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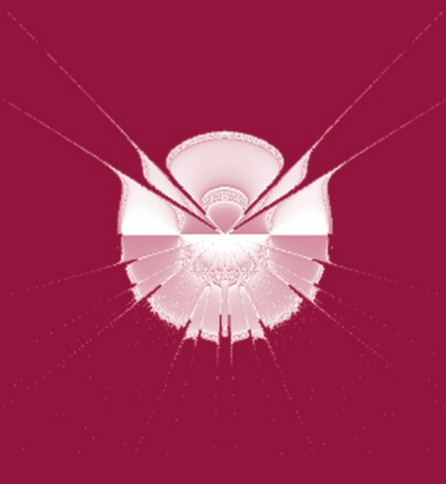

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Evolutionary Monte Carlo Based Techniques for First Passage Time Problems in Credit Risk and Other Applications in Finance

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Abstract. Evolutionary computation techniques are closely connected with Monte Carlo simulations via statistical mechanics. Most practical realizations of such a connection are based on Markov chain Monte Carlo procedures and Markov chain approximation methodologies. However, such realizations face challenges when we have to deal with multivariate situations. In this contribution, we consider the development of evolutionary type Monte Carlo based algorithms for dealing with jump-diffusion stochastic processes. In particular, we focus on the first passage time problems for multivariate correlated jump-diffusion processes in the context of credit risk and the analysis of default correlations. The developed technique can be useful in option pricing as well as in other areas of complex systems analysis.

Keywords: Evolutionary Monte Carlo based techniques; Credit risk; Default correlations; Brownian bridge simulations; Complex systems; Multivariate jump-diffusion processes.

1 Introduction

Evolutionary computation techniques are common in dealing with various optimization, integration, and sampling problems in science and engineering where we attempt to explore the search space with a population that is with a multi-subset of such a subset. The connection between evolutionary algorithms and Monte Carlo methods has been analyzed by a number of authors (e.g., [3,4,8]). It is quite natural to utilize this connection in the multivariate stochastic models where we deal the analysis of more than one statistical variable.

Evolutionary computation techniques usually include both selection and variation where during the selection we replicate an individual in the population based on selection probabilities, while their stochastic perturbations during this process are viewed as variation. The selection mechanism can be based on different distributions and corresponding algorithms including Boltzmann-Gibbs, Tsallis-Stariolo, Metropolis-Hasting and others [6,8]. One of the ways of integrating evolutionary techniques and Monte Carlo is to apply Evolutionary Markov

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Chain Monte Carlo (EMCMC) methodologies [4], deeply rooted in the Markov chain approximations [15].

Several application areas in data mining for EMCMC methods have been recently explored [8,5] and several evolutionary strategy algorithms based on sequential Monte Carlo simulations have been proposed [10]. The interest in this new development has been largely stimulated by the fact that standard MCMC methods can be trapped in a local mode indefinitely [9]. Attempts were made in the past to remedy this difficulty by developing multiple MCMC that can be run in parallel, each of which can, in principle, be characterized by different (yet related) distributions [7]. This followed by more recent interest to interacting MCMC algorithms [2] and by a number of interesting works directed to solving problems with complex multi-modal probability density landscapes, to the analysis of average properties of complex systems, as well as to the development of generic learning strategies [13,8,15,14,18].

This latter set of works inspired us to develop these ideas in the context of credit risk problems. There are two aspects that are intrinsic to the problems we are dealing with in this paper: (a) we deal with the first passage times (FPT) of stochastic processes with jumps, and (b) we are interested in the multivariate (and correlated) case.

2 First Passage Time in Credit Risk Models

Many problems in finance require the information on the FPT of a stochastic process. Mathematically, such problems are often reduced to the evaluation of the probability density of the time for such a process to cross a certain level, a boundary, or to enter a certain region. While in other areas of applications the FPT problem can often be solved analytically, in finance we usually have to resort to the application of numerical procedures, in particular when we deal with jump-diffusion stochastic processes (JDP).

