Deep Generative Models

Lecture 14

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Outline

1. Conditional flow matching
    - Conical gaussian paths (continued)
    - Link with score-based models
    - Linear interpolation
    - Rectified flows
2. Latent space models
    - Score-based models
    - Autoregressive models
3. The worst course overview

Recap of previous lecture

Discrete-in-time objective

$$\mathbb{E}_{\pi(\mathbf{x}_0)}\mathbb{E}_{t\sim U\{1,T\}}\mathbb{E}_{q(\mathbf{x}_t|\mathbf{x}_0)}\left\|\mathbf{s}_{\boldsymbol{\theta},t}(\mathbf{x}_t) - \nabla_{\mathbf{x}_t}\log q(\mathbf{x}_t|\mathbf{x}_0)\right\|_2^2$$

Continuous-in-time objective

$$\mathbb{E}_{\pi(\mathbf{x}(0))}\mathbb{E}_{t\sim U[0,1]}\mathbb{E}_{q(\mathbf{x}(t)|\mathbf{x}(0))}\left\|\mathbf{s}_{\boldsymbol{\theta}}(\mathbf{x}(t), t) - \nabla_{\mathbf{x}(t)}\log q(\mathbf{x}(t)|\mathbf{x}(0))\right\|_2^2$$

NCSN

$$q(\mathbf{x}(t)|\mathbf{x}(0)) = \mathcal{N}\left(\mathbf{x}(0), \left[\sigma^2(t) - \sigma^2(0)\right]\cdot\mathbf{I}\right)$$

DDPM

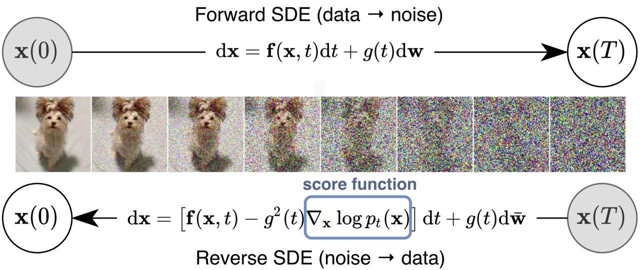
$$q(\mathbf{x}(t)|\mathbf{x}(0)) = \mathcal{N}\left(\mathbf{x}(0)e^{-\frac{1}{2}\int_0^t \beta(s)ds}, \left(1 - e^{-\int_0^t \beta(s)ds}\right)\cdot\mathbf{I}\right)$$

Song Y., et al. Score-Based Generative Modeling through Stochastic Differential Equations, 2020

Recap of previous lecture

Sampling

Solve reverse SDE using numerical solvers (SDESolve).



- Discretization of the reverse SDE gives us the ancestral sampling.
- If we use probability flow instead of SDE than the reverse ODE gives us the DDIM sampling.

Song Y., et al. Score-Based Generative Modeling through Stochastic Differential Equations, 2020

Recap of previous lecture

Let consider ODE dynamic $\mathbf{x}(t)$ in time interval $t \in [0, 1]$ with $\mathbf{x}_0 \sim p_0(\mathbf{x}) = p(\mathbf{x})$, $\mathbf{x}_1 \sim p_1(\mathbf{x}) = \pi(\mathbf{x})$.

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}, t), \quad \text{with initial condition } \mathbf{x}(0) = \mathbf{x}_0.$$

KFP theorem (continuity equation)

$$\frac{\partial p_t(\mathbf{x})}{\partial t} = -\text{div}\left(\mathbf{f}(\mathbf{x}, t) p_t(\mathbf{x})\right) \Leftrightarrow \frac{d \log p_t(\mathbf{x}(t))}{dt} = -\text{tr}\left(\frac{\partial \mathbf{f}(\mathbf{x}(t), t)}{\partial \mathbf{x}(t)}\right)$$

Solving the continuity equation using the adjoint method is complicated and unstable process.

Flow Matching

$$\mathbb{E}_{t \sim U[0,1]} \mathbb{E}_{\mathbf{x} \sim p_t(\mathbf{x})} \left\| \mathbf{f}(\mathbf{x}, t) - \mathbf{f}_{\boldsymbol{\theta}}(\mathbf{x}, t) \right\|^2 \to \min_{\boldsymbol{\theta}}$$

Lipman Y., et al. Flow Matching for Generative Modeling, 2022

Recap of previous lecture

Let introduce the latent variable $\mathbf{z}$:

$$p_t(\mathbf{x}) = \int p_t(\mathbf{x}|\mathbf{z})p(\mathbf{z})d\mathbf{z}$$

$$\frac{\partial p_t(\mathbf{x}|\mathbf{z})}{\partial t} = -\text{div}\left(\mathbf{f}(\mathbf{x}, \mathbf{z}, t)p_t(\mathbf{x}|\mathbf{z})\right).$$

- $p_t(\mathbf{x}|\mathbf{z})$ is a **conditional probability path**;
- $\mathbf{f}(\mathbf{x}, \mathbf{z}, t)$ is a **conditional vector field**.

