Deep Generative Models
Lecture 9

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Summary of previous lecture

- Likelihood is not a perfect criteria to measure quality of generative model.
- Adversarial learning suggest to solve minimax problem with generator and discriminator.
- Vanila GAN tries to optimize (in some sense) Jensen-Shannon divergence.
- Mode collapse and vanishing gradients are the two main problems of vanila GAN.
- Lots of tips and tricks has to be used to make the GAN training is stable and scalable.
- Wasserstein distance is more appropriate objective function for distribution matching problem.

Wasserstein distance

$$W(\pi, p) = \inf_{\gamma \in \prod(\pi, p)} \mathbb{E}_{(\mathbf{x},\mathbf{y})\sim\gamma} \|\mathbf{x}-\mathbf{y}\| = \inf_{\gamma \in \prod(\pi, p)} \int \|\mathbf{x}-\mathbf{y}\| \gamma(\mathbf{x}, \mathbf{y}) d\mathbf{x} d\mathbf{y}$$

- $\prod(\pi, p)$ – the set of all joint distributions $\gamma(\mathbf{x}, \mathbf{y})$ with marginals π and p ($\int \gamma(\mathbf{x}, \mathbf{y}) d\mathbf{x} = p(\mathbf{y})$, $\int \gamma(\mathbf{x}, \mathbf{y}) d\mathbf{y} = \pi(\mathbf{x})$)
- $\gamma(\mathbf{x}, \mathbf{y})$ – transportation plan (the amount of "dirt" that should be transported from point $\mathbf{x}$ to point $\mathbf{y}$).
- $\gamma(\mathbf{x}, \mathbf{y})$ – the amount, $\|\mathbf{x} - \mathbf{y}\|$– the distance.

For better understanding of transportation plan function γ, try to write down the plan for previous discrete case.

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https://arxiv.org/abs/1701.07875

Wasserstein distance vs KL vs JSD

Theorem 1

Let $G(\mathbf{z}, \boldsymbol{\theta})$ be any feedforward neural network, and $p(\mathbf{z})$ a prior over $\mathbf{z}$ such that $\mathbb{E}_{\mathbf{z} \sim p(\mathbf{z})} \|\mathbf{z}\| < \infty$. Then therefore $W(\pi, p)$ is continuous everywhere and differentiable almost everywhere.

Theorem 2

Let π be a distribution on a compact space $\mathcal{X}$ and $\{p_t\}_{t=1}^{\infty}$ be a sequence of distributions on $\mathcal{X}$.

$$KL(\pi||p_t) \to 0 \,(\text{or } KL(p_t||\pi) \to 0) \tag{1}$$

$$JSD(\pi||p_t) \to 0 \tag{2}$$

$$W(\pi||p_t) \to 0 \tag{3}$$

Then, considering limits as $t \to \infty$, (1) implies (2), (2) implies (3).

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Wasserstein GAN

Wasserstein distance

$$W(\pi, p) = \inf_{\gamma \in \prod(\pi, p)} \mathbb{E}_{(\mathbf{x},\mathbf{y})\sim\gamma} \|\mathbf{x}-\mathbf{y}\| = \inf_{\gamma \in \prod(\pi, p)} \int \|\mathbf{x}-\mathbf{y}\| \gamma(\mathbf{x}, \mathbf{y}) d\mathbf{x} d\mathbf{y}$$

The infimum across all possible joint distributions in $\prod(\pi, p)$ is intractable.

Kantorovich-Rubinstein duality

$$W(\pi, p) = \frac{1}{K} \max_{\|f\|_L \leq K} \left[\mathbb{E}_{\mathbf{x}\sim\pi} f(\mathbf{x}) - \mathbb{E}_{\mathbf{x}\sim p} f(\mathbf{x})\right],$$

where $\|f\|_L \leq K$ are $K-$Lipschitz continuous functions $(f : \mathcal{X} \to \mathbb{R})$

$$|f(\mathbf{x}_1) - f(\mathbf{x}_2)| \leq K\|\mathbf{x}_1 - \mathbf{x}_2\|, \quad \text{for all } \mathbf{x}_1, \mathbf{x}_2 \in \mathcal{X}.$$

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Wasserstein GAN

Kantorovich-Rubinstein duality

$$W(\pi, p) = \frac{1}{K} \max_{\|f\|_L \leq K} \left[\mathbb{E}_{\mathbf{x}\sim\pi} f(\mathbf{x}) - \mathbb{E}_{\mathbf{x}\sim p} f(\mathbf{x})\right],$$

- Now we have to ensure that f is K-Lipschitz continuous.
- Let make $f(\mathbf{x}, \phi)$ parametrized by parameters ϕ.
- If parameters ϕ lie in a compact set $\mathbf{\Phi}$ then $f(\mathbf{x}, \phi)$ will be K-Lipschitz continuous function.
- Let clamp the parameters to a fixed box $\mathbf{\Phi} \in [-0.01, 0.01]^d$ after each gradient update.

