

**15<sup>th</sup>**



# **World Congress**

on Scientific Computation,  
Modelling and  
Applied Mathematics

Berlin, August 1997

Volume 6

## **Application in Modelling and Simulation**

Edited by  
Achim Sydow

in cooperation with  
R.-P. Schäfer  
W. Rufeger  
H. Lehmann

UNIVERSITÄTSBIBLIOTHEK  
HANNOVER  
TECHNISCHE  
INFORMATIONSBIBLIOTHEK

**Wissenschaft & Technik Verlag  
Berlin**

<b>Contents</b>	<b>Page</b>
<b>Plenary Paper</b>	
J.R. Rice Future Challenges for Scientific Simulation	1
<b>Environmental Modelling</b>	
W.-A. Flügel Geoinformatics for Integrated Environment Systems Analyses: Application for Regional Catchment Modelling	9
G.H. Leavesley, R.J. Viger, S.M. Markstrom, M.S. Brewer A Modular Approach to Integrating Environmental Modeling and GIS	15
O. David Next Generation of Modelling Systems in Hydrology: System Design Challenges	21
T. Ranchin Wavelets, Remote Sensing and Environmental Modelling	27
V. Hochschild Derivation of Geomorphological Forms and Surface Moisture Conditions from ERS-1 SAR Data for Modelling of the Icefree Antarctic Environment	35
St. Jung, J. Albrecht VGIS: A Visual Tool for Environmental Modelling with GIS	41
U. Rhein Environmental Monitoring Using an Remote Sensing and GIS Approach	47
I. Gerharz, T. Lux, A. Sydow Inclusion of Lagrangian Models in the DYMOS System	53
K.K. Sabelfeld Stochastic Transport Models of Coagulating Aerosol Particles in the Atmosphere	59
K.H. Schlünzen A Validation Concept for Nonhydrostatic Atmospheric Models	65
A.V. Alexandrov, G.C. Hocking, L.R. Townley Modelling of Time-dependent Lake-aquifer Interaction	71
F. Nejari, A. Benhammou, B. Dahhou, G. Roux Predictive Control of a Simulated Biological Wastewater Treatment Process	77
P. Devine, R.C. Paton Individual Based Modelling in an Explicitly Spatio-temporal Ecosystem	83
O. Bröker, K. Cassirer, R. Hess, C. Jablonowski, W. Joppich, S. Pott Contributions to the Design of a Grid Oriented Global Weather Forecast Model	89
V.P. Belogurov Methodology of Synthesis of Systems for Water Quality Control and Monitoring	95
G. Bulitko, V. Bulitko, E. Nikipelova About Oscillations of Production Quality in Liquid Deposit Exploitation	101

D. Prochnow, C. Engelhardt, H. Bungartz Transport and Distribution of the Mean Settling Velocity of Particulate Matter in Rivers	105
S.V. Sheshenin Numerical Simulation of Water Filtration in Porous Rock/Soil Media and the Related Topics	111
R.V.N. Melnik Error Dynamics and Coupling Procedures in Mathematical Climate System Models	117
P.-W. Gräber Modelling and Simulation of Ground Water Processes	123
M. Nishigaki, W. Arnold Modeling of Transport Phenomena of Two-Phase Flow	129
A.S. Sennov, A.V. Yurkov The Use of a Mathematical Models for Groundwater Protection Problems	135
W. Grossmann, C. Vichienson, M. Hitz Modelling Forest Ecosystems - From Theoretical Models to Simulation	141
U. Heller, P. Struss Conceptual Modeling in the Environmental Domain	147
M. Schwab, H. Werthner, G. Guariso Data Structures in Environmental Modelling	153
G. Guariso, S. Muratori Qualitative Behaviour of Ecological Models: The Bifurcation Approach	159
<b>Modelling and Simulation in Engineering</b>	
K. Shimizu, H. Shi Metabolic Systems Engineering Approach for the Control of Bioprocesses	165
K. Mauch, S. Arnold, C. Posten, M. Reuss Computer Algebra Systems in Model-building and Model-analysis for Bioprocesses	171
J.-Q. Yuan, S.R. Guo, K.-H. Bellgardt A Cell Age Model for Penicillin Production	177
W. Wiechert, M. Möllney, M. Wurzel Modelling, Analysis and Simulation of Metabolic Isotopomer Labelling Systems	183
M. Arnold Impasse Points and Descriptor Systems with Unilateral Constraints	189
V.B. Bajic Large-scale Singular Networks with Slow Varying Reactances: Model and Response Properties	195
G. Wiedemann, K.J. Reinschke Stability Tests of a Bicycle Model in Descriptor Form	201
M. Winckler Semi-automatic Discontinuity Treatment in FORTRAN77-coded ODE Models	207
V.B. Bajic, K.M. Przyłuski A Note on Behaviour Testing of Nonlinear Time-discrete Descriptor Systems	213

