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In: Proceedings of the VIth World Congress on Computational Mechanics in Conjunction with APCOM'04, Eds.: Yao, ZH, Yuan, MW, Zhong, WX, ISBN 7-89494-512-9 (CD-ROM), 10 pages, Tsinghua University Press and Springer-Verlag, 2004.

COMPUTAIONAL MECHANICS
WCCM VI in conjunction with APCOM'04,*Sept.5-10, 2004, Beijing, China*
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A Unified Numerical Treatment of Modified Compound KdV-Burgers' Models

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Abstract: In this paper we develop a fully implicit numerical method based on the differential-algebraic approach to solve the general form of the modified compound Korteweg-de Vries-Burgers' equation and its possible variants. The proposed numerical methodology is validated numerically on a series of examples, including Burgers' type models for Reynolds' numbers up to 10 000 and Riemann initial value problems. The effects of time relaxation have been also included into the analysis by considering hyperbolic modifications of Burgers' model, and a combined contribution of dissipation and dispersion effects in the KdV-Burgers' equation has been analysed numerically. The developed methodology is a simple, robust, and efficient tool for solving numerically a wide range of engineering problems pertinent to KdVB models.

Key words: Korteweg-de Vries-Burgers' models, differential-algebraic solvers, backward differentiation formula.

1 Introduction

The analysis of Korteweg-de Vries (KdV) equation has had an enormous impact on the development of modern non-linear science and engineering. The equation has many fundamental applications in fluid mechanics, in particular in the context of modelling non-linear phenomena. The basic research into the KdV equation has led to a deeper insight into the analysis of shock and bifurcation phenomena, solitons, deterministic chaos, and hypercomplex systems ([3] and references therein).

Another equation that is fundamental to the analysis of nonlinear phenomena in science and engineering is Burgers' equation. It is a model equation in investigating many nonlinear phenomena, including convection-diffusion processes and fluid dynamics models. The equation is one of a few nonlinear Partial Differential Equations (PDE) which have been solved analytically for various initial conditions ([6] and references therein), and therefore it serves as a testing ground for other, more complicated models.

Apparently, it is possible to study both these equations within the same framework. In fact, it is well known [9] that these two fundamental models can be combined together to produce a new nonlinear PDE known as the Korteweg-de Vries-Burgers (KdVB) equation. This new model, that involves damping, dispersion, and convective terms, has many important applications in engineering sciences. In particular, it describes the electromagnetic waves in transmission lines [6], the waves propagating through liquid filled elastic tubes , and shallow water waves in a viscous fluid [9]. The steady state solutions of the KdVB model have been shown to model weak plasma shocks propagating perpendicularly to a magnetic field. From the qualitative analysis of steady-state solutions to the KdVB model [9], it is known that when diffusion/dissipative effects are dominating, the solution may exhibit monotonic shocks, while in the case of prevailing dispersion the shocks may become oscillatory. Although a number of procedures for the qualitative analysis of the KdVB model have been developed, in particular for steady-state situations, many engineering applications require the quantitative analysis of solutions to time-dependent KdVB models.

The goal of our present work is to develop and validate a simple, robust, and effective methodology for the solution of the time-dependent KdVB equation. Such a methodology would allow us to treat a wide range of problems of practical interest in engineering sciences pertinent to KdVB models. In particular, we shall focus on nonlinear convection-diffusion problems, the KdV model, relaxation effects in hyperbolic modifications of Burgers' model, and stabilising effects of a combined contribution of dissipation and dispersion in the compound KdV equation. Several methods have been proposed in the literature to treat numerically special cases of the KdVB model, and for some such cases even analytical solutions are known. In particular, in [18], a solitary wave solution to the compound KdVB equation was reported, and in [14], the exact solution to the compound KdVB equation was obtained for some special cases by using a homogeneous balance method. We use some such known results in the current paper as a testing ground for validating our procedure. However, in the general case the KdVB model cannot be solved analytically, and the development of efficient numerical procedures is required to overcome difficulties caused by interactions between convection, dispersion and diffusion, and the higher order nonlinear terms in the compound KdVB models. Prior work in this direction includes [1] (see also references therein), where Burgers' and KdVB models were treated numerically by means of a cubic B-spline finite element scheme based on Galerkin's method with quadratic B-spline interpolation functions over finite elements. In [17] a methodology for the numerical solution of the KdVB equation has been developed based on a B-spline finite element method where Bubnov-Galerkin's method was applied with weight and shape functions taken as cubic B-spline functions over finite elements. The methodology developed in this paper is different from those reported before and provides a simple and efficient way of solving numerically a wide class of problems described by KdVB-type models. The paper is organised as follows. In Section 2, we give the model formulation and identify four special cases of that model on which we focus our attention in the subsequent sections of the paper. In Section 3 we develop a fully implicit numerical method based on the differential algebraic approach for the solution of the general form of the KdVB equation. We test the developed procedure in Section 4, where we consider three examples and solve numerically Burgers' type models with very large Reynolds numbers as well as some representative Riemann initial value problems. In Section 5 we study the effect of time relaxation in hyperbolic modifications of Burgers' models, and in Section 6 we analyse numerically a combined contribution of dispersion and dissipation effects in KdVB models. Concluding remarks are given in Section 7.