$$\max_{\phi\in\mathbf{\Phi}} \left[\mathbb{E}_{\mathbf{x}\sim\pi} f(\mathbf{x}, \phi) - \mathbb{E}_{\mathbf{x}\sim p} f(\mathbf{x}, \phi)\right] \leq \max_{\|f\|_L \leq K} \left[\mathbb{E}_{\mathbf{x}\sim\pi} f(\mathbf{x}) - \mathbb{E}_{\mathbf{x}\sim p} f(\mathbf{x})\right] = K \cdot W(\pi||p)$$

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Wassestein GAN

Vanilla GAN objective

$$\min_G \max_D \mathbb{E}_{\pi(\mathbf{x})} \log D(\mathbf{x}) + \mathbb{E}_{p(\mathbf{z})} \log(1 - D(G(\mathbf{z})))$$

WGAN objective

$$\min_G W(\pi, p) = \min_G \max_{\phi \in \mathbf{\Phi}} \left[\mathbb{E}_{\pi(\mathbf{x})} f(\mathbf{x}, \phi) - \mathbb{E}_{p(\mathbf{z})} f(G(\mathbf{z}), \phi) \right].$$

- Discriminator D is similar to the function f, but not the same (it is not a classifier anymore). In the WGAN model, function f is usually called *critic*.
- "Weight clipping is a clearly terrible way to enforce a Lipschitz constraint". If the clipping parameter is large, it is hard to train the critic till optimality. If the clipping parameter is too small, it could lead to vanishing gradients.

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https://arxiv.org/abs/1701.07875

Wasserstein GAN

Algorithm 1 WGAN, our proposed algorithm. All experiments in the paper used the default values $\alpha = 0.00005$, $c = 0.01$, $m = 64$, $n_{\text{critic}} = 5$.

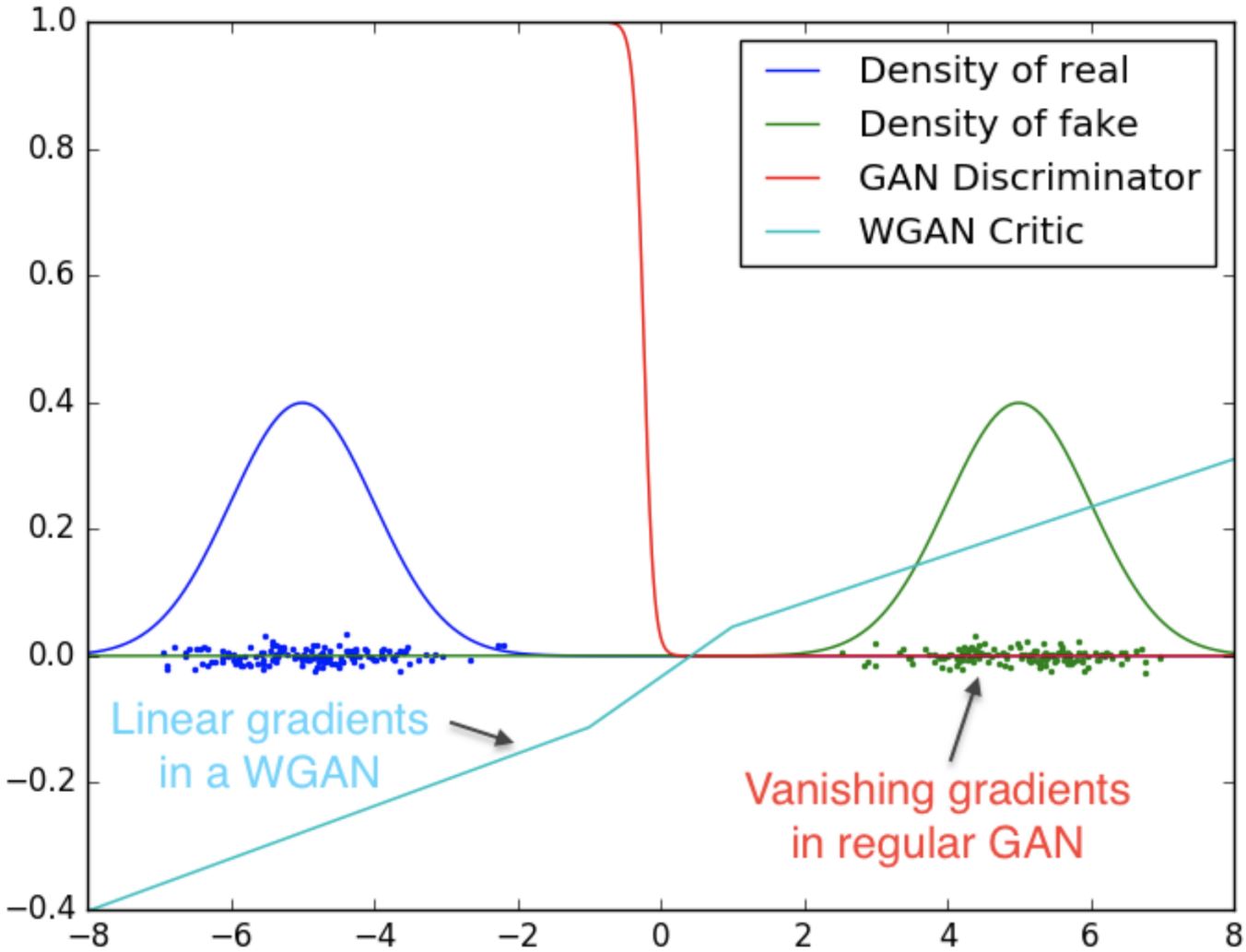
Require: : α, the learning rate. c, the clipping parameter. m, the batch size. n_{critic}, the number of iterations of the critic per generator iteration.