Credit risk can be defined as the possibility of a loss occurring due to the financial failure to meet contractual debt obligations. This is one of the measures of the likelihood that a party will default on a financial agreement.

In structural credit-risk models, a default occurs when a company cannot meet its financial obligations, or in other words, when the firm's value falls below a certain threshold. One of the major problems in credit risk analysis is when a default occurs within a given time period and what is the default rate during such a time period. This problem can be reduced to a FPT problem, that can be formulated mathematically as a certain stochastic differential equation (SDE). It concerns the estimation of the probability density of the time for a random process to cross a specified threshold level. Therefore, it is natural that the FPT problem occurs also frequently in other areas of applications, including many branches of science and engineering [17,19].

An important phenomenon that we account for in our discussion here lies with the fact that, in the market economy, individual companies are inevitably linked together via dynamically changing economic conditions. Therefore, the default events of companies are often correlated, especially in the same industry. Authors of [21] and [11] were the first to incorporate default correlation into the Black-Cox first passage structural model, but they have not included the jumps. As pointed out in [22] and [12], the standard Brownian motion model for market behavior falls short of explaining empirical observations of market returns and their underlying derivative prices. In the meantime, jump-diffusion processes have established themselves as a sound alternative to the standard Brownian motion model [1]. Multivariate jump-diffusion models provide a convenient framework for investigating default correlations with jumps and become more readily accepted in the financial world as an efficient modeling tool.

However, as soon as jumps are incorporated in the model, except for very basic applications where analytical solutions are available, for most practical cases we have to resort to numerical procedures. Examples of known analytical solutions include problems where the jump sizes are doubly exponential or exponentially distributed [12] as well as the jumps can have only nonnegative values (assuming that the crossing boundary is below the process starting value). For other situations, Monte Carlo methods remain a primary candidate for applications.

In a structural model, a firm i defaults when it can not meet its financial obligations, or in other words, when the firm assets value $V_i(t)$ falls below a threshold level $D_{V_i}(t)$. Generally speaking, finding the threshold level $D_{V_i}(t)$ is one of the challenges in using the structural methodology in credit risk modeling, since in reality firms often rearrange their liability structure when they have credit problems. In this contribution, we use an exponential form defining the threshold level $D_{V_i}(t) = \kappa_i \exp(\gamma_i t)$ as proposed by [21], where γ_i can be interpreted as the growth rate of the firm's liabilities. Coefficient κ_i captures the liability structure of the firm and is usually defined as the firm's short-term liability plus 50% of the firm's long-term liability. If we set $X_i(t) = \ln[V_i(t)]$, then the threshold of $X_i(t)$ is $D_i(t) = \gamma_i t + \ln(\kappa_i)$. Our main interest is in the process $X_i(t)$.

Prior to moving further, we define a default correlation that measures the strength of the default relationship between different firms. Take two firms i and j as an example, whose probabilities of default are P_i and P_j, respectively. Then the default correlation can be defined as

$$\rho_{ij} = \frac{P_{ij} - P_i P_j}{\sqrt{P_i(1-P_i)P_j(1-P_j)}}, \tag{1}$$

where P_{ij} is the probability of joint default. From Eq. (1) we have $P_{ij} = P_i P_j + \rho_{ij}\sqrt{P_i(1-P_i)P_j(1-P_j)}$. Let us assume that $P_i = P_j = 5\%$. If these two firms are independent, i.e., the default correlation equals zero, then the probability of joint default is $P_{ij} = 0.25\%$. If the two firms are positively correlated, for example, $\rho_{ij} = 0.4$, then the probability that both firms default becomes $P_{ij} = 2.15\%$ which is almost 10 times higher than in the former case. Thus, the default correlation ρ_{ij} plays a key role in the joint default with important implications in the field of credit analysis.

3 Multivariate Jump-Diffusion Processes and Monte Carlo Simulations

Although for jump-diffusion processes, the closed form solutions are usually unavailable, yet between each two jumps the process is, generally speaking, a Brownian bridge for a univariate jump-diffusion process. Authors of [1] have deduced the one-dimensional first passage time distribution for time period $[0, T]$. In order to evaluate multiple processes, we obtain multi-dimensional formulas and reduce them to computable forms.

