Deep Generative Models

Lecture 11

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 Ozon Masters

Spring, 2022

Recap of previous lecture

WGAN objective

$$\min_G W(\pi||p) = \min_G \max_{\phi \in \mathbf{\Phi}} \left[\mathbb{E}_{\pi(\mathbf{x})} f(\mathbf{x}, \phi) - \mathbb{E}_{p(\mathbf{z})} f(G(\mathbf{z}), \phi) \right].$$

- Function f in WGAN is usually called *critic*.
- If parameters ϕ lie in a compact set $\mathbf{\Phi} \in [-0.01, 0.01]^d$ then $f(\mathbf{x}, \phi)$ will be K-Lipschitz continuous function.

Gradient penalty

$$W(\pi||p) = \underbrace{\mathbb{E}_{\pi(\mathbf{x})} f(\mathbf{x}) - \mathbb{E}_{p(\mathbf{x})} f(\mathbf{x})}_{\text{original critic loss}} + \lambda \underbrace{\mathbb{E}_{U[0,1]} \left[(\|\nabla_{\hat{\mathbf{x}}} f(\hat{\mathbf{x}})\|_2 - 1)^2 \right]}_{\text{gradient penalty}}.$$

Samples $\hat{\mathbf{x}}_t = t\mathbf{x} + (1-t)\mathbf{y}$ with $t \in [0, 1]$ are uniformly sampled along straight lines between pairs of points: $\mathbf{x}$ from the data distribution $\pi(\mathbf{x})$ and $\mathbf{y}$ from the generator distribution $p(\mathbf{x}|\theta)$.

Arjovsky M., Chintala S., Bottou L. Wasserstein GAN, 2017
Gulrajani I. et al. Improved Training of Wasserstein GANs, 2017

Recap of previous lecture

$$f(\mathbf{x}, \phi) = \mathbf{W}_{K+1}\sigma_K(\mathbf{W}_K\sigma_{K-1}(\dots\sigma_1(\mathbf{W}_1\mathbf{x})\dots)).$$

- σ_k is a pointwise nonlinearities. We assume that $\|\sigma_k\|_L = 1$ (it holds for ReLU).
- $\mathbf{g}(\mathbf{x}) = \mathbf{W}\mathbf{x}$ is a linear transformation ($\nabla\mathbf{g}(\mathbf{x}) = \mathbf{W}$).

$$\|\mathbf{g}\|_L = \sup_{\mathbf{x}} \|\nabla\mathbf{g}(\mathbf{x})\|_2 = \|\mathbf{W}\|_2.$$

Critic spectral norm

$$\|f\|_L \leq \|\mathbf{W}_{K+1}\|_2 \cdot \prod_{k=1}^{K} \|\sigma_k\|_L \cdot \|\mathbf{W}_k\|_2 = \prod_{k=1}^{K+1} \|\mathbf{W}_k\|_2.$$

Spectral Normalization GAN

If we replace the weights in the critic $f(\mathbf{x}, \phi)$ by $\mathbf{W}_k^{SN} = \mathbf{W}_k / \|\mathbf{W}_k\|_2$, we will get $\|f\|_L \leq 1$.

Power iteration approximates the value of $\|\mathbf{W}\|_2$.

Miyato T. et al. Spectral Normalization for Generative Adversarial Networks, 2018

Recap of previous lecture

f-divergence minimization

$$D_f(\pi||p) = \mathbb{E}_{p(\mathbf{x})} f\left(\frac{\pi(\mathbf{x})}{p(\mathbf{x})}\right) \to \min_p .$$

Here $f : \mathbb{R}_+ \to \mathbb{R}$ is a convex, lower semicontinuous function satisfying $f(1) = 0$.

Variational divergence estimation

$$D_f(\pi||p) \geq \sup_{T \in \mathcal{T}} \left[\mathbb{E}_\pi T(\mathbf{x}) - \mathbb{E}_p f^*(T(\mathbf{x}))\right],$$

Fenchel conjugate

$$f^*(t) = \sup_{u \in \text{dom}_f} \left(ut - f(u)\right), \quad f(u) = \sup_{t \in \text{dom}_{f^*}} \left(ut - f^*(t)\right)$$

Note: To evaluate lower bound we only need samples from $\pi(\mathbf{x})$ and $p(\mathbf{x})$. Hence, we could fit implicit generative model.

Nowozin S., Cseke B., Tomioka R. f-GAN: Training Generative Neural Samplers using Variational Divergence Minimization, 2016

Outline

1. Evaluation of likelihood-free models
2. Inception score/FID/Precision-Recall
3. Evolution of GANs

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Evaluation of likelihood-free models

How to evaluate generative models?

Likelihood-based models

- Split data to train/val/test.
- Fit model on the train part.
- Tune hyperparameters on the validation part.
- Evaluate generalization by reporting likelihoods on the test set.

Not all models have tractable likelihoods

- VAE: compare ELBO values.
- GAN: ???

Evaluation of likelihood-free models

Let take some pretrained image classification model to get the conditional label distribution $p(y|\mathbf{x})$ (e.g. ImageNet classifier).

What do we want from samples?

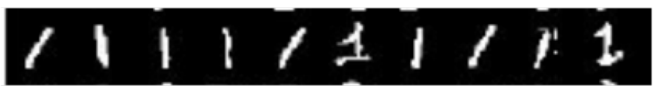
- **Sharpness**

Low sharpness

High sharpness

The conditional distribution $p(y|\mathbf{x})$ should have low entropy (each image $\mathbf{x}$ should have distinctly recognizable object).

- **Diversity**



Low diversity

High diversity

The marginal distribution $p(y) = \int p(y|\mathbf{x})p(\mathbf{x})d\mathbf{x}$ should have high entropy (there should be as many classes generated as possible).

