Deep Generative Models
Lecture 1

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Ozon Masters

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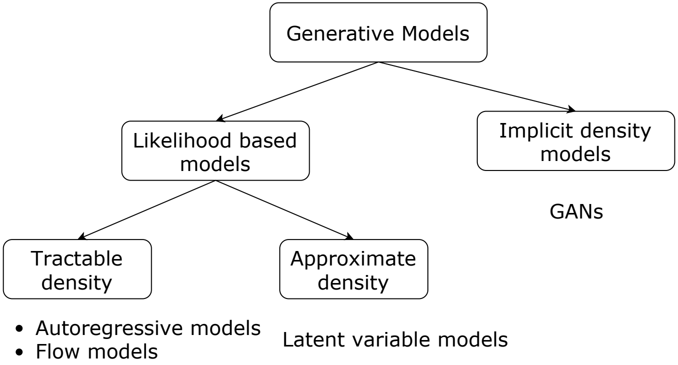
Logistics

- homeworks: 30 points
  - hw1: autoregressive models
  - hw2: latent variable models
  - hw3: flow models
  - hw4: adversarial models
- exam: 30 points
- final project: 40 points

Last year course page: link
Admission: link

Generative models zoo



Motivation

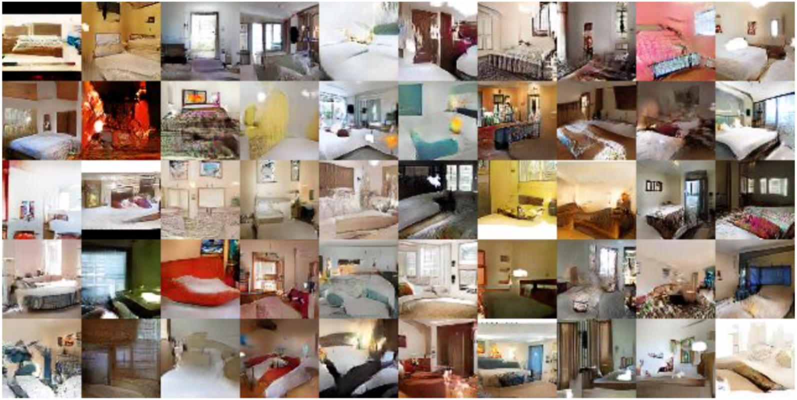
- "Pure" Reinforcement Learning (cherry)
  - The machine predicts a scalar reward given once in a while.
  - **A few bits for some samples**
- Supervised Learning (icing)
  - The machine predicts a category or a few numbers for each input
  - Predicting human-supplied data
  - **10→10,000 bits per sample**
- Unsupervised/Predictive Learning (cake)
  - The machine predicts any part of its input for any observed part.
  - Predicts future frames in videos
  - **Millions of bits per sample**

LeCun, NIPS 2016 Keynote

Applications: Image generation (VAE)

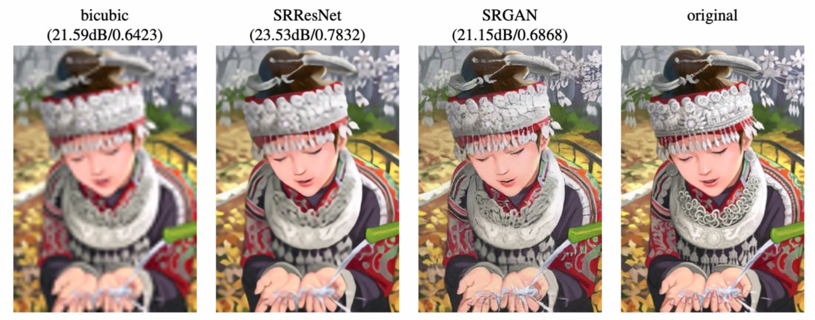
Kingma D. P., Welling M. Auto-encoding variational bayes, 2013

Applications: Image generation (DCGAN)



Radford A., Metz L., Chintala S. Unsupervised representation learning with deep convolutional generative adversarial networks, 2015

Applications: SuperResolution (SRGAN)



Ledig C. et al. Photo-realistic single image super-resolution using a generative adversarial network, 2016

Applications: Domain translation (CycleGAN)

Zhu J. Y. et al. Unpaired image-to-image translation using cycle-consistent adversarial networks, 2017

Applications: Face generation (StyleGAN)

Karras T., Laine S., Aila T. A style-based generator architecture for generative adversarial networks, 2018

Applications: Face generation (VQ-VAE-2)

Razavi A., Oord A., Vinyals O. Generating Diverse High-Fidelity Images with VQ-VAE-2, 2019

Applications

- Audio Generation (WaveNet, ...)
- Video Generation (DVD-GAN)
- NLP (Transformer, BERT, GPT-3, ...)
- Compression

Problem Statement

We are given samples $\{\mathbf{x}_i\}_{i=1}^{n} \in X$ from unknown distribution $\pi(\mathbf{x})$.

