

Deep Generative Models

Lecture 11

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Recap of previous lecture

Kantorovich-Rubinstein duality

$$W(\pi || p) = \frac{1}{K} \max_{\|f\|_L \leq K} [\mathbb{E}_{\pi(x)} f(x) - \mathbb{E}_{p(x)} f(x)],$$

where $\|f\|_L \leq K$ are K -Lipschitz continuous functions
($f : \mathcal{X} \rightarrow \mathbb{R}$).

Gradient penalty

$$W(\pi || p) = \underbrace{\mathbb{E}_{\pi(x)} f(x) - \mathbb{E}_{p(x)} f(x)}_{\text{original critic loss}} + \lambda \underbrace{\mathbb{E}_{U[0,1]} \left[(\|\nabla_{\hat{x}} f(\hat{x})\|_2 - 1)^2 \right]}_{\text{gradient penalty}}.$$

Samples $\hat{x}_t = t\mathbf{x} + (1-t)\mathbf{y}$ with $t \in [0, 1]$ are uniformly sampled along straight lines between pairs of points: \mathbf{x} from the data distribution $\pi(\mathbf{x})$ and \mathbf{y} from the generator distribution $p(\mathbf{x}|\theta)$.

Recap of previous lecture

$$f(\mathbf{x}, \phi) = \mathbf{W}_{K+1} a_K (\mathbf{W}_K a_{K-1} (\dots a_1 (\mathbf{W}_1 \mathbf{x}) \dots)).$$

- ▶ a_k is a pointwise nonlinearities. We assume that $\|a_k\|_L = 1$ (it holds for ReLU).
- ▶ $\mathbf{g}(\mathbf{x}) = \mathbf{W}\mathbf{x}$ is a linear transformation ($\nabla \mathbf{g}(\mathbf{x}) = \mathbf{W}$).

$$\|\mathbf{g}\|_L = \sup_{\mathbf{x}} \sigma(\nabla \mathbf{g}(\mathbf{x})) = \sigma(\mathbf{W}).$$

Critic spectral norm

$$\|f\|_L \leq \|\mathbf{W}_{K+1}\| \cdot \prod_{k=1}^K \|a_k\|_L \cdot \|\mathbf{W}_k\| = \prod_{k=1}^{K+1} \sigma(\mathbf{W}_k).$$

Spectral Normalization GAN

If we replace the weights in the critic $f(\mathbf{x}, \phi)$ by $\mathbf{W}_k^{SN} = \mathbf{W}_k / \sigma(\mathbf{W}_k)$, we will get $\|f\|_L \leq 1$.

Power iteration approximates the value of $\sigma(\mathbf{W})$.

Recap of previous lecture

What is a divergence?

Let \mathcal{S} be the set of all possible probability distributions. Then $D : \mathcal{S} \times \mathcal{S} \rightarrow \mathbb{R}$ is a divergence if

- ▶ $D(\pi || p) \geq 0$ for all $\pi, p \in \mathcal{S}$;
- ▶ $D(\pi || p) = 0$ if and only if $\pi \equiv p$.

f-divergence minimization

$$D_f(\pi || p) = \mathbb{E}_{p(\mathbf{x})} f\left(\frac{\pi(\mathbf{x})}{p(\mathbf{x})}\right) \rightarrow \min_p .$$

Variational divergence estimation

$$D_f(\pi || p) \geq \sup_{T \in \mathcal{T}} [\mathbb{E}_\pi T(\mathbf{x}) - \mathbb{E}_p f^*(T(\mathbf{x}))],$$

where $f^*(t) = \sup_{u \in \text{dom}_f} (ut - f(u))$ – Fenchel conjugate.

Recap of previous lecture

Let's take some pretrained image classification model to get the conditional label distribution $p(y|\mathbf{x})$ (e.g. ImageNet classifier).

Evaluation of likelihood-free models

- ▶ Sharpness \Rightarrow low $H(y|\mathbf{x}) = -\sum_y \int_{\mathbf{x}} p(y, \mathbf{x}) \log p(y|\mathbf{x}) d\mathbf{x}$.
- ▶ Diversity \Rightarrow high $H(y) = -\sum_y p(y) \log p(y)$.