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image credit: https://deepgenerativemodels.github.io

Evaluation of likelihood-free models

What do we want from samples?

- **Sharpness.** The conditional distribution $p(y|\mathbf{x})$ should have low entropy (each image $\mathbf{x}$ should have distinctly recognizable object).
- **Diversity.** The marginal distribution $p(y) = \int p(y|\mathbf{x})p(\mathbf{x})d\mathbf{x}$ should have high entropy (there should be as many classes generated as possible).

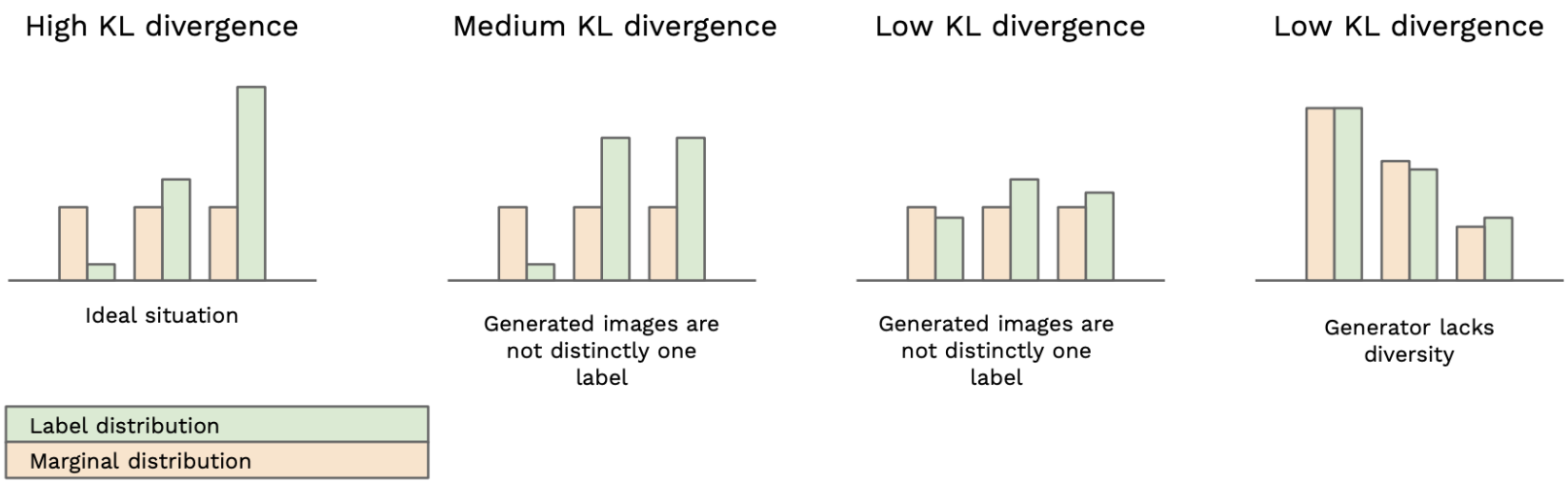

image credit: https://medium.com/octavian-ai/a-simple-explanation-of-the-inception-score-372dff6a8c7a

Evaluation of likelihood-free models

What do we want from samples?

- Sharpness $\Rightarrow$ low $H(y|\mathbf{x}) = -\sum_y \int_{\mathbf{x}} p(y, \mathbf{x}) \log p(y|\mathbf{x}) d\mathbf{x}$.
- Diversity $\Rightarrow$ high $H(y) = -\sum_y p(y) \log p(y)$.

Inception Score

$$
\begin{aligned}
IS &= \exp(H(y) - H(y|\mathbf{x})) \\
&= \exp\left(-\sum_y p(y) \log p(y) + \sum_y \int_{\mathbf{x}} p(y, \mathbf{x}) \log p(y|\mathbf{x}) d\mathbf{x}\right) \\
&= \exp\left(\sum_y \int_{\mathbf{x}} p(y, \mathbf{x}) \log \frac{p(y|\mathbf{x})}{p(y)} d\mathbf{x}\right) \\
&= \exp\left(\mathbb{E}_{\mathbf{x}} \sum_y p(y|\mathbf{x}) \log \frac{p(y|\mathbf{x})}{p(y)}\right) = \exp\left(\mathbb{E}_{\mathbf{x}} KL(p(y|\mathbf{x})||p(y))\right)
\end{aligned}
$$

Salimans T. et al. Improved Techniques for Training GANs, 2016

Outline

1. Evaluation of likelihood-free models
2. Inception score/FID/Precision-Recall
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Evaluation of likelihood-free models

Theorem (informal)

If $\pi(\mathbf{x})$ and $p(\mathbf{x}|\boldsymbol{\theta})$ has moment generation functions then

$$\pi(\mathbf{x}) = p(\mathbf{x}|\boldsymbol{\theta}) \Leftrightarrow \mathbb{E}_{\pi}\mathbf{x}^k = \mathbb{E}_{p}\mathbf{x}^k, \quad \forall k \geq 1.$$

This is intractable to calculate all moments.