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}, t) \quad \Rightarrow \quad \frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}, \mathbf{z}, t)$$

Theorem

The following vector field generates the probability path $p_t(\mathbf{x})$.

$$\mathbf{f}(\mathbf{x}, t) = \mathbb{E}_{p_t(\mathbf{z}|\mathbf{x})}\mathbf{f}(\mathbf{x}, \mathbf{z}, t) = \int \mathbf{f}(\mathbf{x}, \mathbf{z}, t)\frac{p_t(\mathbf{x}|\mathbf{z})p(\mathbf{z})}{p_t(\mathbf{x})}d\mathbf{z}$$

Tong A., et al. Improving and Generalizing Flow-Based Generative Models with Minibatch Optimal Transport, 2023

Recap of previous lecture

Flow Matching (FM)

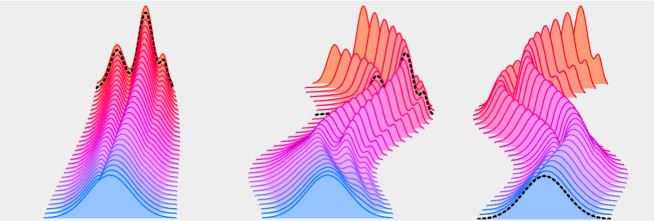
$$\mathbb{E}_{t \sim U[0,1]} \mathbb{E}_{\mathbf{x} \sim p_t(\mathbf{x})} \left\| \mathbf{f}(\mathbf{x}, t) - \mathbf{f}_{\boldsymbol{\theta}}(\mathbf{x}, t) \right\|^2 \to \min_{\boldsymbol{\theta}}$$

Conditional Flow Matching (CFM)

$$\mathbb{E}_{t \sim U[0,1]} \mathbb{E}_{\mathbf{z} \sim p(\mathbf{z})} \mathbb{E}_{\mathbf{x} \sim p_t(\mathbf{x}|\mathbf{z})} \left\| \mathbf{f}(\mathbf{x}, \mathbf{z}, t) - \mathbf{f}_{\boldsymbol{\theta}}(\mathbf{x}, t) \right\|^2 \to \min_{\boldsymbol{\theta}}$$

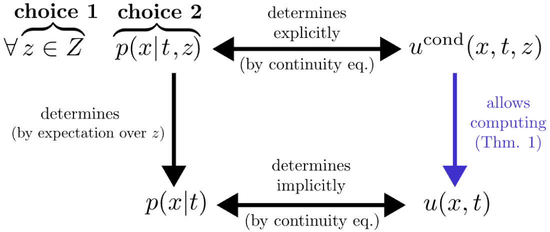
Theorem

If $\text{supp}(p_t(\mathbf{x})) = \mathbb{R}^m$, then the optimal value of FM objective is equal to the optimal value of CFM objective.



Tong A., et al. Improving and Generalizing Flow-Based Generative Models with Minibatch Optimal Transport, 2023

Recap of previous lecture



Constraints

$$p(\mathbf{x}) = \mathcal{N}(0, \mathbf{I}) = \mathbb{E}_{p(\mathbf{z})} p_0(\mathbf{x}|\mathbf{z}); \quad \pi(\mathbf{x}) = \mathbb{E}_{p(\mathbf{z})} p_1(\mathbf{x}|\mathbf{z}).$$

- How to choose the conditioning latent variable $\mathbf{z}$?
- How to define $p_t(\mathbf{x}|\mathbf{z})$ which follows the constraints?

Gaussian conditional probability path

$$p_t(\mathbf{x}|\mathbf{z}) = \mathcal{N}\left(\boldsymbol{\mu}_t(\mathbf{z}), \boldsymbol{\sigma}_t^2(\mathbf{z})\right)$$

$$\mathbf{x}_t = \boldsymbol{\mu}_t(\mathbf{z}) + \boldsymbol{\sigma}_t(\mathbf{z}) \odot \mathbf{x}_0, \quad \mathbf{x}_0 \sim p_0(\mathbf{x}) = \mathcal{N}(0, \mathbf{I})$$

image credit: A Visual Dive into Conditional Flow Matching

Recap of previous lecture

Gaussian conditional probability path

$$p_t(\mathbf{x}|\mathbf{z}) = \mathcal{N}\left(\boldsymbol{\mu}_t(\mathbf{z}), \boldsymbol{\sigma}_t^2(\mathbf{z})\right); \quad \mathbf{x}_t = \boldsymbol{\mu}_t(\mathbf{z}) + \boldsymbol{\sigma}_t(\mathbf{z}) \odot \mathbf{x}_0$$

$$\mathbf{f}(\mathbf{x}, \mathbf{z}, t) = \boldsymbol{\mu}'_t(\mathbf{z}) + \frac{\boldsymbol{\sigma}'_t(\mathbf{z})}{\boldsymbol{\sigma}_t(\mathbf{z})} \odot (\mathbf{x} - \boldsymbol{\mu}_t(\mathbf{z}))$$

Conditioning latent variable

Let choose $\mathbf{z} = \mathbf{x}_1$. Then $p(\mathbf{z}) = p_1(\mathbf{x}_1)$.

$$p_t(\mathbf{x}) = \int p_t(\mathbf{x}|\mathbf{x}_1) p_1(\mathbf{x}_1) d\mathbf{x}_1$$