Goal

We would like to learn a distribution $\pi(\mathbf{x})$ for

- evaluating $\pi(\mathbf{x})$ for new samples;
- sampling from $\pi(\mathbf{x})$.

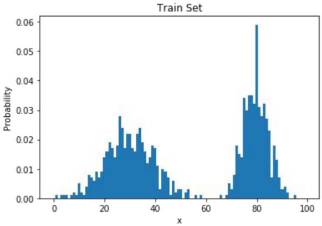
Challenge

Data is complex and high-dimensional (curse of dimensionality).

Histogram as a generative model

The histogram is totally defined by

$$p_k = p(x = k) = \frac{\sum_{i=1}^{k} [x_i = k]}{n}.$$



Problem: curse of dimensionality.

MNIST: 28x28 gray-scaled images
$2^{28\times28} - 1$ parameters to specify $p(\mathbf{x})$

$$p(\mathbf{x}) = p(x_1) \cdot p(x_2|x_1) \cdot \dots \cdot p(x_m|x_{m-1}, \dots, x_1).$$

Question: How many parameters do we need in these cases?

$$p(\mathbf{x}) = p(x_1) \cdot p(x_2) \cdot \dots \cdot p(x_m).$$

$$p(\mathbf{x}) = p(x_1) \cdot p(x_2|x_1) \cdot \dots \cdot p(x_m|x_{m-1}).$$

image credit: https://sites.google.com/view/berkeley-cs294-158-sp20/home

Maximum likelihood

Fix probabilistic model $p(\mathbf{x}|\boldsymbol{\theta})$ – the set of parameterized distributions .
Instead of searching true $\pi(\mathbf{x})$ over all probability distributions, learn function approximation $p(\mathbf{x}|\boldsymbol{\theta}) \approx \pi(\mathbf{x})$.

MLE problem

$$\boldsymbol{\theta}^* = \arg\max_{\boldsymbol{\theta}} p(\mathbf{X}|\boldsymbol{\theta}) = \arg\max_{\boldsymbol{\theta}} \prod_{i=1}^{n} p(\mathbf{x}_i|\boldsymbol{\theta}) = \arg\max_{\boldsymbol{\theta}} \sum_{i=1}^{n} \log p(\mathbf{x}_i|\boldsymbol{\theta}).$$

The problem is solved with SGD.

Requirements

- efficiently compute $\log p(\mathbf{x}|\boldsymbol{\theta})$;
- efficiently compute gradient of $\log p(\mathbf{x}|\boldsymbol{\theta})$.

Autoregressive model

MLE problem

$$\boldsymbol{\theta}^* = \arg\max_{\boldsymbol{\theta}} p(\mathbf{X}|\boldsymbol{\theta}) = \arg\max_{\boldsymbol{\theta}} \prod_{i=1}^{n} p(\mathbf{x}_i|\boldsymbol{\theta}) = \arg\max_{\boldsymbol{\theta}} \sum_{i=1}^{n} \log p(\mathbf{x}_i|\boldsymbol{\theta}).$$

Challenge

$p(\mathbf{x}|\boldsymbol{\theta})$ could be intractable.

Likelihood as product of conditionals

Let $\mathbf{x} = (x_1, \dots, x_m)$, $\mathbf{x}_{1:i} = (x_1, \dots, x_i)$. Then

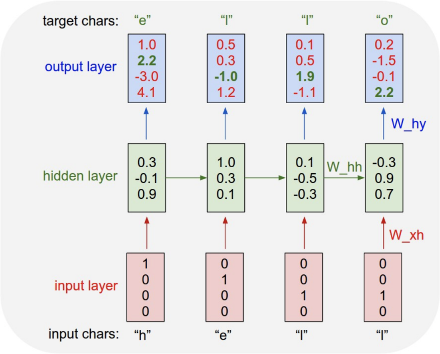
$$p(\mathbf{x}|\boldsymbol{\theta}) = \prod_{i=1}^{m} p(x_i|\mathbf{x}_{1:i-1}, \boldsymbol{\theta}); \quad \log p(\mathbf{x}|\boldsymbol{\theta}) = \sum_{i=1}^{m} \log p(x_i|\mathbf{x}_{1:i-1}, \boldsymbol{\theta}).$$

Autoregressive models

$$\log p(\mathbf{x}|\boldsymbol{\theta}) = \sum_{i=1}^{m} \log p(x_i|\mathbf{x}_{1:i-1}, \boldsymbol{\theta})$$

- Each conditional could be modelled by neural network.
- To extend to high dimensions share parameters across conditionals.
- Sampling is sequential.

Char RNN

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PANDARUS:
Alas, I think he shall be come approached and the day
When little srain would be attain'd into being never fed,
And who is but a chain and subjects of his death,
I should not sleep.

Second Senator:
They are away this miseries, produced upon my soul,
Breaking and strongly should be buried, when I perish
The earth and thoughts of many states.

DUKE VINCENTIO:
Well, your wit is in the care of side and that.

