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Technical Notes

Energetics and Invariants of Axially Deploying Beam with Uniform Velocity

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I. Introduction

THE class of axially moving materials has a wide range of applications in diverse mechanical systems such as serpentine belt systems, band saws, high-speed magnetic tapes, power transmission chains, robotic manipulators, and satellite tethers, among many others. The traveling Bernoulli–Euler beam is one of the most common models of such a type of axially moving media. The pioneering work of axially moving continua is contributed by Mote [1] and Ulsoy et al. [2], who investigated the vibration of a band saw and first introduced the gyroscopic terms into the axially translating system. Wickert and Mote [3] presented the summary work on axially moving continua. A beam moving with time-dependent axial velocity was examined by Öz and Pakdemirli [4] based on the method of multiple scales. Similar dynamical phenomena to those axially moving materials can also be found in the study of pipes conveying fluid [5].

For the studies of an axially moving material with both ends supported, the length of the beam holds a constant. The linear free vibration of such a system is still conservative, although gyroscopic terms may induce some new dynamic phenomena. However, the deploying beam model with a prismatic joint at one end and a moving boundary on the other end, with applications from robot arms to high-rise elevators and the appendages of spacecraft, leads to a new challenging problem. In the deploying cantilever beam model, the length of the beam is time-variant, and the system turns into a moving boundary value problem as the spatial domain changes with time. The dynamical study of time-varying parameter flexible structures poses some distinct technical challenges and dictates the use of concepts, formulations, and analytical methodologies. Unlike linear time-invariant systems, the linear time-varying systems in general do not have a closed-form solution due to parameters changing as a function of time. The transient phenomenon that occurs in the time-varying parameter systems is still not fully understood. For example, without periodicity, even the concepts of Lyapunov asymptotic stability and orbital stability have no meaning in the analysis of the transient phenomenon. The majority of current techniques from steady-state analysis of time-invariant systems are inapplicable. Nevertheless, an analytical study of time-varying parameter systems is highly desirable in order to gain a better understanding of the transient dynamics and to apply it to practical engineering structures, such as robotic manipulators, lifting devices, tethered satellites, etc. The deploying beam model is a basic and classic example of time-varying parameter structures.

The first detailed research on the deploying beam was provided by Tabarrok et al. [6]. They used Newton's second law to derive the governing partial differential equations with moving boundaries. It was shown that, for a constant axial velocity, oscillatory motions dominate the response during the initial stage of deployment and the transverse deflection becomes unbounded with time for the time-varying linear model. The assumed-mode technique has been employed in such a beam model with varying length. Because of the involvement of time-varying parameters and the gyroscopic terms due to the axially moving velocity, the analytical solutions of the transverse vibrations of the deploying beam are not accessible. The gyroscopic terms may be neglected if the axially moving velocity is low. Without the effect of the gyroscopic terms, the analytical investigation can be implemented by focusing only on the time-varying parameters [7,8]. Kalaycioglu and Misra [9] presented approximate analytical solutions and numerical solutions for beam-type appendage deployment and tethered system deployment and verified the accuracy of the approximate method. Bergamaschi and Sinopoli [10] used the Wentzel–Kramers–Brillouin theory to obtain the analytical solutions of the deploying beam, while the gyroscopic term was neglected. Another approximate analytical procedure, the method of multiple scales, was used to study the transient dynamics of deploying and retreating beams by AlBedoor and Khulief [11]. In studies by Stylianou and Tabarrok [12,13], numerical solutions to several variations of the axially moving beam were obtained by finite element analysis.

The nonlinear dynamics of deployable antennas was also studied by Tabarrok and Behdinan [14] via the Galerkin method and the finite element method. Downer and Park [15] used a finite element technique with moving reference grid to deal with large overall motion of the beams. This type of finite element method with fixed element number and varying length as a function of time was used in the study of time-varying beam models [16–18]. The finite element with fixed domain and fixed number was discussed by introducing moving boundary conditions [19]. Behdinan and Tabarrok [20] and Behdinan et al. [21,22] developed the corotational finite element method with variable domain to study the dynamics of the sliding links. Matsuzaki et al. [23] performed experiments deploying or retrieving cantilever beams for the first time and provided data, along with comparisons to the numerical results. Sugiyama et al. [24] employed the finite segment method from multibody dynamics to the vibration analysis of the deploying beam. They found that the numerical results obtained using the finite segment method agree well with the results obtained using the experiment. Tang et al. [25] studied the dynamics of the variable-length satellite tethers using flexible multibody theory based on the absolute nodal coordinate formulation. The dynamics of deploying a slender continuum have also been modeled in the context of what is known as the reverse spaghetti problem [24,26].

For the time-invariant linear Hamiltonian system, the energy imparted at $t = 0$ in the system by the initial conditions is partitioned among the linear modes, and no further energy exchanges between

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modes is allowed for $t > 0$. Each linear mode conserves its own energy and participates accordingly in the response of the system through linear superposition with the responses of the other modes [27]. For the time-varying parameter system, the natural frequencies and corresponding modes are not steady-state and turn out to be time variant. Liu and Deng [28] discussed the pseudonatural frequencies of such a linear time-varying system and verified the results by experiment. Further, Gosselin et al. [29] studied in detail the natural frequencies and damping as functions of time. From the variation of the damping, which denotes the change of energy, one can determine the stability at the corresponding instant. However, over the long run, the transient stability at a number of instants is not meaningful to describe the dynamics of the time-varying parameter system. Wang et al. [30] investigated the total energy of transverse vibrations with respect to time for the deploying beam, from which the tendency of the transient dynamics could be determined. Zhu and Ni [31] proposed the concept of energy density to the dynamics of translating media with varying length. It is concluded that the energy analysis should be helpful in the transient dynamic study of time-varying parameter systems.