We need to ensure boundary constraints:

$$\begin{cases} p(\mathbf{x}) = \mathbb{E}_{p(\mathbf{z})} p_0(\mathbf{x}|\mathbf{z}); (= \mathcal{N}(0, \mathbf{I})) \\ \pi(\mathbf{x}) = \mathbb{E}_{p(\mathbf{z})} p_1(\mathbf{x}|\mathbf{z}). \end{cases} \Rightarrow \quad \begin{cases} p_0(\mathbf{x}|\mathbf{x}_1) = \mathcal{N}(0, \mathbf{I}); \\ p_1(\mathbf{x}|\mathbf{x}_1) = \delta(\mathbf{x} - \mathbf{x}_1). \end{cases}$$

Lipman Y., et al. Flow Matching for Generative Modeling, 2022

Recap of previous lecture

$$p_0(\mathbf{x}|\mathbf{x}_1) = \mathcal{N}(0, \mathbf{I}); \quad p_1(\mathbf{x}|\mathbf{x}_1) = \delta(\mathbf{x} - \mathbf{x}_1).$$

Gaussian conditional probability path

$$p_t(\mathbf{x}|\mathbf{x}_1) = \mathcal{N}\left(\boldsymbol{\mu}_t(\mathbf{x}_1), \boldsymbol{\sigma}_t^2(\mathbf{x}_1)\right); \quad \mathbf{x}_t = \boldsymbol{\mu}_t(\mathbf{x}_1) + \boldsymbol{\sigma}_t^2(\mathbf{x}_1) \odot \mathbf{x}_0.$$

Let consider straight conditional paths

$$\begin{cases} \boldsymbol{\mu}_t(\mathbf{x}_1) = t\mathbf{x}_1; \\ \boldsymbol{\sigma}_t(\mathbf{x}_1) = (1-t). \end{cases} \quad \Rightarrow \quad \begin{cases} p_t(\mathbf{x}|\mathbf{x}_1) = \mathcal{N}\left(t\mathbf{x}_1, (1-t)^2\mathbf{I}\right); \\ \mathbf{x}_t = t\mathbf{x}_1 + (1-t)\mathbf{x}_0. \end{cases}$$

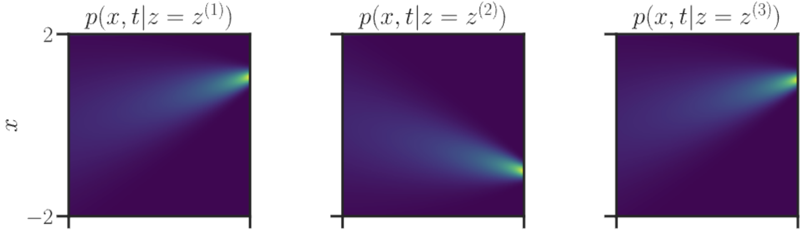


image credit: A Visual Dive into Conditional Flow Matching

Outline

Outline

Conical gaussian paths

$$p_t(\mathbf{x}|\mathbf{x}_1) = \mathcal{N}\left(t\mathbf{x}_1, (1-t)^2\mathbf{I}\right); \quad \mathbf{x}_t = t\mathbf{x}_1 + (1-t)\mathbf{x}_0.$$

Conditional vector field

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}, \mathbf{x}_1, t) = \boldsymbol{\mu}'_t(\mathbf{x}_1) + \frac{\boldsymbol{\sigma}'_t(\mathbf{x}_1)}{\boldsymbol{\sigma}_t(\mathbf{x}_1)} \odot (\mathbf{x} - \boldsymbol{\mu}_t(\mathbf{x}_1))$$

$$\mathbf{f}(\mathbf{x}, \mathbf{x}_1, t) = \mathbf{x}_1 - \frac{1}{1-t} \cdot (\mathbf{x} - t\mathbf{x}_1) = \frac{\mathbf{x}_1 - \mathbf{x}}{1-t} =$$

$$= \frac{\mathbf{x}_1 - t\mathbf{x}_1 + (1-t)\mathbf{x}_0}{1-t} = \mathbf{x}_1 - \mathbf{x}_0$$

Conditional Flow Matching

$$\mathbb{E}_{t\sim U[0,1]}\mathbb{E}_{\mathbf{z}\sim p(\mathbf{z})}\mathbb{E}_{\mathbf{x}\sim p_t(\mathbf{x}|\mathbf{z})} \left\|\mathbf{f}(\mathbf{x}, \mathbf{z}, t) - \mathbf{f}_{\boldsymbol{\theta}}(\mathbf{x}, t)\right\|^2 =$$

$$\mathbb{E}_{t\sim U[0,1]}\mathbb{E}_{\mathbf{x}_1\sim \pi(\mathbf{x})}\mathbb{E}_{\mathbf{x}\sim p_t(\mathbf{x}|\mathbf{x}_1)} \left\|\left(\frac{\mathbf{x}_1 - \mathbf{x}}{1-t}\right) - \mathbf{f}_{\boldsymbol{\theta}}(\mathbf{x}, t)\right\|^2 =$$

$$\mathbb{E}_{t\sim U[0,1]}\mathbb{E}_{\mathbf{x}_1\sim \pi(\mathbf{x})}\mathbb{E}_{\mathbf{x}_0\sim \mathcal{N}(0,\mathbf{I})} \left\|(\mathbf{x}_1 - \mathbf{x}_0) - \mathbf{f}_{\boldsymbol{\theta}}\left(t\mathbf{x}_1 + (1-t)\mathbf{x}_0, t\right)\right\|^2$$

Lipman Y., et al. Flow Matching for Generative Modeling, 2022

Conditional Flow Matching

$$\mathbb{E}_{t\sim U[0,1]}\mathbb{E}_{\mathbf{x}_1\sim\pi(\mathbf{x})}\mathbb{E}_{\mathbf{x}_0\sim\mathcal{N}(0,\mathbf{I})}\left\|(\mathbf{x}_1-\mathbf{x}_0)-\mathbf{f}_{\boldsymbol{\theta}}(\mathbf{x},t)\right\|^2 \to \min_{\boldsymbol{\theta}}$$

We fit straight lines between noise distribution $p(\mathbf{x})$ and the data distribution $\pi(\mathbf{x})$.

