Noye
Teubner
Gill

COMPUTATIONAL TECHNIQUES AND APPLICATIONS: CTAC97

Proceedings of the Eighth Biennial Conference

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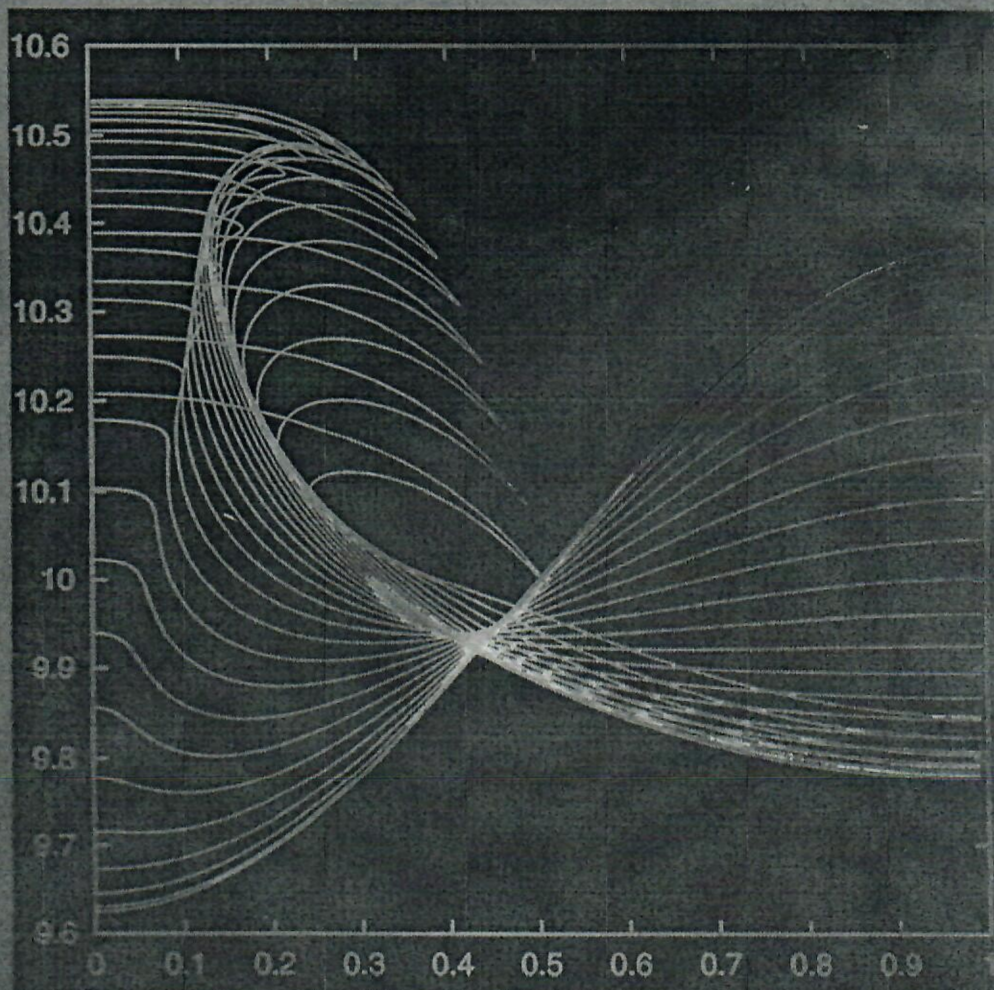
Editors: **John Noye, Michael Teubner & Andrew Gill**



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Computational Techniques and Applications: CTAC97
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On Computational Aspects of Certain Optimal Digital Signal Processing Algorithms

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1. Introduction

An increasing complexity of practical problems requires a comparative analysis of the accuracy and effectiveness of optimal methods (and methods close to optimal with respect to their computational characteristics) that exist for the solution of problems in applied and computational mathematics. In many cases we need improvements in methods for the estimation of computational errors. This paper addresses these issues for the optimisation of the accuracy of the numerical evaluation of certain oscillatory integrals. Such integrals occur as an important part of the modelling of optical systems and automatic control systems. They also occur in digital filtering, image recognition, statistical analysis of experimental data and many other problems [2, 4]. The algorithms described in this paper are designed to solve some practical problems in digital signal processing and image recognition. They deal with the case of inaccurate data and provide estimates of accuracy in the solutions.

