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Effect of aspect ratio of enclosure on free convection from horizontal cylinders in Bingham plastic fluids

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Heat transfer in steady free convection from differentially heated cylinders enclosed in a rectangular duct filled with Bingham plastic fluids has been solved numerically for the ranges of the dimensionless groups as, Rayleigh number, $10^2 \leq \mathrm{Ra} \leq 10^6$; Prandtl number, $10 \leq \mathrm{Pr} \leq 100$ and, Bingham number, $0 \leq \mathrm{Bn} \leq 50$ for aspect ratios AR = 0.5, 0.6, 0.7, 0.8, 0.9 and 2. The streamlines, isotherm contours, yield surfaces, local and average Nusselt numbers were analysed and discussed. It is found that as the aspect ratio of the enclosure increases from 0.5 to 0.9, the average Nusselt number on the surface of the hot cylinder increases as a larger amount of fluid takes part in convection. Moreover, at sufficiently large Bingham numbers, yield stress forces dominate over buoyancy causing the flow to cease and thus the Nusselt number approaches its conduction limit. Finally, the Nusselt number approaches its conduction limit once the maximum Bingham number is reached.

Keywords: Bingham plastic fluids, aspect ratio, Rayleigh number, Nusselt number, natural convection

1. INTRODUCTION

Suspensions, emulsions, foams, gels, etc., such fluids exhibit yield stress (Barnes and Walters, 1985; Bird et al., 1983). The dual nature of these fluids not only poses challenges in homogenization and mixing (Chhabra and Richardson, 2008; Paul et al., 2004) but also impedes heat transfer in such systems (Chhabra and Richardson, 2008). Heat transfer mechanism is conduction and convection in the unyielded (solid-like) and yielded (fluid-like) domains, respectively.

Considerable literature is now available for the enclosure walls at different types of boundary conditions (Turan et al., 2012), geometrical arrangement, e.g., cylinders placed along an x-axis (Park et al., 2012), or on the diagonal duct (Lee et al., 2010), or vertically aligned (Yoon et al., 2014) and enclosure aspect

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ratio (Turan et al., 2011). Recently, the results of the natural convection have been summarised for square enclosures with and without internal bodies in Newtonian fluids. The reported results are for the internal objects that are square, circular, and elliptical cylinders (Pandey et al., 2019). Very few results are available for enclosed horizontal circular cylinders inside a square duct in a Bingham fluid (Sairamu et al., 2013). Also, some results are available for differentially heated system along the vertical and diagonal of the square enclosure maintained at isothermal conditions in Bingham plastic fluids (Mishra and Chhabra, 2020). The rectangular enclosure has been solved for the range of $10^4 \leq \mathrm{Ra} \leq 10^6$, $1/8 \leq AR \leq 8$ at Pr = 7 (Turan et al., 2011). So far, natural convection from two internal cylinders enclosed in the rectangular duct of different aspect ratios is not solved. Albeit, this has many practical applications such as in solar collectors, in heating and preservation of canned foods, in electronic equipment cooling and in energy storage and conservation. The objective is to fill this research gap.

2. PROBLEM STATEMENT AND EQUATIONS

Figure 1 is conducted for different aspect ratio values ($AR = H/L$) of the rectangular enclosure at a constant value of R, i.e., $R/L = 0.2$. Height of the enclosure is varied to achieve a range of aspect ratios keeping width as fixed. One cylinder surface is hot (at temperature T_h) while the other cylinder surface is cold (at temperature T_c). Since the maximum temperature difference (between fluid at initial and hot surface) applied is 5K; therefore, the fluid thermophysical properties are not dependent on temperature. The Boussinesq approximation $\rho_c - \rho = \rho_c \beta (T - T_c)$ is used to model the density. In dimensionless formulations, the mass, momentum and heat transfer equations are (Baranwal and Chhabra, 2017):