Training

1. Get the sample $\mathbf{x}_1 \sim \pi(\mathbf{x})$.
2. Sample timestamp $t \sim U\{1, T\}$ and $\mathbf{x}_0 \sim \mathcal{N}(0, \mathbf{I})$.
3. Get noisy image $\mathbf{x}_t = \sqrt{\bar{\alpha}_t} \cdot \mathbf{x}_0 + \sqrt{1-\bar{\alpha}_t} \cdot \boldsymbol{\epsilon}$.
4. Compute loss $\mathcal{L} = \left\|(\mathbf{x}_1-\mathbf{x}_0)-\mathbf{f}_{\boldsymbol{\theta}}(\mathbf{x},t)\right\|^2$.

Sampling

1. Sample $\mathbf{x}_0 \sim \mathcal{N}(0, \mathbf{I})$.
2. Solve the ODE to get $\mathbf{x}_1$:

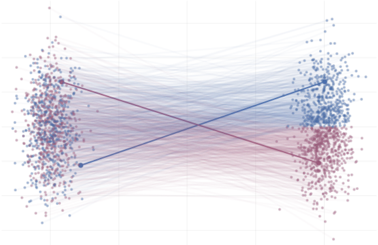
$$\mathbf{x}_1 = \text{ODESolve}_f(\mathbf{x}_0, \boldsymbol{\theta}, t_0 = 0, t_1 = 1).$$

Lipman Y., et al. Flow Matching for Generative Modeling, 2022

Flow Matching

$$\mathbf{x}_t = t\mathbf{x}_1 + (1-t)\mathbf{x}_0$$

- The conditional probability path $p_t(\mathbf{x}|\mathbf{z})$ is an **optimal transport path** from $p_0(\mathbf{x}|\mathbf{z})$ to $p_1(\mathbf{x}|\mathbf{z})$ (in terms of the conditional trajectories straightness).
- The marginal path $p_t(\mathbf{x})$ is not in general an optimal transport path from the standard normal $p_0(\mathbf{x})$ to the data distribution $p_1(\mathbf{x})$.



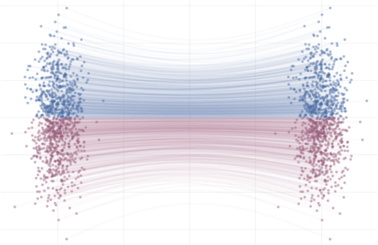


image credit: https://mlg.eng.cam.ac.uk/blog/2024/01/20/flow-matching.html

Outline

Score-based generative models through SDEs

Training

$$\mathbb{E}_{\pi(\mathbf{x}(0))}\mathbb{E}_{t\sim U[0,1]}\mathbb{E}_{q(\mathbf{x}(t)|\mathbf{x}(0))}\left\|\mathbf{s}_{\boldsymbol{\theta}}(\mathbf{x}(t),t)-\nabla_{\mathbf{x}(t)}\log q(\mathbf{x}(t)|\mathbf{x}(0))\right\|_2^2$$

Variance Exploding SDE (NCSN)

$$q(\mathbf{x}(t)|\mathbf{x}(0))=\mathcal{N}\left(\mathbf{x}(0),\left[\sigma^2(t)-\sigma^2(0)\right]\cdot\mathbf{I}\right),\quad \sigma(0)=0.$$

Variance Preserving SDE (DDPM)

$$q(\mathbf{x}(t)|\mathbf{x}(0))=\mathcal{N}\left(\mathbf{x}(0)\alpha(t),\left(1-\alpha(t)^2\right)\cdot\mathbf{I}\right);\quad \alpha(t)=e^{-\frac{1}{2}\int_0^t\beta(s)ds}$$

Flow matching uses the reverse time direction.

$$\textbf{NCSN:}\ p(\mathbf{x}|\mathbf{x}_1)=\mathcal{N}\left(\mathbf{x}_1,\sigma_{1-t}^2\cdot\mathbf{I}\right)$$

$$\textbf{DDPM:}\ p(\mathbf{x}|\mathbf{x}_1)=\mathcal{N}\left(\alpha_{1-t}\mathbf{x}_1,\left(1-\alpha_{1-t}^2\right)\cdot\mathbf{I}\right)$$

Lipman Y., et al. Flow Matching for Generative Modeling, 2022

Flow matching vs score-based SDE models

Flow matching probability path

$$p_t(\mathbf{x}|\mathbf{x}_1) = \mathcal{N}\left(t\mathbf{x}_1, (1-t)^2\mathbf{I}\right); \quad \mathbf{f}(\mathbf{x}, \mathbf{x}_1, t) = \frac{\mathbf{x}_1 - \mathbf{x}}{1-t}$$