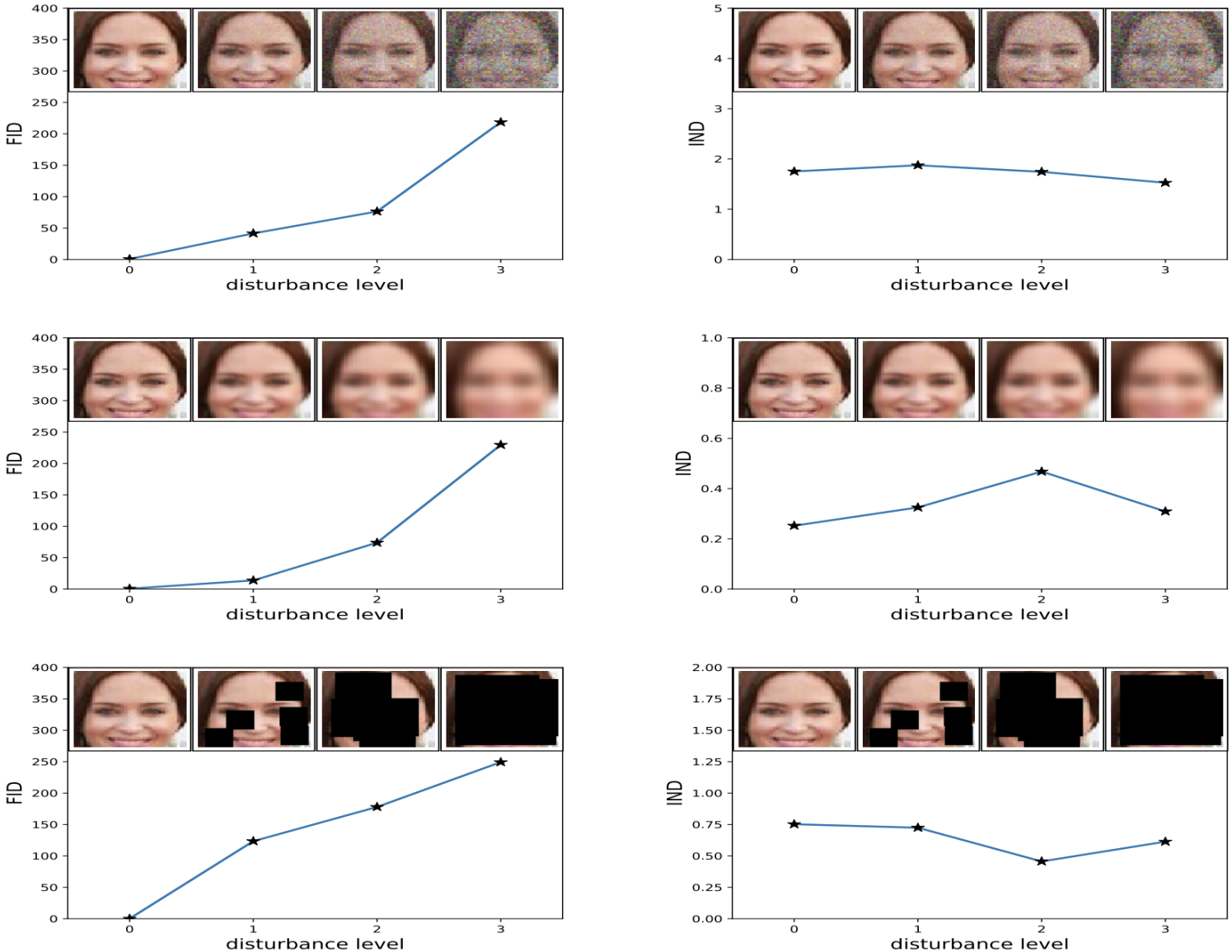
Frechet Inception Distance

$$FID(\pi, p) = \|\mathbf{m}_{\pi} - \mathbf{m}_{p}\|_2^2 + \mathrm{Tr}\left(\boldsymbol{\Sigma}_{\pi} + \boldsymbol{\Sigma}_{p} - 2\sqrt{\boldsymbol{\Sigma}_{\pi}\boldsymbol{\Sigma}_{p}}\right)$$

- Representations are outputs of intermediate layer from pretrained classification model.
- $\mathbf{m}_{\pi}$, $\boldsymbol{\Sigma}_{\pi}$ are mean vector and covariance matrix of feature representations for real samples from $\pi(\mathbf{x})$
- $\mathbf{m}_{p}$, $\boldsymbol{\Sigma}_{p}$ are mean vector and covariance matrix of feature representations for generated samples from $p(\mathbf{x}|\boldsymbol{\theta})$.

Heusel M. et al. GANs Trained by a Two Time-Scale Update Rule Converge to a Local Nash Equilibrium, 2017

Evaluation of likelihood-free models



Heusel M. et al. GANs Trained by a Two Time-Scale Update Rule Converge to a Local Nash Equilibrium, 2017

Limitations

Inception Score

$$IS = \exp\left(\mathbb{E}_{\mathbf{x}} KL(p(y|\mathbf{x})||p(y))\right)$$

- If generator produces images with a different set of labels from the classifier training set, IS will be low.
- If generator produces one image per class, the IS will be perfect (there is no measure of intra-class diversity).

Frechet Inception Distance

$$FID = \|\mathbf{m}_\pi - \mathbf{m}_p\|_2^2 + \mathrm{Tr}\left(\mathbf{\Sigma}_\pi + \mathbf{\Sigma}_p - 2\sqrt{\mathbf{\Sigma}_\pi \mathbf{\Sigma}_p}\right)$$

- Needs a large sample size for evaluation.
- Calculation of FID is slow.
- Estimates only two sample moments.

