Advanced Materials Research Vols 47-50 (2008) pp 69-72 *Online: 2008-06-12*
© (2008) Trans Tech Publications, Switzerland
doi:10.4028/www.scientific.net/AMR.47-50.69

Feedback Linearization of Hysteretic Thermoelastic Dynamics of Shape Memory Alloy Actuators with Phase Transformations

Linxiang Wang[1, a], Rong Liu[1,b], Roderick. V. N. Melnik[2,c]

[1]Institute of Mechatronic Engineering

Hangzhou Dianzi University, Xiasha, Hangzhou, P. R. China, 310037,

[2]Mathematical Modelling and Computational Sciences

Wilfrid Laurier University, Waterloo, 75 University Ave W, Canada N2L 3C5

[a]wanglinxiang@mci.sdu.dk, [b]liurzju@gmail.com, [c]rmelnik@wlu.ca

Keywords: Hysteresis, differential model, Landau theory, feedback linearization.

Abstract. In the current paper, a macroscopic differential model for the hysteretic dynamics in shape memory alloy actuators is constructed by using the modified Landau theory of the first order phase transformation. An intrinsic thermo-mechanical coupling is achieved by constructing the free energy as a function depends on both mechanical deformation and the material temperature. Both shape memory and pseudoelastic effects are modeled. The hysteretic dynamics is linearized by introducing another hysteresis loop via nonlinear feedback strategy, which cancels the original one.

Introduction

For many engineering applications, the unique properties of shape memory alloys (SMAs) are very attractive due to its capability of converting energy between the mechanical and thermal types. In practice, the major obstacle which holds back further application developments of SMAs is the analysis and modeling of its hysteretic dynamics. It is well understood nowadays that the hysteretic dynamics related to the "pseudo-elasticity" and "Shape Memory Effects" in SMAs is a consequency of the thermoelastic phase transformations [1,2]. On the other side, the theories for dynamical system design, analysis, optimization, and control are well established for systems given by ordinary differential equations, therefore it is always desired to capture the hysteretic dynamics of the SMAs by a simple while sufficiently accurate differential model [3,4,5].

In the current paper, a macroscopic differential model is constructed for the modeling of the hysteretic dynamics. The hysteresis is linearized by using nonlinear feedback strategies. A free energy function is constructed as a non-convex function of the mechanical deformation, and linearly depends on the material temperature. Governing equations for the coupled dynamics are formulated by using conservation laws for the linear momentum and internal energy, which are expressed as two nonlinear partial differential equations. The mechanical and thermal fields are coupled in an intrinsic way in the model.

To construct a lumped differential model, the partial differential equations are simplified into two ordinary differential equations for the lumped dynamics of displacement and material temperature. For the convenience of system analysis and control, the dynamics is linearized by introducing a nonlinear feedback into the system. The feedback itself is a hysteretic one. It is shown that by suitably choosing the feedback law, the original hysteretic dynamics induced by the phase transformations can be cancelled by the artificially introduced one via feedback, such that the original system can be transformed into a linearized counterpart.

Shape Memory Alloy Actuators with Phase Transformations

We confine our discussion here within one dimensional SMA actuators, as sketched in Fig.1. The considered dynamics is the movement of the mass block m attached at the right end of the SMA rod. The initial location of the mass block is x_0. Its displacement along the x direction, u_r, is not a monotone function of the applied force f, due to the hysteretic dynamics of the SMA rod.

It has been verified experimentally that, the unique properties of the SMAs are caused by the martensite transformations and variant re-orientations. Therefore an effective way to construct a model for the dynamics is to start the modeling by describing the phase transformations mathematically. The essence of the Landau theory of the first order transformation is to construct a free energy as a non-convex function of the chosen *order parameters* to characterize different phases involved in the transformation. For the one-dimensional actuator considered here, there are two martensite variants and one austenite phase involved, as sketched in Fig.2. Phase transformations can be induced either thermally ($A \Leftrightarrow M_+$ or $A \Leftrightarrow M_-$) or mechanically ($A \Leftrightarrow M$ and/or $M_+ \Leftrightarrow M_-$). Each phase has a specific strain value. Therefore the free energy can be constructed as a non-convex function of the strain, which has three local equilibriums associated with the three phases, respectively, as sketched in Fig.3. The two local equilibriums marked as shaded squares are associated with martensite variants (M_+ and M_-) respectively, while the one marked as shaded circle is associated with the austenite. The phase transformation dynamics thus can be modeled by formulating the system state transformation dynamics from one local equilibrium to another.