2 Compound KdVB Model and Its Special Cases

The starting point of our discussion in this paper is the following modified compound KdVB equation:

$$\frac{\partial u}{\partial t} + \tau\frac{\partial^2 u}{\partial t^2} + ap(u)\frac{\partial u}{\partial x} + b\frac{\partial^2 u}{\partial x^2} + c\frac{\partial^3 u}{\partial x^3} = f(x,t), \tag{1}$$

where τ, a, and c are all positive constants, b is negative, $p(u)$ is a function of u (with some order of nonlinearity), and $f(x,t)$ is a function of x and t, which acts here as a driving force.

Model (1) includes effects of time relaxation, nonlinear convection, dissipation, and dispersion. The influence of these effects on the solution of the model is dependent on the associated coefficients and their relative values. Furthermore, the time relaxation term influences the transient characteristics of the solution (in time domain), and might in some cases make the solution unstable ([12, 11] and references therein); the convective term influences the wave propagation speed in the solution; and the dissipative term tends to suppress possible oscillations in the wave propagation process, while the dispersive term tends to generate oscillations, so that dissipation and dispersion are often should be considered as competitive effects ([9, 17]).

Since it is natural to expect that the solution will have different characteristics with different coefficient values, we consider typical reduction scenarios for model (1). In particular, we focus our attention on four models, upon which our further discussion will be based.

Case 1. If coefficient c is far smaller than other coefficients, the associated dispersion effect is negligible. If further time relaxation effects can be neglected, model (1) is reduced to the classical Burgers equation, a model example for the description of the convection-diffusion processes:

$$\frac{\partial u}{\partial t} + ap(u)\frac{\partial u}{\partial x} + b\frac{\partial^2 u}{\partial x^2} = f(x,t). \tag{2}$$

Case 2. If time relaxation effects are neglected, and coefficient b is far smaller than other coefficients, indicating that dissipation effects are negligible, then model (1) is reduced to the following compound KdV equation:

$$\frac{\partial u}{\partial t} + ap(u)\frac{\partial u}{\partial x} + c\frac{\partial^3 u}{\partial x^3} = f(x,t). \tag{3}$$

Case 3. In order to simulate problems with memory effects [11] and references therein), we keep the time relaxation term, include convection and dissipation effects, while assuming dispersion negligible. This leads to the following model:

$$\frac{\partial u}{\partial t} + \tau\frac{\partial^2 u}{\partial t^2} + ap(u)\frac{\partial u}{\partial x} + b\frac{\partial^2 u}{\partial x^2} = f(x,t). \tag{4}$$

Case 4. If neither dissipation nor dispersion effects are negligible, by neglecting time relaxation effects (τ=0), we reduce model (1) to the compound KdV-Burgers' equation:

$$\frac{\partial u}{\partial t} + ap(u)\frac{\partial u}{\partial x} + b\frac{\partial^2 u}{\partial x^2} + c\frac{\partial^3 u}{\partial x^3} = f(x,t). \tag{5}$$

There are a number of other reduced models derived directly from the generic model (1) which are encountered in engineering applications. Due to a wide range of associated problems, it is important to develop a unified numerical procedure that would allow us to treat all the situations discussed above.

3 Algorithmic Aspects of the Unified Numerical Procedure

In order to treat numerically all the models mentioned above in a unified manner, in what follows we develop a simple and efficient procedure based on the differential-algebraic approach combined with backward differentiation. As a first step, we introduce a new variable and re-write model (1) as a system of partial differential equations (PDEs):

$$\begin{aligned} &\frac{\partial u}{\partial t} = v, \\ &\tau\frac{\partial v}{\partial t} + v + ap(u)\frac{\partial u}{\partial x} + b\frac{\partial^2 u}{\partial x^2} + c\frac{\partial^3 u}{\partial x^3} = f(x,t). \end{aligned} \tag{6}$$

Then, the resulting system of PDEs is converted into a set of Ordinary Differential Equations (ODEs) by means of the method of lines (MOL). Finally, we apply a fully implicit method to solve the obtained ODE system.