Let us consider N_{firm} firms $\boldsymbol{X}_t = [X_1, X_2, ..., X_{N_{\text{firm}}}]^T$, each X_i describes the process of individual firm i. We expect that each process X_i satisfies the following SDE:

$$\begin{aligned} dX_i &= \mu_i dt + \sum_j \sigma_{ij} dW_j + dZ_i \\ &= \mu_i dt + \sigma_i d\tilde{W}_i + dZ_i, \end{aligned} \tag{2}$$

where $\tilde{W}_i$ is a standard Brownian motion and σ_i is:

$$\sigma_i = \sqrt{\sum_j \sigma_{ij}^2}.$$

We assume that in the interval $[0, T]$, the total number of jumps for firm i is M_i. Let the jump instants be $T_1, T_2, \cdots, T_{M_i}$. Let $T_0 = 0$ and $T_{M_i+1} = T$. The quantities τ_j equal interjump times, which are $T_j - T_{j-1}$. Following the notation of [1], let $X_i(T_j^-)$ be the process value immediately before the jth jump, and $X_i(T_j^+)$ be the process value immediately after the jth jump. The jump-size is $X_i(T_j^+) - X_i(T_j^-)$, and we can use such jump-sizes to generate $X_i(T_j^+)$ sequentially.

Let $A_i(t)$ be the event consisting of process X_i crossing the threshold level $D_i(t)$ for the first time in the interval $[t, t + dt]$, then the conditional interjump first passage density is defined as [1]:

$$g_{ij}(t) = P(A_i(t) \in dt | X_i(T_{j-1}^+), X_i(T_j^-)). \tag{3}$$

If we only consider one interval $[T_{j-1}, T_j]$, we can obtain

$$\begin{aligned} g_{ij}(t) = &\frac{X_i(T_{j-1}^+) - D_i(t)}{2y_i \pi \sigma_i^2} (t - T_{j-1})^{-\frac{3}{2}} (T_j - t)^{-\frac{1}{2}} \\ &* \exp\left(-\frac{[X_i(T_j^-) - D_i(t) - \mu_i(T_j - t)]^2}{2(T_j - t)\sigma_i^2} \right) \\ &* \exp\left(-\frac{[X_i(T_{j-1}^+) - D_i(t) + \mu_i(t - T_{j-1})]^2}{2(t - T_{j-1})\sigma_i^2} \right), \end{aligned} \tag{4}$$

where

$$y_i = \frac{1}{\sigma_i\sqrt{2\pi\tau_j}} \exp\left(-\frac{[X_i(T_{j-1}^+) - X_i(T_j^-) + \mu_i\tau_j]^2}{2\tau_j\sigma_i^2}\right).$$

After getting a result in one interval, we combine the results to obtain the density for the whole interval $[0, T]$. In a slight generalization, the process X_i may be viewed as a Brownian bridge $B(s)$ with $B(T_{j-1}^+) = X_i(T_{j-1}^+)$ and $B(T_j^-) = X_i(T_j^-)$ in the interval $[T_{j-1}, T_j]$, i.e. between each of the two successive jumps. If $X_i(T_j^-) > D_i(t)$, then the probability that the minimum of $B(s_i)$ is always above the boundary level is

$$P_{ij} = 1 - \exp\left(-\frac{2[X_i(T_{j-1}^+) - D_i(t)][X_i(T_j^-) - D_i(t)]}{\tau_j\sigma_i^2}\right), \tag{5}$$

and zero otherwise. The event "$B(s_i)$ is below the threshold level" means that the default happens or already happened, and its probability is $1 - P_{ij}$. Let $L(s_i) \equiv L_i$ denote the index of the interjump period in which the time s_i (first passage time) falls in $[T_{L_i-1}, T_{L_i}]$. Also, let I_i represent the index of the first jump, which happened in the simulated jump instant,

$$\begin{aligned} I_i = \min(j : X_i(T_k^-) > D_i(t); k = 1, \ldots, j, \text{ and} \\ X_i(T_k^+) > D_i(t); k = 1, \ldots, j-1, \text{ and} \\ X_i(T_j^+) \leq D_i(t)). \end{aligned} \tag{6}$$

