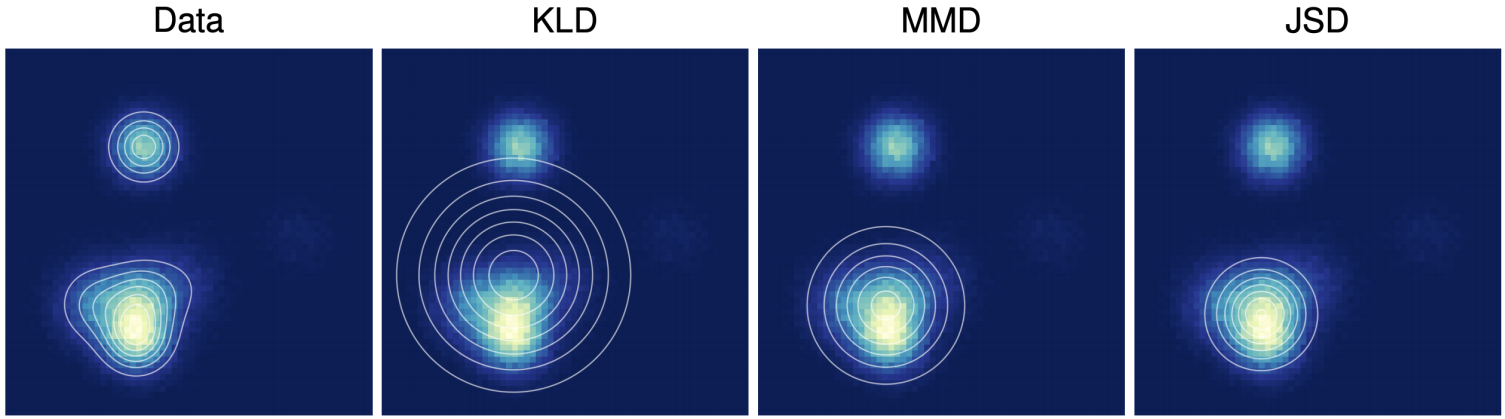
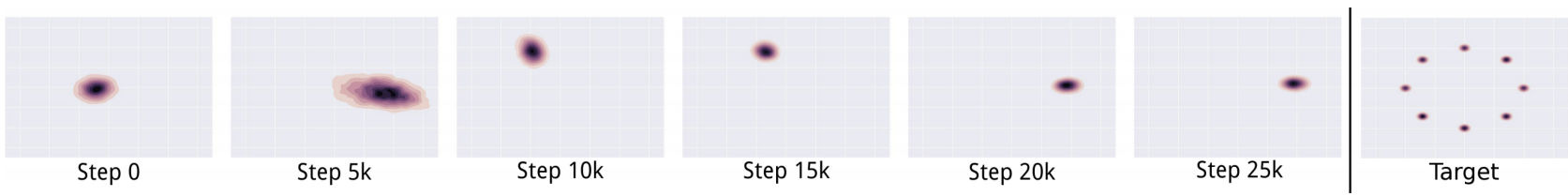
https://arxiv.org/abs/1511.01844

Mode collapse

The phenomena where the generator of a GAN collapses to one or few distribution modes.





Alternate architectures, adding regularization terms, injecting small noise perturbations and other millions bags and tricks are used to avoid the mode collapse.

https://arxiv.org/abs/1406.2661

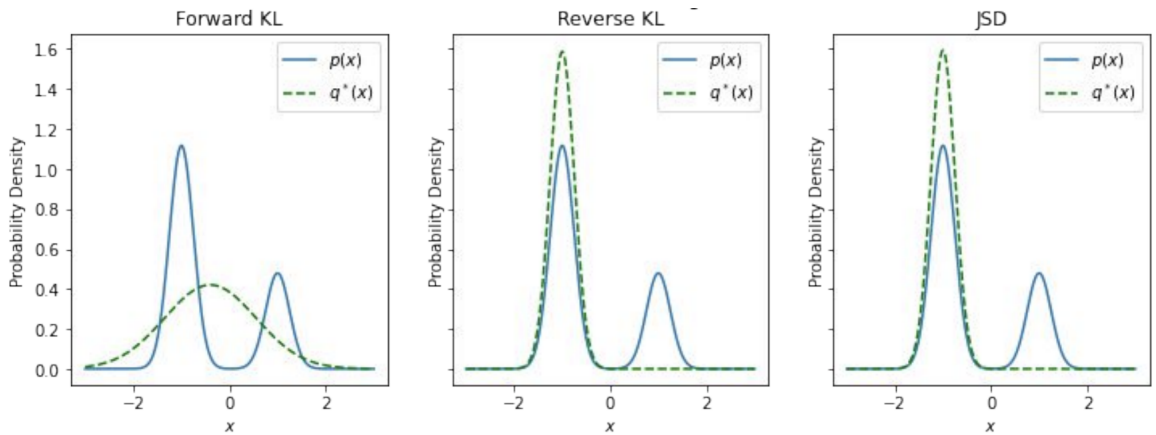
https://arxiv.org/abs/1611.02163

Mode collapse

Mode covering vs mode seeking

$$KL(\pi||p) = \int \pi(\mathbf{x}) \log \frac{\pi(\mathbf{x})}{p(\mathbf{x})} d\mathbf{x}, \quad KL(p||\pi) = \int p(\mathbf{x}) \log \frac{p(\mathbf{x})}{\pi(\mathbf{x})} d\mathbf{x}$$

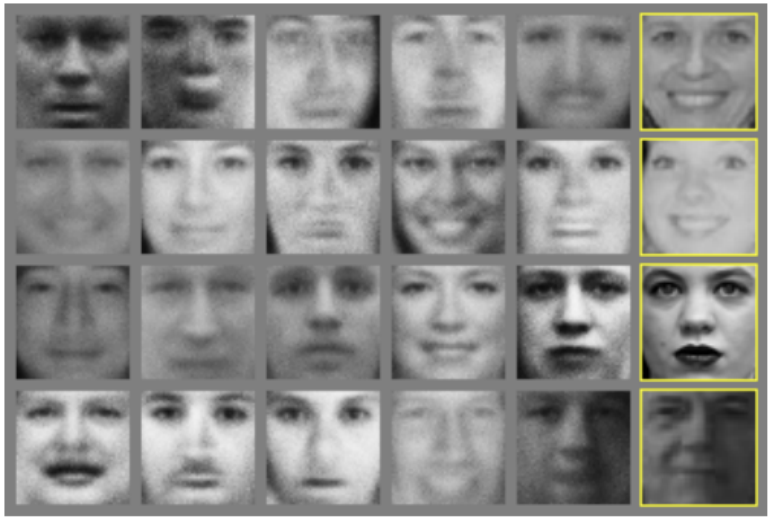
$$JSD(\pi||p) = \frac{1}{2}\left[KL\left(\pi(x)||\frac{\pi(\mathbf{x}) + p(\mathbf{x})}{2}\right) + KL\left(p(x)||\frac{\pi(\mathbf{x}) + p(\mathbf{x})}{2}\right)\right]$$



