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Accounting for Thermal Effects in Dynamics of Piezoelectric Structures

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1 Introduction

An increasing interest in smart material and structure technology generated by many industrial applications has put a special emphasis on a better understanding of the dynamic behaviour of piezoelectrics as an integral part of this technology. Due to the ability to generate a charge in response to a mechanical deformation (the direct piezoelectric effect) and to provide mechanical strains when an electric field is applied (the converse piezoelectric effect), piezoelectric materials are employed both as sensors and actuators in the development of smart structures. In many industrial applications piezoelectric sensors and actuators are used as embedded control elements in advanced composites and smart structures that are subjected to severe mechanical and thermal environments. The influence of thermal effects on the dynamic behaviour of piezoelectrics and the coupling between thermal, mechanical, and electric fields are key issues in the analysis of such structures. Although these issues are also important in many traditional applications of piezo-transformers, acoustic wave generators, transducers, and other devices with an active piezoelectric element, they have not been studied in the literature with the vigour they deserve.

2 Mathematical Models in Dynamic Thermoelectroelasticity

Following earlier work in this direction (see [1, 9, 10, 4, 6, 7, 8] and references therein) we consider a coupled system for piezoelectrics that includes the equation of motion and the Maxwell equation for electric field

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \sigma_{ji,i} + F_i, \quad D_{i,i} = f_2, \quad i,j = 1,2,3, \qquad (1)$$

where u_i are components of the mechanical displacement vector, ρ is the density of the material, σ_{ij} is the stress tensor, D_i are electric displacements, F_i are components of the vector of mass forces, f_2 is the electric charge density. An important new element of this work is that we consider the full thermo-electromechanical (rather then electromechanical as in [6]) coupling by employing the following state equations

$$\sigma_{ij} = c_{ijkl}\epsilon_{kl} - e_{ijm}E_m - \gamma_{ij}\theta, \qquad (2)$$

$$D_i = \varepsilon_{ik}E_k + e_{ikl}\epsilon_{kl} + p_i\theta, \qquad (3)$$

where ϵ_{ij} are strains, $E_i = -\nabla\varphi$ are components of the electric field with the potential φ, θ is the difference between the current, T, and the reference temperature, T_0, c_{ijkl} are elements of the matrix of elastic moduli, e_{ikl} are piezoelectric constants, γ_{ij} are elements of the matrix product $\Lambda \cdot \Xi$ (Λ is the matrix of elastic moduli, Ξ is the thermal expansion matrix), ε_{ik} are dielectric permittivities, and p_i are pyroelectric (thermoelectric coupling) constants. The Cauchy formula is assumed for the strain-displacement relation. The energy balance equation is expressed in the form of the continuity equation for the amount of heat and supplemented by the constitutive equation for the entropy (per unit volume), S,

$$T\frac{\partial S}{\partial t} = -q_{i,i} + Q, \qquad (4)$$

$$S = \gamma_{ij}\epsilon_{ij} + c_\epsilon^V \theta / T_0 + p_i E_i, \qquad (5)$$

where c_ϵ^V is the (volumetric) heat capacity, $\mathbf{q} = (q_1, q_2, q_3)$ is the heat influx through the body surface and Q is the energy due to heat generation inside of the body.

In the general case the complete set of equations is nonlinear. The heat capacity, thermal conductivity, and electric resistance are all functions of temperature. Even assuming that these functions are constants within the range of temperatures under consideration, the nonlinearity of system (1)–(5) comes from the factor $T = T_0(1 + \theta/T_0)$ in the energy balance equation. Formally, the assumption $|\theta/T_0| \ll 1$ will reduce the system (1)–(5) to a linear one. In both cases Joule losses in piezoelectrics (determined via electric resistivity ρ_e and the current density $\mathbf{J}$ as $\rho_e\mathbf{J}^2$) acting as driving terms and a strong thermo-electromechanical coupling (including the coupling via boundary conditions for stress [6]) complicates essentially the solution of this problem.