Inception Score

$$IS = \exp(H(y) - H(y|\mathbf{x})) = \exp(\mathbb{E}_{\mathbf{x}} KL(p(y|\mathbf{x}) || p(y)))$$

Frechet Inception Distance

$$D^2(\pi, p) = \|\mathbf{m}_\pi - \mathbf{m}_p\|_2^2 + \text{Tr} \left(\mathbf{C}_\pi + \mathbf{C}_p - 2\sqrt{\mathbf{C}_\pi \mathbf{C}_p} \right).$$

FID is related to moment matching.

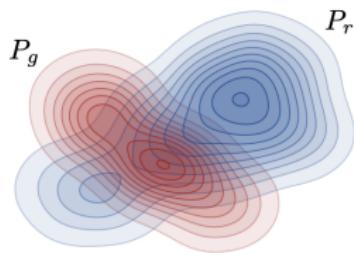
Salimans T. et al. *Improved Techniques for Training GANs*, 2016

Heusel M. et al. *GANs Trained by a Two Time-Scale Update Rule Converge to a Local Nash Equilibrium*, 2017

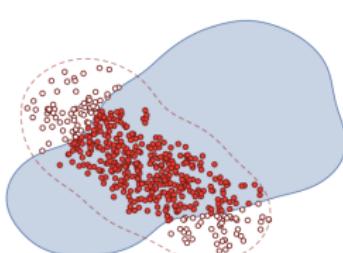
Precision-Recall for Generative Models

What do we want from samples

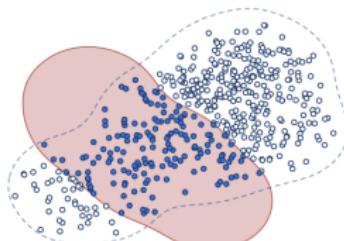
- ▶ **Sharpness:** generated samples should be of high quality.
- ▶ **Diversity:** their variation should match that observed in the training set.



(a) Example distributions



(b) Precision



(c) Recall

- ▶ **Precision** denotes the fraction of generated images that are realistic.
- ▶ **Recall** measures the fraction of the training data manifold covered by the generator.

Precision-Recall for Generative Models

- ▶ $\mathcal{S}_\pi = \{\mathbf{x}_i\}_{i=1}^n \sim \pi(\mathbf{x})$ – real samples;
- ▶ $\mathcal{S}_p = \{\mathbf{x}_i\}_{i=1}^n \sim p(\mathbf{x}|\theta)$ – generated samples.

Embed samples using pretrained classifier network (as previously):

$$\mathcal{G}_\pi = \{\mathbf{g}_i\}_i^n, \quad \mathcal{G}_p = \{\mathbf{g}_i\}_i^n.$$

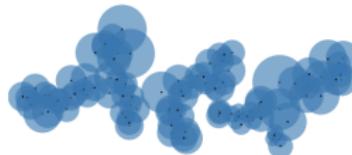
Define binary function:

$$f(\mathbf{g}, \mathcal{G}) = \begin{cases} 1, & \text{if exists } \mathbf{g}' \in \mathcal{G} : \|\mathbf{g} - \mathbf{g}'\|_2 \leq \|\mathbf{g} - \text{NN}_k(\mathbf{g}, \mathcal{G})\|_2; \\ 0, & \text{otherwise.} \end{cases}$$

$$\text{Precision}(\mathcal{G}_\pi, \mathcal{G}_p) = \frac{1}{n} \sum_{\mathbf{g} \in \mathcal{G}_p} f(\mathbf{g}, \mathcal{G}_\pi); \quad \text{Recall}(\mathcal{G}_\pi, \mathcal{G}_p) = \frac{1}{n} \sum_{\mathbf{g} \in \mathcal{G}_\pi} f(\mathbf{g}, \mathcal{G}_p).$$

