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ECHO EFFECTS ON RELATIVISTIC LANDAU LEVELS IN GRAPHENE AND BIGRAPHENE AS A MANIFESTATION OF THE QUANTUM MEMORY

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We consider the echo effects, which can take place in graphene and bigraphene (bilayer graphene), when the system of relativistic Landau levels in a quantizing magnetic field appears. Graphene (bigraphene) is examined theoretically in the long-wave approximation near the Dirac points. We propose to use the echo effects for realization of quantum memory for optical states in the far-infrared region.

Keywords: Graphene; echo effect; quantum memory.

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1. Introduction

Interest in studying the echo phenomena, which were discovered in the mid-20th century,[1,2] has increased significantly in recent years due to the prospects of these phenomena for the application in quantum-memory devices. The concept of quantum memory has arisen in a context of the theory and applications of quantum computation, and its appearance is connected with the need to keep and save the wave function for a long time.[3–5] Suggested devices for experimental study of quantum memory use various physical principles,[6,7] including those using the light echo effect,[8,9] which allows us to combine fast information processing and a sufficiently long-time storage. Note that both the spin and the light echoes are observed in a number of materials and the experimental observation of such effects does not involve any fundamental difficulties.

At the same time, since the recent discovery of graphene,[10–12] the area of its possible applications continuously increases. This is partly due to the fact that electrons in graphene propagate nearly ballistically, so that the effects associated with scattering processes can be neglected in most cases. Relatively long relaxation times of electrons in carbon structures[13,14] also make this material promising for potential information storage. Hence, an interesting question to address would be wether the effect similar to light echo is possible to observe in the electron system of graphene. Note that the usual quantum memory for light echo techniques uses the impurity environment, in which the interstitial atoms are given a pre-defined energy level scheme, which allows to record information. In graphene, the relativistic Landau levels can play a similar role.[15,16] The distance between the energy levels in this case lies in the range of hundred meVs, which allows us to record and read information by using far-infrared lasers.

In this regard the so-called bigraphene (bilayer graphene) becomes very promising too, because of the possibility to control the energy level spacing by applying a controlled voltage between the layers of graphene.[17]

2. Basic Equations

Consider a system of electrons in graphene in the long-wavelength approximation near one of the Dirac points (to be specific near the point K, another point will give a factor of two in the final answer due to symmetry), which is described by the Hamiltonian

$$\mathcal{H} = \nu_F(\sigma_x p_x + \sigma_y p_y)\,, \tag{1}$$

where σ_x and σ_y are the corresponding Pauli matrices and ν_F is the Fermi velocity.

The long-wavelength approximation is justified in our case by the following two reasons. First, the value of the Fermi velocity in the Hamiltonian (1) is sufficiently large ($\nu_F \sim c/100$, where c is the speed of light), so that even at room temperatures, electrons are excited only in a narrow layer near the Fermi level. Secondly, the magnetic field, which will be further discussed, does not change the energy of the electrons, and hence the momentum of the electrons can not escape beyond the approximation, i.e. it cannot reach values near the edges of the Brillouin zone. Also we would note that, to our knowledge, currently there are no experimental data on graphene properties which, for its explanation would require going beyond the long-wave approximation.

In the presence of external magnetic field perpendicular to the graphene layer, and an external alternating electric field directed along the axis y, it is necessary (in accordance with the minimal coupling concept[18]) to implement the replacement

$$\mathbf{p} \to \mathbf{p} - q\mathbf{A}$$

and to add to the Hamiltonian the term corresponding to an alternating electric field $\mathbf{E} = (0, E(t), 0)$, which is uniform within a given graphene layer, namely

$$\mathcal{H} \to \mathcal{H} + qE(t)y\mathcal{I}\,,$$

where we have chosen the gauge $\mathbf{B} = \nabla \times \mathbf{A}$; $\mathbf{A} = (-By, 0, 0)$, and $\mathcal{I}$ is the identity matrix. In this case, the Hamiltonian of the system can be written in terms of the operators[19,20]

$$a = \frac{1}{\sqrt{2}}\left(y + \frac{\partial}{\partial y}\right), \qquad a^\dagger = \frac{1}{\sqrt{2}}\left(y - \frac{\partial}{\partial y}\right), \tag{2}$$

