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Constructing Operator-Difference Schemes for Problems with Matching Boundaries

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Abstract In this paper we construct a system of difference equations resulting from the approximation of a dynamic model based on a strongly coupled system of partial differential equations. The model is an extension of the previously studied case of dynamic electromechanical vibrations of hollow piezoceramic cylinders, but here we account for the acoustic coupling of such cylinders with the surrounding media. The coupling between three media of distinctively different physical natures brings new features in the analysis of the discrete model. We propose an algorithm for the solution of the operator-difference equations resulting from our approximations. Error estimates for our approximations are discussed in the framework of non-smooth solutions.

Keywords Operator-difference schemes, Matching boundaries, Discrete negative norms in space-time.

AMS Subject Classification 35A35, 65M06

1 Introduction

Many important classes of systems of difference equations come from discretization procedures of coupled time-dependent models based on partial differential equations. The systems containing hyperbolic-type equations represent the most challenging and arguably most interesting class of such models. It is often an intrinsic interplay between numerical dissipation and dispersion that create major challenges in investigating discrete approximations for such models. In addition, in a number of applications one has to respond adequately to challenges coming from a strong coupling of the resulting system due to the physical essence of the problem where, for example, the mutual influence of thermal and elastic, or elastic and electric, fields in a solid is very important. The situation becomes even more complicated if we have to derive

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discrete schemes for problems where such solids are surrounded by media with distinctively different physical properties, e.g., by fluids. This would require dynamic internal boundary conditions at the interface between the solid and the fluid. It is these types of applied problems that have led us to the study of systems of difference equations arising from the full discretization of such coupled time-dependent mathematical models. Such difference approximations can often be defined as operator-difference equations in a sense that they are difference equations with respect to time but they are operator equations with respect to space (e.g., [6]). Operators of these schemes are defined in a certain linear normed space H_h dependent on a vector parameter h which is an analogue of the spatial step.

We organize our further discussion as follows. First, we give a brief background of the application area where the class of difference approximations we are interested in came from. Then, we recall some basic results obtained previously in [6] for a simplified problem and derive a system of difference equations approximating the new fully coupled time-dependent problem considered here. Next, we propose an algorithm for the solution of the difference equations obtained. Finally, we discuss the analysis of the operator-difference system in the framework of the negative norm technique.

2 Application area and coupled systems of PDEs

Our major interest in this paper is the solution procedure for a system of difference equations obtained as a result of the discretization of the following problem.

Consider a piezoelectric solid shell. In order to describe the dynamics of this shell, subject to various loading conditions, one has to incorporate some mechanism of *coupling* between mechanical and electric fields. It is due to this electromechanical coupling that piezoelectrics have become very popular materials in such areas as transducer applications, biomedical engineering, smart materials and structure technology, to name just a few. This coupling can be introduced into the model by constitutive equations. In what follows we consider a solid made of piezoelectric ceramics, meaning that it is a polycrystallic solid with dipoles in each crystal. Those dipoles are oriented stochastically in general, but we can order them (in some small domains) by applying an electric field to the sample. This process is called a preliminary polarization, and we note that if the cylindrical geometry is used (as it is the case here where we consider a hollow piezoceramic cylinder surrounded by media with different physical properties), then most typical types of preliminary polarizations are axial, radial, and circular. From a mathematical point of view, all three types can be obtained from a simple re-ordering of axes. We do not want to go into any further technical detail here except for noting that the radial preliminary polarization provides opportunities for the strongest coupling in the case of this geometry [6]. Putting the above discussion in the modelling context, in order to describe the dynamic behavior of our system one has to solve 22 equations with respect to 3 components of displacements $\mathbf{u} = (u_1, u_2, u_3)^T$, the electric field strength $\mathbf{E}$, electric field induction vectors $\mathbf{D}$, 6 components of stress $\vec{\sigma}_v$ and strain tensors $\vec{\epsilon}_v$, and the electrostatic potential φ. More precisely, in what follows we consider the axisymmetric geometry which leads to the following constitutive equations

$$\vec{\sigma}_v = \mathbf{c}\vec{\epsilon}_v - \mathbf{e}^T\mathbf{E}, \quad \mathbf{D} = \vec{\varepsilon}\mathbf{E} + \mathbf{e}\vec{\epsilon}_v, \tag{1}$$

