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A differential algebraic approach for the modeling of polycrystalline ferromagnetic hysteresis with minor loops and frequency dependence

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ABSTRACT

In the current paper, a nonlinear differential algebraic approach is proposed for the modeling of hysteretic dynamics of polycrystalline ferromagnetic materials. The model is constructed by employing a phenomenological theory to the magnetization orientation switching. For the modeling of hysteresis in polycrystalline ferromagnetic materials, the single crystal model is applied to each magnetic domain along its own principal axis. The overall dynamics of the polycrystalline materials is obtained by taking a weighted combination of the dynamics of all magnetic domains. The weight function for the combination is taken as the distribution function of the principal axes. Numerical simulations are performed and comparisons with its experimental counterparts are presented. The hysteretic dynamics caused by orientation switching processes is accurately captured by the proposed model. Minor hysteresis loops associated with partial-amplitude loadings are also captured. Rate dependence of the hysteresis loops are inherently incorporated into the model due to its differential nature.

1. Introduction

Hysteresis is a common phenomenon existing in most magnetic materials and other smart materials, if not all. Static hysteresis loop in the H–M curve (H is the magnetic field, M is the magnetization) was found centuries ago, and its correspondence with the magnetization orientation switching was well accepted [1–3]. At the same time, in other materials such as electromagnetic, ferromagnetic, and magnetoelastic materials, hysteretic behaviors of the magnetic fields have a dramatic influence on the dynamics of other physical fields involved in the systems, due to the complicated coupling effects among them.

Modeling of hysteresis loops in the H–M curves is a classic problem and has attracted a lot attention. Among the models employed for hysteresis investigations, the Preisach model [1,2,4] and the Jiles-Atherton model [5] are the two main models. The Preisach model was first proposed by Preisach based on some assumption of the physical mechanism of magnetization phenomenon in 1935. The basic idea of the model is that magnetic material is assumed to consist of a group of magnetic dipoles which have rectangle hysteric behaviors. Macroscopic hysteric behavior of the material is seen as the sum of hysteric behaviors of these magnetic dipoles and the system output is the weighted sum of all basic hysteresis relay operators in material. In the 1970s, the Russian mathematician Krasnoselskii separated this model from its physical meaning and represented it in a pure mathematical form which is similar to a spectral resolution of operators [6]. The new generalized Preisach model, which is a mathematical model and belongs to phenomenological theory system because of its independence of physical mechanism of hysteresis, can now be used for the mathematical description of hysteresis of any physical nature. The advantage of the mathematical model is that it ensures the closure of the minor hysteresis loop and can be applied to the condition where physical mechanism hasnot been well understood because of its generalized algorithm. It also has high predictive ability of nonlinear hysteresis and good versatility. But the model can represent the relationship only between the input and output and provides no insights into the physical mechanism of hysteresis dynamics of the system. In addition, the complex formulas and the identification of a large number of non-physical parameters are needed which make the model inflexible and time-consuming. Currently there are still many efforts devoted to modifying and improving the Preisach model for the better modeling of the hysteresis dynamics behavior [7].

In 1986, the J-A model was proposed by Jiles and Atherton based on the ferromagnetic theory which believe that the irreversible domain wall motion results in the hysteresis. The effective magnetic field in material is the sum of the excitation field, the interaction field between domains and the field induced by stress, and the magnetization consists of reversible and irreversible