H. Hahn, H.-J. Sommer Causality of Linear Time Invariant Descriptor Systems	219
M. Zhuang, W. Mathis Parallelism of the Backward Differentiation Formula on Workstation Clusters	225
M. Mena, O. Touhami, R. Ibtouen, M. Benhaddadi A Rapid Mathematical Method Applied to Synchronous Machine	231
X. Liu, E.L. Dörr, J. Walter Analytical Thermal Modelling and Simulation of the Cutting Process for Hard Machinable Materials	239
A. Nethe, H.D. Stahlmann Application of Process Models in Production Preparation	245
J. Lewicki Linear Model of Three-phase Transformers with Asymmetrical and Distorted Current-voltage Functions	251
R. Sünkel, C. Pautsch Planning of Forming Processes - Time and Cost Reduction Applying the Simulation System INDEED	257
P. Trehin, N. Heraud Observability Algorithm Applied to an Electrical Network	263
W. Schaefer, D.M. Lipinski, S. Andersen Numerical Modeling of the Filling Sequence, Solidification and Thermal Stresses of Castings	269
W. Kostecki The use of Maple V for Design and Performance Analysis of an SCR-controlled D-C Motor Drive	275
A. Zarifian, A. Nikitenko, P. Kolpachyan, B. Khomenko Computer Modeling of Dynamic Processes in Complex Electromechanical Systems	281
D. Beraha, W. Pointner The Nuclear Plant Analyzer ATLAS: Experience and Benefits	287
K.-H. Martens, H. Fischer Simulation of Contaminant Transport in a Brine-filled Repository System with the Computer Code MARNIE	293
P. Romstedt Numerical Aspects of Nuclear Power Plant Simulation	299
<b>Modelling and Simulation in Chemistry</b>	
G. Yablonskii New Results in Modelling Complex Catalytic Reactions	305
V. Gol'dshtein, V. Panfilov Multistability Control in a Periodically Forced Catalytic Reactor	311
V. Sobolev, I. Andreev, E. Shchepakina Modeling of Critical Phenomena in Autocatalytic Burning Problems	317
I. Goldfarb, V. Goldshtein, G. Kuzmenko, A. Zinoviev Delay Effect of Self-ignition in Multiphase Media	323

M. Lazman	329
Finding all the Roots of Nonlinear Algebraic Equations: A Global Approach and Application to Chemical Problems	
P. Phanawadee, J.T. Gleaves	335
Modelling of High Speed Transient Response Data from Chemical Kinetic Studies	
<b>Parallel and Real-time Simulation</b>	
Y.K. Dimitriev	341
On Models of Robust Self-diagnosable Computer Systems	
A.I. Gerasimov	347
Theoretical Grounds for a Decision Model of Computer Systems and Networks	
B.M. Glinsky	353
Architecture of Computer - Telecommunication Systems for Active Seismology	
V.G. Khoroshevsky	359
Modelling of Large-scale Distributed Computer Systems	
M.S. Tarkov	365
Mapping Parallel Programs Onto Distributed Robust Computer Systems	
A.N. Tomilin	371
Tunable System for Computer Structure Simulation	
V.A. Melentiev, N.G. Gryaznov	375
Analysis of Potential and Structural Robustness of Distributed Computer Systems	
V.G. Khoroshevsky, A.F. Zadorozhny, M.S. Tarkov, B.B. Kobets, A.M. Petrov	381
Architecture of Distributed Robust Computer Systems for Control of Electric Power Systems	
A.V. Zabrodin, V.K. Levin, V.V. Korneev	387
The Structure of MBC-100 Massively Parallel Computer Systems and Large Scale Applications	
M. Georgieva, V. Lazarov, D. Petrov, P. Philipov, M. Ivanova, Z. Zlatev	391
Simulation Modelling of Massively Parallel Computers	
G. Ewing, D. McNickle, K. Pawlikowski	397
Multiple Replications in Parallel: Distributed Generation of Data for Speeding up Quantitative Stochastic Simulation	
M. Rümekasten	403
The Tolerant, Hybrid Synchronization for the Parallel Simulation of Telecommunications Networks	
D. Talia	409
Cellular Automata + Parallel Computing = Computational Simulation	
A. Hirschowitz, D.G. Green, S.H. Brindle	415
Use of HPCN Simulation Techniques in Small and Medium Sized Enterprises	
A. Geiger, U. Lang, R. Rühle	421
Multi-site Collaborative Working-Environments for the Use of HPCN in the Automotive and Aerospace Industry	
J. Marczyk	427
A Meta-computing Approach to Stochastic Mechanics; On New Trends in Modern Engineering	