3 Computing Dynamics of Cylindrical Piezoelectric Elements

In many applications where piezoelectric elements are used as part of engineering devices, the cylindrical topology has a number of advantages compared to other topological designs. In what follows we consider nonstationary axisymmetric oscillations of hollow thin-walled piezoceramic cylinder. If the thickness of this cylinder (i.e. the difference between its external, R_1, and internal, R_0, radii) is considerably smaller than its length (mathematical speaking we call such cylinders "infinitely long"), one dimensional models are sufficient to describe adequately the most important characteristics of piezoelectric-based devices. All such characteristics are functions of the dynamics of coupled thermoelectromechanical fields. Therefore, for the analysis of thermoelectroelastic fields and determination of device characteristics the solution of a general time-dependent (rather than steady-state) problem is required. This brings to light an important and challenging problem the mathematical formulation of which for cylinders preliminary polarised radially follows.

Our main interest in this paper lies with the coupled system of mixed partial differential equations written in the cylindrical system of coordinates

$$\rho\frac{\partial^2 u}{\partial t^2} = \frac{1}{r}\frac{\partial}{\partial r}(r\sigma_r) - \frac{\sigma_\theta}{r} + f_1(r,t), \qquad (6)$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rD_r) = f_2(r,t), \qquad (7)$$

$$T_0\left(1+\frac{\theta}{T_0}\right)\frac{\partial S}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}(kr\nabla\theta) + f_3(r,t), \qquad (8)$$

where f_i, $i = 1,2,3$ are given functions. Following [5] the constitutive equations of the polarised piezoceramics are equivalent to the equations of piezocrystal of *hexagonal 6mm* symmetry. As in the general case, we choose strain, electric field and temperature as independent variables in our constitutive equations. Denoting by $\boldsymbol{\sigma} = (\sigma_r, \sigma_\theta)^T$, $\boldsymbol{\epsilon} = (\epsilon_r, \epsilon_\theta)^T$, and $\mathbf{E} = (E_r, E_\theta)^T$ vectors of stress, strain, and electric field, respectively, the constitutive equation (2) for stress can then be written in the generalised Duhamel-Neumann form

$$\boldsymbol{\sigma} = \mathbf{c}\boldsymbol{\epsilon} - \mathbf{e}^T\mathbf{E} - \boldsymbol{\gamma}\theta, \qquad (9)$$

while the constitutive equation (3) for electric displacement can be written as

$$\mathbf{D} = \mathbf{e}\boldsymbol{\epsilon} - \boldsymbol{\epsilon}\mathbf{E} + \mathbf{p}\theta. \qquad (10)$$

c_{11}	12.1×10^{10}	e_{33}	15.8
c_{13}	7.52×10^{10}	e_{13}	-5.4
c_{33}	11.1×10^{10}	ϵ_0	8.85×10^{-12}
α_{33}	$5. \times 10^{-6}$	p_3	$4. \times 10^{-4}$
α_{11}	$5. \times 10^{-6}$	c_ϵ^S	420
ρ	7.5×10^3	k	1.25

TABLE 1. Characteristics of PZT-5 piezoceramics

The entropy of the thermoelectroelastic system under consideration is

$$S = \boldsymbol{\gamma}^T\boldsymbol{\epsilon} + \mathbf{p}^T\mathbf{E} + c_\epsilon^V\theta/T_0, \qquad (11)$$

where vectors $\boldsymbol{\gamma}$ and $\mathbf{p}$ have specific forms discussed below, and $c_\epsilon^V = \rho c_\epsilon^S$ (c_ϵ^S is the specific heat capacity). The Cauchy relationship between deformations (strains) and (radial) displacements, and the electrostatic form for the electric field (via the electrostatic potential, φ) are

$$\epsilon_r = \frac{\partial u}{\partial r}, \quad \epsilon_\theta = \frac{u}{r}, \quad E_r = -\frac{\partial \varphi}{\partial r}. \qquad (12)$$

