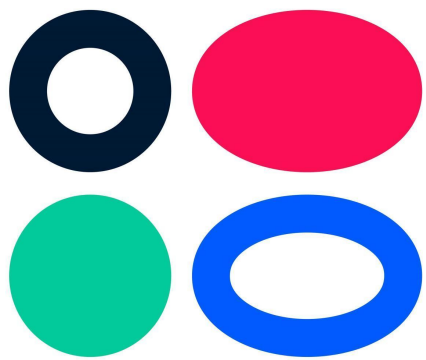
Deep Generative Models
Lecture supp

Roman Isachenko

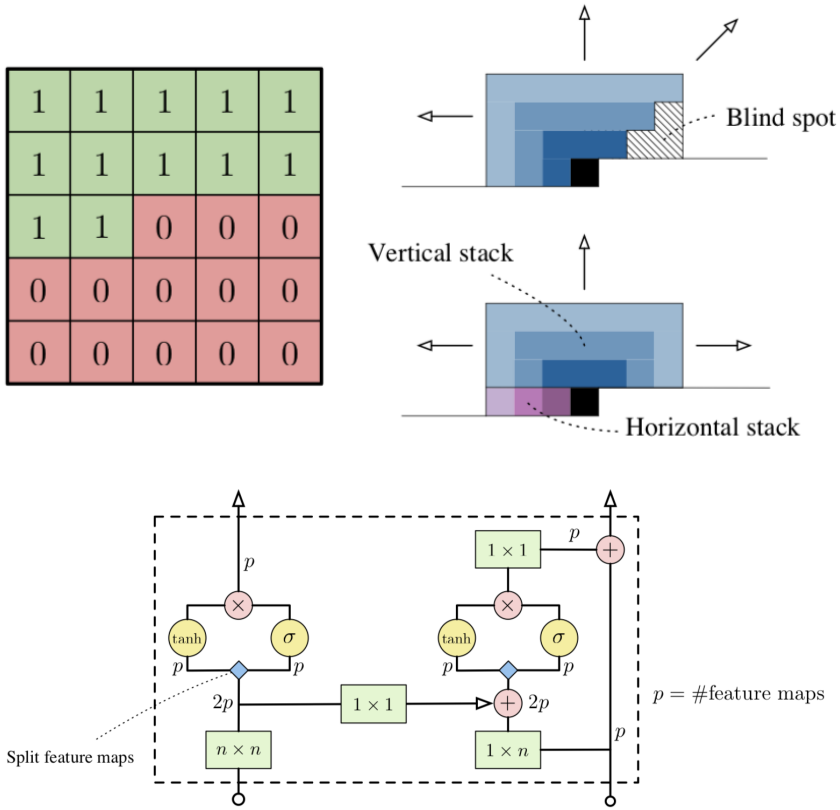


Ozon Masters

Spring, 2021

GatedPixelCNN

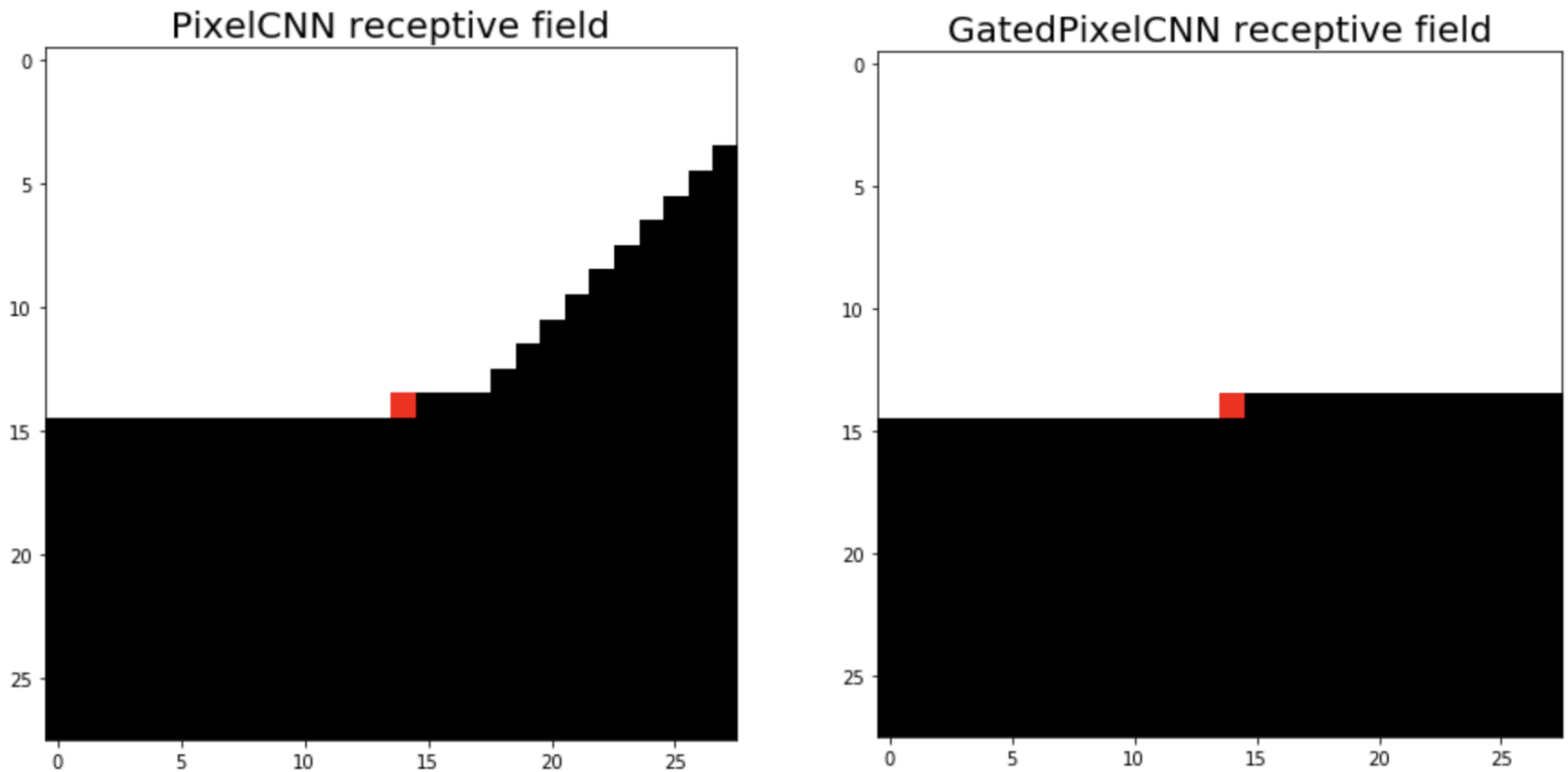
GatedPixelCNN (2016)



Van den Oord A. et al. Conditional image generation with pixelcnn decoders

https://arxiv.org/pdf/1606.05328.pdf

GatedPixelCNN (2016)



Van den Oord A. et al. Conditional image generation with pixelcnn decoders

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Extensions

- **PixelCNN++**: *Improving the PixelCNN with Discretized Logistic Mixture Likelihood and Other Modifications*
https://arxiv.org/pdf/1701.05517.pdf
(mixture of logistics instead of softmax);
- **PixelSNAIL**: *An Improved Autoregressive Generative Model*
https://arxiv.org/pdf/1712.09763.pdf
(self-attention to learn optimal autoregression ordering).