J.-L. Migeot Silent Vehicles through Coupled Noise and Vibration Simulations	433
M. Schlemmer Path-parameterized Time Optimal Trajectory Planning in Real-Time	439
J. Puzicha, T. Hofmann, J.M. Buhmann Deterministic Annealing: Fast Physical Heuristics for Real-time Optimization of Large Systems	445
D.B. Leineweber, H.G. Bock, J.P. Schlöder Fast Direct Methods for Real-time Optimization of Chemical Processes	451
R. Hohmann Experimental Frames in the Hardware-in-the-Loop-Simulation	457
U. Kiffmeier, R. Otterbach, H. Schütte Real-time Simulation of a 3-D Vehicle Dynamics Model on the DEC Alpha Processor	463
H. Heutger On the Structure and Models, Used in the Realtime Simulation of a Fixed Wing Aircraft	469
G. Thiele, H.J. Beestermöller Problem-oriented Real-Time Programming of Embedded Systems with PEARL 90	475
P. Saager Realtime Helicopter System-Simulator: Structures, Models and Programming Aspects	481
<b>Knowledge-based Simulation</b>	
M.H. Breitner, U. Rettig, O. von Stryk Robust Optimal Control with Large Neural Networks Emulated on the Neuro-computer Board SYNAPSE2•PC	487
H. Hopp, L. Prechelt CuPit-2: A Portable Parallel Programming Language for Artificial Neural Networks	493
G. Kock, T. Becher MiND: An Environment for the Development, Integration, and Acceleration of Connectionist Systems	499
A. Strey, J. Riehm Automatic Generation of Efficient Parallel Programs from EpsilonNN Neural Network Specifications	505
L.W. Buchan, A.F. Murray, H.M. Reekie Standard CMOS Floating Gate Memories for Non-volatile Weight Storage in Analogue VLSI Neural Networks	511
L. Larsson Visualization of Backpropagation by Learn Trajectories to Explore Approximations for Hardware Implementations	517
K. Mohraz, U. Schott, M. Pauly Parallel Simulation of Pulse Coded Neural Networks	523
P. Paschke, R. Möller Simulation of Sparse Neural Networks on a CNAPS SIMD Neurocomputer	529
H. Baumgärtel Constraint-based Multi-criteria Optimization for Flow Production Planning	535

M. Bigand, D. Corbeel, J.-P. Bourey The Design of Flexible Manufacturing Systems by using a Knowledge based System	541
H. Boley, M. Perling, M. Sintek Transforming Workpiece Geometries into Lathe-NC Programs by Qualitative Simulation	547
U. Geske, A. Fordan Using Constraint Logic Programming for Modelling and Simulation of Manufacturing Systems	553
U. John Constraint-based Simulation of Configuration Processes	559
<b>General Simulation Aspects</b>	
C.M. Vong, O. Babka Knowledge-based Support of Simulation	565
A. Galuszka Block World - State Space Searching	571
Y.L. Menshikov The Reduction of Initial Data Inaccuracy in Ill-posed Problems	577
T.R. Norton Simalytic Hybrid Modeling: Planning the Capacity of Client/Server Applications	583
L. Pengbo Application of MESA on Validating Simulation Model	589
J. Wu, J. Zeng, H. Chen, H. Li, G. Sun A Recursive Model for Combined Simulation	593
D.A. Harrell Detonating "Cognitive Fission" - Exploring the Entity Task Universe with One Self-expanding Modular Program	599
L.O. Shakunle What If There Are No Set Rules?	605
R. Starkermann Audi, vide, tace, si vis vivere in pace	611
E.W. Wette Intrafinite Complexity of the Elementary Inconsistency-Computation, and Hydrogen Atoms Rectified in the (Deterministic) Ether-Geometry	617
M.J. Corbin Potential Applications of Internet Computing Technologies to Distributed Simulation	623
G.A. Csanády, J.G. Filser A Physiological Toxicokinetic Model for Styrene Together with its Metabolite Styrene - 7, -Oxide in Rodents, and Human	629
A. Kremling, R. Rehner, C. Kammerer, J. Wang, E.D. Gilles A Simulation Tool for Metabolic Structured Models	635
S.A. Miller, N.W. Fussey, R.A. Williams An Integrated Approach to Chassis Tuning	641