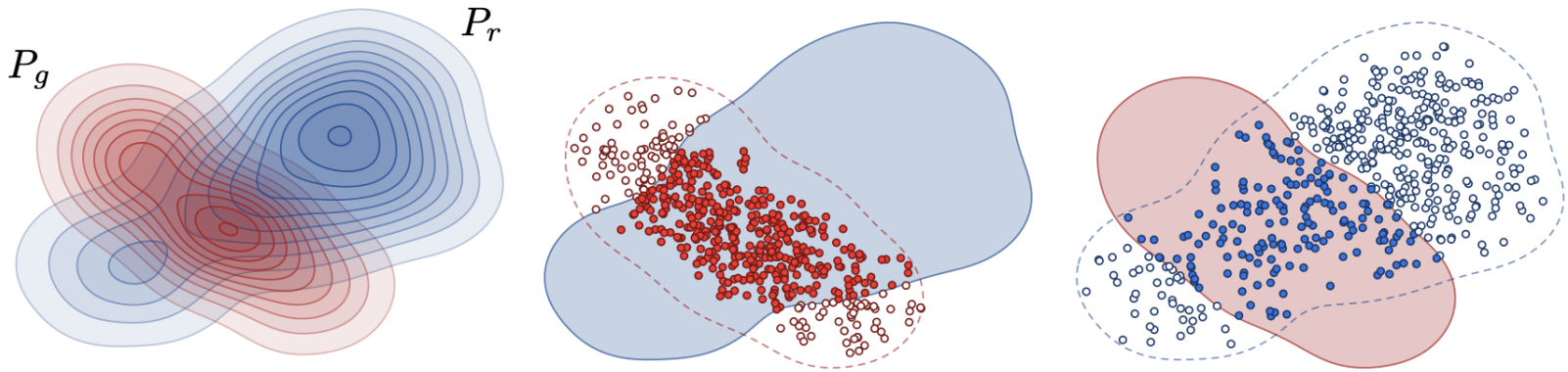
Both scores depend on the pretrained classifier $p(y|\mathbf{x})$.

Barratt S., Sharma R. A Note on the Inception Score, 2018
Heusel M. et al. GANs Trained by a Two Time-Scale Update Rule Converge to a Local Nash Equilibrium, 2017

Precision-Recall for Generative Models

What do we want from samples

- **Sharpness**: generated samples should be of high quality.
- **Diversity**: their variation should match that observed in the training set.



(a) Example distributions (b) Precision (c) Recall

- **Precision** denotes the fraction of generated images that are realistic.
- **Recall** measures the fraction of the training data manifold covered by the generator.

Kynkäänniemi T. et al. Improved precision and recall metric for assessing generative models, 2019

Precision-Recall for generative models

- $\mathcal{S}_\pi = \{\mathbf{x}_i\}_{i=1}^n \sim \pi(\mathbf{x})$ – real samples;
- $\mathcal{S}_p = \{\mathbf{x}_i\}_{i=1}^n \sim p(\mathbf{x}|\theta)$ – generated samples.

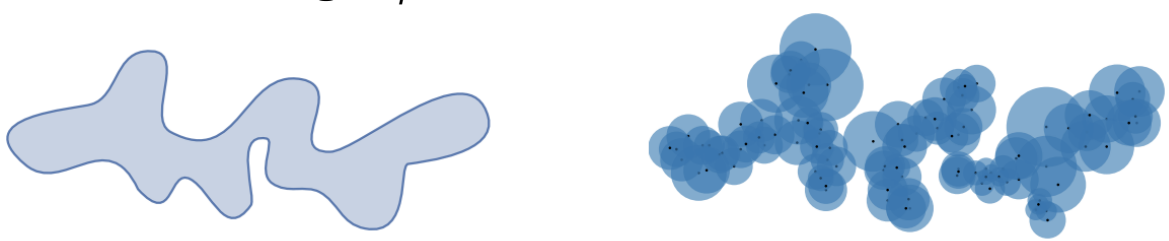
Embed samples using pretrained classifier network (as previously):

$$\mathcal{G}_\pi = \{\mathbf{g}_i\}_{i=1}^n, \quad \mathcal{G}_p = \{\mathbf{g}_i\}_{i=1}^n.$$

Define binary function:

$$f(\mathbf{g}, \mathcal{G}) = \begin{cases} 1, \text{if exists } \mathbf{g}' \in \mathcal{G} : \|\mathbf{g} - \mathbf{g}'\|_2 \leq \|\mathbf{g}' - \mathrm{NN}_k(\mathbf{g}', \mathcal{G})\|_2; \\ 0, \text{otherwise.} \end{cases}$$

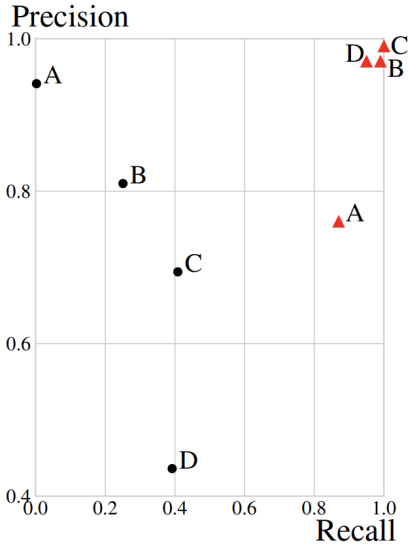
$$\text{Precision}(\mathcal{G}_\pi, \mathcal{G}_p) = \frac{1}{n} \sum_{\mathbf{g} \in \mathcal{G}_p} f(\mathbf{g}, \mathcal{G}_\pi); \quad \text{Recall}(\mathcal{G}_\pi, \mathcal{G}_p) = \frac{1}{n} \sum_{\mathbf{g} \in \mathcal{G}_\pi} f(\mathbf{g}, \mathcal{G}_p).$$



(a) True manifold (b) Approx. manifold

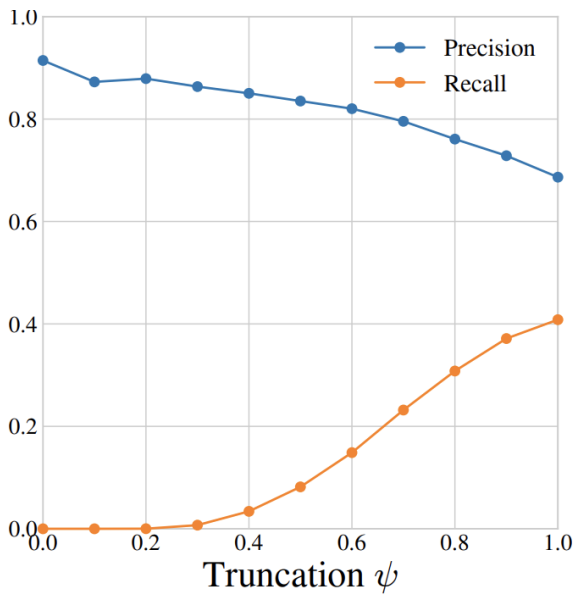
Kynkäänniemi T. et al. Improved precision and recall metric for assessing generative models, 2019