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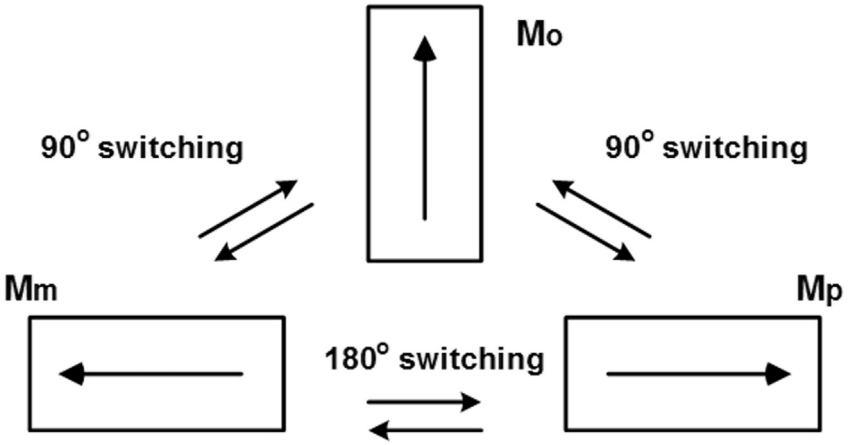

Fig. 1. Sketch of magnetization orientation switching in a one-dimensional analog.

components. The J-A model which is represented by a differential equation can't ensure the closure of the minor hysteresis loop although it has been proved to have good precision on modeling the hysteretic behavior in magnetostrictive material when the change of the magnetic field keeps at a constant rate. In 1992, the original model was extended by Jiles to make the minor hysteresis loop close [8], but this reduces the utility of the model for dynamic control. Furthermore, the J-A model can capture the hysteric behavior only in ferromagnetic materials.

Motivated by engineering applications, the model for hysteresis is desired to incorporate the dynamical behavior of the orientation switching processes, which is responsible for the dependence of the hysteresis loops on the loading rates. At the same time, it should also be capable of capturing minor hysteresis loops caused by partial-amplitude loading cycles [3,9]. Meanwhile, it is also beneficial to construct a differential model for the hysteretic dynamics since classic control and optimization theory are well established on the basis of differential equations. Recently, a unified framework was proposed for the modeling of dynamics involving hysteresis in ferroric materials [9]. The approach was based on the thermodynamic theory and statistic mechanics and the model was formulated as ordinary differential equations. But the application of the model in control systems is not a trivial task at all because it is computationally very expensive [3,9].

In this paper, a macroscopic differential model for the nonlinear dynamics of ferromagnetic materials is proposed on the basis of modeling the magnetization orientation switching, in a one dimensional description. The essence of the model is to associate the hysteretic dynamics to the orientation switching dynamics in the materials. Hysteretic dynamics of single crystal materials is first modeled by using the Landau theory of phase transition [10]. For the modeling of the hysteretic dynamics of polycrystalline materials, the single crystal model is extended by using an assumption that the principal axes of different magnetization domains have a different direction, and has a certain distribution. The hysteretic dynamics in each magnetic domain can be modeled as a single crystal case in its own principal axis direction. The overall dynamics is then modeled by taking a weighted combination of the dynamics of all involved domains [11]. It is shown by comparison with experimental results that the hysteresis dynamics related to magnetization orientation switching can be accurately captured by the proposed differential model. The rate dependence property of the hysteresis loops is inherently incorporated due to the facts that the model is given by ordinary differential equations. Minor hysteresis loops are also successfully captured.

2. Magnetization orientation switching

It is well understood nowadays that the hysteretic behavior of ferromagnetic materials is a consequence of orientation switching of magnetic dipoles (moments) upon employing magnetic fields. If the discussion is confined within a one-dimensional description for the sake of clarification and the material temperature is assumed to be below the Curie temperature, the magnetic dipoles will have two orientations (parallel to its principal axis), and all switchings induced by magnetic fields are 180°. In other cases, dipoles can also be switched 90°, which makes the new orientation perpendicular to the original orientations. To account for the effects of 90° switching in the one dimensional analog, an extra orientation which is perpendicular to the principal axis is introduced here. The perpendicular orientation has no contribute of magnetization to the principal axis direction, but it will change the orientation switching dynamics in many cases [11,12].