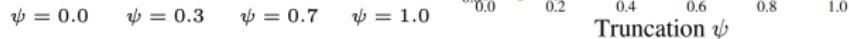
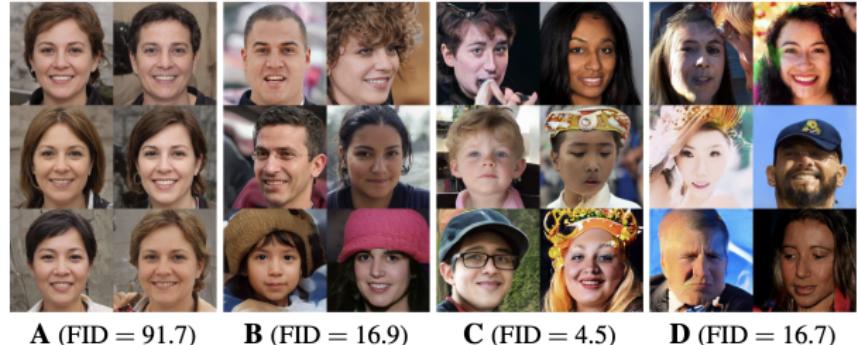
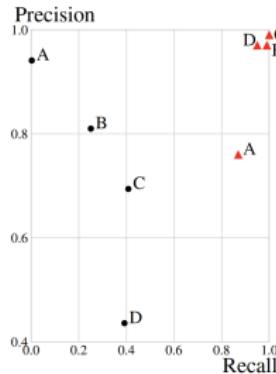


(a) True manifold



(b) Approx. manifold

Precision-Recall for Generative Models



Kynkäanniemi T. et al. Improved precision and recall metric for assessing generative models, 2019

Evolution of GANs



2014



2015



2016



2017



2018

- ▶ **Vanilla GAN** <https://arxiv.org/abs/1406.2661>
- ▶ **DCGAN** <https://arxiv.org/abs/1511.06434>
- ▶ **CoGAN** <https://arxiv.org/abs/1606.07536>
- ▶ **ProGAN** <https://arxiv.org/abs/1710.10196>
- ▶ **StyleGAN** <https://arxiv.org/abs/1812.04948>

Self-Attention GAN

- ▶ Convolutional layers process the information in a local neighborhood.
- ▶ Using convolutional layers alone is computationally inefficient for modeling long-range dependencies in images.

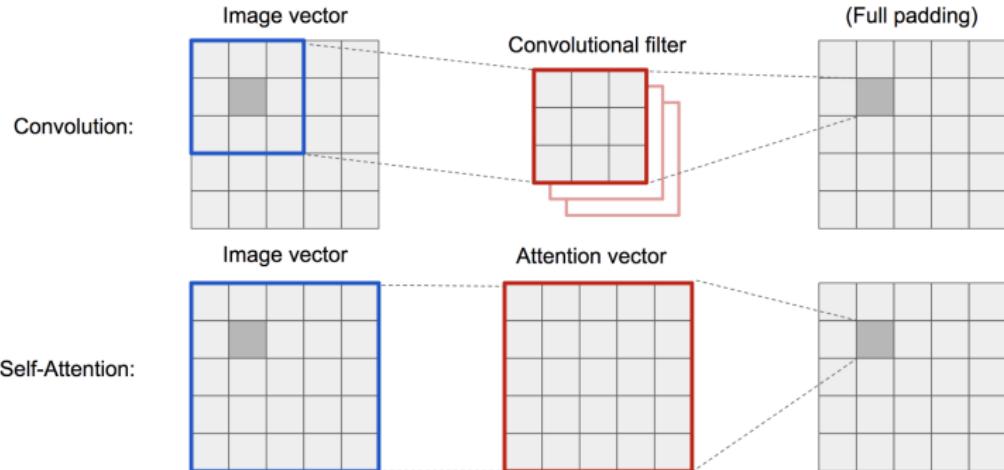
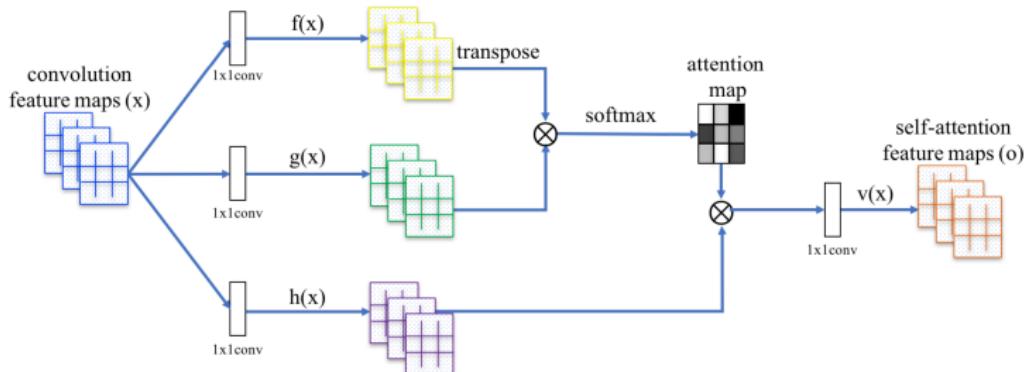


