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Feedback Linearization of Hysteretic Dynamics of Smart Structures and Devices

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Abstract

Smart devices are widely used in engineering systems as embedded sensors and actuators. In the current paper, hysteretic dynamics of smart actuators are modelled by using the Landau theory of phase transformations. The original partial differential equation model is reduced to an ordinary differential equation model for convenience of engineering applications. The hysteretic dynamics is linearized by using nonlinear feedback methods. An example of modelling and linearization of hysteretic dynamics of an actuator made from PZT-5 is presented. By using nonlinear feedback methods, an associated linearized system has been obtained.

Key words: Smart devices, hysteresis, polarization switching, nonlinear feedback linearization.

Introduction

Smart devices such as sensors and actuators are widely used in engineering applications due to their capability of converting energy among different physical fields. For devices made from ferroelectric materials, such as lead zirconate titanate (PZT), input energy can be converted between electrical and mechanical fields by the material itself, due to the inherent property of electro-mechanical coupling in the material. The multi-physical coupling property of the material enable the devices made from it to generate electrical signal upon mechanical input (sensors), or vice versa (actuators) [2].

In most of applications, smart devices are embedded in systems as components or sub-systems. For the purpose of analysis, controlling, and optimization of the overall system, it is essential to understand the dynamics of all the components included in the system. The challenging part of the task is associated with the modelling of nonlinear dynamics of smart devices, since a common feature of the dynamics is that hysteresis occurs during the loading and unloading procedure, which is associated with memory effects in the input-output relations of the system. In most cases, the hysteretic dynamics of smart devices can be attributed to phase transitions (polarization switchings in ferroelectric materials) induced in the material by external loadings. The phase transition process itself is an inherently nonlinear process which may lead to coupling of many physical fields.

To seek for a better overall performance of the system a suitable model for the dynamics of smart devices is essential. Considering the fact that control and analysis techniques for systems whose dynamics can be described by Ordinary Differential Equations (ODE) are well developed, it is always beneficial if the hysteretic dynamics can be formulated in a form of ODEs. At the same time, the dimension of the model should also be kept as low as possible, especially when the considered system is a nonlinear one. Another demand for the model is that it should carry insightful information about the mechanisms of the system nonlinearity. The latter desired property of the model is particularly useful for nonlinear analysis and control, since there is no general theory for this purpose, and it is always instructive for the system analysis and control in nonlinear cases if the mechanisms of the nonlinearity under consideration are well understood [7, 8].

Modelling aspects of hysteresis have been under investigation since the beginning of the last century, particularly for the modelling of hysteresis in magnetics using the Preisach model [1]. In order to incorporate dynamical behaviour into hysteresis, the Bouc-Wen model for hysteresis was constructed and the hysteretic dynamics are described in this case by a few ODEs, also in a pure empirical way without referring to any physics behind the phenomenon. The resultant expression is quite complicated [13] and it is not easy to apply for control and optimization purposes. It is also not a trivial task to identify parameters introduced into the model. Recently a new general differential model for hysteresis in ferroic materials was proposed based on thermodynamic consideration of phase transitions [9, 10]. In the model, a free energy function is constructed and the dynamical hysteretic behaviour is modelled

by simulating the transition process from one physical equilibrium phase to another using thermodynamic principles and statistical mechanics. A second approach is to start from a free energy function employing the Landau theory for the first order phase transformations with phenomenological constants. By using a nonlinear electro-mechanical approach at macroscale, hysteretic dynamics induced by phase transitions can be modelled [11, 12].

In the current paper, the hysteretic dynamics of a smart actuator (made from PZT-5) is modelled by using the Landau theory for the first order phase transformations [3, 4]. Based on this model the dynamics of the actuator is described by a set of nonlinear Partial Differential Equations (PDEs). The PDE system is then simplified into a system of ODEs by taking into account the operating conditions of the considered actuator. The resultant differential model has only one degree of freedom, and a few parameters to be determined by using experimental data. The model is capable of capturing hysteretic dynamics precisely. Due to its inherent nonlinearity, the hysteretic dynamics cannot be approximated by any single linear system. For the purpose of linearization, the hysteresis is compensated by introducing an inner feedback which is designed to cancel the nonlinearity in the original system and make the dynamics of the actuator equivalent to its linear counterpart suitable for the control and system analysis purposes.