Require: : w_0, initial critic parameters. θ_0, initial generator's parameters.

while θ has not converged **do**
  for $t = 0, ..., n_{\text{critic}}$ **do**
    Sample $\{x^{(i)}\}_{i=1}^{m} \sim \mathbb{P}_r$ a batch from the real data.
    Sample $\{z^{(i)}\}_{i=1}^{m} \sim p(z)$ a batch of prior samples.
    $g_w \leftarrow \nabla_w \left[\frac{1}{m}\sum_{i=1}^{m} f_w(x^{(i)}) - \frac{1}{m}\sum_{i=1}^{m} f_w(g_\theta(z^{(i)}))\right]$
    $w \leftarrow w + \alpha \cdot \text{RMSProp}(w, g_w)$
    $w \leftarrow \text{clip}(w, -c, c)$
  end for
  Sample $\{z^{(i)}\}_{i=1}^{m} \sim p(z)$ a batch of prior samples.
  $g_\theta \leftarrow -\nabla_\theta \frac{1}{m}\sum_{i=1}^{m} f_w(g_\theta(z^{(i)}))$
  $\theta \leftarrow \theta - \alpha \cdot \text{RMSProp}(\theta, g_\theta)$
end while

https://arxiv.org/abs/1701.07875

Wasserstein GAN



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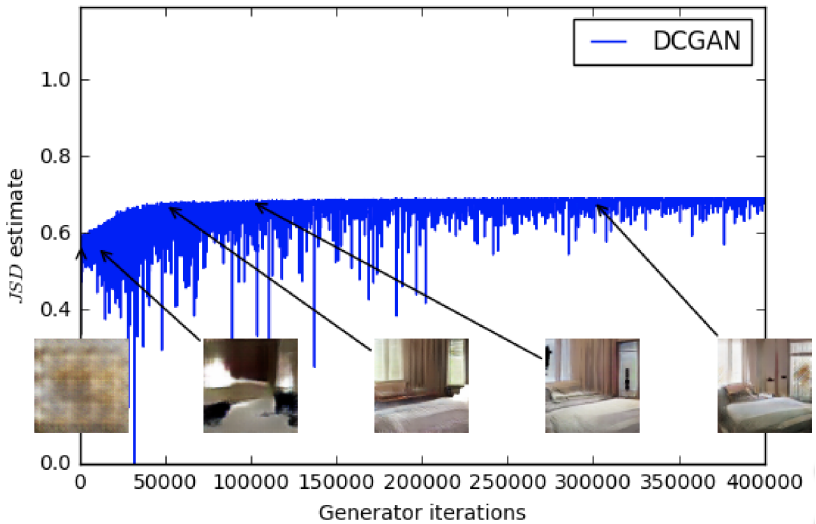
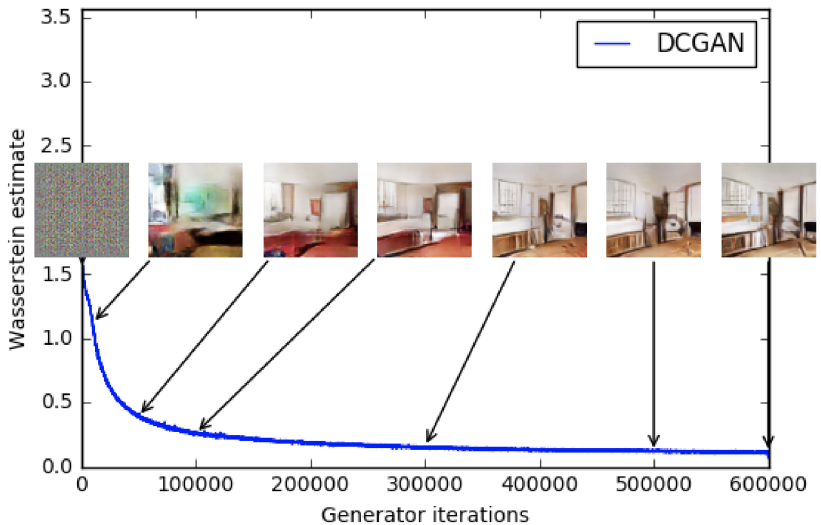
Wasserstein GAN





- *JSD* correlates poorly with the sample quality. Stays contast nearly maximum value $\log 2 \approx 0.69$.
- *W* is highly correlated with the sample quality.