image credit:

<https://lilianweng.github.io/lil-log/2018/06/24/attention-attention.html>

Self-Attention GAN



- ▶ x – feature vector for one feature location.
- ▶ N – number of feature locations.

$$\mathbf{f}(\mathbf{x}) = \mathbf{W}_f \mathbf{x}, \quad \mathbf{g}\mathbf{x} = \mathbf{W}_g \mathbf{x}, \quad \mathbf{h}\mathbf{x} = \mathbf{W}_h \mathbf{x}, \quad \mathbf{v}\mathbf{x} = \mathbf{W}_v \mathbf{x}$$

$$s_{ij} = \mathbf{f}(\mathbf{x}_i)^T \mathbf{g}(\mathbf{x}_j), \quad a_{ij} = \frac{\exp s_{ij}}{\sum_{i=1}^N \exp s_{ij}}, \quad \mathbf{o}_j = \mathbf{v} \left(\sum_{i=1}^N a_{ij} \mathbf{h}(\mathbf{x}_i) \right)$$

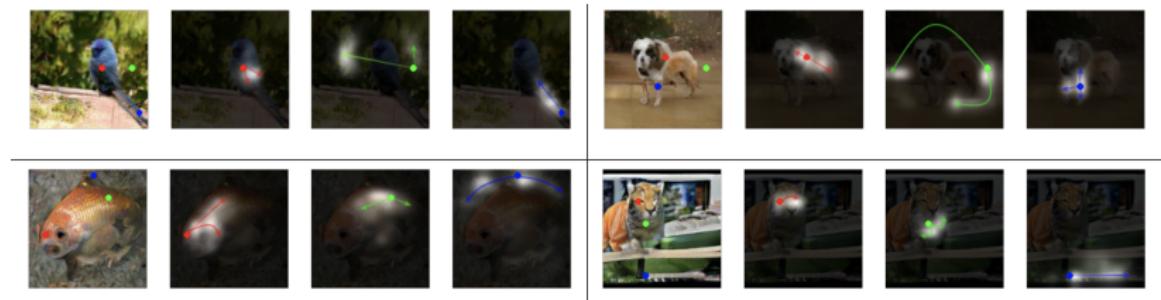
Self-Attention GAN

Technical Details

- ▶ Hinge loss for training.
- ▶ SpectralNorm in both the generator and the discriminator.
- ▶ Separate learning rates for the generator and the discriminator.

Model	Inception Score	Intra FID	FID
AC-GAN (Odena et al., 2017)	28.5	260.0	/
SNGAN-projection (Miyato & Koyama, 2018)	36.8	92.4	27.62*
SAGAN	52.52	83.7	18.65

Visualization of attention maps



BigGAN

Technical Details

- ▶ Hinge loss.
- ▶ Self-Attention GAN baseline.
- ▶ **Orthogonal regularization**

$$\|\mathbf{W}^T \mathbf{W} - \mathbf{I}\|^2 \Rightarrow \|\mathbf{W}^T \mathbf{W} - \text{diag}(\mathbf{W}^T \mathbf{W})\|^2$$

- ▶ **Truncation trick.** Components of $\mathbf{z} \sim \mathcal{N}(0, \mathbf{I})$ which fall outside a predefined range are resampled.