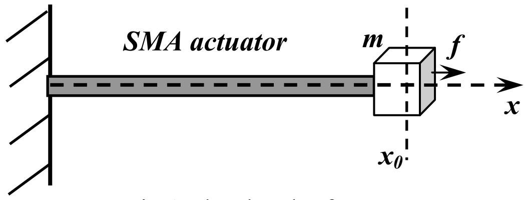

Fig.1 The sketch of a one-dimensional SMA actuator

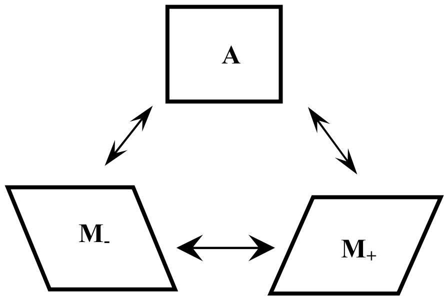

Fig.2 Sketch of the martensite transformation and martensite variants re-orientations

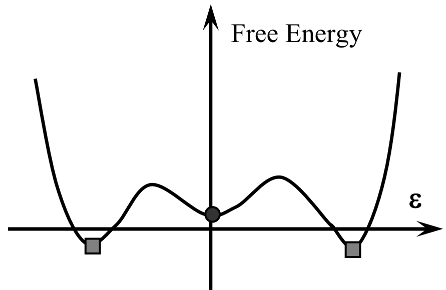

Fig.3 The profile a Landau free energy function for one-dimensional SMA actuators.

For the current consideration, the free energy function can be constructed as a sixth order polynomial of the *order parameter* (the strain) as [6]:

$$F(\theta,\varepsilon)=\frac{a_2(\theta-\theta_c)}{2}\varepsilon^2+\frac{a_4}{4}\varepsilon^4+\frac{a_6}{6}\varepsilon^6, \tag{1}$$

where θ is the material temperature, $\varepsilon = \partial u/\partial x$ is the strain, and a_2, a_4, a_6, and θ_c are all material constants. $u(x)$ is the displacement of particles in the SMA rod. It is clear that the free energy depends on the material temperature. The basic calculus says that, a sixth polynomial may have three local minimum, provided suitable coefficient values are chosen. This is verified in literatures [7,8,9].

Differential Model of Hysteresis

For the description of the dynamics of the mechanical field in the SMA rod, the following Lagrangian φ is introduced as the sum of kinetic and potential energy contributions:

$$\varphi=\int_0^L\left(\frac{\rho}{2}\left(\frac{\partial u}{\partial t}\right)^2-F(\theta,\varepsilon)\right)dx, \tag{2}$$

where ρ is the density of the SMA, and L is the total length. The first part of the integrants models the kinetic contribution, while the second one models the potential contribution. Please note that the non-convex Landau free energy is employed here as the potential energy, which enable the model to describe the hysteretic dynamics related to the phase transformations.

By setting the variation of the functional given in Eq. 2 with respect to u(x, t) (the true path) zero according to the Hamilton's principle, one has the following:

$$\delta\int_0^T \varphi dt = \int_0^T\int_0^L \delta\left(\frac{\rho}{2}\left(\frac{\partial u}{\partial t}\right)^2 - F(\theta,\varepsilon)\right)dxdt = 0\,. \tag{3}$$

A simple calculus of variation using the above equation leads one to the following wave equation for the mechanical field in the SMA rod:

$$\rho\frac{\partial^2 u}{\partial t^2} = \frac{\partial \sigma}{\partial x} = \frac{\partial}{\partial x}\left(a_2(\theta-\theta_c)\varepsilon + a_4\varepsilon^3 + a_6\varepsilon^5\right), \tag{4}$$

where σ is the stress. The above model is capable of modeling nonlinear wave propagations with phase transformation dynamics [6,7,8]. Of course, boundary conditions need to be implemented to complete the model:

$$u = 0 \quad at \quad x = 0, \quad \sigma = m\frac{\partial^2 u_r}{\partial t^2} \quad at \quad x = L\,. \tag{5}$$

At macroscale, the wave propagations inside the actuator considered here is not the major concern. Thus it is assumed that the SMA actuator will behave in a uniform way, which means that the strain and temperature distributions in the actuator are uniform. Therefore ε can be formulated as: $\varepsilon = u_r / L$, by which Eq. 4 and Eq. 5 can be simplified as the following:

$$\frac{d^2 u_r}{dt^2} = k_1 u_r + k_2 u_r^{\ 3} + k_3 u_r^{\ 5} + k_c\frac{du_r}{dt} + f(t)\ , \tag{6}$$

with the coefficients are defined as:

$$k_1 = \frac{a_2(\theta-\theta_c)}{mL}, \quad k_2 = \frac{a_4}{mL^3}, \quad k_3 = \frac{a_6}{mL^5}, \tag{7}$$

where f account for external forces applied on the SMA rod, and k_c is the damping coefficient.

The dynamical response of the thermal field in the actuator can be derived on the basis of the conservation laws of the internal energy. Provided that ε and θ are uniform, the heat equation for the actuator can be formulated as:

$$c_v\frac{d\theta}{dt} = \frac{a_2}{L^2}\theta u_r\frac{du_r}{dt} + g\,, \tag{8}$$

where c_v is the specific heat of the material, and g is the external thermal loadings. It is clear by looking at Eq. 6 and Eq. 8 that the mechanical and thermal fields in the actuator are intrinsically coupled, in a nonlinear way. Eq. 6 and Eq. 8 are two ordinary differential equations. It has been verified that the hysteresis loops induced by phase transformations can be modeled precisely by Eq. 6 and Eq. 8, provided that the parameter values are carefully chosen [6]. More importantly, the model is expressed as a differential style and can be easily modified to take into account rate-dependence of the characteristics [8].

