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We propose a method to flip the spin completely by an adiabatic transport of quantum dots. We show that it is possible to flip the spin by inducing a geometric phase on the spin state of a quantum dot. We estimate the geometric spin flip time (approximately 2 ps) which turned out to be much shorter than the experimentally reported decoherence time (approximately 100 ns) that would provide an alternative means of fliping the spin before reaching decoherence. It is important that both the Rashba coupling and the Dresselhaus coupling are present for inducing a phase necessary for spin flip. If one of them is absent, the induced phase is trivial and irrelevant for spin-flip. © *2014 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4871004]

Manipulation of single electron spins in low dimensional semiconductor nanostructures such as quantum dots (QDs), wells, and nanowires is essential for spintronic and spin-based quantum information processing.[1–4] The electron spin in these nanostructures can be manipulated by several different techniques: for example, electron spin resonance induced by oscillating magnetic fields at the Zeeman frequency, and electric field control of spin through spin-orbit coupling.[5–7] More robust techniques have been suggested for manipulation of the electron spin in a QD by letting the dot to move adiabatically along a closed loop.[8–11] Recently, the geometric phase has been measured experimentally for qubits driven by a microwave pulse in the presence of tilted magnetic fields.[2,12]

In this paper, we let a QD move adiabatically in the two-dimensional (2D) plane of the electron gas under application of a gate controlled periodic lateral electric field. We report that in the presence of both the Rashba and the Dresselhaus spin-orbit couplings, the geometric phase changes its sign over a short period of time on the spin state. This tells us that the complete spin flip is possible by controlling the applied electric field. We point out that the geometric spin flip can be achieved much faster than the decoherence.

We write the total Hamiltonian of a QD formed in the plane of a two-dimensional electron gas with gate potential confined along z-direction at the heterojunction in the presence of externally applied magnetic field along z-direction as: $H = H_0 + H_R + H_D$.[3,13,14] Here

$$H_0 = \frac{\{\mathbf{p} + e\mathbf{A}(\mathbf{r})\}^2}{2m} + \frac{1}{2}m\omega_o^2\mathbf{r}^2 + e\mathbf{E}(t)\cdot\mathbf{r} + \frac{\Delta}{2}\sigma_z, \quad (1)$$

$$H_R = \frac{\alpha}{\hbar}\{\sigma_x(p_y + eA_y) - \sigma_y(p_x + eA_x)\}, \quad (2)$$

$$H_D = \frac{\beta}{\hbar}\{-\sigma_x(p_x + eA_x) + \sigma_y(p_y + eA_y)\}. \quad (3)$$

In (1), $\mathbf{p} = -i\hbar(\partial_x, \partial_y, 0)$ is the canonical momentum, $\mathbf{r} = (x, y, 0)$ is the position vector, e is the electronic charge, m is the effective mass of an electron in the QDs, $\Delta = g_0\mu_B B$ is the Zeeman energy, μ_B is the Bohr magneton, and g_0 is the bulk g-factor of an electron in the QD. Also, $\mathbf{E}(t) = (E_x(t), E_y(t), 0)$ with $E_x(t) = E_0\cos\omega t$ and $E_y(t) = E_0\sin\omega t$ is the electric potential energy due to the applied periodic lateral electric field. Note that $e\mathbf{E}(t)\cdot\mathbf{r}$ is the coupling energy (potential energy) having the dimension of energy. By varying $\mathbf{E}(t)$ very slowly, we treat its two components as adiabatic parameters. The time varying electric field $\mathbf{E}(t)$ also induces a magnetic field which is in practice several order magnitude smaller than the applied magnetic field along z-direction.[14] Thus, ignoring such a small contribution, we use the vector potential of the form $\mathbf{A}(r) = B/2(-y, x, 0)$. In (2), $\alpha = \gamma_R eE_z$ is the Rashba spin-orbit coupling coefficient originating from structural inversion asymmetry. In (3), $\beta = 0.78\gamma_D(2me/\hbar^2)^{2/3}E_z^{2/3}$ is the Dresselhaus spin-orbit coupling coefficient originating from bulk inversion asymmetry. For GaAs QD, we chose $\gamma_R = 4.4$ Å^2 and $\gamma_D = 26$ eV Å^3.