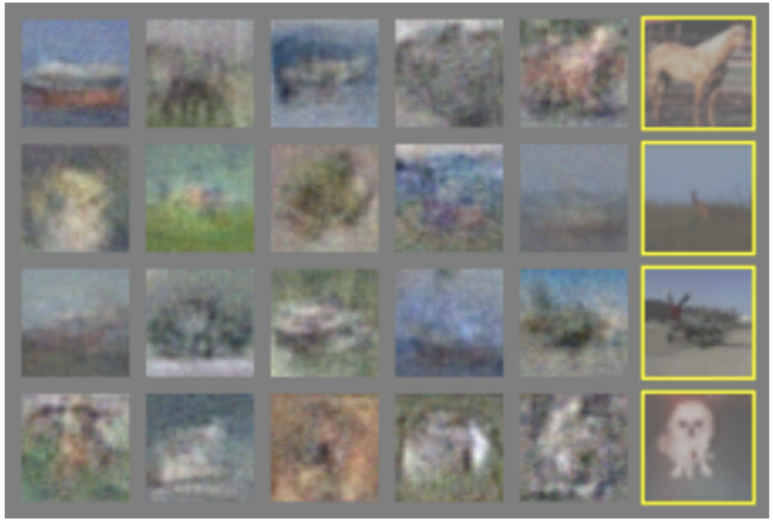
https://sites.google.com/view/berkeley-cs294-158-sp20/home

Vanila GAN results

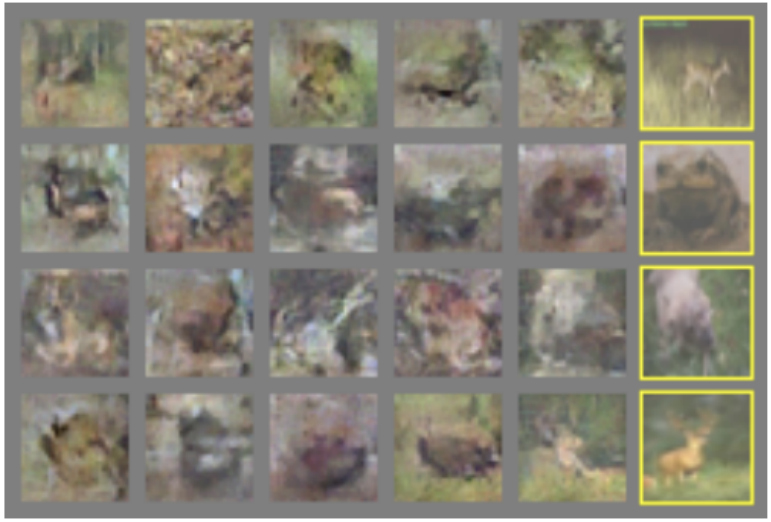
a)



b)



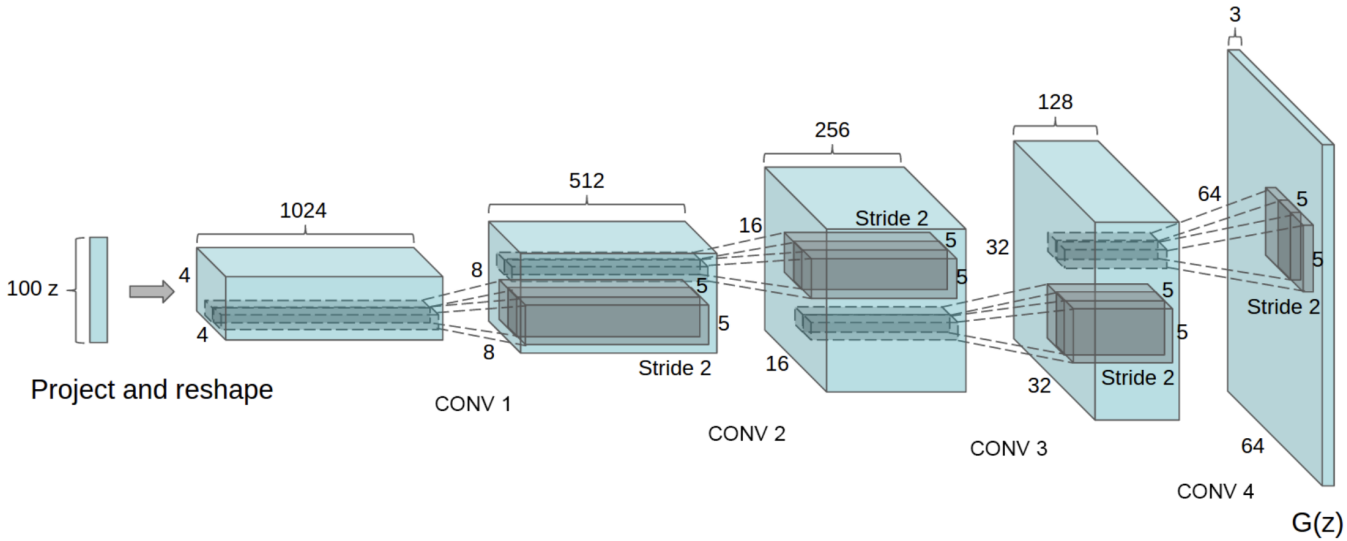
c)



d)

https://arxiv.org/abs/1406.2661

Deep Convolutional GAN



- Mean-pooling instead of max-pooling.
- Transposed convolutions in the generator for upsampling.
- Downsample with strided convolutions and average pooling.
- ReLU for generator, Leaky-ReLU (0.2) for discriminator.
- Output nonlinearity: tanh for Generator, sigmoid for discriminator.
- Batch Normalization used to prevent mode collapse (not applied at the output of G and input of D).
- Adam: small LR = 2e-4; small momentum: 0.5, batch-size: 128.

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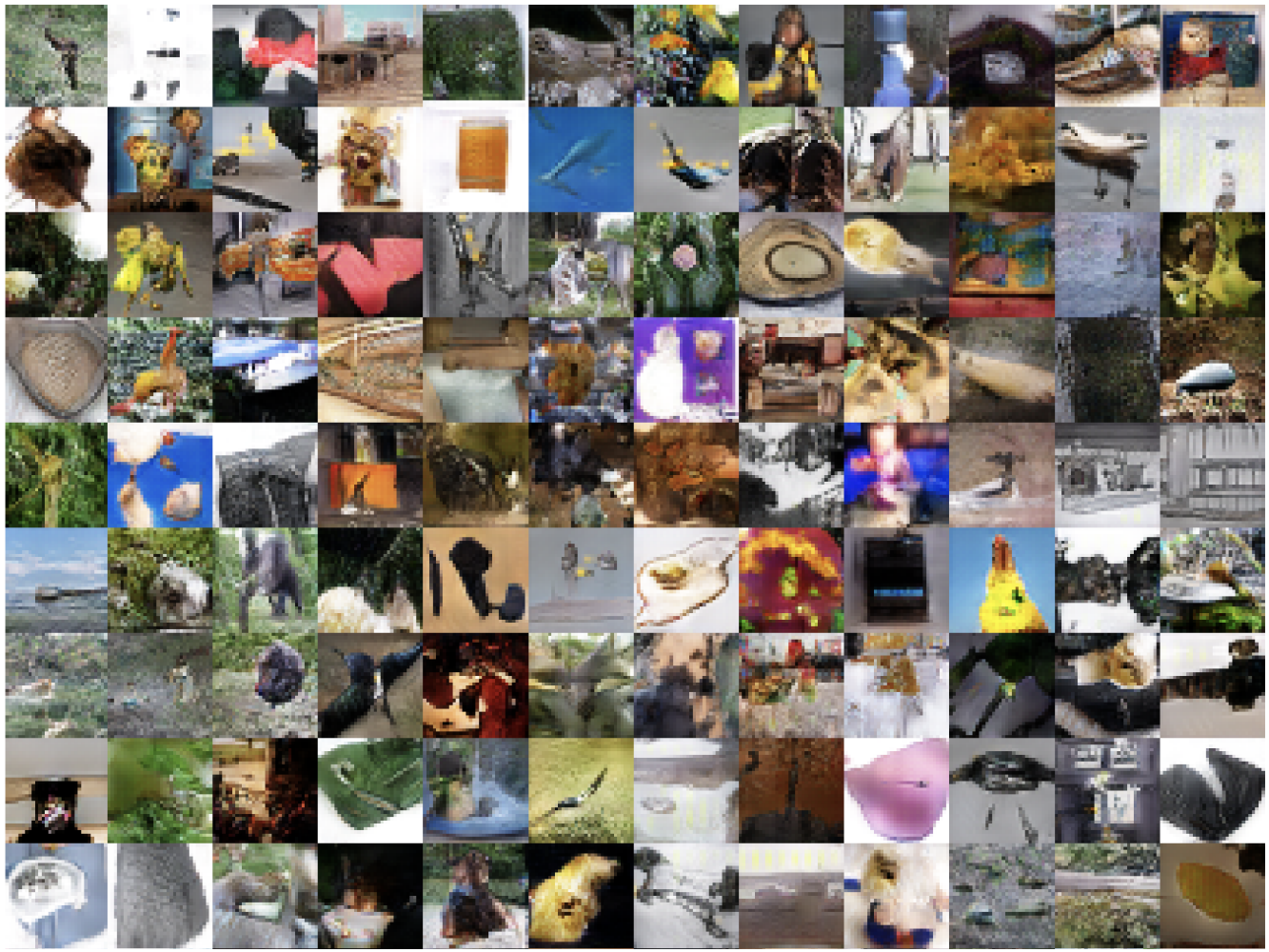
https://arxiv.org/abs/1511.06434