Let us describe this procedure in some detail. Assume that the domain of interest is $x \in [x_a, x_b]$. We discretise model (6) and write the result as follows:

$$\begin{aligned} &\frac{du_i(t)}{dt} - v_i(t) = 0, \\ &\tau\frac{dv_i(t)}{dt} + v_i(t) + ap(u_i(t))D_x(u_i(t)) + bD_{xx}u_i(t) + cD_{xxx}(u_i(t)) = f(x_i,t), \end{aligned} \tag{7}$$

where $i = 1, 2, ..., m$, m is the number of discretization points in $x \in [x_a, x_b]$, $u_i(t)$ (respectively, $v_i(t)$) denotes the value of function u (v, respectively) in spatial grid point x_i at time t. Operators D_x, D_{xx} and D_{xxx} are discretization (difference) operators for the first, second and third order derivatives, respectively. To achieve a high spatial resolution, we apply here the fourth order central difference scheme (the forth order approximation has been also used in approximating third spatial derivatives). System (7) is a system of differential-algebraic equations that can be cast in the following generic form:

$$\mathbf{A}\frac{d\mathbf{U}}{dt} + \mathbf{H}\left(t, \mathbf{X}, \mathbf{U}\right) = \mathbf{0}, \tag{8}$$

where matrix $A = \mathrm{diag}(a_1, a_2, ..., a_{2m})$ has non-zero entries for differential equations and zero entries for algebraic equations (boundary conditions lead to algebraic equations), $\mathbf{U}$ is the vector of unknowns which has dimensionality $2 \times m$, and vector-function $\mathbf{H}$ is defined by collecting all approximations of spatial terms in (7). The procedure described above leads to stiff systems [13] which require efficient time-integration algorithms for the numerical solution. We apply the second order backward differentiation formula for the integration of system (8) [7]

$$\mathbf{A}\left(\frac{3}{2}\mathbf{U}^n - 2\mathbf{U}^{n-1} + \frac{1}{2}\mathbf{U}^{n-2}\right) + \Delta t\mathbf{H}\left(t_n, \mathbf{X}, \mathbf{U}^n\right) = 0, \tag{9}$$

where n denotes the current time layer. Such a fully implicit methodology leads to the nonlinear system of algebraic equations (9) which is then solved in an iterative manner for each time layer. In particular, we use the Newton iterations with the Jacobian matrix $\mathbf{J}$ calculated by the following formula:

$$\mathbf{J} = 1.5\mathbf{A} + \Delta t \times \frac{\partial \mathbf{H}}{\partial \mathbf{U}}, \tag{10}$$

where the size of the resultant Jacobian matrix $\mathbf{J}$ is $2m \times 2m$.

The described procedure has been supplemented by a filtering operation to deal effectively with solution discontinuities and steep gradients that may exist in the solution even for relatively simple problems such as Riemann initial value problems for Burgers' models. To overcome non-physical oscillations in the solution resulted from the Gibbs phenomenon [16, 2], we apply the following low-pass filter in order to remove high frequency oscillations in the solution at each time level:

$$u_i^*(t) = 0.25u_{i-1}(t) + 0.5u_i(t) + 0.25u_{i+1}(t), \tag{11}$$

where $u_i^*(t)$ is the numerical result after applying the described filtering operation. This procedure does not apply to the boundary points, and when no spurious oscillations are expected, our code allows us to switch off the filtering operation.

4 Computational Experiments

In order to validate the numerical approach proposed in Section 3 and demonstrate its effectiveness, we consider two representative problems based on Burgers' equation and the KdV model. In the discussion that follows in this section we assume that $p(u) = u$ and $f(x, t) = 0$.

Example 1. Our first example concerns with the solution of the Riemann initial value problem for Burgers' model which corresponds to infinite Reynolds number $R_e = \inf$. In this case the initial profile of the solution is given by the following discontinuous function :

$$u\left(x, 0\right) = \left\{ \begin{array}{ll} 1, & 0 \leq x \leq 0.2, \\ 0, & 0.2 < x \leq 1. \end{array} \right. \tag{12}$$

The exact solution to system (2) and (12) is a shock front moving at constant speed 0.5, which represents a shock wave of compressive nature. The numerical approach developed in Section 3