For the convenience of the following discussion, the rightward orientation of the magnetic dipoles is denoted as M_p while the leftward one as M_m. The one perpendicular to the principal axis is denoted as M_o. Upon employing external magnetic fields, orientation switching can be induced between the two opposite orientations directly ($M_p \leftrightarrow M_m$). At the same time, 90° orientation switching could also be induced ($M_o \leftrightarrow M_p$ or $M_o \leftrightarrow M_m$). A sketch of the orientation switching in a one-dimensional analog is presented in Fig. 1. Hysteresis is then induced by the orientation switching between these three orientations.

According to the Landau theory, the essential element in the modeling of phase transition dynamics is a free energy function characterizing different phases involved [10]. Here, there are three orientations involved, the free energy function can be constructed as a polynomial retaining the 6th order term of the order parameter [10], such that it is capable of providing three local minima, while being an even function to account for the symmetry properties in physics. In order to construct a macroscale differential model, the potential energy function here is constructed as the same as the Landau free energy function, as follows:

$$F(M) = \frac{a_2}{2}M^2 + \frac{a_4}{4}M^4 + \frac{a_6}{6}M^6 \tag{1}$$

where a_2, a_4, and a_6 are material constants, M is the magnetization which is chosen as the only order parameter. For a specified material, the material constants could be determined by experimental tests. It should be realized that the magnetostatic interaction and the exchange interaction have already incorporated into the Landau free energy function. In Ref. [13], R.C. Smith etc. have deduced the Helmholtz free Energy, which has a similar profile with

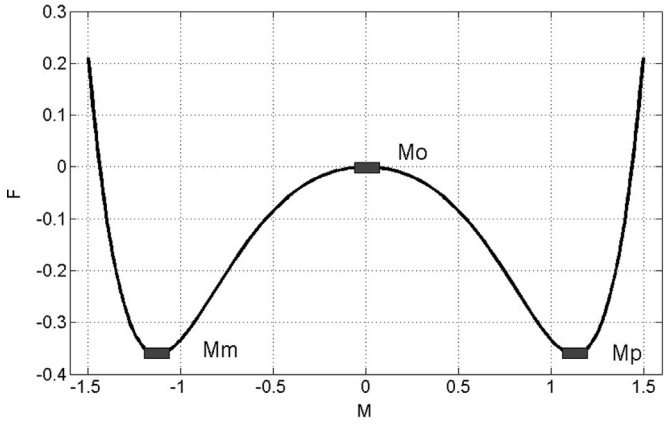

Fig. 2. Schematic plotting of a non-convex potential energy function for magnetization orientation switching.

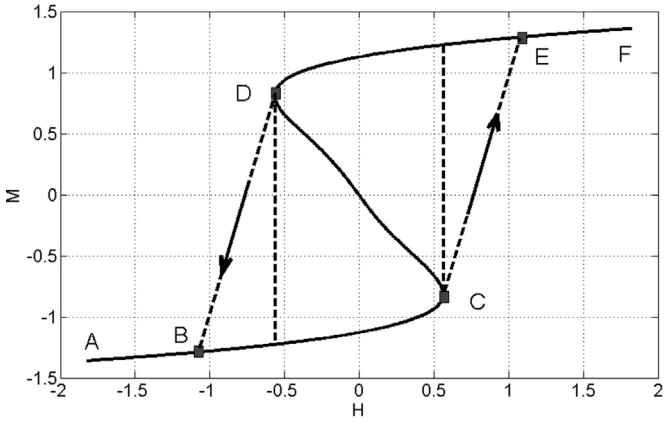

Fig. 3. Schematic plotting of a non-convex constitutive relation of magnetic materials.

the Landau free energy constructed here, from the microscopic energy relations. The Landau free energy can be obtained by taking a Taylor series expansion of the energy derived by Smith. With suitable material constants, there will be at most three local minima of the above potential energy function. The three different orientations can be characterized by the three local minima, by associating each orientation with one local minimum. To clarify the discussion, one example of the potential energy function (non-dimensionalized) is plotted in Fig. 2 with the following parameter values: $a_2=-0.6$, $a_4=-0.8$, $a_6=1$. The plot evidently shows that there are two local minima in the potential energy function, corresponding to the M_p (right), M_m (left), respectively. There is also a local extremum in the center associated with M_o. It is clear that in this case only M_p and M_m are stable and M_o is unstable. It indicates that orientation switching can only take place between M_p and M_m. If one slightly increase the value of a_2, M_o will become locally stable at certain point, thus 90° switching can also be included.

