

# Deep Generative Models

## Lecture 11

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Ozon Masters

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# Spectral Normalization GAN

How else could we enforce Lipschitzness?

Fact 1

$$\|\mathbf{g}\|_L = \sup_{\mathbf{x}} \sigma(\nabla \mathbf{g}(\mathbf{x}))$$

Here  $\sigma(\mathbf{A})$  – spectral norm of matrix  $\mathbf{A}$ .

$$\sigma(\mathbf{A}) = \max_{\mathbf{h} \neq 0} \frac{\|\mathbf{Ah}\|_2}{\|\mathbf{h}\|_2} = \max_{\|\mathbf{h}\|_2 \leq 1} \|\mathbf{Ah}\|_2 = \lambda_{\max}(\mathbf{A}),$$

where  $\lambda_{\max}(\mathbf{A})$  is the largest singular value of  $\mathbf{A}$ .

Fact 2

$$\|\mathbf{g}_1 \circ \mathbf{g}_2\|_L \leq \|\mathbf{g}_1\|_L \cdot \|\mathbf{g}_2\|_L$$

## Spectral Normalization GAN

Let consider the critic  $f(\mathbf{x}, \phi)$  of the following form:

$$f(\mathbf{x}, \phi) = \mathbf{W}_{K+1} a_K (\mathbf{W}_K a_{K-1} (\dots a_1 (\mathbf{W}_1 \mathbf{x}) \dots)).$$

This feedforward network is a superposition of simple functions.

- ▶  $a_k$  is a pointwise nonlinearities. We assume that  $\|a_k\|_L = 1$  (it holds for ReLU).
- ▶  $\mathbf{g}(\mathbf{h}) = \mathbf{W}\mathbf{h}$  is a linear transformation.

$$\|\mathbf{g}\|_L = \sup_{\mathbf{x}} \sigma(\nabla \mathbf{g}(\mathbf{x})) = \sigma(\mathbf{W}).$$

### Critic spectral norm

$$\|f\|_L \leq \prod_{k=1}^{K+1} \sigma(\mathbf{W}_k).$$

If we replace the weights in the critic by  $\mathbf{W}_k^{SN} = \mathbf{W}_k / \sigma(\mathbf{W}_k)$ , we will get  $\|f\|_L \leq 1$ .

# Spectral Normalization GAN

If we apply singular value decomposition to compute the  $\sigma(\mathbf{W})$  at each round of the algorithm, the algorithm becomes computationally heavy.

## Power iteration

- ▶  $\mathbf{u}$  – random vector.
- ▶ repeat

$$\mathbf{v} = \frac{\mathbf{W}^T \mathbf{u}}{\|\mathbf{W}^T \mathbf{u}\|}, \quad \mathbf{u} = \frac{\mathbf{Wv}}{\|\mathbf{Wv}\|}$$

- ▶ approximate the spectral norm

$$\sigma(\mathbf{W}) \approx \mathbf{u}^T \mathbf{Wv}$$

# Spectral Normalization GAN

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**Algorithm 1** SGD with spectral normalization

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- Initialize  $\tilde{\mathbf{u}}_l \in \mathcal{R}^{d_l}$  for  $l = 1, \dots, L$  with a random vector (sampled from isotropic distribution).
- For each update and each layer  $l$ :
  1. Apply power iteration method to a unnormalized weight  $W^l$ :

$$\tilde{\mathbf{v}}_l \leftarrow (W^l)^T \tilde{\mathbf{u}}_l / \| (W^l)^T \tilde{\mathbf{u}}_l \|_2 \quad (20)$$

$$\tilde{\mathbf{u}}_l \leftarrow W^l \tilde{\mathbf{v}}_l / \| W^l \tilde{\mathbf{v}}_l \|_2 \quad (21)$$

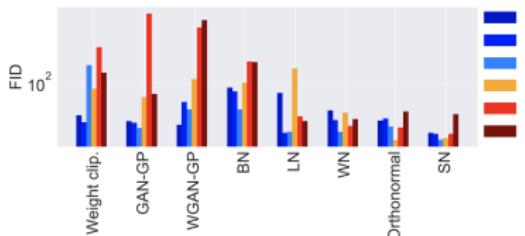
2. Calculate  $\bar{W}_{\text{SN}}$  with the spectral norm:

$$\bar{W}_{\text{SN}}^l(W^l) = W^l / \sigma(W^l), \text{ where } \sigma(W^l) = \tilde{\mathbf{u}}_l^T W^l \tilde{\mathbf{v}}_l \quad (22)$$

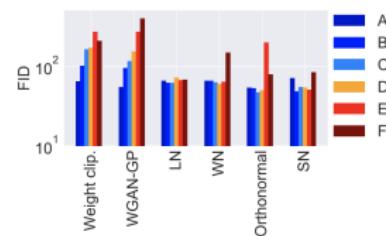
3. Update  $W^l$  with SGD on mini-batch dataset  $\mathcal{D}_M$  with a learning rate  $\alpha$ :

$$W^l \leftarrow W^l - \alpha \nabla_{W^l} \ell(\bar{W}_{\text{SN}}^l(W^l), \mathcal{D}_M) \quad (23)$$

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(a) CIFAR-10



(b) STL-10

# Divergences

## What do we have?

- ▶ Forward KL divergence in maximum likelihood estimation
- ▶ Reverse KL in variational inference
- ▶ JS divergence in vanilla gan
- ▶ Wasserstein distance in WGAN

## Divergence minimization

$$\min_p D(\pi || p)$$

## What is a divergence?

Let  $\mathcal{S}$  be the set of all possible probability distributions. Then  $D : \mathcal{S} \times \mathcal{S} \rightarrow \mathbb{R}$  is a divergence if

- ▶  $D(\pi || p) \geq 0$  for all  $\pi, p \in \mathcal{S}$ ;
- ▶  $D(\pi || p) = 0$  if and only if  $\pi \equiv p$ .