Precision-Recall for generative models



A (FID = 91.7) **B** (FID = 16.9) **C** (FID = 4.5) **D** (FID = 16.7)

$\psi = 0.0$ $\psi = 0.3$ $\psi = 0.7$ $\psi = 1.0$



Kynkäänniemi T. et al. Improved precision and recall metric for assessing generative models, 2019

Outline

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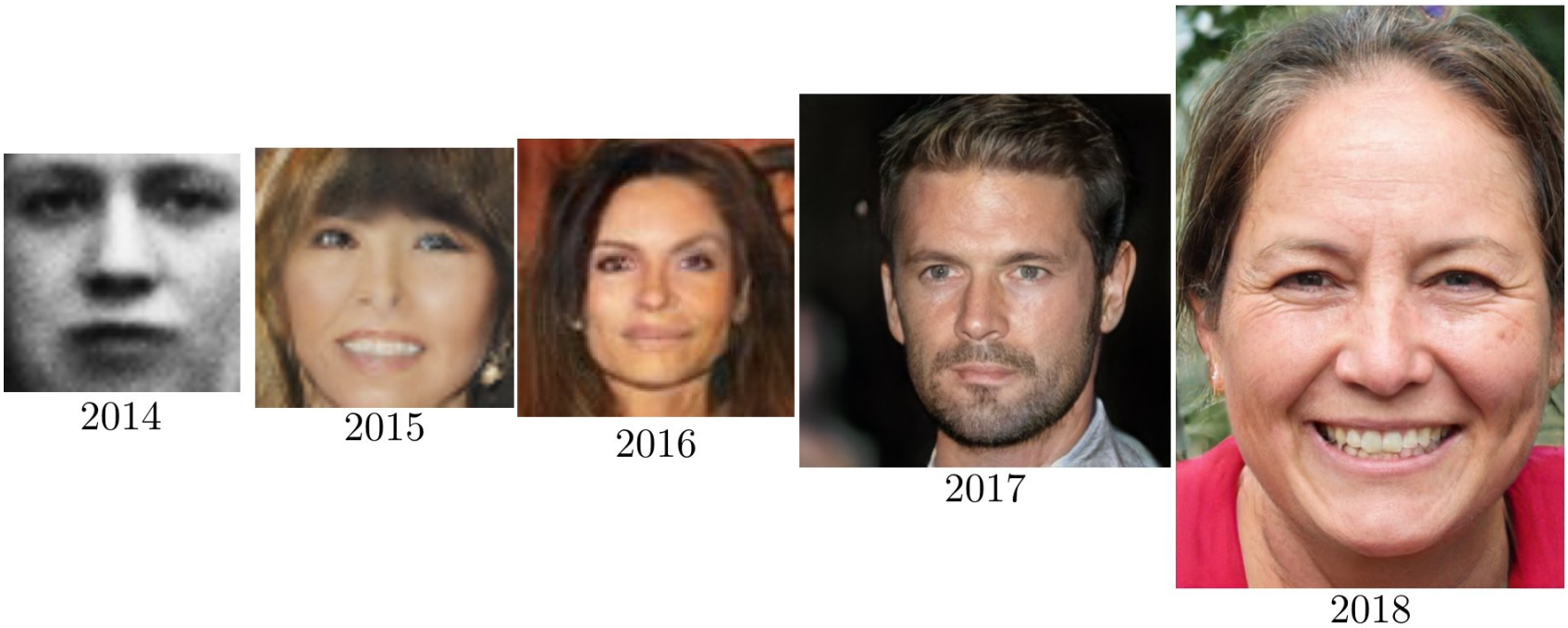
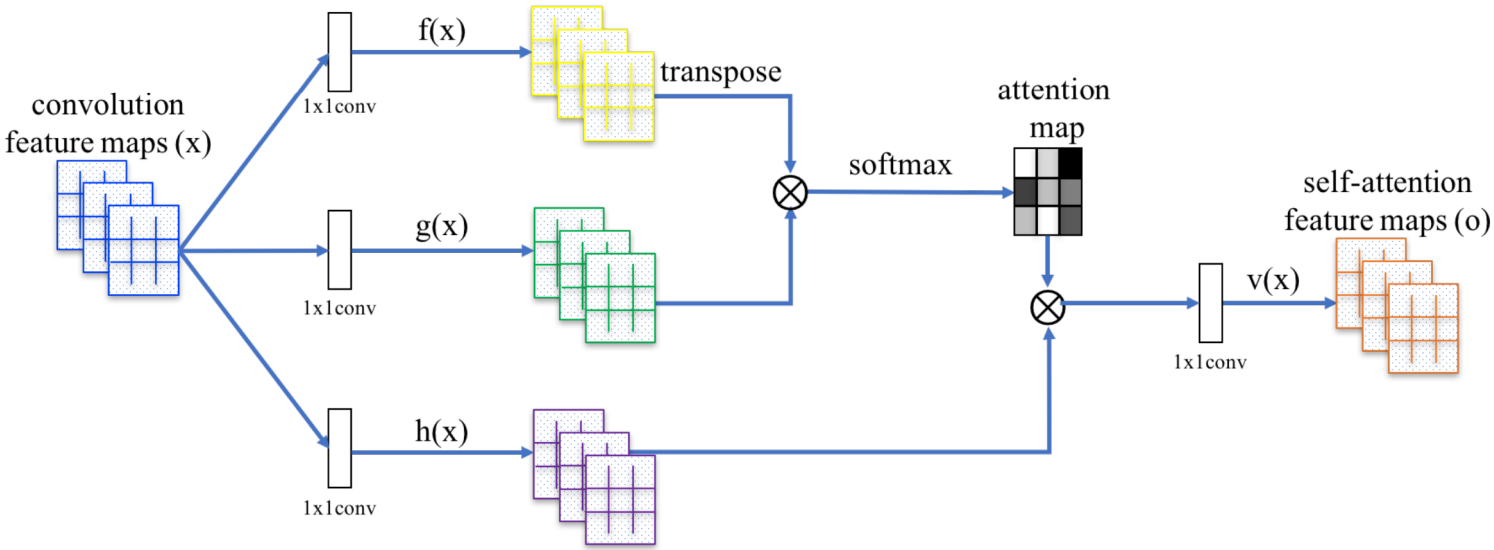
Evolution of GANs