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}, \mathbf{x}_1, t) = \boldsymbol{\mu}'_t(\mathbf{x}_1) + \frac{\boldsymbol{\sigma}'_t(\mathbf{x}_1)}{\boldsymbol{\sigma}_t(\mathbf{x}_1)} \odot (\mathbf{x} - \boldsymbol{\mu}_t(\mathbf{x}_1))$$

Variance Exploding SDE probability path

$$p_t(\mathbf{x}|\mathbf{x}_1) = \mathcal{N}\left(\mathbf{x}_1, \sigma^2_{1-t}\mathbf{I}\right) \quad \Rightarrow \quad \mathbf{f}(\mathbf{x}, \mathbf{x}_1, t) = -\frac{\sigma'_{1-t}}{\sigma_{1-t}} \cdot (\mathbf{x} - \mathbf{x}_1)$$

Variance Preserving SDE probability path

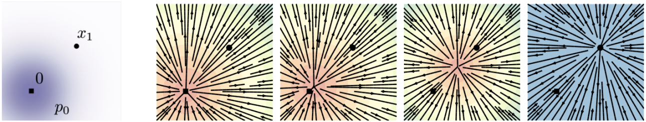
$$p_t(\mathbf{x}|\mathbf{x}_1) = \mathcal{N}\left(\alpha_{1-t}\mathbf{x}_1, (1-\alpha^2_{1-t})\mathbf{I}\right) \Rightarrow \mathbf{f}(\mathbf{x}, \mathbf{x}_1, t) = \frac{\alpha'_{1-t}}{1-\alpha^2_{1-t}} \cdot (\alpha_{1-t}\mathbf{x} - \mathbf{x}_1)$$

Lipman Y., et al. Flow Matching for Generative Modeling, 2022

Flow matching vs score-based SDE models

SDE vector field

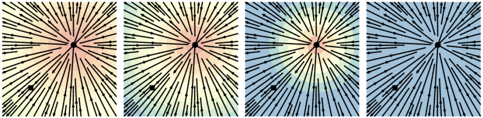
$$\mathbf{f}(\mathbf{x}, \mathbf{x}_1, t) = \frac{\alpha'_{1-t}}{1 - \alpha^2_{1-t}} \cdot (\alpha_{1-t}\mathbf{x} - \mathbf{x}_1)$$



$t = 0.0$ $t = 1/3$ $t = 2/3$ $t = 1.0$

FM vector field

$$\mathbf{f}(\mathbf{x}, \mathbf{x}_1, t) = -\frac{\sigma'_{1-t}}{\sigma_{1-t}} \cdot (\mathbf{x} - \mathbf{x}_1)$$

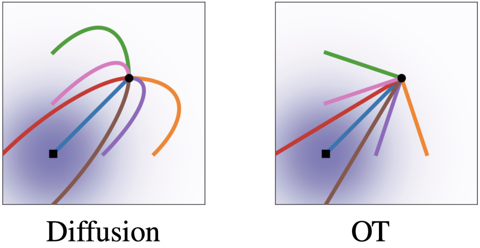


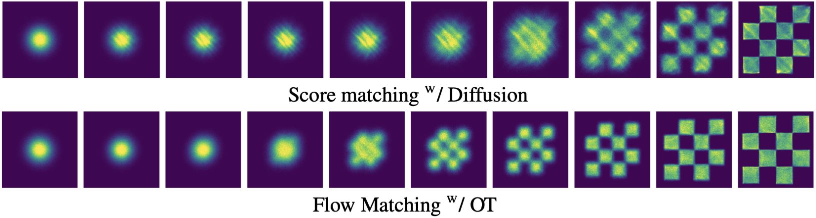
$t = 0.0$ $t = 1/3$ $t = 2/3$ $t = 1.0$

Lipman Y., et al. Flow Matching for Generative Modeling, 2022

Flow matching vs score-based SDE models

Trajectories





Lipman Y., et al. *Flow Matching for Generative Modeling*, 2022

Outline

Pair conditioning

Conditional Flow Matching

$$\mathbb{E}_{t \sim U[0,1]} \mathbb{E}_{\mathbf{z} \sim p(\mathbf{z})} \mathbb{E}_{\mathbf{x} \sim p_t(\mathbf{x}|\mathbf{z})} \left\| \mathbf{f}(\mathbf{x}, \mathbf{z}, t) - \mathbf{f}_{\boldsymbol{\theta}}(\mathbf{x}, t) \right\|^2 \rightarrow \min_{\boldsymbol{\theta}}$$

Conditioning latent variable

Let choose $\mathbf{z} = (\mathbf{x}_0, \mathbf{x}_1)$. Then $p(\mathbf{z}) = p_0(\mathbf{x}_0) p_1(\mathbf{x}_1)$.

$$p_t(\mathbf{x}) = \int p_t(\mathbf{x}|\mathbf{x}_0, \mathbf{x}_1) p_0(\mathbf{x}_0) p_1(\mathbf{x}_1) d\mathbf{x}_0 d\mathbf{x}_1$$