If no such I_i exists, then we set $I_i = 0$. By combining Eq. (4), (5) and (6), we get the probability of X_i crossing the boundary level in the whole interval $[0, T]$ as

$$\begin{aligned} &P(A_i(s_i) \in ds | X_i(T_{j-1}^+), X_i(T_j^-), j = 1, \ldots, M_i + 1) \\ &= \begin{cases} g_{iL_i}(s_i)\prod_{k=1}^{L_i-1} P_{ik} & \text{if } L_i < I_i \text{ or } I_i = 0, \\ g_{iL_i}(s_i)\prod_{k=1}^{L_i-1} P_{ik} + \prod_{k=1}^{L_i} P_{ik}\delta(s_i - T_{I_i}) & \text{if } L_i = I_i, \\ 0 & \text{if } L_i > I_i, \end{cases} \end{aligned} \tag{7}$$

where δ is the Dirac's delta function.

For firm i, after generating a series of first passage times s_i, we use a kernel density estimator with Gaussian kernel to estimate the first passage time density (FPTD) f. The kernel density estimator is based on centering a kernel function of a bandwidth (similar to [16,1]).

Next, we develop a procedure for generating beforejump and postjump values $X_i(T_j^-)$ and $X_i(T_j^+)$, respectively. Here $j = 1, \cdots, M$ where M is the total number of jumps for all the firms. We compute P_{ij} according to Eq. (5). To recur the first passage time density $f_i(t)$, we have to consider three possible cases that may occur for each non-default firm i:

1. **First passage happens inside the interval.** We know that if $X_i(T_{j-1}^+) > D_i(T_{j-1})$ and $X_i(T_j^-) < D_i(T_j)$, then the first passage happened in the time

interval $[T_{j-1}, T_j]$. To evaluate when the first passage happened, we introduce a new variable b_{ij} as $b_{ij} = \frac{T_j - T_{j-1}}{1-P_{ij}}$. We generate several correlated uniform numbers Y_i by using the Sum-Of-Uniforms (SOU) method, then compute $s_i = b_{ij}Y_i + T_{j-1}$. If s_i belongs to interval $[T_{j-1}, T_j]$, then the first passage time occurred in this interval. We set $\texttt{IsDefault}(i) = 1$ to indicate that firm i has defaulted and compute the conditional boundary crossing density $g_{ij}(s_i)$ according to Eq. (4). To get the density for the entire interval $[0, T]$, we use $\widehat{f}_{i,n}(t) = \left(\frac{T_j - T_{j-1}}{1-P_{ij}}\right) g_{ij}(s_i) * K(h_{opt}, t - s_i)$, where n is the iteration number of the Monte Carlo cycle.

2. **First passage does not happen in this interval.** If s_i does not belong to interval $[T_{j-1}, T_j]$, then the first passage time has not yet occurred in this interval.
3. **First passage happens at the right boundary of the interval.** If $X_i(T_j^+) < D_i(T_j)$ and $X_i(T_j^-) > D_i(T_j)$ (see Eq. (6)), then T_{I_i} is the first passage time and $I_i = j$, we evaluate the density function using kernel function $\widehat{f}_{i,n}(t) = K(h_{opt}, t - T_{I_i})$, and set $\texttt{IsDefault}(i) = 1$.

Next, we increase j and examine the next interval and analyze the above three cases for each non-default firm again. After running N times the Monte Carlo cycle, we get the FPTD of firm i as $\widehat{f}_i(t) = \frac{1}{N}\sum_{n=1}^{N} \widehat{f}_{i,n}(t)$.

In order to provide a reasonable credit analysis, we need to calibrate the developed model or, in other words, to numerically choose or optimize the parameters, such as drift, volatility and jumps to fit the most liquid market data. We have used the historical default data to optimize the parameters in the model based on the least-square methodology.