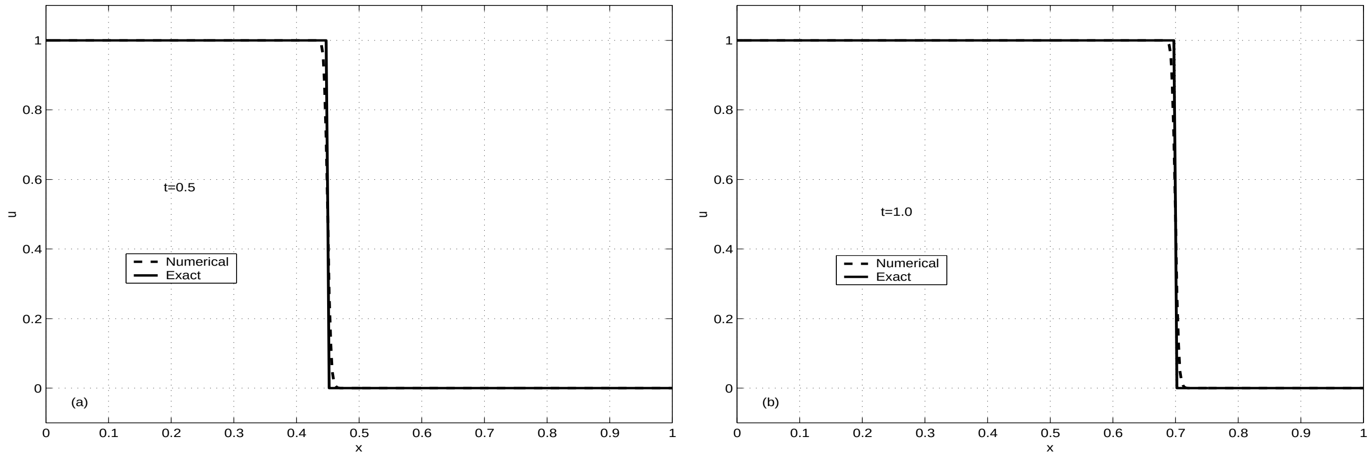

Figure 1: Solution to Burgers' equation: Riemann initial value problem

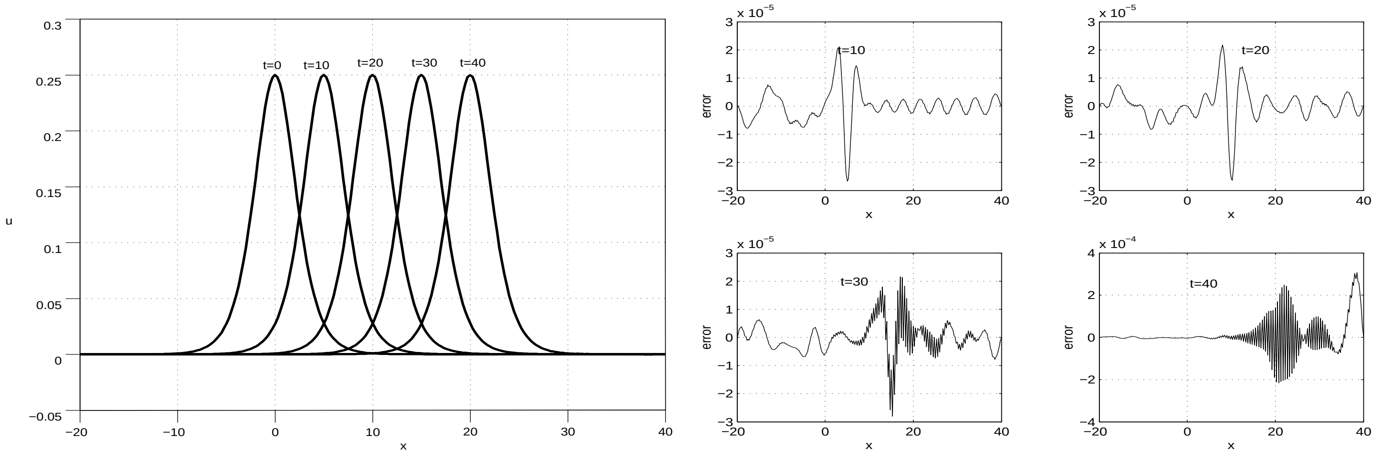

Figure 2: Solution to the KdV equation: (a) wave propagation along the x direction; (b) absolute error between numerical and analytical results.

reproduces this solution very well. To demonstrate this, we present here the results of our computations for time moments $t = 0.5$ and $t = 1.0$, and compare them with their exact counterparts in Fig.1(a) (left) and Fig.1(b) (right), respectively.

Example 2. Our second example in this section concerns the KdV equation, which in the classification of Section 2 corresponds to Case 2. This model has been studied intensively by means of both analytical and numerical tools ([3] and references therein):

$$\frac{\partial u}{\partial t} + 6u\frac{\partial u}{\partial x} + \frac{\partial^3 u}{\partial x^3} = 0. \tag{13}$$

The exact solution to model (13) can be obtained by the inverse scattering method [3]. The simplest one-soliton solution is the sech^2 solution, which is employed here to assess the accuracy of our numerical results:

$$u\,(x,t) = \frac{1}{2}c_w\ \mathrm{sech}^2\left(\frac{1}{2}\sqrt{c_w}\,(x - c_w t + x_0)\right), \tag{14}$$

where x_0 is the initial position of the soliton, and c_w is a characteristic related to the amplitude of the soliton. In particular, the factor $\sqrt{c_w}$ under the secant function implies that the propagation speed of the wave depends on the amplitude, which is one of the important features of solitons.

We perform numerical simulations with the initial condition deduced from (14) at $t = 0$, setting x_0 to 0, and choosing c_w as 0.5. The interval of interest has been chosen as $x \in [-20, 40]$. We apply homogeneous Dirichlet boundary conditions, $u = 0$, which also follow formally from (14) taken into account the assumptions made above. The spatial and temporal step-sizes used here are 0.25 and

0.05, respectively. Since no spurious oscillations are expected in this situation, the filtering procedure (11) should be switched off, otherwise the the amplitude of the soliton will be whittled. The developed code allows us to capture correctly the wave propagation in long time range intervals. In Fig.2(a) (left) we present the evolution of a single soliton obtained numerically with the procedure described in Section 3. In Fig.2(b) (right) we present the dynamics of error $u_{numerical} - u_{analytical}$, demonstrating that even for large time intervals the maximum absolute error does not exceed 0.4 percent, and that the numerical solution agrees very well with the exact solution.

