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Error Dynamics and Coupling Procedures in Mathematical Climate System Models

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ABSTRACT

The mathematical hypothesis concerning the external character of perturbations is critically examined in the special case of a climate system model. The importance of embedding such models into a perturbation space with L^1-structure for the analysis of error and model stability is demonstrated.

1. TOPOLOGICAL EMBEDDING OF MATHEMATICAL MODELS

Since there are many real dynamic systems on which controlled experiments cannot be carried out, it is important to construct an appropriate model of a "proxy system" on which such experiments can be conducted. A classic example of such a dynamic system is the atmosphere-active-layers (AAL) system. As a result of increasing computational power of modern computers, improvements in measuring techniques and achievements in the development of effective numerical procedures, a number of mathematical models for the proxy AAL system are currently available (see references in [4]). As a rule, such models require a large datasets of initial conditions (in comparison to fast-wave-filtered models) and an appropriate definition of boundary conditions in order to avoid possible instability. In this study the Community Climate Model CCM3 developed in NCAR was chosen as a model for the proxy AAL system. Figures 1 and 2 show examples of the computation of the meridional component of wind and water vapour on the vertical level 15 when a hybrid sigma-pressure system is used for the vertical coordinate (see details in [4] and references therein).

Can we claim that the *total error* in the description of the climate system by a model that consists of a finite set of partial differential equations and approximate initial and boundary conditions is small? One may observe that although the results presented in figures 1 and 2 were obtained from a deterministic mathematical model, there is a temptation to say that both figures exhibit some degree of stochasticity. If this is the case then is it real randomness, low-dimensional attractors or distorted pictures of climatic fields due to accumulated error of computation? To answer these questions rigorously we have to know to what extent, if any, the original system is deterministic. Furthermore, even if climate is a deterministic system, in mathematical models based on a finite set of nonlinear equations *the uniqueness* of long-term statistics which define initial and boundary conditions for the problem is not guaranteed. Indeed, mathematical theory is typically applied to dynamic systems which are *closed* in the sense that perturbations of the system are *external* to it and are not influenced by the system itself. This view may be effectively applied only to a relatively narrow class of dynamic systems. Since climate does not belong to this class, in order to get its formalized description we have to include all parts of the climatic environment, which are influenced by climate itself into the model, a task which is hardly tractable in the rigorous mathematical sense.

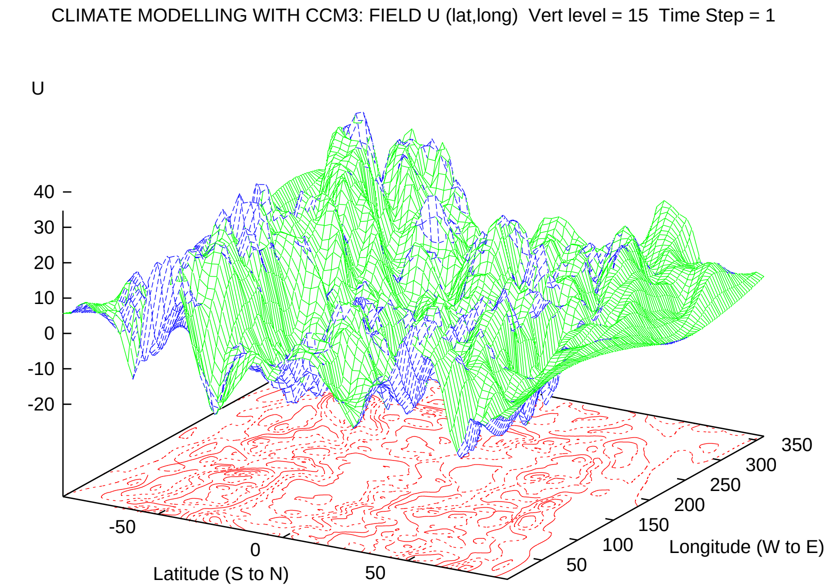

Figure 1: Meridional component of the wind (V, vertical level 15).