Deep Convolutional GAN

ImageNet samples



https://arxiv.org/abs/1511.06434

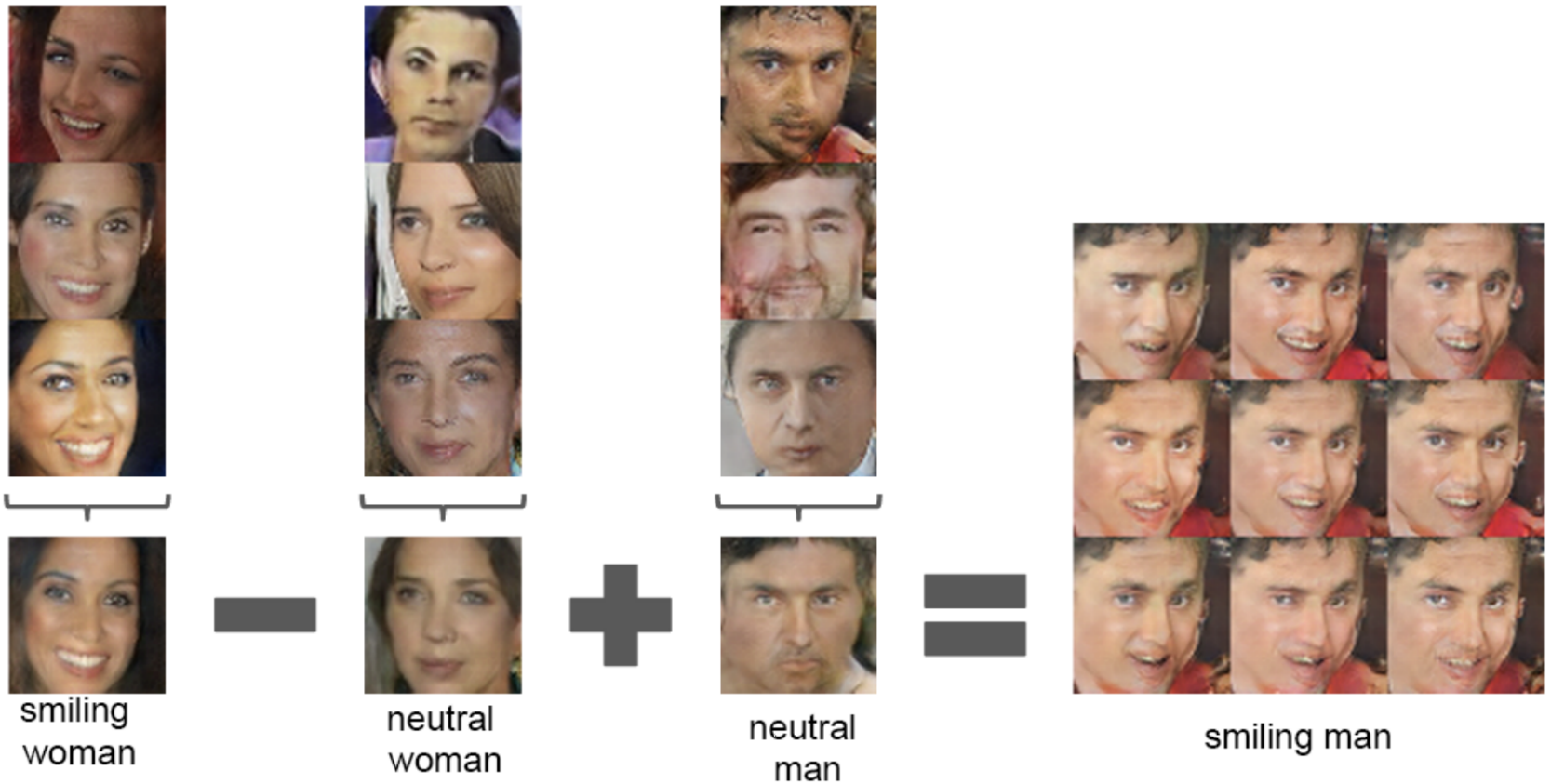
Deep Convolutional GAN

Smooth interpolations

https://arxiv.org/abs/1511.06434

Deep Convolutional GAN

Vector arithmetic



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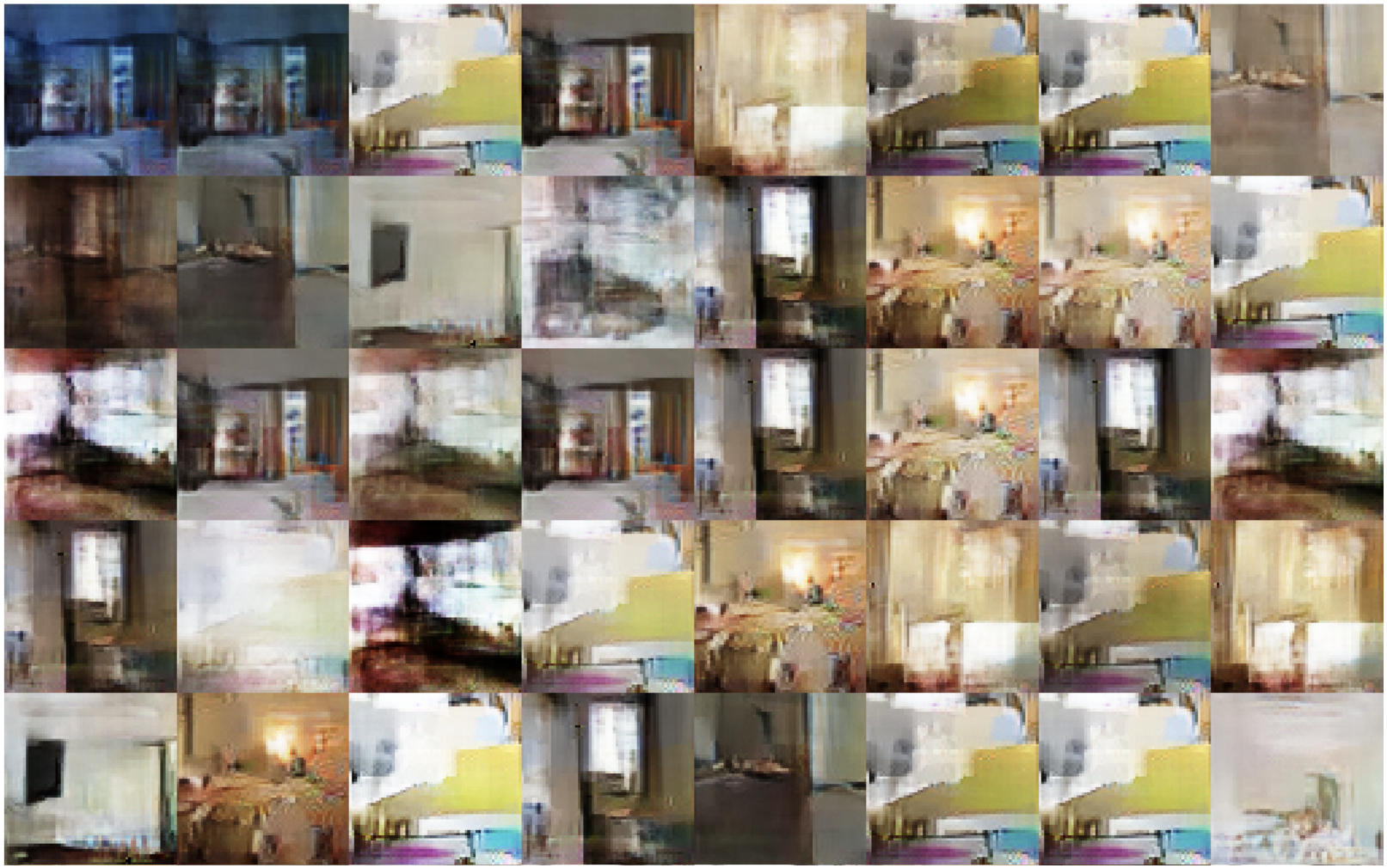
Deep Convolutional GAN

Mode collapse



https://arxiv.org/abs/1701.07875

Improved techniques for training GANs

- Feature matching

$$\mathcal{L}_G = \|\mathbb{E}_{\pi(\mathbf{x})}\mathbf{d}(\mathbf{x}) - \mathbb{E}_{p(\mathbf{z})}\mathbf{d}(G(\mathbf{z}))\|_2^2$$

  Here $\mathbf{d}(\mathbf{x})$ – intermediate layer of discriminator. Matching the learned discriminator statistics instead of the output of the discriminator. Helps to avoid the vanishing gradients for sufficiently good discriminator.

- Historical averaging adds extra loss term for generator and discriminator losses

$$\|\boldsymbol{\theta} - \sum_{t=1}^{T} \boldsymbol{\theta}_t\|_2^2.$$

  Here $\boldsymbol{\theta}_t$ – value of parameters at the previous step t. It allows to stabilize training procedure.

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https://arxiv.org/abs/1606.03498

Improved techniques for training GANs

- One-sided label smoothing. Instead of using one-hot labels in classification, use $(1 - \alpha)$ for real data (the generated samples are not smoothed).