Feedback Linearization

In applications, nonlinear systems are often approximated by corresponding linear systems via truncating the Taylor expansion series. For the current model given by Eq. 6 and Eq. 8, the nonlinearity occurs in thermo-mechanical coupling effects (k_1 term and the heat equation) are able to be linearized by this conventional methods, since there are no structure nonlinearities. But the nonlinearity related to the hysteresis loops, which is a structure nonlinearity, is not able to be approximated by its Taylor expansion series since bifurcations occur in the dynamics [8,9]. For the linearization purpose of the hysteretic dynamics, a nonlinear feedback is introduced to cancel the original nonlinearities. To clarify the discussion, only Eq. 6 is linearized here.

The essential idea of the feedback linearization is to introduce a feedback which is a nonlinear function of the system states. To show the idea for the purpose here, Eq. 6 is recast into the following state space form with separated linear and nonlinear parts:

$$\frac{du_r}{dt} = v, \quad \frac{dv}{dt} = k_1 u_r + k_2 u_r^{\ 3} + k_3 u_r^{\ 5} + k_c v + f(t)\,. \tag{9}$$

Assume that the input *f(t)* is the feedback to the system, and can be constructed as a nonlinear function of the system state u_r and v, together with a new input signal $f_n(t)$:

$$f(t) = K_A(u_r, v) + K_B(u_r, v) f_n(t)\,, \tag{10}$$

where K_A and K_B are two matrices to be determined. The goal of nonlinear feedback is then to transform the nonlinear system given by Eq. 6 to a linear one as follows:

$$\frac{du_r}{dt} = v, \quad \frac{dv}{dt} = K_a u_r + K_b v + M f_n(t), \tag{11}$$

where $K_a = k_1$, $K_b = c$, and M is a constant can be chosen arbitrarily for the convenience of system design. By substituting the feedback law into Eq. 8, it is easy to get the following relation:

$$\frac{dv}{dt} = K_a u_r + K_b v + M K_B^{\ -1}(u_r, v)\left(f(t) - K_A(u_r, v)\right), \tag{12}$$

Where K_B^{-1} is the inverse of the operator K_B. A simple calculation gives the following feedback law for the linearization purpose:.

$$K_A = -\left(k_2 x^3 + k_3 x^5\right), \quad K_B = M\,. \tag{13}$$

It is easy to show that the introduced feedback is actually a hysteretic one. The original hysteretic dynamics is canceled by the one introduced via the feedback, such that the original hysteretic system is transformed into a linear one. The pre-requirement of the feedback strategy for the linearization purpose here is that, the hysteretic dynamics is modeled as a differential system.

Conclusion

In the present paper, the hysteretic dynamics of a shape memory alloy actuator is modeled by using the Landau theory of the first order phase transformations. The thermo-mechanical dynamics is described by two coupled nonlinear ordinary differential equations. A nonlinear feedback mechanism is introduced into the dynamics to cancel the original hysteretic input-output relations. The nonlinear system is finally converted into a linear one by using the nonlinear transformation.

References

[1] E. Patoor, D. C. Lagoudas, P. B. Entchev, L. C. Brinson, X. Gao. , Mechanics of materials, Vol.38 , (2006), 391-429.
[2] L. X. Wang, R. V. N. Melnik. Applied Mathematical Modelling, Vol.31 (2007), pp 2008-2018.
[3] Visintin, *Differential Models of Hysteresis*,Springer, Berlin, 1995.
[4] Hassan K, Khalil, *Nonlinear systems*, Prentice-Hall Inc. New Jersey, 2004.
[5] S, Sastry, *Nonlinear systems: analysis, stability,and control*, Springer, New York, 1999.
[6] L. X. Wang, K. Hemmant, Smart Materials and Structures,Vol.15 (2006), 1725-1733.
[7] F. Falk, P. Konopka, J.Phys.:Condens.Matter, Vol.2, (1990), 61-77.
[8] R.C. Smith, *Smart Material Systems: Model Development,* SIAM Publishers, Philadelphia, 2005.
[9] Wen, Y. K. Journal of Engineering Mechanics, Vol. 102, (1976), 249-263.

Multi-functional Materials and Structures
10.4028/www.scientific.net/AMR.47-50

Feedback Linearization of Hysteretic Thermoelastic Dynamics of Shape Memory Alloy Actuators with Phase Transformations
10.4028/www.scientific.net/AMR.47-50.69

DOI References
[2] L. X. Wang, R. V. N. Melnik. Applied Mathematical Modelling, Vol.31 (2007), pp 2008-2018.
doi:10.1016/j.apm.2006.08.019
[7] F. Falk, P. Konopka, J.Phys.:Condens.Matter, Vol.2, (1990), 61-77.
doi:10.1088/0953-8984/2/1/005