ELBO gradient, Log derivative trick

ELBO gradient (E-step, $\nabla_\phi \mathcal{L}(\phi, \theta)$)

$$\mathcal{L}(\phi, \theta) = \mathbb{E}_q \log p(\mathbf{X}|\mathbf{Z}, \theta) - KL(q(\mathbf{Z}|\mathbf{X}, \phi)||p(\mathbf{Z})) \to \max_{\phi, \theta}.$$

Difference from M-step: density function $q(\mathbf{z}|\mathbf{x}, \phi)$ depends on the parameters ϕ, it is impossible to use Monte-Carlo estimation:

$$\nabla_\phi \mathcal{L}(\phi, \theta) = \int \nabla_\phi q(\mathbf{Z}|\mathbf{X}, \phi) \log p(\mathbf{X}|\mathbf{Z}, \theta) d\mathbf{Z} - \nabla_\phi KL$$

Log-derivative trick

$$\nabla_\xi q(\eta|\xi) = q(\eta|\xi) \left(\frac{\nabla_\xi q(\eta|\xi)}{q(\eta|\xi)} \right) = q(\eta|\xi) \nabla_\xi \log q(\eta|\xi).$$

$$\nabla_\phi q(\mathbf{Z}|\mathbf{X}, \phi) = q(\mathbf{Z}|\mathbf{X}, \phi) \nabla_\phi \log q(\mathbf{Z}|\mathbf{X}, \phi).$$

ELBO gradient (E-step, $\nabla_\phi \mathcal{L}(\phi, \theta)$)

$$\nabla_\phi \mathcal{L}(\phi, \theta) = \int \nabla_\phi q(\mathbf{Z}|\mathbf{X}, \phi) \log p(\mathbf{X}|\mathbf{Z}, \theta) d\mathbf{Z} - \nabla_\phi KL =$$

$$= \int q(\mathbf{Z}|\mathbf{X}, \phi) \left[\nabla_\phi \log q(\mathbf{Z}|\mathbf{X}, \phi) \log p(\mathbf{X}|\mathbf{Z}, \theta)\right] d\mathbf{Z} - \nabla_\phi KL$$

After applying log-reparametrization trick, we are able to use Monte-Carlo estimation:

$$\nabla_\phi \mathcal{L}(\phi, \theta) \approx n \nabla_\phi \log q(\mathbf{z}_i^*|\mathbf{x}_i, \phi) \log p(\mathbf{x}_i|\mathbf{z}_i^*, \theta) - \nabla_\phi KL,$$

$$\mathbf{z}_i^* \sim q(\mathbf{z}_i|\mathbf{x}_i, \phi).$$

Problem

Unstable solution with huge variance.

Solution

Reparametrization trick

IWAE, active units, posterior collapse

IWAE, 2015

How to determine whether all VAE latent variables are informative?

$$A_i = \mathrm{cov}_{\mathbf{x}}\left(\mathbb{E}_{q(z_i|\mathbf{x})}[z_i]\right) > 0.01 \quad \Leftrightarrow \quad z_i \text{ is active}$$

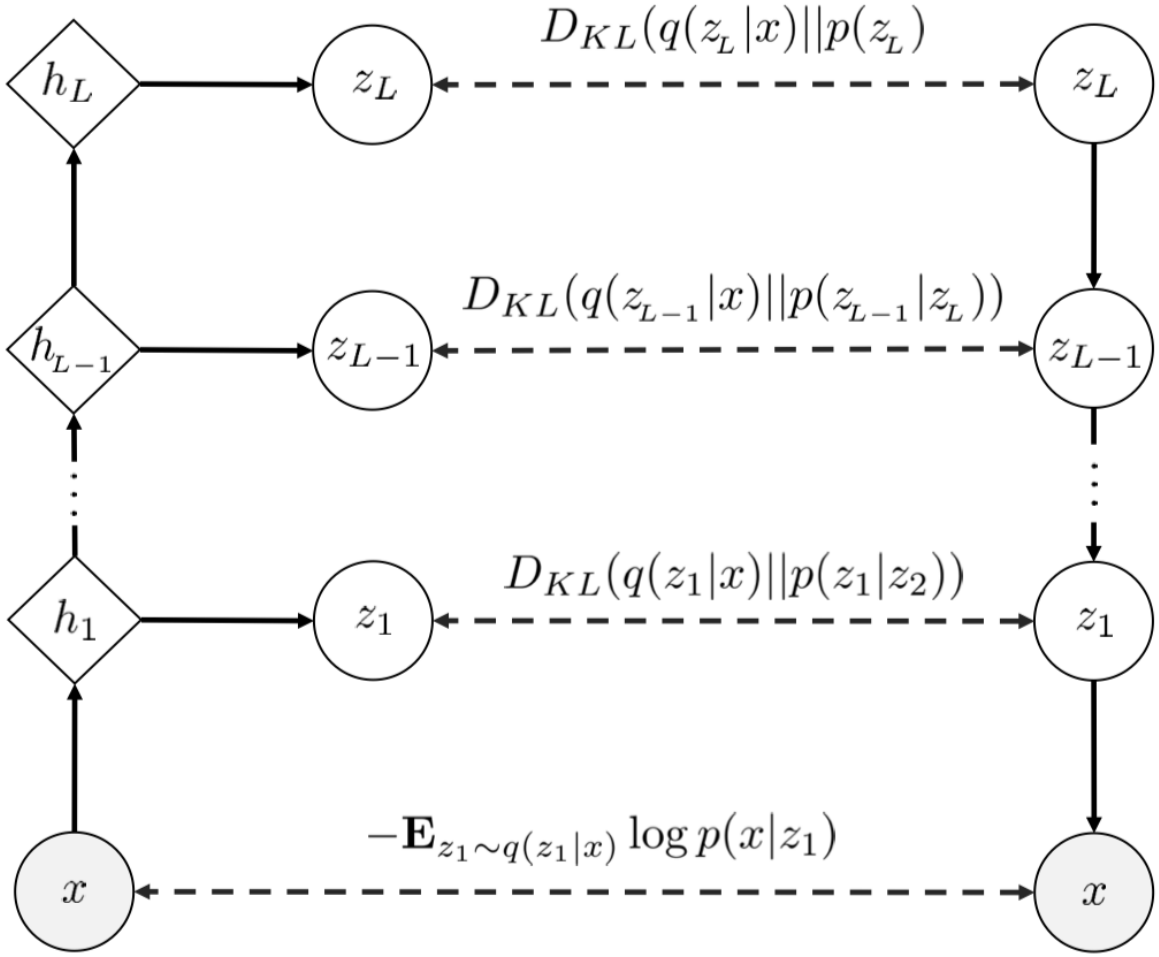
		MNIST				OMNIGLOT			
		VAE		IWAE		VAE		IWAE	
# stoch. layers	k	NLL	active units	NLL	active units	NLL	active units	NLL	active units
1	1	86.76	19	86.76	19	108.11	28	108.11	28
	5	86.47	20	85.54	22	107.62	28	106.12	34
	50	86.35	20	84.78	25	107.80	28	104.67	41
2	1	85.33	16+5	85.33	16+5	107.58	28+4	107.56	30+5
	5	85.01	17+5	83.89	21+5	106.31	30+5	104.79	38+6
	50	84.78	17+5	82.90	26+7	106.30	30+5	103.38	44+7

https://arxiv.org/pdf/1509.00519.pdf

PixelVAE, Hierarchical VAE

PixelVAE, 2016

Hierarchical VAE



https://arxiv.org/pdf/1611.05013.pdf

PixelVAE, 2016

Hierarchical decomposition

$$p(\mathbf{z}_1, \dots, \mathbf{z}_L) = p(\mathbf{z}_L)p(\mathbf{z}_{L-1}|\mathbf{z}_L) \dots p(\mathbf{z}_1, \mathbf{z}_2);$$
$$q(\mathbf{z}_1, \dots, \mathbf{z}_L|\mathbf{x}) = q(\mathbf{z}_1|\mathbf{x}) \dots q(\mathbf{z}_L|\mathbf{x}).$$

ELBO

$$\begin{aligned}
\mathcal{L}(q, \boldsymbol{\theta}) &= \mathbb{E}_{q(\mathbf{z}_1|\mathbf{x})} \log p(\mathbf{x}|\mathbf{z}_1, \boldsymbol{\theta}) - KL(q(\mathbf{z}_1, \dots, \mathbf{z}_L|\mathbf{x})||p(\mathbf{z}_1, \dots, \mathbf{z}_L)) \\
&= \mathbb{E}_{q(\mathbf{z}_1|\mathbf{x})} \log p(\mathbf{x}|\mathbf{z}_1, \boldsymbol{\theta}) - \int \prod_{j=1}^{L} q(\mathbf{z}_j|\mathbf{x}) \sum_{i=1}^{L} \log \frac{q(\mathbf{z}_i|\mathbf{x})}{p(\mathbf{z}_i|\mathbf{z}_{i+1})} d\mathbf{z}_1 \dots d\mathbf{z}_L \\
&= \mathbb{E}_{q(\mathbf{z}_1|\mathbf{x})} \log p(\mathbf{x}|\mathbf{z}_1, \boldsymbol{\theta}) - \sum_{i=1}^{L} \int \prod_{j=1}^{L} q(\mathbf{z}_j|\mathbf{x}) \log \frac{q(\mathbf{z}_i|\mathbf{x})}{p(\mathbf{z}_i|\mathbf{z}_{i+1})} d\mathbf{z}_1 \dots d\mathbf{z}_L \\
&= \mathbb{E}_{q(\mathbf{z}_1|\mathbf{x})} \log p(\mathbf{x}|\mathbf{z}_1, \boldsymbol{\theta}) - \sum_{i=1}^{L} \int q(\mathbf{z}_{i+1}|\mathbf{x}) q(\mathbf{z}_i|\mathbf{x}) \log \frac{q(\mathbf{z}_i|\mathbf{x})}{p(\mathbf{z}_i|\mathbf{z}_{i+1})} d\mathbf{z}_i d\mathbf{z}_{i+1} \\
&= \mathbb{E}_{q(\mathbf{z}_1|\mathbf{x})} \log p(\mathbf{x}|\mathbf{z}_1, \boldsymbol{\theta}) - \sum_{i=1}^{L} \mathbb{E}_{q(\mathbf{z}_{i+1}|\mathbf{x})} \left[KL(q(\mathbf{z}_i|\mathbf{x})||p(\mathbf{z}_i|\mathbf{z}_{i+1})) \right]
\end{aligned}$$