$$D^*(\mathbf{x}) = \frac{(1-\alpha)\pi(\mathbf{x})}{\pi(\mathbf{x}) + p(\mathbf{x}|\boldsymbol{\theta})}$$

- Virtual batch normalization. BatchNorm makes samples within minibatch are highly correlated.

Use reference fixed batch to compute the normalization statistics. To avoid overfitting construct batch with the reference batch and the current sample.

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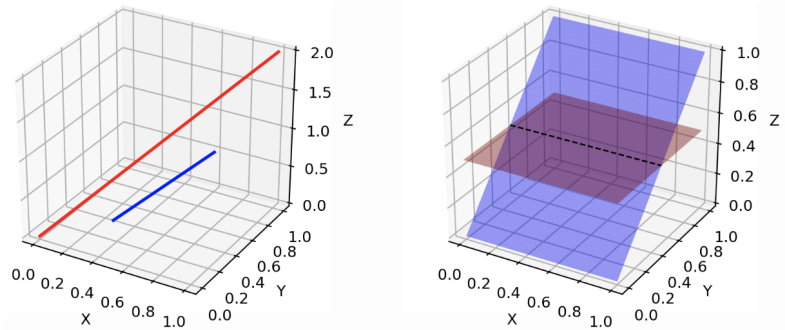
https://arxiv.org/abs/1606.03498

Informal theoretical results

- Since $\mathbf{z}$ usually has lower dimensionality compared to $\mathbf{x}$, manifold $G(\mathbf{z})$ has a measure 0 in $\mathbf{x}$ space. Hence, support of $p(\mathbf{x}|\boldsymbol{\theta})$ lies on low-dimensional manifold.
- Distribution of real images $\pi(\mathbf{x})$ is also concentrated on a low dimensional manifold.



- If $\pi(\mathbf{x})$ and $p(\mathbf{x}|\boldsymbol{\theta})$ have disjoint supports, then there is a smooth optimal discrimator. In this case we won't really be able to learn anything by backproping through it.
- For such low-dimensional disjoint manifolds

$$KL(\pi||p) = KL(p||\pi) = \infty, \quad JSD(\pi||p) = \log 2$$

- Adding continuous noise to the inputs of the discriminator smoothes the distributions of the probability mass.

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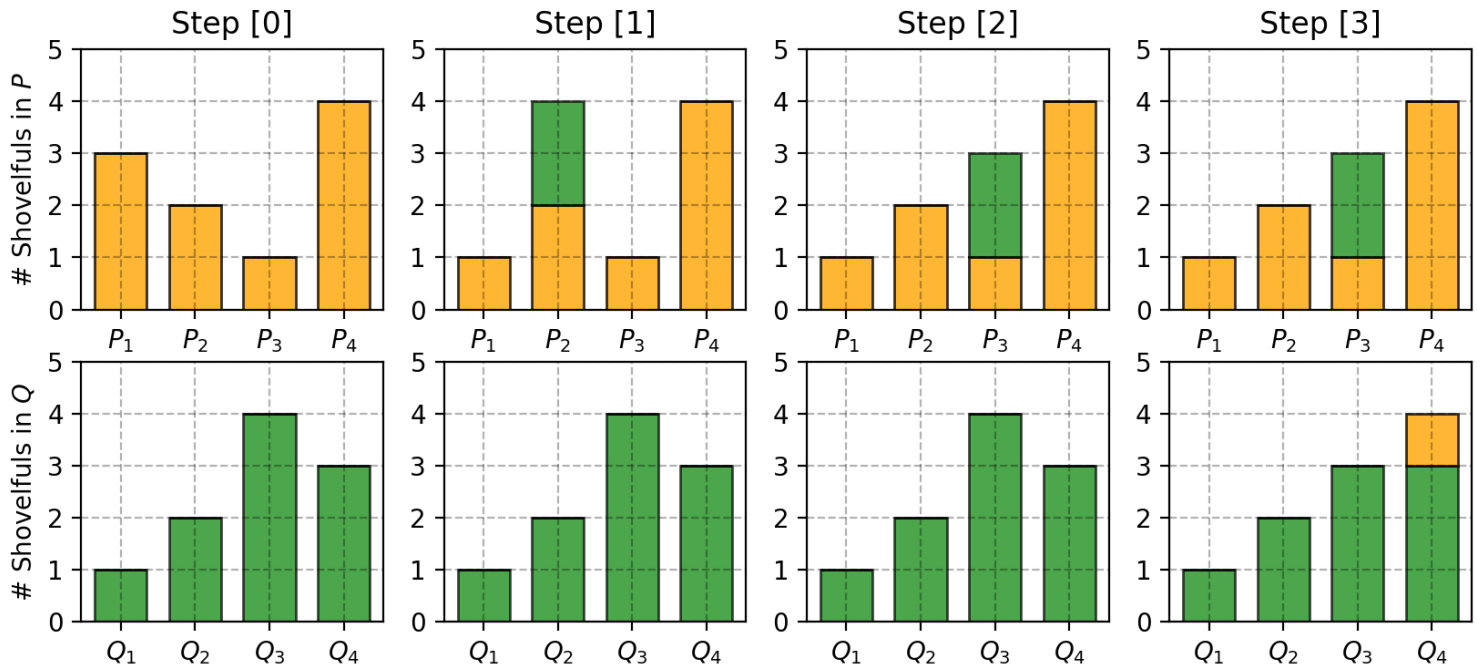
https://lilianweng.github.io/lil-log/2017/08/20/from-GAN-to-WGAN.html

