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# The dispersion and damping of the Dirac plasmon polariton of graphene in water

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**Abstract**—We study theoretically the dispersion and damping of the Dirac plasmon polariton in graphene in the range of frequencies from the microwave to the near infrared in the presence of a thick layer of pure water at room temperature.

**Index Terms**—graphene, Dirac plasmon polariton, water, dielectric permittivity

## I. INTRODUCTION

Graphene has been recently studied with increasing interest for applications in nanophotonic and nanoplasmonic devices that operate in the terahertz (THz) to the near infrared range of frequencies [1]–[5]. Further applications of graphene have emerged quite recently at lower frequency ranges, such as tunable radiofrequency (RF) plasmonic antennas [6], or biochemical sensors, where graphene operates at the RF while being exposed to a liquid electrolyte [7].

The main feature of graphene’s dynamic response to the electromagnetic fields in those applications is its sheet plasmon, or Dirac plasmon polariton (DPP). This collective electronic mode arises in doped graphene due to the intraband electronic excitations within graphene’s  $\pi$  electron band treated in the Dirac cone approximation. The dispersion relation of the DPP is highly tunable by changing the doping density in graphene by applying the potential to external gate(s). The DPP can therefore exhibit quite long propagation distances along samples of clean graphene, whereas the associated electromagnetic fields exhibit strong confinement in the directions perpendicular to graphene [3]–[5].

However, in “dirty” graphene samples, the presence of the nearby charged impurities or atomic defects in the atomic lattice of graphene can cause significant dissipative processes that can reduce the propagation distances of the DPP. On the other hand, the media surrounding graphene in various devices may exhibit strong dissipative channels as well. Such “lossy” media are known to affect the operation of standard nanoplasmonic devices based noble metals [8], [9], but comparatively less is known about the effects of such media on the DPP in graphene. In particular, the dielectric permittivity of water is known to exhibit large dissipative features at sub-THz frequencies [10], which can strongly affect the operation of graphene devices in biochemical sensors [7]. Therefore, we provide here an initial

study of the dispersion and damping of the DPP in a large sheet of graphene immersed in water.

## II. MODEL

By placing graphene in the  $xy$  plane of a Cartesian coordinate system, we can write the electric and magnetic fields of a TM polarized DPP in the regions  $m = 1$  (for  $z > 0$ ) and  $m = 2$  (for  $z < 0$ ) as [11]

$$\begin{aligned} \mathbf{E}_m &= (E_{m,x}, 0, E_{m,z}) e^{-i\omega t} e^{ikx} e^{-q_m|z|}, \\ \mathbf{B}_m &= (0, B_{m,y}, 0) e^{-i\omega t} e^{ikx} e^{-q_m|z|}, \end{aligned}$$

where  $q_m^2 = k^2 - \omega^2 \epsilon_m / c^2$ , with  $k$  being the in-plane wavenumber of the DPP in graphene and  $\epsilon_m$  the relative dielectric permittivity of the material in the region  $m$ . By applying the usual electromagnetic boundary conditions in the  $xy$  plane, where the in-plane current in graphene may be expressed in terms of the in-plane electric field via a two-dimensional Ohm’s law with the graphene conductivity  $\sigma$ , one arrives at the relation [11]

$$\frac{\epsilon_1}{q_1} + \frac{\epsilon_2}{q_2} + \frac{i}{\epsilon_0} \frac{\sigma}{\omega} = 0, \quad (1)$$

which may be used to deduce the dispersion relation,  $k(\omega)$ , for the DPP mode for a given frequency  $\omega$  of the electromagnetic fields.

It was shown that the conductivity of graphene at low frequencies may be well described by the standard Drude model [2],

$$\sigma(\omega) = i \frac{v_B}{\pi} \frac{\omega_F}{\omega + i\gamma} \quad (2)$$

where  $v_B = e^2 / \hbar \approx c / 137$  is the Bohr velocity,  $\gamma$  an intrinsic damping rate of graphene, and  $\omega_F = v_F k_F$ , with  $v_F \approx c / 300$  being the Fermi speed in graphene and  $k_F = \sqrt{\pi |n|}$  its Fermi wavenumber for the doping density  $n$ . We shall take  $n = 2.35 \times 10^{13} \text{ cm}^{-2}$  as a typical value.