Landau Theory

In modelling of the dynamics of various devices made from different smart materials, most of their hysteretic dynamics can be directly connected to the Landau phase transformation theory, and the approach for model construction in these cases are similar. Here we consider the modelling of hysteretic dynamics of a PZT-5 actuator (one-dimensional case) as an example. However, the same strategy can be employed while studying hysteretic dynamics in other devices and structures and the corresponding modification in such cases are straightforward.

In a PZT-5 actuator, polarization switchings (phase transitions) are induced by external electrical fields. As sketched in Fig. (1), the actuator has a cross-section area of A, thickness L. Assume there is an electrical field applied to the actuator along the x direction, the polarization inside the material will be changed correspondingly. The direction of the alignment of dipoles will follow the direction of the electrical field. The change of polarization will, in its turn, induce changes of strain and stress in the actuator, which can be used for actuation purposes. A perovskite-type material, like PZT, has a cubic crystalline structure when its temperature is above the Curie temperature θ_c while the crystal structure becomes tetragonal (or orthorhombic) at temperatures lower than θ_c. In the one-dimensional description only two polarization directions exist when
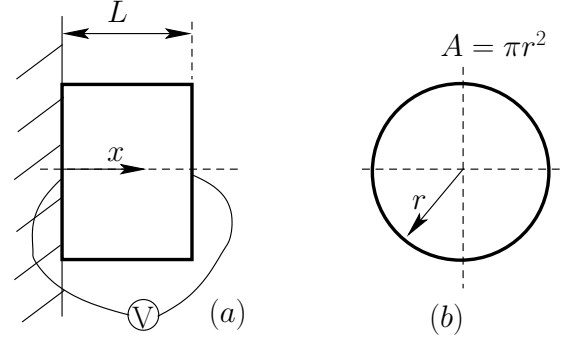

Figure 1: Sketch of a PZT actuator work with full range of applied electrical fields. (a) side view; (b) front view.

$\theta < \theta_c$ when only electrical loadings along the x direction are considered. We denote the tetragonal structure as $P+$ ($P-$), depending on its orientation. Hysteresis in the PZT material is due to the switching between these two orientations. When the material temperature is slightly higher than θ_c, the cubic structure may coexist with tetragonal ones. A one-dimensional model can be made by introducing another dipole orientation (upwards) as sketched in Fig. (2). It is clear that the upward orientation dipoles has no contribution to the electric displacement in the x direction.

In order to make the model thermodynamically consistent, we start from a general Helmholtz free energy function as follows :

$$\Psi(\theta, P, \epsilon) = \psi_\theta(\theta) + \psi_e(P) + \phi_m(\epsilon) + \psi_{me}(P, \epsilon), \quad (1)$$

where P is the polarization, ϵ is the strain, θ is the temperature. In the above energy function, $\psi_\theta(\theta)$ models thermal contributions, $\psi_e(P)$ models electric contributions, $\phi_m(\epsilon)$ models elastic contributions, and $\psi_{me}(P, \epsilon)$ models contributions due to electric-mechanic couplings. Since we are not interested in the dynamics of thermal fields in the current paper, $\psi_\theta(\theta)$ will be neglected. Other fields may still be dependent on the material temperature.

According to the Landau theory, the essential element in the modelling of phase transition dynamics is a free energy function characterizing different phases in the transition (the Landau free energy function). In order to model the polarization switching for the current problem, the Helmholtz free energy function can be constructed at a given temperature θ as [9, 12]:

$$\Psi(P, \epsilon) = \frac{a_2(\theta - \theta_0)}{2} P^2 + \frac{a_4}{4} P^4 + \frac{a_6}{6} P^6 + \frac{k_1 \epsilon^2}{2} - b_1 \epsilon P, \quad (2)$$

in which the Landau free energy function takes the following form :

