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Computational analysis of coupled physical fields in piezothermoelastic media

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Abstract

The dynamics of piezoelectric solids is studied computationally with a mathematical model coupling the equation of motion, the Maxwell equation for electric field and the energy balance equation. The main results are reported for (hollow) cylindrical piezoelectric elements with radial preliminary polarization where a strong effect of thermo-electromechanical coupling is amplified by the mechanical boundary conditions for stresses. Such elements (and the associated models allowing computations of their dynamics under different thermo-electromechanical loading conditions) become increasingly important not only in the traditional applications of piezoelectrics such as piezotransducers, but also in the context of smart materials and structures applications. Based on the variational approach and theory of generalized solutions efficient numerical schemes for a fully coupled model have been developed. Computational results are presented for a hollow cylindrical element made of PZT-5A piezoceramics. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

Many new challenging problems in science, engineering and industry cannot be solved efficiently without invoking a unification of two or more physical theories that have been traditionally considered separately [1]. A classical example of such a situation is dynamic thermoelasticity where on the basis of thermodynamic theory of irreversible processes two physical theories, the elasticity theory and theory of heat conduction, are treated as a unified whole [2]. The work of many technical devices are often based on the mutual interaction of two or more different physical fields, e.g., electric and mechanical fields, magnetic and mechanical fields, mechanical and acoustic fields, etc. [1,3]. In response to a huge potential of electromechanical systems in aerospace, automotive, medical, and consumer-goods manufacturing industries, and due to an increasing interest in applications of smart materials and structures in a variety of fields [4–9], the development and applications of physical models that account for the coupling between thermal, mechanical, and electric fields become essential prerequisites for further technological advances in these important areas. Due to their ability to generate a charge in response to a mechanical deformation and to provide mechanical strains when an electric field is applied piezoelectrics

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become an integral part of smart material and structure technology [6,7,10,11]. Although piezoelectric devices such as sensors and actuators are already used widely as embedded control elements in advanced composites and smart structures [5,7,12] that could be subjected to severe mechanical and thermal environments, coupled thermoelectromechanical models describing adequately the dynamics of these devices and efficient numerical methods for their solution are at the stage of intensive development [6,10,11,13].

The present paper is a further development of a general methodology to coupled problems of electroelasticity originally proposed by the author (see [1,14] and references therein). In contrast to many other papers in this field (see, e.g., [15,16] and references therein) we consider the full thermo-electromechanical coupling amplified by mechanical boundary conditions for stresses. Such situations are not amenable to analytical treatments. Note that analytical treatments have been developed for some special cases of non-radial preliminary polarizations and/or the case of harmonic oscillations (see [14,17,18] and references therein). In the general case the development of efficient numerical methods are required. The main ideas for this development, pursuit in this paper, are based on the variational approach and generalized solutions theory. These ideas have much in common with the finite element approach used in the literature for coupled field problems for such structural elements as laminates, shells, and plates (see [9,19,20] for further details and references). The main interest in the present paper is different and lies with the general non-stationary behaviour of piezoelectric elements of hollow cylindrical shapes which are becoming increasingly important in applications ranging from the design of piezotransducers to the vibration control of flexible structures with piezo-sensors/actuators.

2. Mathematical models for fully coupled physical fields in thermopiezoelectric media

Although the development of models capable of describing unsteady behaviour of “smart” materials such as piezoelectrics has fundamental importance in many areas of applications, most studies performed in this field hitherto have been limited to stationary problems only [16] and/or to the description of purely electromechanical fields, without taking into account the influence of the thermal field [9,16]. During recent years the interest to modeling static and dynamic behaviour of piezoelectric solids with coupled models have increased dramatically (e.g., [9,13–15,19] and references therein). The model under consideration in this paper is an extension of a coupled dynamic system for piezoelectrics [9] and includes the equation of motion, the Maxwell equation for electric field, and the energy balance equation