A number of different problems in applied mathematics require the evaluation of integrals of the following form:

$$I^n(f) = \int_{a_1}^{b_1} \dots \int_{a_n}^{b_n} f(x_1, \dots, x_n)\varphi_1(x_1)\dots\varphi_n(x_n)dx_1\dots dx_n, \tag{1.1}$$

where $\varphi_k(x_k), k = 1, \dots, n$ are known integrable functions, and $f(X)$ belongs to some predefined class of functions F, in which information about $f(X)$ is defined by its values at N points $X = (x_1, \dots, x_n) \in \pi_n, \pi_n = \{a_k \leq x_k \leq b_k, k = 1, \dots, n\}$. An important special case of this problem is the numerical evaluation of integrals of the following forms:

$$I_1^n(f) = \int_{a_1}^{b_1} \dots \int_{a_n}^{b_n} f(x_1, \dots, x_n)dx_1\dots dx_n, \tag{1.2}$$

$$I_2^n(f) = \int_{a_1}^{b_1} \dots \int_{a_n}^{b_n} f(x_1, \dots, x_n)\sin(\omega_1 x_1)\dots\sin(\omega_n x_n)dx_1\dots dx_n, \tag{1.3}$$

$$I_3^n(f) = \int_{a_1}^{b_1} \dots \int_{a_n}^{b_n} f(x_1, \dots, x_n)\cos(\omega_1 x_1)\dots\cos(\omega_n x_n)dx_1\dots dx_n, \tag{1.4}$$

where ω_k are arbitrary real numbers, $|\omega_k| \geq 2\pi/(b_k - a_k)$, $k = 1, ..., n$ with $n = 1$ and $n = 2$. The construction of algorithms for numerical evaluation of $I_i^n(f)$, $i = 1, 2, 3$ for $f(X)$ from some function classes, and the analysis and optimization of the accuracy and effectiveness of these algorithms, are important tasks with diverse application areas.

In constructing methods of approximate integration quite broad function classes are usually considered. In this paper it is assumed that function $f(X)$ (say, $f(X) \in F$) is determined by the set of its values $f_i, i = 1, ..., N$ in N fixed points $X_i, i = 1, ..., N$ from its domain of definition. Such a way of definition of initial information leads to an essential narrowing of the corresponding class F. The introduction of a new, narrower class F_N called the interpolation class, allows us to use all available information about the function, and to improve the quality of algorithms constructed for numerical evaluation of $I^n(f)$. Moreover, it puts us closer to the real situations that appear in applications.

In practice, instead of exact initial data $f_i, i = 1, ..., N$ only approximate values $\tilde{f}_i, i = 1, ..., N$ are known. In this case we will consider functions f which satisfy the inequality $|\tilde{f}_i - f(x_i)| \leq \varepsilon_i$, $i = 1, ..., N$, where $\varepsilon = (\varepsilon_1, ..., \varepsilon_N)$ is a certain fixed vector.

Let us consider the function f from some class F_N and let M be a specified set of integration algorithms. The integration error $v(F_N, A, f)$ of a function $f \in F_N$ by algorithm $A \in M$ can be chosen as a criterion for estimating the quality of algorithm. One of the main tools of our construction of integration formulae is the *limit functions method*. This method consists of the construction of the best algorithm for the worst function from the class. The worst function from the class F_N for a given algorithm is considered to be a function which provides $\sup_{f \in F_N} v(F_N, A, f)$.