After Monte Carlo simulation we obtain the estimated density $\hat{f}_i(t)$ by using the kernel estimator method (with Gaussian kernel). The cumulative default rates for firm i in our model is defined as:

$$P_i(t) = \int_0^t \widehat{f}_i(\tau) d\tau, \tag{8}$$

where the kernel density estimator here is chosen from centering a kernel function of a bandwidth [16,20]. Recall that EMCMC combine Evolutionary Computation techniques and (often parallel) MCMC algorithms in order to design new algorithms for sampling or optimizing complex distribution functions. In a sense, we pursue here the same goal as we minimize the difference between our model and historical default data $\widetilde{A}_i(t)$ to obtain the optimized parameters in the model (such as σ_{ij}, arrival intensity λ in Eq. (2)):

$$\operatorname{argmin}\left(\sum_i \sqrt{\sum_{t_j}\left(\frac{P_i(t_j) - \widetilde{A}_i(t_j)}{t_j}\right)^2}\right). \tag{9}$$

For example, theoretical (taken as in [21]) and simulated default correlations of two A-rated firms (A,A) for one-, two-, five, and ten-year periods would be

(0.00, 0.00), (0.02, 2.47), (1.65, 6.58), (7.75, 9.28), respectively. Note that simulated results here are obtained by the UNIForm sampling (UNIF) method as explained in the next section.

4 Density Functions, Default Rates, and Correlated Default

First, we considered a set of historical default data on differently rated firms (including the one presented in [21]) and described the FPTD functions and default rates of these firms. The optimized parameters (including optimal bandwidths) were obtained according to the procedure described in Section 3. Historical, theoretical (obtained with closed form solutions as in [21]), and simulated cumulative default rates for differently rated firms were compared. These results will be published elsewhere and one example of such a comparison can be found in [20].

However, here we focus on an example concerning the default correlation of two firms. If we do not include jumps in the model, the default correlation can easily be calculated. In Tables 1 and 2 we present comparisons of our results with those based on closed form solutions provided by [21] with $\rho = 0.4$. Next, let us consider the default correlations under the multivariate jump-diffusion processes. We use the following conditions in our multivariate UNIF method:

1. Setting $X_i(0) = 2$ and $\ln(\kappa_i) = 0$ for all firms.
2. Setting the growth rate of debt value equivalent to the growth rate of the firm's value: $\gamma_i = \mu_i$ and $\mu_i = -0.001$ for all firms.
3. Since we are considering two correlated firms, we choose $\boldsymbol{\sigma}$ as,

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}, \tag{10}$$

where $\boldsymbol{\sigma}\boldsymbol{\sigma}^\top = \boldsymbol{H}_0$ such that,

$$\boldsymbol{\sigma}\boldsymbol{\sigma}^\top = \boldsymbol{H}_0 = \begin{bmatrix} \sigma_1^2 & \rho_{12}\sigma_1\sigma_2 \\ \rho_{12}\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix},$$

and

$$\begin{cases} \sigma_1^2 = \sigma_{11}^2 + \sigma_{12}^2, \\ \sigma_2^2 = \sigma_{21}^2 + \sigma_{22}^2, \\ \rho_{12} = \dfrac{\sigma_{11}\sigma_{21} + \sigma_{12}\sigma_{22}}{\sigma_1\sigma_2}. \end{cases} \tag{11}$$

In Eq. (11), ρ_{12} reflects the correlation of the diffusion parts of the processes of the two firms. In order to compare with the standard Brownian motion and to evaluate the default correlations between different firms, we set all the $\rho_{12} = 0.4$ as in [21]. Furthermore, we use the optimized σ_1 and σ_2 for firm 1 and 2, respectively. Assuming $\sigma_{12} = 0$, we get,

$$\begin{cases} \sigma_{11} = \sigma_1, \\ \sigma_{12} = 0, \\ \sigma_{21} = \rho_{12}\sigma_2, \\ \sigma_{22} = \sqrt{1-\rho_{12}^2}\sigma_2. \end{cases}$$

4. The arrival rate for jumps satisfies the Poisson distribution with intensity parameter $\lambda = 0.1$ for all firms. The jump size is a normal distribution $Z_i \sim N(\mu_{Z_i}, \sigma_{Z_i})$, where μ_{Z_i} and σ_{Z_i} can be different for different firms to reflect specifics of the jump process for each firm. We adopt the optimized parameters.
5. As before, we generate the same interjump times $(T_j - T_{j-1})$ that satisfy an exponential distribution with mean value equal to 1 for each of the two firms.