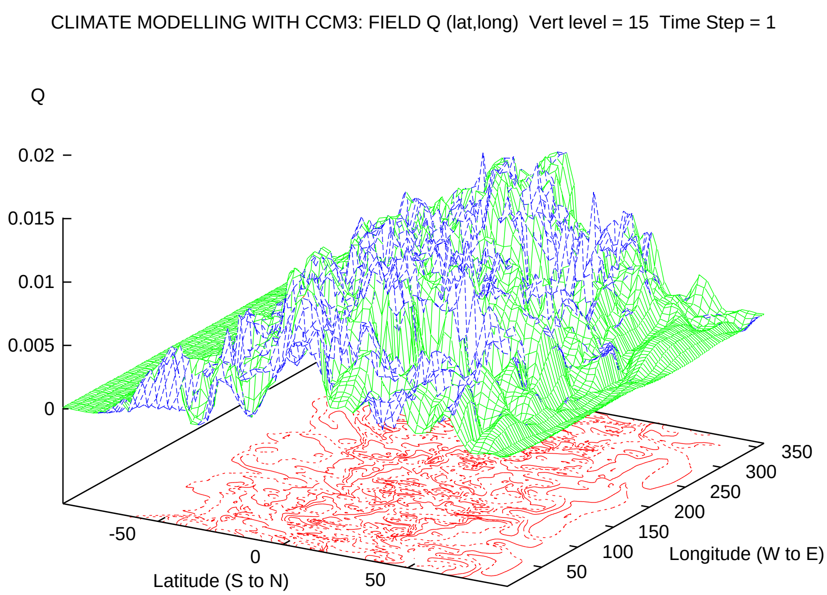

Figure 2: Water vapour at vertical level 15.

In this paper I propose an alternative approach. It is well-understood that those original difficulties connected with the structure of turbulence (see references in [4]) that led to reversion from models based on filtering procedures (for meteorological "noises") those based on primitive hydrodynamic equations *have not been removed*. They rather were gradually fuzzyfied in computational aspects of the problem (see [4] and references therein). These difficulties are intrinsic to mathematical models of complex dynamic systems that require investigation of *phase transitions*.

The crux of the problem appears to be in the appropriate choice of *the topological space for perturbations* in which the dynamic system can be embedded. The increasing quality of initial datasets for climate simulation implies an adequate treatment of the phenomenon of wave instability on frontal interfaces that mathematically may be essentially simplified by the assumption of the given "exact" boundary conditions. From the physical point of view such a simplification can be seen as an approximation of a fundamentally *open dynamic system* by a *closed dynamical system*. It is this step of mathematical modeling that allows us to apply *conservation laws for different physical quantities such as mass, momentum, energy*. This produces an avoidable error in the results obtained by the mathematical models with respect to the real system, but it may be effectively applied in a wide range of physical situations. Now the natural question arises of how such models can be improved.

2. PARAMETERIZATION OF PHYSICAL PROCESSES

Since an extreme sensitivity to physical parameterization is an integral part of mathematical models for the proxy AAL system [2], the improvement of such models can be achieved by *embedding into them* mathematical models of new active layers or by improving models of layers included on earlier stages of the model development (for example, embedding into coupled atmosphere-ocean models a thermodynamic sea-ice model, improving solar and terrestrial radiative heating models, cloudiness parameterization etc). Essentially, these improvements are achieved by a more careful examination of the top-of-atmosphere, surface energy budgets etc. As soon as physical parameterization is chosen, and hence, the mathematical model is "frozen", such an examination can be effectively performed using tools of mathematical modeling and computational experiment. In this case physical quantities such as mass and energy may be conserved only approximately. Indeed, due to coupling to the physical parameterization, the AAL system can be formalized mathematically only through procedures with incomplete information. Therefore, when constructing computational models from "frozen" mathematical models, the associated uncertainty in knowledge bases should be taken into account. In numerical analysis of mathematical models this leads to the necessity of relaxing a-priori excessive assumptions on the solution smoothness [3]. Mathematical and computational models become strongly coupled by the chosen *physical model* and by the *algorithm* of the problem solution. Mathematically this coupling can be formalized through the concept of perturbations.