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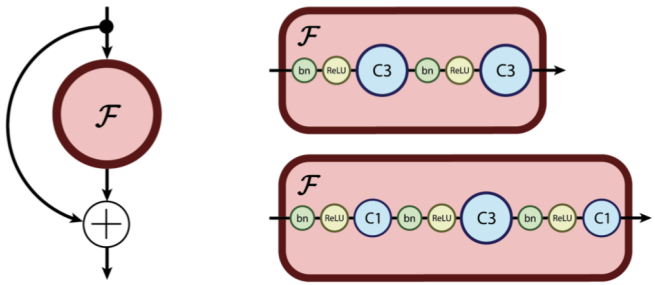
https://arxiv.org/pdf/1611.05013.pdf

RevNet, i-RevNet

RevNets, 2017

- Modern neural networks are trained via backpropagation.
- Residual networks are state of the art in image classification.
- Backpropagation requires storing the network activations.



Problem

Storing the activations imposes an increasing memory burden. GPUs have limited memory capacity, leading to constraints often exceeded by state-of-the-art architectures (with thousand layers).

Gomez A. N. et al. The Reversible Residual Network: Backpropagation Without Storing Activations, 2017

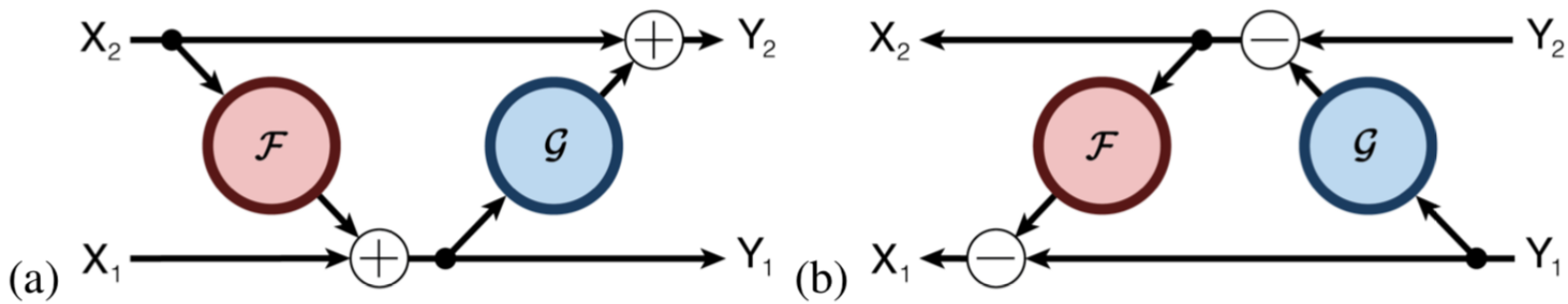
RevNets, 2017

NICE

$$\begin{cases} \mathbf{z}_1 = \mathbf{x}_1; \\ \mathbf{z}_2 = \mathbf{x}_2 + \mathcal{F}(\mathbf{x}_1, \boldsymbol{\theta}); \end{cases} \quad \Leftrightarrow \quad \begin{cases} \mathbf{x}_1 = \mathbf{z}_1; \\ \mathbf{x}_2 = \mathbf{z}_2 - \mathcal{F}(\mathbf{z}_1, \boldsymbol{\theta}). \end{cases}$$

RevNet

$$\begin{cases} \mathbf{y}_1 = \mathbf{x}_1 + \mathcal{F}(\mathbf{x}_2, \boldsymbol{\theta}); \\ \mathbf{y}_2 = \mathbf{x}_2 + \mathcal{G}(\mathbf{y}_1, \boldsymbol{\theta}); \end{cases} \quad \Leftrightarrow \quad \begin{cases} \mathbf{x}_2 = \mathbf{y}_2 - \mathcal{F}(\mathbf{y}_1, \boldsymbol{\theta}); \\ \mathbf{x}_1 = \mathbf{y}_1 - \mathcal{G}(\mathbf{x}_2, \boldsymbol{\theta}). \end{cases}$$



Gomez A. N. et al. The Reversible Residual Network: Backpropagation Without Storing Activations, 2017

RevNets, 2017

Architecture	CIFAR-10 [15]		CIFAR-100 [15]	
	ResNet	RevNet	ResNet	RevNet
32 (38)	**7.14%**	7.24%	29.95%	**28.96%**
110	**5.74%**	5.76%	26.44%	**25.40%**
164	5.24%	**5.17%**	**23.37%**	23.69%

- If the network contains non-reversible blocks (poolings, strides), activations for these blocks should be stored.
- To avoid storing activations in the modern frameworks, the backward pass should be manually redefined.

Gomez A. N. et al. The Reversible Residual Network: Backpropagation Without Storing Activations, 2017

i-RevNet, 2018

Hypothesis

The success of deep convolutional networks is based on progressively discarding uninformative variability about the input with respect to the problem at hand.

- It is difficult to recover images from their hidden representations.
- Information bottleneck principle: an optimal representation must reduce the MI between an input and its representation to reduce uninformative variability + maximize the MI between the output and its representation to preserve each class from collapsing onto other classes.

Jacobsen J. H., Smeulders A., Oyallon E. i-RevNet: Deep Invertible Networks, 2018

i-RevNet, 2018

Hypothesis

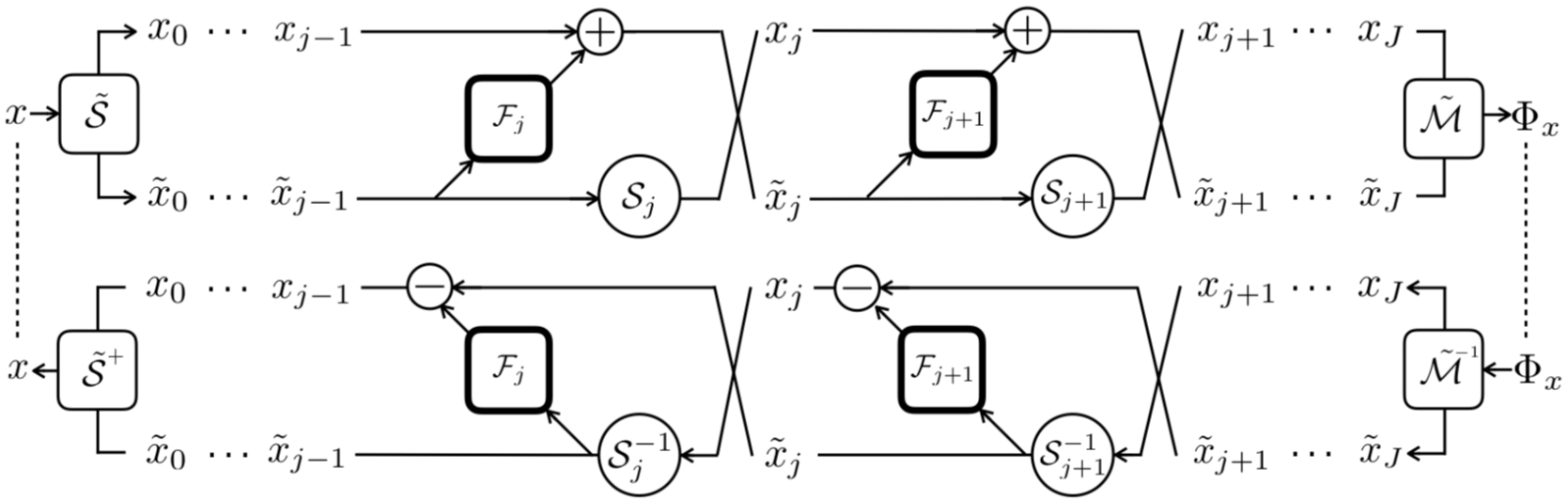
The success of deep convolutional networks is based on progressively discarding uninformative variability about the input with respect to the problem at hand.

Idea

Build a cascade of homeomorphic layers (i-RevNet), a network that can be fully inverted up to the final projection onto the classes, i.e. no information is discarded.