# f-divergence family

## f-divergence

$$D_f(\pi || p) = \mathbb{E}_{p(\mathbf{x})} f\left(\frac{\pi(\mathbf{x})}{p(\mathbf{x})}\right) = \int p(\mathbf{x}) f\left(\frac{\pi(\mathbf{x})}{p(\mathbf{x})}\right) d\mathbf{x}.$$

Here  $f : \mathbb{R}_+ \rightarrow \mathbb{R}$  is a convex, lower-semicontinuous function satisfying  $f(1) = 0$ .

## Fenchel conjugate

$$f^*(t) = \sup_{u \in \text{dom}_f} (ut - f(u)), \quad f^{**} = f, \quad f(u) = \sup_{t \in \text{dom}_{f^*}} (ut - f^*(t))$$

Name	$D_f(P  Q)$	Generator $f(u)$	$T^*(x)$
Kullback-Leibler	$\int p(x) \log \frac{p(x)}{q(x)} dx$	$u \log u$	$1 + \log \frac{p(x)}{q(x)}$
Reverse KL	$\int q(x) \log \frac{q(x)}{p(x)} dx$	$-\log u$	$-\frac{q(x)}{p(x)}$
Pearson $\chi^2$	$\int \frac{(q(x)-p(x))^2}{p(x)} dx$	$(u-1)^2$	$2\left(\frac{p(x)}{q(x)} - 1\right)$
Squared Hellinger	$\int \left(\sqrt{p(x)} - \sqrt{q(x)}\right)^2 dx$	$(\sqrt{u}-1)^2$	$(\sqrt{\frac{p(x)}{q(x)}} - 1) \cdot \sqrt{\frac{q(x)}{p(x)}}$
Jensen-Shannon	$\frac{1}{2} \int p(x) \log \frac{2p(x)}{p(x)+q(x)} + q(x) \log \frac{2q(x)}{p(x)+q(x)} dx$	$-(u+1) \log \frac{1+u}{2} + u \log u$	$\log \frac{2p(x)}{p(x)+q(x)}$
GAN	$\int p(x) \log \frac{2p(x)}{p(x)+q(x)} + q(x) \log \frac{2q(x)}{p(x)+q(x)} dx - \log(4)$	$u \log u - (u+1) \log(u+1)$	$\log \frac{p(x)}{p(x)+q(x)}$

## f-divergence family

### Variational divergence estimation

$$\begin{aligned} D_f(\pi || p) &= \int p(\mathbf{x}) f\left(\frac{\pi(\mathbf{x})}{p(\mathbf{x})}\right) d\mathbf{x} \\ &= \int p(\mathbf{x}) \sup_{t \in \text{dom}_{f^*}} \left( \frac{\pi(\mathbf{x})}{p(\mathbf{x})} t - f^*(t) \right) d\mathbf{x} \\ &= \int \sup_{t \in \text{dom}_{f^*}} (\pi(\mathbf{x}) t - p(\mathbf{x}) f^*(t)) d\mathbf{x} \\ &\geq \sup_{T \in \mathcal{T}} \int (\pi(\mathbf{x}) T(\mathbf{x}) - p(\mathbf{x}) f^*(T(\mathbf{x}))) d\mathbf{x} \\ &= \sup_{T \in \mathcal{T}} [\mathbb{E}_\pi T(\mathbf{x}) - \mathbb{E}_p f^*(T(\mathbf{x}))] \end{aligned}$$

Here  $\mathcal{T} : \mathcal{X} \rightarrow \mathbb{R}$  is an arbitrary class of functions.

The lower bound is tight for  $T^*(\mathbf{x}) = f'\left(\frac{\pi(\mathbf{x})}{p(\mathbf{x})}\right)$ .

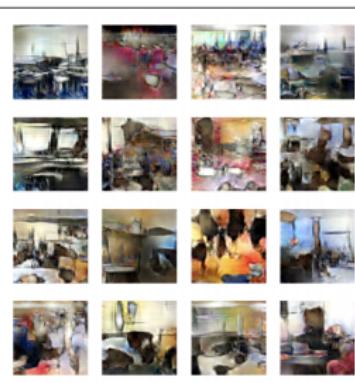
# f-divergence family

## Variational divergence estimation

$$D_f(\pi || p) \geq \sup_{T \in \mathcal{T}} [\mathbb{E}_\pi T(\mathbf{x}) - \mathbb{E}_p f^*(T(\mathbf{x}))]$$



(a) GAN



(b) KL



(c) Squared Hellinger

# Evaluation of likelihood-free models

How to evaluate generative models?

## Likelihood-based models

- ▶ Split data to train/val/test.
- ▶ Fit model on the train part.
- ▶ Tune hyperparameters on the validation part.
- ▶ Evaluate generalization by reporting likelihoods on the test set.

Not all models have tractable likelihoods

- ▶ VAE: compare ELBO values.
- ▶ GAN: ???

# Evaluation of likelihood-free models

Let's take some pretrained image classification model to get the conditional label distribution  $p(y|x)$  (e.g. ImageNet classifier).

What do we want from samples?

- ▶ Sharpness



The conditional distribution  $p(y|x)$  should have low entropy (each image  $x$  should have distinctly recognizable object).

- ▶ Diversity



The marginal distribution  $p(y) = \int p(y|x)p(x)dx$  should have high entropy (there should be as many classes generated as possible).

# Evaluation of likelihood-free models

## What do we want from samples?

- ▶ **Sharpness.** The conditional distribution  $p(y|x)$  should have low entropy (each image  $x$  should have distinctly recognizable object).
- ▶ **Diversity.** The marginal distribution  $p(y) = \int p(y|x)p(x)dx$  should have high entropy (there should be as many classes generated as possible).