Batch	Ch.	Param (M)	Shared	Skip- z	Ortho.	Itr $\times 10^3$	FID	IS
256	64	81.5		SA-GAN Baseline			1000	18.65
512	64	81.5	✗	✗	✗	1000	15.30	58.77(± 1.18)
1024	64	81.5	✗	✗	✗	1000	14.88	63.03(± 1.42)
2048	64	81.5	✗	✗	✗	732	12.39	76.85(± 3.83)
2048	96	173.5	✗	✗	✗	295(± 18)	9.54(± 0.62)	92.98(± 4.27)
2048	96	160.6	✓	✗	✗	185(± 11)	9.18(± 0.13)	94.94(± 1.32)
2048	96	158.3	✓	✓	✗	152(± 7)	8.73(± 0.45)	98.76(± 2.84)
2048	96	158.3	✓	✓	✓	165(± 13)	8.51(± 0.32)	99.31(± 2.10)
2048	64	71.3	✓	✓	✓	371(± 7)	10.48(± 0.10)	86.90(± 0.61)

BigGAN

Samples (512x512)



Interpolations



Brock A., Donahue J., Simonyan K. *Large Scale GAN Training for High Fidelity Natural Image Synthesis*, 2018

Progressive Growing GAN

Problems with HR image generation

- ▶ Disjoint manifolds \Rightarrow gradient problem.
- ▶ Small minibatch \Rightarrow training instability.

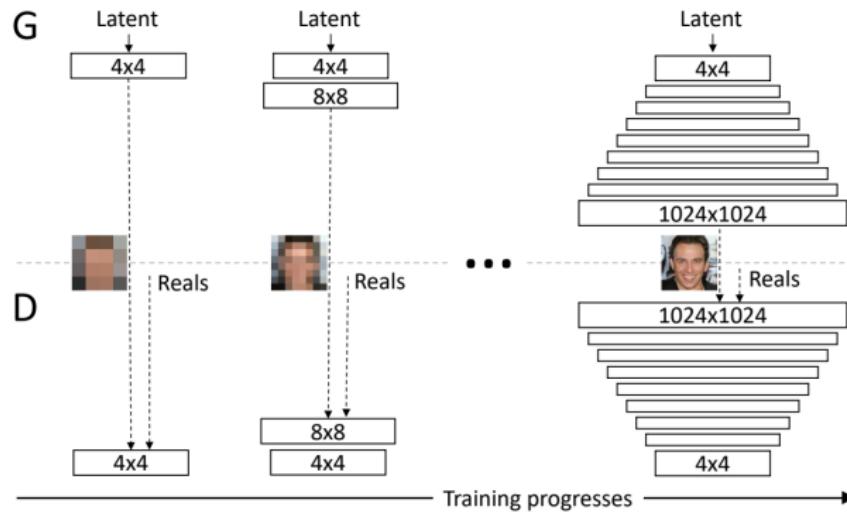
Samples (1024x1024)



Progressive Growing GAN

Grow both the generator and discriminator progressively, new layers will introduce higher-resolution details as the training progresses.

- ▶ Train GAN which generate 4x4 images (2 convs for G and D).
- ▶ Add upsampling layers to G, downsampling layers to D.
- ▶ Train GAN which generate 8x8 images.
- ▶ etc.



Karras T. et al. *Progressive Growing of GANs for Improved Quality, Stability, and Variation*, 2017

StyleGAN

- ▶ Generating of HR images is hard.
- ▶ Progressive growing greatly simplifies the task.
- ▶ The ability to control specific features of the generated image is very limited.