P.G. Thomasson Modelling the Dynamics of Ram Air Parachutes	647
T. Ernst, S. Jähnichen, M. Klose The Architecture of the Smile/M Simulation Environment	653
C. Klein-Robbenhaar Numerical Methods for Dynamic Simulation of Thermal Energy Systems: A Case Study	659
G. Bartsch, P. Jochum, M. Kloas Cost Efficiency at the Modernization of East German "Plattenbauten"	665
H. Tummescheit Simulation of a Solar Thermal Central Receiver Power Plant	671
<b>Traffic Simulation</b>	
A. Boulmakoul, S. Sellam, A. Dussauchoy A Free Distributive Lattice for Urban Traffic Modelling	677
A. Alessandri, A. Di Febbraro, A. Ferrara, E. Punta Simulation Analysis to Design Control Strategies for Freeway Systems	683
L. Penhouët, G. Thomas, R. Fondacci A Short Term Air Traffic Flow Control Model using Optimal Control Technique	689
<b>Modelling for Intelligent Diagnosis</b>	
P. Amann, P.M. Frank On Fuzzy Model-building in Observers for Fault Diagnosis	695
J. Kicinski, R. Drozdowski, P. Materny The Computer Simulation of Rotor Machine Dynamics Applied to the Construction of "Intelligent" Diagnostic Systems	701
A.K.A. Toguyeni, E. Craye, J.C. Gentina <i>Time and Reasoning for the On-line Diagnosis of Failures in Flexible Manufacturing Systems</i>	709
W. Cholewa Real-time Expert Systems for Technical Diagnostics	715
E. Czogala, G. Drwal, J. Leski Decision Support System "Fuzzy-Flou" in Diagnosis Problems	721
J.E. Larsson Recent Developments in Multilevel Flow Modeling Diagnosis	727
<b>Modelling in Economics</b>	
F. Lemke, J.-A. Müller Self-organizing Modelling in Financial Risk Control	733
R. Neck, S. Karbuz Optimal Control of the Austrian Economy under Alternative Assumptions about Exogenous Variables	739



I. Ståhl	745
The Dynamic, Stochastic World View of Simulation Applied to Corporate Modelling: A Comparison of Relative Strengths and Weaknesses	
E. Kremer	751
Mathematics of Largest Claims Reinsurance	
E. Chattoe	757
What Simulation has Done for Economics and What it Might Do	
<b>Author Index</b>	<b>763</b>

# Error Dynamics and Coupling Procedures in Mathematical Climate System Models

R. V. N. Melnik,  
Department of Mathematics and Computing,  
University of Southern Queensland, QLD 4350, AUSTRALIA  
E-mail: melnik @usq.edu.au

**Keywords:** complexity of coupling, computational efficiency, effective viscosity coefficient, open dynamic systems, nonreflexive topological spaces.

## ABSTRACT

The mathematical hypothesis concerning the external character of perturbations is critically examined in the special case of a climate system model. The importance of embedding such models into a perturbation space with  $L^1$ -structure for the analysis of error and model stability is demonstrated.

## 1. TOPOLOGICAL EMBEDDING OF MATHEMATICAL MODELS

Since there are many real dynamic systems on which controlled experiments cannot be carried out, it is important to construct an appropriate model of a “proxy system” on which such experiments can be conducted. A classic example of such a dynamic system is the atmosphere-active-layers (AAL) system. As a result of increasing computational power of modern computers, improvements in measuring techniques and achievements in the development of effective numerical procedures, a number of mathematical models for the proxy AAL system are currently available (see references in [4]). As a rule, such models require a large datasets of initial conditions (in comparison to fast-wave-filtered models) and an appropriate definition of boundary conditions in order to avoid possible instability. In this study the Community Climate Model CCM3 developed in NCAR was chosen as a model for the proxy AAL system. Figures 1 and 2 show examples of the computation of the meridional component of wind and water vapour on the vertical level 15 when a hybrid sigma-pressure system is used for the vertical coordinate (see details in [4] and references therein).