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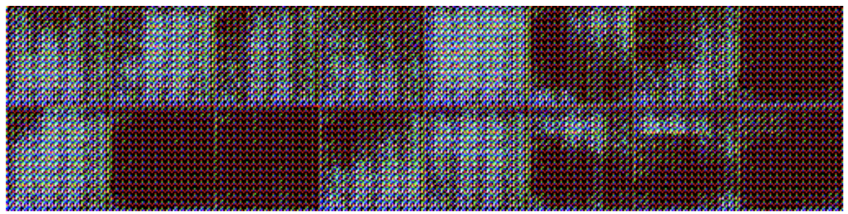
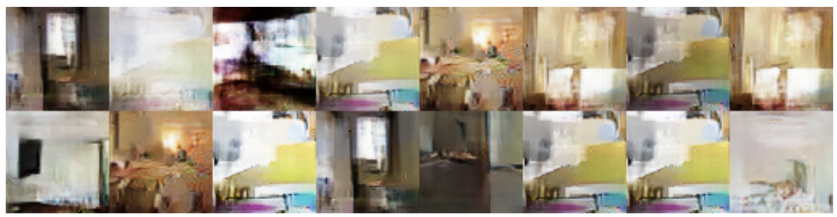
https://arxiv.org/abs/1701.07875

Wasserstein GAN

WGAN converged without batch norm and constant number of filters



"In no experiment did we see evidence of mode collapse for the WGAN algorithm."



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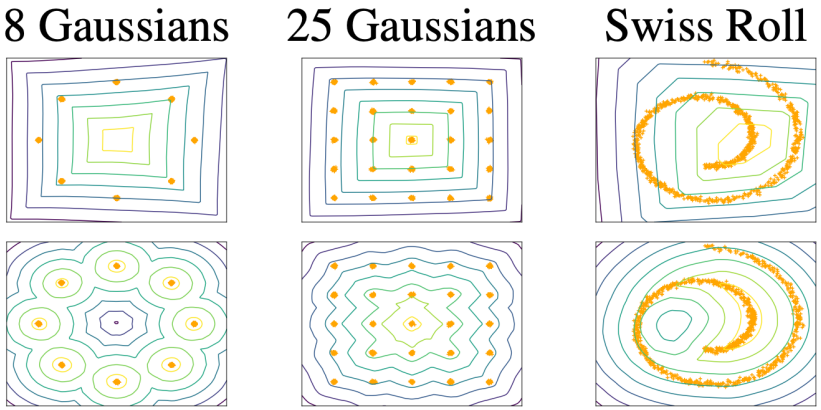
https://arxiv.org/abs/1701.07875

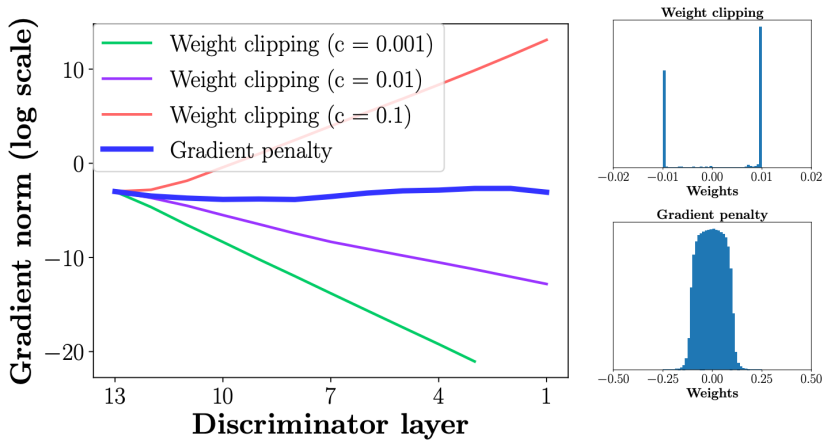
Wasserstein GAN with Gradient Penalty

The generator distribution is fixed and equals to the real distribution + Gaussian noise.
Problems with weight clipping:

- The critic ignores higher moments of the data distribution.
- The gradient either grow or decay exponentially.

Gradient penalty makes the gradients more stable.





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Wasserstein GAN with Gradient Penalty

Theorem

Let $\pi(\mathbf{x})$ and $p(\mathbf{x})$ be two distribution in $\mathcal{X}$, a compact metric space. Then, there is 1-Lipschitz function f^* which is the optimal solution of

$$\max_{\|f\|_L \leq 1} \left[\mathbb{E}_{\mathbf{x}\sim\pi} f(\mathbf{x}) - \mathbb{E}_{\mathbf{x}\sim p} f(\mathbf{x})\right].$$

Let γ be the optimal transportation plan between $\pi(\mathbf{x})$ and $p(\mathbf{x})$. Then, if f^* is differentiable $\gamma(x = y) = 0$ and $\mathbf{x}_t = t\mathbf{x} + (1-t)\mathbf{y}$ with $t \in [0, 1]$ it holds that

$$\mathbb{P}_{(\mathbf{x},\mathbf{y})\sim\gamma}\left[\nabla f^*(\mathbf{x}_t) = \frac{\mathbf{y} - \mathbf{x}_t}{\|\mathbf{y} - \mathbf{x}_t\|}\right] = 1.$$

Corollary

f^* has gradient norm 1 almost everywhere under $\pi(\mathbf{x})$ and $p(\mathbf{x})$.