- **Standard GAN** https://arxiv.org/abs/1406.2661
- **DCGAN** https://arxiv.org/abs/1511.06434
- **CoGAN** https://arxiv.org/abs/1606.07536
- **ProGAN** https://arxiv.org/abs/1710.10196
- **StyleGAN** https://arxiv.org/abs/1812.04948

Self-Attention GAN

Convolutional layers process the information in a local neighborhood $\Rightarrow$ inefficient for modeling long-range dependencies in images.



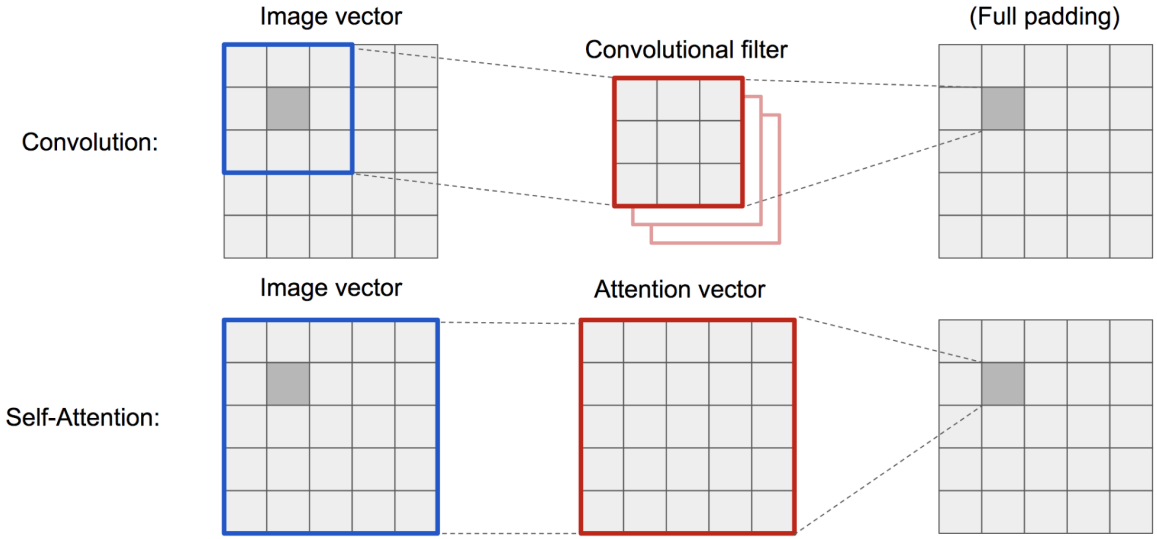
$$\mathbf{f}(\mathbf{x}) = \mathbf{W}_f\mathbf{x}, \quad \mathbf{g}(\mathbf{x}) = \mathbf{W}_g\mathbf{x}, \quad \mathbf{h}(\mathbf{x}) = \mathbf{W}_h\mathbf{x}, \quad \mathbf{v}(\mathbf{x}) = \mathbf{W}_v\mathbf{x}$$

$$s_{ij} = \mathbf{f}(\mathbf{x}_i)^T\mathbf{g}(\mathbf{x}_j), \quad a_{ij} = \frac{\exp s_{ij}}{\sum_{i=1}^{N}\exp s_{ij}}, \quad \mathbf{o}_j = \mathbf{v}\left(\sum_{i=1}^{N} a_{ij}\mathbf{h}(\mathbf{x}_i)\right)$$