5 Hyperbolic modifications of Burgers' equation

Models based on the classical Burgers equation such as (2) serve as a good testing ground for studying Navier-Stokes turbulence, shock waves, and convection-diffusion processes. However, such models lose their applicability in extended media where relaxation time effects become important ([12, 11] and references therein). The classical Fourier law connecting the heat flux and the temperature gradient leads to infinite speed of propagating waves, and from a physical point of view should be modified for the description of transient processes to account for memory effects ([10, 12] and reference therein). Such memory effects are responsible for the influence of the pre-history of the process, and are associated with the time of essential influence of past states of the system on its current state. One of the most common way to account for such memory effects is to introduce a hyperbolic correction into the model, a new term containing the second temporal derivative, acting as a time relaxation factor [10, 11, 12]. After such a modification, model (1) can be reduced to model (4) discussed in Section 2. Our computational experiments in this section concerns a variant of that model written in the following form:

$$\frac{\partial u}{\partial t} + \tau \frac{\partial^2 u}{\partial t^2} + u \frac{\partial u}{\partial x} - \frac{1}{R_e} \frac{\partial^2 u}{\partial x^2} = 0. \tag{15}$$

The basic feature of this modified Burgers' equation with hyperbolic term is that its solution can be qualitatively different from the solution of the classical Burgers equation (2). In order to show this, first we introduce the Mach number as $M = |u|\sqrt{\tau R_e}$. The analysis of the linearized counterpart of model (15) shows that when $M < 1$ the model does not have growing solutions. However, as soon as $M > 1$, there is a branch of harmonics with exponential growth [12], which indicates the possibility of blow-up solutions in systems described by model (15). Such qualitatively different behaviours are confirmed here numerically by using the procedure developed in Section 3. Computational experiments, reported below, have been carried out on a mesh containing 101 nodes in interval $[0, 1]$, and the time step discretisation have been chosen as 0.005.

In order to demonstrate the effects described above, we analyse numerically model (15) with single-soliton initial conditions chosen as $u(x, 0) = \exp[-100 * (x - 0.5)^2]$ and $\partial u(x, 0)/\partial t = 0$. Homogeneous Dirichlet boundary conditions, $u(x, t) = 0$, are assumed at both ends of the spatial interval of interest. We present numerical results for three different Mach numbers in Fig.3(a), Fig.3(b), and Fig.3(c). In Fig.3(a) (top left), the Mach number is set at $M = 1 \times 10^{-2}$ by choosing $\tau = 3 \times 10^{-5}$, $a = 1$ and $R_e = 3.333$. The obtained numerical solution is practically indistinguishable from that for the classical Burgers equation, without hyperbolic modification. This should not come as a surprise since relaxation effects are negligible in this case due to our choice of parameters. The situation changes in Fig.3(b) (top right), where the parameters have been chosen as $\tau = 0.3$, $a = 1$, and $R_e = 3.333$. According to the stability analysis carried out for the linearized model [12], $M = 1$ is a critical value. As it is demonstrated by Fig.3(b), in the nonlinear case the situation becomes more complicated.

Although we do not observe growing solutions (so in this sense, qualitatively the solution remains similar to that of $M < 1$), the initial-profile soliton is branched and dissipated in an oscillating manner. The transient process presented in Fig.3(b) captures quite well dissipative oscillations of two branches of the original wave. Such a behaviour is due to the inclusion of memory effects, represented by the second order time derivative, which make the system behave like a second order system in