Continuity equation:

$$\frac{\partial U_x}{\partial x} + \frac{\partial U_y}{\partial y} = 0 \tag{1}$$

x-momentum equation:

$$\left(\frac{\partial \left(U_x U_x \right)}{\partial x} + \frac{\partial \left(U_x U_y \right)}{\partial y} \right) = -\frac{\partial p}{\partial x} + \sqrt{\frac{\mathrm{Pr}}{\mathrm{Ra}}} \left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} \right) \tag{2}$$

y-momentum equation:

$$\left(\frac{\partial \left(U_y U_x \right)}{\partial x} + \frac{\partial \left(U_y U_y \right)}{\partial y} \right) = -\frac{\partial p}{\partial y} + \theta + \sqrt{\frac{\mathrm{Pr}}{\mathrm{Ra}}} \left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} \right) \tag{3}$$

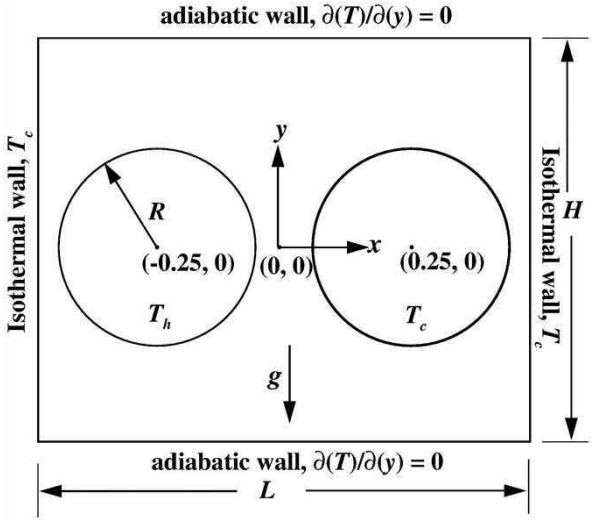

Fig. 1. Geometric representation of the physical domain

Energy equation:

$$\frac{\partial\left(U_x\theta\right)}{\partial x}+\frac{\partial\left(U_y\theta\right)}{\partial y}=\frac{1}{\sqrt{\mathrm{Pr\,Ra}}}\left(\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right) \tag{4}$$

The extra stress tensor in non-dimensional form is expressed as (Bird et al., 1987):

$$\tau=\left(1+\frac{\mathrm{Bn}}{|\dot{\gamma}|}\right)\dot{\gamma}, \qquad \text{if } |\tau|>\mathrm{Bn} \tag{5a}$$

$$\dot{\gamma}=0, \qquad \text{if } |\tau|\leq\mathrm{Bn} \tag{5b}$$

Because Eqs. (5a) and (5b) together are non-differentiable and discontinuous, it is important to use an appropriate continuous form of this equation (Glowinski and Wachs, 2011). Two regularization approaches are widely used (O'Donovan and Tanner, 1984; Papanastasiou, 1987) to overcome this difficulty. These approaches can be written as follows:

Papanastasiou model:

$$\tau=\left(1+\frac{\left[1-\exp\left(-M\,|\dot{\gamma}|\right)\right]\mathrm{Bn}}{|\dot{\gamma}|}\right)\dot{\gamma} \tag{6}$$

where M is the regularization parameter.

Bi-viscous model:

$$\tau=\frac{\mu_y}{\mu_B}\dot{\gamma}, \qquad \text{if } |\tau|<\mathrm{Bn} \tag{7a}$$

$$\tau=1+\left[1-\left(\frac{1}{\frac{\mu_y}{\mu_B}}\right)\right]\frac{\mathrm{Bn}}{|\dot{\gamma}|}, \qquad \text{if } |\tau|\geq\mathrm{Bn} \tag{7b}$$

where μ_y is the yielding viscosity.

The no-slip boundary condition is used on all solid boundaries. For temperature, $\theta=0$ is on the cold surface and the vertical walls, $\partial\theta/\partial y=0$ is on the horizontal walls, and $\theta=1$ is on the hot surface. U_r $(=\sqrt{Lg\beta\Delta T})$ and L are used for the reference scales. Thus, the stress, shear rate, and pressure are all scaled as $\mu_B\,(U_r/L)$, U_r/L and $\rho_c U_r^2$, respectively.