We carry out the UNIF method to evaluate the default correlations via the following formula:

$$\rho_{12}(t) = \frac{1}{N}\sum_{n=1}^{N} \frac{P_{12,n}(t) - P_{1,n}(t)P_{2,n}(t)}{\sqrt{P_{1,n}(t)(1-P_{1,n}(t))P_{2,n}(t)(1-P_{2,n}(t))}}, \tag{12}$$

where $P_{12,n}(t)$ is the probability of joint default for firms 1 and 2 in each Monte Carlo cycle, $P_{1,n}(t)$ and $P_{2,n}(t)$ are the cumulative default rates of firm 1 and 2, respectively, in each Monte Carlo cycle.

The simulated default correlations for one and ten year periods are given in Tables 1 and 2, respectively. All the simulations were performed with the Monte Carlo runs $N = 500,000$. Comparing simulated default correlations with the theoretical data for standard Brownian motions, we can conclude that

Table 1. One year default correlations (%). All the simulations are performed with Monte Carlo runs $N = 500,000$.

	UNIF				[21]			
	A	Baa	Ba	B	A	Baa	Ba	B
A	-0.01				0.00			
Baa	-0.02	3.69			0.00	0.00		
Ba	2.37	4.95	19.75		0.00	0.01	1.32	
B	2.80	6.63	22.57	26.40	0.00	0.00	2.47	12.46

Table 2. Ten year default correlations (%). All the simulations are performed with the Monte Carlo runs $N = 500,000$.

	UNIF				[21]			
	A	Baa	Ba	B	A	Baa	Ba	B
A	8.79				7.75			
Baa	10.51	13.80			9.63	13.12		
Ba	9.87	14.23	22.50		9.48	14.98	22.51	
B	8.50	12.54	20.49	24.98	7.21	12.28	21.80	24.37

1. Similarly to conclusions of [21], the default correlations of same rated firms are usually large compared to differently rated firms. Furthermore, the default correlations tend to increase over long time and may converge to a stable value.
2. In our simulations, the one year default correlations of (A,A) and (A,Baa) are negative. This is because they seldom default jointly during one year. Note, however, that the default correlations of other firms are positive and usually larger than in the results presented by [21].
3. For two and five years, the default correlations of different firms increase. This can be explained by the fact that their individual first passage time density functions increase during these time periods, hence the probability of joint default increases.
4. As for ten year default correlations, our simulated results are almost identical to the theoretical data for standard Brownian motions. The differences are that the default correlations of (Ba,Ba), (Ba,B) and (B,B) decrease from the fifth year to tenth year in our simulations. The reason is that the first passage time density functions of Ba- and B-rated firms begin to decrease from the fifth year, hence the probability of joint default may increase slowly.

5 Conclusion

In this contribution, we have analyzed the credit risk problems of multiple correlated firms in a structural model framework, where we incorporated jumps to reflect the external shocks or other unpredicted events. By combining the fast Monte-Carlo method for one-dimensional jump-diffusion processes and the generation of correlated multidimensional variates, we have developed a fast evolutionary type Monte-Carlo type procedure for the analysis of multivariate and correlated jump-diffusion processes. The developed approach generalizes previously discussed non-correlated jump-diffusion cases for multivariate and correlated jump-diffusion processes. Finally, we have applied the developed technique to analyze the default events of multiple correlated firms via a set of historical default data. The developed methodology provides an efficient computational technique that is applicable in other areas of credit risk and for the pricing of options.