https://arxiv.org/abs/1701.04862

Wasserstein distance (discrete)

Also called Earth Mover's distance. The minimum cost of moving and transforming a pile of dirt in the shape of one probability distribution to the shape of the other distribution.



$$W(P, Q) = 2(\text{step } 1) + 2(\text{step } 2) + 1(\text{step } 3) = 5$$

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https://lilianweng.github.io/lil-log/2017/08/20/from-GAN-to-WGAN.html

Wasserstein distance

$$W(\pi, p) = \inf_{\gamma \in \prod(\pi, p)} \mathbb{E}_{(\mathbf{x},\mathbf{y})\sim\gamma} \|\mathbf{x}-\mathbf{y}\| = \inf_{\gamma \in \prod(\pi, p)} \int \|\mathbf{x}-\mathbf{y}\| \gamma(\mathbf{x}, \mathbf{y}) d\mathbf{x} d\mathbf{y}$$

- $\prod(\pi, p)$ – the set of all joint distributions $\gamma(\mathbf{x}, \mathbf{y})$ with marginals π and p ($\int \gamma(\mathbf{x}, \mathbf{y}) d\mathbf{x} = p(\mathbf{y})$, $\int \gamma(\mathbf{x}, \mathbf{y}) d\mathbf{y} = \pi(\mathbf{x})$)
- $\gamma(\mathbf{x}, \mathbf{y})$ – transportation plan (the amount of "dirt" that should be transported from point $\mathbf{x}$ to point $\mathbf{y}$).
- $\gamma(\mathbf{x}, \mathbf{y})$ – the amount, $\|\mathbf{x} - \mathbf{y}\|$– the distance.

For better understanding of transportation plan function γ, try to write down the plan for previous discrete case.

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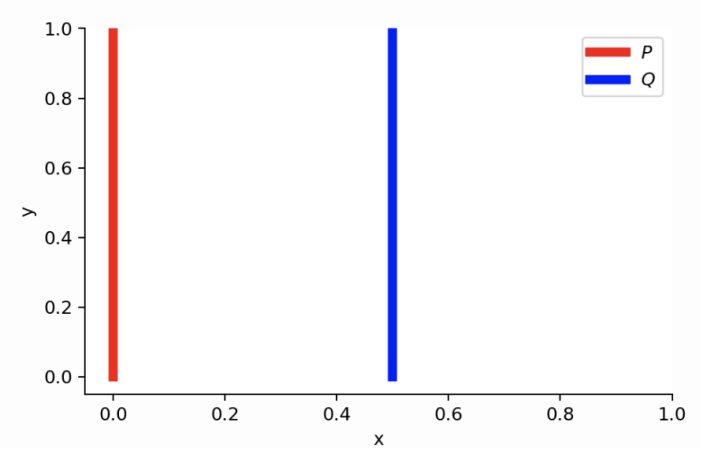
https://arxiv.org/abs/1701.07875

Wasserstein distance vs KL vs JSD

Consider 2d distributions

$$\pi(x, y) = (0, U[0, 1])$$

$$p(x, y|\theta) = (\theta, U[0, 1])$$



- $\theta = 0$. Distributions are the same

$$KL(\pi||p) = KL(p||\pi) = JSD(p||pi) = W(\pi, p) = 0$$

- $\theta \neq 0$

$$KL(\pi||p) = \int_{U[0,1]} 1 \log \frac{1}{0} dy = \infty = KL(\pi||p)$$

$$JSD(\pi||p) = \frac{1}{2}\left(\int_{U[0,1]} 1 \log \frac{1}{1/2} dy + \int_{U[0,1]} 1 \log \frac{1}{1/2} dy\right) = \log 2$$

$$W(\pi, p) = |\theta|$$

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https://arxiv.org/abs/1701.07875

Wasserstein distance vs KL vs JSD

Theorem 1

Let $G(\mathbf{z}, \boldsymbol{\theta})$ be any feedforward neural network, and $p(\mathbf{z})$ a prior over $\mathbf{z}$ such that $\mathbb{E}_{\mathbf{z} \sim p(\mathbf{z})} \|\mathbf{z}\| < \infty$. Then therefore $W(\pi, p)$ is continuous everywhere and differentiable almost everywhere.

Theorem 2

Let π be a distribution on a compact space $\mathcal{X}$ and $\{p_t\}_{t=1}^{\infty}$ be a sequence of distributions on $\mathcal{X}$.

$$KL(\pi||p_t) \to 0 \,(\text{or } KL(p_t||\pi) \to 0) \tag{1}$$

$$JSD(\pi||p_t) \to 0 \tag{2}$$

$$W(\pi||p_t) \to 0 \tag{3}$$

Then, considering limits as $t \to \infty$, (1) implies (2), (2) implies (3).

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https://arxiv.org/abs/1701.07875

Wasserstein GAN

Wasserstein distance

$$W(\pi, p) = \inf_{\gamma \in \prod(\pi, p)} \mathbb{E}_{(\mathbf{x},\mathbf{y})\sim\gamma} \|\mathbf{x}-\mathbf{y}\| = \inf_{\gamma \in \prod(\pi, p)} \int \|\mathbf{x}-\mathbf{y}\| \gamma(\mathbf{x}, \mathbf{y}) d\mathbf{x} d\mathbf{y}$$

The infimum across all possible joint distributions in $\prod(\pi, p)$ is intractable.