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \sigma_{ji,i} + F_i, \qquad D_{i,i} = f_2, \qquad T\frac{\partial S}{\partial t} = -q_{i,i} + Q, \quad i, j = 1, 2, 3, \tag{2.1}$$

where u_i are components of the mechanical displacement vector $\mathbf{u}$, ρ is the density of the material, $\boldsymbol{\sigma} = \{\sigma_{ij}\}$ is the stress tensor, $\mathbf{D} = \{D_i\}$ are electric displacements, θ is the difference between the current temperature, T, and the reference temperature, T_0 (i.e. $T = T_0(1 + \theta/T_0)$), $\mathbf{F} = \{F_i\}$ are components of the vector of mass forces, f_2 is the electric charge density, S is the system entropy, $\mathbf{q} = (q_1, q_2, q_3)$ is the heat influx through the system surface, and Q is the energy due to heat generation inside of the body. As a development of [9], the full thermo-electromechanical (rather then electromechanical only) coupling is considered by employing a general form of the state equations where we choose strain, electric field and temperature as independent variables. In particular, the constitutive equation for stress is considered in this paper in the generalized Duhamel–Neumann form

$$\boldsymbol{\sigma} = \mathbf{c}\boldsymbol{\epsilon} - \mathbf{e}^{\mathrm{T}}\mathbf{E} - \boldsymbol{\gamma}\theta \quad \text{or} \quad \sigma_{ij} = c_{ijkl}\epsilon_{kl} - e_{ijm}E_m - \gamma_{ij}\theta. \tag{2.2}$$

The constitutive equation for electric displacement is taken as

$$\mathbf{D} = \mathbf{e}\boldsymbol{\epsilon} + \boldsymbol{\varepsilon}\mathbf{E} + \mathbf{p}\theta \quad \text{or} \quad D_i = \varepsilon_{ik}E_k + e_{ikl}\epsilon_{kl} + p_i\theta. \tag{2.3}$$

And the entropy of the thermoelectroelastic system under consideration is

$$S = \boldsymbol{\gamma}^{\mathrm{T}}\boldsymbol{\epsilon} + \mathbf{p}^{\mathrm{T}}\mathbf{E} + c_{\epsilon}^{V}\theta/T_0 \quad \text{or} \quad S = \gamma_{ij}\epsilon_{ij} + c_{\epsilon}^{V}\theta/T_0 + p_iE_i. \tag{2.4}$$

In constitutive equations (2.2)–(2.4) $\boldsymbol{\epsilon} = \{\epsilon_{ij}\}$ denotes strains, $\mathbf{E} = \{E_i\} = -\nabla\varphi$ denotes the electric field with the potential φ, $\mathbf{c} = \{c_{ijkl}\}$ are elements of the matrix of elastic moduli, $\mathbf{e} = \{e_{ikl}\}$ are piezoelectric constants, $\boldsymbol{\gamma} = \{\gamma_{ij}\}$ are elements of the matrix product $\Lambda \cdot \Xi$ (Λ is the matrix of elastic moduli, Ξ is the thermal expansion matrix), $\boldsymbol{\varepsilon} = \{\varepsilon_{ik}\}$ are dielectric permittivities, and $\mathbf{p} = \{p_i\}$ are pyroelectric (thermoelectric coupling) constants. The Cauchy formula is assumed for the strain-displacement relation. Other notations are standard and explained in the text when required. Similar models have been considered in the literature for laminates, shells, and plates (e.g., [13,19]). The main focus of the present paper is piezoelectric elements of hollow cylindrical shapes.

3. Computing dynamics of cylindrical piezoelectric elements

In many applications where piezoelectric elements are used as part of engineering devices, the cylindrical topology has a number of advantages compared to other topological designs [9,21]. Motivated by this fact, the effects of dynamic coupling and thermally-induced vibrations are demonstrated for finite piezoelectric elements of cylindrical shape with hexagonal 6 mm symmetry. In this case we specify components of vectors $\boldsymbol{\sigma}$, $\boldsymbol{\epsilon}$, $\mathbf{E}$, $\mathbf{D}$, and $\boldsymbol{\gamma}$ as follows

$$\begin{aligned}&\boldsymbol{\sigma} = (\sigma_z, \sigma_\theta, \sigma_r, \sigma_{rz}, 0, 0)^{\mathrm{T}}, \quad \boldsymbol{\epsilon} = (\epsilon_z, \epsilon_\theta, \epsilon_r, \epsilon_{rz}, 0, 0)^{\mathrm{T}}, \quad \mathbf{E} = (E_z, E_\theta, E_r)^{\mathrm{T}},\\ &\mathbf{D} = (D_z, D_\theta, D_r)^{\mathrm{T}}, \quad \boldsymbol{\gamma} = (\gamma_{11}, \gamma_{11}, \gamma_{33}, 0, 0, 0)^{\mathrm{T}}.\end{aligned} \tag{3.1}$$