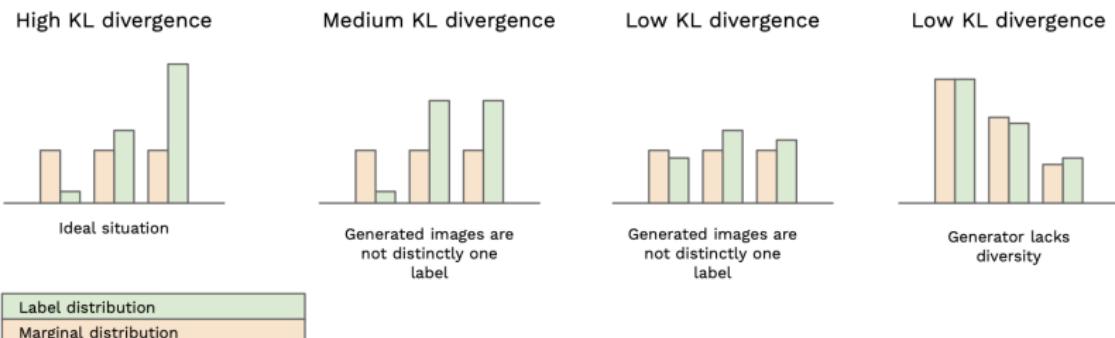


image credit: <https://medium.com/octavian-ai/a-simple-explanation-of-the-inception-score-372dff6a8c7a>

# Evaluation of likelihood-free models

What do we want from samples?

- ▶ Sharpness  $\Rightarrow$  low  $H(y|\mathbf{x}) = - \sum_y \int_{\mathbf{x}} p(y, \mathbf{x}) \log p(y|\mathbf{x}) d\mathbf{x}$ .
- ▶ Diversity  $\Rightarrow$  high  $H(y) = - \sum_y p(y) \log p(y)$ .

Inception Score

$$\begin{aligned} IS &= \exp(H(y) - H(y|\mathbf{x})) \\ &= \exp \left( - \sum_y p(y) \log p(y) + \sum_y \int_{\mathbf{x}} p(y, \mathbf{x}) \log p(y|\mathbf{x}) d\mathbf{x} \right) \\ &= \exp \left( \sum_y \int_{\mathbf{x}} p(y, \mathbf{x}) \log \frac{p(y|\mathbf{x})}{p(y)} d\mathbf{x} \right) \\ &= \exp \left( \mathbb{E}_{\mathbf{x}} \sum_y p(y|\mathbf{x}) \log \frac{p(y|\mathbf{x})}{p(y)} \right) = \exp (\mathbb{E}_{\mathbf{x}} KL(p(y|\mathbf{x}) || p(y))) \end{aligned}$$

# Evaluation of likelihood-free models

## Inception Score

$$IS = \exp(\mathbb{E}_{\mathbf{x}} KL(p(y|\mathbf{x}) || p(y)))$$

### IS limitations

- ▶ Inception score depends on the quality of the pretrained classifier  $p(y|\mathbf{x})$ .
- ▶ If generator produces images with a different set of labels from the classifier training set, IS will be low.
- ▶ If the generator produces one image per class, the IS will be perfect (there is no measure of intra-class diversity).
- ▶ IS only require samples from the generator and do not take into account the desired data distribution  $\pi(\mathbf{x})$  directly (only implicitly via a classifier).

# Evaluation of likelihood-free models

## Theorem

If  $\pi(\mathbf{x})$  and  $p(\mathbf{x}|\theta)$  has moment generation functions then

$$\pi(\mathbf{x}) = p(\mathbf{x}|\theta) \Leftrightarrow \mathbb{E}_\pi \mathbf{x}^k = \mathbb{E}_p \mathbf{x}^k, \quad \forall k \geq 1.$$

This is intractable to calculate all moments.

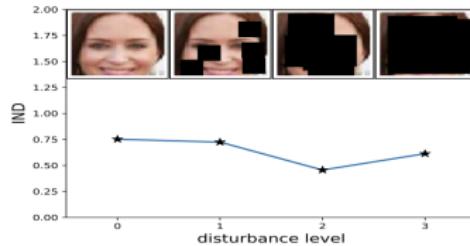
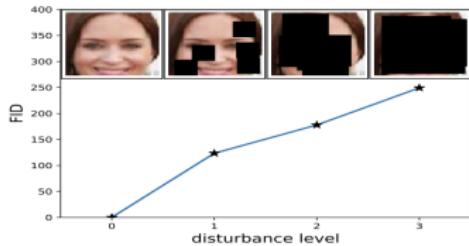
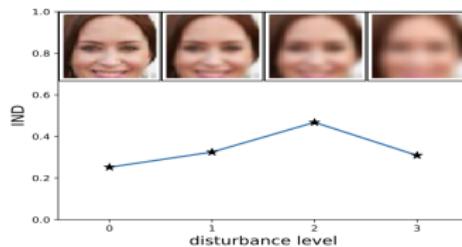
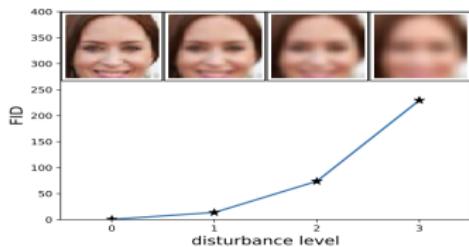
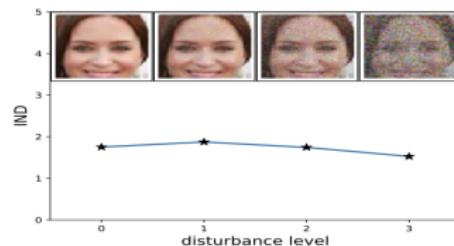
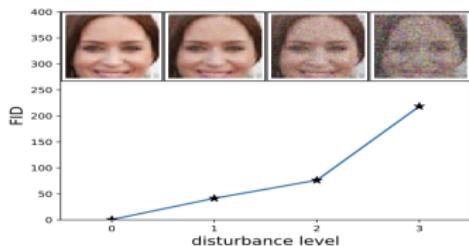
## Frechet Inception Distance

$$D^2(\pi, p) = \|\mathbf{m}_\pi - \mathbf{m}_p\|_2^2 + \text{Tr} \left( \mathbf{C}_\pi + \mathbf{C}_p - 2\sqrt{\mathbf{C}_\pi \mathbf{C}_p} \right)$$

- ▶  $\mathbf{m}_\pi, \mathbf{C}_\pi$  are mean vector and covariance matrix of feature representations for real samples from  $\pi(\mathbf{x})$
- ▶  $\mathbf{m}_p, \mathbf{C}_p$  are mean vector and covariance matrix of feature representations for generated samples from  $p(\mathbf{x}|\theta)$ .
- ▶ Representations are outputs of intermediate layer from ~~pretrained classification model~~.