Kantorovich-Rubinstein duality

$$W(\pi, p) = \frac{1}{K} \max_{\|f\|_L \leq K} \left[\mathbb{E}_{\mathbf{x}\sim\pi} f(\mathbf{x}) - \mathbb{E}_{\mathbf{x}\sim p} f(\mathbf{x})\right],$$

where $\|f\|_L \leq K$ are $K-$Lipschitz continuous functions $(f : \mathcal{X} \to \mathbb{R})$

$$|f(\mathbf{x}_1) - f(\mathbf{x}_2) \leq K\|\mathbf{x}_1 - \mathbf{x}_2\|, \quad \text{for all } \mathbf{x}_1, \mathbf{x}_2 \in \mathcal{X}.$$

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https://arxiv.org/abs/1701.07875

Wasserstein GAN

Kantorovich-Rubinstein duality

$$W(\pi, p) = \frac{1}{K} \max_{\|f\|_L \leq K} \left[\mathbb{E}_{\mathbf{x}\sim\pi} f(\mathbf{x}) - \mathbb{E}_{\mathbf{x}\sim p} f(\mathbf{x})\right],$$

- Now we have to ensure that f is K-Lipschitz continuous.
- Let make $f(\mathbf{x}, \phi)$ parametrized by parameters ϕ.
- If parameters ϕ lies in a compact set $\mathbf{\Phi}$ then $f(\mathbf{x}, \phi)$ will be K-Lipschitz continuous function.
- Let clamp the parameters to a fixed box $\mathbf{\Phi} \in [-0.01, 0.01]^d$ after each gradient update.

$$\max_{\phi\in\mathbf{\Phi}} \left[\mathbb{E}_{\mathbf{x}\sim\pi} f(\mathbf{x}, \phi) - \mathbb{E}_{\mathbf{x}\sim p} f(\mathbf{x}, \phi)\right] \leq \max_{\|f\|_L \leq K} \left[\mathbb{E}_{\mathbf{x}\sim\pi} f(\mathbf{x}) - \mathbb{E}_{\mathbf{x}\sim p} f(\mathbf{x})\right] = KW(\pi||p)$$

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https://arxiv.org/abs/1701.07875

Wassestein GAN

Vanilla GAN objective

$$\min_G \max_D \mathbb{E}_{\pi(\mathbf{x})} \log D(\mathbf{x}) + \mathbb{E}_{p(\mathbf{z})} \log(1 - D(G(\mathbf{z})))$$

WGAN objective

$$\min_G W(\pi, p) = \min_G \frac{1}{K} \max_{\phi \in \mathbf{\Phi}} \left[\mathbb{E}_{\mathbf{x} \sim \pi} f(\mathbf{x}, \phi) - \mathbb{E}_{\mathbf{z} \sim p} f(G(\mathbf{z}), \phi) \right].$$

- Discriminator D is similar to the function f, but not the same (it is not a classifier anymore). In the WGAN model, function f is usually called *critic*.
- "Weight clipping is a clearly terrible way to enforce a Lipschitz constraint". If the clipping parameter is large, it is hard to train the critic till optimality. If the clipping parameter is too small, it could lead to vanishing gradients.

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https://arxiv.org/abs/1701.07875

Wasserstein GAN

Algorithm 1 WGAN, our proposed algorithm. All experiments in the paper used the default values $\alpha = 0.00005$, $c = 0.01$, $m = 64$, $n_{\text{critic}} = 5$.

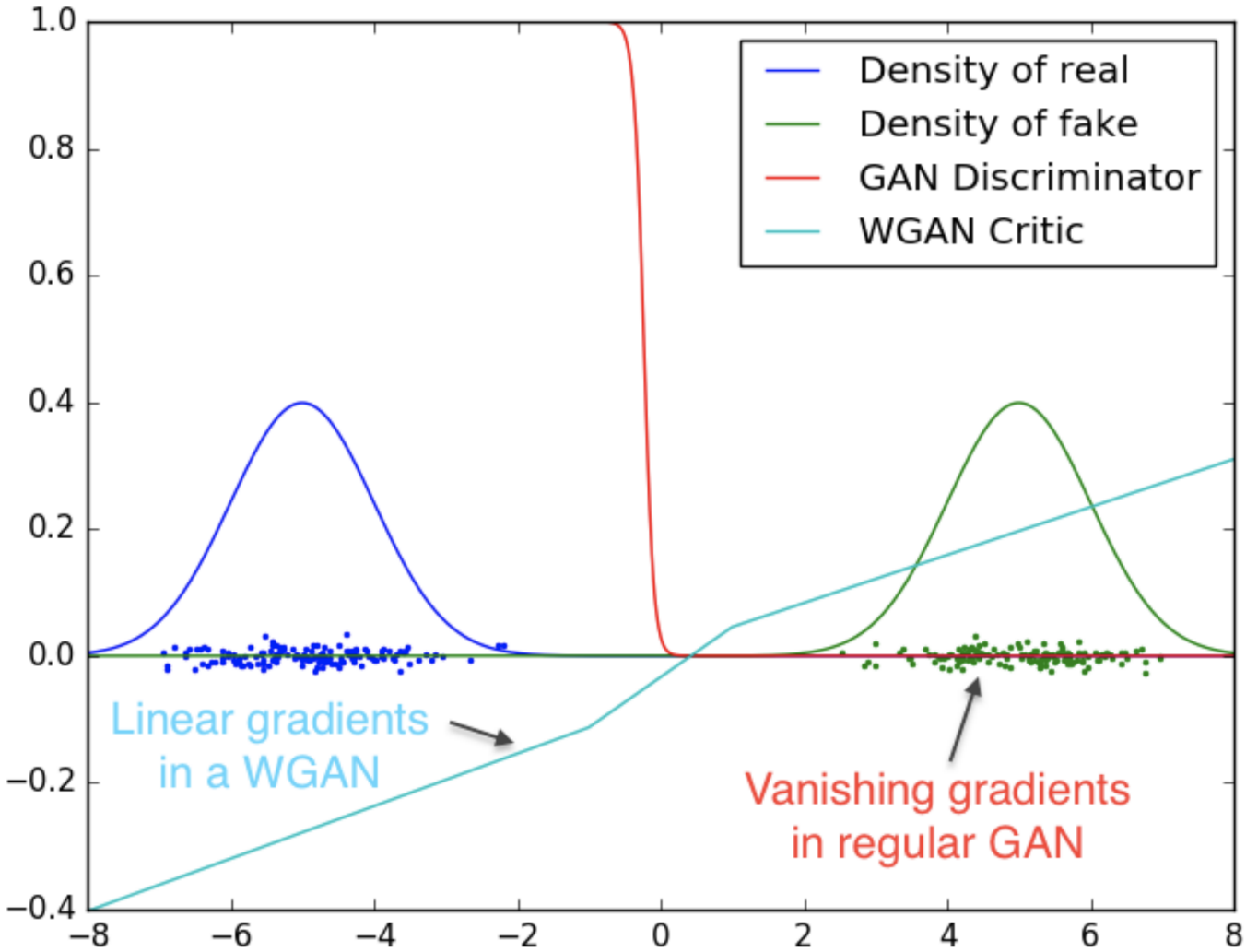
Require: : α, the learning rate. c, the clipping parameter. m, the batch size. n_{critic}, the number of iterations of the critic per generator iteration.

Require: : w_0, initial critic parameters. θ_0, initial generator's parameters.

while θ has not converged **do**
  for $t = 0, ..., n_{\text{critic}}$ **do**
    Sample $\{x^{(i)}\}_{i=1}^{m} \sim \mathbb{P}_r$ a batch from the real data.
    Sample $\{z^{(i)}\}_{i=1}^{m} \sim p(z)$ a batch of prior samples.
    $g_w \leftarrow \nabla_w \left[\frac{1}{m}\sum_{i=1}^{m} f_w(x^{(i)}) - \frac{1}{m}\sum_{i=1}^{m} f_w(g_\theta(z^{(i)}))\right]$
    $w \leftarrow w + \alpha \cdot \text{RMSProp}(w, g_w)$
    $w \leftarrow \text{clip}(w, -c, c)$
  end for
  Sample $\{z^{(i)}\}_{i=1}^{m} \sim p(z)$ a batch of prior samples.
  $g_\theta \leftarrow -\nabla_\theta \frac{1}{m}\sum_{i=1}^{m} f_w(g_\theta(z^{(i)}))$
  $\theta \leftarrow \theta - \alpha \cdot \text{RMSProp}(\theta, g_\theta)$
end while