$$F_l = \frac{a_2(\theta - \theta_0)}{2} P^2 + \frac{a_4}{4} P^4 + \frac{a_6}{6} P^6, \quad (3)$$
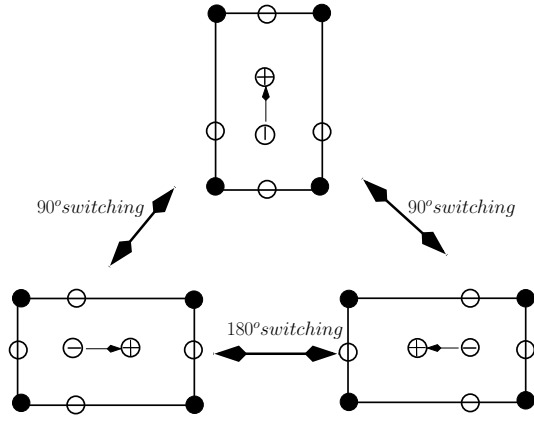
Figure 2: Schematics of phase transformations between cubic and tetragonal structures.

where the constants a_2, a_4, a_6, k_1, b_1, and θ_0 are material constants. Electrostrictive effects are not taken into account in the present consideration. The Ginzburg term, which is introduced to account for the capillary effect (interfacial energy contributions) is also ignored since it is small compared to other terms [4].

By using thermodynamic equilibrium conditions, constitutive laws for the material can be easily obtained on the basis of the above free energy function:

$$E = \frac{\partial \Psi}{\partial P}, \quad \sigma = \frac{\partial \Psi}{\partial \epsilon}, \quad (4)$$

which lead to the following constitutive laws:

$$E = a_2(\theta - \theta_0)P + a_4 P^3 + a_6 P^5 - b_1 \epsilon, \quad (5)$$
$$\sigma = k_1 \epsilon - b_1 P, \quad (6)$$

where σ is the stress, E is the electric field. It is clearly shown by Eq.(5) and (6) that both polarization and strain contribute to the electric field and stress due to the electro-mechanic couplings.

In many engineering applications, it is more convenient to use the electric displacement D instead of polarization P. The difference between D and P is the following :

$$D - P = \epsilon_0 E. \quad (7)$$

Considering the fact that the dielectric constant in vacuum is $\epsilon_0 = 8.859 \cdot 10^{-12}$ F/m, it is clear that D is almost the same as P when E is at the order of 10^6 V/m. This implies that the constitutive laws can be written in terms of the electric displacement instead of the polarization as follows:

$$E = K_1 D + K_2 D^3 + K_3 D^5 - h\epsilon, \quad (8)$$
$$\sigma = -hD + c^D \epsilon, \quad (9)$$

where h and c^D are the piezoelectric constant and stiffness of the material, respectively. Note that the
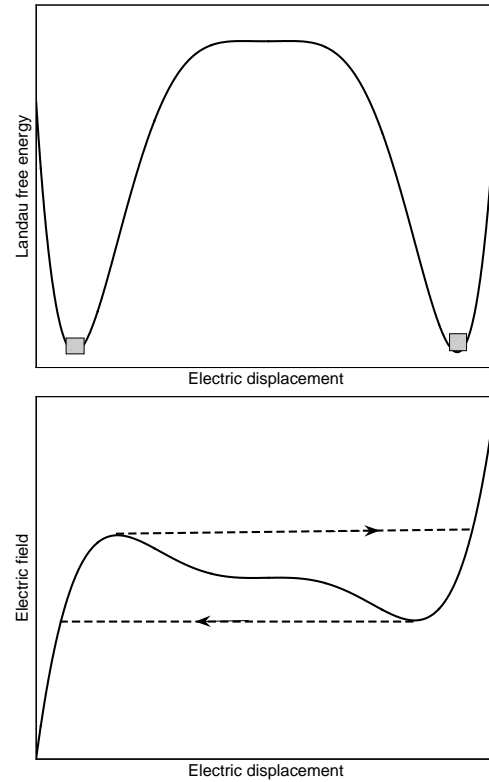

Figure 3: Schematics of the Landau free energy function (top) and the resultant non-convex constitutive $E - D$ curve (bottom). The used parameter values are $K_1 = -1.5249, K_2 = 204.6, K_3 = 1500.8$.

coefficients in Eq.(8) and (9) slightly vary from those in Eq.(5) and (6) due to the replacement of P by D. The coefficients K_2 and K_3 account for nonlinear effects, and $K_1 \approx a_2(\theta - \theta_0)$ includes the temperature dependence (linear) of the constitutive behaviour. The above mentioned coefficients must be determined by fitting to experimental data.