Jacobsen J. H., Smeulders A., Oyallon E. i-RevNet: Deep Invertible Networks, 2018

i-RevNet, 2018



Architecture	Injective	Bijective	Top-1 error	Parameters
ResNet	-	-	24.7	26M
RevNet	-	-	25.2	28M
i-RevNet (a)	yes	-	24.7	181M
i-RevNet (b)	yes	yes	26.7	29M

Jacobsen J. H., Smeulders A., Oyallon E. i-RevNet: Deep Invertible Networks, 2018

Posterior collapse, toy example

Posterior collapse: toy example

Let define latent variable model in the following way:

$$p(\mathbf{x}|\boldsymbol{\theta}) = \int p(\mathbf{x}, \mathbf{z}|\boldsymbol{\theta}) d\mathbf{z} = \int p(\mathbf{x}|\mathbf{z}, \boldsymbol{\theta}) p(\mathbf{z}) d\mathbf{z}$$

- prior distribution $p(\mathbf{z}) = \mathcal{N}(\mathbf{z}|0, \mathbf{I})$;
- probabilistic model $p(\mathbf{x}|\mathbf{z}, \boldsymbol{\theta}) = \mathcal{N}(\mathbf{x}|\boldsymbol{\mu}_{\boldsymbol{\theta}}(\mathbf{z}), \boldsymbol{\sigma}_{\boldsymbol{\theta}}(\mathbf{z}))$ (diagonal covariance);
- variational posterior $q(\mathbf{z}|\mathbf{x}, \boldsymbol{\phi}) = \mathcal{N}(\mathbf{x}|\boldsymbol{\mu}_{\boldsymbol{\phi}}(\mathbf{x}), \boldsymbol{\sigma}_{\phi}(\mathbf{x}))$ (diagonal covariance).

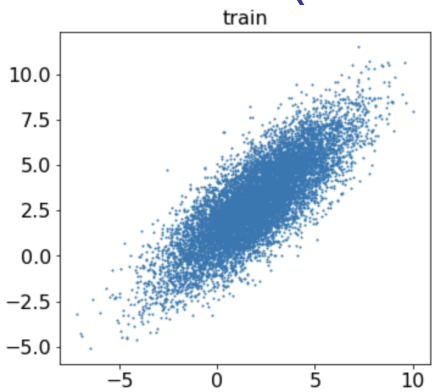
Let data distribution is $\pi(\mathbf{x}) = \mathcal{N}(\mathbf{x}|\boldsymbol{\mu}, \boldsymbol{\Sigma})$. Possible cases:

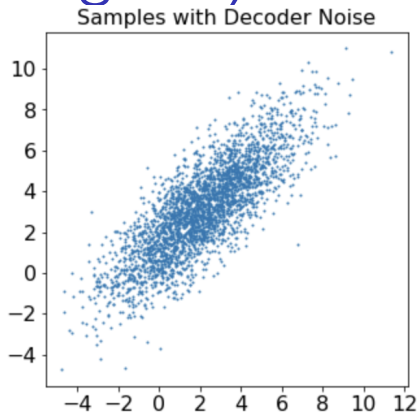
- covariance matrix $\boldsymbol{\Sigma}$ is diagonal (univariate case);
- covariance matrix $\boldsymbol{\Sigma}$ is **not** diagonal (multivariate case).

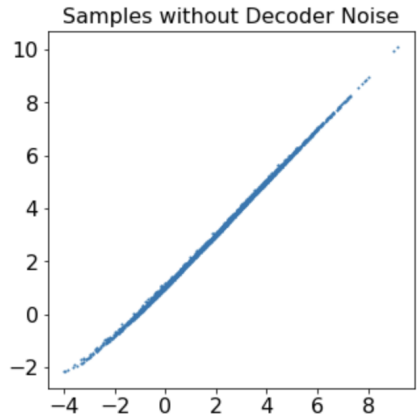
What is the difference?

Posterior collapse: toy example

Multivariate ($\mathbf{\Sigma}$ is non-diagonal)

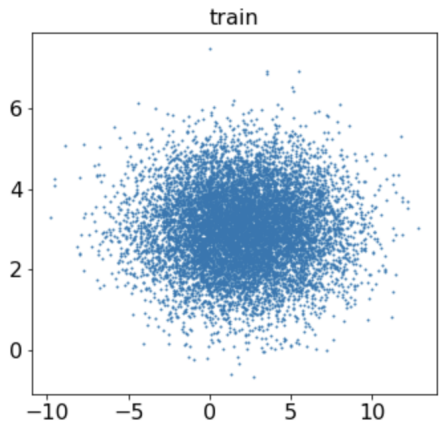


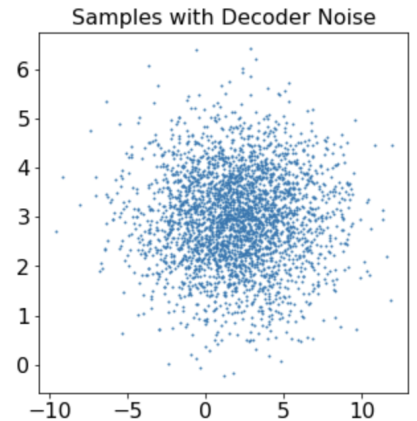


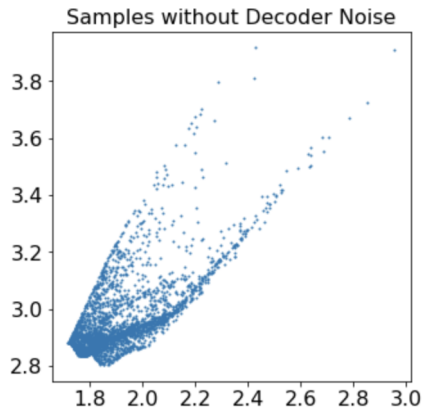


The encoder uses latent variables to model data.

Univariate ($\mathbf{\Sigma}$ is diagonal)







Latent variables are not used, since the decoder could model the data without the encoder.

Posterior collapse, VLAE

Variational Lossy AutoEncoder

Lossy code via explicit information placement

$$p(\mathbf{x}|\mathbf{z},\theta) = \prod_{i=1}^{m} p(x_i|\mathbf{z}, \mathbf{x}_{\text{WindowAround}(i)}, \theta).$$

- WindowAround(i) restricts the receptive field (it forbids to represent arbitrarily complex distribution over $\mathbf{x}$ without dependence on $\mathbf{z}$).
- Local statistics of 2D images (texture) will be modeled by a small local window.
- Global structural information (shapes) is long-range dependency that can only be communicated through latent code $\mathbf{z}$.

Chen X. et al. Variational Lossy Autoencoder, 2016

Variational Lossy AutoEncoder

- Can VLAE learn lossy codes that encode global statistics?
- Does using AF priors improves upon using IAF posteriors as predicted by theory?
- Does using autoregressive decoding distributions improve density estimation performance?

MNIST

Model	**NLL Test**
Normalizing flows (Rezende & Mohamed, 2015)	85.10
DRAW (Gregor et al., 2015)	< 80.97
Discrete VAE (Rolfe, 2016)	81.01
PixelRNN (van den Oord et al., 2016a)	79.20
IAF VAE (Kingma et al., 2016)	79.88
AF VAE	79.30
VLAE	**79.03**

CIFAR10

Method	bits/dim $\leq$
Results with tractable likelihood models:	
Uniform distribution [1]	8.00
Multivariate Gaussian [1]	4.70
NICE [2]	4.48
Deep GMMs [3]	4.00
Real NVP [4]	3.49
PixelCNN [1]	3.14
Gated PixelCNN [5]	3.03
PixelRNN [1]	3.00
PixelCNN++ [6]	**2.92**
Results with variationally trained latent-variable models:	
Deep Diffusion [7]	5.40
Convolutional DRAW [8]	3.58
ResNet VAE with IAF [9]	3.11
ResNet VLAE	3.04
DenseNet VLAE	**2.95**

Chen X. et al. Variational Lossy Autoencoder, 2016

Disentanglement, InfoGAN

InfoGAN

GAN objective

$$\min_G \max_D V(G, D)$$

$$V(G, D) = \mathbb{E}_{\pi(\mathbf{x})} \log D(\mathbf{x}) + \mathbb{E}_{p(\mathbf{z})} \log(1 - D(G(\mathbf{z})))$$

Latent vector $\mathbf{z}$ is not imposed to be disentangled.
InfoGAN decomposes input vector:

- $\mathbf{z}$ – incompressible noise;
- $\mathbf{c}$ – structured latent code $p(\mathbf{c}) = \prod_{j=1}^{d} p(c_j)$.

Information-theoretic regularization

$$\max I(\mathbf{c}, G(\mathbf{z}, \mathbf{c}))$$

Information in the latent code $\mathbf{c}$ should not be lost in the generation process.