References

1. Atiya, A.F., Metwally, S.A.K.: Efficient Estimation of First Passage Time Density Function for Jump-Diffusion Processes. SIAM Journal on Scientific Computing 26, 1760–1775 (2005)
2. Campillo, F., Rakotozafy, R., Rossi, V.: Parallel and interacting Markov chain Monte Carlo algorithm. Mathematics and Computers in Simulation 79, 3424–3433 (2009)
3. Cercueil, A., Francois, O.: Monte Carlo Simulation and Population-Based Optimization. In: Int. Congress on Evolutionary Computation, CEC 2001, pp. 191–198. IEEE Press, Los Alamitos (2001)

4. Drugan, M.M., Thierens, D.: Evolutionary Markov chain Monte Carlo. In: Liardet, P., Collet, P., Fonlupt, C., Lutton, E., Schoenauer, M. (eds.) EA 2003. LNCS, vol. 2936, pp. 63–76. Springer, Heidelberg (2004)
5. Dugan, N., Erkoc, S.: Genetic algorithm-Monte Carlo hybrid geometry optimization method for atomic clusters. Computational Materials Science 45(1), 127–132 (2009)
6. Dukkipati, A., Bhatnagar, S., Murty, M.N.: Gelfand-Yaglom-Perez theorem for generalized relative entropy functionals. Information Sciences 177(24), 5707–5714 (2007)
7. Geyer, C.J.: Practical Markov Chain Monte Carlo. Statistical Science 7, 473–483 (1992)
8. Goswami, G., Liu, J.S., Wong, W.H.: Evolutionary Monte Carlo methods for clustering. J. of Computational and Graphical Statistics 16(4), 855–876 (2007)
9. Goswami, G., Liu, J.S.: On learning strategies for evolutionary Monte Carlo. Statistics and Computing 17(1), 23–38 (2007)
10. Johansson, A.M., Lehmann, E.A.: Evolutionary Optimization of Dynamics Models in Sequential Monte Carlo Target Tracking. IEEE Trans. on Evolutionary Computation 13(4), 879–894 (2009)
11. Hull, J., White, A.: Valuing Credit Default Swaps II: Modeling Default Correlations. Journal of Derivatives 8, 12–22 (2001)
12. Kou, S.G., Wang, H.: First passage times of a jump diffusion process. Adv. Appl. Probab. 35, 504–531 (2003)
13. Laskey, K.B., Myers, J.W.: Population Markov Chain Monte Carlo. Machine Learning 50(1-2), 175–196 (2003)
14. Melnik, R.V.N.: Coupling control and human factors in mathematical models of complex systems. Engin. Appl. of Artificial Intelligence 22(3), 351–362 (2009)
15. Melnik, R.V.N.: Markov chain network training and conservation law approximations: Linking microscopic and macroscopic models for evolution. Applied Mathematics and Computation 199(1), 315–333 (2008)
16. Silverman, B.W.: Density Estimation for Statistics and Data Analysis. Chapman & Hall, London (1986)
17. Song, J.H., Kiureghian, A.D.: Joint First-Passage Probability and Reliability of Systems under Stochastic Excitation. Journal of Engineering Mechanics 132(1), 65–77 (2006)
18. Vrugt, J.A., et al.: Accelerating Markov Chain Monte Carlo Simulation by Differential Evolution with Self-Adaptive Randomized Subspace Sampling. Int. J. of Nonlinear Sciences and Numer. Simul. 10(3), 273–290 (2009)
19. Wang, J., Zhang, W.-J., Liang, J.-R., et al.: Fractional nonlinear diffusion equation and first passage time. Physica A 387(1), 764–772 (2008)
20. Zhang, D., Melnik, R.V.N.: First passage time for multivariate jump-diffusion processes in finance and other areas of applications. Applied Stochastic Models in Business and Industry 25(5), 565–582 (2009)
21. Zhou, C.: An analysis of default correlation and multiple defaults. Review of Financial Studies 14, 555–576 (2001)
22. Zhou, C.: The Term Structure of Credit Spreads with Jump Risk. Journal of Banking and Finance 25, 2015–2040 (2001)

EvoMUSART Contributions