where the symmetric mechanical stress and strain tensors are presented here in schematic vector forms as $\vec{\sigma} = (\sigma_\theta, \sigma_z, \sigma_r, \sigma_{rz}, \sigma_{r\theta}, \sigma_{\theta z})^T$, $\vec{\epsilon} = (\epsilon_\theta, \epsilon_z, \epsilon_r, 2\epsilon_{rz}, 2\epsilon_{r\theta}, 2\epsilon_{\theta z})^T$, electric field strength and electric induction are given as $\mathbf{E} = (E_\theta, E_z, E_r)^T$, $\mathbf{D}^r = (D_\theta, D_z, D_r)^T$, respectively, $\mathbf{E} = -\nabla\varphi$, and the matrices of elastic, dielectric, and piezoelectric coefficients are given in the forms

$$\mathbf{c} = \begin{pmatrix} c_{33} & c_{13} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{11} & c_{12} & 0 & 0 & 0 \\ c_{13} & c_{12} & c_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{55} \end{pmatrix}, \tag{2}$$

$$\vec{\varepsilon} = \begin{pmatrix} \varepsilon_{33} & 0 & 0 \\ 0 & \varepsilon_{11} & 0 \\ 0 & 0 & \varepsilon_{11} \end{pmatrix}, \quad \mathbf{e} = \begin{pmatrix} e_{33} & e_{31} & e_{31} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & e_{15} \\ 0 & 0 & 0 & e_{15} & 0 & 0 \end{pmatrix}, \tag{3}$$

respectively. These constitutive equations couple two partial differential equations describing the electromechanical dynamics of the hollow cylinder (the equation of motion and the Maxwell equation):

$$\rho\frac{\partial^2 \mathbf{u}}{\partial t^2} = \nabla \cdot \vec{\sigma} + \mathbf{F}, \quad \mathrm{div}\mathbf{D} = \mathbf{G}. \tag{4}$$

In (4) ρ is the density of piezoelectric material, $\mathbf{G}$ and $\mathbf{F}$ are electric and body forces on the piezoelectric.

Discrete schemes for this model have been already studied in [6, 7]. The present paper is not merely a contribution to a further development of the results for systems of difference equations obtained from space-time discretizations of model (1)–(4). Here we consider a much more complicated case where one has to account for the coupling of the solid to the surrounding media, for example fluids, in addition to a strong electromechanical coupling exhibited by the piezoelectric cylinder itself. Let ρ_i, c_i, and p_i, $i = 1, 2$ denote densities, speeds of sound, and acoustic pressures in the internal and external media, surrounding the piezoelectric body. Then, in terms of the differential model this means that (1)–(4) has to be coupled to two pairs (for the interior and exterior of the shell) of three equations which we consider here with respect to components of the fluid particle velocity $\mathbf{v}^{(i)} = (v_r^{(i)}, v_z^{(i)})$, $i = 1, 2$

$$\frac{1}{\rho_i c_i^2}\frac{\partial p_i}{\partial t} + \frac{1}{r}\left(r\frac{\partial v_r^{(i)}}{\partial r}\right) + \frac{\partial v_z^{(i)}}{\partial z} = 0, \tag{5}$$

$$\rho_i\frac{\partial v_r^{(i)}}{\partial t} + \frac{\partial p_i}{\partial r} = 0, \quad \rho_i\frac{\partial v_z^{(i)}}{\partial t} + \frac{\partial p_i}{\partial z} = 0. \tag{6}$$

Equations (5)–(6) are coupled to (1)–(4) by the *matching boundary conditions*. These conditions are dynamic in a sense that they require (a) pressure continuity at the interface, leading to the time derivative of the velocity potentials, and (b) continuity of the radial velocity, leading to time derivatives of the displacements at the interface between the solid and the fluid.