Chen X. et al. InfoGAN: Interpretable Representation Learning by Information Maximizing Generative Adversarial Nets, 2016

InfoGAN

Objective

$$\min_G \max_D V(G, D) - \lambda I(\mathbf{c}, G(\mathbf{z}, \mathbf{c}))$$

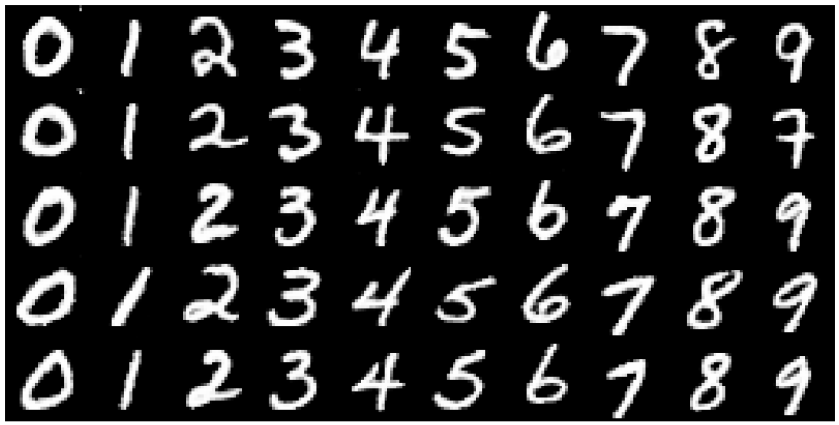
Variational Information Maximization

$$\begin{aligned} I(\mathbf{c}, G(\mathbf{z}, \mathbf{c})) &= H(\mathbf{c}) - H(\mathbf{c}|G(\mathbf{z}, \mathbf{c})) = \\ &= H(\mathbf{c}) + \mathbb{E}_{\mathbf{x}\sim G(\mathbf{z},\mathbf{c})}\left[\mathbb{E}_{\mathbf{c}'\sim p(\mathbf{c}|\mathbf{x})} \log p(\mathbf{c}'|\mathbf{x})\right] = \\ &= H(\mathbf{c}) + \mathbb{E}_{\mathbf{x}\sim G(\mathbf{z},\mathbf{c})} KL(p(\mathbf{c}'|\mathbf{x})||q(\mathbf{z}'|\mathbf{x})) + \\ &\quad + \mathbb{E}_{\mathbf{x}\sim G(\mathbf{z},\mathbf{c})}\mathbb{E}_{\mathbf{c}'\sim p(\mathbf{c}|\mathbf{x})} \log q(\mathbf{c}'|\mathbf{x}) \geq \\ &\geq H(\mathbf{c}) + \mathbb{E}_{\mathbf{x}\sim G(\mathbf{z},\mathbf{c})}\mathbb{E}_{\mathbf{c}'\sim p(\mathbf{c}|\mathbf{x})} \log q(\mathbf{c}'|\mathbf{x}) = \\ &\quad H(\mathbf{c}) + \mathbb{E}_{\mathbf{c}\sim p(\mathbf{c})}\mathbb{E}_{\mathbf{x}\sim G(\mathbf{z},\mathbf{c})} \log q(\mathbf{c}|\mathbf{x}) \end{aligned}$$

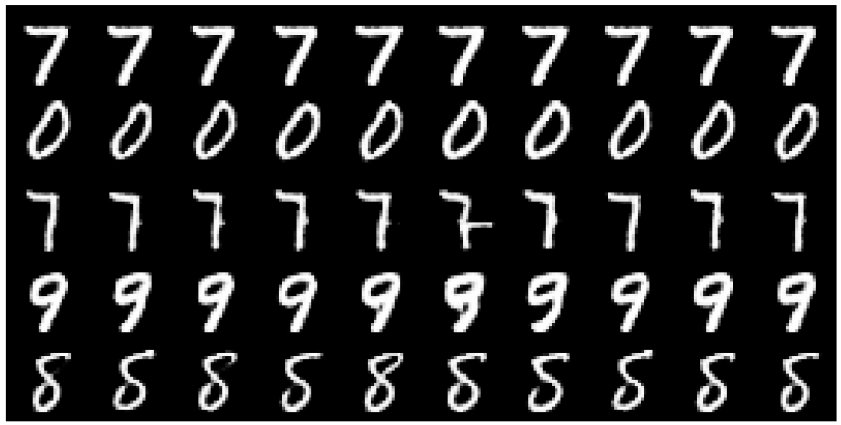
Chen X. et al. InfoGAN: Interpretable Representation Learning by Information Maximizing Generative Adversarial Nets, 2016

InfoGAN

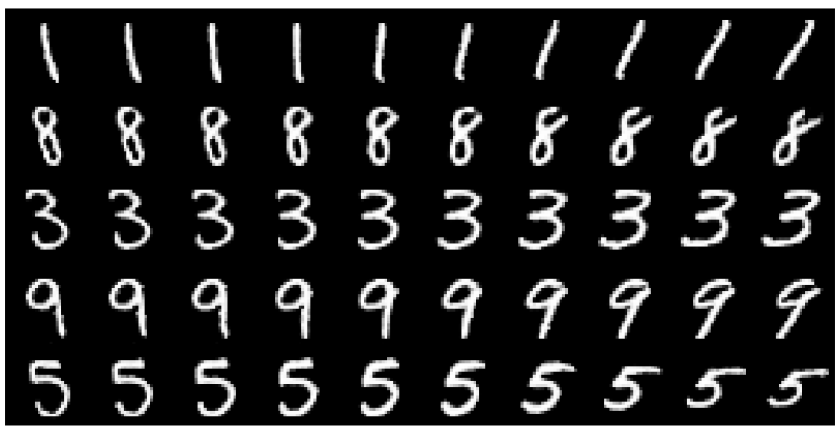
Latent codes on MNIST



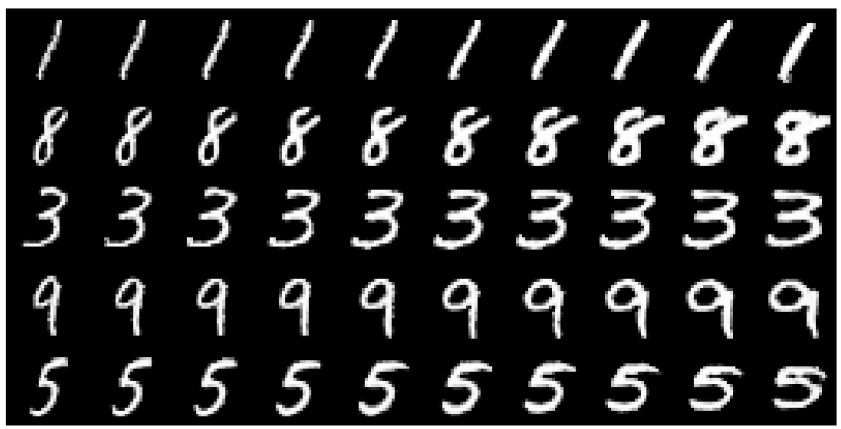
(a) Varying c_1 on InfoGAN (Digit type)



(b) Varying c_1 on regular GAN (No clear meaning)



(c) Varying c_2 from -2 to 2 on InfoGAN (Rotation)



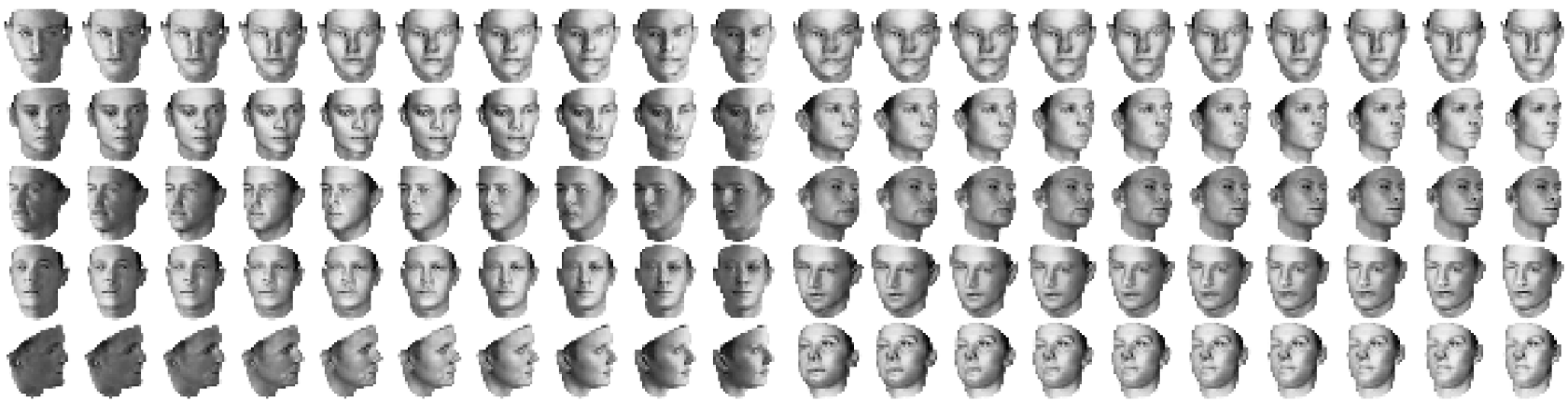
(d) Varying c_3 from -2 to 2 on InfoGAN (Width)