Although the conservative analysis of the moving beam on supported ends was studied by Chen and Zhao [32] and Chen [33], the invariant investigation has not been found for the time-varying parameter system in the literature. In this Note, the energetics and invariants of the deploying beam with given initial conditions are studied by the assumed-mode method. The transfer of energy among different mode orders is discussed for several types of initial conditions. The energy variation and an adiabatic invariant for the transient response are proposed for the fundamental mode.

II. Governing Equations and Assumed-Mode Truncation

Consider a flexible uniform beam deploying axially from a prismatic joint (Fig. 1); the moving beam has cross-sectional area A, area moment of inertia I, Young's modulus E, mass density ρ, and length $L(t)$ and velocity $U(t)$ at time t. Viewing the sliding beam as a system of changing mass, one assumes that the part of the beam inside the prismatic joint is nondeformable and has a prescribed axial motion. The axial velocity is uniform along the entire beam because the beam is assumed inextensible (i.e., the velocity is dependent on time t but not on spatial coordinate x). Thus, the task is to determine the transverse motion of the beam in a plane as it emerges from the prismatic joint.

By studying the Euler–Bernoulli beam segment at an arbitrary distance x, according to Newton's second law in the transverse direction, we have

$$\rho A\left(\frac{\partial w^2}{\partial t^2}+2U\frac{\partial w^2}{\partial x\partial t}+\left[U^2+(L-x)\frac{\partial U}{\partial t}\right]\frac{\partial w^2}{\partial x^2}\right)+EI\frac{\partial^4 w}{\partial x^4}=0 \tag{1}$$

in which w denotes the transverse displacement.

Equation (1) was derived by Tabarrok et al. [33] using both the momentum equilibrium method and Hamilton's principle. Behdinan et al. [34] discussed the geometrically nonlinear formulations, governing the sliding beams for different descriptions, via the extended Hamilton's principle. Recently, Park et al. [35] proposed a set of coupled equations governing the transverse and longitudinal motions of the deploying and retracting beams. The governing linear time-varying equation used in this study can be obtained from the aforementioned references by eliminating the higher-order and coupled terms. For this case, Newton's second law provides the most convenient way to derive the equation governing the transverse motion of the deploying or retracting beam.

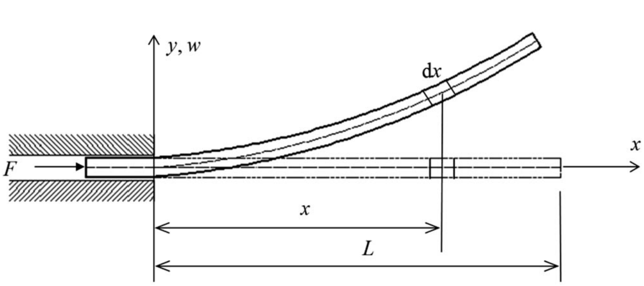


Fig. 1 Configuration of axially inextensible sliding beam.

The time-varying parameter partial differential equation can be solved by the Galerkin truncation method (i.e., the assumed-mode method). The Galerkin method has been widely used in the vibration analysis of time-independent gyroscopic systems, such as axially moving continua and pipes conveying fluid on two supports [36–39]. The time-independent assumed-mode is usually adopted as the corresponding mode function of the static case. In the analysis of transient dynamics of time-varying parameter systems, it has also been proven that the assumed-mode method is effective, although the assumed mode is time dependent [6,29,35,40]. In the current study, the assumed-mode method will be used for the study of the deploying beam, and its convergence will be demonstrated.

The transverse displacement is expanded in a series form, in terms of a set of assumed-mode functions:

$$w(x,t)=\sum_{n=1}^{N}\varphi_n(x,L)f_n(t) \tag{2}$$

in which the f_n denote the temporal generalized coordinates and the assumed-mode functions φ_n denote spatial variables. However, because the length of the beam is varying, the transient assumed-mode function φ_n is dependent on length L and further dependent on time t. The following cantilevered beam eigenfunctions are used

$$\phi_i(x,L)=\cosh\beta_i\frac{x}{L}-\cos\beta_i\frac{x}{L}-\frac{\cos\beta_i+\cosh\beta_i}{\sin\beta_i+\sinh\beta_i}\left(\sinh\beta_i\frac{x}{L}-\sin\beta_i\frac{x}{L}\right) \tag{3}$$

By the assumed-mode method, the transverse vibration of the deploying beam can be studied by the superposition of the time-varying assumed-modes. The time-varying assumed-modes after truncation will hold their contour shape, even the length is varying with time.

Substituting Eq. (2) into Eq. (1), applying Galerkin's procedure, and performing some manipulation, the discretized time-varying parameter differential equation of motion becomes

$$\ddot{\boldsymbol{f}}+2(U/L)\boldsymbol{A}\dot{\boldsymbol{f}}+[(\dot{U}/L)\boldsymbol{A}-(U/L)^2(\boldsymbol{A}+\boldsymbol{B})+\Lambda]\boldsymbol{f}=0 \tag{4}$$

in which the vectors and matrices are defined as

$$\begin{aligned}&\boldsymbol{f}=\{f_1\ f_2\ f_3\ \cdots\}^T,\quad \boldsymbol{\Lambda}_{ij}=(EI/\rho AL^4)\beta_i^4\delta_{ij},\\ &A_{ij}=\int_0^1(1-\xi)\phi_i\phi_j'\,\mathrm{d}\xi-\frac{1}{2}\delta_{ij}\quad B_{ij}=\int_0^1(1-\xi)^2\phi_i'\phi_j'\,\mathrm{d}\xi-\frac{1}{4}\delta_{ij},\\ &\phi_i=\cosh(\beta_i\xi)-\cos(\beta_i\xi)-\alpha_i[\sinh(\beta_i\xi)-\sin(\beta_i\xi)],\quad i,j=1,\ldots,N\end{aligned} \tag{5}$$

These integrations of assumed-mode functions can be obtained by numerical calculations [6], and some analytical results can be found in the references [5,29].