Can we claim that the *total error* in the description of the climate system by a model that consists of a finite set of partial differential equations and approximate initial and boundary conditions is small? One may observe that although the results presented in figures 1 and 2 were obtained from a deterministic mathematical model, there is a temptation to say that both figures exhibit some degree of stochasticity. If this is the case then is it real randomness, low-dimensional attractors or distorted pictures of climatic fields due to accumulated error of computation? To answer these questions rigorously we have to know to what extent, if any, the original system is deterministic. Furthermore, even if climate is a deterministic system, in mathematical models based on a finite set of nonlinear equations *the uniqueness* of long-term statistics which define initial and boundary conditions for the problem is not guaranteed. Indeed, mathematical theory is typically applied to dynamic systems which are *closed* in the sense that perturbations of the system are *external* to it and are not influenced by the system itself. This view may be effectively applied only to a relatively narrow class of dynamic systems. Since climate does not belong to this class, in order to get its formalized description we have to include all parts of the climatic environment, which are influenced by climate itself into the model, a task which is hardly tractable in the rigorous mathematical sense.

CLIMATE MODELLING WITH CCM3: FIELD U (lat,long) Vert level = 15 Time Step = 1

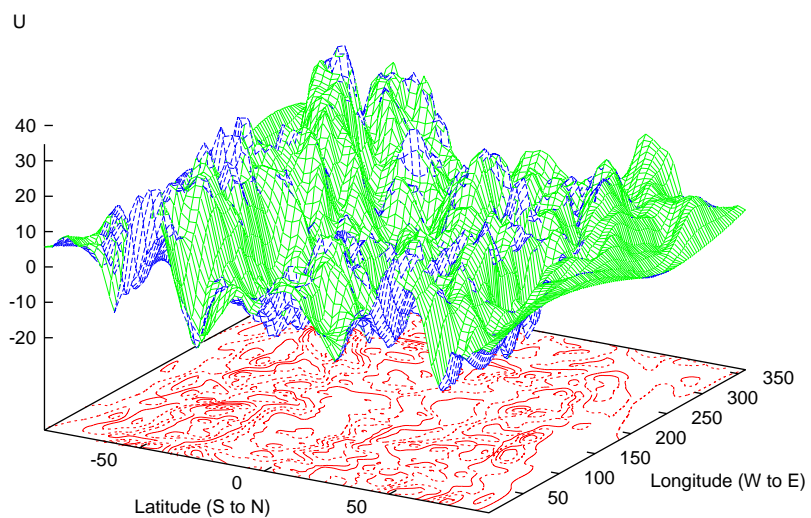


Figure 1: Meridional component of the wind (V, vertical level 15).

CLIMATE MODELLING WITH CCM3: FIELD Q (lat,long) Vert level = 15 Time Step = 1

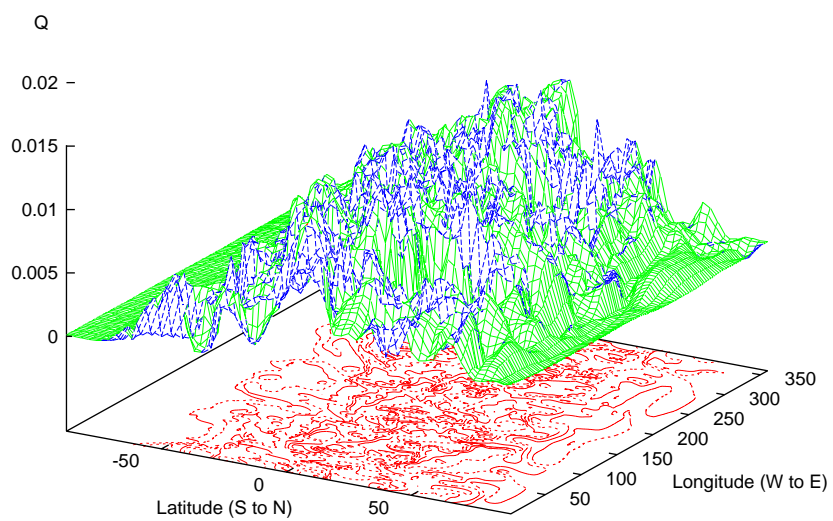


Figure 2: Water vapour at vertical level 15.

