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NONLOCAL EFFECTS IN ELECTROMECHANICAL NANO-BEAMS VIA THE TIMOSHENKO MODEL

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Abstract. Taking into account the coupling of strain gradient to polarization, a closed-form nonlocal solution is obtained for a Timoshenko clamped-clamped beam. The solution is analyzed and compared with known results such as those obtained for Euler-Bernoulli beams.

1 INTRODUCTION

Nano-switches, used frequently in nanotechnology as integral parts of nanoelectromechanical systems (NEMS) and devices, can often be simulated as Timoshenko (T) beams. The material they are fabricated from may be piezoelectric or flexoelectric. Therefore, in order to analyze electromechanical properties of these nanostructures we need to take into account the coupling between the strain and the polarization as well as the strain gradient effect. This effect can arise even in non-piezoelectric nanostructures at inhomogeneous strain. The authors of[1] were among the first to account for the variation of strain on the electric field in a narrow piezoelectric beam. This was followed by further developments in the context of the Euler-Bernoulli (EB) model for nanostructures[2] where coupling the strain to polarization has been included in the energy density of deformation and polarization. By now it is known that the coupling of strain gradient to polarization may have a strong size dependency due to the scaling of strain gradient with structural feature size. However, up until now the analysis of nonlocal effects in beams were predominantly based on the EB model where the deflection of the beam is described in the classical conventional form, but with the bending rigidity additionally dependent on the coupling coefficient[3], terms connected with polarization gradient, and a strain coupling constant, as well as the flexoelectric coefficient. In the present work we analyze the properties of nano-beams by developing a more refined model based on the nonlocal T beam model.

2 NONLOCAL TIMOSHENKO MODEL FOR ELECTROMECHANICAL NANO-BEAMS

We consider a dielectric slender clamped-clamped beam loaded mechanically by a force q and a moment m distributed along the length of the beam. Figure 1 shows the coordinate system and beam configuration.

Figure 1: (a) Mechanical loading along the dielectric beam. (b) Electric loading across the dielectric beam.

According to[4], when the deflection of a beam is considered in the $x-z$ plane in T beam theory, we have

$$u_1(x,y,z,t) \approx z\psi(x,t), \quad u_2(x,y,z,t) \approx 0, \quad u_3(x,y,z,t) \approx w(x,t),$$

where $w = w(x)$ is the transverse displacement of the point of the centroidal axis $(y = z = 0)$ and $\psi = \psi(x)$ is the rotation of the beam cross-section about the positive $y-$axis. The strain components are

$$\varepsilon_x = -z\psi_x^{'}, \qquad \gamma_{xz} = \psi + w_x^{'}. \tag{1}$$

We assume that the beam possesses a plane of material symmetry parallel to the $x-y$ plane. Taking into account the stress gradient acting in the x-direction and admitting flexoelectricity in the material[2] of the beam we can present the constitutive equations as

$$\begin{aligned}
&\sigma_x = Q_{11}\varepsilon_x - e_{31}E_z + d_{31}f^{'}\varepsilon_z + (e-f)\varepsilon_0\chi E_{z,z}, \\
&\tau_{xz} = Q_{55}\gamma_{xz} - e_{15}E_x, \quad D_x = e_{15}\gamma_{xz} - \epsilon_{11}E_x, \\
&D_z = e_{31}\varepsilon_x + \epsilon_{33}E_z + \gamma_{xz}f_{55}(1 - f^{'}\frac{d_{31}e_{31}}{Q_{11}}) + (e-f)\frac{e_{31}}{Q_{11}}E_{z,z}
\end{aligned} \tag{2}$$

where σ_x, τ_{xz} are stress components, ε_x, γ_{xz} are strain components, e_{ij} are the piezoelectric constants, D_x, D_z are the electric displacement vector components, ϵ_{ij} are the dielectric constants, $e = e^*_{13}$, $f = f_{13}$, e^*_{ij} are the components of the fourth order tensor corresponding to polarization gradient and strain coupling, f_{ij} are the components of the fourth order flexoelectric tensor, $f^{'} = f_{55}\varepsilon_0\chi$, ε_0 is permittivity, χ is the dielectric susceptibility, and the Q_{ii} are the reduced elastic constants of the beam $Q_{11} = \frac{E_{11}}{1-\nu_{12}\nu_{21}}$, $\quad Q_{55} = G_{13}$, E_{11} is the Young's modulus and G_{13} is the shear modulus.