We introduce the characteristic

$$\delta(F_N) = \inf_{A \in M} \sup_{f \in F_N} v(F_N, A, f), \tag{1.5}$$

where

$$v(F_N, A, f) = |I^n(f) - r(F_N, A, f)|, \tag{1.6}$$

$r(F_N, A, f)$ is the result of algorithm A applied to function f, and M is the set of all integration formulae using the information description of class F_N. The integration formula for which $\delta(F_N)$ can be reached (provided it exists) is considered to be *optimal-by-accuracy* for a given function class.

If for some other integration formula $\bar{A}$

$$v(F_N, \bar{A}, f) \leq \delta(F_N) + \eta, \ \eta \geq 0, \tag{1.7}$$

then $\bar{A}$ is called an *optimal integration formula on class F_N with accuracy up to η*. Finally, $\bar{A}$ is known as an *asymptotically optimal* integration formula when $\eta = o(\delta(F_N))$ and an *optimal-by-order* integration formula when $\eta = O(\delta(F_N))$.

It is known (see, for example, [1, 4] and references therein) that for certain function classes the optimal-by-accuracy value of the integral $I^n(f)$ is given by the expression

$$I^*(F_N) = 1/2\left(I^+(F_N) + I^-(F_N)\right), \tag{1.8}$$

where

$$I^+(F_N) = \sup_{f \in F_N} I^n(f), \quad I^-(F_N) = \inf_{f \in F_N} I^n(f) \tag{1.9}$$

are respectively upper and lower limits of possible values for integrals of the forms (1.1)–(1.4) in the integration domain on functions from class F_N. Then

$$\delta(F_N) = 1/2\left(I^+(F_N) - I^-(F_N)\right). \tag{1.10}$$

In this case $I^*(F_N)$ is a Chebyshev centre for the undefinability domain of values $I^n(f)$ on the class F_N and the Chebyshev radius is equal to $\delta(F_N)$.

In the Sections that follow we construct integration formulae for numerical evaluation of integrals from the classes described above using the limit functions method when the various ways to determine *a priori* information are applied.

2. Optimal Integration Formulae for Certain Oscillatory Integrals

In many applied problems we have to evaluate integrals of the form

$$I^1(f) = \int_a^b f(x)\varphi(x)dx, \tag{2.1}$$

assuming that $\varphi(x)$ is a known integrable function and $f(x)$ belongs to a certain given function class F_N. In practical cases $\varphi(x)$ might be equal to $1, \sin(\omega x), \cos(\omega x), e^{-i\omega x}$. This type of integral often occurs as an important part of Fourier transform evaluation, function expansion in Fourier series and so on. Although a number of methods for numerical integration of (2.1) are available, it is rarely the case that inaccuracy of *a priori* information is taken into account. In this paper we propose some optimal-by-order integration formulae in order to evaluate integral (2.1) under the assumption of inaccurate *a priori* information.

Let us consider the following function classes:

• $C^1_{1,L,N}$ is the class of continuous functions $f(x)$ defined on the interval $[a, b]$, which have function values and first derivatives satisfying the conditions

$$f(x_i) = f_i, \ i = 1, ..., N, \tag{2.2}$$
$$|f'(x)| \le L, \ \forall x \in [a, b], \tag{2.3}$$

where $x_1, ...x_N$ are fixed points in $[a, b]$.

• $C^1_{1,L,N,\varepsilon}$ is the class of continuous functions $f(x)$ defined on the interval $[a, b]$ with first derivatives satisfying (2.3) and approximate values $\tilde{f}_i, i = 1, ..., N$ satisfying

$$|f(x_i) - \tilde{f}_i| \le \varepsilon_i, \ \varepsilon_i \ge 0, \ i = 1, ..., N, \tag{2.4}$$

where $\varepsilon = (\varepsilon_1, ..., \varepsilon_N)$ is a certain fixed vector. In what follows we assume that all considered function classes are non-empty.