Chen X. et al. InfoGAN: Interpretable Representation Learning by Information Maximizing Generative Adversarial Nets, 2016

InfoGAN

Latent codes on 3D Faces

(a) Azimuth (pose) (b) Elevation



(c) Lighting (d) Wide or Narrow

Chen X. et al. InfoGAN: Interpretable Representation Learning by Information Maximizing Generative Adversarial Nets, 2016

β-VAE, Disentanglement metric

β-VAE

Disentangling metric

1. Generate two sets of objects

$$\mathbf{x}_{li} \sim \text{Sim}(\mathbf{v}_{li}, \mathbf{w}_{li}); \quad \mathbf{x}_{lj} \sim \text{Sim}(\mathbf{v}_{lj}, \mathbf{w}_{lj}); \quad y_{ij} \sim U[1, d].$$

$$\mathbf{v}_{li} \sim p(\mathbf{v}); \quad \mathbf{v}_{lj} \sim p(\mathbf{v})\,([v_{li}]_y = [v_{lj}]_y); \quad \mathbf{w}_{li}, \mathbf{w}_{lj} \sim p(\mathbf{w}).$$

2. Find representations

$$q(\mathbf{z}|\mathbf{x}) = \mathcal{N}\left(\mu(\mathbf{x})|\sigma^2(\mathbf{x})\right); \quad \mathbf{z}_{li} = \mu(\mathbf{x}_{li}); \quad \mathbf{z}_{lj} = \mu(\mathbf{x}_{lj}).$$

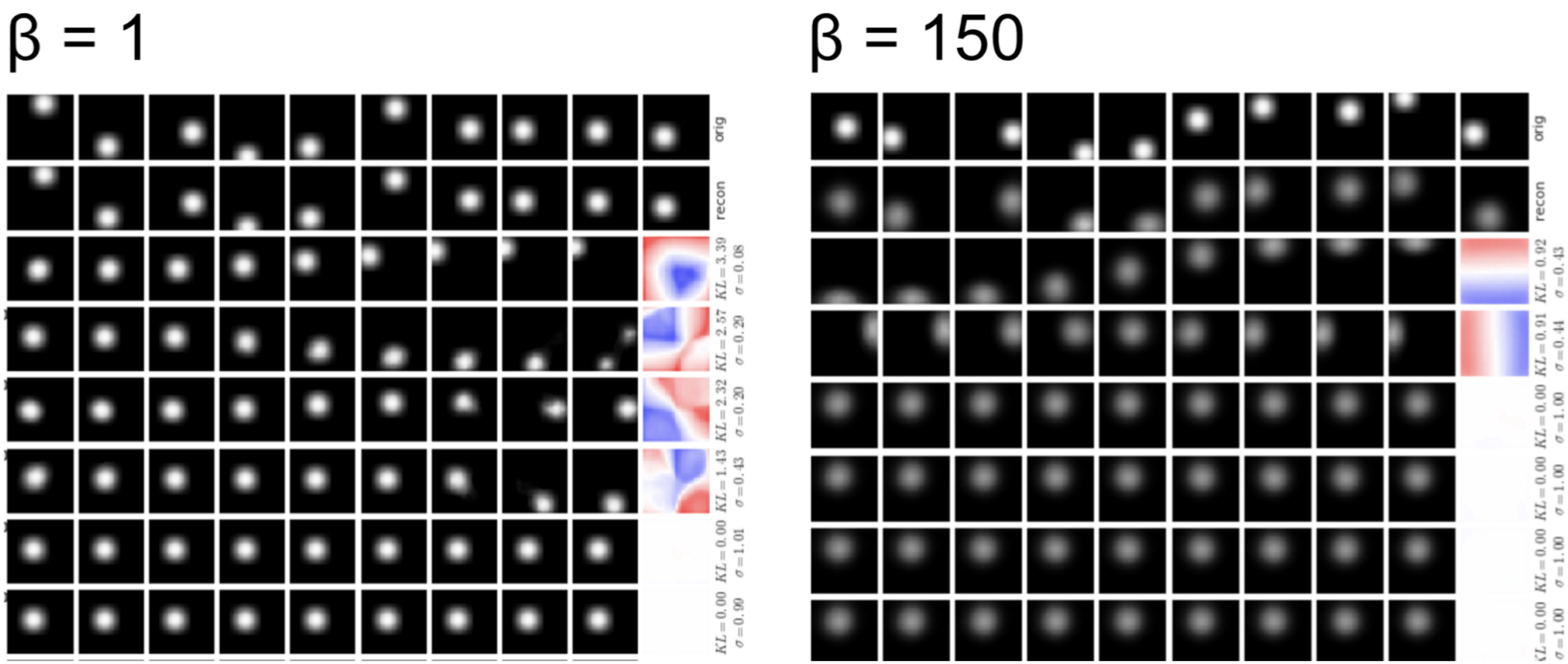
3. Use accuracy of classifier $p(y|\mathbf{z}_{\text{diff}})$ with a low VC-dimension as metric of disentanglement

$$\mathbf{z}_{\text{diff}} = \frac{1}{L}\sum_{l=1}^{L} |\mathbf{z}_{li} - \mathbf{z}_{lj}|.$$

Higgins I. et al. beta-VAE: Learning Basic Visual Concepts with a Constrained Variational Framework, 2017

β-VAE

- **Top row:** original images.
- **Second row:** the corresponding reconstructions.
- **Remaining rows:** latent traversals ordered by KL divergence with the prior.
- **Heatmaps:** latent activations for each 2D position.

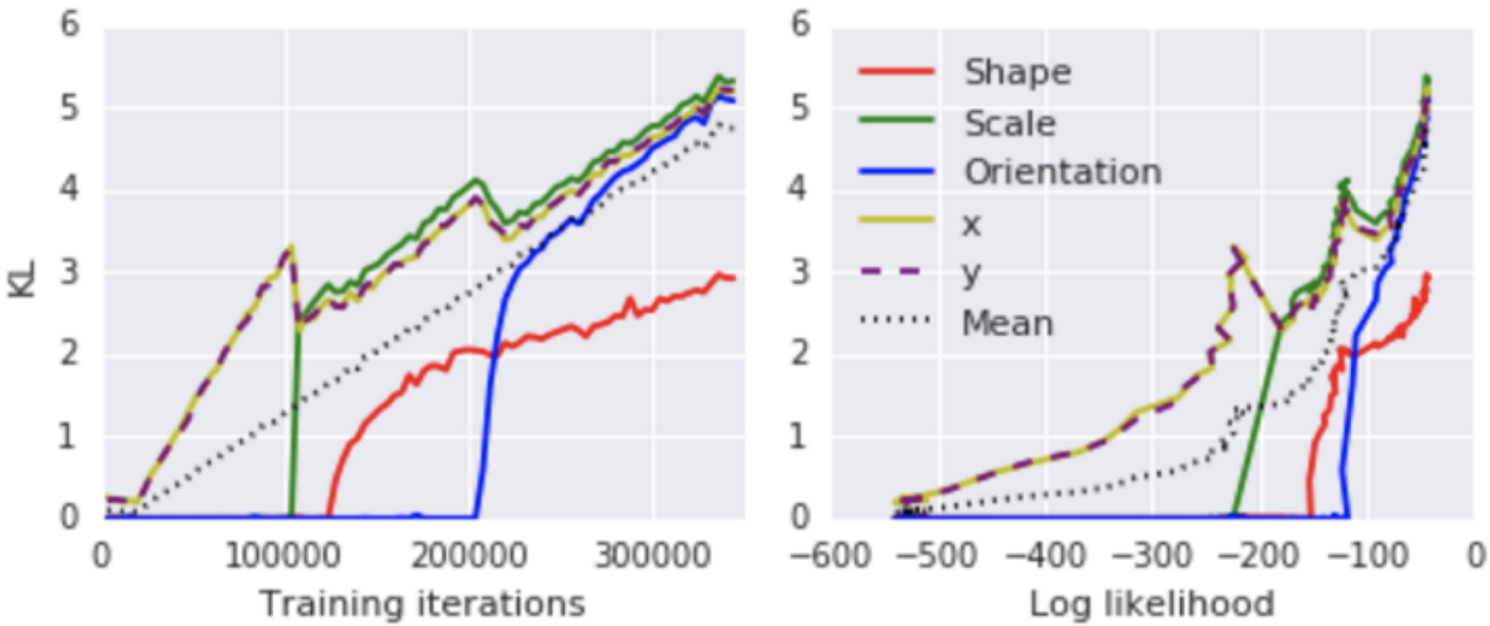


Burgess C. P. et al. Understanding disentangling in β-VAE, 2018

β-VAE

Controlled encoding capacity

$$\mathcal{L}(q, \boldsymbol{\theta}, \beta) = \mathbb{E}_{q(\mathbf{z}|\mathbf{x})} \log p(\mathbf{x}|\mathbf{z}, \boldsymbol{\theta}) - |KL(q(\mathbf{z}|\mathbf{x})||p(\mathbf{z})) - C|.$$



The early capacity is allocated to positional latents only, followed by a scale latent, then shape and orientation latents.