III. Transient Response to Initial Conditions and Energy Variation

Equation (4) is integrated using a Runge–Kutta algorithm with error control. This algorithm is implemented in the standard ode45 function from the MATLAB software (The MathWorks Inc., Natick, MA, 2012). Calculations have been performed with the values of the parameters adopted from [9]: mass/unit length $(\rho A)=0.599$ kg/m, bending stiffness $(EI)=3798\ \mathrm{N\cdot m^2}$, and initial length $(L_0)=2$ m.

A. Instantaneous Natural Frequencies

It is known that the natural frequencies of a static cantilever beam decrease with the increase of length while other parameters hold the same values. On the other hand, the introduction of the axial motion also makes the natural frequencies higher, even for the axially moving beams on two fixed supports [41,42]. In Fig. 2, the instantaneous natural frequencies are plotted with the time for the axially deploying cantilever beam with constant velocity $U = 0.8\ \mathrm{m/s}$. Because the deployment is continuous, the total length of the cantilever beam is increasing, which results in decreasing of the instantaneous natural frequencies.

B. Contour Shape Holding

Now we study the contour shape of the vibrations imparted from different-order initial mode. The plots in Fig. 3 demonstrate the snapshots of the transverse vibrations of the axially deploying beam with different-order initial mode. The snapshot shape of the beam released from the first-order initial condition holds the same shape as the first-order mode. While imparted from the second- and third-mode initial conditions, the second-and the third-mode vibrations will dominate respectively.

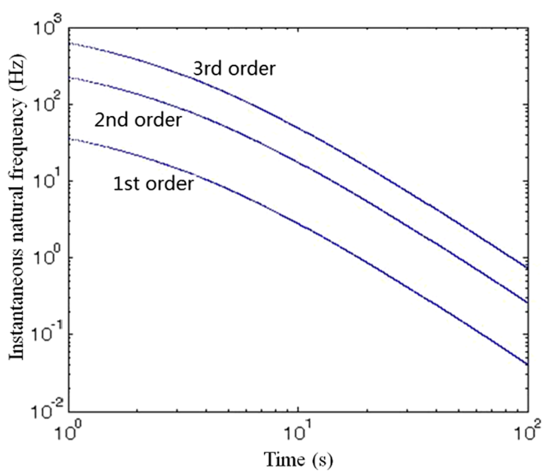


Fig. 2 Transient natural frequencies vs time.

In every plot in Fig. 3, the contour shape is like being stretched while the overall mode shape holds the same as the beam is being deployed.

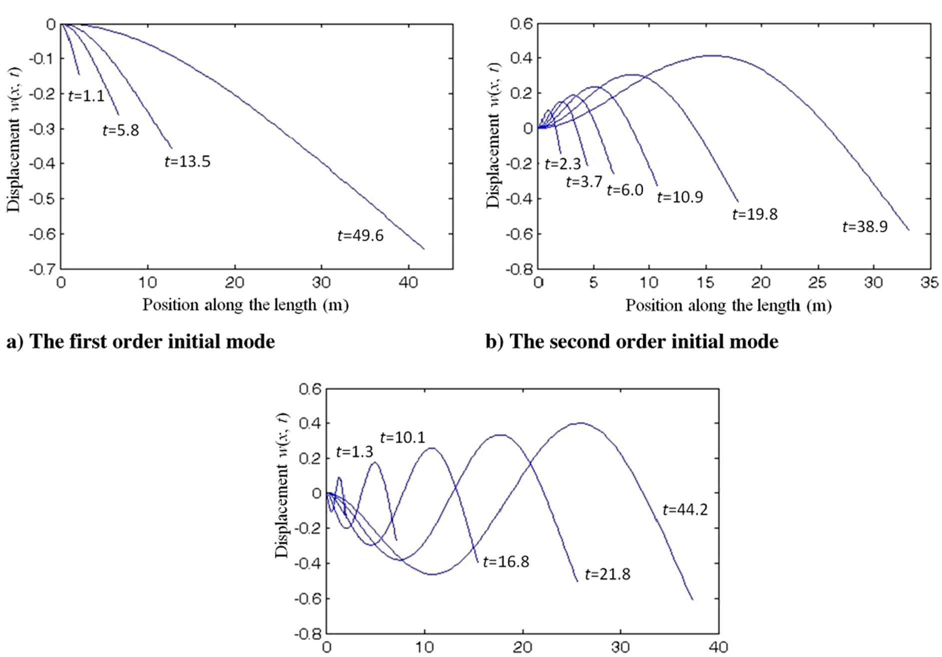
C. Energy Variation

By close inspection of the truncated ordinary differential Eq. (4), we obtain the kinetic energy E_k, potential energy E_p, and total energy E_t:

$$E_p = \frac{1}{2} \boldsymbol{f}^T [(\dot{U}/L)\boldsymbol{A} - (U/L)^2(\boldsymbol{A} + \boldsymbol{B}) + \boldsymbol{\Lambda}]\boldsymbol{f},$$
$$E_k = \frac{1}{2} \dot{\boldsymbol{f}}^T \dot{\boldsymbol{f}},$$
$$E_t = E_p + E_k \tag{6}$$

Based on Eq. (6), the change of energy with time is described in Fig. 4. Without deploying velocity, the total energy is a constant as presented in Fig. 4a, which is well known for the free vibrations of undamped linear systems. For the deploying case, $U = 1.0\ \mathrm{m/s}$, the change of the energy is depicted in Fig. 4b. If the beam is pushed out with a constant velocity, the pushing force should be zero when the transverse vibration does not occur and the damping is neglected. However, if transverse vibrations exist, the beam may be in tension. Hence, the "pushing" force is actually "pulling" force in some cases to keep the deploying velocity constant. One can conclude that when the beam is pushed out, the boundary must do negative work to remove energy from the beam. It can be found numerically that the rate of decrease in the energy varies when the beam is decreasing in length with a constant speed. Chen and Ferguson [43] found similar conclusions when investigating the string with varying length. They also verified that the change in total energy, when the length is varying, has a linear relationship with the fundamental natural frequency of the string system, but it is not linear as a function of time. Inspired by the relationship between the total energy and the instantaneous natural frequency, the adiabatic invariant will be discussed in Sec. IV.