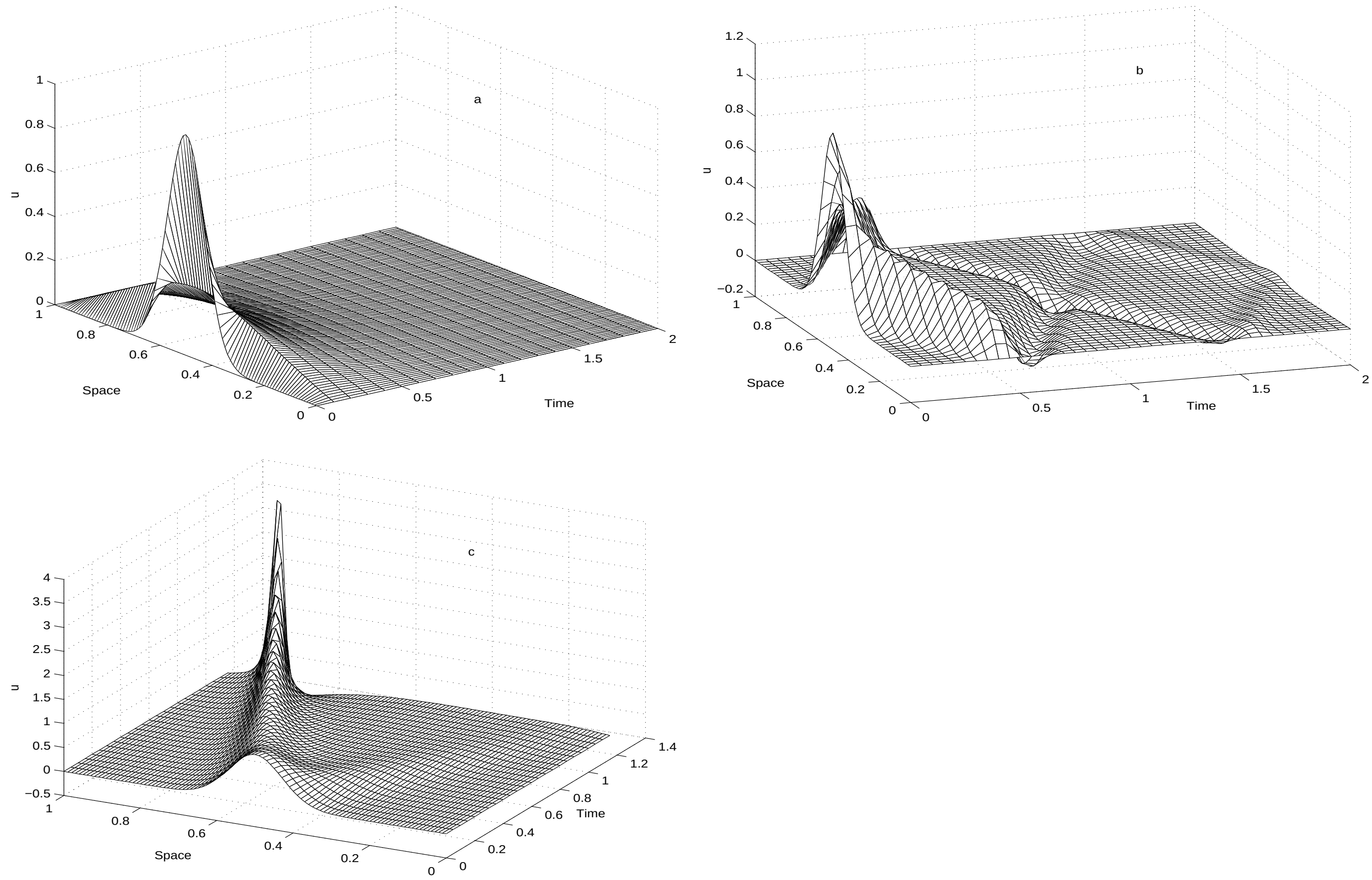

Figure 3: Solution to Burgers' equation with hyperbolic modification: (a) $M = 1 \times 10^{-2}$; (b) $M = 1$; (c)$M = \sqrt{10}$.

the time domain. Finally, in Fig.3(c) (bottom), we present the result of computation for Mach number $M = \sqrt{10}$, set by choosing $\tau = 0.3$, $a = 1$, and $R_e = 33.33$. The solution is qualitatively different from the previous cases. The profile of the initial soliton becomes self-accelerating and its amplitude increases exponentially. The hydrodynamic interpretation of such blow-up solutions and their relationship to turbulence can be found in the existing literature (e.g., [12] and references therein).

6 Competing Effects of Dissipation and Dispersion

In some cases, dissipation and dispersion effects can be partially balanced in KdVB models. Let us consider a reduction of model (1) to form (5) with no driving force:

$$\frac{\partial u}{\partial t} + au\frac{\partial u}{\partial x} + b\frac{\partial^2 u}{\partial x^2} + c\frac{\partial^3 u}{\partial x^3} = 0. \tag{16}$$

Let us further supplement (16) by the following initial conditions:

$$u(x, 0) = 1 - \tanh(x) \quad , \quad x \in [-20, 20], \tag{17}$$

and assume homogeneous Dirichlet boundary conditions as an example. In this case, there is a step-like shock front in the initial distribution, before and after the front. The profile of the initial distribution has been chosen in such a way that any possible oscillations would be easily observable. Moreover, in the absence of dissipation and dispersion, the initial shock wave would simply propagate along the positive x direction.