Let us consider the case, when $\varphi(x)$ is assumed to be $\sin(\omega x)$ or $\cos(\omega x)$, where ω is a real number that determines an oscillatory factor of the integrand, $|\omega| \geq 2\pi/(b-a)$ and $f(x)$ belongs to $C^1_{1,L,N}$ or $C^1_{1,L,N,\varepsilon}$. For such functional classes optimal-by-accuracy algorithms for the numerical evaluation of integral (2.1) are known (see, for example, [1, 4] and references therein). Although such algorithms are suitable for a wide range of oscillation patterns of the integrand (with the assumption that the values of L and ε used in these algorithms are given accurately), in many practical cases it is not possible to apply these algorithms. Indeed, in practice it is typical that numerical *a priori* information (used for the definition of function classes) is inaccurate. Instead of exact values of L and ε, we rather have some estimations of these values. Below we propose efficient algorithms for such situations. We construct optimal-by-order, rather than optimal-by-accuracy, algorithms that are based on methods of quasi-solutions and residual minimization .

In the numerical integration algorithm, constructed on the basis of the quasi-solutions method, integrand $f(x)$ is approximated by a function which is the solution of the following problem [1]:

$$\min_{f\in F} \max_{i=1,...,N} |f(x_i) - \tilde{f}_i|. \tag{2.5}$$

The quasi-solutions method (2.5) consists of the determination of a function that has the least possible deviation from a given set of points $(x_i, \tilde{f}_i), i = 1, ..., N$. The solution of (2.5) in class $C^1_{1,L,N,\varepsilon}$ is a linear spline $S(x, L)$ for which maximal deviation from a given set of points is minimal. Hence, we have

$$S(x,L) = \hat{f}_i + \frac{x - x_i}{x_{i+1} - x_i}(\hat{f}_{i+1} - \hat{f}_i),\ x \in [x_i,\ x_{i+1}],\ i = 1, ..., N-1, \tag{2.6}$$

$$\hat{f}_i = (\tilde{f}_i^+ + \tilde{f}_i^-)/2, \tag{2.7}$$

$$\tilde{f}_i^+ = \max_{1\leq j\leq N}(\tilde{f}_j - L|x_j - x_i|), \tag{2.8}$$

$$\tilde{f}_i^- = -\max_{1\leq j\leq N}(-\tilde{f}_j - L|x_j - x_i|),\ i = 1, ..., N. \tag{2.9}$$

Quite often numerical *a priori* information about class F is given as a set of restrictions on a certain functional $\phi(f)$. For classes $C^1_{1,L,N}$ and $C^1_{1,L,N,\varepsilon}$ such a functional $\phi(f)$ is a uniform norm of a derivative. In the numerical integration algorithm constructed by means of the residual minimization method the integrand $f(x)$ is approximated by a function, which is the solution of the following problem:

$$\min_{f\in F} \phi(f) \tag{2.10}$$

The solution of (2.10) is a linear spline $S(x,M)$ which is determined by formulae (2.6)-(2.9) when we replace the constant L by constant M, where

$$M = \max_{1\leq i\leq N}\left(0, \max_{j>i}\frac{|\tilde{f}_j - \tilde{f}_i| - \varepsilon_j - \varepsilon_i}{x_j - x_i}\right). \tag{2.11}$$

Then integration formulae for integral (2.1) constructed by means of quasi-solutions and residual minimization methods have the following forms

$$\bar{R}(\varphi, S) = \int_a^b S(x,L)\varphi(x)dx, \tag{2.12}$$

$$\bar{\bar{R}}(\varphi, S) = \int_a^b S(x,M)\varphi(x)dx. \tag{2.13}$$

Formulas (2.12) and (2.13) are optimal-by-order with a constant not exceeding two (even compared to the case of accurately determined L and ε; see also [3]). Of course, the application of the proposed algorithms is most efficient under the assumption of inaccurate *a priori* information about the problem. In such situations the quasi-solutions or residual minimization techniques, described above, provide an effective tool for the accuracy optimization of numerical integration formulae.