*Heusel M. et al. GANs Trained by a Two Time-Scale Update Rule Converge to a Local Nash Equilibrium, 2017*

# Evaluation of likelihood-free models



# Evaluation of likelihood-free models

## Frechet Inception Distance

$$D^2(\pi, p) = \|\mathbf{m}_\pi - \mathbf{m}_p\|_2^2 + \text{Tr} \left( \mathbf{C}_\pi + \mathbf{C}_p - 2\sqrt{\mathbf{C}_\pi \mathbf{C}_p} \right)$$

## FID limitations

- ▶ FID depends on the pretrained classification model.
- ▶ FID needs a large samples size for evaluation.
- ▶ Calculation of FID is slow.
- ▶ FID estimates only two sample moments.

## Summary

- ▶ Wasserstein GAN uses Kantorovich-Rubinstein duality to estimate Wasserstein distance.
- ▶ Gradient Penalty proposes the regularizer to enforce Lipschitzness.
- ▶ Spectral normalization is a weight normalization technique to enforce Lipshitzness.
- ▶ f-divergence family is a unified framework for divergence minimization.
- ▶ Inception Score and Frechet Inception Distance are the common metrics for GAN evaluation.

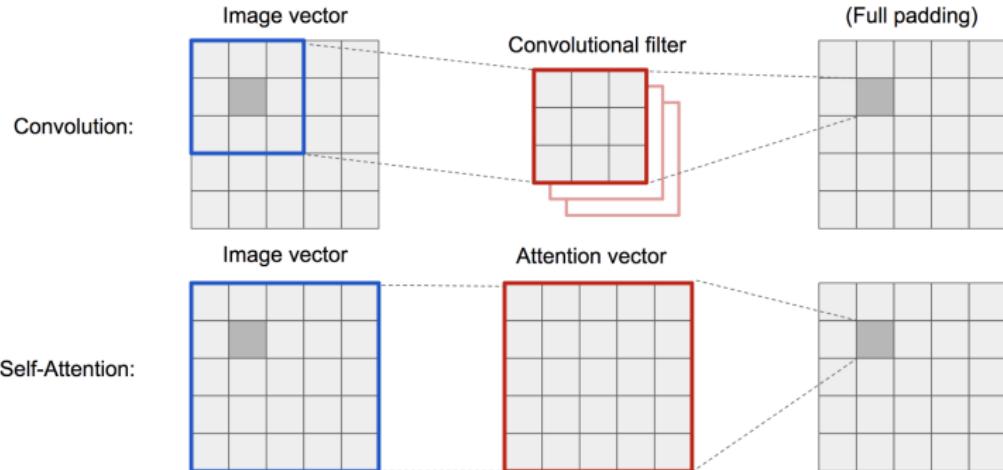
# Evolution of GANs



- ▶ **Vanilla GAN** <https://arxiv.org/abs/1406.2661>
- ▶ **DCGAN** <https://arxiv.org/abs/1511.06434>
- ▶ **CoGAN** <https://arxiv.org/abs/1606.07536>
- ▶ **ProGAN** <https://arxiv.org/abs/1710.10196>
- ▶ **StyleGAN** <https://arxiv.org/abs/1812.04948>

# Self-Attention GAN

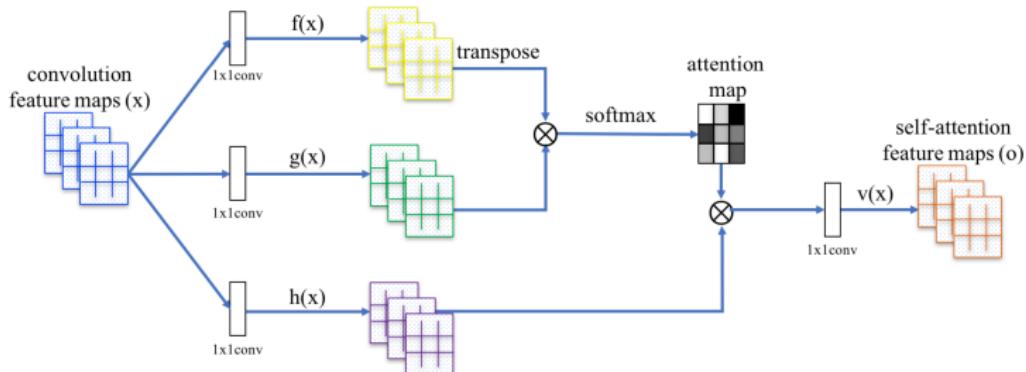
- ▶ Convolutional layers process the information in a local neighborhood.
- ▶ Using convolutional layers alone is computationally inefficient for modeling long-range dependencies in images.