3. Single crystal model

In order to formulate the dynamics into a differential style at macroscale, the time dependent Ginzburg-Landau equation is employed here for the transition of the system state. Using the potential energy function, the governing equation for the evolution of M can be formulated as follows:

$$\tau\frac{dM}{dt} = -\frac{\partial F}{\partial M} + H \tag{2}$$

where τ is a coefficient accounting for the generalized damping effects of the orientation switching processes [11]. By a simple substitution, the dynamics of the orientation switching can be described by the following equation:

$$\tau\frac{dM}{dt} + a_2M + a_4M^3 + a_6M^5 = H \tag{3}$$

It is important to note that the generalized inertial effects of the orientation switching is assumed to be much smaller than the generalized damping effects, hence only the damping term is kept in the governing equation. When the loading H varies very slowly such that the system has enough time to achieve its equilibrium state, the above equation will give the constitutive relation of the material, which is given as follows by setting the time derivative term zero in Eq. (3):

$$H = a_2M + a_4M^3 + a_6M^5 \tag{4}$$

An H–M relation is sketched in Fig. 3 with the same parameter values as those in Fig. 2. When a cyclic magnetic field H is applied, hysteresis loops will be formed due to the bifurcation induced in the dynamics, as sketched by point C and D in the plot. In practice, the 6th order polynomial given in Eq. (1) for the potential energy function is overly restrictive. It can be conveniently replaced by a piecewise spline, which should also be non-convex and has the same number of local minima, and necessary symmetry property Fig. 4.

4. Polycrystal model

In the single crystal model, the orientation switching takes place uniformly in the entire considered material, whose dynamics can be described by Eq. (3). In this case, the jump from C to E (from D to B) will be a straight line, and can not be interrupted by reducing the H value during the switching processes. This means that there will be no minor loops (inner loops) induced inside the major hysteresis loop. The orientation switching in polycrystalline materials behaves in a different way. The mechanism is that there are multiple magnetic domains exist in polycrystalline materials, and the principal axes of different magnetic domains are different. When an external magnetic field H is applied, the effective magnetic field component in the principal axis direction in different magnetic domains will be different. For a specific magnetic domain whose principal axis has an angle of θ ($\theta \in [0, \pi/2]$ is representative) with the direction of the applied magnetic field, the effective magnetic field component in its principal axis direction will be $H\cos\theta$. Since the orientation switching can be induced only in its principal axis direction, its switching dynamics can be modeled by using Eq. (3):

$$\tau\frac{dM_\theta}{dt} + a_2M_\theta + a_4{M_\theta}^3 + a_6{M_\theta}^5 = H\cos\theta \tag{5}$$

where M_θ is the magnetization in the considered domain measured in its principal axis direction. The function $\cos\theta \in [0, 1]$ is to model the effective magnetic field component for the orientation switching in the direction parallel to its principal axis. It is actually a projection of the applied magnetic field on the principal axis of the considered magnetic domain.

The effect of $\cos\theta$ is clear that, for a certain applied magnetic field H on the considered magnetic materials, dipole orientations in some magnetic domains (whose principal axis have a θ value closer to zero) will be switched earlier since the effective field component in its principal axis direction is stronger. There might be some domains in which the orientation switching will not be induced at all if the domains have a θ value close to or equal to $\pi/2$,