The key challenges can be demonstrated by considering an infinitely long cylinder, in which case the one-dimensional (in the radial direction) solution is sufficient. In the acoustic approximation limit equations (5)–(6) can be transformed into wave equations with respect to acoustic pressures of the surrounding media. If we assume that the particle fluid velocities can be represented as $v_i = -\nabla\varphi_i$, and that $p_i = -\rho_i\frac{\partial\varphi_i}{\partial t}$, $i = 1, 2$, we arrive at the following acoustic approximations for the surrounding media

$$\frac{\partial^2\varphi_i}{\partial t^2} - c_i^2\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\varphi_i}{\partial r}\right) = 0, \quad i = 1, 2. \tag{7}$$

These equations are coupled to the electromechanical part of the system in the following way. The continuity conditions for pressures and velocities require $\sigma_r = -p_i(t)$, $\frac{\partial u}{\partial t} = \frac{\partial\varphi_i}{\partial r}$. Recall that we have confined ourselves to the infinitely-long-cylinder case, so our geometry is the spatial-temporal region $Q_T = G \times [0, T]$ with $G = [R_0, R_3]$, where $R_0 \leq R_1 < R_2 < R_3$ (in (R_0, R_1) we have a fluid region denoted by index 1, in (R_1, R_2) we have a piezoelectric solid region, and in (R_2, R_3) we have a fluid region denoted by index 2). This means that in the above interfacial boundary conditions $i = 1$ for $r = R_1$, and $i = 2$ for $r = R_2$. Other conditions (electrical boundary conditions, initial conditions, and the limiting conditions for the behavior of the surrounding fluids) will not be discussed here; they are analogous to those considered in [6, 7]. The electromechanical part of the system is described by (1)–(4) which in this specific case will have the form (for region $(R_1, R_2) \times [0, T]$)

$$\rho_0\frac{\partial^2 u}{\partial t^2} = \frac{1}{r}\frac{\partial}{\partial r}(r\sigma_r) - \frac{\sigma_\theta}{r} + F(r, t), \quad \frac{1}{r}\frac{\partial}{\partial r}(rD_r) = G(r, t). \tag{8}$$

Equations (8) are coupled by the constitutive equations for the radial preliminary polarisation $\sigma_r = c_{33}\epsilon_r + c_{13}\epsilon_\theta - e_{33}E_r$, $\sigma_\theta = c_{13}\epsilon_r + c_{11}\epsilon_\theta - e_{31}E_r$,

$D_r = \varepsilon_{33}E_r + e_{31}\epsilon_\theta + e_{33}\epsilon_r$. This case has not been considered in the literature before in such a general setting as it is proposed here. At the same time, the development of discrete models based on operator-difference schemes represents an important task in theory and applications of difference equations arising from approximations of time-dependent coupled systems of PDEs.

3 Discretization procedures and the solution of the resulting system of difference equations

We aim at developing an efficient procedure for the solution of the problem formulated in Section 2.

For this purpose we make use of the already developed discrete model applicable in region (R_1, R_2) of our problem (see [6]). The resulting system of difference equations can be written in the following form

$$\begin{cases} \rho_2 \dfrac{\Delta y^n}{\tau^2} = \dfrac{1}{r}\dfrac{(\bar{r}\bar{\sigma}_r)^{(+1)} - \bar{r}\bar{\sigma}_r}{h_2} - \dfrac{(\sigma_\theta)^{(+1)} + \sigma_\theta}{2r} + F, \\ \dfrac{1}{r}\dfrac{(\bar{r}\bar{D}_r)^{(+1)} - \bar{r}\bar{D}_r}{h_2} = G. \end{cases} \tag{9}$$

This system is a result of the application of the variational approach to system (8) on computational grid $\bar{\omega}_\tau \times \omega_2$ in $[R_1, R_2] \times [0, T]$, where, for example, for the locally uniform grids we have $\omega_2 = \{r_i^{(2)} = R_1 + ih_2; i = 0, 1, ..., N_2; h_2 = (R_2 - R_1)/N_2\}$, and $\bar{\omega}_\tau = \{t^n = n\tau, n = 0, 1, ..., K\}$, where $K\tau = T$. In a similar way, the grids are introduced in the fluid regions $\bar{\omega}_k = \{r_i^{(k)} = R_{k-1} + ih_k; i = 0, 1, ..., N_k; h_k = (R_k - R_{k-1})/N_k\}$, $k = 1, 3$, where $r_{N_1}^{(1)} = r_0^{(2)}$, and $r_{N_2}^{(1)} = r_0^{(3)}$. In (9) we take approximations of the constitutive equations in the "flux" grid points as $\bar{\epsilon}_r = (y - y^{(-1)})/h_2$, $\bar{\epsilon}_\theta = (y + y^{(-1)})/2r$, $\Delta y^n = y^{n+1} - 2y^n + y^{n-1}$, super-indices "±" indicate the right/left shift for one grid point in the spatial direction, $\bar{E}_r = -(\mu - \mu^{(-1)})/h_2$, $\bar{\sigma}_r = c_{33}\bar{\epsilon}_r + c_{13}\bar{\epsilon}_\theta - e_{33}\bar{E}_r$, $\bar{\sigma}_\theta = c_{13}\bar{\epsilon}_r + c_{11}\bar{\epsilon}_\theta - e_{31}\bar{E}_r$, $\bar{D}_r = \varepsilon_{33}\bar{E}_r + e_{31}\bar{\epsilon}_\theta + e_{33}\bar{\epsilon}_r$. Approximations of hyperbolic equations (7) are carried out in a standard manner, while the matching boundary conditions are approximated by using the integro-interpolational approach (e.g., [2, 11]). In particular, to get difference equations for the continuity of pressure at the exterior of the cylinder we integrate (7) in $[R_2, R_2 + h_3/2]$) and then use second-order approximations for all terms in the resulting expression, which, after some additional work omitted here, leads to the following approximation