3. CONSERVATION LAWS AND NONREFLEXIVE TOPOLOGICAL SPACES

A limiting mathematical case of the situations discussed above is represented by the Cauchy problem for mathematical conservation laws. Although it is still widely accepted that the solution of this problem may be well-approximated in the class of piecewise constant functions using the solutions of auxiliary Riemann problems, examples when this may not be the case have been recently constructed [6].

Without going into details it is important to emphasize that the main mathematical difficulty in the field of mathematical conservation laws stem from *the* **continuous** *dependency* *of the solution on perturbations* that cannot be eliminated. This leads to the situation where the solution itself can be defined in practice only with respect to *certain level of perturbations*. It means that with respect to the original physical system mathematical models are *a priori* perturbed. Even if we formally consider a mathematical model as unperturbed, we have to investigate its stability to perturbation in a typically infinite dimensional space wider than state space of the system. The natural choice for such a space is L^1. However, most investigations consider only L^2-perturbations. The mathematical techniques used in such investigations depend crucially on the analysis of nearest points and rely heavily on the Hilbertian structure or the perturbation space or even on occasion on the finite dimensional structure. This restricted the possible application of such results to mathematical models for complex dynamic systems such as climate. The inadequacy of the description of perturbations in terms of time-independent functions for complex dynamic systems such as climate necessitates the choice of L^1structure in the perturbation space.

4. CLIMATIC DETERMINISM AND SCALING LAWS

Modern Climate System Models consist of *relatively independent* components that are responsible for interconnected parts of climate (such as atmosphere, ocean, land surface, sea ice). This makes possible to apply effective parallelization procedures. Nevertheless, in order to obtain approximate solutions, such models still require substantial computational power which often is not available. Since different computers and different compilers may give *noticeably* different results some authors have concentrated on the round off error resulting from the approximate solution of a system of PDEs with given initial and boundary conditions. However, the rounding error *alone* may not be an adequate reflection of the total error which also includes initial datasets approximate in nature and the approximate physical parameterization "built" in to a finite set of differential equations.

In order to assess the quality of simulation, we have to assess the quality of initial datasets and the physical parameterization. Indeed, the quality of our approximate solutions depend on *the consistency between the mathematical model and the real climate* rather than on *the consistency between two mathematical models*. Since the improvement of mathematical models can be achieved by improved initial datasets and physical parameterizations, *the concept of coupling between different components of the model* becomes straightforward. The dilemma lies in the fact that the refinement of the approach based on coupling procedures may continue indefinitely, yet it does not necessarily lead to the absence of the error in mathematical/computational implementation of conservation laws. In fact, there may not exist *a finite time procedure* that allows us to solve the problem of the interaction between small-scale convection and large-scale motion in a rigorous mathematical sense. However, if the *relative error for the solution of this problem* is given, such a procedure can be found at least in principle. In modeling complex dynamic systems that require the solution of the problem of interaction between different space-time scale processes, the error becomes a time-dependent function. The specified level of error implicitly defines the time-range within which the model can be effectively applied.

The "timeless" features of many mathematical models stem from a simplified mathematical assumption that smaller-scale turbulence may be regarded as a *dissipative factor* which might be characterized by an *effective viscosity coefficient* μ. For practical applications *the law of viscosity* has to be supplemented by *the law of energy dissipation*. In climate modeling the Prandtl mixing-length hypothesis is used in order to find a reasonable analogy between the motion of molecules and the motion of macroscopic elements in turbulent fluids (see references in [4]). A connection between these two physical laws, the law of viscosity and the law of energy dissipation, has to be defined by a *scaling law*. Many current investigations in including climate study are based on the Karman-Prandtl logarithmic scaling law, which in some cases may be inappropriate as an adequate description of turbulent processes (see references in [4]). In order to ensure consistency between mathematical and physical models we have to take into account *the time dependency of spatial averaging procedures* that become important in large-scale modeling. This requires the formulation of the scaling law to be made on the basis of both informational parts about the solution of the problem, *a priori* information and *a posteriori* information. In turn, this dictates more restrictive assumptions on the solution regularity, and L^1-error bounds or at least L^k-bounds where $k \in (1,2)$ become an important measure of the quality of associated with the mathematical model numerical schemes. For linearized models L^2-error bounds provide sufficient information on the quality of approximate solutions.