(so that $[a, a^\dagger] = 1$) as

$$\mathcal{H} = \mathcal{H}_0 + V + \frac{kE(t)}{B}\mathcal{I}, \tag{3}$$

where

$$\mathcal{H}_0 = \begin{vmatrix} 0 & ha \\ ha^\dagger & 0 \end{vmatrix}, \qquad V = r(t)(a^\dagger + a)\mathcal{I},$$

$h = \nu_F\sqrt{2qB}$, and $r(t) = E(t)\sqrt{q/2B}$ (here and thereafter we will set $\hbar = c = 1$). When obtaining Eq. (3) we used the following representation for the bispinor $\psi(x, y)$ describing the electrons near the Dirac point K:

$$\psi(x, y) = \psi(y)\exp(ikx). \tag{4}$$

In the case of bigraphene, similarly looking Hamiltonian (3) contains

$$\mathcal{H}_0 = \begin{vmatrix} U & ha^\dagger & t_0 & 0 \\ ha & U & 0 & 0 \\ t_0 & 0 & -U & ha^\dagger \\ 0 & 0 & ha & -U \end{vmatrix} \tag{5}$$

The terms proportional to the identity matrix $\mathcal{I}$ in Eqs. (3) and (5) plays no role in our problem; the spectrum and wave functions of the operator $\mathcal{H}_0$ form a system of relativistic Landau levels in graphene. The eigenvalues of the operator $\mathcal{H}_0$ for graphene (3) are

$$e_n = \pm h\sqrt{n+1}. \tag{6}$$

The corresponding eigenfunctions have the form

$$|\psi_n\rangle = \begin{pmatrix} |n\rangle \\ \pm|n+1\rangle \end{pmatrix} \quad \text{for} \quad n \neq 0; \quad |\psi_0\rangle = \begin{pmatrix} |0\rangle \\ |0\rangle \end{pmatrix}, \tag{7}$$

where $|n\rangle$ is the harmonic oscillator eigenfunction, corresponding to the nth level.

Quite similarly, in the case of bilayer graphene eigenvalues are determined from the equation

$$\det\begin{vmatrix} U-E & h\sqrt{n} & t_0 & 0 \\ h\sqrt{n} & U-E & 0 & 0 \\ t_0 & 0 & -U-E & h\sqrt{n} \\ 0 & 0 & h\sqrt{n} & -U-E \end{vmatrix} = 0, \tag{8}$$

where n is the level number, and the eigenfunctions have the form

$$|\psi_n\rangle = \begin{pmatrix} |n\rangle \\ c_1|n+1\rangle \\ c_2|n\rangle \\ c_3|n+1\rangle \end{pmatrix} . \tag{9}$$

Here c_i $(i = 1, 2, 3)$ are the constants, and the vectors $|n\rangle$ are defined in the same way as for graphene.

Next we consider the problem of the effect of perturbation V. For definiteness we will consider graphene here, keeping in mind that the calculations for bigraphene are basically similar.

Let an alternating electric field be the pulse with envelope $a(t)$, and the filling rate chosen to be $\omega = h(\sqrt{3} - \sqrt{2})$. Let us also assume that only the lowest Landau level with $n = 1$ is originally occupied, and the population of the level with $n = 2$ can be neglected initially. Note that, since the spectrum of (6) is not equidistant, only transitions between the first and second Landau levels are allowed.

Let us expand the wave function of a non-stationary problem

$$i\frac{\partial}{\partial t}|\psi\rangle = (\mathcal{H}_0 + V)|\psi\rangle\,, \tag{10}$$

accounting for the perturbation V, as

$$|\psi\rangle = \sum_n b_n(t)|\psi_n\rangle\,, \tag{11}$$

and the external electric field as

$$r(t) = A(t)\exp(i\omega t) + \text{c.c.}\,, \tag{12}$$

where the function $A(t)$ changes slowly in comparison with the exponent $\exp(i\omega t)$, i.e.

$$\left|\frac{1}{A(t)}\frac{dA(t)}{dt}\right| \ll \omega\,.$$

Substituting Eqs. (11) and (12) into the Schrödinger equation (10) with the Hamiltonian (3), and multiplying on the left bra-vector $\langle\psi_k|$, we come to the following system of equations:

$$\begin{aligned} \frac{idb_1}{dt} &= e_1 b_1 + r(t)(\sqrt{3} + \sqrt{2}d)b_2\,, \\ \frac{idb_2}{dt} &= e_2 b_2 + r(t)(\sqrt{3} + \sqrt{2})b_1\,, \end{aligned} \tag{13}$$

where e_1 and e_1 are the first and second levels of the spectrum (6), respectively.

