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# Intelligent Structures and Coupling in Mathematical Models: Examples from Dynamic Electroelasticity

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**Abstract** - Macroscopic mathematical models for intelligent structures are discussed. Due to a fundamental connection between densities of drift and diffusion currents and inter-dependency of relaxing particles such mathematical models acquire non-local features. This requires adequate mathematical tools in description of underlying physical and chemical processes. Some such tools are considered.

**1. Introduction.** In mathematical modelling of physical and chemical processes in dielectrics many traditional assumptions appear to be incompatible with new technologies. Such assumptions are often connected with *the concept of relaxation time* [3,4]. Since macroscopic mathematical models can only approximately reflect processes in question, a continuous improvement of existing mathematical models is necessary to accommodate new knowledge about the material from which a specific device or structure consists of, as well as additional knowledge about geometrical properties of this device or structure. Uncertainties associated with the environment, in which this device or structure will operate, should also be "built-in" into the model. Since the environment is continuously changing the resulting mathematical models gain *non-stationary features.*

When uncertainties of the environment and incomplete knowledge about the material are formalized mathematically using tools of probability theory, uncertainties connected with statistical variability of parameters and *fuzziness of information* about these parameters are mixed. Although this often allows a convenient mathematical framework for justification of mathematical models under a-priori assumptions on the solution smoothness, it does not answer the question of adequateness of these mathematical models to the real physical situation.

**2. Motivation.** Many processes in nature significantly depart from classical physical laws [12]. In theory of dielectrics such a departure can be seen on the example of the Debye law [4]. Nevertheless, in many cases classical mathematical models based on the theory of differential equeations can still be applied through the perturbation technique or series expansion method. These mathematical methods can be extended, at least in principle, to stochastic mathematical models when perturbations are not necessarily small. However, if information on the uncertain parameters can not be assumed to be completely known (either in the deterministic sense or with probability 1), these methods can only be used in a heuristic rather than in a rigorous mathematical sense. This heuristic approach, which is widely used in mathematical literature, requires some additional assumptions on distribution laws of unknown parameters as well as regularity assumptions on unknown functions. Such assumptions are often excessive in practical situations when a given mathematical model is applied.

**3. The Einstein relationship and the dependency of relaxing entities.** Although the implementing additional information in macroscopic mathematical models is natural, it has some *statistical limitations.* To better approximate "tails" of state densities we are required some knowledge of the microscopic structure. In such situations the Fermi-Dirak statistics can provide an adequate approximation [8,4]. If this approximation is used then the application of existing mathematical models in many branches of material sciences is based on the generalized Einstein relationship that gives a connection between diffusion and drift through the gradient of the Fermi level. For solids it has the form $D = \mu(-\partial\varphi/\partial \ln c)$, where $D$ and $\mu$ are the diffusion and the mobility coefficients, $c$ is the carrier concentration and $\varphi$ is the Fermi quasi-level (this relationship is widely used in semiconductor device theory). The mobility of carriers can be defined from experiments through the averaged relaxation time $< \tau >$ and the effective mass $m^*$ by the formula $\mu = q < \tau > /m^*$, where $q$ is the carrier charge. If we assume that all relaxing entities are independent and relaxing spontaneously with the same probability, mathematical models gain some "autonomous" features induced by the "constant" time delay of these entities. In reality such entities are not independent of each other [4]. For dielectrics this implies the time-dependency of the rate of depolarization after removal of a field. More generally, any specific dependency of

the mobility on the field is somewhat artificial since in such cases the relaxation time should also change with the field. The averaging time $< \tau >$ can be interpreted as the "return-to-equilibrium" time, whereas the mechanism of such a return is provided by the dispersion of carriers on *non-homogeneities of the chemical structure of the material.* If the structure is non-degenerate the use of the Boltzmann statistics simplifies the overall picture (then $D = \mu kT/q$, where $T$ is the effective temperature, $k$ is the Boltzmann constant). Whatever the particular case is, in order to construct the macroscopic mathematical model for dielectrics we should be able to *estimate* either the effective temperature (for weakly interacting system) or the gradient of quasi-level Fermi (for strongly interacting system). Since in many cases ion mobilities are small, the model for dielectrics can be reduced to the fundamental system applied in semiconductor device theory. For those structures that are designed for the long-term information storage (like MDS), this simplification is not appropriate.

The approximate character of mathematical models in material sciences stems from an approximation of a dependency between densities of drift and diffusion currents. The statistical nature of this dependency implies that both deterministic and stochastic mathematical models provide approximations the quality of which is application-specific.