In this paper I propose an alternative approach. It is well-understood that those original difficulties connected with the structure of turbulence (see references in [4]) that led to reversion from models based on filtering procedures (for meteorological “noises”) those based on primitive hydrodynamic equations *have not been removed*. They rather were gradually fuzzyfied in computational aspects of the problem (see [4] and references therein). These difficulties are intrinsic to mathematical models of complex dynamic systems that require investigation of *phase transitions*.

The crux of the problem appears to be in the appropriate choice of *the topological space for perturbations* in which the dynamic system can be embedded. The increasing quality of initial datasets for climate simulation implies an adequate treatment of the phenomenon of wave instability on frontal interfaces that mathematically may be essentially simplified by the assumption of the given “exact” boundary conditions. From the physical point of view such a simplification can be seen as an approximation of a fundamentally *open dynamic system* by a *closed dynamical system*. It is this step of mathematical modeling that allows us to apply *conservation laws for different physical quantities such as mass, momentum, energy*. This produces an avoidable error in the results obtained by the mathematical models with respect to the real system, but it may be effectively applied in a wide range of physical situations. Now the natural question arises of how such models can be improved.

## 2. PARAMETERIZATION OF PHYSICAL PROCESSES

Since an extreme sensitivity to physical parameterization is an integral part of mathematical models for the proxy AAL system [2], the improvement of such models can be achieved by *embedding into them* mathematical models of new active layers or by improving models of layers included on earlier stages of the model development (for example, embedding into coupled atmosphere-ocean models a thermodynamic sea-ice model, improving solar and terrestrial radiative heating models, cloudiness parameterization etc). Essentially, these improvements are achieved by a more careful examination of the top-of-atmosphere, surface energy budgets etc. As soon as physical parameterization is chosen, and hence, the mathematical model is “frozen”, such an examination can be effectively performed using tools of mathematical modeling and computational experiment. In this case physical quantities such as mass and energy may be conserved only approximately. Indeed, due to coupling to the physical parameterization, the AAL system can be formalized mathematically only through procedures with incomplete information. Therefore, when constructing computational models from “frozen” mathematical models, the associated uncertainty in knowledge bases should be taken into account. In numerical analysis of mathematical models this leads to the necessity of relaxing a-priori excessive assumptions on the solution smoothness [3]. Mathematical and computational models become strongly coupled by the chosen *physical model* and by the *algorithm* of the problem solution. Mathematically this coupling can be formalized through the concept of perturbations.

## 3. CONSERVATION LAWS AND NONREFLEXIVE TOPOLOGICAL SPACES

A limiting mathematical case of the situations discussed above is represented by the Cauchy problem for mathematical conservation laws. Although it is still widely accepted that the solution of this problem may be well-approximated in the class of piecewise constant functions using the solutions of auxiliary Riemann problems, examples when this may not be the case have been recently constructed [6].

Without going into details it is important to emphasize that the main mathematical difficulty in the field of mathematical conservation laws stem from *the continuous dependency*

of the solution on perturbations that cannot be eliminated. This leads to the situation where the solution itself can be defined in practice only with respect to *certain level of perturbations*. It means that with respect to the original physical system mathematical models are *a priori* perturbed. Even if we formally consider a mathematical model as unperturbed, we have to investigate its stability to perturbation in a typically infinite dimensional space wider than state space of the system. The natural choice for such a space is  $L^1$ . However, most investigations consider only  $L^2$ -perturbations. The mathematical techniques used in such investigations depend crucially on the analysis of nearest points and rely heavily on the Hilbertian structure or the perturbation space or even on occasion on the finite dimensional structure. This restricted the possible application of such results to mathematical models for complex dynamic systems such as climate. The inadequacy of the description of perturbations in terms of time-independent functions for complex dynamic systems such as climate necessitates the choice of  $L^1$  structure in the perturbation space.

#### 4. CLIMATIC DETERMINISM AND SCALING LAWS

Modern Climate System Models consist of *relatively independent* components that are responsible for interconnected parts of climate (such as atmosphere, ocean, land surface, sea ice). This makes possible to apply effective parallelization procedures. Nevertheless, in order to obtain approximate solutions, such models still require substantial computational power which often is not available. Since different computers and different compilers may give *noticeably* different results some authors have concentrated on the round off error resulting from the approximate solution of a system of PDEs with given initial and boundary conditions. However, the rounding error *alone* may not be an adequate reflection of the total error which also includes initial datasets approximate in nature and the approximate physical parameterization “built” in to a finite set of differential equations.