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https://arxiv.org/abs/1704.00028

Wasserstein GAN with Gradient Penalty

A differentiable function is 1-Lipschtiz if and only if it has gradients with norm at most 1 everywhere.

Gradient penalty

$$W(\pi, p) = \underbrace{\mathbb{E}_{\mathbf{x}\sim\pi} f(\mathbf{x}) - \mathbb{E}_{\mathbf{x}\sim p} f(\mathbf{x})}_{\text{original critic loss}} + \lambda \underbrace{\mathbb{E}_{\hat{\mathbf{x}}} \left[(\|\nabla_{\hat{\mathbf{x}}} f(\hat{\mathbf{x}})\|_2 - 1)^2 \right]}_{\text{gradient penalty}},$$

- Samples $\hat{\mathbf{x}}$ are uniformly sampled along straight lines between pairs of points from the data distribution $\pi(\mathbf{x})$ and the generator distribution $p(\mathbf{x}|\boldsymbol{\theta})$.
- Enforcing the unit gradient norm constraint everywhere is intractable, it turns out sifficient to enforce it only along these straight lines.

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https://arxiv.org/abs/1704.00028

Wasserstein GAN with Gradient Penalty

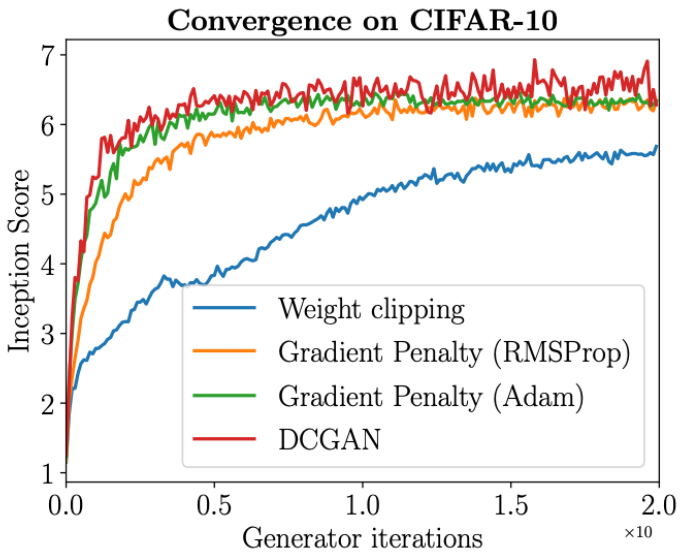
Algorithm 1 WGAN with gradient penalty. We use default values of $\lambda = 10$, $n_{\text{critic}} = 5$, $\alpha = 0.0001$, $\beta_1 = 0$, $\beta_2 = 0.9$.

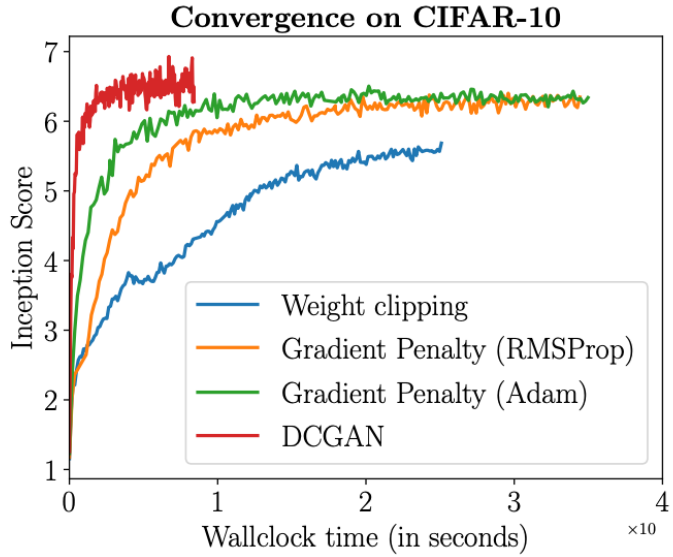
Require: The gradient penalty coefficient λ, the number of critic iterations per generator iteration n_{critic}, the batch size m, Adam hyperparameters α, β_1, β_2.
Require: initial critic parameters w_0, initial generator parameters θ_0.

while θ has not converged **do**
  for $t = 1, ..., n_{\text{critic}}$ **do**
    for $i = 1, ..., m$ **do**
      Sample real data $\boldsymbol{x} \sim \mathbb{P}_r$, latent variable $\boldsymbol{z} \sim p(\boldsymbol{z})$, a random number $\epsilon \sim U[0, 1]$.
      $\tilde{\boldsymbol{x}} \leftarrow G_\theta(\boldsymbol{z})$
      $\hat{\boldsymbol{x}} \leftarrow \epsilon \boldsymbol{x} + (1 - \epsilon)\tilde{\boldsymbol{x}}$
      $L^{(i)} \leftarrow D_w(\tilde{\boldsymbol{x}}) - D_w(\boldsymbol{x}) + \lambda(\|\nabla_{\hat{\boldsymbol{x}}} D_w(\hat{\boldsymbol{x}})\|_2 - 1)^2$
    end for
    $w \leftarrow \text{Adam}(\nabla_w \frac{1}{m} \sum_{i=1}^{m} L^{(i)}, w, \alpha, \beta_1, \beta_2)$
  end for
  Sample a batch of latent variables $\{\boldsymbol{z}^{(i)}\}_{i=1}^{m} \sim p(\boldsymbol{z})$.
  $\theta \leftarrow \text{Adam}(\nabla_\theta \frac{1}{m} \sum_{i=1}^{m} -D_w(G_\theta(\boldsymbol{z})), \theta, \alpha, \beta_1, \beta_2)$
end while