*image credit:*

<https://lilianweng.github.io/lil-log/2018/06/24/attention-attention.html>

# Self-Attention GAN



- ▶  $x$  – feature vector for one feature location.
- ▶  $N$  – number of feature locations.

$$\mathbf{f}(\mathbf{x}) = \mathbf{W}_f(\mathbf{x}), \quad \mathbf{g}(\mathbf{x}) = \mathbf{W}_g(\mathbf{x}), \quad \mathbf{h}(\mathbf{x}) = \mathbf{W}_h(\mathbf{x}), \quad \mathbf{v}(\mathbf{x}) = \mathbf{W}_v(\mathbf{x})$$

$$s_{ij} = \mathbf{f}(\mathbf{x}_i)^T \mathbf{g}(\mathbf{x}_j), \quad a_{ij} = \frac{\exp s_{ij}}{\sum_{i=1}^N \exp s_{ij}}, \quad \mathbf{o}_j = \mathbf{v} \left( \sum_{i=1}^N a_{ij} \mathbf{h}(\mathbf{x}_i) \right)$$

# Self-Attention GAN

## Technical Details

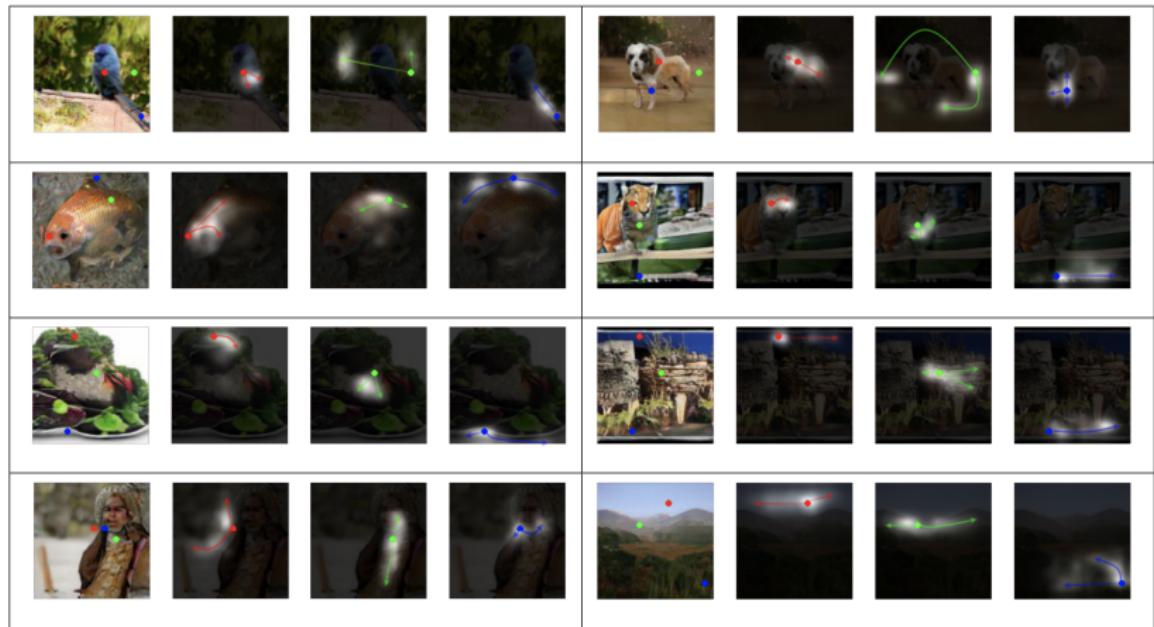
- ▶ Hinge loss for training.
- ▶ Spectral Normalization in both the generator and the discriminator.
- ▶ Separate learning rates for the generator and the discriminator.

Model	no attention	SAGAN				Residual			
		$feat_8$	$feat_{16}$	$feat_{32}$	$feat_{64}$	$feat_8$	$feat_{16}$	$feat_{32}$	$feat_{64}$
FID	22.96	22.98	22.14	<b>18.28</b>	18.65	42.13	22.40	27.33	28.82
IS	42.87	43.15	45.94	51.43	<b>52.52</b>	23.17	44.49	38.50	38.96

Model	Inception Score	Intra FID	FID
AC-GAN ( <a href="#">Odena et al., 2017</a> )	28.5	260.0	/
SNGAN-projection ( <a href="#">Miyato &amp; Koyama, 2018</a> )	36.8	92.4	27.62*
SAGAN	<b>52.52</b>	<b>83.7</b>	<b>18.65</b>

# Self-Attention GAN

## Visualization of attention maps



# BigGAN

## Model description

- ▶ Self-Attention GAN baseline.
- ▶ Class-conditional generator.
- ▶ Increasing batch size gives tremendous benefit (allows to cover more modes).
- ▶ Increasing model size is helpful (wider helps as much as deeper).
- ▶ Hinge loss for training.
- ▶ Orthogonal regularization for smoothness the generator output.
- ▶ Truncation trick for balancing between diversity and fidelity.

# BigGAN

## ► Orthogonal regularization

$$\|\mathbf{W}^T \mathbf{W} - \mathbf{I}\|^2 \Rightarrow \|\mathbf{W}^T \mathbf{W} - \text{diag}(\mathbf{W}^T \mathbf{W})\|^2$$

- **Truncation trick.** Coordinates of samples  $\mathbf{z} \sim \mathcal{N}(0, \mathbf{I})$  which fall outside a predefined range are resampled to fall inside that range.

Batch	Ch.	Param (M)	Shared	Skip-z	Ortho.	Itr $\times 10^3$	FID	IS
256	64	81.5		SA-GAN Baseline		1000	18.65	52.52
512	64	81.5	✗	✗	✗	1000	15.30	58.77( $\pm 1.18$ )
1024	64	81.5	✗	✗	✗	1000	14.88	63.03( $\pm 1.42$ )
2048	64	81.5	✗	✗	✗	732	12.39	76.85( $\pm 3.83$ )
2048	96	173.5	✗	✗	✗	295( $\pm 18$ )	9.54( $\pm 0.62$ )	92.98( $\pm 4.27$ )
2048	96	160.6	✓	✗	✗	185( $\pm 11$ )	9.18( $\pm 0.13$ )	94.94( $\pm 1.32$ )
2048	96	158.3	✓	✓	✗	152( $\pm 7$ )	8.73( $\pm 0.45$ )	98.76( $\pm 2.84$ )
2048	96	158.3	✓	✓	✓	165( $\pm 13$ )	8.51( $\pm 0.32$ )	99.31( $\pm 2.10$ )
2048	64	71.3	✓	✓	✓	371( $\pm 7$ )	10.48( $\pm 0.10$ )	86.90( $\pm 0.61$ )

# BigGAN

Samples (512x512)