Equation (6) provides the total energy variation of all the modes with respect to time. Now we will study the energy variation of each assumed mode with different initial conditions. Because Eq. (4) couples all the assumed modes, we cannot obtain the explicit expressions for the total energy. However, we can use a numerical method to study the variation of the generalized coordinates $\boldsymbol{f}$. As before, we use the Runge–Kutta method with error control to obtain

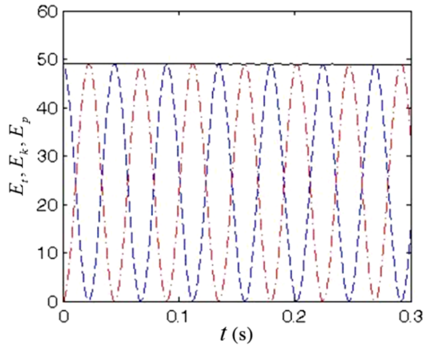


a) The first order initial mode

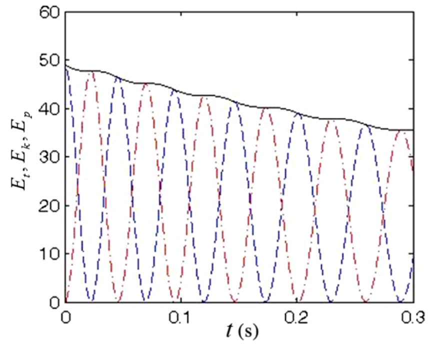
b) The second order initial mode

c) The third order initial mode

Fig. 3 Contour shapes of the axially moving beam with different initial mode.



a) Zero velocity case



b) Non-zero velocity case

Fig. 4 Energy variation with time. Black solid lines denote total energy E_t, red dashed-dotted lines denote kinetic energy E_k, and blue dashed lines denote potential energy E_p.

solutions numerically. The response to the first-order initial condition is plotted in Fig. 5. The generalized coordinates of all modes are increasing with time from the first-order mode initial condition

$$\{f_1, f_2, \ldots, f_N\}_{t=0} = \{0.1, 0, \ldots, 0\}$$
$$\{\dot{f}_1, \dot{f}_2, \ldots, \dot{f}_N\}_{t=0} = \{0, 0, \ldots, 0\} \tag{7}$$

in which the first mode initial generalized deflection is set to a nonzero value while the other mode deflections and the initial generalized velocities are all set to zero.

The first mode initial condition rises gradually the response of other assumed modes with increasing time, the first order initial mode condition gradually increases the response of other assumed modes, as can be seen in Fig. 5. The increasing coordinates of the other mode imply an "energy leakage" from the initially excited first-order mode to the other order modes. By studying the responses of the first six assumed modes due to the different initial modes of orders, it can be concluded that the energy leakage occurs for every case and that the vibration energy, due to the initial mode, is spreading to the other order modes. From the variations of the generalized coordinates, it also can be found that the modes more adjacent to the initial excited mode absorb more energy than those that are farther away.

By inspection of Eq. (4), one finds that the phenomenon of energy transfer among assumed modes is caused by the velocity-dependent term $-(U/L)^2(\boldsymbol{A}+\boldsymbol{B})$ for the constant velocity case. Matrix $\boldsymbol{A}$ is skew symmetric, and matrix $\boldsymbol{B}$ is symmetric. Numerical integrations show that the summation matrix $(\boldsymbol{A}+\boldsymbol{B})$ is diagonally dominant, which indicates that the coupling of the assumed modes is weak when the axially deploying velocity is low. By assuming the deploying velocity to be zero, we can obtain a decoupled equation governing the transverse vibrations of the cantilever beam with constant length.

IV. Adiabatic Invariant of the Decoupled System

The first-order truncation is usually effective for the case of the first-order mode initial condition. Now we study the first-order truncation of Eq. (4) with constant velocity

$$\ddot{f}_1 + [-(U/L)^2 B_{11} + \Lambda_{11}] f_1 = 0 \tag{8}$$

It should be noted that the diagonal elements of matrix $\boldsymbol{A}$ in Eq. (4) are zero.