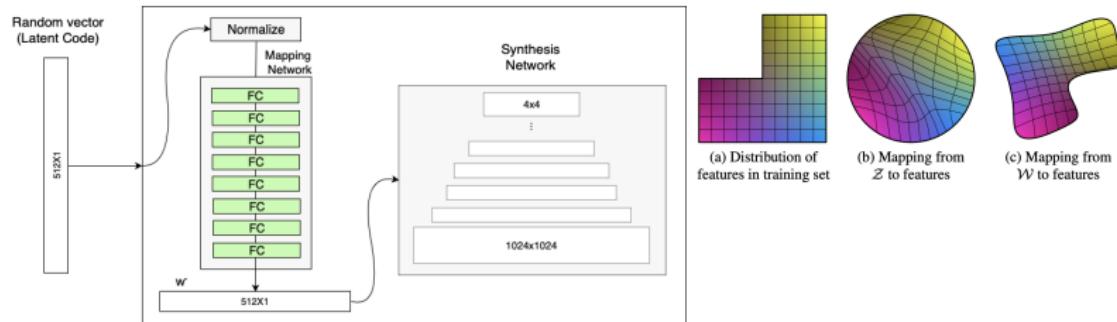
Face image features

- ▶ Coarse (pose, general hair style, face shape). Resolution $4^2 - 8^2$.
- ▶ Middle (finer facial features, hair style, eyes open/closed). Resolution $16^2 - 32^2$.
- ▶ Fine (color scheme (eye, hair and skin) and micro features). Resolution $64^2 - 1024^2$.

StyleGAN

Mapping Network

- ▶ Generator input is likely to be **disentangled**. Each component of input vector \mathbf{z} should be responsible for one generative factor.
- ▶ Mapping network $f : \mathcal{Z} \rightarrow \mathcal{W}$ is used to reduce correlations between components of \mathbf{z} .

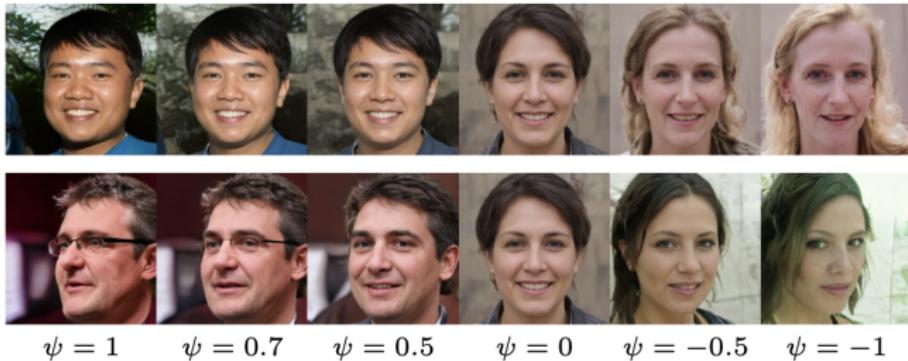


StyleGAN

Truncation trick

$$\mathbf{w}' = \hat{\mathbf{w}} + \psi \cdot (\mathbf{w} - \hat{\mathbf{w}}), \quad \hat{\mathbf{w}} = \mathbb{E}_{\mathbf{z}} p(f(\mathbf{z}))$$

- ▶ Constant ψ is a tradeoff between diversity and fidelity.
- ▶ $\psi = 0.7$ is used for most of the results.
- ▶ Truncation is done only at the low-resolution layers.