The equations of motion of the beam are derived via Hamilton's principle[5]:

$$\delta\int_{t_1}^{t_2}(T - H + W)dt = 0 \tag{3}$$

where $\delta(\cdot)$ denotes the first variation, T is the kinetic energy, H is the electric enthalpy, and W is the work done by the external forces and moments.

The kinetic energy of the beam can be written as

$$T = \frac{1}{2}\int_{V_b} \varrho\{\dot{u}\}^T\{\dot{u}\}dV_b = \frac{1}{2}\int_{V_b} \varrho[z\dot{\psi}^2 + \dot{w}^2]dV_b = \frac{1}{2}\int_{-l}^{l}[I_1(\dot{w}) + I_3(\dot{\psi}^2)]dx, \tag{4}$$

where $(I_1, I_3) = \int_A \varrho(1, z^2)dA$, A is the area of the beam's cross section, V_b is the volume of the beam, and the length of the beam is equal to $2l$.

Let us consider the simplest case when the voltage V is applied only in the thickness direction, i.e., $E = \{0, 0, E_z\}^T$. For the quasistatic electric field[5], $D_{i,i} = 0$, $D_x = D_x(x)$, and since $\partial D_x/\partial x = -\partial D_z/\partial z$, $D_z = D_z(x)$.

Following the procedure described in[1,2], we obtain

$$\sigma_x = \frac{e_{31}}{h}V + Q_{11}(1+\xi)z\psi_x^{'} + \xi(e-f)\varepsilon_0\chi\frac{24V}{h^2} + \xi(e-f)\varepsilon_0\chi E_{z,z} + (d_{31}f^{'} - \frac{\xi^2 Q_{11}}{d_{31}^2}), \tag{5}$$

where $\xi = \frac{e_{31}^2}{Q_{11}\epsilon_{33}} = \frac{k^2}{1-k^2}$ is the square of the expedient coupling coefficient and $k^2 = \frac{e_{31}^2}{e_{31}^2 + Q_{11}\epsilon_{33}}$ is the electromechanical coupling coefficient[3].

Now, the electric enthalpy of the beam can be presented as[5]

$$H = \frac{1}{2}\int_{V_b}(\sigma_x\varepsilon_x + \tau_{xz}\gamma_{xz} - e_{31}E_z\varepsilon_x - \epsilon_{33}E_z^2)dV_b \tag{6}$$

Let us denote $EI = \int_A z^2 Q_{11}dA$, $\quad GA = k\int_A Q_{55}dA$, $\quad M_{el} = \int_A ze_{31}E_z dA$, where EI and GA are the generalized elastic constants of the beam, M_{el} is the electric bending moment, and $k = 5/6$ is the shear correction factor.

The work done by the external forces may be written as

$$W = \int_{-l}^{l}(qw + m\psi)dx. \tag{7}$$

After substituting equations (1), (2) (4)-(7) into (3) and integrating by parts , we obtain two equations of motion

$$\begin{aligned} &\frac{\partial}{\partial x}[GA(\psi + \frac{\partial w}{\partial x})] + q = I_1\frac{\partial^2 w}{\partial t^2}, \\ &\frac{\partial}{\partial x}[EI(1 + \xi + \frac{S_A f^{'} d_{31}}{EI} + \frac{\xi^2 Q_{11}}{d_{31}})\frac{\partial \psi}{\partial x} - M_{el}] - (GA)(\psi + \frac{\partial w}{\partial x}) + m = I_3\frac{\partial^2\psi}{\partial t^2}. \end{aligned} \tag{8}$$

In the static case, the system of two differential equations (8) for w and ψ is equivalent to the one equation for w

$$w^{IV} = \frac{1}{EIZ}q - \frac{1}{kGA}q^{''} - \frac{M_{el}}{EIZ} + \frac{m^{'}}{EIZ}, \tag{9}$$

where $Z = 1 + \xi + \frac{Af^{'}d_{31}}{EI} + \frac{\xi^2 Q_{11}A}{d_{31}}$. Similar to[2] note that the second term in this expression appears due to the electromechanical coupling, the third term - due to flexoelectric effect, and the last term - due to nonlinear interaction between flexoelectricity and piezoelectricity. For the clamped-clamped beam, the boundary conditions have the form