The volumetric heat capacity is $c_\epsilon^V = \rho c_\epsilon^S$, where c_ϵ^S is the specific heat capacity. The thermoelastic pressure vector is defined via the vector of thermal expansion $\boldsymbol{\alpha}$ as

$$\boldsymbol{\gamma} = \mathbf{c}\boldsymbol{\alpha}, \quad \text{and the pyroelectric vector is} \quad \mathbf{p} = (0, 0, p_3)^{\mathrm{T}}.$$

The elastic, piezoelectric, and dielectric permittivity matrices in the case under consideration have the following forms

$$\mathbf{c} = \begin{pmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0\\ c_{12} & c_{11} & c_{13} & 0 & 0 & 0\\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & c_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\end{pmatrix}, \qquad \mathbf{e}^{\mathrm{T}} = \begin{pmatrix} 0 & 0 & e_{13}\\ 0 & 0 & e_{13}\\ 0 & 0 & e_{33}\\ e_{15} & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}, \qquad \boldsymbol{\varepsilon} = \begin{pmatrix} \varepsilon_{11} & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & \varepsilon_{33}\end{pmatrix}. \tag{3.2}$$

In what follows we concern with nonstationary axisymmetric oscillations of hollow thin-walled piezoceramic cylinders. If the thickness of such cylinders is comparable with their length, one dimensional models (see R.V.N. Melnik, Int. Comm. Heat Mass Transfer and [22]), cannot describe adequately most important characteristics of piezoelectric-based devices, and the solution of a general 2D time-dependent (rather than steady-state) problem is required. The problem at hand includes the equations of motion coupled to the Maxwell equation and to the energy balance equation

$$\begin{cases} \rho \dfrac{\partial^2 u_r}{\partial t^2} = \frac{\partial \sigma_r}{\partial r} + \dfrac{\partial \sigma_{rz}}{\partial z} + \dfrac{\sigma_r - \sigma_\theta}{r} + F_1,\\ \rho \dfrac{\partial^2 u_z}{\partial t^2} = \frac{\partial \sigma_{rz}}{\partial r} + \dfrac{\partial \sigma_z}{\partial z} + \dfrac{\partial \sigma_{rz}}{r} + F_2,\\ \dfrac{1}{r}\dfrac{\partial}{\partial r}(r D_r) + \dfrac{\partial D_z}{\partial z} = f_2,\\ T_0\left(1 + \dfrac{\theta}{T_0}\right)\frac{\partial S}{\partial t} = \dfrac{1}{r}\dfrac{\partial}{\partial r}\left(kr\dfrac{\partial \theta}{\partial r}\right) + \dfrac{\partial}{\partial z}\left(k\dfrac{\partial \theta}{\partial z}\right) + Q(r, z, t).\end{cases} \tag{3.3}$$

This system is supplemented by the initial and boundary conditions given in the form

$$\mathbf{u}(r,z,0)=\mathbf{u}^{(0)}(r,z), \qquad \frac{\mathbf{u}(r,z,0)}{\partial t}=\mathbf{u}^{(1)}(r,z), \tag{3.4}$$

with $\mathbf{u}=(u_r,u_z,0)$, $\mathbf{u}^{(0)}=(u_r^{(0)},u_z^{(0)},0)$, $\mathbf{u}^{(1)}=(u_r^{(1)},u_z^{(1)},0)$,

$$\begin{aligned}
&\sigma_r(R_i,z,t)=p_r^{(i)}(z,t), \qquad &&\sigma_z(r,Z_i,t)=p_z^{(i)}(r,t),\\
&\sigma_{rz}(R_i,z,t)=p_{zt}^{(i)}(z,t), \qquad &&\sigma_{rz}(r,Z_i,t)=p_{rt}^{(i)}(r,t),\\
&\varphi(R_i,z,t)=\pm V(z,t), \qquad &&D_z(r,Z_i,t)=0, \quad i=0,1.
\end{aligned} \tag{3.5}$$

In what follows we briefly outline some important points in the reduction of the model to a computationally amenable form and the implementation of the numerical scheme for the solution of (3.3)–(3.5).

