Deep Generative Models
Lecture 1

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Ozon Masters

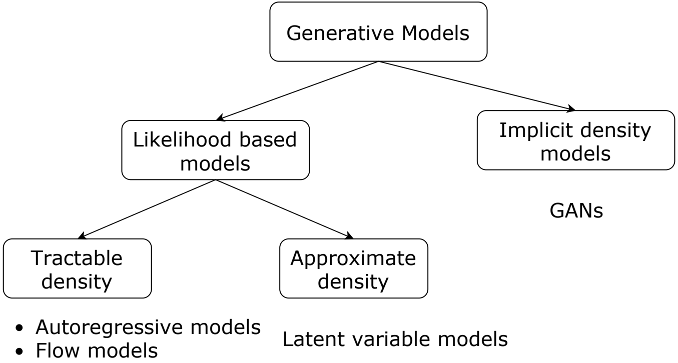
Spring, 2021

Logistics

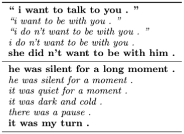
- homeworks: 30 points
  - hw1: autoregressive models
  - hw2: latent variable models
  - hw3: flow models
  - hw4: adversarial models
- exam: 30 points
- final project: 40 points

Last year course page: link
Admission: link

Generative models zoo

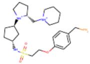


Applications

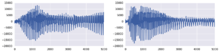


Text analysis

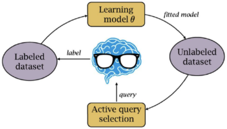
Image analysis



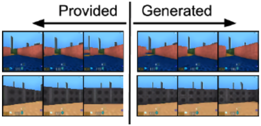
Graph analysis



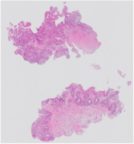
Audio analysis



Active Learning



Reinforcement Learning



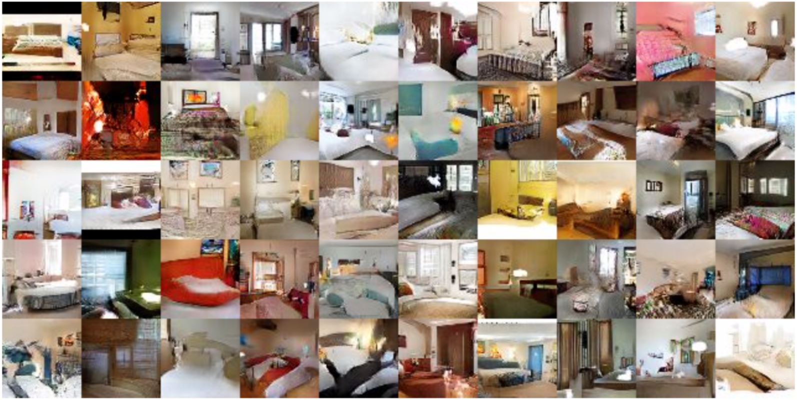
Medical data

and more...

Applications: Image generation (VAE)

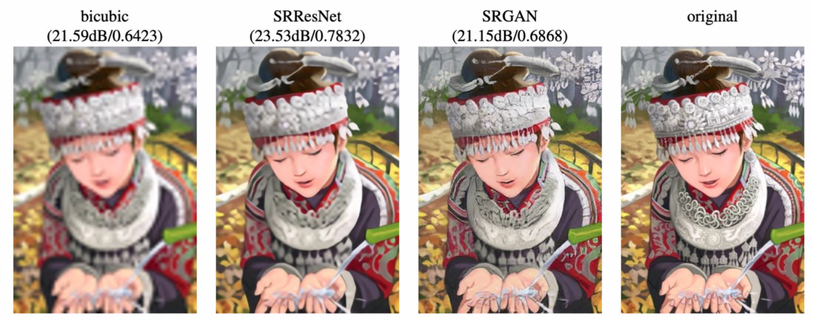
Kingma D. P., Welling M. Auto-encoding variational bayes, 2013

Applications: Image generation (DCGAN)