# BigGAN

## Interpolations



# Progressive Growing GAN

## Problems with HR image generation

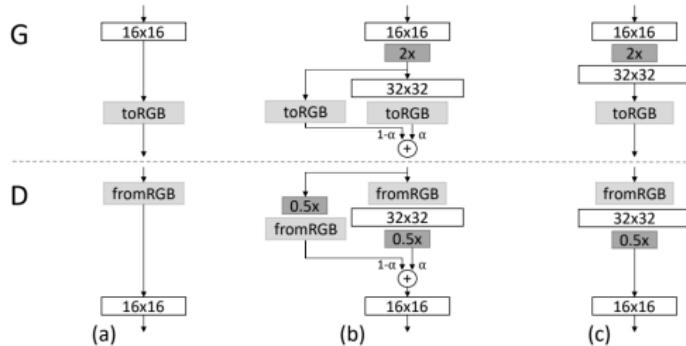
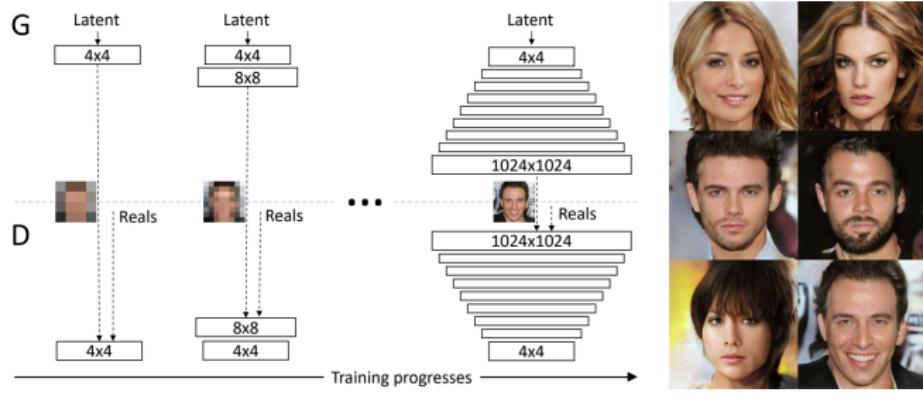
- ▶ Disjoint manifolds  $\Rightarrow$  gradient problem.
- ▶ Small minibatch  $\Rightarrow$  training instability.

## Solution

Grow both the generator and discriminator progressively, starting from LR images, and add new layers that introduce higher-resolution details as the training progresses.

- ▶ Train GAN which generate 4x4 images (just 2 convolutions for G and D).
- ▶ Add upsampling layers to G, downsampling layers to D.
- ▶ Train GAN which generate 8x8 images.
- ▶ etc.

# Progressive Growing GAN



Karras T. et al. *Progressive Growing of GANs for Improved Quality, Stability, and Variation*, 2017

# Progressive Growing GAN

Samples (1024x1024)



# StyleGAN

- ▶ Generating of HR images is hard.
- ▶ Progressive growing greatly simplifies the task.
- ▶ The ability to control specific features of the generated image is very limited.

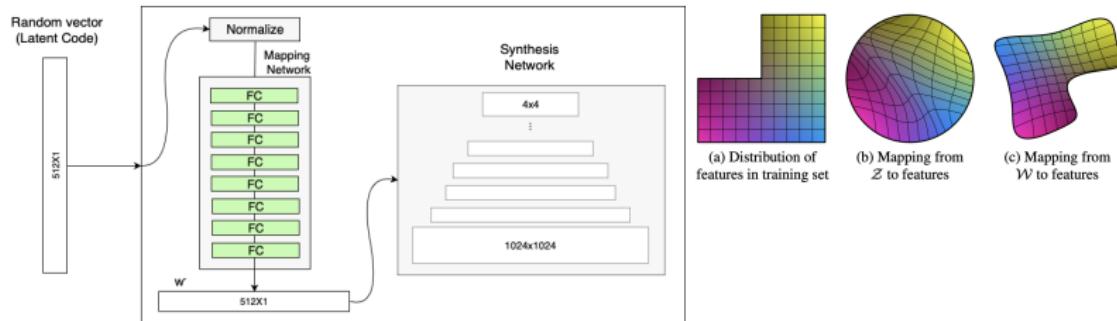
## Face image features

- ▶ Coarse (pose, general hair style, face shape). Resolution  $4^2 - 8^2$ .
- ▶ Middle (finer facial features, hair style, eyes open/closed). Resolution  $16^2 - 32^2$ .
- ▶ Fine (color scheme (eye, hair and skin) and micro features). Resolution  $64^2 - 1024^2$ .

# StyleGAN

## Step 1: Mapping Network

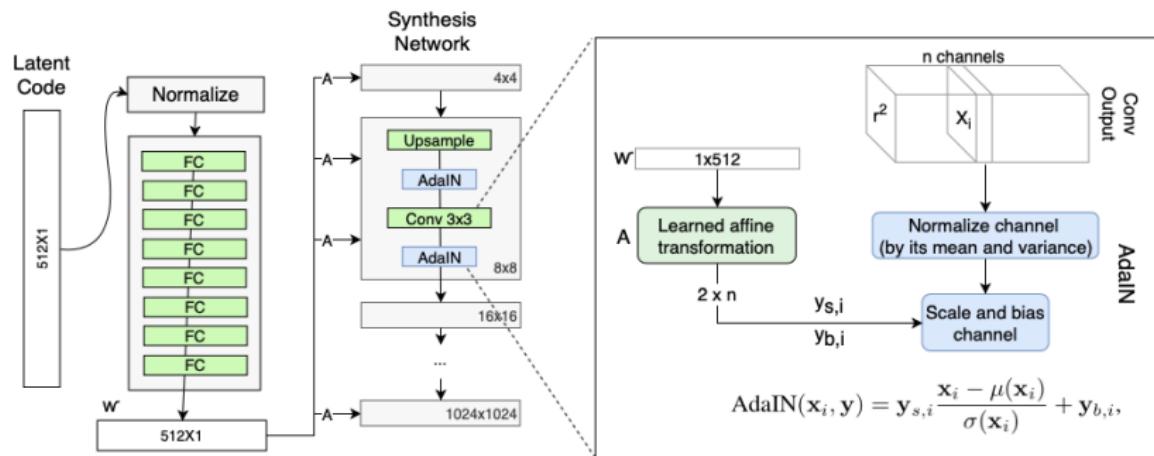
- ▶ Generator input is likely to be **disentangled**. Each component of input vector  $\mathbf{z}$  should be responsible for one generative factor.
- ▶ Mapping network  $f : \mathcal{Z} \rightarrow \mathcal{W}$  is used to reduce correlations between components of  $\mathbf{z}$ .