Due to a substantial difference in parameter magnitudes we applied a scaling procedure to problem (3.3)–(3.5) prior to its numerical solution. The procedure was a direct extension of the scaling procedure used in [22] for the 1D case. Then the energy balance equation was reduced to the following form

$$\frac{\partial\tilde\theta}{\partial\tilde t}=\frac{A}{\omega c_\epsilon^V}\left[\frac{1}{\tilde r}\frac{\partial}{\partial\tilde r}(k\tilde r\nabla\tilde\theta)+\frac{\partial}{\partial\tilde z}\left(k\frac{\partial\tilde\theta}{\partial\tilde z}\right)+\tilde Q\right]-\frac{\partial}{\partial\tilde t}\left(\tilde\gamma_{33}^{\mathcal E}\tilde\epsilon_r+\tilde\gamma_{11}^{\mathcal E}\tilde\epsilon_\theta+\tilde\gamma_{11}^{\mathcal E}\tilde\epsilon_z+\tilde p_3^{\mathcal E}\tilde E_r\right), \tag{3.6}$$

with

$$\widetilde Q=R_*^2Q/\theta_*, \qquad A=t_*/R_*^2, \qquad \omega=1+\theta_*\tilde\theta/T_0. \tag{3.7}$$

In (3.6)–(3.7) all dimensionless variables and associated with them coefficients (denoted by tilde) are determined via normalized factors (denoted by star), e.g.,

$$\tilde\gamma_{ii}^{\mathcal E}=\gamma_{ii}B\frac{u_*}{R_*}, \qquad \tilde p_i^{\mathcal E}=p_iB\frac{\varphi_*}{R_*}, \quad i=1,3 \quad \text{with } B=\frac{T_0}{\theta_*c_\epsilon^V}. \tag{3.8}$$

The representation of the energy balance equation in form (3.6) allows us to extend numerical procedures described and analyzed in details in [9] to the present problem. The importance of coupling in the modeling piezoelectric-based devices have been emphasized in a number of recent papers [3,9,13,14,19] and the last section of this paper is devoted to the quantification of the coupling effect for finite length hollow cylinders made of PZT-5A piezoceramics (see Table 1 for characteristics of this material). This material is known as an excellent material to use in applications of acoustic wave generation, piezo-transducers/transformers design, and piezo-sensors/actuators in the vibration control of flexible structures. All reported results have been obtained with the numerical scheme derived by using the techniques of the theory of generalized solutions linked to the variational approach. The rigorous analysis of the scheme (including the analysis of the scheme accuracy), based on these techniques, is a relatively straightforward extension of the methodology described in [9].

Table 1
Characteristics of PZT-5A piezoceramics

c_{11}	12.1×10^{10}	N/m^2	e_{33}	15.8	C/m^2
c_{12}	7.54×10^{10}	N/m^2	e_{15}	12.3	C/m^2
c_{13}	7.52×10^{10}	N/m^2	e_{13}	-5.4	C/m^2
c_{33}	11.1×10^{10}	N/m^2	ϵ_0	8.85×10^{-12}	F/m
c_{44}	2.11×10^{10}	N/m^2	p_3	$4.\times10^{-4}$	C/(m^{2}K)
α_{33}	2.5×10^{-6}	1/°C	p_3	$4.\times10^{-4}$	C/(m^{2}K)
α_{11}	2.5×10^{-6}	1/°C	c_ϵ^S	420	J/(kg°C)
ρ	7.75×10^3	kg/m^3	k	1.25	W/(kgK)

4. Computational experiments on the effects of dynamic thermoelectromechanical coupling

Many efforts have been devoted to vibration suppression of piezoelectric-based structures subjected to mechanical loading (e.g., [10]). However, since many such structures work under fairly large variations in temperature in the presence of electric fields, our experiments concern primarily the influence of thermal and electrical loading patterns on mechanical characteristics of piezoelectric-based devices. For this reason, in all experiments we assume stress-free boundary conditions and neglect any mass forces (that is $p_r^{(i)} = p_z^{(i)} = p_{zt}^{(i)} = p_{rt}^{(i)} = 0$, $F_i = 0$, $i = 1, 2$).

First, by considering a cylinder with dimensionless thickness 0.05, applied potential difference $2V = 1$, and thermal loading $Q = A\sin(t)$, we change the value of A in the range from 0 to 20. The results of computing radial displacements on the external surface of the cylinder confirmed that the effect of thermal field is practically negligible in this case and our results were practically identical to those obtained in the framework of electroelasticity theory (see [9,22] and references therein). However, we found that with increasing further the values of A the effect of full thermoelectromechanical coupling becomes essential. In Fig. 1 we present the results of computing radial displacements as a function of time, as well as spatial distributions of displacements at the dimensionless moment of time $t = 12$ (results are presented for the constant voltage case where $V = \pm 0.5$ and $A = 40$).