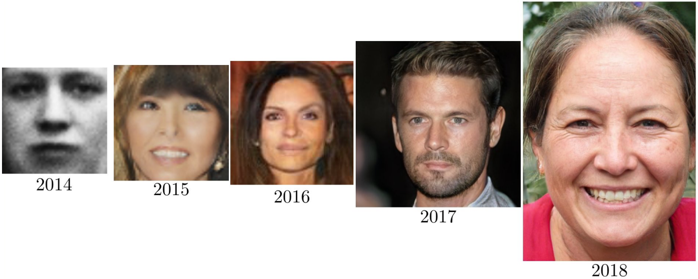
Radford A., Metz L., Chintala S. Unsupervised representation learning with deep convolutional generative adversarial networks, 2015

Applications: SuperResolution (SRGAN)



Ledig C. et al. Photo-realistic single image super-resolution using a generative adversarial network, 2016

Applications: Face generation (StyleGAN)

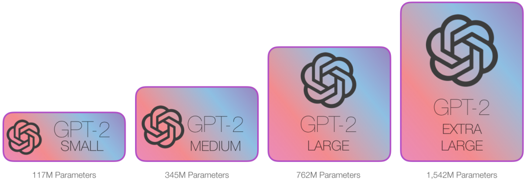


Karras T., Laine S., Aila T. *A style-based generator architecture for generative adversarial networks, 2018*

Applications: Face generation (VQ-VAE-2)

Razavi A., Oord A., Vinyals O. *Generating Diverse High-Fidelity Images with VQ-VAE-2*, 2019

Applications: Language Modelling



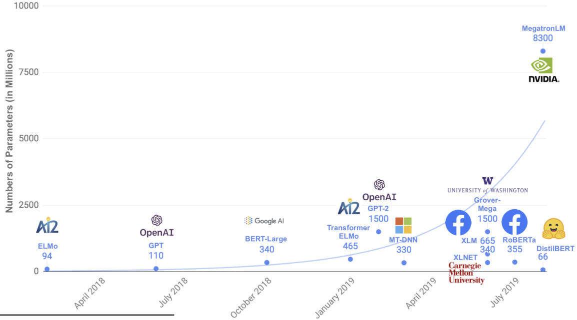


image credit: http://jalammar.github.io/illustrated-gpt2
Sanh V. et al. DistilBERT, a distilled version of BERT: smaller, faster, cheaper and lighter, 2019.

Problem Statement

We are given i.i.d. samples $\{\mathbf{x}_i\}_{i=1}^{n} \in X$ (e.g. $X = \mathbb{R}^m$) from unknown distribution $\pi(\mathbf{x})$.

Goal

We would like to learn a distribution $\pi(\mathbf{x})$ for

- evaluating $\pi(\mathbf{x})$ for new samples (how likely to get object $\mathbf{x}$?);
- sampling from $\pi(\mathbf{x})$ (to get new objects $\mathbf{x} \sim \pi(\mathbf{x})$).

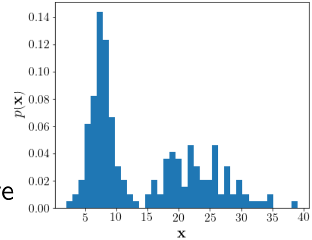
Challenge

Data is complex and high-dimensional. Imagine the dataset of images which lies in the space $X \subset \mathbb{R}^{\text{width} \times \text{height}}$.

Histogram as a generative model

Let $x \sim$ Categorical. The histogram is totally defined by

$$\pi_k = \pi(x = k) = \frac{\sum_{i=1}^{k} [x_i = k]}{n}.$$



MNIST: 28x28 gray-scaled images each image is $\mathbf{x} = (x_1, \dots, x_{784})$, where $x_i \sim \mathrm{Be}(p_i)$.
$2^{28\times 28} - 1$ parameters to specify $\pi(\mathbf{x})$

$$\pi(\mathbf{x}) = \pi(x_1) \cdot \pi(x_2|x_1) \cdot \dots \cdot \pi(x_m|x_{m-1}, \dots, x_1).$$

Question: How many parameters do we need in these cases?

$$\pi(\mathbf{x}) = \pi(x_1) \cdot \pi(x_2) \cdot \dots \cdot \pi(x_m).$$

$$\pi(\mathbf{x}) = \pi(x_1) \cdot \pi(x_2|x_1) \cdot \dots \cdot \pi(x_m|x_{m-1}).$$

Maximum likelihood

Fix probabilistic model $p(\mathbf{x}|\boldsymbol{\theta})$ – the set of parameterized distributions .
Instead of searching true $\pi(\mathbf{x})$ over all probability distributions, learn function approximation $p(\mathbf{x}|\boldsymbol{\theta}) \approx \pi(\mathbf{x})$.