Other things to note about (9)–(11) is the specific forms of the vector of pyroelectric coefficients, $\mathbf{p} = (p_3, 0)^T$, and the elastic (stiffness) and electric constant matrices

$$\mathbf{c} = \begin{pmatrix} c_{33} & c_{13} \\ c_{13} & c_{11} \end{pmatrix} \quad \mathbf{e} = \begin{pmatrix} e_{33} & e_{13} \\ 0 & 0 \end{pmatrix}. \qquad (13)$$

Finally, the thermoelastic pressure vector $\boldsymbol{\gamma} = (\gamma_{33}, \gamma_{11})^T$ in (9) and (11) is defined as

$$\boldsymbol{\gamma} = \mathbf{c}\boldsymbol{\alpha}, \qquad (14)$$

where $\boldsymbol{\alpha} = (\alpha_{33}, \alpha_{11})^T$ is the vector of thermal expansion coefficients (taken as in [3]). The system (6)–(12) has been supplemented by the initial

$$u = u_0(r), \quad \frac{\partial u}{\partial t} = u_1(r), \quad \theta = \theta_0(r), \qquad (15)$$

and boundary conditions

$$\sigma_r = p_i(t), \varphi = \pm V(t), \frac{\partial \theta}{\partial r} = 0, r = R_0, R_1 \quad (16)$$

with given functions $p_i(t), i = 1,2$, $V(t)$, $u_0(r)$, $u_1(r)$, and θ_0.

4 Material Properties and Scaling Procedures

In many applications of acoustic wave generation, piezo-transducers/transformers design, and piezo-sensors/actuators in the vibration control of flexible structures, piezoceramics such as lead zirconate titanates (PZT) are known to be an excellent material to use. In this paper all computations were performed for piezoceramic PZT-5. Characteristics of this material are presented in Table 1.

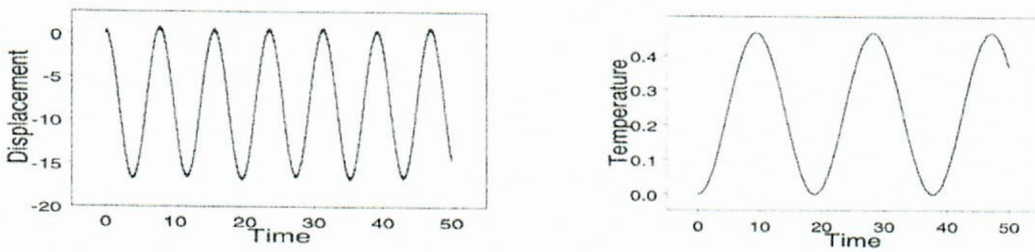

FIGURE 1. A negligible effect of temperature on mechanical characteristics of devices.

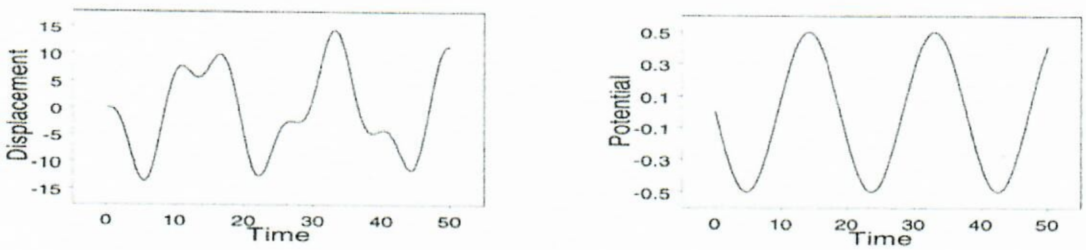

FIGURE 2. Voltage as a control variable of thermomechanical characteristics of devices.

All constants in Table 1 are given in the SI base units (consult [3] on pyroelectric and thermal expansion coefficients, and [1] for other characteristics). It is easy to find that $c_\epsilon^V = 3.255 \times 10^6 \mathrm{J}/(\mathrm{m}^3\mathrm{K})$, and that $\gamma = (9.31, 9.81)^T \times 10^5 \mathrm{N}/(\mathrm{m}^2\mathrm{K})$ (the latter follows from (14)). In all computations we assumed $T_0 = 300°K$ and $\epsilon_{33} = 830\epsilon_0$. Note also that coefficients in the matrix $\mathbf{c}$ were taken at constant electric field $\mathbf{E}$ (for alternative considerations the reader should consult [1] e.g., where constants measured at constant electric displacement are also given).