In order to assess the quality of simulation, we have to assess the quality of initial datasets and the physical parameterization. Indeed, the quality of our approximate solutions depend on *the consistency between the mathematical model and the real climate* rather than on *the consistency between two mathematical models*. Since the improvement of mathematical models can be achieved by improved initial datasets and physical parameterizations, *the concept of coupling between different components of the model* becomes straightforward. The dilemma lies in the fact that the refinement of the approach based on coupling procedures may continue indefinitely, yet it does not necessarily lead to the absence of the error in mathematical/computational implementation of conservation laws. In fact, there may not exist a *finite time procedure* that allows us to solve the problem of the interaction between small-scale convection and large-scale motion in a rigorous mathematical sense. However, if the *relative error for the solution of this problem* is given, such a procedure can be found at least in principle. In modeling complex dynamic systems that require the solution of the problem of interaction between different space-time scale processes, the error becomes a time-dependent function. The specified level of error implicitly defines the time-range within which the model can be effectively applied.

The “timeless” features of many mathematical models stem from a simplified mathematical assumption that smaller-scale turbulence may be regarded as a *dissipative factor* which might be characterized by an *effective viscosity coefficient*  $\mu$ . For practical applications *the law of viscosity* has to be supplemented by *the law of energy dissipation*. In climate modeling the Prandtl mixing-length hypothesis is used in order to find a reasonable analogy between the motion of molecules and the motion of macroscopic elements in turbulent fluids (see references in [4]). A connection between these two physical laws, the law of viscosity and the law of energy dissipation, has to be defined by a *scaling law*. Many current investigations in including

climate study are based on the Karman-Prandtl logarithmic scaling law, which in some cases may be inappropriate as an adequate description of turbulent processes (see references in [4]). In order to ensure consistency between mathematical and physical models we have to take into account *the time dependency of spatial averaging procedures* that become important in large-scale modeling. This requires the formulation of the scaling law to be made on the basis of both informational parts about the solution of the problem, *a priori* information and *a posteriori* information. In turn, this dictates more restrictive assumptions on the solution regularity, and  $L^1$ -error bounds or at least  $L^k$ -bounds where  $k \in (1, 2)$  become an important measure of the quality of associated with the mathematical model numerical schemes. For linearized models  $L^2$ -error bounds provide sufficient information on the quality of approximate solutions.

## 5. STOCHASTICITY AND VANISHING VISCOSITY

Somewhat wider mathematical freedom is allowed when the evolution of states of the atmosphere is regarded as a random process  $m(t)$  [5]. In this case one may attempt to use statistical extrapolation of the process using the *Kolmogorov's hypothesis*. Namely, a random process  $m(t)$  describing the evolution of the turbulent flow in an *environment with vanishing viscosity* asymptotically approaches a Markov process for large  $t$ . Eventually, it is this hypothesis that allows one to keep the assumption of  $L^2$ -structure of perturbation space as a possibility. From such a consideration it follows that the distribution of probabilities  $P^t(dm)$  for  $t > t_0$  may, in principle, be uniquely determined by the state  $m(t_0)$ , and not be dependent on the remote history of the process when  $t < t_0$ . This approach may be effectively applied in practice bearing in mind that the quality of the definition of the probabilities  $P^t(dm)$  become unsatisfactory when  $t$  exceeds a finite (possibly very large) time known as the Lyapunov horizon. In order to increase this horizon we have to increase the accuracy of initial datasets and to improve physical parameterization. Of course, the assumption of *the negligible viscosity approximation* can be reasonably justified numerically on a finite grid. However, no matter how dense such a grid is, the validation of the original mathematical model is conducted under the processing of incomplete information that in turn requires an adequate formulation of *time-dependent scaling laws*. Indeed, we need to *continuously improve* mathematical models of dynamic systems by including additional information. Although it is unreasonable to expect that such a law may be formulated for all practically important cases, it is possible to formulate such laws on classes of mathematical problems. This allows us to predict the error-bound in modeling for an arbitrary representative of the class.

