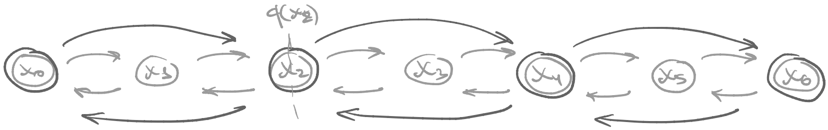

1. $s_\theta(x_t; t) = -\dfrac{\epsilon_\theta(x_t; t)}{\sqrt{1-\bar{\alpha}_t}}$

$\simeq$

$\nabla_x \log q(x_t)$

$$q(x_t) = \int \pi(x_0)\, q(x_t | x_0)\, dx_0$$

2. $q^{new}(x_i | x_0) = q(x_{t_i} | x_0) = \mathcal{N}\{x_{t_i} | \sqrt{\bar{\alpha}_{t_i}}\, x_0 ; (1-\bar{\alpha}_{t_i}) I\}$

$x_{t_1} \dots x_{t_{T'}}$, $T' < T$

p_i

PDIM

1. $q_\sigma(x_{t-1}|x_t;x_0) = \mathcal{N}\{\sqrt{\bar\alpha_{t-1}}\,x_0 + \sqrt{1-\bar\alpha_{t-1}-\sigma_t^2}\,\frac{x_t-\sqrt{\bar\alpha_t}\,x_0}{\sqrt{1-\bar\alpha_t}};\ \sigma_t^2 I\}$

$\hookrightarrow$ $q_\sigma(x_t|x_0)$ не изменится

$q_\sigma(x_t|x_{t-1};x_0) \sim \mathcal{N}$

$\neq$

$q_\sigma(x_t|x_{t-1})$

2. $p_\theta(x_{t-1}|x_t) = \begin{cases} \mathcal{N}\{x_0(x_1;1);\ \sigma_1\}, & t=1 \\ q_\sigma(x_{t-1}|x_t;\ x_0(x_t;t)), & t>1 \end{cases}$

$x_t = \sqrt{\bar\alpha_t}\,x_0 + \sqrt{1-\bar\alpha_t}\,\varepsilon \ \mapsto\ x_0 = \frac{x_t - \sqrt{1-\bar\alpha_t}\,\varepsilon_\theta(x_t;t)}{\sqrt{1-\bar\alpha_t}}$

$\varepsilon_\theta(x_t;\theta)$

3. $J_\sigma(\theta) = \mathbb{E}_{q(x_{0:T})}\{\log q(x_{1:T}|x_0) - \log p_\theta(x_{0:T})\}$

$= \mathbb{E}_{q(x_{0:T})}\{\log q(x_T|x_0) + \sum_{t=2}^{T}\log q_\sigma(x_{t-1}|x_t;x_0) - \sum_{t=1}^{T}\log p_\theta(x_{t-1}|x_t) - \log p_\theta(x_T)\}$

4. Th: $\forall \sigma>0\ \exists \gamma\in\mathbb{R}^T_{>0}$: $J_\sigma = L_\gamma + C$, $L_\gamma(\theta) = \sum \gamma_t \|\varepsilon_\theta(x_t)-\varepsilon\|_2^2$

5. $\{\tau_1 \dots \tau_T\} \subseteq \{1 \dots T\}$

$q(x_{\tau_i}|x_0) = \mathcal{N}\{\sqrt{\bar\alpha_{\tau_i}}\,x_0;\ (1-\bar\alpha_{\tau_i}) I\}$

$\Rightarrow$ лосс, для короткого процесса тоже эквив. лоссу в DDPM.

6. Семплирование:

$x_{t-1} = \sqrt{\bar\alpha_{t-1}}\left\{\frac{x_t - \sqrt{1-\bar\alpha_t}\,\varepsilon_\theta(x_t)}{\sqrt{\bar\alpha_t}}\right\} + \sqrt{1-\bar\alpha_{t-1}-\sigma_t^2}\,\varepsilon_\theta(x_t) + \sigma_t\cdot\varepsilon,\quad \varepsilon\sim\mathcal{N}\{0;1\}$

7. Заметим, что если $\sigma_t^2 = \tilde\beta_t$, то мы получим DDPM

если $\sigma_t^2 = 0$, то мы получаем детерминир. семплирование

$\sigma_t^2 = \eta\cdot\tilde\beta_t$, $\eta\in[0;1]$

$\boxed{\eta = 0 \mapsto \text{DDIM}}$