We need to ensure boundary constraints:

$$\begin{cases} p(\mathbf{x}) = \mathbb{E}_{p(\mathbf{z})} p_0(\mathbf{x}|\mathbf{z}); (= \mathcal{N}(0, \mathbf{I})) \\ \pi(\mathbf{x}) = \mathbb{E}_{p(\mathbf{z})} p_1(\mathbf{x}|\mathbf{z}). \end{cases} \Rightarrow \begin{cases} p_0(\mathbf{x}|\mathbf{x}_0, \mathbf{x}_1) = \delta(\mathbf{x} - \mathbf{x}_0); \\ p_1(\mathbf{x}|\mathbf{x}_0, \mathbf{x}_1) = \delta(\mathbf{x} - \mathbf{x}_1). \end{cases}$$

Lipman Y., et al. Flow Matching for Generative Modeling, 2022

Linear interpolation

$$p_0(\mathbf{x}|\mathbf{x}_0, \mathbf{x}_1) = \delta(\mathbf{x} - \mathbf{x}_0); \quad p_1(\mathbf{x}|\mathbf{x}_0, \mathbf{x}_1) = \delta(\mathbf{x} - \mathbf{x}_1).$$

Gaussian conditional probability path

$$p_t(\mathbf{x}|\mathbf{x}_0, \mathbf{x}_1) = \mathcal{N}\left(\boldsymbol{\mu}_t(\mathbf{x}_0, \mathbf{x}_1), \boldsymbol{\sigma}_t^2(\mathbf{x}_0, \mathbf{x}_1)\right); \quad \mathbf{x}_t = \boldsymbol{\mu}_t(\mathbf{x}_0, \mathbf{x}_1) + \boldsymbol{\sigma}_t^2(\mathbf{x}_0, \mathbf{x}_1) \odot \mathbf{x}_0.$$

Let consider straight conditional paths

$$\begin{cases} \boldsymbol{\mu}_t(\mathbf{x}_1) = t\mathbf{x}_1 + (1-t)\mathbf{x}_0; \\ \boldsymbol{\sigma}_t(\mathbf{x}_1) = 0. \end{cases} \Rightarrow \begin{cases} p_0(\mathbf{x}|\mathbf{x}_0, \mathbf{x}_1) = \delta(\mathbf{x} - \mathbf{x}_0); \\ p_1(\mathbf{x}|\mathbf{x}_0, \mathbf{x}_1) = \delta(\mathbf{x} - \mathbf{x}_1). \end{cases}$$

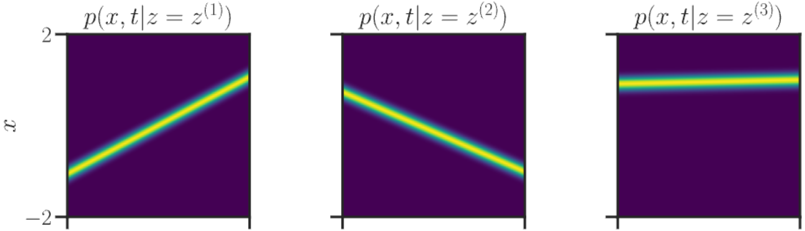


image credit: A Visual Dive into Conditional Flow Matching

Flow Matching

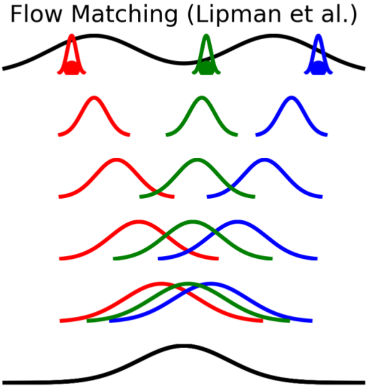
Let consider straight conditional paths

$$\begin{cases} \boldsymbol{\mu}_t(\mathbf{x}_0, \mathbf{x}_1) = t\mathbf{x}_1 + (1-t)\mathbf{x}_0; \\ \boldsymbol{\sigma}_t(\mathbf{x}_0, \mathbf{x}_1) = 0. \end{cases} \quad \Rightarrow \begin{cases} p_t(\mathbf{x}|\mathbf{x}_0, \mathbf{x}_1) = \mathcal{N}\left(t\mathbf{x}_1 + (1-t)\mathbf{x}_0 \right. \\ \mathbf{x}_t = t\mathbf{x}_1 + (1-t)\mathbf{x}_0. \end{cases}$$

$$\frac{d\mathbf{x}}{dt} = \mathbf{x}_1 - \mathbf{x}_0.$$

Lipman Y., et al. Flow Matching for Generative Modeling, 2022

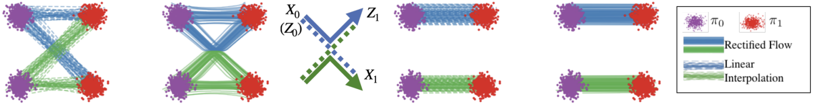
Flow Matching



Tong A., et al. *Improving and Generalizing Flow-Based Generative Models with Minibatch Optimal Transport*, 2023

Outline

Rectified flows

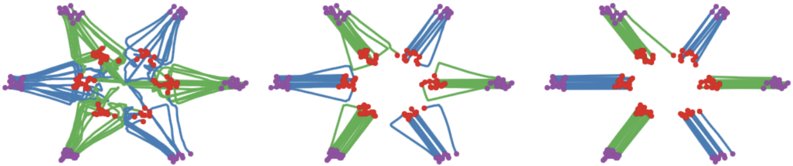


(a) Linear interpolation $X_t = tX_1 + (1-t)X_0$

(b) Rectified flow Z_t induced by (X_0, X_1)