## 6. COMPUTATIONAL EFFICIENCY AND COMPLEXITY OF COUPLING

The total modeling error  $\varepsilon$  for a complex dynamic system such as climate cannot be reduced to zero due to the presence of unremovable error between the system and its model. This error can be interpreted as a perturbation parameter that is intrinsic to the chosen law of scaling. For any given model this parameter defines the upper bound,  $\tau$ , for time-range, as a function of a set  $\mathcal{M}$  that characterizes the model

$$\tau = f(\mathcal{M}, \varepsilon).$$

This time bound can be interpreted through the Lyapunov horizon. If we assume that  $\mathcal{M} \neq \emptyset$  and  $\varepsilon > 0$ , then the function  $f$  defines the complexity of coupling procedures. Any specific choice of positive  $\varepsilon$  implies a simplification of the original problem and provides a measure of

the discrepancy between system and model. The simplification is also unavoidable in the case when  $\varepsilon \rightarrow 0^+$ . Indeed, in this case the system has to be considered as a unified whole that often makes the problem computationally untractable in practice. Numerical aspects of the problem are defined by the connection between the set  $\mathcal{M}$  and the value of  $\varepsilon$  and it may not be possible to reduce the error without associated changes in the structure of  $\mathcal{M}$ . Whenever the structure of  $\mathcal{M}$  is fixed (as it is the case for the CCM3 model) and the existence of  $\lim_{\varepsilon \rightarrow 0^+} f(\mathcal{M}, \varepsilon)$  is assumed *a priori*, the solution of the problem consists of a numerical interplay between the time variable  $\tau$  and the function of complexity of coupling  $f$ . Even for a high-level-coupling “proxy system”, smaller-scale phenomena may still substantially influence the larger-scale properties of the system, yet they cannot be extracted from the latter using modern computational resources.

This observation leads to the conclusion that it is necessary to maintain a balance between *computational efficiency* and the *complexity* of coupling procedures. Such a balance has to be maintained for both deterministic and random mathematical models. Both types of model may provide appropriate descriptions of the same system but with different  $f$  and  $\mathcal{M}$ . It is important, however, to consider both models as perturbed mathematical models that are embedded into the topological space for perturbations with  $L^1$ -structure. This idea has been recently developed from different directions in [1,4,6] where links to the recent advances in control theory as well as numerical procedures can also be found.

## ACKNOWLEDGMENTS

This research was supported by the Centre for Industrial & Applied Mathematics at the University of South Australia. The results of computation have been obtained from a SUN station at the CIAM and from the VPP-300 supercomputer at the Australian Supercomputer Facility. The author thanks Dr Murray Dow for the permission to use a vectorised version of the CCM3 model and Dr G. Gross for the help in graphical interface.

## REFERENCES

- [1 ] J. Borwein, Q.J. Zhu, *Variational Analysis in Nonreflexive Spaces and Applications to Control Problems with  $L^1$  Perturbations*, Nonlinear Analysis, TAM 28, p.889 (1997).
- [2 ] C.C. Ma et al, *Sensitivity of a Coupled Ocean-Atmosphere Model to Physical Parameterizations*, Journal of Climate, 7, p.1883 (1994).
- [3 ] V.N. Melnik, *Non-conservation Law Equation in Mathematical Modelling: Aspects of Approximation*, Proc. of the International Conference AEMC'96, Sydney, pp423-430 (1996).
- [4 ] R.V.N. Melnik, *Mathematical Models for Climate as a Link between Coupled Physical Processes and Computational Decoupling*, CIAM, Maths, University of South Australia, TR 1997/1, 42p.
- [5 ] A.S. Monin, Weather Forecasting as a Problem in Physics, *The Massachusetts Institute of Technology*, 1972.
- [6 ] A. Tveito, R. Winther, *The Solution of Nonstrictly Hyperbolic Conservation Laws May Be Hard to Compute*, SIAM. J. Sci.Comput. 16, p.320 (1995).

**Error Dynamics and Coupling Procedures in  
Mathematical Climate System Models**

**Melnik, (R.)V.N.**

**In: Proceedings of the XVth World Congress on Scientific Computation,  
Modelling and Applied Mathematics, Germany, 1997;  
Vol. 6: Applications in Modelling and Simulation, Ed. A. Sydow,  
Wissenschaft and Technik Verlag, Berlin, 117--122, 1997.**

**Publisher: Wiss.-&-Technik-Verlag, Berlin**

**ISBN: 3896855565, 3896855506**