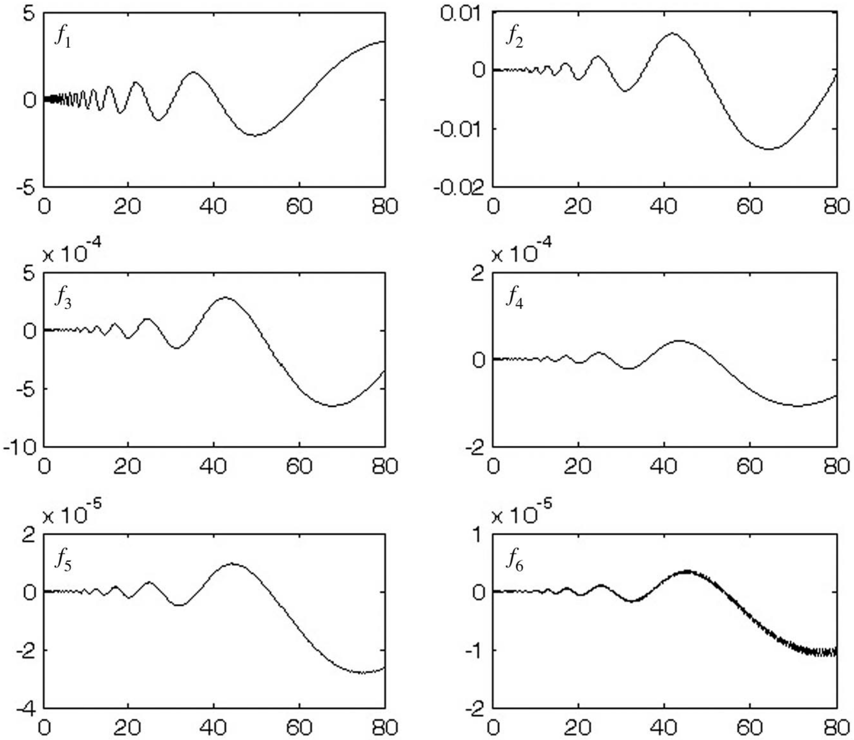


Fig. 5 Truncated generalized coordinates with the first initial mode.

A. Averaging Method

Because $B_{11} \ll \Lambda_{11}$, for low deploying velocities, Eq. (8) can be rewritten as an oscillator with slowly varying frequency

$$\ddot{f}_1 + \omega^2(t) f_1 = 0 \tag{9}$$

in which

$$\omega^2(t) = -\left(\frac{U}{L}\right)^2 B_{11} + \Lambda_{11} \tag{10}$$

Now we employ the averaging method to the truncated system. We put $\dot{f}_1 = \omega(t) g_1$ and transform $f_1, g_1 \to r, \varphi$ by

$$f_1 = r \sin\varphi, \qquad g_1 = r\cos\varphi \tag{11}$$

Substituting Eqs. (11) and their derivatives into Eq. (9), after some manipulations we obtain

$$\begin{aligned} \dot{r} &= -\frac{1}{\omega(t)}\frac{\mathrm{d}\omega}{\mathrm{d}t} r\cos^2\varphi \\ \dot{\varphi} &= \omega(t) + t\frac{1}{\omega(t)}\frac{\mathrm{d}\omega}{\mathrm{d}t} r\sin\varphi\cos\varphi \end{aligned} \tag{12}$$

By integrating the right side of the first equation of (12) with respect to φ over $[0, 2\pi]$, the averaged expression can be obtained as

$$\frac{\mathrm{d}r_a}{\mathrm{d}t} = -\frac{1}{2\omega(t)}\frac{\mathrm{d}\omega}{\mathrm{d}t} r_a \tag{13}$$

After integration of Eq. (13), we get

$$r_a\sqrt{\omega(t)} = C \tag{14}$$

with constant C determined by initial conditions. In the original variables x and $\dot{x}$, Eq. (14) gives

$$\omega(t) f_1^2 + \frac{1}{\omega(t)}\dot{f}_1^2 = C \tag{15}$$

The energy-like quantity defined by Eq. (15) is reserved in a finite time. This quantity, conserved with a varying parameter, is called an adiabatic invariant [44]. An adiabatic invariant is a property of a physical system that stays constant when changes occur slowly.

Figure 6a presents the phase map calculated numerically with the Runge–Kutta method based on the truncated governing Eq. (4) from the first-order mode initial condition. The phase map depicts the curve from the initial point A, where $t = 0$, to the end point B, where $t = 4.8$ s. Apparently, the transverse deflection is increasing while the transverse velocity is decreasing.

The ellipses in Fig. 6b are calculated based on Eq. (15) with the same initial conditions for every 0.3 s from $t = 0$ to 4.8 s. Because the adiabatic invariant (15) holds for the time-varying parameter system, all the ellipses share the same area determined by the initial condition.

B. Bessel Equation Method

With the assumption of constant deploying speed

$$L = Ut \tag{16}$$

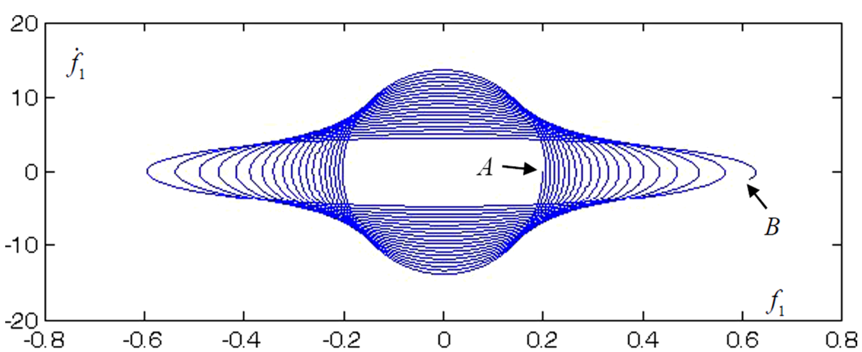
the decoupled time-varying parameter Ordinary Differential Equation (ODE) (8) can be cast into a standard Bessel equation. Tabarrok et al. [6] and Kalaycioglu and Misra [9] derived the solution to Eq. (8)

$$f_1 = \sqrt{t}\left[D_1 J_\nu\left(\frac{\kappa}{t}\right) + D_2 J_{-\nu}\left(\frac{\kappa}{t}\right)\right] \tag{17}$$