# StyleGAN

## Step 2: Style modulation

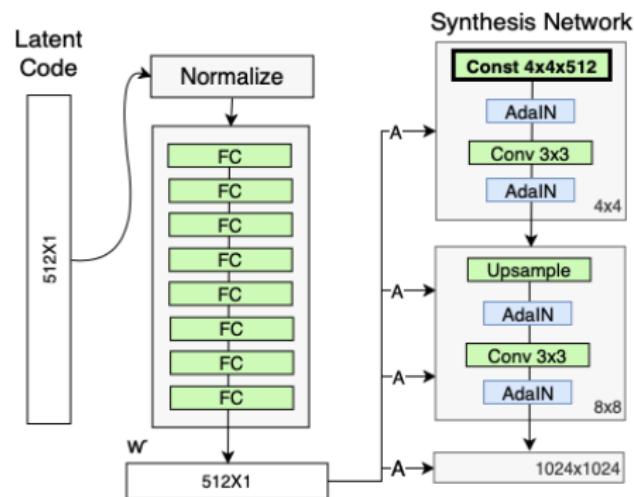
- ▶ Adaptive Instance Normalization transfers the  $\mathbf{w}$  vector to the synthesis Network.
- ▶ The module is added to each resolution to define the visual expression of the features.



# StyleGAN

## Step 3: Remove traditional input

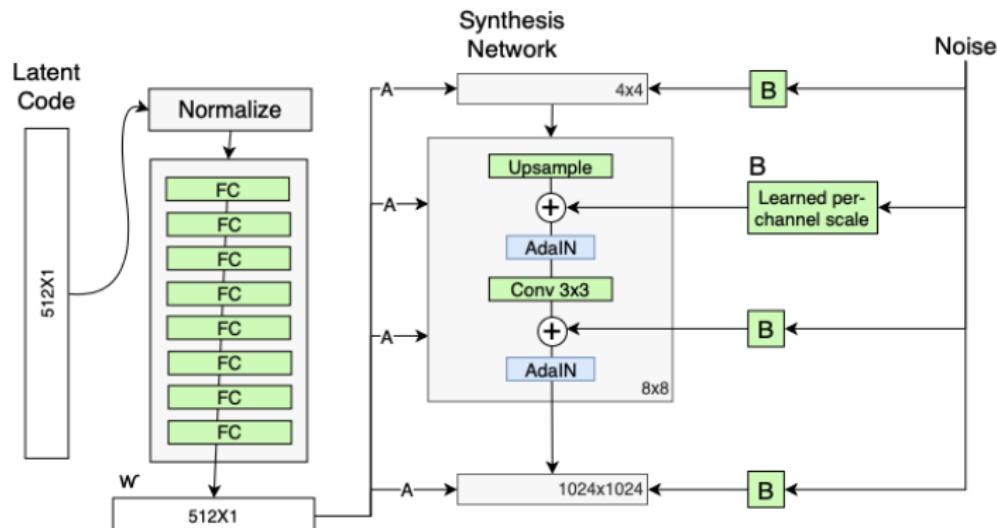
Mapping network provides stochasticity to different stages of the synthesis network. Input of the synthesis network is a trainable vector.



# StyleGAN

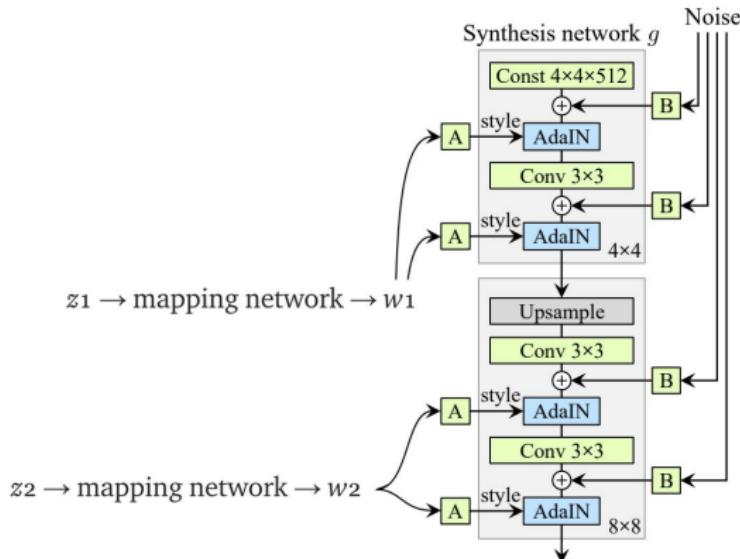
## Step 4: Stochastic variation

Inject random noise to add small aspects, such as freckles, exact placement of hairs, wrinkles, features which make the image more realistic and increase the variety of outputs.