$$\frac{h_3}{2}(R_2 + h_3/4)\Delta\omega_0^n/\tau^2 = c_2^2\left((R_2 + h_3/2)\frac{w_1 - w_0}{h_3} - R_2\frac{y_{N_2}^{n+1} - y_{N_2}^{n-1}}{2\tau}\right). \tag{10}$$

In a similar manner, we get approximations for the second matching boundary conditions by integrating the equation of motion in the appropriate limits, and by using second order approximations for the resulting terms.

The complete set of difference equations is written in terms of discrete functions (v, y, μ, w) giving approximations to $(\varphi_1, u, \varphi, \varphi_2)$, as it is explained in Section 2. It is important to notice that function v is defined on $\bar{\omega}_\tau \times \bar{\omega}_1$ and is coupled to function y defined on $\bar{\omega}_\tau \times \bar{\omega}_2$ by matching conditions at $r^{(1)}_{N_1} = r^{(2)}_0$. Functions y and μ are coupled on $\bar{\omega}_\tau \times \bar{\omega}_2$. In addition, function y is coupled to function w defined on $\omega_\tau \times \bar{\omega}_3$ by matching conditions at $r^{(1)}_{N_2} = r^{(3)}_0$. To resolve these difficulties, one can apply the idea similar to that proposed in [2] for the uncoupled case of circular preliminary polarization. In particular, four approximations derived from approximations of the matching interface boundary conditions represent a pair of systems of linear equations $A\mathbf{x} = \mathbf{b}$ with $\mathbf{x} \in \mathbb{R}^2$, $\mathbf{b} \in \mathbb{R}^2$. The first system of this pair should be solved with respect to $(v^{n+1}_{N_1}, y^{n+1}_0)^T$ for matrix $A = A_1$ where

$$A_1 = \begin{pmatrix} \frac{h_1}{2\tau^2}(R_1 - \frac{h_1}{4}) & -\frac{c_1^2 R_1}{2\tau} \\ \frac{R_1}{2\tau} & \rho_2 \frac{h_2}{2\tau^2}(R_1 + \frac{h_2}{4}) \end{pmatrix}, \tag{11}$$

and the second system should be solved with respect to $(y^{n+1}_{N_2}, w^{n+1}_0)^T$ for matrix $A = A_2$ where

$$A_2 = \begin{pmatrix} c_2^2 \frac{R_2}{2\tau} & \frac{h_3}{2\tau^2}(R_2 + \frac{h_3}{4}) \\ \rho_2 \frac{h_2}{2\tau^2}(R_2 - \frac{h_2}{4}) & \frac{R_2}{2\tau} \end{pmatrix}. \tag{12}$$

The resulting systems have positive determinants and their solution, supplemented by the procedure described in [6], allows us to find all unknown functions on the time-layer $n+1$, which in its turn allows us to compute the stresses and the acoustic pressure, and proceed to the next time level. Computational experiments with the resulting system of difference equations will be reported in a separate paper [7].