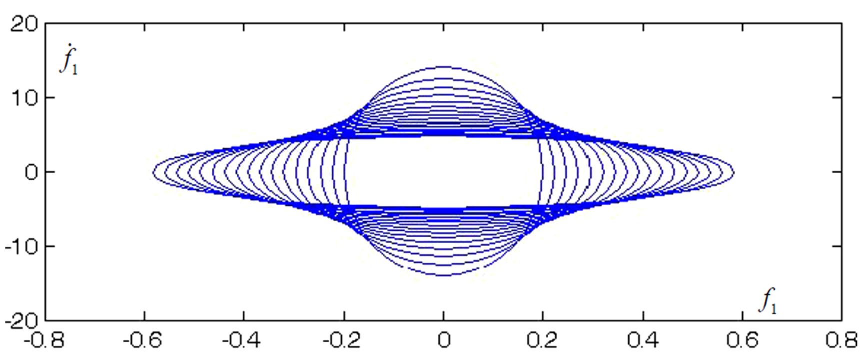
in which J is a Bessel function of order ν and the constants D1 and D2 depend on the initial conditions. The new parameters are defined as

$$\begin{aligned} \kappa &= \sqrt{\Lambda_{11} t^4} = \sqrt{(EI/\rho A U^4)\lambda_1^4} \\ \nu &= \sqrt{B_{11} + \frac{1}{4}} \end{aligned} \tag{18}$$

The qualitative behavior of the deploying beam for small and large values of t can be determined by the asymptotic values of the Bessel function



a) Phase map by numerical method



b) Ellipses determined by the adiabatic invariant

Fig. 6 Comparison of variables determined by numerical method and adiabatic invariant.

$$t \to 0, \qquad \left(\frac{\kappa}{t}\right) \to \infty, \qquad J_{\pm v} \to \sqrt{\frac{2t}{\pi\beta}}{}^{\cos}_{\sin}\left(\frac{\kappa}{t} - \frac{\pi}{4} - \frac{\nu\pi}{2}\right) \tag{19}$$

and

$$t \to \infty, \qquad \left(\frac{\kappa}{t}\right) \to 0, \qquad J_{\pm v} \to \frac{(\kappa/2t)^{\pm\nu}}{\Gamma(1 \pm \nu)} \tag{20}$$

in which Γ is the gamma function. The asymptotic values imply that, at the initial stage of the axial deployment, the oscillatory motions will be dominant. At a later stage, an unbounded motion will dominate the lateral motion [6].

Now we pay attention to the earlier stage of deployment to check the energy-like invariant. Substituting functions (19) into Eq. (17) yields

$$\begin{aligned} f_1 &= \frac{\sqrt{2}Ut}{\sqrt{\pi\kappa}}\left[D_1 \cos\left(\frac{\kappa}{t} - \frac{\pi}{4} - \frac{\nu\pi}{2}\right) + D_2 \sin\left(\frac{\kappa}{t} - \frac{\pi}{4} - \frac{\nu\pi}{2}\right)\right] \\ &= \frac{t}{\sqrt{\kappa}} A \cos\left(\frac{\kappa}{t} + \psi\right) \end{aligned} \tag{21}$$

in which the constants A and ψ can be determined from the initial condition and the velocity.

Considering the asymptotic condition when $t \to 0$, the first derivative of f_1 can be obtained approximately as

$$\dot{f}_1 = \frac{-\sqrt{\kappa}}{t} A \sin\left(\frac{\kappa}{t} + \psi\right) \tag{22}$$

The readers can refer to Eqs. [6] for detained discussion of the Bessel function method. Comparing Eq. (21) and Eq. (22), we have the following equation for

$$\frac{\kappa}{t^2} f_1^2 + \frac{\dot{f}_1}{\kappa/t^2} = C \tag{23}$$

Substituting Eq. (16) into Eq. (10) and using the first equation of (18), we find

$$t \to 0, \qquad \omega(t) \to \frac{\kappa}{t^2} \tag{24}$$

Apparently, the adiabatic invariant (23) obtained by the Bessel function method, agrees with the one obtained by the averaging method (15) in the earlier stage of deploying.

V. Conclusions

In this study, the linear time-varying parameter system of deploying beams has been investigated by the assumed-mode truncation method. The configurations of the assumed-modes have been studied for different initial-order modes. The transfer of energy among different mode orders has been discussed. The energy-like invariant for the transient response have been proposed. The main conclusions are as follows.

1) When set free from one given initial mode, the contour shape of the corresponding mode will be held. The contour shape is being stretched during deploying, while the overall mode shape holds the same configuration as the initial mode.

2) During deployment, the vibration energy due to the initial mode is spreading to the other order modes. It also can be found that the modes more adjacent to the initial excited mode absorb more energy than those that are farther away.

3) For the truncated first-order time-varying system, an energy-like invariant, the adiabatic invariant, was derived by both the averaging method and the Bessel function method. When the beam is being deployed with constant speed, the adiabatic invariant may be kept a constant.

Acknowledgments

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References

[1] Mote, C. D., Jr., "A Study of Band Saw Vibrations," *Journal of the Franklin Institute*, Vol. 279, No. 6, 1965, pp. 430–444. doi:10.1016/0016-0032(65)90273-5

[2] Ulsoy, A. G., Mote, C. D., and Szymani, R., "Principal Developments in Band Saw Vibration and Stability Research," *Holz Als Roh-Und Werkstoff*, Vol. 36, No. 7, 1978, pp. 273–280. doi:10.1007/Bf02610748

[3] Wickert, J. A., and Mote, C. D., "Classical Vibration Analysis of Axially Moving Continua," *Journal of Applied Mechanics*, Vol. 57, No. 3, 1990, pp. 738–744. doi:10.1115/1.2897085

[4] Öz, H. R., and Pakdemirli, M., "Vibrations of an Axially Moving Beam with Time-Dependent Velocity," *Journal of Sound and Vibration*, Vol. 227, No. 2, 1999, pp. 239–257. doi:10.1006/jsvi.1999.2247

[5] Païdoussis, M. P., *Fluid-Structure Interactions: Slender Structures and Axial Flow*, Vol. 1, Elsevier Academic Press, London, 1998, pp. 328–333.