Let us further assume that the pulses have a rectangular shape with the amplitude E_i and duration t_i $(i = 1, 2)$. Considering that during the pulse we can use the rotating wave approximation,[21] the solution of Eqs. (13) with the initial conditions $b_1|_{t=0} = b_{10}$ and $b_2|_{t=0} = b_{20}$ can be obtained in the following form

$$\begin{aligned} b_1(t) &= (b_{10}\cos S_i + b_{20}\sin S_i)\exp(ie_1t_i)\,, \\ b_2(t) &= (-b_{10}\sin S_i + b_{20}\cos S_i)\exp(ie_2t_i)\,, \\ S_i &= E_it_i(\sqrt{3}+\sqrt{2}), \quad i = 1,2\,, \end{aligned} \tag{14}$$

where the first two equations are written in the same manner for the first and the second pulses. In the case of the absence of an alternating electric field it is sufficient to put $S_i = 0$ in Eq. (14).

3. Response. Zitterbewegung and Double-Pulse Echo Signal

The response in our problem is connected with the appearance of a current along the axis y, whose operator is given by[22,23]

$$\hat{j} = q\nu_F\sigma_y\,. \tag{15}$$

This operator has to be averaged over the wave function of the system under consideration, which in our case is not stationary. Before proceeding to the calculation of the echo signal, we consider the wave function corresponding to a superposition state when the system occupies two Landau levels, i.e.

$$|\psi\rangle = b_1\begin{pmatrix}|1\rangle\\|2\rangle\end{pmatrix}\exp(ie_1t) + b_2\begin{pmatrix}|2\rangle\\|3\rangle\end{pmatrix}\exp(ie_2t), \tag{16}$$

where as usual, $|n\rangle$ is the eigenfunction of the harmonic oscillator corresponding to the nth level, i.e. $a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$ ($a^\dagger$ is the creation operator for an elementary oscillator excitation).

In this case, the current is given by the formula

$$j = 4qi(b_1^*b_2 - b_2^*b_1)\cos\{(e_2-e_1)t\}\,, \tag{17}$$

i.e. periodic oscillations of the current appear. They are related to the nature of the spinor wave function and are similar to the earlier considered zitterbewegung effect[23,24] in graphene. Note that the experimental observation of this effect is most probably impossible due to the fact that the magnetic field, which is used by experimentalists, is inhomogeneous. It means that Eq. (17) should be averaged over the magnetic field B, which is generally random.[25] Furthermore, there are also so-called pseudo-magnetic fields in graphene, related to the fact that in this material, due to the displacements u_{xx}, u_{xy}, and u_{yy} (defined, e.g., by the fields of lattice defects[26,27]), gauge fields arise: $A_x \propto u_{xx} - u_{yy}$ and $A_y \propto -u_{xy}$. The latter gauge fields act as an effective magnetic field directed perpendicular to the graphene layer:

$$B_{\text{eff}} = \frac{\partial A_x}{\partial y} - \frac{\partial A_y}{\partial x}\,.$$

In this case, the time dependence of the signal given by (17) is replaced by

$$\int \cos\{\nu_F \sqrt{2\hbar q(B+B_{\text{eff}})}(\sqrt{3}-\sqrt{2})t\} F(B_{\text{eff}}) dB_{\text{eff}}\,, \tag{18}$$

where $F(B_{\text{eff}})$ stands for the normalized distribution of local effective magnetic fields. This will determine the decrease in the signal (17). Furthermore, an increase in dispersion of the distribution $F(B_{\text{eff}})$ will increase the rate of the signal attenuation, determined by Eqs. (17) and (18).

After discussing the effects associated with the presence of electrons in the second Landau level, we thereafter will assume that only the first level is inhabited, while the second Landau level is empty, i.e. $b_2|_{t=0} = 0$. This can be achieved, e.g., by cooling the sample till the population of the second Landau level can be neglected. Another way to achieve this is to transfer electrons from the second level to the third one applying a pulse with the appropriate frequency and duration.