https://arxiv.org/abs/1704.00028

Wasserstein GAN with Gradient Penalty



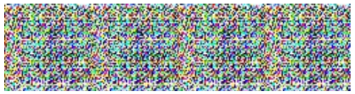
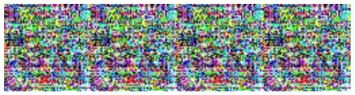
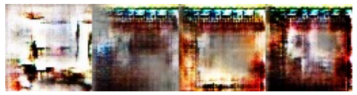
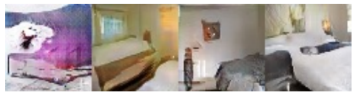
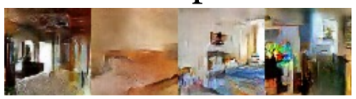
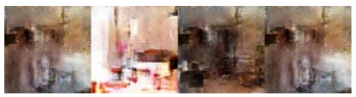
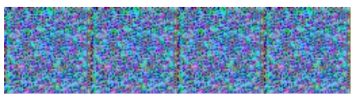
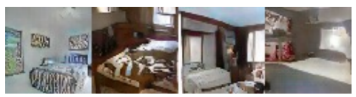


Nonlinearity (G)	[ReLU, LeakyReLU, $\frac{\text{softplus}(2x+2)}{2} - 1$, tanh]
Nonlinearity (D)	[ReLU, LeakyReLU, $\frac{\text{softplus}(2x+2)}{2} - 1$, tanh]
Depth (G)	[4, 8, 12, 20]
Depth (D)	[4, 8, 12, 20]
Batch norm (G)	[True, False]
Batch norm (D; layer norm for WGAN-GP)	[True, False]
Base filter count (G)	[32, 64, 128]
Base filter count (D)	[32, 64, 128]

Min. score	**Only GAN**	**Only WGAN-GP**	**Both succeeded**	**Both failed**
1.0	0	8	192	0
3.0	1	88	110	1
5.0	0	147	42	11
7.0	1	104	5	90
9.0	0	0	0	200

https://arxiv.org/abs/1704.00028

Wasserstein GAN with Gradient Penalty

DCGAN	LSGAN	WGAN (clipping)	WGAN-GP (ours)
Baseline (G: DCGAN, D: DCGAN)				
G: No BN and a constant number of filters, D: DCGAN				
G: 4-layer 512-dim ReLU MLP, D: DCGAN				
No normalization in either G or D				
Gated multiplicative nonlinearities everywhere in G and D				
tanh nonlinearities everywhere in G and D				
101-layer ResNet G and D				

https://arxiv.org/abs/1704.00028

Spectral Normalization GAN

How else could we enforce Lipschitzness?

Fact 1

$$\|\mathbf{g}\|_L = \sup_{\mathbf{x}} \sigma(\nabla \mathbf{g}(\mathbf{x}))$$

Here $\sigma(\mathbf{A})$ – spectral norm of matrix $\mathbf{A}$.

$$\sigma(\mathbf{A}) = \max_{\mathbf{h} \neq 0} \frac{\|\mathbf{A}\mathbf{h}\|_2}{\|\mathbf{h}\|_2} = \max_{\|\mathbf{h}\|_2 \leq 1} \|\mathbf{A}\mathbf{h}\|_2 = \lambda_{\max}(\mathbf{A}),$$

where $\lambda_{\max}(\mathbf{A})$ is the largest singular value of $\mathbf{A}$.

Fact 2

$$\|\mathbf{g}_1 \circ \mathbf{g}_2\|_L \leq \|\mathbf{g}_1\|_L \cdot \|\mathbf{g}_2\|_L$$

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https://arxiv.org/abs/1802.05957

Spectral Normalization GAN

Let consider the critic $f(\mathbf{x}, \phi)$ of the following form:

$$f(\mathbf{x}, \phi) = \mathbf{W}_{K+1} a_K(\mathbf{W}_K a_{K-1}(\dots a_1(\mathbf{W}_1 \mathbf{x}) \dots)).$$

This feedforward network is a superposition of simple functions.

- a_k is a pointwise nonlinearities. We assume that $\|a_k\|_L = 1$ (it holds for ReLU).
- $\mathbf{g}(\mathbf{h}) = \mathbf{W}\mathbf{h}$ is a linear transformation.

$$\|\mathbf{g}\|_L = \sup_{\mathbf{x}} \sigma(\nabla \mathbf{g}(\mathbf{x})) = \sigma(\mathbf{W}).$$

Critic spectral norm

$$\|f\|_L \leq \prod_{k=1}^{K+1} \sigma(\mathbf{W}_k).$$

If we replace the weights in the critic by $\mathbf{W}_k^{SN} = \mathbf{W}_k / \sigma(\mathbf{W}_k)$, we will get $\|f\|_L \leq 1$.