[6] Tabarrok, B., Leech, C. M., and Kim, Y. I., "On the Dynamics of an Axially Moving Beam," *Journal of the Franklin Institute*, Vol. 297, No. 3, 1974, pp. 201–220.

[7] Weeks, G. E., "Dynamic Analysis of a Deployable Space Structure," *Journal of Spacecraft and Rockets*, Vol. 23, No. 1, 1986, pp. 102–107. doi:10.2514/3.25090

[8] Tadikonda, S. S. K., and Baruh, H., "Dynamics and Control of a Translating Flexible Beam with a Prismatic Joint," *Journal of Dynamic Systems Measurement and Control-Transactions of the Asme*, Vol. 114, No. 3, 1992, pp. 422–427. doi:10.1115/1.2897364

[9] Kalaycioglu, S., and Misra, A. K., "Approximate Solutions for Vibrations of Deploying Appendages," *Journal of Guidance Control and Dynamics*, Vol. 14, No. 2, 1991, pp. 287–293.

[10] Bergamaschi, S., and Sinopoli, A., "On the Flexural Vibrations of Arms with Variable Length–An Exact Solution," *Mechanics Research Communications*, Vol. 10, No. 6, 1983, pp. 341–345. doi:10.1016/0093-6413(83)90062-9

[11] AlBedoor, B. O., and Khulief, Y. A., "An Approximate Analytical Solution of Beam Vibrations During Axial Motion," *Journal of Sound and Vibration*, Vol. 192, No. 1, 1996, pp. 159–171. doi:10.1006/jsvi.1996.0181

[12] Stylianou, M., and Tabarrok, B., "Finite-Element Analysis of an Axially Moving Beam 1. Time Integration," *Journal of Sound and Vibration*, Vol. 178, No. 4, 1994, pp. 433–453. doi:10.1006/jsvi.1994.1497

[13] Stylianou, M., and Tabarrok, B., "Finite-Element Analysis of an Axially Moving Beam 2. Stability Analysis," *Journal of Sound and Vibration*, Vol. 178, No. 4, 1994, pp. 455–481. doi:10.1006/jsvi.1994.1498

[14] Tabarrok, B., and Behdinan, K., "Dynamics of Spacecraft Antenna During Deployment," *Proceedings of IUTAM Symposium on Deployable Structures: Theory and Applications*, edited by Pellegrino, S., and Guest, S. D., Kluwer, Dordrecht, 2000, pp. 415–424.

[15] Downer, J. D., and Park, K. C., "Formulation and Solution of Inverse Spaghetti Problem–Application to Beam Deployment Dynamics," *AIAA Journal*, Vol. 31, No. 2, 1993, pp. 339–347. doi:10.2514/3.61535

[16] Pan, Y. C., Scott, R. A., and Ulsoy, A. G., "Dynamic Modeling and Simulation of Flexible Robots with Prismatic Joints," *Journal of Mechanical Design*, Vol. 112, No. 3, 1990, pp. 307–314. doi:10.1115/1.2912609

[17] Kim, Y. K., and Gibson, J. S., "A Variable-Order Adaptive Controller for a Manipulator with a Sliding Flexible Link," *IEEE Transactions on Robotics and Automation*, Vol. 7, No. 6, 1991, pp. 818–827. doi:10.1109/70.105390

[18] Vuquoc, L., and Li, S., "Dynamics of Sliding Geometrically-Exact Beams–Large-Angle Maneuver and Parametric Resonance," *Computer Methods in Applied Mechanics and Engineering*, Vol. 120, Nos. 1–2, 1995, pp. 65–118. doi:10.1016/0045-7825(94)00051-N

[19] AlBedoor, B. O., and Khulief, Y. A., "General Planar Dynamics of a Sliding Flexible Link," *Journal of Sound and Vibration*, Vol. 206, No. 5, 1997, pp. 641–661. doi:10.1006/jsvi.1997.1129

[20] Behdinan, K., and Tabarrok, B., "A Finite Element Formulation for Sliding Beams, Part I," *International Journal for Numerical Methods in Engineering*, Vol. 43, No. 7, 1998, pp. 1309–1333. doi:10.1002/(Sici)1097-0207(19981215)43:7<1309::Aid-Nme456>3.0.Co;2-2

[21] Behdinan, K., Stylianou, M. C., and Tabarrok, B., "Sliding Beams, Part II: Time Integration," *International Journal for Numerical Methods in Engineering*, Vol. 43, No. 7, 1998, pp. 1335–1363. doi:10.1002/(SICI)1097-0207(19981215)43:7<1335::AID-NME472>3.0.CO;2-L

[22] Behdinan, K., Stylianou, M. C., and Tabarrok, B., "Co-Rotational Dynamic Analysis of Flexible Beams," *Computer Methods in Applied Mechanics and Engineering*, Vol. 154, Nos. 3–4, 1998, pp. 151–161. doi:10.1016/S0045-7825(97)00124-2

[23] Matsuzaki, Y., Toyama, M., and Taki, Y., "Vibration of a Cantilevered Beam During Deployment and Retrieval–Analysis and Experiment," *Smart Structures and Materials 1994 Conference — Smart Structures and Intelligent Systems*, Vol. 2190, edited by Hagood, N. W., Proceedings of the Society of Photo-Optical Instrumentation Engineers (SPIE), Orlando, FL, Feb. 1994, pp. 705–716. doi:10.1117/12.175225

[24] Sugiyama, H., Kobayashi, N., and Komaki, Y., "Modeling and Experimental Methods for Dynamic Analysis of the Spaghetti Problem," *Journal of Vibration and Acoustics*, Vol. 127, No. 1, 2005, pp. 44–51. doi:10.1115/1.1857919

[25] Tang, J. L., Ren, G. X., Zhu, W. D., and Ren, H., "Dynamics of Variable-Length Tethers with Application to Tethered Satellite Deployment," *Communications in Nonlinear Science and Numerical Simulation*, Vol. 16, No. 8, 2011, pp. 3411–3424. doi:10.1016/j.cnsns.2010.11.026

[26] Kobayashi, N., and Watanabe, M., "Dynamics and Stability of Spaghetti and Reverse Spaghetti Problems Coupled with Fluid Force," *Multibody System Dynamics*, Vol. 11, No. 2, 2004, pp. 111–125. doi:10.1023/B:Mubo.0000025392.77757.26

[27] Vakakis, A. F., Gendelman, O. V., Bergman, L. A., McFarland, D. M., Kerschen, G., and Lee, Y. S., *Nonlinear Targeted Energy Transfer in Mechanical and Structural Systems*, Springer, Berlin, 2009, pp. 7–9.