Second Lord:
They would be ruled after this chamber, and
my fair nues begun out of the fact, to be conveyed,
Whose noble souls I'll have the heart of the wars.

Clown:
Come, sir, I will make did behold your worship.

VIOLA:
I'll drink it.
```

Drawback

Sequential computation of all conditionals $p(x_i | \mathbf{x}_{1:i-1}, \boldsymbol{\theta})$.

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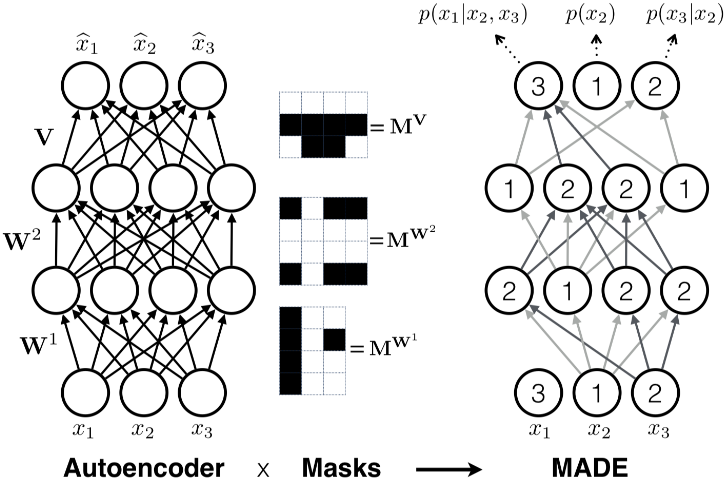
image credit: http://karpathy.github.io/2015/05/21/rnn-effectiveness

MADE



Germain M. et al. *Made: Masked autoencoder for distribution estimation, 2015*

WaveNet

Goal

Efficient generation of raw audio waveforms with natural sounds.

Solution

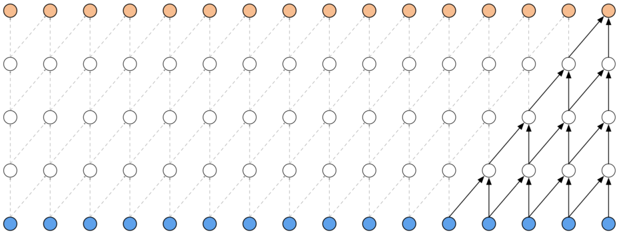
Autoregressive model

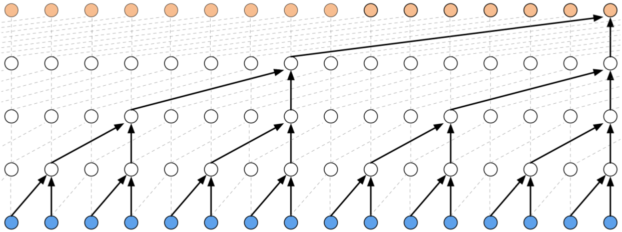
$$p(\mathbf{x}|\boldsymbol{\theta}) = \prod_{t=1}^{T} p(x_t|\mathbf{x}_{1:t-1}, \boldsymbol{\theta}).$$

The model uses causal dilated convolutions.

Oord A. et al. Wavenet: A generative model for raw audio, 2016

WaveNet (2016)





Oord A. et al. Wavenet: A generative model for raw audio, 2016

PixelCNN

Goal

Model a distribution of natural images.

Solution

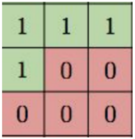
Autoregressive model

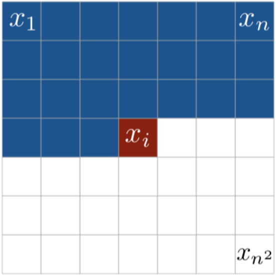
$$p(\mathbf{x}|\boldsymbol{\theta}) = \prod_{i=1}^{n^2} p(x_i|\mathbf{x}_{1:i-1}, \boldsymbol{\theta}).$$

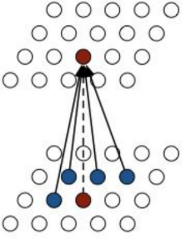
- masked convolutions;
- dependencies over RGB channels.

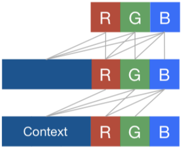
Oord A., Kalchbrenner N., Kavukcuoglu K. Pixel recurrent neural networks, 2016

PixelCNN (2016)









Oord A., Kalchbrenner N., Kavukcuoglu K. Pixel recurrent neural networks, 2016

Summary

- Sampling from autoregressive models is trivial, but sequential
  - sample $x_0 \sim p(x_0)$;
  - sample $x_1 \sim p(x_1|x_0)$;
  - $\dots$.
- Estimating probability:

$$p(\mathbf{x}) = \prod_{i=1}^{m} p(x_i|\mathbf{x}_{1:i-1}).$$

- Work on both continuous and discrete data.
- There is no natural way to do unsupervised learning.

Summary