At a fixed time t_0, the electric field shifts the center of QDs from $\mathbf{r} = 0$ to $\mathbf{r} = \mathbf{r_0}(t_0)$, where $\mathbf{r}_0 = -e\mathbf{E}(t_0)/m\omega_0^2$. Hence, we write Hamiltonian (1) as

$$H_0 = \frac{\{\mathbf{p} + e\mathbf{A}(\mathbf{r})\}^2}{2m} + \frac{1}{2}m\omega_o^2(\mathbf{r} - \mathbf{r_0})^2 - G + \frac{\Delta}{2}\sigma_z, \quad (4)$$

where $G = r_0 eE_0/2$ is an unimportant constant. As the applied **E**–field varies, the QD will be adiabatically transported along a circle of radius $r_0 = |\mathbf{r}_0| = eE_0/m\omega_0^2$.

Now, we write relative coordinate $\mathbf{R} = \mathbf{r} - \mathbf{r_0}$ and relative momentum $\mathbf{P} = \mathbf{p} - \mathbf{p_0}$, where $\mathbf{p_0}$ is the momentum of the slowly moving dot which may be classically given by $m\dot{\mathbf{r}}_0$. We can show that the adiabatic variables $\mathbf{p}_0$ and $\mathbf{r}_0$ will be gauged away from the Hamiltonian by the transformation $\tilde{H} = UHU^{-1}$ and $\tilde{\psi} = U\psi$ with $U = \exp\{(i/\hbar)(\mathbf{p_0} + e\mathbf{A}(\mathbf{r}_0))\cdot\mathbf{R}\}$, so that

$$\tilde{H}_0 = \frac{1}{2m}\{\mathbf{P} + e\mathbf{A}(\mathbf{R})\}^2 + \frac{1}{2}m\omega_o^2R^2 - G + \frac{\Delta}{2}\sigma_z, \quad (5)$$

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$$\tilde{H}_{so} = \tilde{H}_R + \tilde{H}_D = UH_{so}(\mathbf{p},\mathbf{r})U^{-1} = H_{so}(\mathbf{P},\mathbf{R}), \quad (6)$$

where $\mathbf{A}(\mathbf{R}) = (B/2)(-Y, X, 0)$. This means the electron in the shifted dot obeys a quasi-static eigenequation, $\tilde{H}(\mathbf{P},\mathbf{R})\tilde{\psi}_n(\mathbf{R}) = \tilde{\varepsilon}_n\tilde{\psi}_n(\mathbf{R})$, where $\tilde{H} = \tilde{H}_0 + \tilde{H}_{so}$. By an adiabatic transport of the dot, the eigenfunction $\tilde{\psi}_n$ will acquire the geometric phase as well as the usual dynamical phase. Namely, $\psi_n(\mathbf{r},t) = e^{i\gamma_n(t)}e^{i\theta_n(t)}U^{-1}\tilde{\psi}_n(R)$, where γ_n is the geometric phase and θ_n is the dynamical phase.

In order to evaluate the geometric phase explicitly, we write the original Hamiltonian H in the form

$$H = \tilde{H}_0(\mathbf{P},\mathbf{R}) + H_{so}(\mathbf{P},\mathbf{R}) + H_{ad}(\mathbf{P},\mathbf{R};\mathbf{p_0},\mathbf{r_0}), \quad (7)$$

where

$$H_{ad}(\mathbf{P},\mathbf{R};\mathbf{p}_0,\mathbf{r}_0) = \frac{1}{m}\{\mathbf{P} + e\mathbf{A}(\mathbf{R})\}\cdot\{\mathbf{P} + e\mathbf{A}(\mathbf{R})\} + H_{so}(\mathbf{p}_0,\mathbf{r}_0) + G', \quad (8)$$

with another unimportant constant $G' = r_0^2\omega_+^2/4$, where $\omega_+ = \omega + \omega_c/2$ and $\omega_c = eB/m$ is the cyclotron frequency.