It has been verified that when the values of coefficients are suitably chosen, the above Landau free energy function will have two local minima at low temperature, which can be used for characterization of two polarization orientations. The resultant constitutive relation will account for hysteretic behaviour when the dynamics of the material is considered [12]. To clarify the discussion, one example of the Landau free energy function is plotted in Fig. (3) together with the corresponding constitutive $E - D$ relation (with $\sigma = 0$).

When the temperature is slightly higher than the Curie temperature, the value of $K_1 = a_2(\theta - \theta_0)$ will be different, F_l has three local minima associated with three polarization orientations in the one-dimensional case, and the hysteretic dynamics will be changed. A similar plot is also presented for this case in Fig. (4).
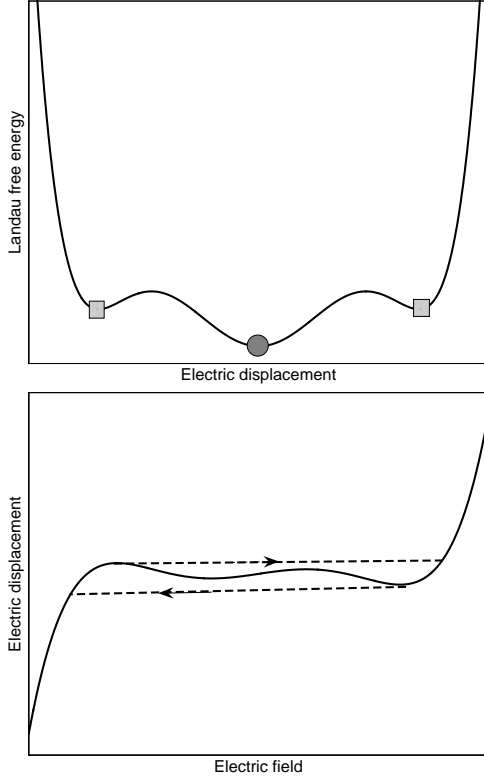
Figure 4: Schematics of the Landau free energy function (top) and the resultant non-convex constitutive $E - D$ curve (bottom). The used parameter values are $K_1 = -3.5249, K_2 = 204.6, K_3 = 1500.8$.

Nonlinear Dynamics

In order to model the electro-mechanic dynamics, the governing equations for the actuator under investigation can be formulated by using conservation laws for a one-dimensional structure. The mechanical part of the dynamics can be modelled by the wave equation [12, 14]:

$$\rho \frac{\partial^2 u}{\partial t^2} = \frac{\partial \sigma}{\partial x} + f_x, \qquad (10)$$

where u, ρ are the particle displacement, and the mass density, respectively. f_x is a distributed body force which is set identically to zero in the current problem. We assume that the actuator is connected in series with an electric impedance Z_{el} and a voltage generator $V(t)$, i.e., Ohm's Law reads:

$$V(t) = Z_{el}I + \int E dz, \qquad (11)$$

where I is the electric current and the spatial integration is performed from actuator electrode 1 to electrode 2 (as sketched in Fig.(1)). Since drift currents in piezoelectric-dielectric transducers are insignificant as compared to displacement currents, it is a good approximation to write for the current:

$$I = A \frac{\partial D}{\partial t}, \qquad (12)$$

with the cross-sectional area of the actuator A .

In order to construct the relationship between the mechanic and electric fields, Eqs. (8) and (12) are substituted into Eq.(11) and the following differential equation is obtained:

$$
\begin{aligned}
\frac{\partial D}{\partial t} &= \left[- \left(K_1 D + K_2 D^3 + K_3 D^5 - \frac{h^2}{c^D} D \right) L \right. \\
&\left. + V(t) + \int \frac{h}{c^D} \sigma(x) dx \right] \frac{1}{Z_{el}A},
\end{aligned}
\qquad (13)
$$

with L being the thickness of the actuator. Eq.(13) constitutes the electric part of the dynamics of the actuator.