StyleGAN

Results

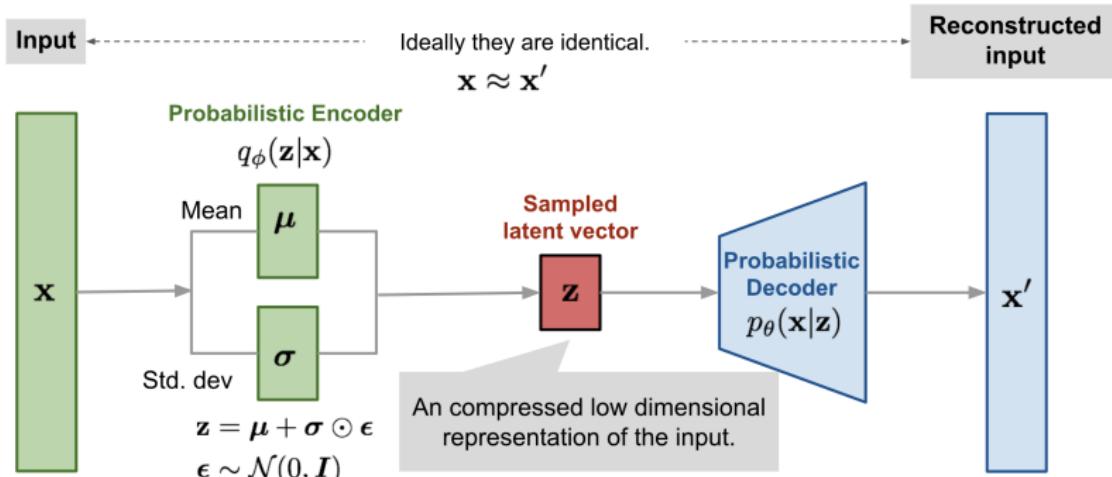
Method	CelebA-HQ	FFHQ
A Baseline Progressive GAN [30]	7.79	8.04
B + Tuning (incl. bilinear up/down)	6.11	5.25
C + Add mapping and styles	5.34	4.85
D + Remove traditional input	5.07	4.88
E + Add noise inputs	5.06	4.42
F + Mixing regularization	5.17	4.40

Samples (1024x1024)



Karras T., Laine S., Aila T. A Style-Based Generator Architecture for Generative Adversarial Networks, 2018

VAE recap



- ▶ Encoder $q(\mathbf{z}|\mathbf{x}, \phi) = \mathcal{N}(\mathbf{z}|\mu_\phi(\mathbf{x}), \sigma_\phi(\mathbf{x}))$.
- ▶ Variational posterior $q(\mathbf{z}|\mathbf{x}, \phi)$ originally approximates the true posterior $p(\mathbf{z}|\mathbf{x}, \theta)$.
- ▶ Which methods are you already familiar with to make the posterior more flexible?

image credit:

<https://lilianweng.github.io/lil-log/2018/08/12/from-autoencoder-to-beta-vae.html>

Adversarial Variational Bayes

ELBO objective

$$\mathcal{L}(\phi, \theta) = \mathbb{E}_{q(z|x, \phi)} [\log p(x|z, \theta) + \log p(z) - \log q(z|x, \phi)] \rightarrow \max_{\phi, \theta} .$$

What is the problem to make the variational posterior model an implicit model?

- ▶ The first term is reconstruction loss that needs only samples from $q(z|x, \phi)$ to evaluate.
- ▶ Reparametrization trick allows to get gradients of reconstruction loss

$$\begin{aligned}\nabla_{\phi} \int q(z|x, \phi) f(z) dz &= \nabla_{\phi} \int r(\epsilon) f(z) d\epsilon \\ &= \int r(\epsilon) \nabla_{\phi} f(g(x, \epsilon, \phi)) d\epsilon \approx \nabla_{\phi} f(g(x, \epsilon^*, \phi)),\end{aligned}$$

where $\epsilon^* \sim r(\epsilon)$, $z = g(x, \epsilon, \phi)$, $z \sim q(z|x, \phi)$.

Adversarial Variational Bayes

ELBO objective

$$\mathcal{L}(\phi, \theta) = \mathbb{E}_{q(z|x, \phi)} [\log p(x|z, \theta) + \log p(z) - \log q(z|x, \phi)] \rightarrow \max_{\phi, \theta} .$$

What is the problem to make the variational posterior model an implicit model?