A substantial difference in parameter magnitudes requires paying due attention to scaling procedures performed with respect to system (6)–(16) prior to its numerical solution. Introducing dimensionless variables $\tilde{t} = t/t_*$, $\tilde{r} = r/R_*$, $\tilde{u} = u/u_*$, $\tilde{\theta} = \theta/\theta_*$, $\tilde{\varphi} = \varphi/\varphi_*$, $\tilde{D}_r = D_r/D_*$, $\tilde{\sigma} = \sigma/\sigma_*$ and setting $\rho u_* R_*/(t_*^2\sigma_*) = 1$, $c_{11}u_*/(R_*\sigma_*) = 1$, $\epsilon_{33}\varphi_*/(D_*R_*) = 1$, and $\gamma_{33}\theta_*/\sigma_* = 1$ we reduce the system (6)–(16) to a dimensionless form where new coefficients take computationally convenient values. For example, the scaling law

$$\tilde{c}_{ij} = c_{ij}/c_{11}, \quad \tilde{e}_{kl} = e_{kl}/\sqrt{\epsilon_{33}c_{11}} \tag{17}$$

leads to the following matrices of coefficients

$$\tilde{\mathbf{c}} = \begin{pmatrix} 0.917 & 0.621 \\ 0.621 & 1. \end{pmatrix} \quad \mathbf{e} = \begin{pmatrix} 0.530 & -0.181 \\ 0 & 0 \end{pmatrix}.$$

In this case $\tilde{\rho} = 1$ and $c = R_*/t_* = \sqrt{c_{11}/\rho}$. As for the energy balance equation, it was reduced to the following form

$$\frac{\partial\tilde{\theta}}{\partial\tilde{t}} = \frac{\kappa}{\omega c_\epsilon^V}\left[\frac{1}{\tilde{r}}\frac{\partial}{\partial\tilde{r}}(k\tilde{r}\nabla\tilde{\theta}) + \tilde{f}_3\right] - \frac{\partial}{\partial\tilde{t}}\left(\tilde{\gamma}_{33}\tilde{\epsilon}_r + \tilde{\gamma}_{11}\tilde{\gamma}_{11} + \tilde{p}_3\tilde{E}_r\right), \tag{18}$$

where $\tilde{f}_3 = R_*^2 f_3/\theta_*$, $\kappa = t_*/R_*^2$, $\omega = 1 + \theta_*\tilde{\theta}/T_0$. This form allows us to extend numerical procedures described and analysed in detail in [6, 7] to problem (6)–(12).

5 Computational Experiments

Up to now, many efforts have been devoted to vibration suppression of piezoelectric-based structures subjected to mechanical loading. Since many such structures work under fairly large variations in temperature in the presence of electric fields, our experiments concern the influence of thermal and electrical loading patterns on mechanical characteristics of piezoelectric-based devices. Therefore, in all experiments we assume stress-free boundary conditions and neglect any mass forces (that is $p_i = 0$, $i = 1, 2$, $f_1 = 0$).

Figure 1 (left) presents the result of computation of radial displacements on the external surface of the cylinder. The thickness of the cylinder is 0.1 in the dimensionless units. The temperature field distribution is given in Fig 1 (right). In this case, where the potential difference was kept constant (equal to 1 in the dimensionless units), the effect of thermal field is practically negligible and these results are identical to those obtained in the framework of electroelasticity theory [6]. If observed oscillations are to be controlled (as is the case in many piezo-sensor/actuator applications), this can be achieved by using the voltage (supplied to the actuators) as a control variable. This, however, is a nontrivial task because the input voltage depends on the effective surface electrodes and the