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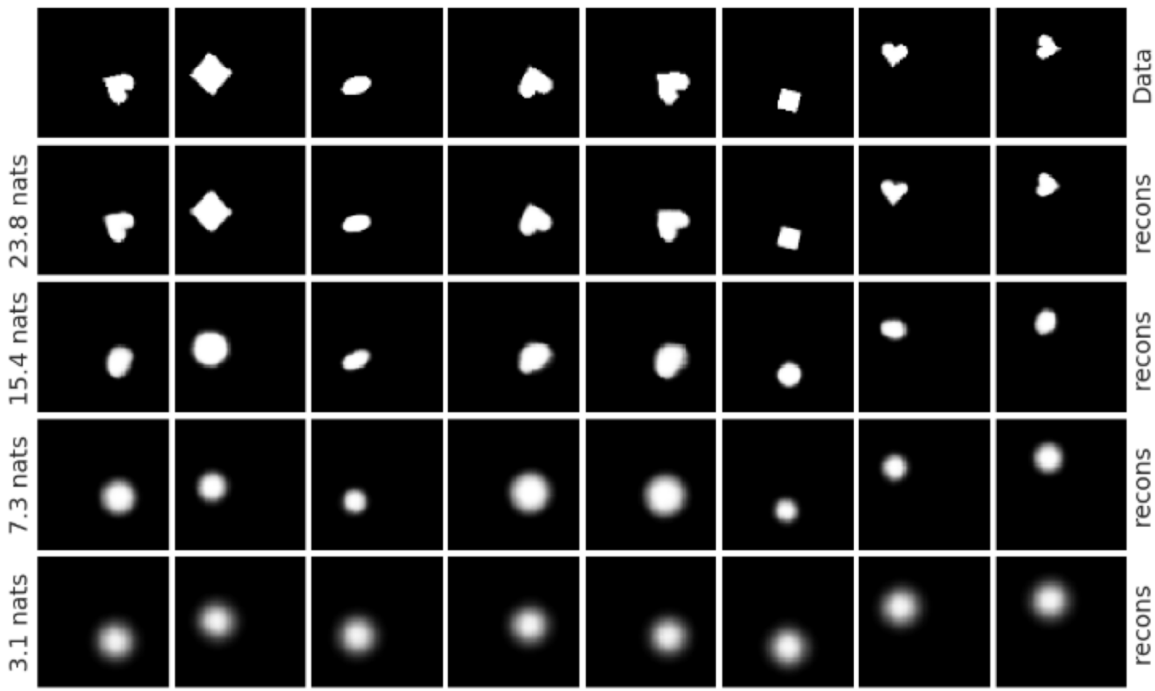
β-VAE

Controlled encoding capacity

$$\mathcal{L}(q, \boldsymbol{\theta}, \beta) = \mathbb{E}_{q(\mathbf{z}|\mathbf{x})} \log p(\mathbf{x}|\mathbf{z}, \boldsymbol{\theta}) - |KL(q(\mathbf{z}|\mathbf{x})||p(\mathbf{z})) - C|.$$



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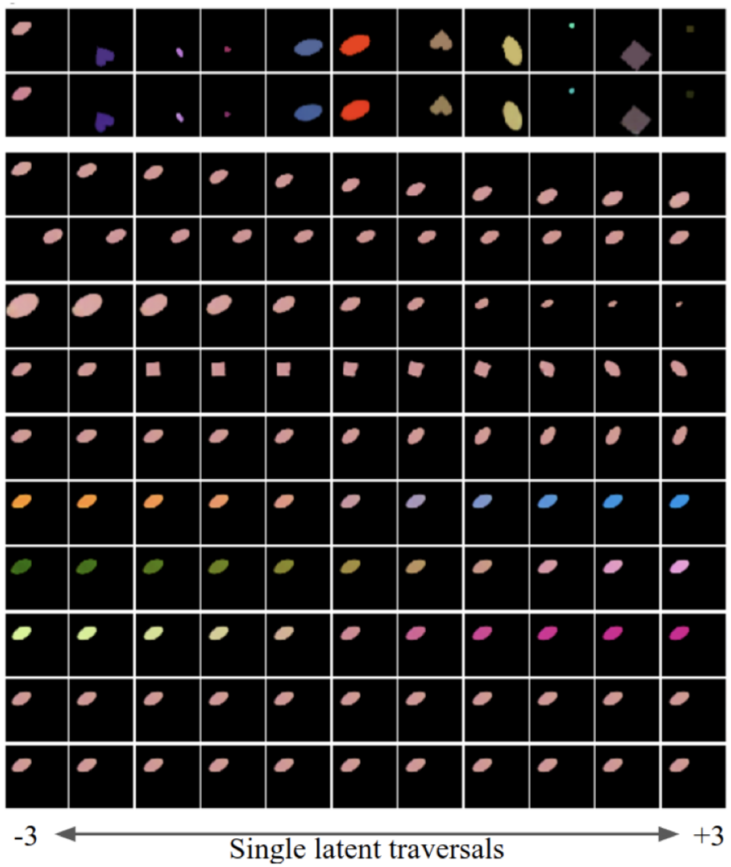
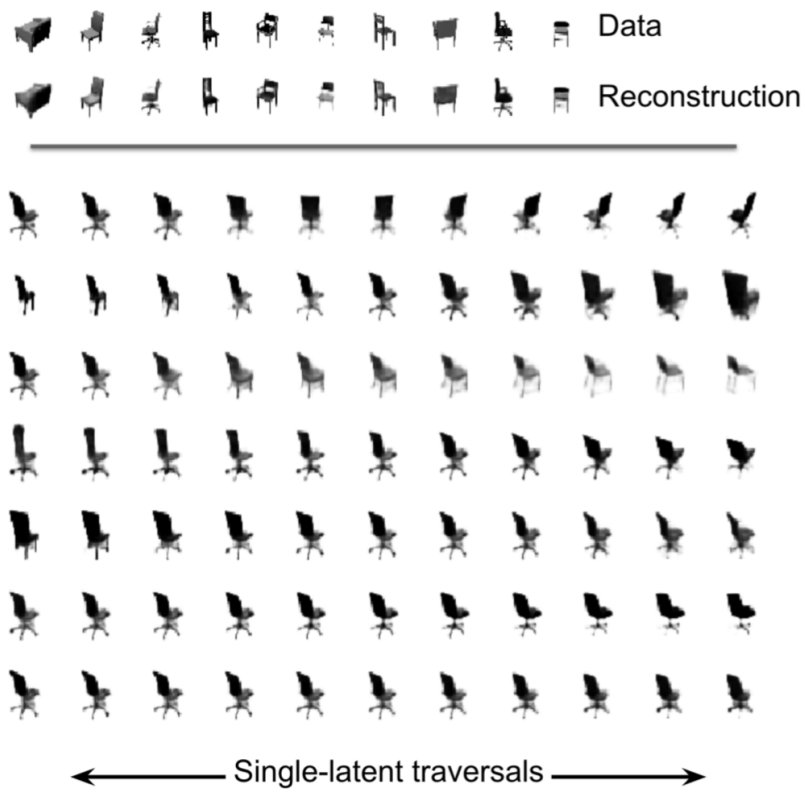
β-VAE

(a) Coloured dSprites



(b) 3D Chairs



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FactorVAE

FactorVAE

Disentangled aggregated variational posterior

$$q(\mathbf{z}) = \frac{1}{n}\sum_{i=1}^{n} q(\mathbf{z}|\mathbf{x}) = \prod_{j=1}^{d} q(z_j)$$

Total correlation regularizer

$$\min KL(q(\mathbf{z})||\prod_{j=1}^{d} q(z_j))$$

FactorVAE objective

$$\min_{\phi,\boldsymbol{\theta}} \mathcal{L}(\phi,\boldsymbol{\theta}) - \gamma \cdot KL(q(\mathbf{z})||\prod_{j=1}^{d} q(z_j))$$

- The last term is intractable.
- FactorVAE uses density ratio trick for estimation.