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https://arxiv.org/abs/1802.05957

Spectral Normalization GAN

If we apply singular value decomposition to compute the $\sigma(\mathbf{W})$ at each round of the algorithm, the algorithm becomes computationally heavy.

Power iteration

- $\mathbf{u}$ – random vector.
- repeat

$$\mathbf{v} = \frac{\mathbf{W}^T\mathbf{u}}{\|\mathbf{W}^T\mathbf{u}\|}, \quad \mathbf{u} = \frac{\mathbf{W}\mathbf{v}}{\|\mathbf{W}\mathbf{v}\|}$$

- approximate the spectral norm

$$\sigma(\mathbf{W}) \approx \mathbf{u}^T\mathbf{W}\mathbf{v}$$

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https://arxiv.org/abs/1802.05957

Spectral Normalization GAN

Algorithm 1 SGD with spectral normalization

- Initialize $\tilde{\boldsymbol{u}}_l \in \mathcal{R}^{d_l}$ for $l = 1, \ldots, L$ with a random vector (sampled from isotropic distribution).
- For each update and each layer l:
  1. Apply power iteration method to a unnormalized weight W^l:

$$\tilde{\boldsymbol{v}}_l \leftarrow (W^l)^{\mathrm{T}} \tilde{\boldsymbol{u}}_l / \|(W^l)^{\mathrm{T}} \tilde{\boldsymbol{u}}_l\|_2 \tag{20}$$

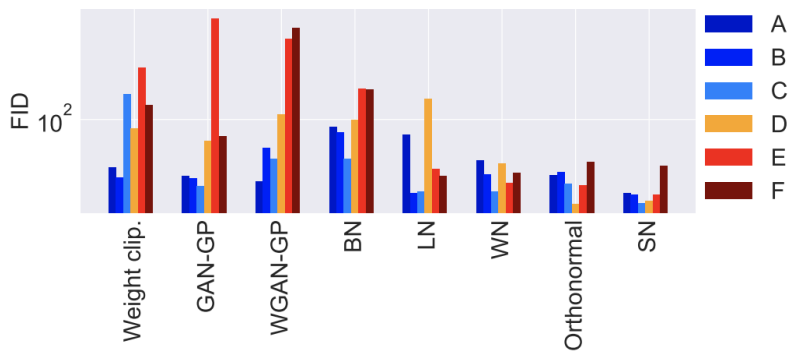
$$\tilde{\boldsymbol{u}}_l \leftarrow W^l \tilde{\boldsymbol{v}}_l / \|W^l \tilde{\boldsymbol{v}}_l\|_2 \tag{21}$$

  2. Calculate $\bar{W}_{\mathrm{SN}}$ with the spectral norm:

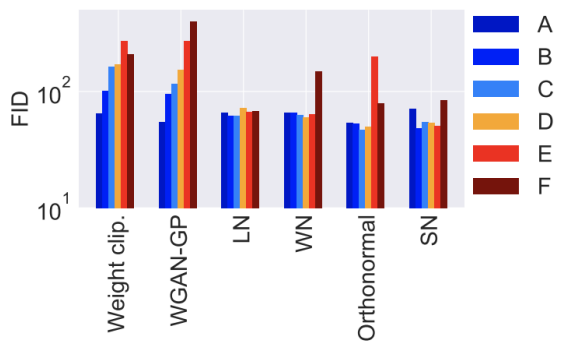
$$\bar{W}^l_{\mathrm{SN}}(W^l) = W^l / \sigma(W^l), \text{ where } \sigma(W^l) = \tilde{\boldsymbol{u}}_l^{\mathrm{T}} W^l \tilde{\boldsymbol{v}}_l \tag{22}$$

  3. Update W^l with SGD on mini-batch dataset $\mathcal{D}_M$ with a learning rate α:

$$W^l \leftarrow W^l - \alpha \nabla_{W^l} \ell(\bar{W}^l_{\mathrm{SN}}(W^l), \mathcal{D}_M) \tag{23}$$



(a) CIFAR-10



(b) STL-10

Divergences

What do we have?

- Forward KL divergence in maximum likelihood estimation
- Reverse KL in variational inference
- JS divergence in vanilla gan
- Wasserstein distance in WGAN

Divergence minimization

$$\min_{p} D(\pi||p)$$

What is a divergence?

Let $\mathcal{S}$ be the set of all possible probability distributions. Then $D : \mathcal{S} \times \mathcal{S} \to \mathbb{R}$ is a divergence if

- $D(\pi||p) \geq 0$ for all $\pi, p \in \mathcal{S}$;
- $D(\pi||p) = 0$ if and only if $\pi \equiv p$.

f-divergence family

f-divergence

$$D_f(\pi||p) = \mathbb{E}_p f\left(\frac{\pi(\mathbf{x})}{p(\mathbf{x})}\right) = \int p(\mathbf{x}) f\left(\frac{\pi(\mathbf{x})}{p(\mathbf{x})}\right) d\mathbf{x}.$$

Here $f : \mathbb{R}_+ \to \mathbb{R}$ is a convex, lower-semicontinuous function satisfying $f(1) = 0$.