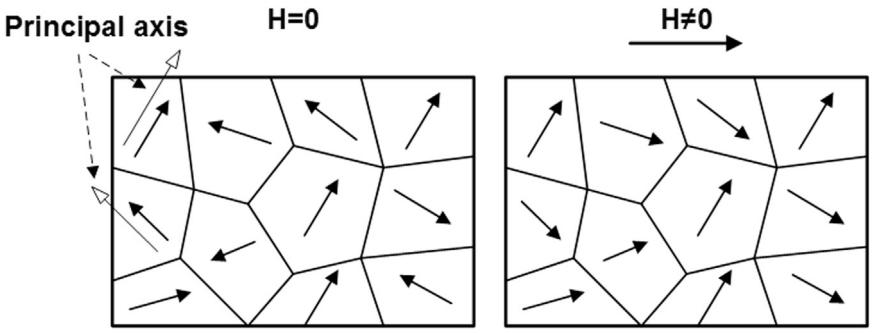

Fig. 4. Outline of distributions of magnetic domains and its principal axes in polycrystalline materials.

since the effective field component could be too small. The distribution of θ varies with materials and its fabrication processes.

Also due to the fact that different magnetic domains have different principal axis directions, the dynamics in each domain will have a different contribute to the overall dynamics in the direction of the applied external magnetic field. The overall dynamics of the material can be assumed as a combination of the dynamics of all the magnetic domains, which can be easily obtained as a weighted combination of M_θ for modeling purpose as follows:

$$M = \int_0^{\pi/2} M_\theta \cos\theta \lambda_\theta d\theta \tag{6}$$

where λ_θ is the weight function accounting for the distribution of the principal axis of different magnetic domains. Thus, the governing equations for the polycrystalline materials are given by Eqs. (5) and (6) together. Furthermore, the above model can be easily re-formulated as a dynamical descrete form by taking the derivative of M with respect to time and applying the Gaussian quadrature technique:

$$\frac{dM}{dt} = \int_0^{\pi/2} \frac{dM_\theta}{dt} \cos\theta\lambda_\theta d\theta = \sum_{k=1}^{N} w_k \lambda_k \cos\theta_k \frac{dM_k}{dt} = \sum_{k=1}^{N} W_k \frac{dM_k}{dt}$$

$$\tau\frac{dM_k}{dt} + a_2 M_k + a_4 {M_k}^3 + a_6 {M_k}^5 = H\cos\theta_k \tag{7}$$

where w_k is the kth weight coefficient for the Gaussian quadrature formulation approximating integral operator, N is the number of quadrature points, θ_k is the value of θ on the kth quadrature point, and M_k and λ_k are the values of M_θ and λ_θ on the kth quadrature point, respectively. For clarification, w_k is merged with θ_k and λ_k into a new coefficient W_k, which has to be determined by fitting the model with experimental data.

It's easily noticed that the weight function or the density function plays an important role in the proposed model. The weight function λ_θ in the original model (represented by Eqs. (5) and (6)) accounts for the distribution of the principal axis of different magnetic domains. As pointed out above, this distribution varies with materials and its fabrication processes, and sometimes it even accommodates with the external operation history. Some explicit function may be deduced from the physical mechanism when a specific kind of material is considered. But in this paper, the weight function is obtained in a pure phenomenological way by comparing the model with experimental data for simplicity. Detailed discussion about the construction of the density function for a specific material will be presented in a future work. In the discrete form model (represented by Eq. (7)), the influence of the distribution $\lambda_k$is merged with the projection factor $\cos\theta_k$ and the integral operator w_k in the Gaussian quadrature formula into a new coefficient W_k. And W_k will be fitted directly in the parameter estimation procedure.

It is worth to note that, in order to get the overall dynamics given by Eq. (7), it is not necessary to simulate Eq. (7) with k different ordinary differential equations for k different θ values. In fact, there is a similarity among the evolutions ofM_k as they are all modeled by Eq. (5).

5. Numerical simulation and validation

To demonstrate that the proposed model is capable of modeling the hysteretic dynamics of polycrystalline magnetic materials, the dynamics of magnetization of the semi-processed nonoriented electrical steel Polycore *M*420-50*D* is used for model validation. The experimental hysteresis loops of the considered materials are taken from Ref. [7], and all the units are kept the same and ignored here.