3. NUMERICAL METHODOLOGY

Here, COMSOL Multiphysics software is used to solve the problem numerically. The flow domain is meshed with the free triangular elements of different refinements (G1, G2 and G3). 10^{-5} is used as the relative convergence criterion to terminate the simulations. The results of drag and $\mathrm{Nu_{avg}}$ with the grids G2 and G3 are obtained identical, and therefore, G2 grid elements 39200 and 95040 have been used for $AR=0.5$ and 2 respectively.

4. RESULTS AND DISCUSSION

4.1. Validation of results

Figure 2 shows a comparison for $\mathrm{Nu_{avg}}$ along the hot wall and a temperature distribution along the midline of the enclosure, parallel to the x-axis in a Bingham plastic fluid (Turan et al., 2011). The present and literature values are seen to superimpose each other showing the new results to be independent of numerical artefacts.

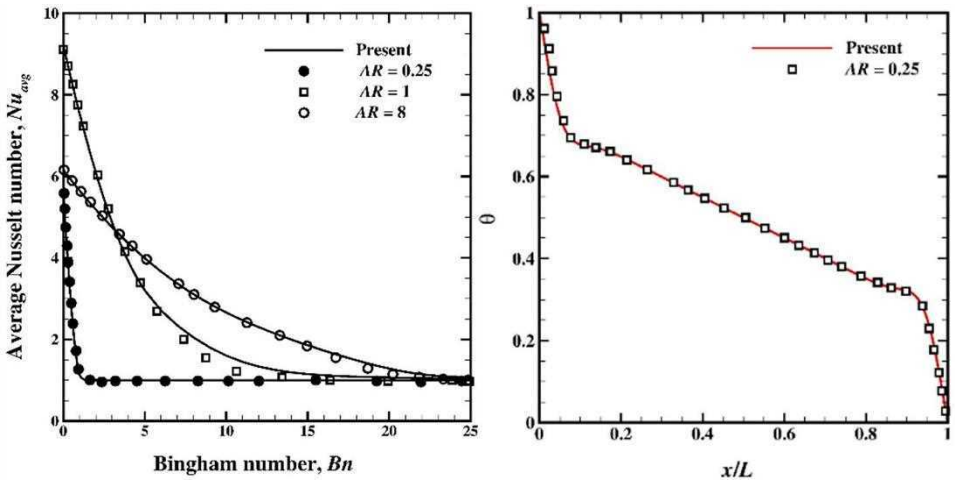

Fig. 2. Comparison of the present results; (lines) with literature (Turan et al., 2011) and (symbols) at Ra = 10^6, Pr = 7 in Bingham plastic fluids

4.2. Streamlines and isotherm contours

For the extreme case of AR = 0.5, Fig. 3(i)(a), the four recirculating regions are formed even at very low Bn. Everywhere flow domain is unyielded for Bn ≥ 0.05 at Ra = 10^2. Bn_{max} value for Ra = 10^6 is