For convenience, the governing equations describing the electro-mechanic dynamics are collected and recast into the following system:

$$
\begin{aligned}
\frac{\partial u}{\partial t} &= v, \qquad \epsilon = \frac{\partial u}{\partial x}, \\
\rho \frac{\partial v}{\partial t} &= \frac{\partial}{\partial x} \left(-hD + c^D \epsilon \right), \\
\frac{\partial D}{\partial t} &= \left[- \left(K_1 D + K_2 D^3 + K_3 D^5 - \frac{h^2}{c^D} D \right) L \right. \\
&\left. + V(t) + \int \frac{h}{c^D} \sigma(x) dx \right] \frac{1}{Z_{el}A},
\end{aligned}
\qquad (14)
$$

where v is the particle velocity. Thus, the problem involves solving four dynamic equations for u, v, σ, and D. Boundary conditions are problem-specific for the actuator and will be specified in the next section.

Low Dimensional Model

The model given by Eq. (14) for the dynamics of PZT devices is capable of capturing wave propagations in the device. It can be used for simulation of dynamical response of acoustic transducers [12, 14]. For the modelling of electro-mechanic actuators dynamics of actuators, the resonance in the device is not of major interest. We assume that strain distribution in the actuator along the x direction is uniform, so does the stress and the electrical field. However, we note that is important to know the displacement change of the actuator as a lumped system. In other words, our main focus is in the analysis of the displacement responses of the actuator upon applied electric field. For the actuator, the displacement responses can be characterized by using the displacement responses on one end ($x = L$) while the other end ($x = 0$) is fixed. It is also assumed that the stress on the actuator due to external mechanical loadings is given as $\sigma_0(t)$.

By using the above assumptions, the governing equations can be simplified as follows:

$$\frac{du}{dt} = v, \qquad \rho \frac{dv}{dt} = 0, \qquad (15)$$

$$\frac{dD}{dt} = \left[- \left(K_1 D + K_2 D^3 + K_3 D^5 - \frac{h^2}{c^D} D \right) L \right.$$

87

$$+V(t) + \frac{h}{c^D}\sigma_0 L \Bigg] \frac{1}{Z_{el}A},$$

where the velocity of the actuator becomes a constant. Since stress, strain, and electric field distributions are all uniform along the thickness of the actuator, the following relation is obtained according to Eq. (9) :

$$\sigma_0 = -hD + c^D\epsilon, \qquad (16)$$
$$\epsilon = \frac{\sigma_0 + hD}{c^D}.$$

According to the above assumptions, the description of the dynamics of the mechanical field will be given by an algebraic relation. The physical consequence is that the mechanical-field responds to the input signal much more faster than that of the electric field, hence it can be regarded as always being in its equilibrium state.

By substituting the above equation into Eq. (15), the following equations suitable for control purpose are obtained:

$$\frac{dD}{dt} = \frac{1}{Z_{el}A}\Bigg[-\left(K_1 D + K_2 D^3 + K_3 D^5 - \frac{h^2}{c^D}D\right)L$$
$$+V(t) + \frac{h}{c^D}\sigma_0 L \Bigg], \qquad (17)$$

$$U_L = \int_0^L \epsilon dx = L\frac{\sigma_0 + hD}{c^D}, \qquad (18)$$

where U_L is the output displacement of the considered actuator, which is actually the displacement difference between the two ends of the actuator.

It is clear from the above equations that the dynamics of the actuator now can be easily expressed in the state-space format which is convenient for control purposes. The only system state is the electric displacement D, the only output is U_L, the input is $V(t)$, and σ_0 is the mechanical loading of the actuator. Again, the electric impedance is chosen simply as a resistance $Z_{el} = R$. The model for the dynamics of the actuator is recast into the following nonlinear state-space equation:

$$\dot{D} = \frac{-L}{AR}\left(K_1 D + K_2 D^3 + K_3 D^5 - \frac{h^2}{c^D}D\right)$$
$$+\frac{1}{AR}\left(\frac{Lh}{c^D}\sigma_0 + V(t)\right),$$
$$U_L = \frac{L}{c^D}\sigma_0 + \frac{Lh}{c^D}D. \qquad (19)$$

The nonlinearity included in the above model is sufficient to induce hysteretic dynamics, when the parameter values are suitably chosen as investigated in Ref. [11, 12].