MLE problem

$$\boldsymbol{\theta}^* = \arg\max_{\boldsymbol{\theta}} p(\mathbf{X}|\boldsymbol{\theta}) = \arg\max_{\boldsymbol{\theta}} \prod_{i=1}^{n} p(\mathbf{x}_i|\boldsymbol{\theta}) = \arg\max_{\boldsymbol{\theta}} \sum_{i=1}^{n} \log p(\mathbf{x}_i|\boldsymbol{\theta}).$$

The problem is solved with SGD.

Requirements

- efficiently compute $\log p(\mathbf{x}|\boldsymbol{\theta})$;
- efficiently compute gradient of $\log p(\mathbf{x}|\boldsymbol{\theta})$.

Autoregressive model

MLE problem

$$\boldsymbol{\theta}^* = \arg\max_{\boldsymbol{\theta}} p(\mathbf{X}|\boldsymbol{\theta}) = \arg\max_{\boldsymbol{\theta}} \prod_{i=1}^{n} p(\mathbf{x}_i|\boldsymbol{\theta}) = \arg\max_{\boldsymbol{\theta}} \sum_{i=1}^{n} \log p(\mathbf{x}_i|\boldsymbol{\theta}).$$

Challenge

$p(\mathbf{x}|\boldsymbol{\theta})$ could be intractable.

Likelihood as product of conditionals

Let $\mathbf{x} = (x_1, \dots, x_m)$, $\mathbf{x}_{1:i} = (x_1, \dots, x_i)$. Then

$$p(\mathbf{x}|\boldsymbol{\theta}) = \prod_{i=1}^{m} p(x_i|\mathbf{x}_{1:i-1}, \boldsymbol{\theta}); \quad \log p(\mathbf{x}|\boldsymbol{\theta}) = \sum_{i=1}^{m} \log p(x_i|\mathbf{x}_{1:i-1}, \boldsymbol{\theta}).$$

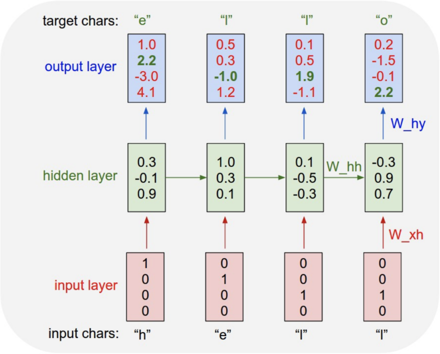
Example: $p(x_1, x_2, x_3) = p(x_2) \cdot p(x_1|x_2) \cdot p(x_3|x_1, x_2)$.

Autoregressive models

$$\log p(\mathbf{x}|\boldsymbol{\theta}) = \sum_{i=1}^{m} \log p(x_i|\mathbf{x}_{1:i-1}, \boldsymbol{\theta})$$

- Each conditional could be modelled by neural network.
- To extend to high dimensions share parameters across conditionals.
- Sampling is sequential.

Char RNN




```
PANDARUS:
Alas, I think he shall be come approached and the day
When little srain would be attain'd into being never fed,
And who is but a chain and subjects of his death,
I should not sleep.

Second Senator:
They are away this miseries, produced upon my soul,
Breaking and strongly should be buried, when I perish
The earth and thoughts of many states.

DUKE VINCENTIO:
Well, your wit is in the care of side and that.

Second Lord:
They would be ruled after this chamber, and
my fair nues begun out of the fact, to be conveyed,
Whose noble souls I'll have the heart of the wars.

Clown:
Come, sir, I will make did behold your worship.

VIOLA:
I'll drink it.
```

Drawback

Sequential computation of all conditionals $p(x_i | \mathbf{x}_{1:i-1}, \boldsymbol{\theta})$.