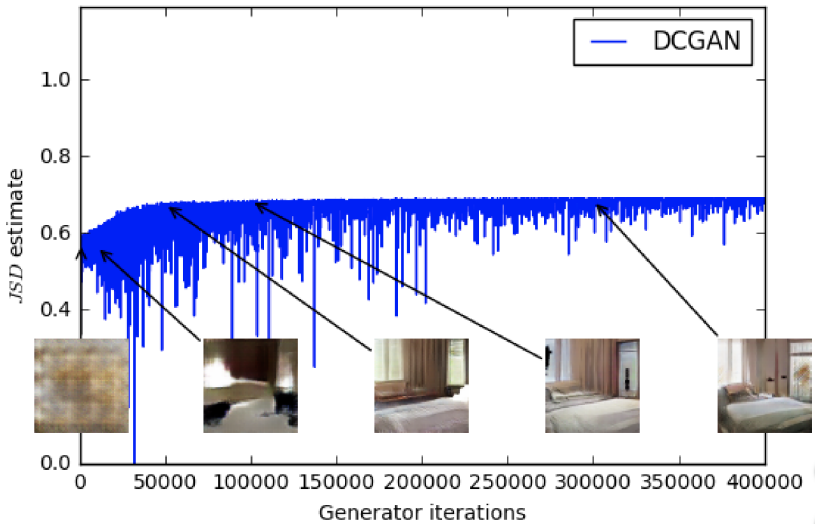
https://arxiv.org/abs/1701.07875

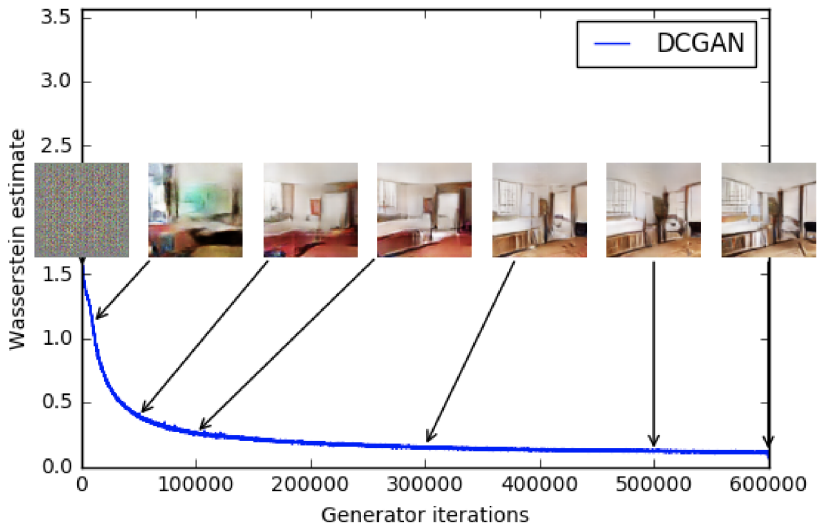
Wasserstein GAN



https://arxiv.org/abs/1701.07875

Wasserstein GAN





- *JSD* correlates poorly with the sample quality. Stays contast nearly maximum value $\log 2 \approx 0.69$.
- *W* is highly correlated with the sample quality.

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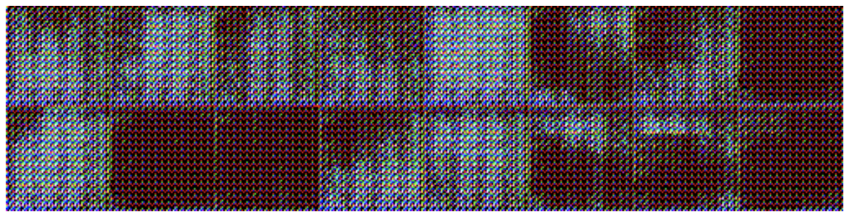
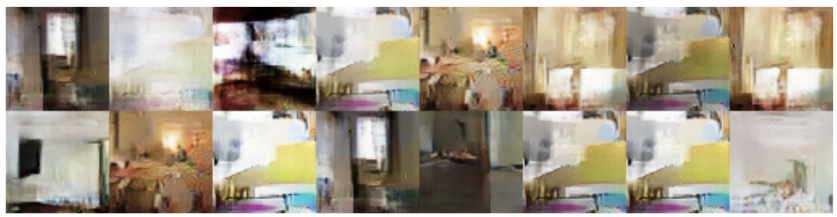
https://arxiv.org/abs/1701.07875

Wasserstein GAN

WGAN converged without batch norm and constant number of filters



"In no experiment did we see evidence of mode collapse for the WGAN algorithm."



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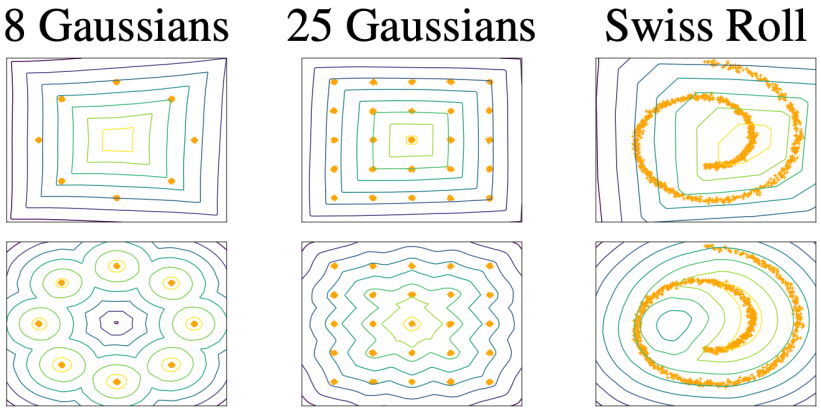
https://arxiv.org/abs/1701.07875

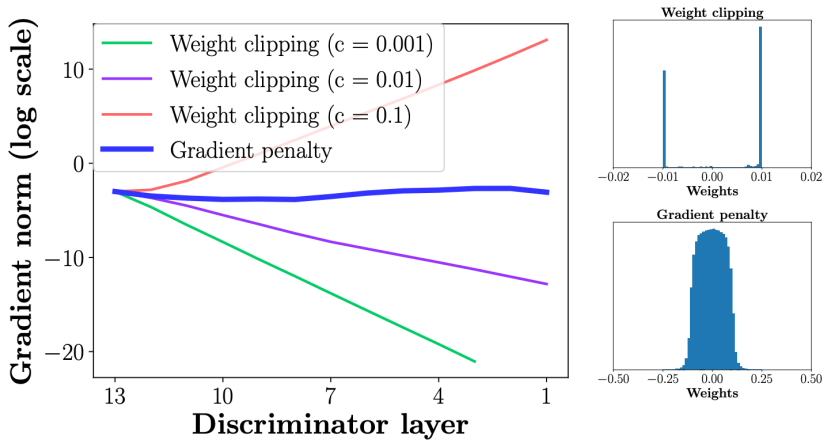
Wasserstein GAN with Gradient Penalty

The generator distribution is fixed and equals to the real distribution + Gaussian noise.
Problems with weight clipping:

- The critic ignores higher moments of the data distribution.
- The gradient either grow or decay exponentially.