The quasi-static Hamiltonian $\tilde{H}_0(\mathbf{P},\mathbf{R})$ can be diagonalized on the basis of the number states $|n_+, n_-, \pm 1\rangle$

$$\tilde{H}_0 = \left(N_+ + \frac{1}{2}\right)\hbar\Omega_+ + \left(N_- + \frac{1}{2}\right)\hbar\Omega_- - G + \frac{\Delta}{2}\sigma_z, \quad (9)$$

with $N_\pm = a_\pm^\dagger a_\pm$ are the number operators with eigenvalues $n_\pm \in N_0$. The other terms in (8) can also be expressed in terms of the raising and lowering operators

$$H_{so}(\mathbf{P},\mathbf{R}) = \alpha(\xi_+\sigma_+a_+ - \xi_-\sigma_-a_-) + i\beta(\xi_+\sigma_-a_+ + \xi_-\sigma_+a_-) + H.c., \quad (10)$$

$$H_{ad}(\mathbf{P},\mathbf{R};\mathbf{p_0},\mathbf{r_0}) = \frac{\hbar}{2}(\xi_+z_+a_+ - \xi_-z_-a_-)\omega_+ + \frac{1}{\hbar}(\alpha z_- - i\beta z_+)m\omega_+\sigma_+ + H.c. \quad (11)$$

In the above, we have used the notations, $z_\pm = x_0 \pm iy_0$, $\xi_\pm = \sqrt{m\Omega/\hbar} \pm eB/\sqrt{4m\hbar\Omega}$, $\sigma_\pm = (\sigma_x \pm i\sigma_y)/2$, $\Omega_\pm = \Omega \pm \omega_c/2$, and $\Omega = \sqrt{\omega_0^2 + \omega_c^2/4}$. In (10) and (11), $H.c.$ signifies the Hermitian conjugate.

To investigate the geometric phase during the adiabatic movement of the dots in the 2D plane, we write the quasi-spin Hamiltonian as

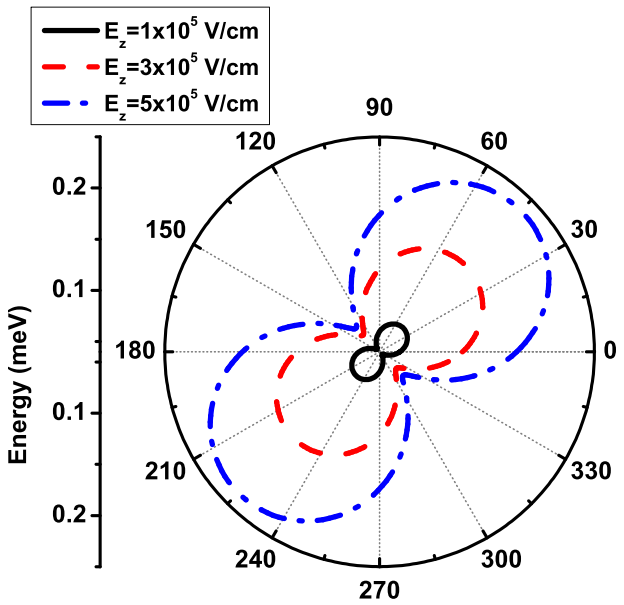

FIG. 1. Adiabatic control of eigenenergy $\Delta\lambda = \lambda_+ - \lambda_-$ (i.e., the energy difference between spin up and down states) vs rotation angle. Here, we chose $E_0 = 10^3$ V/cm, $\ell_0 = 20$ nm, $B = 1$ T.

$$h_{ad} = \kappa(Z_1 - iZ_2)s_+ + \kappa(Z_1 + iZ_2)s_- + \Delta s_z, \quad (12)$$

where $\kappa = m\omega_+/\hbar$, $Z_1 = \alpha x_0 + \beta y_0$, $Z_2 = \beta x_0 + \alpha y_0$, and $s_\pm = s_x \pm is_y$ with $s_x = \sigma_x/2$, $s_y = \sigma_y/2$ and $s_z = \sigma_z/2$. This is the only part of the complete Hamiltonian that contributes to the geometric phase. The eigenvalues of (12) are $\lambda_\pm = \pm\kappa F$, where $F = \left(Z^2 + Z_1^2 + Z_2^2\right)^{1/2}$ with $Z = \Delta/2\kappa$. In Fig. 1, we have plotted the energy difference between λ_+ and λ_- vs rotation angle. Modulation in the energy spectrum can be seen due to the fact that the energy spectrum of the dots depends on the adiabatic control parameters x_0 and y_0.