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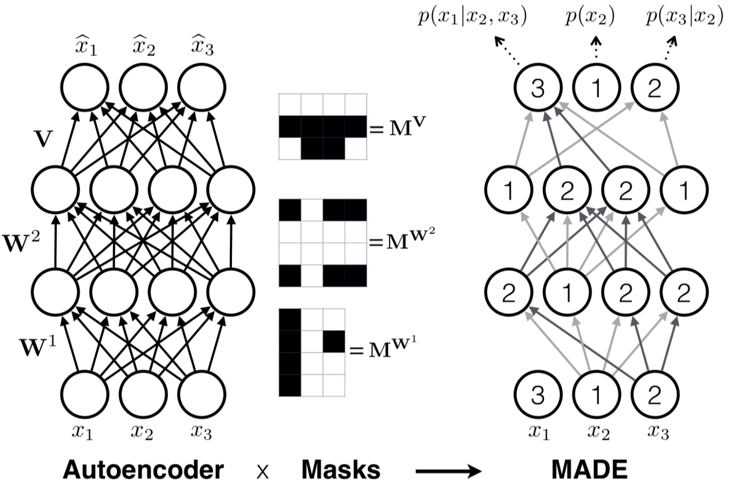
image credit: http://karpathy.github.io/2015/05/21/rnn-effectiveness

MADE



Germain M. et al. Made: Masked autoencoder for distribution estimation, 2015

WaveNet

Goal

Efficient generation of raw audio waveforms with natural sounds.

Solution

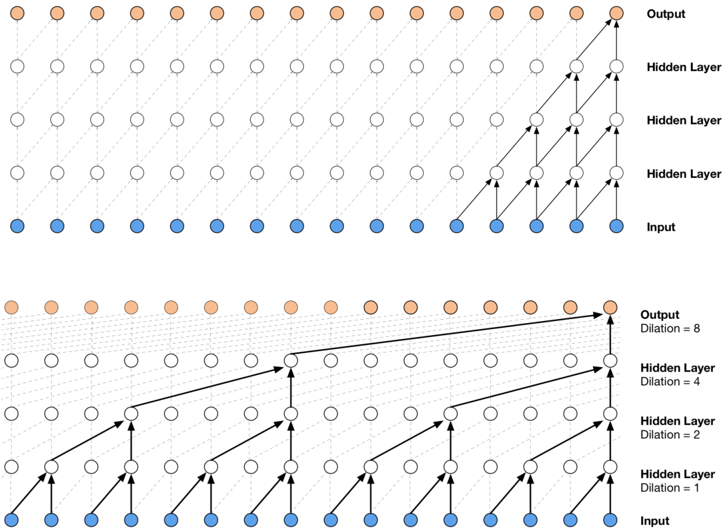
Autoregressive model

$$p(\mathbf{x}|\boldsymbol{\theta}) = \prod_{t=1}^{T} p(x_t|\mathbf{x}_{1:t-1}, \boldsymbol{\theta}).$$

The model uses causal dilated convolutions.

Oord A. et al. Wavenet: A generative model for raw audio, 2016

WaveNet (2016)



Oord A. et al. Wavenet: A generative model for raw audio, 2016

PixelCNN

Goal

Model a distribution of natural images.

Solution

Autoregressive model

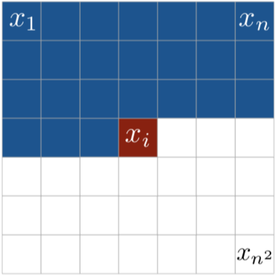
$$p(\mathbf{x}|\boldsymbol{\theta}) = \prod_{i=1}^{n^2} p(x_i|\mathbf{x}_{1:i-1}, \boldsymbol{\theta}).$$

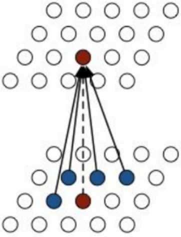
- masked convolutions;
- dependencies over RGB channels.

Oord A., Kalchbrenner N., Kavukcuoglu K. Pixel recurrent neural networks, 2016

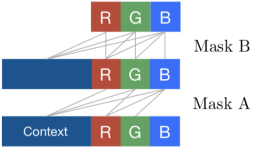
PixelCNN (2016)

1	1	1
1	0	0
0	0	0





PixelCNN



Oord A., Kalchbrenner N., Kavukcuoglu K. Pixel recurrent neural networks, 2016

Summary

- Sampling from autoregressive models is trivial, but sequential
  - sample $x_0 \sim p(x_0)$;
  - sample $x_1 \sim p(x_1|x_0)$;
  - $\dots$.
- Estimating probability:

$$p(\mathbf{x}) = \prod_{i=1}^{m} p(x_i|\mathbf{x}_{1:i-1}).$$

- Work on both continuous and discrete data.
- There is no natural way to do unsupervised learning.

Summary