In many piezo-sensor/actuator applications thermally/thermomechanically induced oscillations should be controlled. This can be achieved by using the voltage (e.g., supplied to the actuators) as a control variable. The effect of variable voltage on the displacements is demonstrated in Fig. 2. Due to a strong coupling between electric and elastic fields, this effect is easy to observe even for relatively small values of temperature (results presented in Fig. 2 were obtained for $A = 10$).

The effect of increasing temperature on the mechanical characteristics of the device (the constant voltage case) is demonstrated in Fig. 3 where the time distribution of the radial displacements is given along with their spatial distribution (at the dimensionless moment of time $t = 42$). Finally, the results of typical calculations for the variable voltage case are presented in Fig. 4 obtained with $A = 80$.

Further increase in temperature, leads to the situation where nonlinear effects become increasingly important due to violation of the condition $|\theta/T_0| \ll 1$.

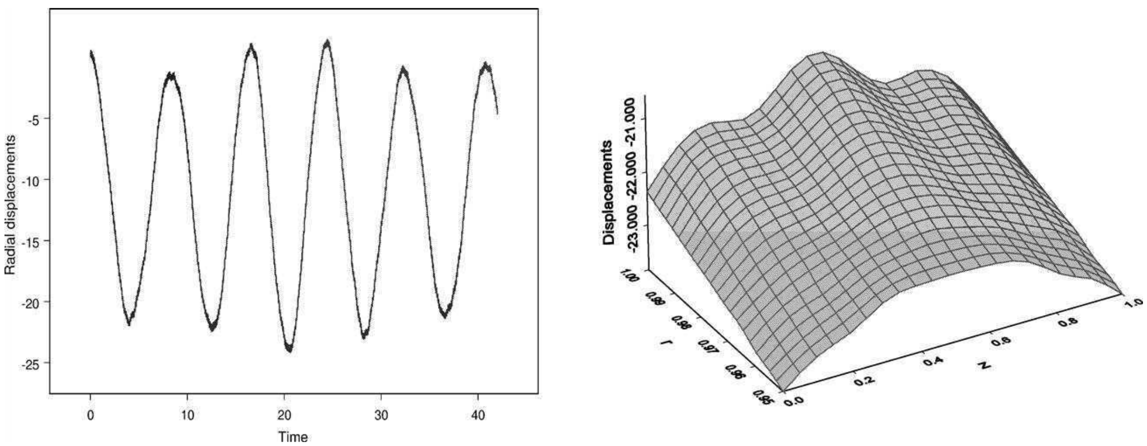

Fig. 1. Mechanical characteristics in a fully coupled thermoelectroelastic problem.

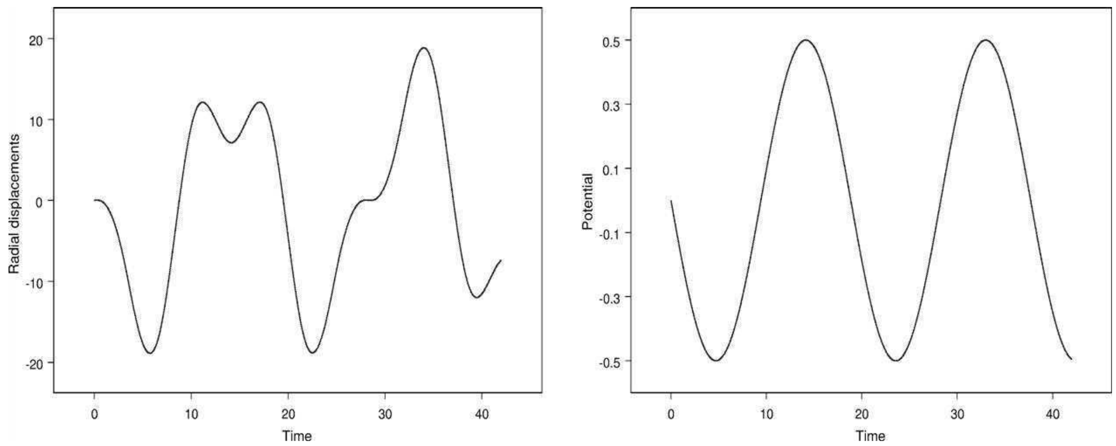

Fig. 2. Voltage as a control variable of thermomechanical characteristics of devices.