We only consider a simplified structure of a graphene sheet immersed in a bulk water, so that  $\epsilon_1 = \epsilon_2 = \epsilon_w(\omega)$ , allowing us to solve Eq. (1) for  $k$  in terms of  $\omega$ . We use the analytical expression for a frequency-dependent relative dielectric permittivity of water,  $\epsilon_w(\omega)$ , given as the sum

of terms representing damped harmonic oscillators, with the parameters listed in the table L2.1. of the Ref. [10]. We are primarily interested in the properties of this function in the gigahertz (GHz) to the THz range, where the imaginary part of  $\epsilon_w(\omega)$  takes large positive values, and exhibits rather intricate behaviour due to variety of dissipative processes, as shown in Fig. 1.

### III. RESULTS AND DISCUSSION

Considering the requirement that the DPP mode remains localized on graphene, that is, the real part of the perpendicular wavenumber  $q$  in water being positive, yields from Eq. (1) a constraint on the intrinsic damping rate of graphene,  $\gamma < \omega \Re\{\epsilon_w(\omega)\} / \Im\{\epsilon_w(\omega)\}$ . It turns out that, for frequencies  $f = \omega/(2\pi) \lesssim 20$  THz, this gives a quite severe constraint,  $\gamma \lesssim \omega$ . Leaving aside the issue of the existence of the DPP when large intrinsic damping in graphene clashes with dissipative processes in water, we take here the limit of ideal graphene,  $\gamma = 0$ , and show in Fig. 2 the real and imaginary parts of the resulting wavevector in graphene in the presence of water,  $k_w$ . One notices that  $\Re\{k_w\}$  is always larger than the corresponding value for graphene in the air,  $\Re\{k_{air}\}$ , and in the GHz range it is much larger, indicating that the DPP wavelength is significantly shorter for microwave radiation.

At the same time, the imaginary part,  $\Im\{k_w\}$ , is always smaller than its real counterpart in Fig. 2, but is close to it, especially in the range of frequency  $f \sim 10$  THz. This indicates that the most attractive range of frequencies for graphene's nanoplasmonic and nanophotonic applications may be diversely affected by the presence of water, which can significantly reduce the propagation distances of the DPP compared to the case of graphene surrounded by a material without significant dielectric losses. In future work, we shall explore a more realistic model of a multilayered structure, including graphene, a finite layer of water, as well as other dielectric materials.

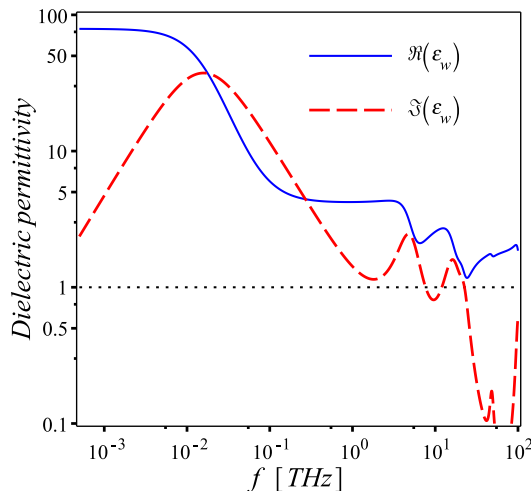


Fig. 1. Real and imaginary parts of the dielectric permittivity of water.

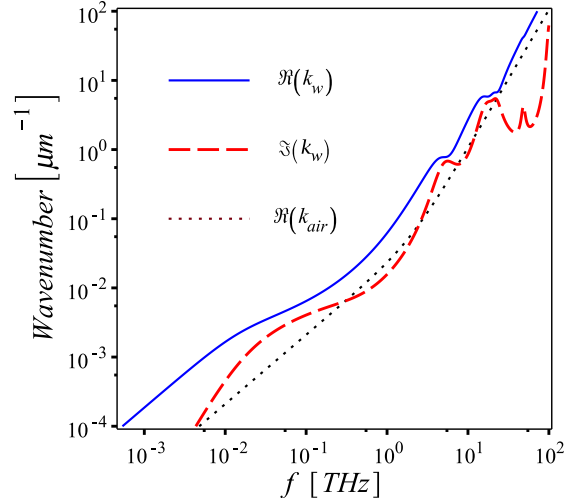


Fig. 2. Real and imaginary parts of the plasmon polariton wavenumber in water, and its real part in the air.

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