Zhang H. et al. Self-Attention Generative Adversarial Networks, 2018

Self-Attention GAN

Convolution vs Attention



Visualization of attention maps

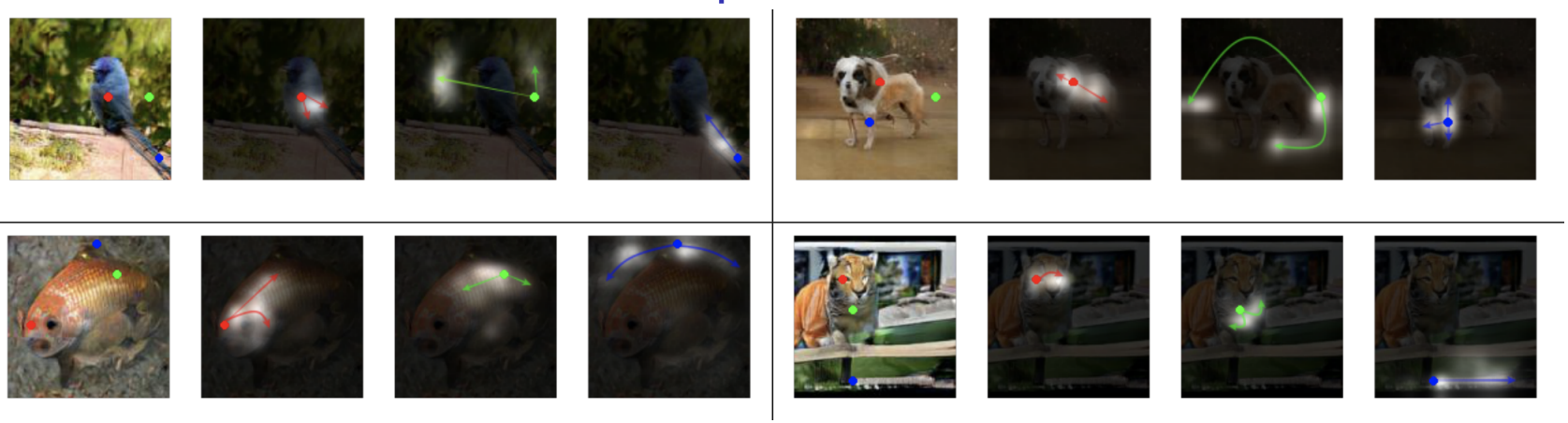


image credit: https://lilianweng.github.io/lil-log/2018/06/24/attention-attention.html
Zhang H. et al. Self-Attention Generative Adversarial Networks, 2018

BigGAN

Batch-size is matter

Batch	Ch.	Param (M)	Shared	Skip-z	Ortho.	Itr $\times 10^3$	FID	IS
256	64	81.5	SA-GAN Baseline			1000	18.65	52.52
512	64	81.5	✗	✗	✗	1000	15.30	58.77($\pm$1.18)
1024	64	81.5	✗	✗	✗	1000	14.88	63.03($\pm$1.42)
2048	64	81.5	✗	✗	✗	732	12.39	76.85($\pm$3.83)
2048	96	173.5	✗	✗	✗	295($\pm$18)	9.54($\pm$0.62)	92.98($\pm$4.27)

Samples (512x512)

Brock A., Donahue J., Simonyan K. Large Scale GAN Training for High Fidelity Natural Image Synthesis, 2018

Progressive Growing GAN

Problems with HR image generation

- Disjoint manifolds $\Rightarrow$ gradient problem.
- Small minibatch $\Rightarrow$ training instability.

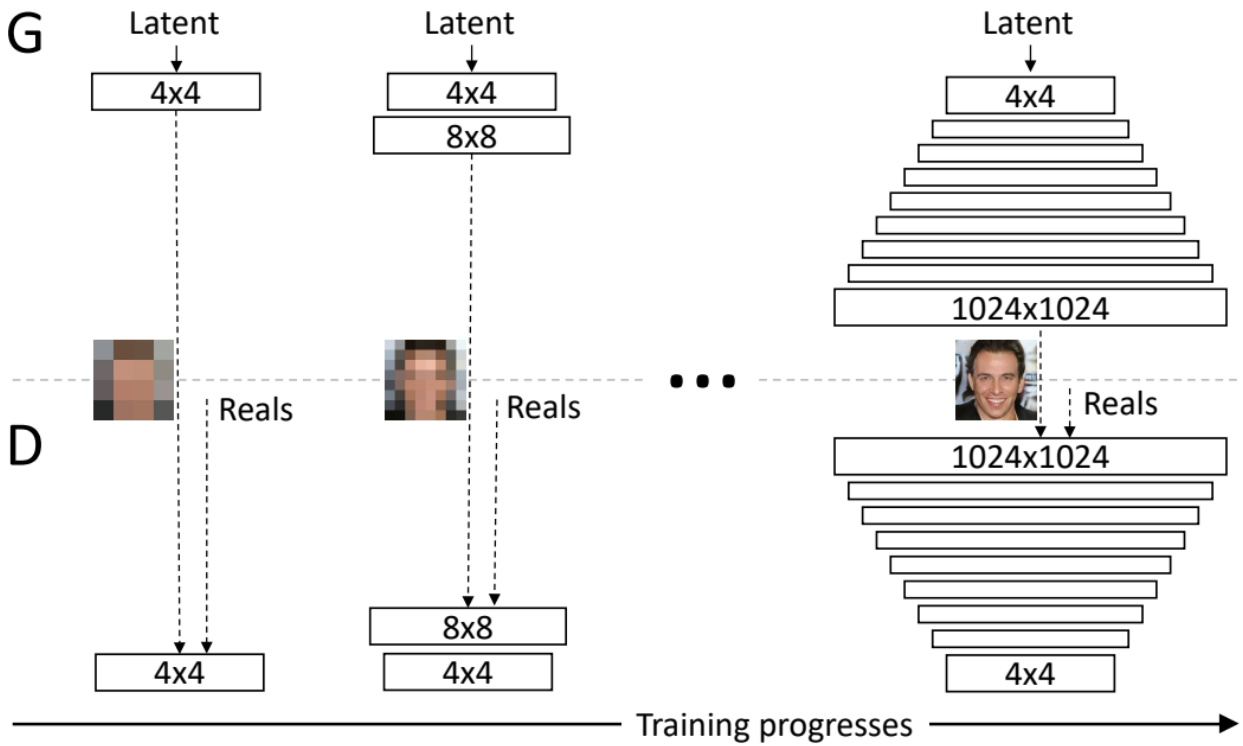
Samples (1024x1024)

Karras T. et al. Progressive Growing of GANs for Improved Quality, Stability, and Variation, 2017

Progressive Growing GAN

Grow both the generator and discriminator progressively, new layers will introduce higher-resolution details as the training progresses.

- Train GAN which generate 4x4 images (2 convs for G and D).
- Add upsampling layers to G, downsampling layers to D.
- Train GAN which generate 8x8 images.
- etc.



Karras T. et al. Progressive Growing of GANs for Improved Quality, Stability, and Variation, 2017

StyleGAN

- Generating of HR images is hard.
- Progressive growing greatly simplifies the task.
- The ability to control specific features of the generated image is very limited.