- ▶ The third term requires the ability to calculate the value of $q(z|x, \phi)$.
- ▶ We could join second and third terms:

$$\mathbb{E}_{q(z|x, \phi)} \log \frac{p(z)}{q(z|x, \phi)} = \mathbb{E}_{q(z|x, \phi)} \log \frac{p(z)\pi(x)}{q(z|x, \phi)\pi(x)}.$$

- ▶ We have to estimate density ratio

$$r(x, z) = \frac{q_1(x, z)}{q_2(x, z)} = \frac{p(z)\pi(x)}{q(z|x, \phi)\pi(x)}.$$

Density ratio trick

Consider two distributions $q_1(\mathbf{x})$, $q_2(\mathbf{x})$ and probabilistic model

$$p(\mathbf{x}|y) = \begin{cases} q_1(\mathbf{x}), & \text{if } y = 1, \\ q_2(\mathbf{x}), & \text{if } y = 0, \end{cases} \quad y \sim \text{Bern}(0.5).$$

Density ratio

$$\begin{aligned} \frac{q_1(\mathbf{x})}{q_2(\mathbf{x})} &= \frac{p(\mathbf{x}|y=1)}{p(\mathbf{x}|y=0)} = \frac{p(y=1|\mathbf{x})p(\mathbf{x})}{p(y=1)} \Big/ \frac{p(y=0|\mathbf{x})p(\mathbf{x})}{p(y=0)} = \\ &= \frac{p(y=1|\mathbf{x})}{p(y=0|\mathbf{x})} = \frac{p(y=1|\mathbf{x})}{1 - p(y=1|\mathbf{x})} = \frac{D(\mathbf{x})}{1 - D(\mathbf{x})} \end{aligned}$$

Here $D(\mathbf{x})$ is a discriminator model the output of which is a probability that \mathbf{x} is a sample from $q_1(\mathbf{x})$ rather than from $q_2(\mathbf{x})$.

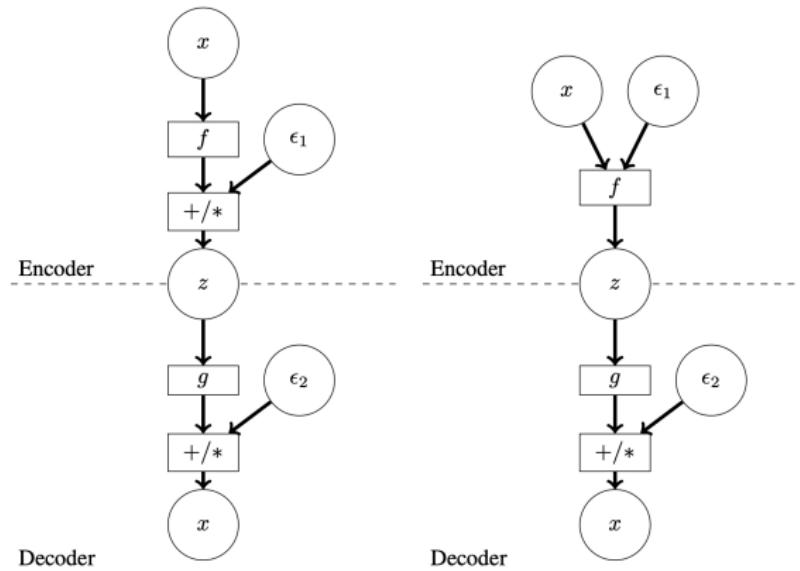
Adversarial Variational Bayes

$$\max_D \left[\mathbb{E}_{\pi(\mathbf{x})} \mathbb{E}_{q(\mathbf{z}|\mathbf{x}, \phi)} \log D(\mathbf{x}, \mathbf{z}) + \mathbb{E}_{\pi(\mathbf{x})} \mathbb{E}_{p(\mathbf{z})} \log(1 - D(\mathbf{x}, \mathbf{z})) \right]$$

Adversarial Variational Bayes

ELBO objective

$$\mathcal{L}(\phi, \theta) = \mathbb{E}_{q(z|x, \phi)} \left[\log p(x|z, \theta) + \log \frac{p(z)}{q(z|x, \phi)} \right] \rightarrow \max_{\phi, \theta} .$$



Summary