Feedback Linearization

In most applications, nonlinear models are often approximated by corresponding linear systems via linearization methods, since various approaches for system analysis and control have been established for
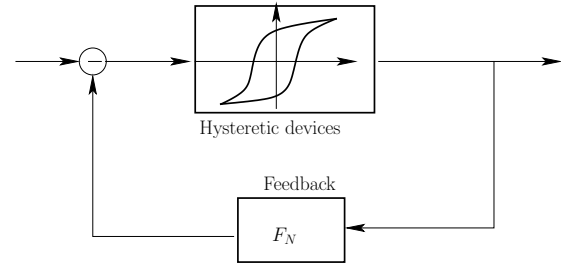

Figure 5: Block diagram of feedback linearization of smart devices with hysteresis.

linear systems and verified via practical applications. For the current problem, it is not possible to linearize the nonlinear model given by Eq. (19) by truncating the Taylor expansion series around a chosen operating point as is done in many other cases. The reason is that there are bifurcations occur in the dynamics [11, 12]. Therefore, to linearize the model, a nonlinear feedback is introduced to cancel the original nonlinearities, and to transform the nonlinear model into a linear model.

The essential idea of feedback linearization is sketched in Fig.(5). Note further that the control problem of the actuator is to tune the output U_L in a desirable way by seeking an appropriate control law $V(t)$. When $V(t)$ is constructed by using the information of system state D, it becomes a feedback control problem. To show the idea of nonlinear feedback, the state space model can be recast into the following form with separated linear and nonlinear parts:

$$\dot{D} = f_L(D) + f_N(D) + g(D)V(t)$$
$$U_o = CD, \qquad (20)$$

where

$$f_L = -\frac{-K_1 L}{AR}D, \quad f_N = -\frac{L(K_2 D^3 + K_3 D^5)}{AR}$$
$$U_o = U_L - \frac{L}{c^D}\sigma_0, \quad C = \frac{Lh}{c^D}, \qquad (21)$$
$$g(D) = \frac{1}{AR}.$$

Assume that a nonlinear feedback law can be constructed for $V(t)$ as a nonlinear function of the system state D and a new input signal $V_l(t)$:

$$V(t) = \alpha(D) + \beta(D)V_l(t). \qquad (22)$$

The goal of the nonlinear feedback is then to transfer the system (20) to a linear one as follows :

$$\dot{D} = M_a D + M_b V_l$$
$$U_o = M_c D, \qquad (23)$$

where M_a, M_b and M_c are constants (for higher dimensional systems they are matrices). In the current problem, M_b can be an arbitrary constant, while

$$M_a = -\frac{-K_1 L}{AR}, \qquad M_c = \frac{Lh}{c^D}.$$

By substituting the control law (22) into Eq.(23), the following relations can be obtained :

$$\dot{D} = M_a D + M_b \beta^{-1} \left((V(t) - \alpha(D)) \right), \quad (24)$$

where β^{-1} is the inverse operator of matrix $\beta(D)$ for every D. By comparing the above relation with Eq.(20), the following two relations must be satisfied by an appropriate choice of α and β :

$$-M_b \beta^{-1} \alpha(D) = f_N(D) = -\frac{L}{AR} \left(K_2 D^3 + K_3 D^5 \right)$$

$$M_b \beta^{-1} = g(D) = \frac{1}{AR}, \qquad (25)$$

which gives the following formula:

$$\alpha(D) = L(K_2 D^3 + K_3 D^5), \quad \beta = ARM_b \qquad (26)$$

It is easy to check that the nonlinear feedback given by Eq.(26) will convert Eq.(20) into a linear one given by Eq.(23), which is suitable for applying linear control theory for controller design. The relationship between $V(t)$ and $V_l(t)$ is not a one-to-one map due to the contribution of the nonlinear function $\alpha(D)$ which is also a non-convex function of D. This non-convexity will introduce nonlinear and hysteretic behaviour to cancel the original hysteretic behaviour in the model and make the new input-output relation a linear one.

Conclusion and Discussion

In the present work, the hysteretic dynamics of a PZT-5 actuator is modelled by using the Landau theory for the first order phase transformations. The full electro-mechanical dynamics is described by a set of coupled nonlinear partial differential equations. The PDE model is reduced to a system of ODEs by taking into account the operating conditions of the actuator. A nonlinear feedback mechanism is introduced into the dynamics to cancel the hysteretic input-output relation. The nonlinear system is finally converted into a linear one by using a nonlinear transformation.

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