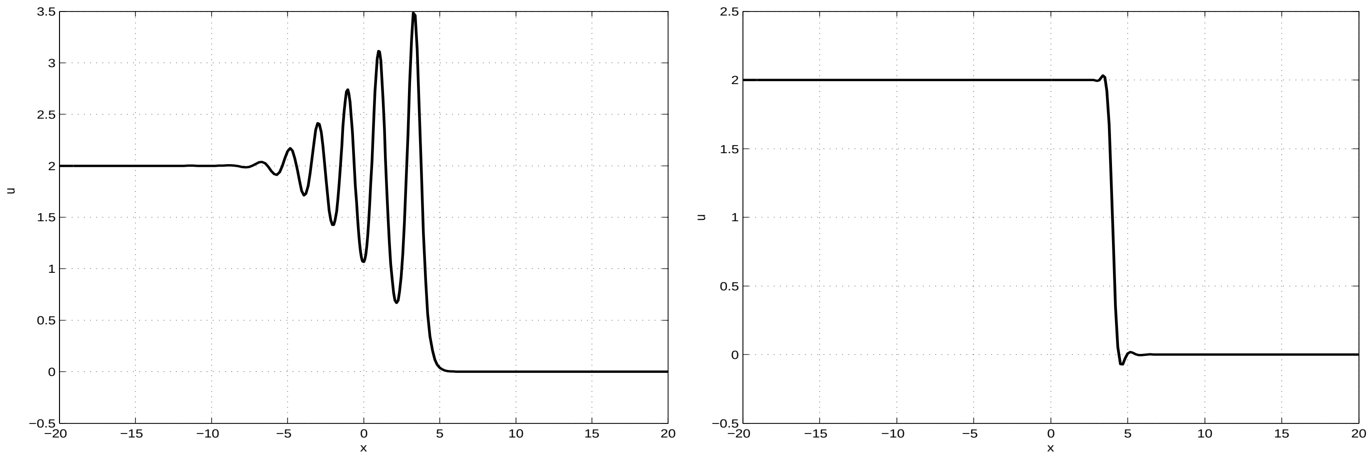

Figure 4: Solution to the KdVB equation: competition between dissipation and dispersion, $a = 1$, $t = 4$; (a) $b = -0.0001$, $c = 0.1$; (b) $b = -0.1$, $c = 0.0001$.

The above problem has been solved numerically on a spatial grid consisting of 201 nodes with the time step-size equal to 0.005 on the time interval $t \in [0, 4]$. The discussion that follows concerns the final moment of time, $t = 4$. The spatial grid in the case considered in this section is non-uniform, so that the spatial step-size in the center of the interval is smaller compared to that near the boundaries in order to capture the wave shock in the center area.

It is expected [9] that in the case where dispersion effects are prevailing in the steady solution, the shock wave has an oscillatory behaviour. On the other hand, in the case where dissipation effects are prevailing the shock wave exhibits a monotonic behaviour. We verify this claim by the following two simulations. Firstly, we consider the dissipation dominant case by setting coefficients $b = -0.0001$ and $c = 0.1$. In all computational experiments reported below we assume that the wave front in the initial profile propagates in the x direction with speed $v = 1$. This is achieved by setting the convection coefficient, a, to 1. The result of simulation is presented in Fig.4(a) (left). It demonstrates clearly the existence of a train of waves (six wave peaks). In our specific case, the leading wave peak is the one near the initial front with a magnitude close to 3.5. Secondly, we consider the dissipation dominant case by assuming the following values of the coefficients: $a = 1$, $b = -0.1$, and $c = 0.0001$. We present the result of computation (for $t = 4$) in Fig.4(b) (right). In this case, we clearly observe that the wave front propagates along the positive x direction with very small oscillations near the front due to the fact that dispersion effects are negligible in this case.

Since the KdVB equation is a combination of Burgers' equation and the KdV equation, it is natural to expect that the competition between dissipation and dispersion effects may play a decisive role in the solution behaviour. We analyse the situation numerically with the following set of experiments. In the first two experiments, we fix the value of b at -0.1 and increase c from 0.01 to 0.1. The results of our computations are presented in Fig.5(a) (top left), and Fig.5(b) (top right), respectively. As it is seen, there is no any substantial differences in the behaviour of the solution given in Fig.4(b), Fig.5(a). Note also that all of them have small oscillations near the front which indicates the "presence" of Burgers' equation in system (16). Although the dispersion coefficient has been increased to $c = 0.01$, dissipation effects are still dominant in this case. A qualitatively different situation is observed in Fig.5(b) where we see the existence of a train of wave peaks in the solution. However, the number and the amplitudes of those wave peaks are smaller compared to their respective values in Fig.4(a), which indicates that the dispersion process becomes prevailing in this situation despite the fact that dissipation effect contributions are still essential.

Our concluding computational experiments concern the situation where the dispersion coefficient fixed at $c = 0.1$. The dissipation effects, represented by the diffusion coefficient b, are controlled by changing the value of b from -0.01 to -0.001. The results of computations are shown in Fig.5(c) (bottom left) and Fig.5(d) (bottom right), respectively. A key observation is that both of these