With a numerical experiment, it is easy to find that the difference between the dynamics of Eq. (5) with different θ values mainly lies in the different values of the coercive field (the field where the magnetization starts to jump, such as the field of point C in Fig. 2). And as illustrated in Ref. [14], with θ values evenly distributed in $[0, \pi/2]$, the coercive fields will not distributed evenly along the field axis. For simplicity, we simply change the coercive fields in the constitutive relation instead of the θ values in Eq. (5). Just as mentioned above, the six order polynomial is too restrictive for the simulation. So, we firstly choose a piecewise square spline as done by Smith [3,9]. In the numerical simulation and validation, constitutive relations with 80 different coercive fields which are evenly distributed in the interval [0, 240] are used. It can be easily noted that the three parts of the constitutive relation are all line-type because a piecewise square spline potential energy is used. Thus there are 81 parameters, 80 densities associated with 80 constitutive relations plus the ratio η of the upper part of constitutive relation (which equals to that of the lower part), that need to be estimated. Furthermore, the time constant is assumed to be zero for simplicity, which means the magnetization will jump instantly at the coercive field. For the estimation of the densities and ratio η, the least square approximation strategy is chosen to formulate the estimation problem as a nonlinear optimization problem as follows:

$$\min_{densities,\eta} G = \sum_{i=1}^{N} \left(\tilde{B}_i - B_i\right)^2 \tag{8}$$

where N is the number of experimental data samples, $\tilde{B}_i$ experimental values of B at the ith time instant, and B_i are the simulated values at the ith time instant.

By using the given experimental data, the estimated densities is shown in Fig. 5a and the ratio η is estimated as 2.1*10^ −4. The comparison between the experimental loop and the numerical simulation is presented in Fig. 5b by plotting the corresponding hysteresis loop in the same figure. A good fitment is obtained except for the part before dipole reorientation begins, which ascribes to a linear constitutive relation chosen in the simulation.

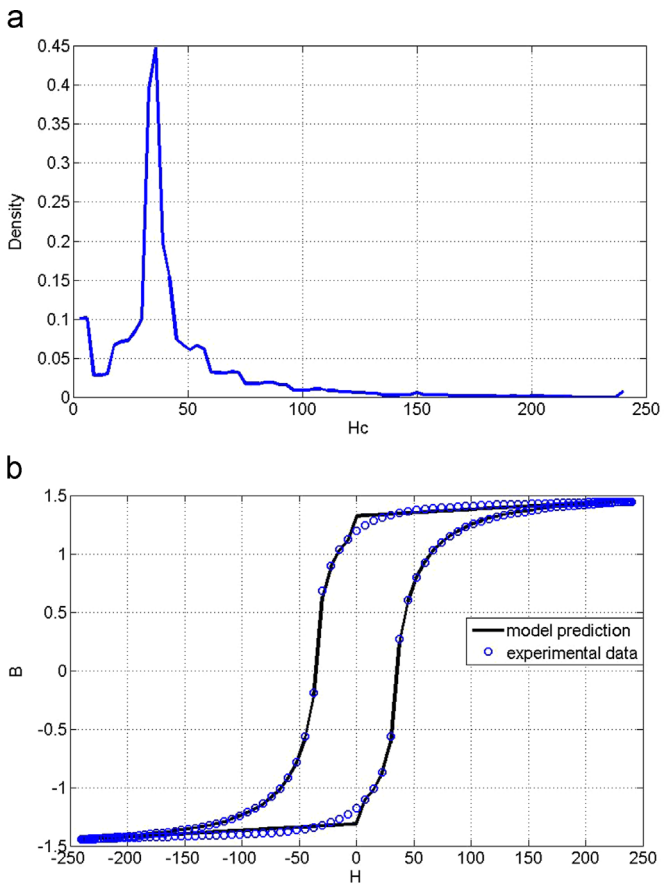

Fig. 5. Simulated results with the line-type constitutive relation. (a) estimated density function; (b) comparison of the simulated result with the experimental data.