Note that the described procedure puts us into the framework of operator-difference scheme

$$\begin{cases} D_1 \Delta y^n/\tau^2 + A_1 y + C_1 \mu = \varphi_1, \\ A_2 \mu + C_2 y = \varphi_2, \\ y = y_0,\ D_1(y^1 - y^0)/\tau = y_1,\ t = 0, \end{cases} \tag{13}$$

with operators of the scheme determined in a way similar to that described in [6]. The stability condition for this scheme is a generalization of the CFL stability condition to the case of coupled electromechanical waves (see detail in [6]). A scale of a priori estimates in Sobolev spaces has been obtained for (13) in the above cited paper. We also obtained an estimate in class $V(\bar{Q}_T) = C(\bar{Q}_T) \cap Q_1(Q_T)$ of continuous functions with piecewise first derivatives which have square integrable generalized derivatives in the continuity region. This result can be improved further by considering the negative norm technique which becomes popular in the context of post-processing of numerical algorithms.

4 Discrete negative norms and further development

In a number of application areas the problems described above require dealing with non-smooth or generalized data, and when a variational-type approach used for the construction of systems of difference equations approximating the original differential problem, this can lead to norm equivalent functionals that are meaningful for less regular solutions than L^2-type functionals traditionally used in obtaining error estimates. In such situations negative norms become a very useful tool for the analysis because $L^2(\Omega) \subseteq H^s(\Omega)$ for $s < 0$, where $H^s(\Omega)$ is the space of linear functionals with the finite norm, e.g. for $s = -1$ it is $\|v\|_{-1} = \sup\limits_{\varphi \in W} \dfrac{(v, \varphi)}{\|\varphi\|_1}$ with W being a closure space with respect to norm in $H^1(\Omega)$. The interest to such negative norms have been recently revitalized in the context of effective post-processing of approximations resulting from discretizations of PDEs due to the possibility of devising preconditioners for discretized equations, and of improving error estimates under relaxed regularity assumptions (e.g., [1, 4, 3]). In most situations considered in the literature the negative norm technique has been applied for stationary problems only, with just a few exceptions (e.g. [1, 9]) in which, however, negative norms were considered with respect to the space but not time. For discrete models considered in this paper it seems natural to apply negative norms in both space and time. Such results for difference schemes approximating the wave equations go back to work [8] and have been recently discussed by the authors of [10]. One of the basic results can be formulated as follows. Consider the operator-difference scheme in the form $D\Delta y^j/\tau^2 + B(y^{j+1} - y^{j-1})/2\tau + Ay^j = \varphi$ given the same initial conditions as in (13). Then, as soon as $A^* = A > 0$, $D^* = D > \beta E$, $\quad \beta > 0$, $B \geq 0$, $BD^{-1}A \geq 0$, $D \geq (1+\epsilon)/4\tau^2 A$, $\quad \epsilon > 0$, by applying the negative norm technique we can obtain the following estimate for the solution of this system of difference equations $\|y(t)\| \leq M \left\{ \|y(0)\|_D + \|Dy_t(0)\|_{A^{-1}} + \|\varphi_1\|_{0,-1} + \|\varphi_2\|_{A^{-1},0} \right\}$, where $y(t) = \xi_{\bar{t}}$, $\xi(0) = 0$, $\|y(t)\|_{A^{-1},0} = \sum_{t'=0}^{T} \tau \|y(t')\|$, $\|y(t)\|^2_{0,-1} = \sum_{t'=\tau}^{T} \tau \|\xi(t')\|$. This result can be relaxed further since in some special cases the condition of self-adjointness of A and D can be dropped. In conclusion, we note that in a special case where the electromechanical coupling is assumed negligible at the interface boundaries of the cylinder (see [5]) the operator-difference schemes considered here can be reduced to the above form, and therefore the general theory for obtaining error estimates in negative norms in time and space can be applied.

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New Progress *in* Difference Equations

Since 1994, the series of International Conferences on Difference Equations and Applications has established a tradition within the mathematics community, bringing together scientists from many different areas of research to discuss current interests, challenges, and unresolved problems.

This volume contains a selection of invited, peer-reviewed papers on difference equations presented at the 2001 conference held in Augsburg, Germany. It covers recent progress in topics ranging from the classical to the contemporary and from the theory of difference equations to its applications. The contributions are organized into sections focused on discrete dynamical systems, dynamic equations of time scales, and miscellaneous difference equations. This collection also includes the opening lecture on difference equations and boundary value problems presented by Alexander N. Sharkovsky and two general interest discussions on the discrete nature of physical laws and real analysis as a degenerate case of discrete analysis.

Proceedings of the Sixth International Conference on Difference Equations; Augsburg, Germany 2001: New Progress in Difference Equations forms a valuable reference for researchers and graduate students working in discrete mathematics, differential equations, and nonlinearity.

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