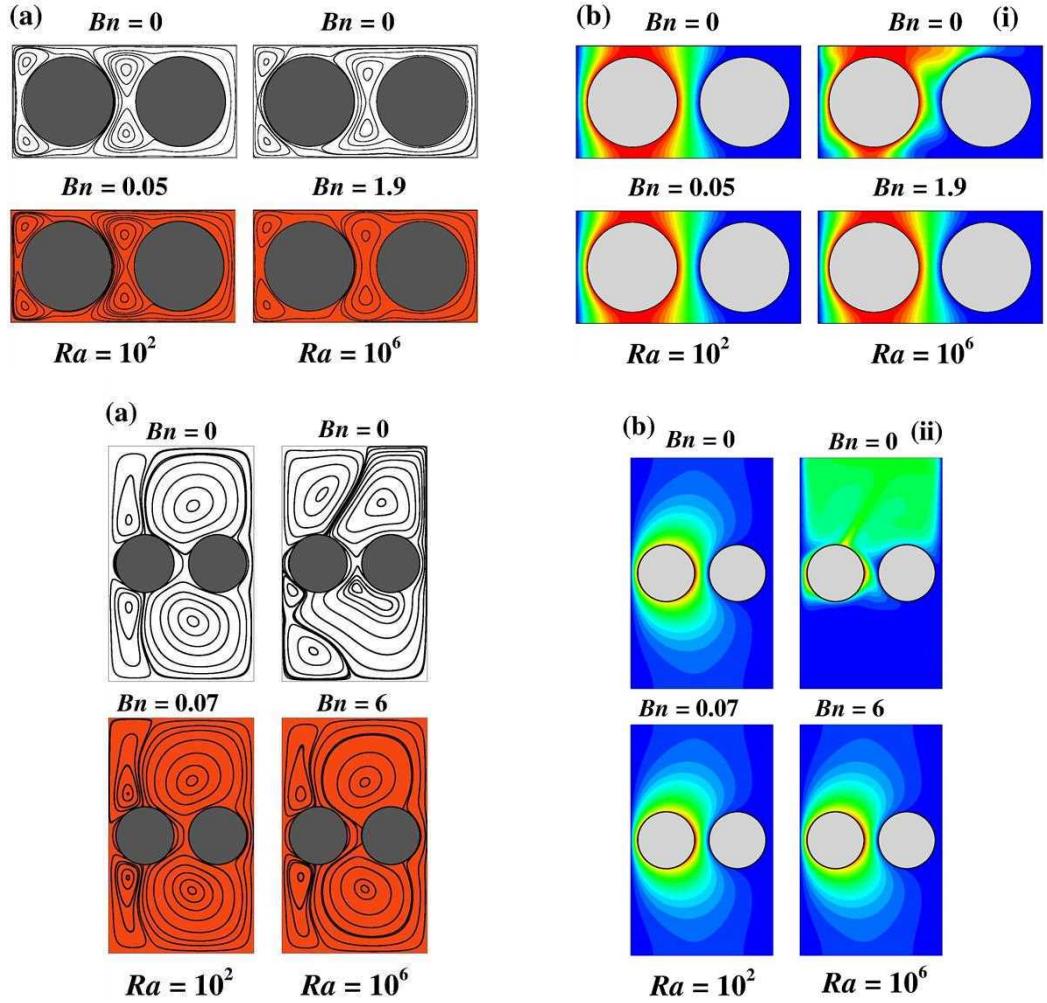

Fig. 3. Representative streamlines (a) and isotherm contours; (b) at Pr = 10 (shaded part-unyielded portions) for (i) AR = 0.5 and (ii) AR = 2

~ 1.9. The recirculating zones are same for $AR = 0.9$ as well except that a larger proportion of the fluid participates in heat transfer. For the $AR = 2$, one can observe similar trends only the different thing is that as increase the AR, the recirculation zone becomes bigger. The value of Bn_{max} increases as AR increases for all Rayleigh number. Effect of AR is more pronounced on Bn_{max} at $Ra = 10^6$ than that of at $Ra = 10^2$. In the isotherms above and below, the cold cylinder mass is almost at $\theta = 0$ that indicating negligible heat transfer. Heat transfer is facilitated when the aspect ratio increases for $AR \leq 1$ because more amount of fluid participates in the convection.

4.3. Average Nusselt number

For all cases of AR, at $Ra = 100$, the results are not affected by Bn because of poor convection as shown in Fig. 4. At another aspect ratio, as the size of the enclosure increases, which promotes heat transfer at least up to $AR = 0.9$. Even at smaller AR (= 0.5), a sufficient amount of fluid is present which modifies the heat transfer pattern. Bn differs in each case indicating that the fluid yield stress has a significant impact on rate of heat transfer. Moreover, effect of the Bn is more pronounced at larger aspect ratio values. Heat transfer rate decreases with rising Bn at fixed values of Ra, Pr, and AR, and it approaches the limit for pure conduction when $Bn = Bn_{max}$.