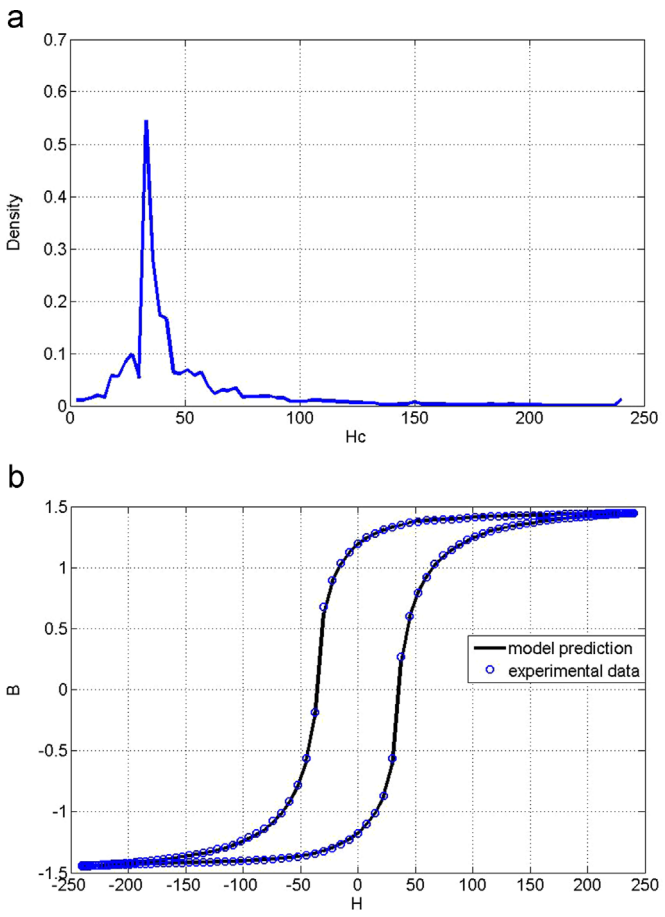

Fig. 6. Simulated results with the improved constitutive relation. (a) estimated density function; (b) comparison of the simulated result with the experimental data.

Thus an improvement is made by replacing the upper and lower part of the constitutive relation by curve that is similar to the experimental data. And the same parameter estimation strategy is applied. The improve results are plotted in Fig. 6, with discrepancy hardly noticeable. The residual is reduced from 0.0963 to 0.0132.

It is worth much to note that, there are more than eighty parameters which need to be estimated for the current model by fitting with experimental data. The number of the parameters will be largely reduced with the choosing of certain particular density functions like the normal distribution function which is chosen by Smith [3]. Further work will be done about this elsewhere.

6. Minor hysteresis loops

Another differentiating feature of the hysteretic dynamics of polycrystalline materials is that there are minor hysteresis loops occur. When the applied magnetic field is increased to switch partially the magnetization orientations and then reduced before the magnetization achieving its saturation value, minor hysteresis loops will be formed. To demonstrate the capability of the current model of capturing the minor hysteresis loops, the field as plotted in Fig. 7a is employed as the input to the model. All the hysteresis loops are presented in Fig. 7b by plotting them together with the experimental loops for comparison purpose. It is clearly seen from the plot that the minor hysteresis loops are captured very well by the provided model.

7. Frequency dependency

From Eqs. (5) and (6), it is very clear that the proposed model is inherently dynamic, since the model is given by an ordinary differential equation and an algebraic equation. Therefore it is natural that the dynamical response of the model will be dependent on the loading rates. For the sake of illustration, the dynamical response of the same material is simulated by using the model with three different loading frequencies (1 Hz, 20 Hz, and 100 Hz, respectively). The simulated responses are plotted together in Fig. 8. It is seen clearly that, when the loading frequencies are increase, the hysteresis loops become larger. This can be easily explained by the fact that, with higher frequency, the orientation switching might not be able to following the change of the loadings since it needs time to finish the switching processes. One can expect that, when the loading frequencies are high enough, orientation switching will not be induced at all due to its frequency dependency property.

