

Deep Generative Models

Lecture 9

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Recap of previous lecture

WGAN objective

$$\min_{\theta} W(\pi||p) = \min_{\theta} \max_{\phi \in \Phi} [\mathbb{E}_{\pi(x)} f_{\phi}(x) - \mathbb{E}_{p(z)} f_{\phi}(\mathbf{G}_{\theta}(z))].$$

- ▶ Function f in WGAN is usually called *critic*.
- ▶ If parameters ϕ lie in a compact set $\Phi \in [-c, c]^d$ then $f(x, \phi)$ will be K -Lipschitz continuous function.

$$\begin{aligned} K \cdot W(\pi||p) &= \max_{\|f\|_L \leq K} [\mathbb{E}_{\pi(x)} f(x) - \mathbb{E}_{p(x)} f(x)] \geq \\ &\geq \max_{\phi \in \Phi} [\mathbb{E}_{\pi(x)} f_{\phi}(x) - \mathbb{E}_{p(x)} f_{\phi}(x)] \end{aligned}$$

"Weight clipping is a clearly terrible way to enforce a Lipschitz constraint"

Recap of previous lecture

Theorem

Let $\pi(\mathbf{x})$ and $p(\mathbf{x})$ be two distributions in \mathcal{X} , a compact metric space. Let γ be the optimal transportation plan between $\pi(\mathbf{x})$ and $p(\mathbf{x})$. Then

$$\mathbb{P}_{(\mathbf{y}, \mathbf{z}) \sim \gamma} \left[\nabla f^*(\hat{\mathbf{x}}_t) = \frac{\mathbf{z} - \hat{\mathbf{x}}_t}{\|\mathbf{z} - \hat{\mathbf{x}}_t\|} \right] = 1.$$

Gradient penalty

$$W(\pi || p) = \underbrace{\mathbb{E}_{\pi(\mathbf{x})} f(\mathbf{x}) - \mathbb{E}_{p(\mathbf{x})} f(\mathbf{x})}_{\text{original critic loss}} + \lambda \underbrace{\mathbb{E}_{U[0,1]} \left[(\|\nabla f(\hat{\mathbf{x}})\|_2 - 1)^2 \right]}_{\text{gradient penalty}}.$$

Samples $\hat{\mathbf{x}}_t = t \cdot \mathbf{y} + (1 - t) \cdot \mathbf{z}$ with $t \in [0, 1]$ are uniformly sampled along straight lines between pairs of points: \mathbf{y} from the data distribution $\pi(\mathbf{x})$ and \mathbf{z} from the generator distribution $p(\mathbf{x}|\theta)$.

Recap of previous lecture

f-divergence minimization

$$D_f(\pi || p) = \mathbb{E}_{p(\mathbf{x})} f\left(\frac{\pi(\mathbf{x})}{p(\mathbf{x})}\right) \rightarrow \min_p .$$

Here $f : \mathbb{R}_+ \rightarrow \mathbb{R}$ is a convex, lower semicontinuous function satisfying $f(1) = 0$.

Variational divergence estimation

$$D_f(\pi || p) \geq \sup_{T \in \mathcal{T}} [\mathbb{E}_\pi T(\mathbf{x}) - \mathbb{E}_p f^*(T(\mathbf{x}))],$$

Fenchel conjugate

$$f^*(t) = \sup_{u \in \text{dom}_f} (ut - f(u)), \quad f(u) = \sup_{t \in \text{dom}_{f^*}} (ut - f^*(t))$$

Note: To evaluate lower bound we only need samples from $\pi(\mathbf{x})$ and $p(\mathbf{x})$. Hence, we could fit implicit generative model.

Recap of previous lecture

How to evaluate likelihood-free models?

$p(y|x)$ – pretrained image classification model (e.g. ImageNet classifier).

What do we want from samples?

- ▶ Sharpness



$p(y|x)$ has low entropy (each image x should have distinctly recognizable object).

- ▶ Diversity



$p(y) = \int p(y|x)p(x)dx$ has high entropy (there should be as many classes generated as possible).

Outline

1. Evaluation of likelihood-free models

Maximum Mean Discrepancy (MMD)

Precision-Recall

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Maximum Mean Discrepancy (MMD)

Theorem

$\pi(\mathbf{x}) = p(\mathbf{y})$ if and only if $\mathbb{E}_{\pi(\mathbf{x})} f(\mathbf{x}) = \mathbb{E}_{p(\mathbf{y})} f(\mathbf{y})$ for any bounded and continuous f .

$$\text{MMD}(\pi, p) = \sup_f [\mathbb{E}_{\pi(\mathbf{x})} f(\mathbf{x}) - \mathbb{E}_{p(\mathbf{x})} f(\mathbf{x})]$$

Theorem (Reproducing Kernel Hilbert Space)

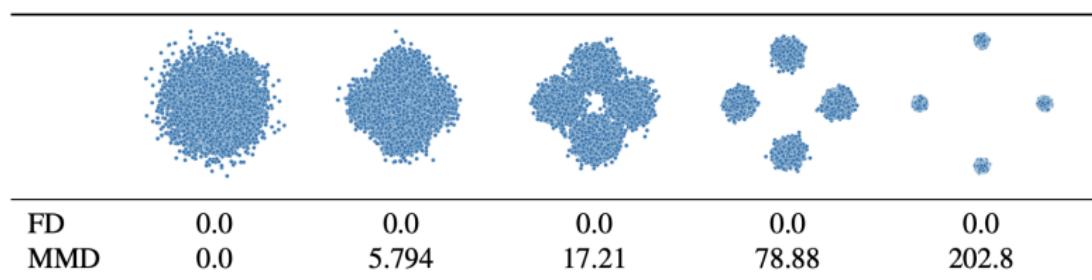
$$\text{MMD}^2(\pi, p) = \mathbb{E}_{(\mathbf{x}, \mathbf{x}')} k(\mathbf{x}, \mathbf{x}') + \mathbb{E}_{(\mathbf{y}, \mathbf{y}')} k(\mathbf{y}, \mathbf{y}') - 2\mathbb{E}_{(\mathbf{x}, \mathbf{y})} k(\mathbf{x}, \mathbf{y}),$$

Here

- ▶ $k(\mathbf{x}, \mathbf{y})$ is a positive definite, symmetric kernel function (for example $k(\mathbf{x}, \mathbf{y}) = \frac{\exp(-\|\mathbf{x}-\mathbf{y}\|^2)}{\sigma^2}$);
- ▶ $f(\mathbf{x}) = \langle f, \phi(\mathbf{x}) \rangle_{\mathcal{H}}$;
- ▶ $\phi(\mathbf{x}) = k(\cdot, \mathbf{x})$.

Maximum Mean Discrepancy (MMD)

$$\text{MMD}^2(\pi, p) = \mathbb{E}_{(\mathbf{x}, \mathbf{x}')} k(\mathbf{x}, \mathbf{x}') + \mathbb{E}_{(\mathbf{y}, \mathbf{y}')} k(\mathbf{y}, \mathbf{y}') - 2\mathbb{E}_{(\mathbf{x}, \mathbf{y})} k(\mathbf{x}, \mathbf{y}),$$



- ▶ Needs less sample size for evaluation.
- ▶ High dependence on the pretrained classification model.
- ▶ Works with any distribution!

Outline

1. Evaluation of likelihood-free models

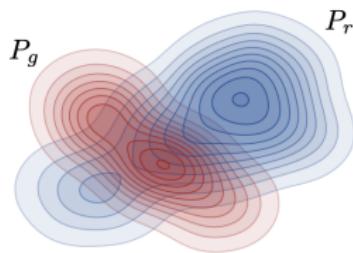
Maximum Mean Discrepancy (MMD)

Precision-Recall

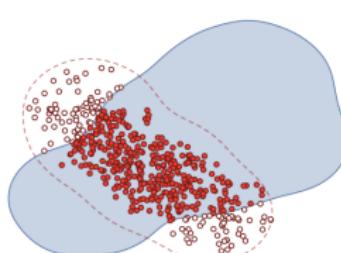
Precision-Recall

What do we want from samples

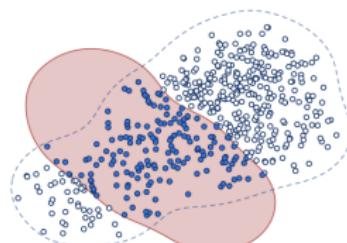
- ▶ **Sharpness:** generated samples should be of high quality.
- ▶ **Diversity:** their variation should match that observed in the training set.



(a) Example distributions



(b) Precision



(c) Recall

- ▶ **Precision** denotes the fraction of generated images that are realistic.
- ▶ **Recall** measures the fraction of the training data manifold covered by the generator.

Precision-Recall

- ▶ $\mathcal{S}_\pi = \{\mathbf{x}_i\}_{i=1}^n \sim \pi(\mathbf{x})$ – real samples;
- ▶ $\mathcal{S}_p = \{\mathbf{x}_i\}_{i=1}^n \sim p(\mathbf{x}|\theta)$ – generated samples.

Embed samples using pretrained classifier network (as previously):

$$\mathcal{G}_\pi = \{\mathbf{g}_i\}_{i=1}^n, \quad \mathcal{G}_p = \{\mathbf{g}_i\}_{i=1}^n.$$

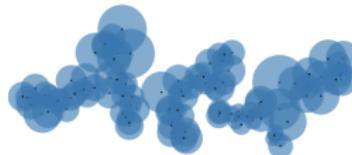
Define binary function:

$$f(\mathbf{g}, \mathcal{G}) = \begin{cases} 1, & \text{if exists } \mathbf{g}' \in \mathcal{G} : \|\mathbf{g} - \mathbf{g}'\|_2 \leq \|\mathbf{g}' - \text{NN}_k(\mathbf{g}', \mathcal{G})\|_2; \\ 0, & \text{otherwise.} \end{cases}$$

$$\text{Precision}(\mathcal{G}_\pi, \mathcal{G}_p) = \frac{1}{n} \sum_{\mathbf{g} \in \mathcal{G}_p} f(\mathbf{g}, \mathcal{G}_\pi); \quad \text{Recall}(\mathcal{G}_\pi, \mathcal{G}_p) = \frac{1}{n} \sum_{\mathbf{g} \in \mathcal{G}_\pi} f(\mathbf{g}, \mathcal{G}_p).$$

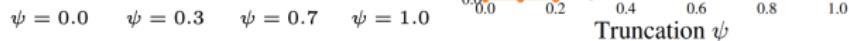
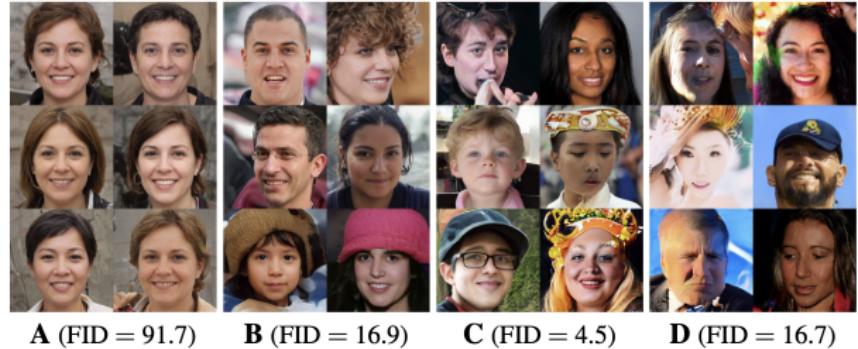


(a) True manifold



(b) Approx. manifold

Precision-Recall



Truncation trick

BigGAN: truncated normal sampling

$$p(\mathbf{z}|\psi) = \mathcal{N}(\mathbf{z}|0, \mathbf{I}) / \int_{-\infty}^{\psi} \mathcal{N}(\mathbf{z}'|0, \mathbf{I}) d\mathbf{z}'$$

Components of $\mathbf{z} \sim \mathcal{N}(0, \mathbf{I})$ which fall outside a predefined range are resampled.

StyleGAN

$$\mathbf{z}' = \hat{\mathbf{z}} + \psi \cdot (\mathbf{z} - \hat{\mathbf{z}}), \quad \hat{\mathbf{z}} = \mathbb{E}_{\mathbf{z}} \mathbf{z}$$

- ▶ Constant ψ is a tradeoff between diversity and fidelity.
- ▶ $\psi = 0.7$ is used for most of the results.

Brock A., Donahue J., Simonyan K. Large Scale GAN Training for High Fidelity Natural Image Synthesis, 2018

Karras T., Laine S., Aila T. A Style-Based Generator Architecture for Generative Adversarial Networks, 2018

Summary

- ▶ Maximum Mean Discrepancy tries to fix some of the FID drawbacks.
- ▶ Precision-recall allow to select model that compromises the sample quality and the sample diversity.
- ▶ Truncation tricks help to select model with compromised samples: diverse and sharp.