5. STOCHASTICITY AND VANISHING VISCOSITY

Somewhat wider mathematical freedom is allowed when the evolution of states of the atmosphere is regarded as a random process $m(t)$ [5]. In this case one may attempt to use statistical extrapolation of the process using the *Kolmogorov's hypothesis*. Namely, a random process $m(t)$ describing the evolution of the turbulent flow in an *environment with vanishing viscosity* asymptotically approaches a Markov process for large t. Eventually, it is this hypothesis that allows one to keep the assumption of L^2-structure of perturbation space as a possibility. From such a consideration it follows that the distribution of probabilities $P^t(dm)$ for $t > t_0$ may, in principle, be uniquely determined by the state $m(t_0)$, and not be dependent on the remote history of the process when $t < t_0$. This approach may be effectively applied in practice bearing in mind that the quality of the definition of the probabilities $P^t(dm)$ become unsatisfactory when t exceeds a finite (possibly very large) time known as the Lyapunov horizon. In order to increase this horizon we have to increase the accuracy of initial datasets and to improve physical parameterization. Of course, the assumption of *the negligible viscosity approximation* can be reasonably justified numerically on a finite grid. However, no matter how dense such a grid is, the validation of the original mathematical model is conducted under the processing of incomplete information that in turn requires an adequate formulation of *time-dependent scaling laws*. Indeed, we need to *continuously improve* mathematical models of dynamic systems by including additional information. Although it is unreasonable to expect that such a law may be formulated for all practically important cases, it is possible to formulate such laws on classes of mathematical problems. This allows us to predict the error-bound in modeling for an arbitrary representative of the class.

6. COMPUTATIONAL EFFICIENCY AND COMPLEXITY OF COUPLING

The total modeling error ε for a complex dynamic system such as climate cannot be reduced to zero due to the presence of unremovable error between the system and its model. This error can be interpreted as a perturbation parameter that is intrinsic to the chosen law of scaling. For any given model this parameter defines the upper bound, τ, for time-range, as a function of a set $\mathcal{M}$ that characterizes the model

$$\tau = f(\mathcal{M}, \varepsilon).$$

This time bound can be interpreted through the Lyapunov horizon. If we assume that $\mathcal{M} \neq \emptyset$ and $\varepsilon > 0$, then the function f defines the complexity of coupling procedures. Any specific choice of positive ε implies a simplification of the original problem and provides a measure of the discrepancy between system and model. The simplification is also unavoidable in the case when $\varepsilon \to 0^+$. Indeed, in this case the system has to be considered as a unified whole that often makes the problem computationally untractable in practice. Numerical aspects of the problem are defined by the connection between the set $\mathcal{M}$ and the value of ε and it may not be possible to reduce the error without associated changes in the structure of $\mathcal{M}$. Whenever the structure of $\mathcal{M}$ is fixed (as it is the case for the CCM3 model) and the existence of $\lim_{\varepsilon\to 0^+} f(\mathcal{M}, \varepsilon)$ is assumed *a priori*, the solution of the problem consists of a numerical interplay between the time variable τ and the function of complexity of coupling f. Even for a high-level-coupling "proxy system", smaller-scale phenomena may still substantially influence the larger-scale properties of the system, yet they cannot be extracted from the latter using modern computational resources.

This observation leads to the conclusion that it is necessary to maintain a balance between *computational efficiency* and the *complexity* of coupling procedures. Such a balance has to be maintained for both deterministic and random mathematical models. Both types of model may provide appropriate descriptions of the same system but with different f and $\mathcal{M}$. It is important, however, to consider both models as perturbed mathematical models that are embedded into the topological space for perturbations with L^1-structure. This idea has been recently developed from different directions in [1,4,6] where links to the recent advances in control theory as well as numerical procedures can also be found.

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