In this case, when the sample is exposed to two rectangular pulses with the amplitude E_i, duration t_i, and carrier frequency $\omega = h(\sqrt{3}-\sqrt{2})$, separated by a time interval τ, proceeding from Eq. (14), the response defined by Eq. (15) can be calculated as follows

$$\begin{aligned} j(t) &= 2b_{10}^2 \mathcal{G}(t) \sin S_1 \cos S_1 \cos^2 S_2 + 2b_{10}^2 \mathcal{G}(t-\tau)(\cos^2 S_1 - \sin^2 S_1) \sin S_2 \cos S_2 \\ &\quad + 2b_{10}^2 \mathcal{G}(t-2\tau) \sin S_1 \cos S_1 \sin^2 S_2\,, \end{aligned} \tag{19}$$

where $\mathcal{G}(t-t') = \int \cos\{\nu_F \sqrt{2\hbar q(B+B_{\text{eff}})}(\sqrt{3}-\sqrt{2})(t-t')\} F(B_{\text{eff}}) dB_{\text{eff}}$ is the function describing the decrease in a signal, which arises from the distribution of local fields. It is already taken into account in Eq. (19) that each Landau level corresponds to two wave functions, with signs + and − in Eq. (6), respectively. The first two terms in Eq. (19) describe the evolution of the system of electrons on the Landau levels after the first and second pulses, respectively, while the latter is just responsible for the echo signal. Note that the amplitude of the echo signal depends on the amplitudes and durations of the first and second pulses (in the case of small quantities) as

$$A_{\text{echo}} \propto E_1 E_2^2 t_1 t_2^2\,,$$

which is in agreement with the corresponding dependence for the Hahn echo.[1] This is due to the mechanism of echo formation, which occurs because of the time dependence of the electron wave functions described by bispinors according to Eq. (9). The dependences obtained for bigraphene basically coincide with Eq. (19) except for a numerical factor related to the fact that the wave function is now represented by a pair of bispinors similar to Eq. (9).

We can also give estimates for the possibility to observe this effect. Given that the relaxation times of electrons in graphene are of the order of 10^{-11} s, we can choose the frequency of the pulse of the order of 10^{14} s^{-1} and its duration of the order of 10^{-13} s. That allows us to observe the effect at times of about three orders of magnitude larger than the pulse duration. Note that in this study we deal with the system of units $\hbar = c = 1$. The magnetic field has to be typical for an experimental observation of Landau levels in graphene, i.e. of the order of one Tesla. For the electric field we expect a typical value for laser pulses, 10^6 V/m, which makes it possible to achieve quite measurable currents in the signal echo of the order of nano-ampere.

4. Conclusions

The irradiation of graphene in a magnetic field by two pulses of alternating electric field with the carrier frequency equal to the frequency of transitions between Landau levels of electronic system can lead to the appearance of the echo effect. Despite a similar expression for the echo signals, this effect is different from both the plasma echo and the echo on the atomic levels. In contrast to the echo on the atomic levels, in our case the electrons are not localized in the atom or even within a few lattice cells. Unlike the plasma echo, we have a fundamentally different mechanism for the formation of the attenuation of signals arising from the action of the pulse and echo itself (i.e. the attenuation is not associated with the Landau damping). Moreover, in our problem, the echo effect appears due to the phase memory of electrons associated with certain energy levels.

The effect we have revealed can be efficiently used in a variety of storage and information processing devices. The advantage of this effect, in particular, is that the response is simply a current flowing through the graphene. Thus, it is possible to combine the processing of optical information in the far-infrared range simultaneously with its transformation into pulses of electrical current. The latter is certainly important for the development of hybrid devices, which combine the speed attainable in optical transmission and data processing, and the advanced circuitry of electronic devices. Note also that this effect allows us to use all the advanced information processing methods based on the spin echo, which is associated with a similar dependence of the echo signal on the pulse amplitudes of the pump.

In addition, the stimulated three-pulse echo effect must also take place in the system under discussion. The latter would be practically useful for quantum memory devices. Note that in this case, we have to emit the pulses with carrier frequencies equal to the frequencies of transitions between different Landau levels, and the choice of the Landau level can be determined from the requirement that this level would have the greatest lifetime.

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