**4. Model-based algorithms and a-posteriori information.** Since the quality of any specific algorithm depends on the consistency of the model to the real-world problem, the algorithm of solution should be *adaptive* and *error control should be undertaken in different norms*, not only energy norms. In the general non-stationary case, only self-adjointness of the spatial operator can essentially justify the use of energy norms. In each specific application for which a given model is appropriate, a scale of a-priori estimates should be obtained with respect to different requirements on the solution smoothness. This allows to compensate for an approximate nature of the model within reasonable statistical variability of parameters (see [11] for details).

In many cases it is not clear *a-priori* whether the model is appropriate for the given application. In such situations the choice of norms in the error control is problem-specific and the mathematical analysis of the problem should include both a-priori and a-posteriori estimates [2]. It is also important to note that in material science applications many of known a-posteriori estimates are based on stress recovery (i.e. on the calculation of stresses by other techniques than the direct differentiation of the computed displacement field). The accuracy of such estimates strongly depends on the accuracy of the recovered stress field. Since a-posteriori errors should be integrated with respect to time, an alternative approach lies with the use of more general topologies than it is allowed by applications of Hilbert functional spaces [9,10].

**5. Control, identification and estimation.** Although the uncertainty associated with any algorithm can be mathematically modelled using the concept of reliability or entropy [5], the most reliable algorithm is not necessarily computationally feasible. Hence both reliability and computational cost measures should be combined into *a unified whole.* This precludes the assumption that information about the problem may be complete, exact and free (no charge to know the distances as in Hausdorff topological spaces). In applications of mathematics to material sciences, robotics and complex dynamic systems this approach (often associated with the combinatorial complexity of the problem) is incompatible with the information-based complexity approach [14]. The desire to reduce uncertainty in mathematical models and associated algorithms and to improve performance measures is expressed mathematically in optimal control theory. The theory is essentially based on the assumption of *a-priori knowledge of the model for the object and its environment.* Of course, this knowledge can be *gradually* accumulated through human experience, experimentation, identification, estimation, etc. However, obtained information is always partial, noisy and costly. As a result, an effective algorithm for the solution of optimal control problem may prompt the way to improve the model for dynamics, rather than provide the complete solution of the problem. At present time, most existing controllers are based on the use of feedback in order to reduce uncertainty. They mix the two sources of uncertainty, not being able to distinguish between the feedback related to decision-making and the feedback related to uncertainty about the environment. In reality, any specific algorithm chosen by the decision making *interacts* with the environment which includes a set of monitoring algorithms. This naturally leads to the idea of the construction of *a feedback hierarchy.* In turn, this requires a lower bound for the performance-measure evolution as well as the introduction of some learning rules.

**6. Rule-based algorithms and a-priori information.** With increasing complexity of applications the conventional control approach becomes less competitive with approaches that incorporate on-line learning mechanisms in control. Such approaches belong to the field of *intelligent control systems* [1]. Using a-priori information from the environment and simulating the existing forms and decision making in biological systems, engineers often heuristically construct non-linear adaptive controllers with robust properties. The degree of autonomy of such controllers depends not only on limitations of today's computers and conventional mathematical models, but also on our incomplete knowledge of the process of cognition. This implies that new tools in the theory of systems control such as Hierarchical Learning Stochastic Automata (HLSA) may be efficient

in approximations of arising problems. Such automata should have variable structures since their *state probabilities are time-dependent.* In this case for each pair, <environment state, HLSA action>, we define a performance measure function that allows both a learning process and an evaluation of performance.

The design of a technical device or a structure requires taking into account uncertainties, modelling errors, unexpected inputs and variations in parameters. From a control point of view, such a device (or a structure) is an object which requires control systems with an efficient mechanism of disturbance rejections and/or accommodations. Under the framework of the conventional mathematical control almost all of these control systems require some form of exact knowledge of the object. It should be emphasized, however, that such knowledge is required *only* for for the feedback control. Amongst other control actions feedforward and open-loop predictive control actions can also be used. In theory, a combination of all three types of control actions can provide the best result. In applications, the quality of the result is crucially dependent on the ability of the controller to be "intelligent" in a sense of the ability to learn a-priori unknown parameters in dynamics.

**7. Intelligent structures and model-based coupling.** The additional functionality of a technical device, a structure or a system can not be obtained without paying for it in some form. In designing a system that will operate in a dangerous, uncertain or remote environment it is important to *increase precision and decrease "intelligence"* in the sense of minimization of system interaction with a human operator. In such cases the system itself should be able to deal with incomplete, uncertain and rapidly changing information. It should be able to allocate resources between *thinking, acting and monitoring the results.* Hence, the system *itself* should be *intelligent or smart.* Material sciences play a fundamental role in achieving this technological task.