$$w(-l) = w(l) = 0,\ \psi(-l) = \psi(l) = 0,\ \text{or}\ w^{'''}(\pm l) + Kw^{'}(\pm l) = 0, \tag{10}$$

where $K = kGA/EI$. The general solution to problem (9), (10) is straightforward, but what is of main interest here is how shear deformation and the last terms in the expression for Z influence that solution. As an example, we take $q = e^x$, and calculate the coefficient near q to compare it with EI in the classic formulation, as well as with EIZ in the nonlocal formulation of the problem. From Table 1 (we neglect M_{el} for simplicity and take the values of Z for PZT G1195N material), we can see that accounting for the shear deformation results in increasing the effective bending rigidity (equal to EIZ for the EB model and to $bh/(12/Eh^2 - 1/kGl^2)$ for the T model). It is true for both classical and strain gradient models. For smaller aspect ratios, this effect is even more pronounced. We

nm		EB Classic	EB Nonlocal	T Classic	T nonlocal
$h = 20; 2l = 100$	$Z = 2$	1.68	3.36	1.69	3.43
$h = 30; 2l = 100$	$Z = 1.6$	8.50	13.60	8.70	14.13

Table 1: Effective bending rigidity (with the factor of 10^{21}Nm^2).

have also analyzed nano-beams made of flexoelectric materials such as $BaTiO_3$. Finally, we note that an extension of the present analysis to account for higher order nonlinear effects such as electrostriction has been initiated in[6].

3 CONCLUSIONS

Nano-beams characteristics have been analyzed with the nonlocal Timoshenko beam model accounting for the coupling of strain gradient to polarization and allowing to deal with both piezoelectric and flexolectric materials under inhomogeneous strain.

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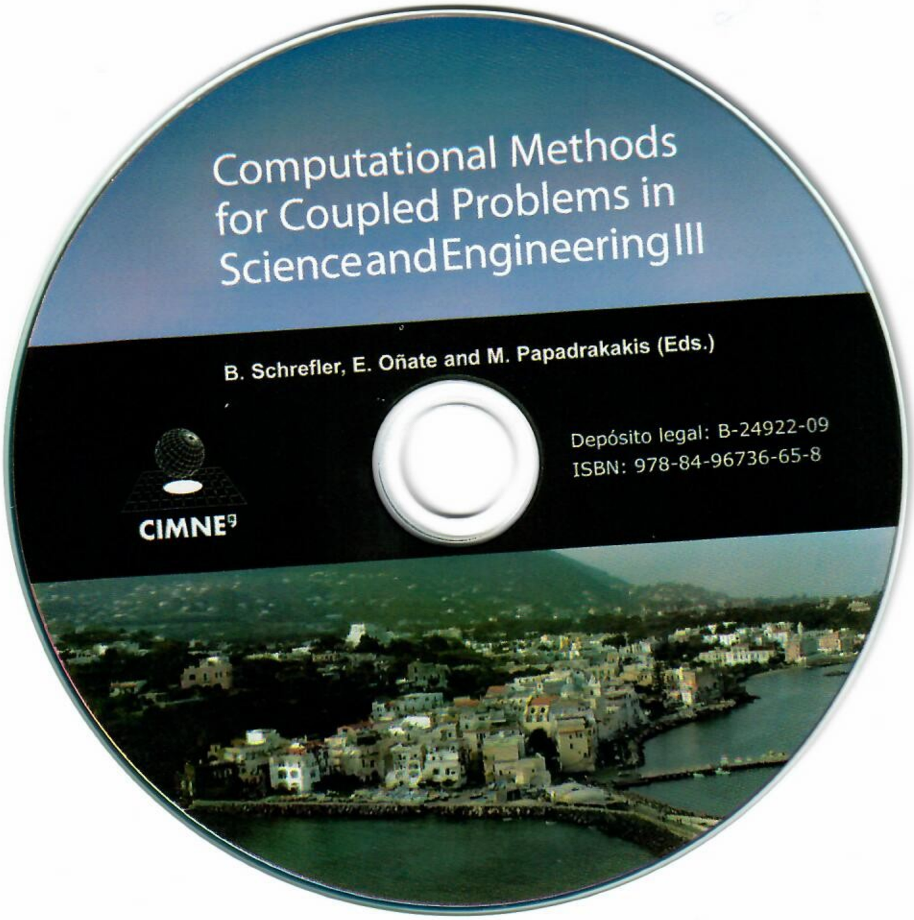
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