8. Conclusion and discussion

In the current paper, the hysteretic dynamics of polycrystalline magnetic materials is modeled by an ordinary differential model. The Landau theory of phase transition is employed for the single crystal model. A weight combination of various single crystal models is used to model the hysteretic dynamics of polycrystalline materials. The proposed model is validated via comparison with experimental counterpart reported in literatures. Rate dependent

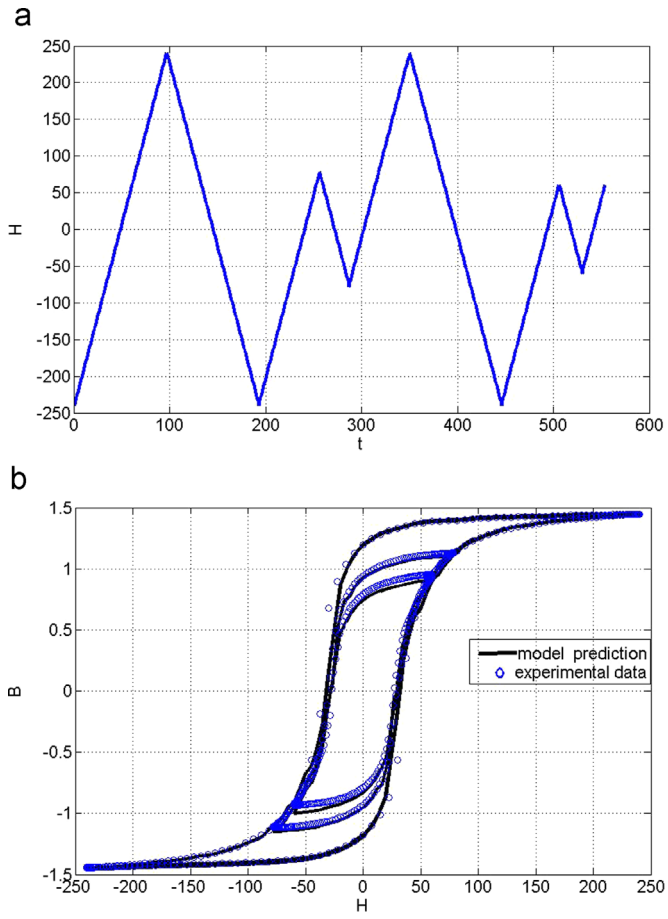

Fig. 7. Simulated minor hysteresis loops in the polycrystalline materials. (a) input H signal in the time domain; (b) comparison of the simulated result with the experimental data.

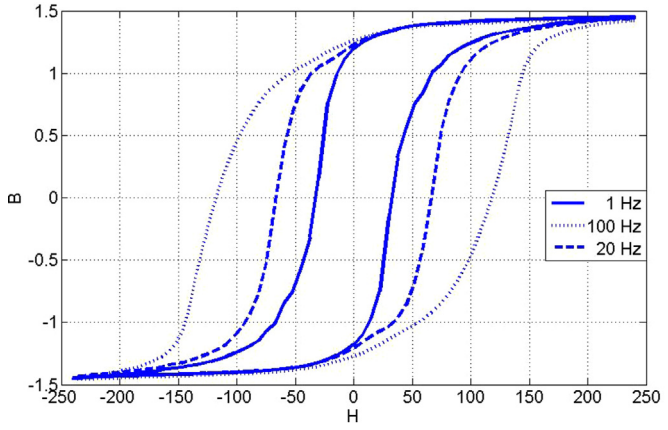

Fig. 8. Simulated dynamic responses of the material with three different loading frequencies.

property and minor hysteresis loops are successfully modeled by the model.

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