3. Error Estimation of the Optimal Numerical Integration Algorithms in the Classes $C^1_{1,L,N}$ and $C^1_{1,L,N,\varepsilon}$

In this section we consider some special cases of (2.1), namely the integrals

$$I^1_2(\omega, f) = \int_a^b f(x)\sin(\omega x)dx, \tag{3.1}$$

$$I^1_3(\omega, f) = \int_a^b f(x)\cos(\omega x)dx, \tag{3.2}$$

where ω is some real number, $|\omega| \geq 2\pi/(b-a)$. For integrals (3.1) and (3.2) integration formulae (2.12) and (2.13) take the form

$$\begin{aligned} R_2(\omega, S) &= \sum_{i=1}^{N-1}\left(\frac{\hat{f}'_i}{\omega^2}(\sin(\omega x_{i+1}) - \sin(\omega x_i))\right) - \\ &\quad \frac{1}{\omega}(\hat{f}_N\cos(\omega x_N) - \hat{f}_1\cos(\omega x_1)), \end{aligned} \tag{3.3}$$

$$\begin{aligned} R_3(\omega, S) &= \sum_{i=1}^{N-1}\left(\frac{\hat{f}'_i}{\omega^2}(\cos(\omega x_{i+1}) - \cos(\omega x_i))\right) + \\ &\quad \frac{1}{\omega}(\hat{f}_N\sin(\omega x_N) - \hat{f}_1\sin(\omega x_1)), \end{aligned} \tag{3.4}$$

where $\hat{f}'_i = \frac{\hat{f}_{i+1} - \hat{f}_i}{x_{i+1} - x_i}$ ($\hat{f}_i, i = 1, ..., N$ is determined by formula (2.7)).

Optimal-by-accuracy numerical integration algorithms for integrals (3.1) and (3.2) in classes $C^1_{1,L,N}$ and $C^1_{1,L,N,\varepsilon}$ are valid under certain conditions, which are imposed on the mutual arrangement of points $x_i, i = 1, ..., N$ and zeros of $\sin(\omega x)$ (or $\cos(\omega x)$) on $[a, b]$. Such an arrangement is required in order to obtain an error estimation for the constructed algorithms.

In order to obtain error estimations for integration formulae (3.3) and (3.4) in class $C^1_{1,L,N}$, we note that the solutions of (2.5) and (2.10) coincide for this class, and the linear spline $S(x, L)$ connects points $(x_i, f_i), i = 1, ..., N$:

$$\begin{aligned} S(x,L) &= f_i + \frac{x - x_i}{x_{i+1} - x_i}(f_{i+1} - f_i), \\ & \quad x \in [x_i, x_{i+1}],\ i = 1, ..., N-1. \end{aligned} \tag{3.5}$$

In fact, it is easily verified that $\hat{f}_i = f_i$ if $f(x) \in C^1_{1,L,N}$. Taking into account that $\tilde{f}_i = f_i,\ i = 1, ..., N$, from formulae (2.8), (2.9) we have

$$f_i^+ = \max_{1 \le j \le N}(f_j - L|x_j - x_i|)),\ i = 1, ..., N. \tag{3.6}$$

$$f_i^- = - \max_{1 \le j \le N}(-f_j - L|x_j - x_i|)),\ i = 1, ..., N. \tag{3.7}$$

For $f(x) \in C_{1,L,N}$ we have the chain of inequalities $|f_j - f_i| \le L|x_j - x_i|$, $-L|x_j - x_i| \le f_j - f_i \le L|x_j - x_i|$ and $-f_j - L|x_j - x_i| \le -f_i \le -f_j + L|x_j - x_i|$, $i = 1, ..., N$, $j = 1, ..., N$. Since $f_i \ge f_j - L|x_j - x_i|$ and $-f_i \ge -f_j - L|x_j - x_i|$, this implies that $\max_{1 \le j \le N}(f_j - L|x_j - x_i|) = f_i$ and $\max_{1 \le j \le N}(-f_j - L|x_j - x_i|) = -f_i$, and formula (3.5) follows.

We assume that $N \ge |\omega|$ and that $[\frac{|\omega|}{\pi}(b - a)] + 1$ *zeros of function* $\sin(\omega x)$ *(or* $\cos(\omega x)$*) appear in the set of points* $x_i, i = 1, ..., N$ *[condition C1]*. This leads us to the following result.