We construct a normalized orthogonal set of eigenspinors of Hamiltonian (12) as

$$\psi_+(\theta) = \frac{1}{\sqrt{2F(Z+F)}}\begin{pmatrix} Z+F \\ Z_1 + iZ_2 \end{pmatrix}, \quad (13)$$

$$\psi_-(\theta) = \frac{1}{\sqrt{2F(Z+F)}}\begin{pmatrix} Z_1 - iZ_2 \\ -Z - F \end{pmatrix}. \quad (14)$$

Let us now calculate the geometric phase

$$\gamma_+(\theta) = i\int_\theta \langle\psi_+|\partial_\theta|\psi_+\rangle d\theta = i\int_\theta \left[\sqrt{\frac{Z+F}{2F}}\partial_\theta\left\{\sqrt{\frac{Z+F}{2F}}\right\} + \frac{Z_1 - iZ_2}{\sqrt{2F(Z+F)}}\partial_\theta\left\{\frac{Z_1 - iZ_2}{\sqrt{2F(Z+F)}}\right\}\right]d\theta. \quad (15)$$

To find the exact analytical expression for the geometric phase of (15), we write Z_1 and Z_2 in a more convenient form

$$Z_1 = D\cos(\theta - \delta),\ Z_2 = D\sin(\theta + \delta), \quad (16)$$

where $D = -r_0\left(\alpha^2 + \beta^2\right)^{1/2}$ and $\tan\delta = \beta/\alpha$. By recognizing $\sin 2\delta = 2\alpha\beta/\left(\alpha^2 + \beta^2\right)$ and

$$F^2 = Z^2 + D^2\left\{1 + \frac{2\alpha\beta}{\alpha^2 + \beta^2}\sin(2\theta)\right\}, \quad (17)$$

we write the expression for the geometric phase (15) as

$$\gamma_+(\theta) = -\frac{r_0^2}{2}|\alpha^2 - \beta^2|\int_0^\theta \frac{1}{\sqrt{Z^2 + D^2(1+\gamma_{RD}\sin 2\theta)}\left\{Z + \sqrt{Z^2 + D^2(1+\gamma_{RD}\sin 2\theta)}\right\}}d\theta, \tag{18}$$

where

$$\gamma_{RD} = \frac{2\alpha\beta}{\alpha^2+\beta^2}, \quad \cos(2\delta) = \frac{\alpha^2-\beta^2}{\alpha^2+\beta^2}. \tag{19}$$

We clearly see that the Rashba-Dresselhaus spin-orbit coupling coefficients γ_{RD} couple to the adiabatic control parameters E_x and E_y. Thus, it is only possible to modulate the geometric phase on spin up and down spinors for the case of mixed Rashba-Dresselhaus spin-orbit couplings. The integral of (18) does not have a closed form. However, for the special cases such as (i) for the pure Rashba case ($\beta = 0$), (ii) for the pure Dresselhaus case ($\alpha = 0$), and (iii) for $Z = 0$, the exact geometric phase as a function of rotation angle (θ) is given by

(i) for the pure Rashba case:

$$\gamma_+(\theta) = -\frac{r_0^2\alpha^2\theta}{2\sqrt{Z^2+D^2}\left\{Z+\sqrt{Z^2+D^2}\right\}}, \tag{20}$$

(ii) for the pure Dresselhaus case:

$$\gamma_+(\theta) = -\frac{r_0^2\beta^2\theta}{2\sqrt{Z^2+D^2}\left\{Z+\sqrt{Z^2+D^2}\right\}}, \tag{21}$$

(iii) for $Z = 0$, i.e., *without Zeeman energy:*

$$\gamma_+(\theta) = \frac{1}{2}\left[\tan^{-1}\left\{\frac{\alpha^2+\beta^2}{|\alpha^2-\beta^2|}\tan\theta + \frac{2\alpha\beta}{|\alpha^2-\beta^2|}\right\}\right]_0^\theta. \tag{22}$$

