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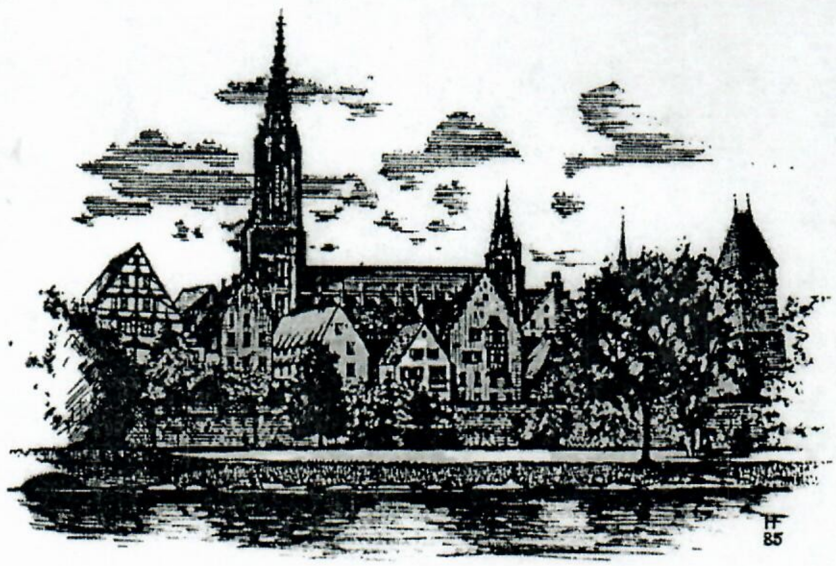

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A Hierarchy of Hyperbolic Macrodynamic Equations as a Model for Network Training

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Abstract — **The author proposes mathematical models of hyperbolic type for training of neural networks, and its computational implementation using the Markov Chain approximation method.**

I. Introduction

Let a mapping $x_t : \mathcal{R} \to \Sigma$ be a sigmoidal function (activator) coupled to a neural network defined by its neurons

$$x_t \circ \mu : T \otimes \Sigma \otimes U_T \to \mathcal{R}, \tag{1}$$

where $\mu : T \otimes \Sigma \otimes U_T \to \mathcal{R}$ will be referred to as the decision making function. In (1) we assume that the network is trained by a dynamic system with the state space Σ during time defined by a set T, and U_T is a set of all permissible training strategies. Let H be a mapping $X_T \to R^M$, and $H_T : \mathcal{N} \to \mathcal{N}$ is a T-computable function. Then H can be in principle arbitrarily well approximated by a network implementing H_T due to the Gödel numeration procedure. On the one hand if X_T is a compact Borel set and $H \in L^1(X_T)$ then for any arbitrary small $\epsilon > 0$ there exists a feedforward neural network (FNN) $\tilde{H}$ such that $||H - \tilde{H}||_{L^1(X_T)} < \epsilon$. On the other hand there exists a block feedback network (BFN) $\tilde{H}_T$ such that for any input $n \in M \subset \mathcal{N}$ a finite number of network steps produces $H(n)$ [2]. The main purpose of this paper is to show how both of the above approaches are connected by a training algorithm that can change weight values and the network structure.

II. Networks Training as a Problem in Optimal Control

Let us choose as a function H the Hamiltonian of the dynamic system associated with the network defined above. Then the process of approximation of the function H can be seen as a construction of a training strategy for a network $\tilde{H}_n^\epsilon$ such that $||H - \tilde{H}_n^\epsilon||_{L^1(X_T)} \to min$, where $H \in L^1(X_T)$, ϵ is an arbitrary small positive number, and n is the number of sigmoidal nodes of the network. Though we do not know *a-priori* the character of dependency of the network on parameters which determine the function μ, the author observes that the problem of constructing the mapping μ is intrinsically connected with the definition of dynamic rules in *singular stochastic control problems.* Mathematically speaking, we expect that if $\epsilon \to 0^+$ and $n \to \infty$ then $\tilde{H}_n^\epsilon \to H$, but to prove this conjecture rigorously we need a connection rule between ϵ, n, and the topology specified in X_T. Such a connection defines the system stability. Similarly, BFN can be applied to the approximation of H on an arbitrary set X_T, but questions of network dimension and architecture should be addressed [2]. Using this analogy it is proposed to analyse the network performance using a family of Discrete Markovian Decision processes. In the constructed hyperbolic mathematical model the training process depends on Markov Chain parameters with possible discontinuities that are dependent on values of the sigmoidal function x_t. If $H \in L^1(X_T)$ the process of training is associated with the generalized energy equation [1]:

$$(1+v_1)\left[\frac{\partial \mu}{\partial x} + \frac{1}{v_1}\left(\frac{\partial \mu}{\partial t} + f_0\right)\right] = 0, \tag{2}$$

where v_1 is the velocity of information transmission between neurons, and f_0 is a training goal defined by a-priori knowledge on X_T and the function H. Algorithmically the function μ is required to be adapted to such *a-priori* given information.

III. Networks and Markov Chains

Many mathematical problems in optimal control and information theory is reducible to the modelling of the training process for networks. The main results of this paper is based on the idea of the Markovian character of the process of training (x_t, μ). The author develops some previously obtained results for the Generalized Dynamic System (GDS) evolution [1]. Using conditional probabilities of the Markov Chain jumps the notion of the GDS velocity function between two macroscopic events is introduced as a measure of changes which take place on the microscopic level with respect to the macroscopic behaviour of the system. It is proposed to view a class of neural networks with built-in training ability as Infinite Length Perturbed Markov Chains (ILPMC). The practical implementation of training requires an approximation of the network activator by a *piecewise-deterministic stochastic process.* Such non-diffusion stochastic models have been previously studied by mathematicians in the theory of DMDP and recently attracted the attention of theoretical physicists. Since in practice information on the function H and topology of X_T are often subject to *a-priori* assumptions, the author proposes an approximation of the training process by an appropriately constructed Markov Chain. Stability of such approximation has been derived when the decision making function is modeled by (2).

IV. Conclusions.

This study proposed a hyperbolic mathematical model for the process of network training. An analogy of this process with the solution of optimal control problems has been revealed and new results in this direction have been obtained. A connection of trained neural networks with Perturbed Markov Chains allows us to derive computational models for the process of training. Finally, stability of the process has been discussed.

References

[1] V.N. Melnik, "Nonconservation law equation in mathematical modelling: aspects of approximation," *Proc. of the International Conference AEMC'96, Sydney*, pp. 423-430, 1996.

[2] S. Santini and A. Del Bimbo, "Properties of block feedback neural networks," *Neural Networks*, Vol.8, No.4, pp. 579–596, 1995.

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Schriftliches Grußwort von Herrn Minister
Klaus von Trotha
für den Tagungsband „I S I T '97" in Ulm

It is my pleasure to welcome the IEEE Information Theory Society to Baden-Württemberg for its International Symposium on Information Theory. It shows that the German scientists working in this field have major contributions to this growing area of science. The presence of so many well known researchers in the field will definitely stimulate our students to start or continue their studies in this direction of Information Sciences on a high level. The IEEE German Chapter on Information Theory has about 200 members, highlighting the importance of the Information Theory Society for the local scientific community. Germany has a great tradition in research, and Information theory plays an important role in the fields of Mathematics and Communications. Several publications by Claude Shannon have appeared in German technical journals. A well-known meeting point for Information theorists is the Oberwolfach conference center in the beautiful Black Forest, not far from Stuttgart. I am sure that many new contacts will be made at this Symposium, not only between scientists, but also between science and industry. The choice of the city of Ulm as venue is an excellent one from several points of view.

Ulm is a city dedicated to science with many research establishments and a young university. It has been chosen to be the city where research in multimedia applications will be concentrated in the near future. Ulm is in the middle of a number of very exciting attractions. I hope that you will take the opportunity to visit a few of these and that your stay will be a profitable one, both from a scientific as well as from a cultural point of view. I conclude using the famous Ulmer saying that it is „In Ulm, um Ulm, und um Ulm herum", where the action is!

Klaus von Trotha

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