Fenchel conjugate

$$f^*(t) = \sup_{u \in \mathrm{dom}_f} (ut - f(u)), \quad f^{**} = f, \quad f(u) = \sup_{t \in \mathrm{dom}_{f^*}} (ut - f^*(t))$$

Name	$D_f(P\|Q)$	Generator $f(u)$	$T^*(x)$
Kullback-Leibler	$\int p(x) \log \frac{p(x)}{q(x)} \mathrm{d}x$	$u \log u$	$1 + \log \frac{p(x)}{q(x)}$
Reverse KL	$\int q(x) \log \frac{q(x)}{p(x)} \mathrm{d}x$	$-\log u$	$-\frac{q(x)}{p(x)}$
Pearson χ^2	$\int \frac{(q(x)-p(x))^2}{p(x)} \mathrm{d}x$	$(u-1)^2$	$2(\frac{p(x)}{q(x)} - 1)$
Squared Hellinger	$\int \left(\sqrt{p(x)} - \sqrt{q(x)}\right)^2 \mathrm{d}x$	$\left(\sqrt{u}-1\right)^2$	$(\sqrt{\frac{p(x)}{q(x)}} - 1) \cdot \sqrt{\frac{q(x)}{p(x)}}$
Jensen-Shannon	$\frac{1}{2} \int p(x) \log \frac{2p(x)}{p(x)+q(x)} + q(x) \log \frac{2q(x)}{p(x)+q(x)} \mathrm{d}x$	$-(u+1) \log \frac{1+u}{2} + u \log u$	$\log \frac{2p(x)}{p(x)+q(x)}$
GAN	$\int p(x) \log \frac{2p(x)}{p(x)+q(x)} + q(x) \log \frac{2q(x)}{p(x)+q(x)} \mathrm{d}x - \log(4)$	$u \log u - (u+1) \log(u+1)$	$\log \frac{p(x)}{p(x)+q(x)}$

https://arxiv.org/abs/1606.00709

f-divergence family

Variational divergence estimation

$$
\begin{aligned}
D_f(\pi||p) &= \int p(\mathbf{x}) f\left(\frac{\pi(\mathbf{x})}{p(\mathbf{x})}\right) d\mathbf{x} \\
&= \int p(\mathbf{x}) \sup_{t \in \mathrm{dom}_{f^*}} \left(\frac{\pi(\mathbf{x})}{p(\mathbf{x})} t - f^*(t)\right) d\mathbf{x} \\
&= \int \sup_{t \in \mathrm{dom}_{f^*}} \left(\pi(\mathbf{x}) t - p(\mathbf{x}) f^*(t)\right) d\mathbf{x} \\
&\geq \sup_{T \in \mathcal{T}} \int \left(\pi(\mathbf{x}) T(\mathbf{x}) - p(\mathbf{x}) f^*(T(\mathbf{x}))\right) d\mathbf{x} \\
&= \sup_{T \in \mathcal{T}} \left[\mathbb{E}_\pi T(\mathbf{x}) - \mathbb{E}_p f^*(T(\mathbf{x}))\right]
\end{aligned}
$$

Here $\mathcal{T} : \mathcal{X} \to \mathbb{R}$ is an arbitrary class of functions.

The lower bound is tight for $T^*(\mathbf{x}) = f'\left(\frac{\pi(\mathbf{x})}{p(\mathbf{x})}\right)$.

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f-divergence family

Variational divergence estimation

$$D_f(\pi||p) \geq \sup_{T \in \mathcal{T}} [\mathbb{E}_\pi T(\mathbf{x}) - \mathbb{E}_p f^*(T(\mathbf{x}))]$$

(a) GAN (b) KL (c) Squared Hellinger

https://arxiv.org/abs/1606.00709

References

- **WGAN**: Wasserstein GAN
  https://arxiv.org/abs/1701.07875
  Summary: Proposed Earth-Mover distance as divergence measure. It shows theoretically that EM distance could be better
  than KL, JS and TV measures. Kantorovich-Rubinstein duality gives the new objective. All we need to enforce is Lipschitz
  continuity (e.x. by gradient clipping).
- **WGAN-GP**: Improved Training of Wasserstein GANs
  https://arxiv.org/abs/1704.00028
  Summary: Proves that optimal critic has unit gradient almost everywhere. Gradient penalty proposed to enforce Lipchitzness and constraint the critic norm. It greatly improves stability.
- **SN-GAN**: Spectral Normalization for Generative Adversarial Networks
  https://arxiv.org/abs/1802.05957
  Summary: Constrain Lipschitz norm by spectral norm pf weights. Spectral norm of superpositionis less or equal to spectral norms product. Usually works better than WGAN-GP.
- **f-GAN**: Training generative neural samplers using variational divergence minimization
  https://arxiv.org/abs/1606.00709
  Summary: Extend adversarial learning pipeline to any f-divergence. Variational divergence minimization frameworkis derived.