[28] Liu, K. F., and Deng, L. Y., "Identification of Pseudo-Natural Frequencies of an Axially Moving Cantilever Beam Using a Subspace-Based Algorithm," *Mechanical Systems and Signal Processing*, Vol. 20, No. 1, 2006, pp. 94–113. doi:10.1016/j.ymssp.2004.10.003

[29] Gosselin, F., Paidoussis, M. P., and Misra, A. K., "Stability of a Deploying/Extruding Beam in Dense Fluid," *Journal of Sound and Vibration*, Vol. 299, Nos. 1–2, 2007, pp. 123–142. doi:10.1016/j.jsv.2006.06.050

[30] Wang, L. H., Hu, Z. D., Zhong, Z., and Ju, J. W., "Dynamic Analysis of an Axially Translating Viscoelastic Beam with an Arbitrarily Varying Length," *Acta Mechanica*, Vol. 214, Nos. 3–4, 2010, pp. 225–244. doi:10.1007/s00707-010-0287-8

[31] Zhu, W. D., and Ni, J., "Energetics and Stability of Translating Media with an Arbitrarily Varying Length," *Journal of Vibration and Acoustics*, Vol. 122, No. 3, 2000, pp. 295–304. doi:10.1115/1.1303003

[32] Chen, L. Q., and Zhao, W. J., "A Conserved Quantity and the Stability of Axially Moving Nonlinear Beams," *Journal of Sound and Vibration*, Vol. 286, No. 3, 2005, pp. 663–668. doi:10.1016/j.jsv.2005.01.011

[33] Chen, L.-Q., "The Energetics and the Stability of Axially Moving Strings Undergoing Planar Motion," *International Journal of Engineering Science*, Vol. 44, Nos. 18–19, 2006, pp. 1346–1352. doi:10.1016/j.ijengsci.2006.08.007

[34] Behdinan, K., Stylianou, M. C., and Tabarrok, B., "Dynamics of Flexible Sliding Beams–Non-Linear Analysis Part I: Formulation," *Journal of Sound and Vibration*, Vol. 208, No. 4, 1997, pp. 517–539. doi:10.1006/jsvi.1997.1167

[35] Park, S., Yoo, H. H., and Chung, J., "Vibrations of an Axially Moving Beam with Deployment or Retraction," *AIAA Journal*, Vol. 51, No. 3, 2013, pp. 686–696. doi:10.2514/1.j052059

[36] Ramu, S. A., and Ganesan, R., "A Galerkin Finite Element Technique for Stochastic Field Problems," *Computer Methods in Applied Mechanics and Engineering*, Vol. 105, No. 3, 1993, pp. 315–331. doi:10.1016/0045-7825(93)90061-2

[37] Ghayesh, M. H., Kafiabad, H. A., and Reid, T., "Sub- and Super-Critical Nonlinear Dynamics of a Harmonically Excited Axially Moving Beam," *International Journal of Solids and Structures*, Vol. 49, No. 1, 2012, pp. 227–243. doi:10.1016/j.ijsolstr.2011.10.007

[38] Tang, Y.-Q., and Chen, L.-Q., "Nonlinear Free Transverse Vibrations of In-Plane Moving Plates: Without and with Internal Resonances," *Journal of Sound and Vibration*, Vol. 330, No. 1, 2011, pp. 110–126. doi:10.1016/j.jsv.2010.07.005

[39] Chen, L.-Q., Zhang, Y.-L., Zhang, G.-C., and Ding, H., "Evolution of the Double-Jumping in Pipes Conveying Fluid Flowing at the Supercritical Speed," *International Journal of Non-Linear Mechanics*, Vol. 58, Jan. 2014, pp. 11–21. doi:10.1016/j.ijnonlinmec.2013.08.012

[40] Wang, P. K. C., and Wei, J. D., "Vibrations in a Moving Flexible Robot Arm," *Journal of Sound and Vibration*, Vol. 116, No. 1, 1987, pp. 149–160.

[41] Marynowski, K., "Dynamic Analysis of an Axially Moving Sandwich Beam with Viscoelastic Core," *Composite Structures*, Vol. 94, No. 9, 2012, pp. 2931–2936. doi:10.1016/j.compstruct.2012.03.040

[42] Yang, X. D., Chen, L. Q., and Zu, J. W., "Vibrations and Stability of an Axially Moving Rectangular Composite Plate," *Journal of Applied Mechanics*, Vol. 78, No. 1, 2011, Paper 011018. doi:10.1115/1.4002002

[43] Chen, E. W., and Ferguson, N. S., "Analysis of Energy Dissipation in an Elastic Moving String with a Viscous Damper at One End," *Journal of Sound and Vibration*, Vol. 333, No. 9, 2014, pp. 2556–2570. doi:10.1016/j.jsv.2013.12.024

[44] Verhulst, F., *Nonlinear Differential Equations and Dynamical Systems*, Springer–Verlag, Berlin, 1996, pp. 147–149.

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