Face image features

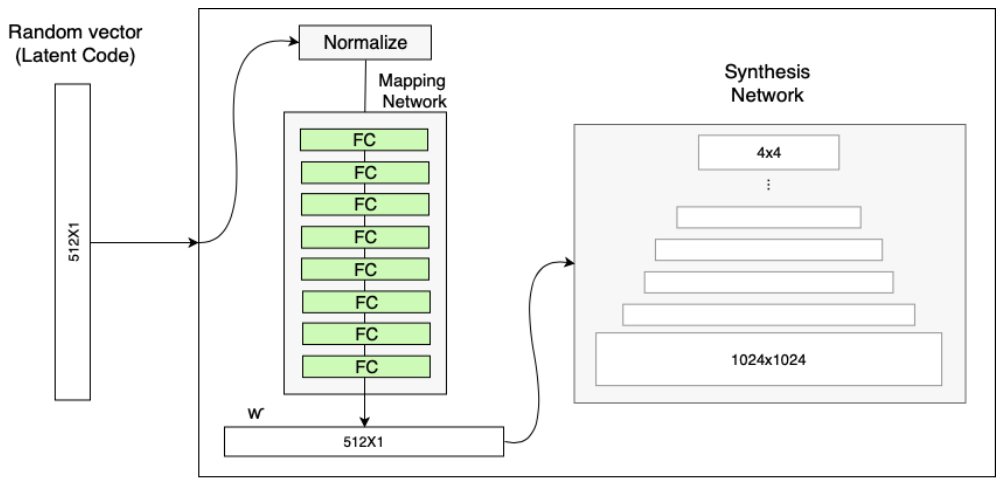
- Coarse (pose, general hair style, face shape). Resolution $4^2 - 8^2$.
- Middle (finer facial features, hair style, eyes open/closed). Resolution $16^2 - 32^2$.
- Fine (color scheme (eye, hair and skin) and micro features). Resolution $64^2 - 1024^2$.

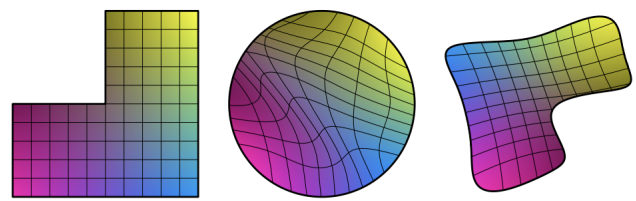
Karras T., Laine S., Aila T. A Style-Based Generator Architecture for Generative Adversarial Networks, 2018

StyleGAN

Mapping Network

- Generator input is likely to be **disentangled**. Each component of input vector **z** should be responsible for one generative factor.
- Mapping network $f : \mathcal{Z} \to \mathcal{W}$ is used to reduce correlations between components of **z**.





(a) Distribution of features in training set

(b) Mapping from $\mathcal{Z}$ to features

(c) Mapping from $\mathcal{W}$ to features

Karras T., Laine S., Aila T. A Style-Based Generator Architecture for Generative Adversarial Networks, 2018

Truncation trick

BigGAN: truncated normal sampling

$$p(\mathbf{z}|b) = \mathcal{N}(\mathbf{z}|0,1)/\int_{-\infty}^{b} \mathcal{N}(\mathbf{z}|0,1)d\mathbf{z}$$

Components of $\mathbf{z} \sim \mathcal{N}(0, \mathbf{I})$ which fall outside a predefined range are resampled.

StyleGAN

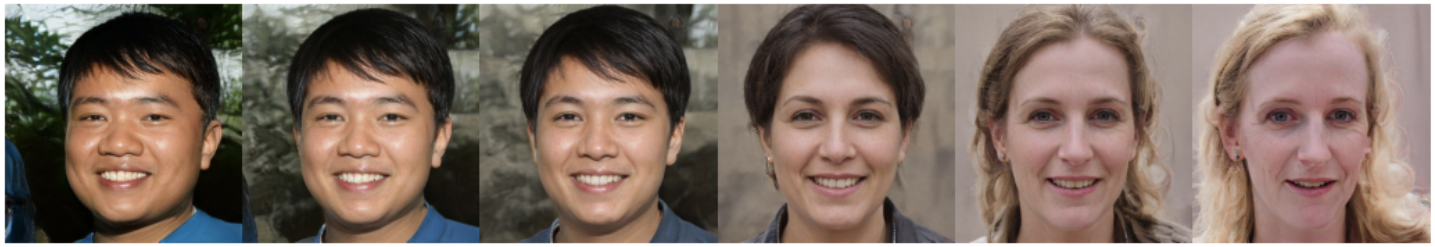
$$\mathbf{w}' = \hat{\mathbf{w}} + \psi \cdot (\mathbf{w} - \hat{\mathbf{w}}), \quad \hat{\mathbf{w}} = \mathbb{E}_{\mathbf{z}} p(f(\mathbf{z}))$$

- Constant ψ is a tradeoff between diversity and fidelity.
- $\psi = 0.7$ is used for most of the results.
- Truncation is done only at the low-resolution layers.

Karras T., Laine S., Aila T. A Style-Based Generator Architecture for Generative Adversarial Networks, 2018

StyleGAN

Truncation trick



Samples (1024x1024)

Karras T., Laine S., Aila T. A Style-Based Generator Architecture for Generative Adversarial Networks, 2018

Summary

- Inception Score and Frechet Inception Distance are the common metrics for GAN evaluation, but both of them have drawbacks.
- Precision-recall allows to select model with compromise with sample quality and sample diversity.
- Self-Attention GAN allows to make huge receptive field and reduce convolution inductive bias.
- BigGAN shows that large batch size increase model quality gradually.
- Progressive growing for GAN learning allows to make training more stable.
- StyleGAN introduces mapping network to get more disentangled latent representation.