Gradient penalty makes the gradients more stable.





https://arxiv.org/abs/1704.00028

Wasserstein GAN with Gradient Penalty

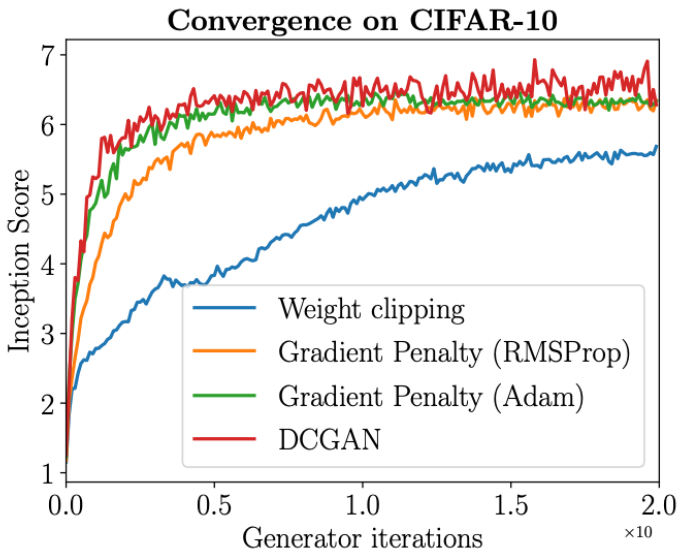
Algorithm 1 WGAN with gradient penalty. We use default values of $\lambda = 10$, $n_{\text{critic}} = 5$, $\alpha = 0.0001$, $\beta_1 = 0$, $\beta_2 = 0.9$.

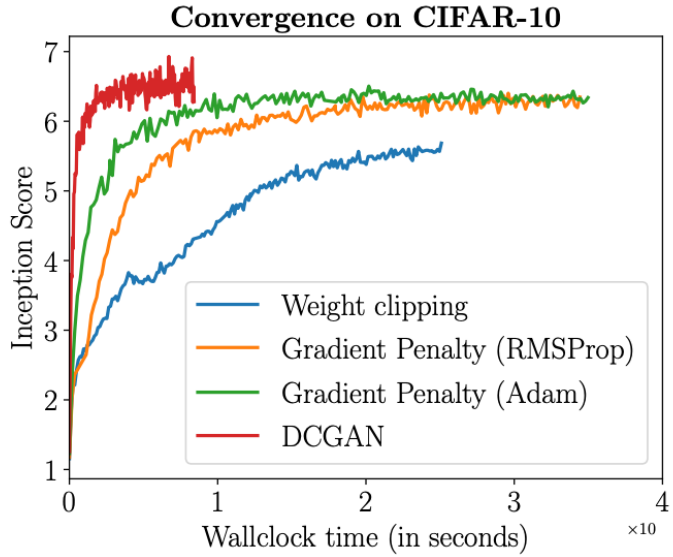
Require: The gradient penalty coefficient λ, the number of critic iterations per generator iteration n_{critic}, the batch size m, Adam hyperparameters α, β_1, β_2.
Require: initial critic parameters w_0, initial generator parameters θ_0.

while θ has not converged **do**
  for $t = 1, ..., n_{\text{critic}}$ **do**
    for $i = 1, ..., m$ **do**
      Sample real data $\boldsymbol{x} \sim \mathbb{P}_r$, latent variable $\boldsymbol{z} \sim p(\boldsymbol{z})$, a random number $\epsilon \sim U[0, 1]$.
      $\tilde{\boldsymbol{x}} \leftarrow G_\theta(\boldsymbol{z})$
      $\hat{\boldsymbol{x}} \leftarrow \epsilon \boldsymbol{x} + (1 - \epsilon)\tilde{\boldsymbol{x}}$
      $L^{(i)} \leftarrow D_w(\tilde{\boldsymbol{x}}) - D_w(\boldsymbol{x}) + \lambda(\|\nabla_{\hat{\boldsymbol{x}}} D_w(\hat{\boldsymbol{x}})\|_2 - 1)^2$
    end for
    $w \leftarrow \text{Adam}(\nabla_w \frac{1}{m} \sum_{i=1}^{m} L^{(i)}, w, \alpha, \beta_1, \beta_2)$
  end for
  Sample a batch of latent variables $\{\boldsymbol{z}^{(i)}\}_{i=1}^{m} \sim p(\boldsymbol{z})$.
  $\theta \leftarrow \text{Adam}(\nabla_\theta \frac{1}{m} \sum_{i=1}^{m} -D_w(G_\theta(\boldsymbol{z})), \theta, \alpha, \beta_1, \beta_2)$
end while

https://arxiv.org/abs/1704.00028

Wasserstein GAN with Gradient Penalty



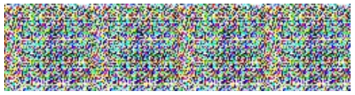
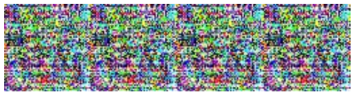
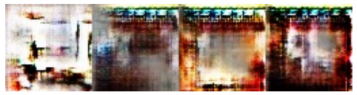
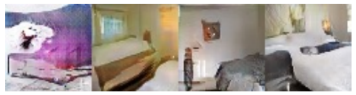
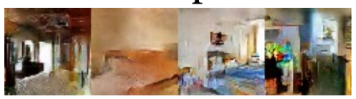
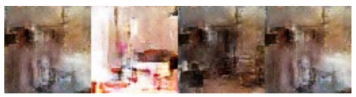
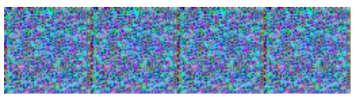
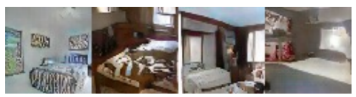


Nonlinearity (G)	[ReLU, LeakyReLU, $\frac{\text{softplus}(2x+2)}{2} - 1$, tanh]
Nonlinearity (D)	[ReLU, LeakyReLU, $\frac{\text{softplus}(2x+2)}{2} - 1$, tanh]
Depth (G)	[4, 8, 12, 20]
Depth (D)	[4, 8, 12, 20]
Batch norm (G)	[True, False]
Batch norm (D; layer norm for WGAN-GP)	[True, False]
Base filter count (G)	[32, 64, 128]
Base filter count (D)	[32, 64, 128]

Min. score	Only GAN	Only WGAN-GP	Both succeeded	Both failed
1.0	0	8	192	0
3.0	1	88	110	1
5.0	0	147	42	11
7.0	1	104	5	90
9.0	0	0	0	200

https://arxiv.org/abs/1704.00028

Wasserstein GAN with Gradient Penalty

DCGAN	LSGAN	WGAN (clipping)	WGAN-GP (ours)
Baseline (G: DCGAN, D: DCGAN)				
G: No BN and a constant number of filters, D: DCGAN				
G: 4-layer 512-dim ReLU MLP, D: DCGAN				
No normalization in either G or D				
Gated multiplicative nonlinearities everywhere in G and D				
tanh nonlinearities everywhere in G and D				
101-layer ResNet G and D				

https://arxiv.org/abs/1704.00028

Summary

References

- A note on the evaluation of generative models
  https://arxiv.org/abs/1511.01844
  Summary:
- Generative Adversarial Nets
  https://arxiv.org/abs/1406.2661
  Summary:
- Unsupervised Representation Learning with Deep Convolutional Generative Adversarial Networks
  https://arxiv.org/abs/1511.06434
  Summary:
- Improved techniques for training GANs
  https://arxiv.org/abs/1606.03498
  Summary:
- Towards principled methods for training generative adversarial networks
  https://arxiv.org/abs/1701.04862
  Summary:
- Wasserstein GAN
  https://arxiv.org/abs/1701.07875
  Summary:
- Improved Training of Wasserstein GANs
  https://arxiv.org/abs/1704.00028
  Summary: