# Семинар 4

2024 курс: Генеративные модели

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$$\underline{P(X|\theta)} = \sum_{k=1}^{K} P(X|z,\theta)\cdot P(z) = \sum_{k=1}^{K} \pi_k \underbrace{N(X|\mu_k,\Sigma_k)}_{N_k(X)}$$

$$P(\underset{\text{dim}}{X}, \underset{1..K}{z}|\theta) \quad \times \quad \overbrace{\qquad\qquad}^{K}$$

$$\underline{P(X,z)_{nk}} = P(X_n, z=k) = P(X_n|z=k)\cdot P(z=k) =$$

$$= N_k(X)\cdot \pi_k$$

$$\underline{P(z|X,\theta)} = \frac{P(X|z)\cdot P(z)}{P(X)} = \frac{\pi_k N_k(X)}{\sum_{k=1}^{K} \pi_k N_k(X)}$$

$$\theta = ? \quad [\mu, \Sigma, \pi]$$

E-шаг $q = p(z=k \mid x_n, \theta) = \frac{\pi_k \mathcal{N}_k(x_n)}{\sum \pi_k \mathcal{N}_k(x_n)} = q_{nk}$

$z \sim q = (q_1 \dots q_k)$

M-шаг $\arg\max_{\theta} \mathbb{E}_q \log p(x_n, z \mid \theta) =$

$= \sum_{k=1}^{k} p_{q^*}(z=k) \cdot \log p(x_n, z=k \mid \theta) =$

$= \sum_{k=1}^{k} q_{nk} \log \mathcal{N}_k(x_n) \cdot \pi_k$

$A$

$\frac{\partial A}{\partial \pi_k} = 0 \; ; \; \frac{\partial A}{\partial \mu_k} ; \frac{\partial A}{\partial \Sigma_k}$