(c) Linear interpolation $Z_t = tZ_1 + (1-t)Z_0$

(d) Rectified flow Z'_t induced by (Z_0, Z_1)



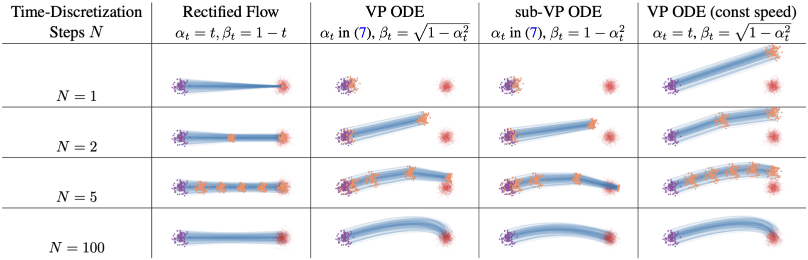
(a)The 1st rectified flow $\boldsymbol{Z}^1$ $\boldsymbol{Z}^1 = \texttt{RectFlow}((X_0, X_1))$

(b) Reflow $\boldsymbol{Z}^2$ $\boldsymbol{Z}^2 = \texttt{RectFlow}((Z_0^1, Z_1^1))$

(c) Reflow $\boldsymbol{Z}^3$ $\boldsymbol{Z}^3 = \texttt{RectFlow}((Z_0^2, Z_1^2))$

Liu X., et al. Flow Straight and Fast: Learning to Generate and Transfer Data with Rectified Flow, 2022

Rectified flows



Liu X., et al. Flow Straight and Fast: Learning to Generate and Transfer Data with Rectified Flow, 2022

Rectified flows: Stable Diffusion 3

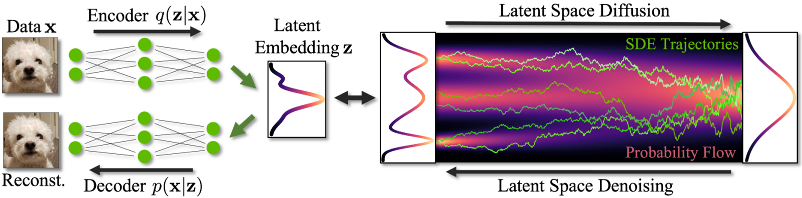
Esser P., et al. Scaling Rectified Flow Transformers for High-Resolution Image Synthesis, 2024

Outline

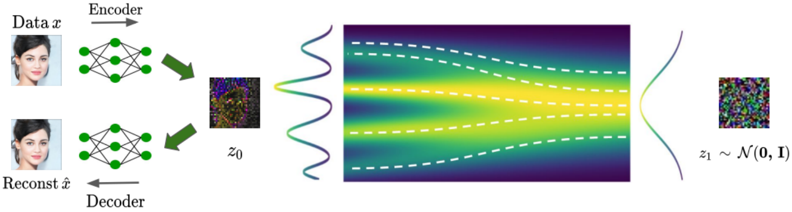
Outline

Latent space models

Score-based models (diffusion)



Flow matching



Dao Q. et al. Flow Matching in Latent Space, 2023
NeurIPS 2023 Tutorial: Latent Diffusion Models: Is the Generative AI Revolution Happening in Latent Space?

Outline

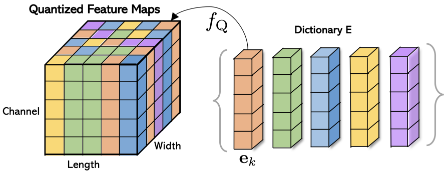
Vector Quantized VAE (VQ-VAE)

Vector quantization

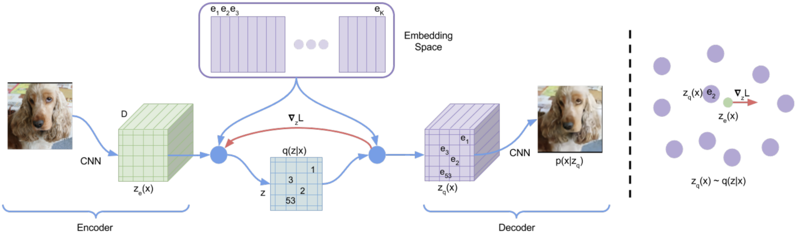
Define the dictionary space $\{\mathbf{e}_k\}_{k=1}^K$, where $\mathbf{e}_k \in \mathbb{R}^C$, K is the size of the dictionary.

$$\mathbf{z}_q = \mathbf{q}(\mathbf{z}) = \mathbf{e}_{k^*}$$

Here $k^* = \arg\min_k \|\mathbf{z} - \mathbf{e}_k\|$.