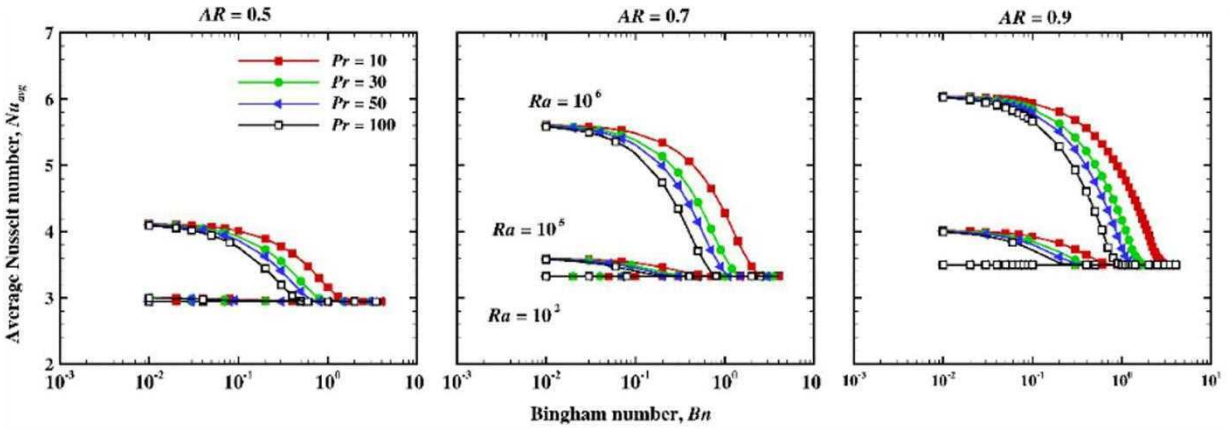

Fig. 4. Dependence of the Nu_{avg} on Bn, Pr, and Ra for the heated cylinder

This pattern is obtained for all AR, except that Ra and Pr modulate rate of approach towards pure conduction. In addition, with rising Bn, Nu_{avg} drops from its value at Bn = 0 to pure conduction depending on the value of AR.

5. CONCLUSIONS

The numerical analysis of two-cylinder system inside a rectangular duct with various aspect ratios has been carried out. The streamlines, isotherm contours, yield surfaces, local and average Nusselt numbers are included in the results. The free convection heat transfer results have been presented to elucidate the effect of the dimensionless numbers. The variation in the aspect ratio of the enclosure is sufficient to modify the heat transfer pattern. The mode of heat transfer in solid zones is primarily by conduction while convection occurs in fluid-like zones. Nu_{avg} follows an opposite dependence on the Bn and Pr, whereas it confirms positive support on Ra. Also, the geometrical configuration of the enclosure increasingly promotes heat transfer at least up to $AR = 0.9$. It is seen that the effect of the Bn is more considerable at larger aspect ratio values.

SYMBOLS

AR aspect ratio,
c_p specific heat capacity of fluid, J/(kg·K)
d diameter of cylinder, m
h convective heat transfer coefficient, W/(m^2·K)
M growth rate parameter, –
Nu_L local Nusselt number, $\dfrac{h \cdot L}{k}$
$\mathrm{Nu}_{\mathrm{avg}}$ average Nusselt number, $\dfrac{1}{s}\int_s \mathrm{Nu}\,\mathrm{d}S$
p pressure, –
U_x, U_y x- and y-components of the velocity, –
S surface area, m^2

Greek symbols

β coefficient of volume expansion, 1/K
$\dot{\gamma}$ rate of strain tensor, –
μ_B Bingham plastic viscosity, Pa·s
ρ density of the fluid, kg/m^3
ρ_c fluid density, T_c, kg/m^3
θ temperature, $\theta = (T - T_c)/(T_h - T_c)$, –
τ yield stress, Pa

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