It is clear that for both the pure Rashba case (20) and the pure Dresselhaus case (21), the geometric phase is linearly proportional to the adiabatic parameter θ resulting only in Berry phases which induce no spin flip. In calculating the Berry phase for $Z = 0$ case (22), care must be taken (see Ref. 15). It is more appropriate to divide the square-bracket into three portions as

$$[\cdot]_0^\pi = \lim_{\epsilon\to 0}\left([\cdot]_0^{\pi/2-\epsilon} + [\cdot]_{\pi/2+\epsilon}^{3\pi/2-\epsilon} + [\cdot]_{3\pi/2+\epsilon}^{2\pi}\right). \tag{23}$$

Notice that $\tan(\pi/2-\epsilon) > 0$ whereas $\tan(\pi/2+\epsilon) < 0$. As a result of this calculation, we can get the non-vanishing Berry phase

$$\gamma_+(\theta) = \pi. \tag{24}$$

To find the total geometric phase on the superposed states of $|\psi_+\rangle$ and $|\psi_-\rangle$, the evolution operator of (12) is needed. Following Refs. 9 and 16, the exact evolution operator of (12) for a spin-1/2 particle can be written as

$$U(t) = \begin{pmatrix} \exp\left\{\frac{b}{2}\right\} + ac\exp\left\{-\frac{b}{2}\right\} & a\exp\left\{-\frac{b}{2}\right\} \\ c\exp\left\{-\frac{b}{2}\right\} & \exp\left\{-\frac{b}{2}\right\} \end{pmatrix}.$$

The functions $a(t)$, $b(t)$, and $c(t)$ are given by

$$\frac{da}{dt} = \frac{k}{i\hbar}\left\{(Z_1 - iZ_2) - a^2(Z_1+iZ_2) + 2Za\right\}, \tag{25}$$

$$\frac{db}{dt} = \frac{2k}{i\hbar}\left\{Z - (Z_1+iZ_2)a\right\}, \tag{26}$$

$$\frac{dc}{dt} = \frac{k}{i\hbar}(Z_1+iZ_2)e^b. \tag{27}$$

At $\theta = 0$, we use the initial condition

$$\psi(0) = \frac{1}{\sqrt{2}}\begin{pmatrix} \left\{\frac{Z+\sqrt{Z^2+D^2}}{\sqrt{Z^2+D^2}}\right\}^{1/2} \\ \frac{r_0(\alpha+i\beta)}{\left\{\sqrt{Z^2+D^2}\left(Z+\sqrt{Z^2+D^2}\right)\right\}^{1/2}} \end{pmatrix}, \tag{28}$$

and write $\psi(\theta) = U(t,0)\psi(0)$ as

$$\psi(\theta) = \frac{1}{\sqrt{2}}\begin{pmatrix} [\exp\{b/2\} + ac\exp\{-b/2\}]\left\{\frac{Z+\sqrt{Z^2+D^2}}{\sqrt{Z^2+D^2}}\right\}^{1/2} + a\exp\{-b/2\}\frac{r_0(\alpha+i\beta)}{\left\{\sqrt{Z^2+D^2}\left(Z+\sqrt{Z^2+D^2}\right)\right\}^{1/2}} \\ c\exp\{-b/2\}\left\{\frac{Z+\sqrt{Z^2+D^2}}{\sqrt{Z^2+D^2}}\right\}^{1/2} + \exp\{-b/2\}\frac{r_0(\alpha+i\beta)}{\left\{\sqrt{Z^2+D^2}\left(Z+\sqrt{Z^2+D^2}\right)\right\}^{1/2}} \end{pmatrix}.$$