Theorem 1 *Let* $f(x) \in C^1_{1,L,N}$, $N \ge |\omega|$, *with condition C1 satisfied, and the values* f_i *determined on* $[a, b]$ *at the points* x_i, $i = 1, ..., N$, *where* $x_1 = a$, $x_{N+1} = b$. *Then integration formula (3.3) for the numerical evaluation of integral (3.1) is optimal-by-order with the error estimation*

$$\begin{aligned} & v(C^1_{1,L,N}, R_2(\omega, S), f) \le \\ & \quad \frac{L}{\omega}\left(\sum_{i=1}^{N-1}\left|\sin(\frac{\omega}{2}(x_{i+1} + x_i))\right|\left(\frac{4}{\omega}(\sin^2\frac{\omega \Delta x_i}{4} - \sin^2\frac{\omega|\Delta f_i|}{4L})\right) + \right. \\ & \quad \frac{2}{\omega}\left|\sin\frac{\omega \Delta x_n}{2}\cos(\omega(b - \frac{\Delta x_N}{2})) - \Delta x_N \cos(\omega b)\right| + \\ & \quad \left.\frac{2}{\omega}\left|\sum_{i=1}^{N-1} sign(\Delta f_i)\cos(\frac{\omega}{2}(x_{i+1} + x_i))(\sin\frac{\omega|\Delta f_i|}{2L} - \frac{|\Delta f_i|}{L\Delta x_i}\sin\frac{\omega \Delta x_i}{2})\right|\right), \end{aligned} \tag{3.8}$$

where $\Delta f_i = f_{i+1} - f_i$, $\Delta x_i = x_{i+1} - x_i$, $i = 1, ..., N$.

The case of strong oscillation is the most difficult for investigation because it has to be considered with regard to function $\sin(\omega x)$ (or $\cos(\omega x)$) oscillations on each elementary segment $[x_i, x_{i+1}]$, $i = 1, ..., N$. Let $f(x) \in C^1_{1,L,N}$ and N *points* x_i *appear in the set of zeros of function* $\sin(\omega x)$ *(or* $\cos(\omega x)$*)* [*condition C2*]. We assume that on segments $[x_i, x_{i+1}]$ there are k_i oscillations of function $\sin(\omega x)$ (or $\cos(\omega x)$), $i = 1, ..., N$. Then we have the following result.

Theorem 2 *Let* $f(x) \in C^1_{1,L,N}$, $N < |\omega|$, *with condition C2 satisfied and the values* f_i *determined on* $[a,b]$ *at the points* x_i, $i = 1, ..., N$, *where* $x_1 = a$, $x_{N+1} = b$. *Then integration formula (3.3) for numerical evaluation of integral (3.1) is optimal-by-order with the error estimation*

$$\begin{aligned}\bar{v}(C^1_{1,L,N}, R_2(\omega,S), f) \leq \frac{2L}{\omega^2}\left(\left[\frac{|\omega|}{\pi}\right] - \sum_{i=1}^{N-1}\left(\left[\frac{|\omega|}{\pi}x_{i+1}\right] - \left[\frac{|\omega|}{\pi}x_i\right]\right) \times\right. \\ \left.\sin^2\frac{\pi|\Delta f_i|}{4L\Delta x_i}\right) + \frac{L}{|\omega|}\left|\frac{2}{\omega}\sin(\frac{\omega}{2}(b - \left[\frac{|\omega|}{\pi}\right]\frac{\pi}{|\omega|}))\cos(\frac{\omega}{2}(2 + \left[\frac{|\omega|}{\pi}\right] \times\right. \\ \left.\frac{\pi}{|\omega|} - b)) - (1 - \left[\frac{|\omega|}{\pi}\right]\frac{\pi}{|\omega|})\cos\omega b\right| + \frac{2L}{\omega^2}\left|\sum_{i=1}^{N-1} sign(\Delta f_i) \times\right. \\ \left.\sum_{k=0}^{k_i-1}\cos(\frac{\omega}{2}(x_{i,k+1} + x_{i,k}))(\sin(\frac{\omega}{2}\frac{\pi|\Delta f_i|}{L\Delta x_i|\omega|}) - \frac{|\Delta f_i|}{L\Delta x_i}(\sin\frac{\omega\pi}{2|\omega|}))\right|, \qquad (3.9)\end{aligned}$$