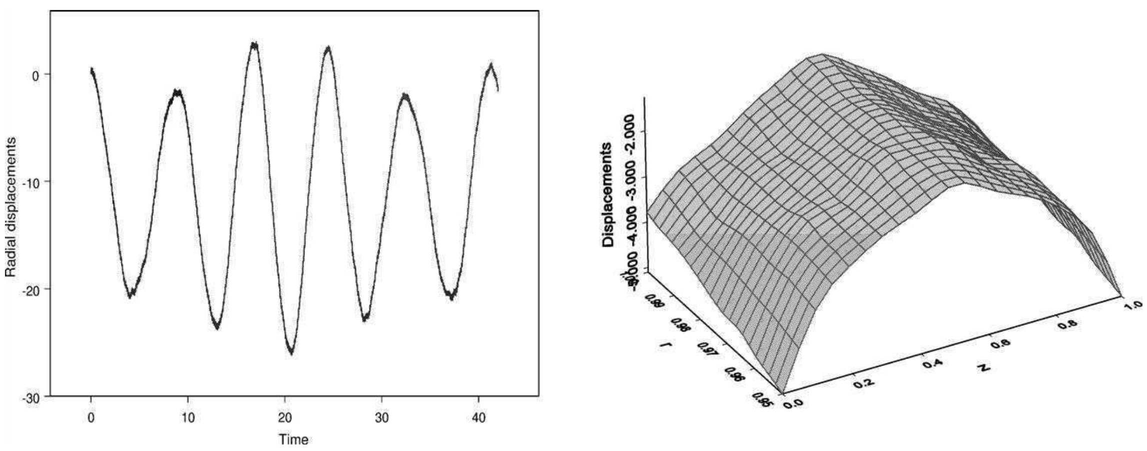

Fig. 3. Fully coupled thermoelectroelastic problem (constant voltage).

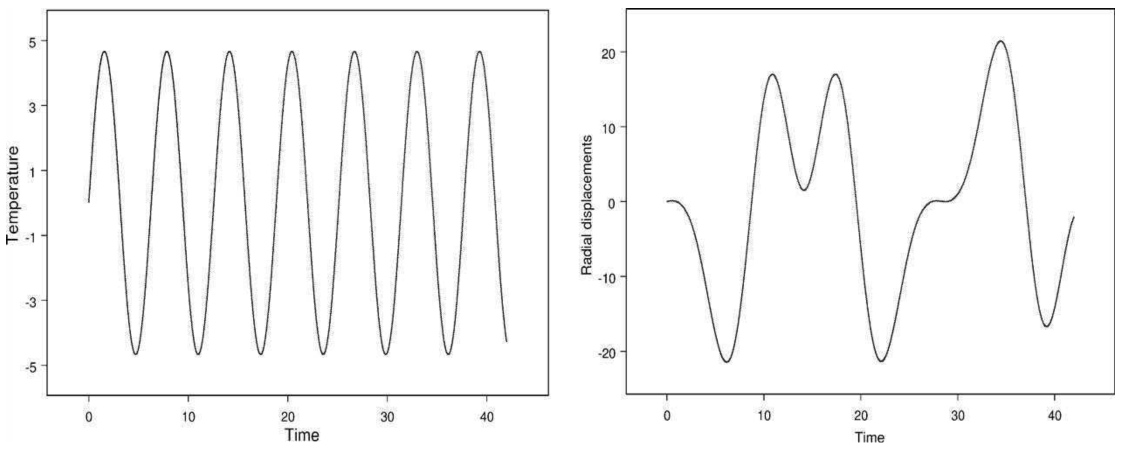

Fig. 4. Full coupling: temperature and radial displacements (variable voltage).

5. Conclusions

The effects of coupling between physical fields in piezothermoelectric media have been analyzed computationally with a special emphasis given to the influence of the coupled thermoelectric field on mechanical characteristics of piezoceramic-based devices. In particular, on the example of axisymmetric oscillations of hollow thin-walled piezoceramic cylinders it was shown that the thermal field can contribute substantially to such important characteristics of devices as radial displacements. The observed effects become increasingly important in such cases where a piezoelectric has to be considered as a part of smart structure (especially when such a structure/substrate contains a polymeric composite material) whose response piezoelectric sensors should monitor, or where the piezoelectric-based device is imbedded into an acoustic interacting media.

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