Kim H., Mnih A. Disentangling by Factorising, 2018

FactorVAE

Consider two distributions $q_1(\mathbf{x})$, $q_2(\mathbf{x})$ and probilistic model

$$p(\mathbf{x}|y) = \begin{cases} q_1(\mathbf{x}), & \text{if } y = 1, \\ q_2(\mathbf{x}), & \text{if } y = 0, \end{cases} \qquad y \sim \text{Bern}(0.5).$$

Density ratio trick

$$\frac{q_1(\mathbf{x})}{q_2(\mathbf{x})} = \frac{p(\mathbf{x}|y=1)}{p(\mathbf{x}|y=0)} = \frac{p(y=1|\mathbf{x})p(\mathbf{x})}{p(y=1)} \Big/ \frac{p(y=0|\mathbf{x})p(\mathbf{x})}{p(y=0)} =$$

$$= \frac{p(y=1|\mathbf{x})}{p(y=0|\mathbf{x})} = \frac{p(y=1|\mathbf{x})}{1-p(y=1|\mathbf{x})} = \frac{D(\mathbf{x})}{1-D(\mathbf{x})}$$

Here $D(\mathbf{x})$ could be treated as a discriminator a model the output of which is a probability that $\mathbf{x}$ is a sample from $q_1(\mathbf{x})$ rather than from $q_2(\mathbf{x})$.

Kim H., Mnih A. Disentangling by Factorising, 2018

FactorVAE

FactorVAE objective

$$\min_{\boldsymbol{\theta},\boldsymbol{\phi}} \text{ELBO}(\boldsymbol{\theta},\boldsymbol{\phi}) - \gamma \cdot KL(q(\mathbf{z})||\prod_{j=1}^{d} q(z_j))$$

Total correlation regularizer

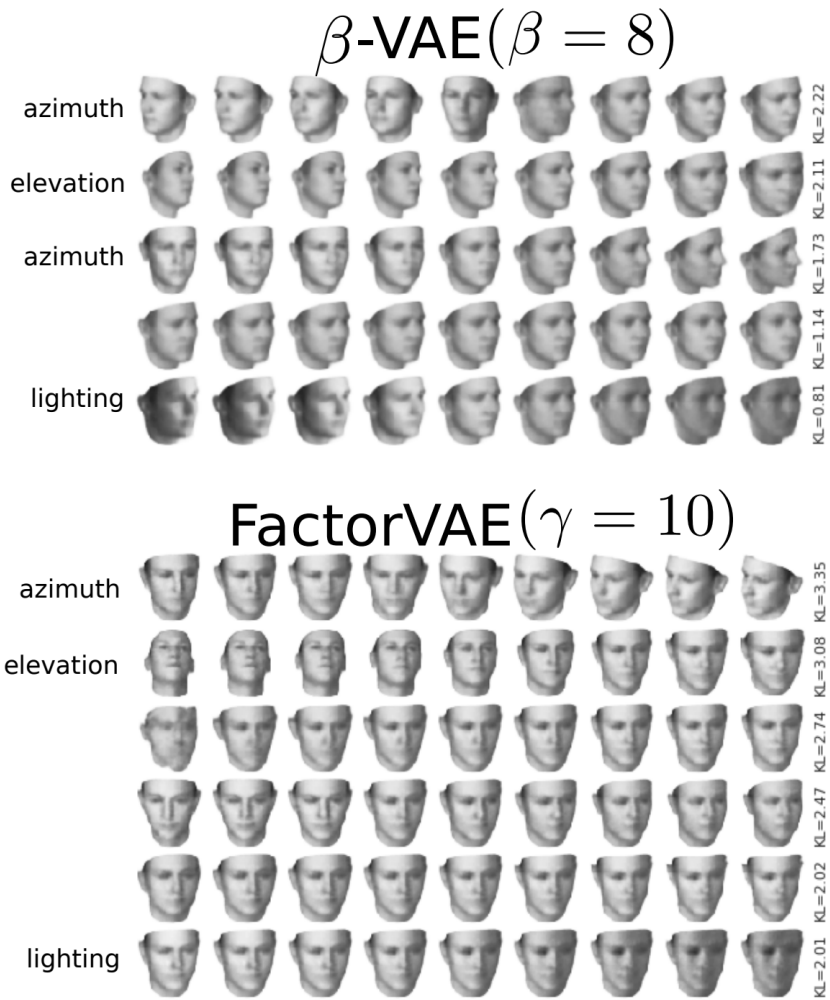
$$KL(q(\mathbf{z})||\prod_{j=1}^{d} q(z_j)) = KL(q(\mathbf{z})||\bar{q}(\mathbf{z})) =$$

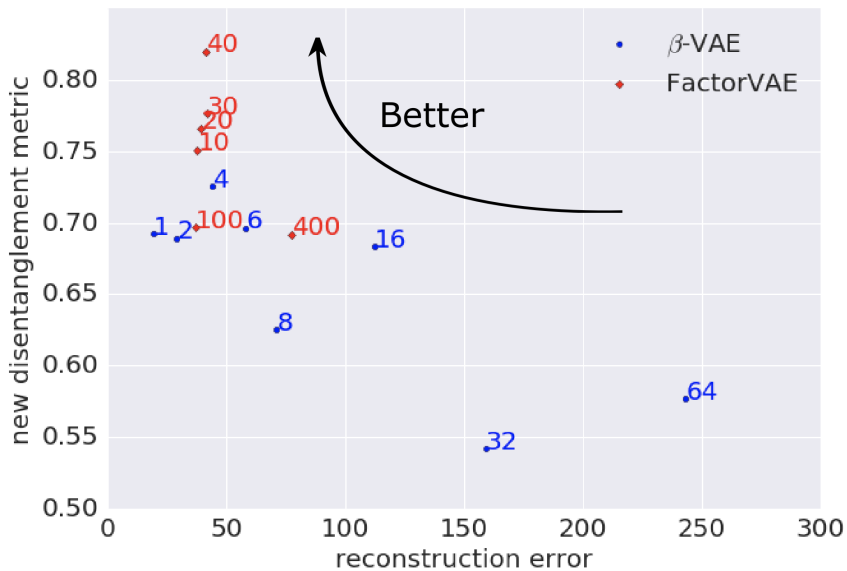
$$= \mathbb{E}_{q(\mathbf{z})} \log \frac{q(\mathbf{z})}{\bar{q}(\mathbf{z})} \approx \mathbb{E}_{q(\mathbf{z})} \log \frac{D(\mathbf{z})}{1 - D(\mathbf{z})}$$

VAE and GAN are trained simultaneously.

Kim H., Mnih A. Disentangling by Factorising, 2018

FactorVAE





Kim H., Mnih A. Disentangling by Factorising, 2018

Improved techniques for training GANs

Improved techniques for training GANs

- Feature matching

$$\mathcal{L}_G = \|\mathbb{E}_{\pi(\mathbf{x})}\mathbf{d}(\mathbf{x}) - \mathbb{E}_{p(\mathbf{z})}\mathbf{d}(G(\mathbf{z}))\|_2^2$$

  Here $\mathbf{d}(\mathbf{x})$ – intermediate layer of discriminator. Matching the learned discriminator statistics instead of the output of the discriminator. Helps to avoid the vanishing gradients for sufficiently good discriminator.

- Historical averaging adds extra loss term for generator and discriminator losses

$$\|\boldsymbol{\theta} - \frac{1}{T}\sum_{t=1}^{T}\boldsymbol{\theta}_t\|_2^2.$$

  Here $\boldsymbol{\theta}_t$ – value of parameters at the previous step t. It allows to stabilize training procedure.

Salimans T. et al. Improved Techniques for Training GANs, 2016

Improved techniques for training GANs

- One-sided label smoothing. Instead of using one-hot labels in classification, use $(1 - \alpha)$ for real data (the generated samples are not smoothed).

$$D^*(\mathbf{x}) = \frac{(1-\alpha)\pi(\mathbf{x})}{\pi(\mathbf{x}) + p(\mathbf{x}|\boldsymbol{\theta})}$$

- Virtual batch normalization. BatchNorm makes samples within minibatch are highly correlated.

Use reference fixed batch to compute the normalization statistics. To avoid overfitting construct batch with the reference batch and the current sample.

Salimans T. et al. Improved Techniques for Training GANs, 2016