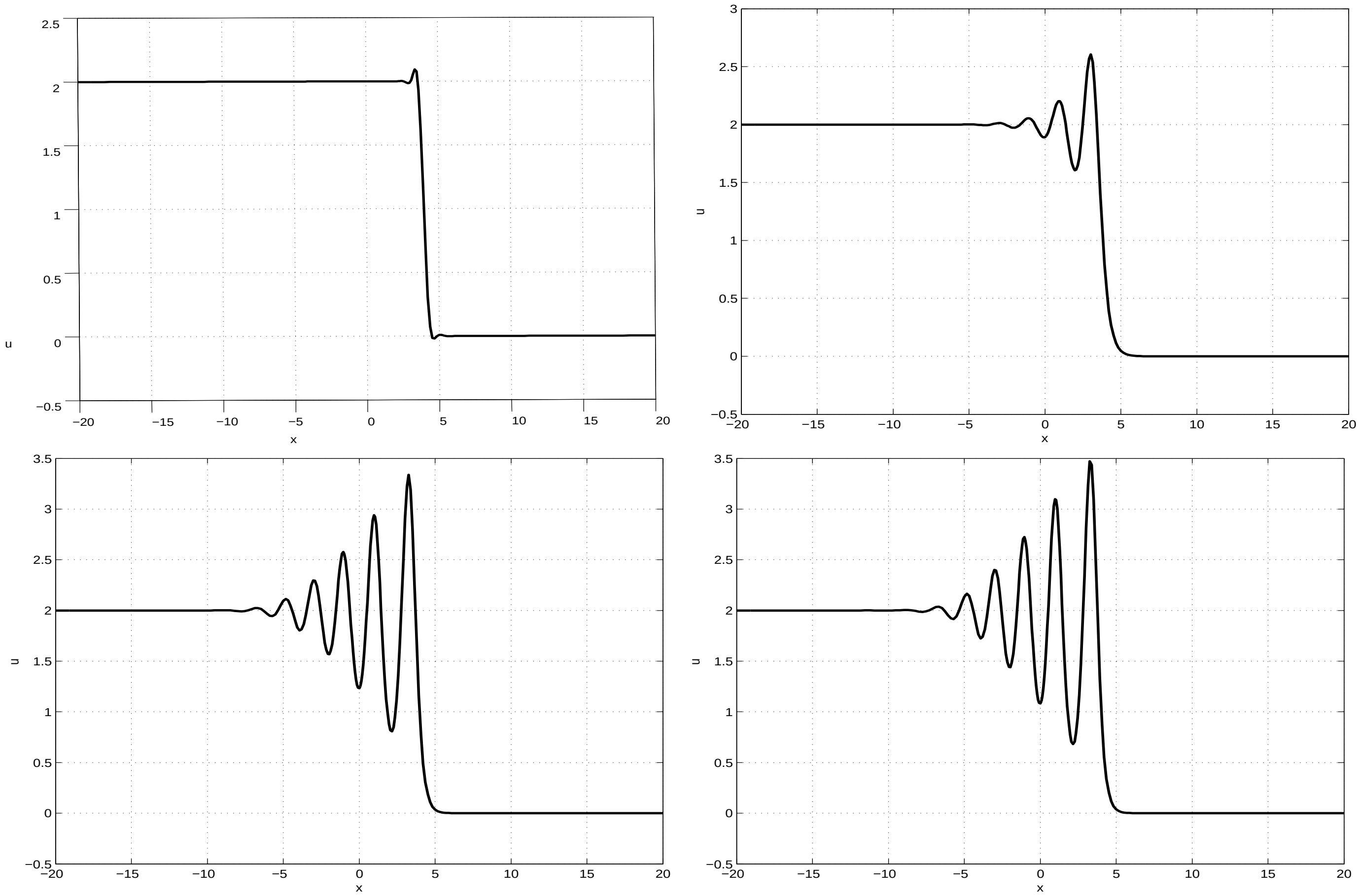

Figure 5: Solution to the KdVB equation: partial balance between dissipation and dispersion, $a = 1$, $t = 4$; (a) $b = -0.1$, $c = 0.001$; (b) $b = -0.1$, $c = 0.01$; (c)$b = -0.1$, $c = 0.1$; (e) $b = -0.01$, $c = 0.1$; (e) $b = -0.001$, $c = 0.1$

solutions have the same train of wave peaks as those presented in Fig.4(a). As expected, the amplitude of the leading wave decreases when dissipation effects, controlled by the diffusion coefficient b, are amplified.

The results of our experiments demonstrate a clearly pronounced pattern in the solution behaviour determined by a competition between the dissipation and the dispersion effects. Namely, if dispersion effects are prevailing in the system described by model (16), the solution exhibits a train of waves, behaving similarly to what is typically observed for the KdV equation. The amplitude of such waves decreases with the dissipation effects suppressed by decreasing coefficient b. On the other hand, if dissipation effects are prevailing in the system, the solution behaves as a monotonic shock wave, reflecting the "presence" of Burgers' equation. Small oscillations will accompany such a behaviour when the dispersion coefficient c is increased. An actual balance between dissipation and dispersion effects in the solution behaviour depends on both actual values of coefficients in model (16) and the initial profile of the solution.

7 Conclusion

In this paper, a simple and efficient methodology has been developed for the analysis of compound modified KdV-Burgers' models in a unified manner. A series of computational experiments have been carried out, demonstrating robustness of the proposed methodology in investigating a wide range of phenomena governed by KdVB' models. Such experiments have included Burgers' models for Reynolds' numbers up to $R_e = 10000$, as well as Riemann initial value problems. We have also analysed hyperbolic modifications of Burgers' models and showed that for large Mach numbers solution blow ups may be induced. Finally, we have analysed a combined contribution of dissipation and dispersion effects to qualitative changes in the solution behaviour for the general KdVB model.

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