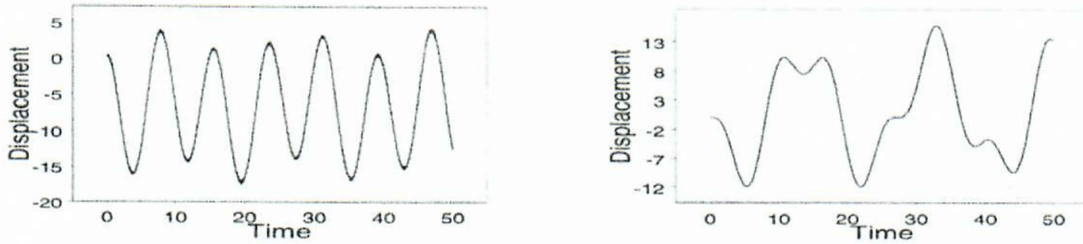

FIGURE 3. The effect of temperature on mechanical characteristics of devices.

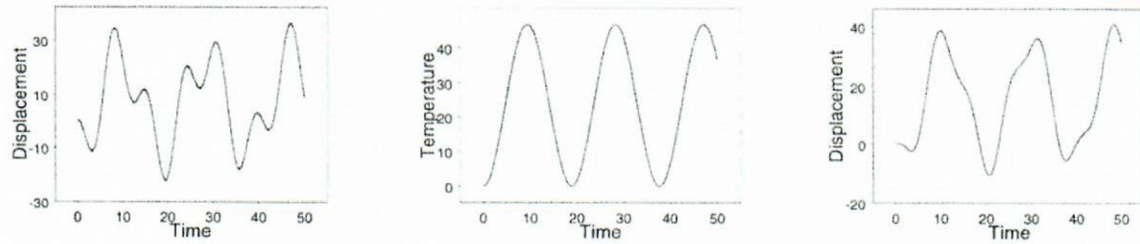

FIGURE 4. Thermally induced nonlinear effects in piezoelectric devices.

electrical displacement [3]. Under the same temperature conditions as in the previous example, the effect of variable voltage (Fig. 2 (right)) on the displacements is demonstrated by Fig. 1 (left). The effect of the increasing temperature (in 10 times) on the mechanical characteristics of the device is demonstrated in Fig. 3, obtained under the constant (Fig. 3(left)) and variable (Fig. 3(right)) potential difference. Further increase in temperature (Fig. 4), leads to the situation where nonlinear effects become increasingly important due to violation of the condition $|\theta/T_0| \ll 1$.

6 Conclusions and Future Directions

In this paper the influence of coupled thermal and electric fields on mechanical characteristics of piezoceramic-based devices has been analysed numerically. On the example of axisymmetric oscillations of hollow thin-walled piezoceramic cylinders it was shown that the thermal field can contribute substantially to such important characteristics of devices as radial displacements.

The observed effects become increasingly important in such cases where a piezoelectric has to be considered as a part of smart structure (especially when such a structure/substrate made a polymeric composite material) whose response piezoelectric sensors should monitor, or where the piezoelectric-based device is imbedded into an acoustic interacting media. Although technically non-trivial, mathematically it is relatively straightforward to generalise our numerical schemes to these cases.

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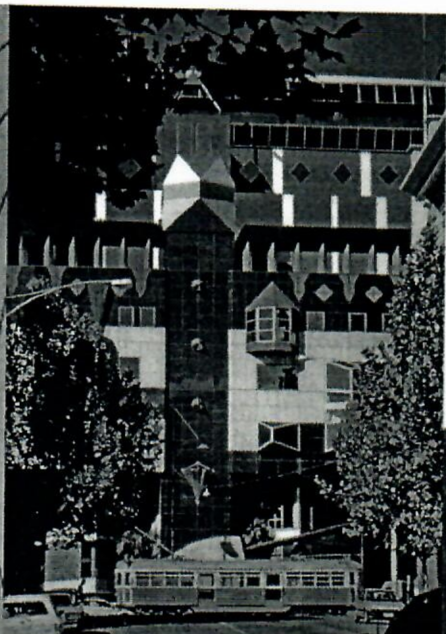

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