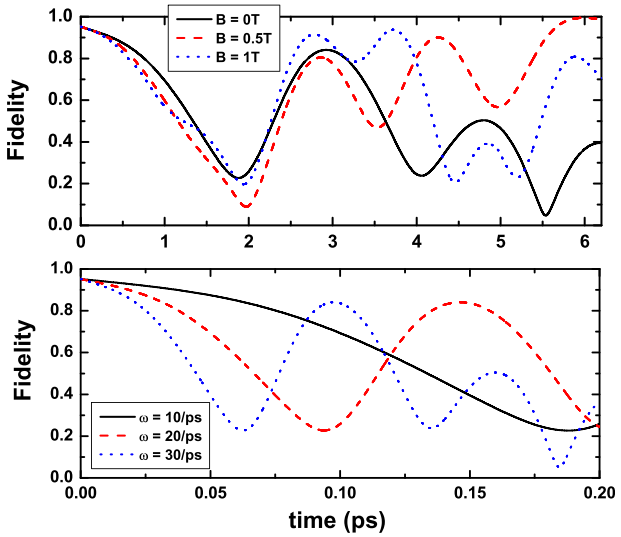

FIG. 2. Fidelity $= |\langle\psi_{-}(\theta)|\psi(\theta)\rangle|$ vs time. For the case with no magnetic fields (solid line, upper panel), the spin Hamiltonian reduces to those of Ref. 9. In the lower panel (B = 0.01 T), much faster spin flip is achieved with increasing adiabatic control frequency $\omega < \lambda_{\pm}/\hbar$. Here, we chose $E_0 = 5 \times 10^4$ V/cm, $\ell_0 = 20$ nm, and $E_z = 10^5$ V/cm.

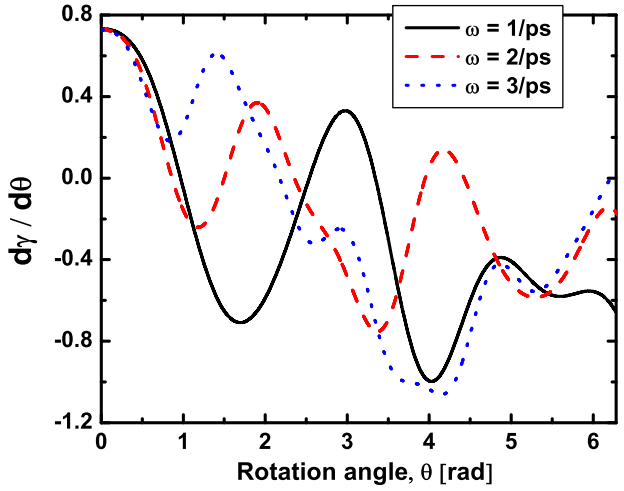

FIG. 3. Geometric phase $(d\gamma/d\theta)$ vs rotation angle on the superposed state $|\psi(\theta)\rangle$. We see that the geometric phase changes from positive to negative regime. This tells us that the complete spin flip occurs only on the superposed state during the adiabatic transport of the dots. Here, we chose $E_0 = 10^3$ V/cm, $\ell_0 = 20$ nm, $B = 1$ T, and $E_z = 10^5$ V/cm.

In Fig. 2, we see that the fidelity is enhanced with magnetic field which means that the spin flip is much faster than for the case with no magnetic field during the adiabatic transport of the dots in the 2D plane. We also notice that the geometric spin flip time (less than 2 ps) is much faster than the experimentally reported decoherence time ($\sim$100 ns)[1] that might overcome the main danger for the low-temperature spin manipulation in QDs coming from the hyperfine coupling to the nuclear spins as earlier reported by Ban *et al.* in Ref. 10.

In Fig. 3, we have plotted the geometric phase $d\gamma/d\theta = i\langle\psi(\theta)|\partial_\theta\psi(\theta)\rangle$ as a function of the rotation angle. As can be seen, the geometric phase can change from positive to negative values which tells us that the spin is completely flipped much faster than the decoherence time during the adiabatic transport of the dots.

To conclude, we have provided an alternative approach to flip the spin completely via the geometric phase. Our designed oscillating electric pulse is the smooth functions of $\sin\theta$ and $\cos\theta$ that might provide the simplest way to measure the geometric phase in QDs. For the experimental set up in the laboratory, see Refs. 1 and 2. In Fig. 2, we have shown that the fidelity is enhanced with increasing magnetic fields and frequency of the control pulse. The spin flip time (less than 2 ps) is much faster than the experimentally reported decoherence time ($\approx$100 ns). In Fig. 3, we have shown that the sign change in the geometric phase indicates that the spin is completely flipped during the adiabatic transport of the dots. This result, yet to be experimentally verified, may provide an alternative for the manipulation of spins before reaching to decoherence. Either for the pure Rashba or the pure Dresselhaus spin-orbit coupling cases, the geometric phase on spin up and down spinors is linear in the rotation angle (see Eqs. (20) and (21)) which is not useful if we are interested in flipping the spin.

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