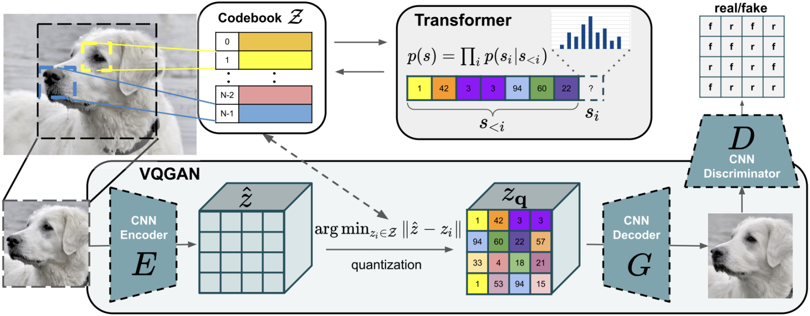
$$\mathcal{L}_{\phi,\theta}(\mathbf{x}) = \log p(\mathbf{x}|\mathbf{z}_q, \boldsymbol{\theta}) - \log K$$



Zhao Y. et al. Feature Quantization Improves GAN Training, 2020
Oord A., Vinyals O., Kavukcuoglu K. Neural Discrete Representation Learning, 2017

Vector Quantized GAN

- Use VQVAE model and objective.
- Add adversarial loss between generated and real images to improve the visual quality of the reconstructions.



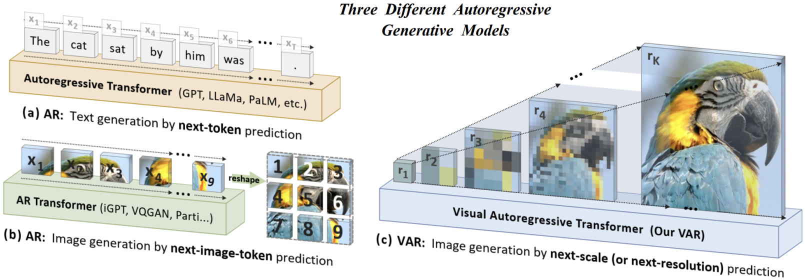
Esser P. et al. Taming Transformers for High-Resolution Image Synthesis, 2020

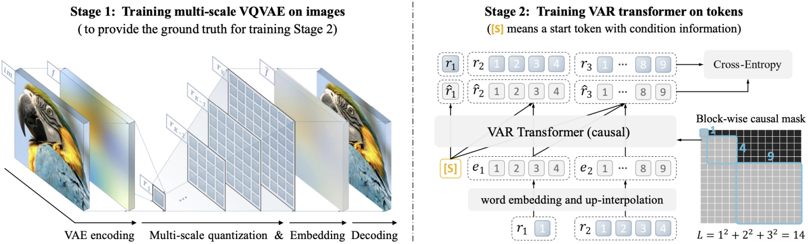
LlamaGen: pure autoregression

- Use VQGAN encoder to map images to the discrete latent space of the codebook vectors.
- Learn the pure autoregression model (Llama-based) in the latent space.
- Use VQGAN decoder to map the discrete space tokens to the image space.

Sun P. et al. Autoregressive Model Beats Diffusion: Llama for Scalable Image Generation, 2024

Visual Autoregressive Modeling (VAR)





Tean K. et al. *Visual Autoregressive Modeling: Scalable Image Generation via Next-Scale Prediction*, 2024

Outline

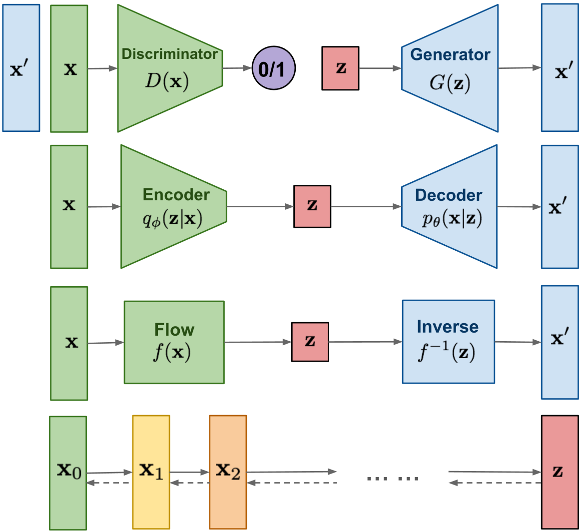
The worst course overview :)

GAN: Adversarial training

VAE: maximize variational lower bound

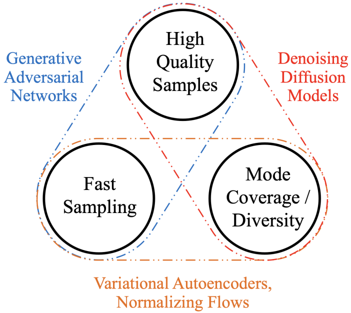
Flow-based models: Invertible transform of distributions

Diffusion models: Gradually add Gaussian noise and then reverse



Weng L. What are Diffusion Models?, blog post, 2021

The worst course overview :)



Model	Efficient	Sample quality	Coverage	Well-behaved latent space	Disentangled latent space	Efficient likelihood
GANs	✓	✓	✗	✓	?	n/a
VAEs	✓	✗	?	✓	?	✗
Flows	✓	✗	?	✓	?	✓
Diffusion	✗	✓	?	✗	✗	✗

Xiao Z., Kreis K., Vahdat A. Tackling the generative learning trilemma with denoising diffusion GANs, 2021
Simon J.D. Prince. Understanding Deep Learning, 2023

Summary

- Conical gaussian paths is the example of the effective FM technique.
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