# StyleGAN

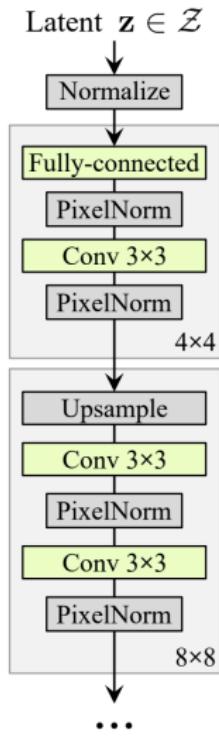
## Step 4: Style Mixing



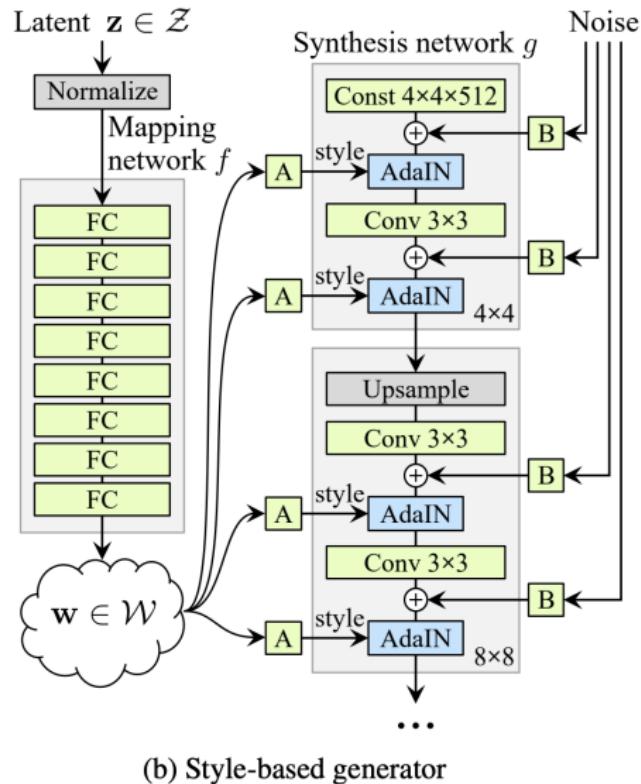
- ▶ Makes different levels of synthesis network to be independent.
- ▶ Allows to couple different styles.

Karras T., Laine S., Aila T. A Style-Based Generator Architecture for Generative Adversarial Networks, 2018

# StyleGAN



(a) Traditional



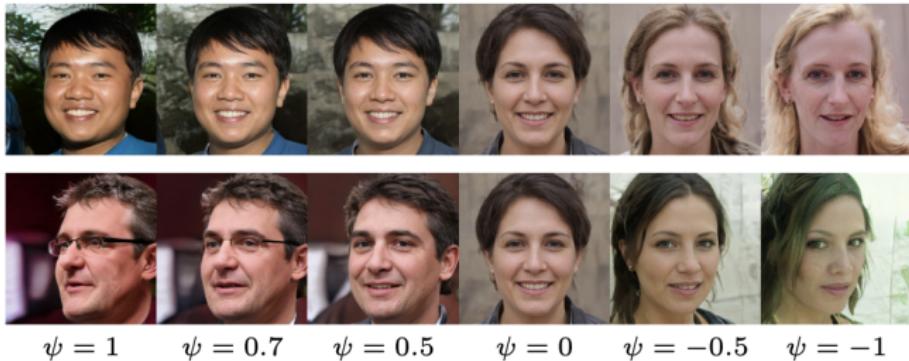
(b) Style-based generator

# StyleGAN

## Truncation trick

$$\mathbf{w}' = \hat{\mathbf{w}} - \psi \cdot (\mathbf{w} - \hat{\mathbf{w}}), \quad \hat{\mathbf{w}} = \mathbb{E}_{\mathbf{z}} p(f(\mathbf{z}))$$

- ▶ Constant  $\psi$  is a tradeoff between diversity and fidelity.
- ▶  $\psi = 0.7$  is used for most of the results.
- ▶ Truncation is done only at the low-resolution layers.



# StyleGAN

## Results

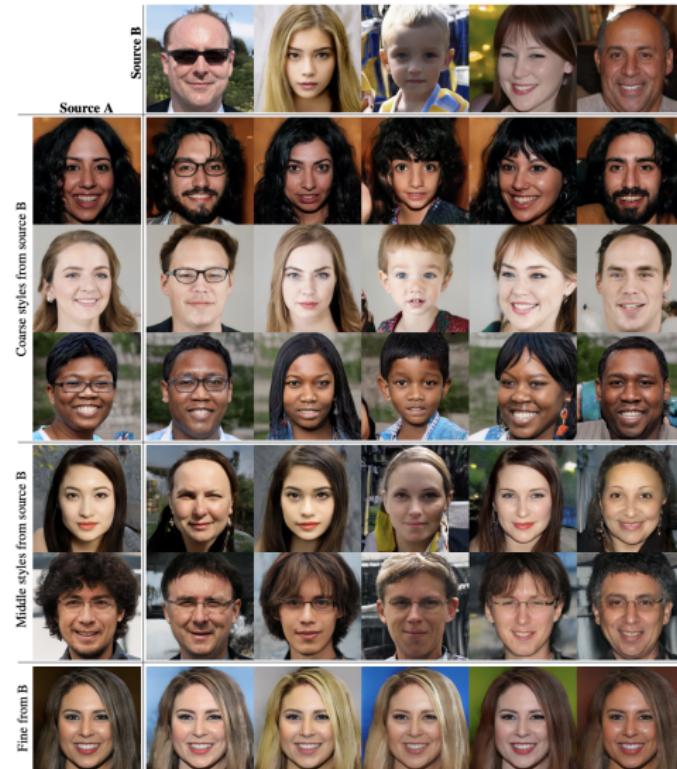
Method	CelebA-HQ	FFHQ
A Baseline Progressive GAN [30]	7.79	8.04
B + Tuning (incl. bilinear up/down)	6.11	5.25
C + Add mapping and styles	5.34	4.85
D + Remove traditional input	5.07	4.88
E + Add noise inputs	<b>5.06</b>	4.42
F + Mixing regularization	5.17	<b>4.40</b>

## Samples (1024x1024)



Karras T., Laine S., Aila T. A Style-Based Generator Architecture for Generative Adversarial Networks, 2018

# StyleGAN



*Karras T., Laine S., Aila T. A Style-Based Generator Architecture for Generative Adversarial Networks, 2018*

# Summary