where $\Delta f_i = f_{i+1} - f_i$, $\Delta x_i = x_{i+1} - x_i$, $k_i = [|\omega|x_{i+1}/\pi] - [|\omega|x_i/\pi]$, $x_{i,k} = \frac{\pi}{2|\omega|}(2[\frac{|\omega|}{\pi}x_i + \frac{1}{2}] + 2k + 1)$ *are zeros of function* $\sin(\omega x)$ *on* $[x_i, x_{i+1}]$, $x_{i,0} = x_i$, $x_{i,k_i} = x_{i+1}$, $k = 0, ..., k_i$, $i = 1, ..., N$.

Remark 1 *The error estimations provided by (3.8) and (3.9) do not exceed by more than two times the error estimations provided by the optimal-by-accuracy algorithms (see also [3]). The results analogous to Theorems 1 and 2 are also valid for integration formula (3.4) applied to integral (3.2).*

In order to construct integration formulae for numerical evaluation of integrals (3.1) and (3.2) in class $C^1_{1,L,N,\varepsilon}$ by the quasi-solutions method it is assumed that the constant L and some estimate of the accuracy for approximate values of function $f(x)$ at N points of an arbitrary grid are given. Due to the application of the linear spline $S(x, L)$ as an integrand approximation, initial data is smoothed, and ε is determined more precisely. In the case when the constant L is not known, but the accuracy of the given values of function $f(x)$ is known, it is useful to apply integration formulae obtained by the residual minimization method, where the value of L is not involved.

It is not difficult to see that the integration formula for numerical evaluation of integral (3.1) in class $C^1_{1,L,N,\varepsilon}$ constructed by the quasi-solutions method, has the form of (3.3). The same conclusion can be made for the integration formula constructed by the residual minimization method. The important difference is that in the quasi-solutions method values $\hat{f}_i$, $i = 1, ..., N$ are calculated by formulae (2.7), (2.8) and

(2.9) using constant L. In the residual minimization method, in order to define $\hat{f}_i, i = 1, ..., N$, constant M is determined by formula (2.11), and the substitution of constant M for constant L is made in formulae (2.8) and (2.9). We will discuss this case in detail elsewhere.

We emphasize that in solving real problems we usually do not know the accurate values of L and ε, and only some of their estimates are available. In this situation the real error of the numerical evaluation of integrals (3.1) and (3.2) with formulae (3.3) and (3.4) could be considerably less than the error estimates obtained using inaccurate *a priori* information.

4. Conclusions

Problems of numerical integration with fast oscillatory integrands constitute an intrinsic part of a wide class of challenging applied problems in digital signal processing. In this paper we proposed effective algorithms that guarantee optimal-by-order solution of the problem on computing integrals with fast oscillatory functions in classes $C^1_{1,L,N}$ and $C^1_{1,L,N,\varepsilon}$. The proposed algorithms provide an effective tool for the solution of such numerical integration problems, especially in the case when *a priori* information about the problem is given inaccurately. Such a consideration puts us closer to situations which are typical in the solution of practical problems.

The application of the proposed algorithms in the case when *a priori* information is assumed to be given precisely gives the results analogous to those obtained in this paper. In such a case the error of our results will not exceed optimal-by-accuracy results by more than two times.

Another advantage of the algorithms proposed in this paper is their simple computational implementation, which is an important feature for the design of software application packages. The importance of the design and implementation of optimal algorithms (and algorithms close to optimal with respect to their computational characteristics) is comparable with the importance of the design and implementation of new computer hardware. The algorithms and methods proposed in this paper offer a flexible choice of options depending on practical aspects of the problems to be solved and permit the application of these techniques in a wide variety of circumstances.

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