At a smaller scale of consideration any smart material can be considered as a smart structure [13]. A smart structure is a non-biological physical structure having a definite purpose, means and imperative to achieve it, and a biological or a Turing machine pattern of functioning. Such structures can adapt to changing conditions, both within itself and in its environment, using a *minimum amount of available energy*, to achieve its purpose. The degree and number of functions will depend on the purpose of the integrated system. A smart structure can be thought of as an intelligent structure at a smaller scale. Since the smart structure bases its actions upon optimally adapting to present conditions which are continually changing, it will never be intelligent in a rigorous sense. The decisions and adaptations are made through the use of *feedback and memory.* However, since smart structures basically mimic biological system, it is convenient to call such structures intelligent. Of course, they need only those biological functions that are required to meet the functional purpose for which the system is designed (see [13] and references therein for further details).

Among materials that can be classified as intelligent are shape memory alloys, magnetostrictive materials, magnetorheological and electrorheological fluids and many others. Our main interest in the next section is *piezoelectric materials.* In order to appropriately describe technical devices and structures based on these materials one should take into account the coupling phenomenon of different-nature physical fields.

**8. Dynamic electroelasticity: which model is appropriate?** Piezoelectric materials are widely used to achieve self-monitoring and control capabilities of many technical devices and structures. With the intelligent structure technology complex functions can be carried out within the material itself. Anisotropic properties of these materials (such as PZT, PVDF) allow to use them as sensors/actuators in the vibration control, vibrator gyroscopes, the marine antifouling protection, acoustics, control of helicopter rotors, structural design, aerospace engineering and many other applications. All these applications require an adequate modelling of *coupled electroelastic effects in piezoelectric materials.* On the other hand, the design of control systems also influences the structural response. Hence, in principle, a coupled controls/structures optimization is necessary. Mathematically this leads to a discrete rather than continuous formulation of the problem.

In principle, an active material system can be simulated by a *network* due to the transformation of different types of energy. Alternatively, if we construct a model on the basis of conservation laws, an algorithm for the problem solution can always be derived from the resulting model without attributing structural materials some "smart properties". However, in the latter case the algorithm should necessarily be "smart" reflecting conservative properties numerically. Of course, in such cases the quality of the algorithm will decisively depend on the quality of the model. This requires the intelligent control of computational process and the implementation of some learning rules into the designed system. This approach appeals to engineers and its mathematical features were described in Section 4. In [11] the author applied this approach to the investigation of coupled electromechanical processes in piezoelectrics. However in spite of its theoretical generality, the practical implementation of the conservation-law approach is often limited [8].

The energy dissipation within piezoelectric structures results mainly from mechanical and electrical losses. Recent research show that *hysteresis* does not introduce a new dissipation source and the dissipation in the form of heat or sound is often negligible. Nevertheless, there are many applications where the

dissipation mechanism in piezoelectrics should be described with a higher precision. For example, it is often important to suppress vibrations of devices, structures or systems subjected not only mechanical loading but thermal loading as well. Temperature can also affect measurements obtained using piezoelectric sensors. Another example is provided by optical engineering applications. Actuators that are driven optically rather than electrically allow to avoid electric noise from hard-wire connections. In such cases the photostrictive effect (a combination of the photovoltaic and piezoelectric effects) is important. Therefore, in many applications of macroscopic mathematical models for dielectrics we need an adequate description not only the coupled elecromechanical field, but the opto-thermo-electro-mechanical behaviour of material. Such macroscopic models for dielectrics require addressing issues discussed in Section 3.

In practice, piezoelectric materials and other intelligent structures for which coupling phenomena are essential may be subjected to mechanical, electric or other stresses that cause their failures due to different types of material discontinuities. Discontinuities may also happen on the interface between piezoelectric and other types of materials such as metal electrodes or polymers. In these cases the accommodation of uncertainties connected with such discontinuities within the model is especially important. If we use variational principles then unknowns have to be understood as field variables and Lagrange multipliers have to be appropriately interpreted [12]. In the general nonstationary case the definition of the solution of the problem becomes closely connected with the formulation of a singular control problem since conservation laws can be preserved only approximately [8]. From the numerical point of view the algorithm for the solution of the problem should acquire adaptive properties based on learning rules obtained from both a-priori and a-posteriori estimates as discussed in Sections 4-6.

Numerical results and computational procedures discussed in this paper can be found in [11]. Theoretical issues addressed here have also been studied in [6]–[10].

**9. Conclusion.** Material sciences in general and piezoelectrics in particular provide an important area of applications of advanced mathematical tools associated with intelligent control of technical devices, structures and systems. The key role in such mathematical tools belongs to algorithms. As for mathematical models, they provide a link between physico-chemical and computational models. The quality of such a link completely depends on its consistency to the real-world problem. This requires a connection between the model and the algorithm through on-line learning rules. Mathematical models in such cases can be formulated in the form of singular control problems for which continuity of unknown functions can not always be guaranteed [10].

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