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# A Preisach-type model based on differential operators for rate-dependent hysteretic dynamics

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ABSTRACT

In the current paper, the classical Preisach model is modified for modeling the rate-dependent hysteresis loops by employing a series of elementary differential Preisach operators. The elementary hysteresis loop is well characterized by a differential operator and the rate-dependence feature is inherently embedded in the operator. By introducing a specific density function, the overall output of the model can be formulated as a weighted sum of the dynamic responses of all the differential operators. Comparisons between the numerical results and the experimental counterparts are presented. A perfect fit of the model results with the experimental measurements is obtained with all input frequencies considered.


## 1. Introduction

Smart materials and structures have been intensively investigated due to their broad application potential. One of the remaining challenging problems in the application developments of smart materials and structures is the modeling, analysis and control of their hysteretic dynamics. Although the severity of hysteresis can be mitigated via feedback mechanisms and judicious design of electronic drivers, it is necessary to quantify the hysteresis in a manner conducive to controller design to achieve the full capabilities of these materials [8]. For this purpose, a suitable model which is capable of capturing the hysteretic dynamics and is easy to implement with practically reasonable computational resources is essential.

Among the models employed for the investigations on the hysteresis loops, the Preisach model has received considerable attention, because it encompasses the basic features of hysteresis in a conceptually simple and mathematically elegant way [4]. The original Preisach model characterizes the hysteresis loops through a superposition of hysteresis operators $\left[R_{\alpha,\beta}(u)\right](t)$, where $\alpha$ and $\beta$ $(\alpha \ge \beta)$ provide thresholds at which the operator switches between $+1$ and $-1$. The overall output of the model is constructed as the weighted sum of the outputs of all such operators

$$v(t) = \iint \omega(\alpha, \beta)\left[R_{\alpha,\beta}(u)\right](t)\,d\alpha\, d\beta, \tag{1}$$

where $\omega(\alpha, \beta)$ is the weighting function or density function, which is related to particular material properties. Due to its mathematical basis, the model can be applied to any application where the underling mechanism is poorly understood or difficult to quantify. However, the classical Preisach model is an intrinsic rate-independent hysteresis model while the hysteresis phenomena in many smart materials are rate-dependent [4,5,8].

To make the classical Preisach model capable of describing rate-dependent hysteresis, some extensions have been made on the model, but the performance is still far from satisfactory. Aiming at dynamic variants of the classical Preisach model, Mayergoyz [5] extended the model by introducing the variation rate of the input into the density function as one of its independent variables. Normally, the derivative of the input with respect to time could be very large, particularly at the instant when the input reverses its direction. In order to confine the amplitude of the input variation rate, Yu et al. [12] did further improvement based on Mayergoyz's extension by introducing an intermediate function of the input variation rate, which has a saturation feature. The density function is then constructed by using the tuned intermediate function instead of the input variation rate directly. But the model predicts the rate-dependence only in a very small frequency range. In modeling the hysteretic dynamics of a piezoceramic actuator, Ben Mrad et al. [3] proposed another improvement on the classical Preisach model by introducing a time-dependent average of the change rate of the input into the density

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function. Experimental verification on a piezoceramic actuator shows a good agreement between the model results and the experimental measurements with an input for the frequency range up to 800 Hz, but with a very small amplitude. Xiao et al. [11] modeled the rate-dependent hysteresis by constructing the overall density function as a weighted sum of a series of elementary density functions, where each elementary density function is associated with a specific frequency component of the input. The limitation of this approach is that it might be infeasible in many cases to approximate the input and output as harmonic series. In the above mentioned generalization strategies, the rate-dependence of the output is implemented by making the density function rate-dependent. A general difficulty in making the density function rate-dependent is caused by the fact that, even in rate-independent cases, the density function is not that easy to be determined. Once it becomes rate-dependent, the determination of density function will become more computationally demanding and complicated.

Since the overall output of the classical Preisach model is a weighted sum of a series of elemental hysteresis operators, it is natural to expect that the model could be constructed as a rate-dependent one by making the elemental hysteresis operators rate-dependent, alternatively. In this approach, Aljanaideh et al. [1] and Ang et al. [2] introduced the frequency dependence into the elementary Preisach operators, where the classical operators are replaced by Prandtl–Ishlinskii operators. The output of Prandtl–Ishlinskii operator is constructed to be dependent on the rate of the input by introducing a dynamical threshold. Based on the original elementary Preisach operator, Bertotti [4] introduced the rate dependence into the model by replacing the instaneous switching behavior with a kinetic switching process with a finite transition rate. But the effect of the transition rate is explicitly confined within the switching area. In the generalization proposed by Smith [7,8], a more comprehensive dynamics of the switching process is incorporated into the elementary Preisach operator by simulating the kinetics of the switching process. Smith's model can predict the dynamic hysteresis phenomena very well, but it is computationally very expensive.

Aiming at modeling the hysteretic dynamics with a much simpler implementation, a novel Preisach-type model for the rate-dependent hysteresis is constructed in this paper based on a differential model for hysteresis loops. The elementary differential model proposed by Wang and Willatzen [9] for modeling phase transitions in single crystal materials is used as the elementary Preisach operator, which is inherently dynamic since it is formulated as a first order ordinary differential equation, and it is easy to implement. By introducing a simple distribution function which can be treated as the density function for the operators, the overall output of the model can be formulated as a weighted sum of the dynamic responses of the differential Preisach operators. The proposed model quantifies dynamic hysteresis phenomena in a reliable way. By incorporating a few assumptions based on the experimental observations, the density functions can be rephrased and the computational resource required is largely reduced.

## 2. The differential Preisach operator

As follows from Eq. (1), the basic idea for modeling the hysteresis using Preisach model is to combine the output of all the hysteresis operators with a certain weight. As in Ref. [9], a differential equation could be employed to characterize the elementary hysteresis loops. In general, for the modeling of hysteresis behavior in physical systems, a double-well potential energy function could be constructed in such a way that its two local minima correspond to the two stable states in the hysteresis loop. By applying the Lagrangian equation, the governing equation of the system can be formulated as

$$\mu \dot{v} + V'(v) = u, \tag{2}$$

where $V(v)$ is the potential energy function and $V'(v)$ is the derivative of $V$ with respect to the output $v$. $u$ is the input and $\mu$ is the generalized viscosity which plays the role of time constant in the current different equation model. In Eq. (2) the first order ordinary differential equation is an approximation of the original second order differential equation. The function $V'(v)$ could be regarded as a generalized constitutive curve for the considered system.

The relationship between the input and the output given by Eq. (2) can be regarded as a dynamical hysteresis operator. As shown in Fig. 1a and b, the elementary hysteresis loops can be modeled by the differential model, provided that the potential energy function is non-convex and bifurcations are induced in the dynamics. It is easy to see that the branch BC is unstable while the branches AB and CD are stable. When the external field increases from state A, the output will switch to branch CD at point B. When the external field decreases from state D, the output will switch to branch AB at point C. The parameter $\mu$ represents the time constant for the switching kinetics as shown in Fig. 1c. Thus a dynamic hysteresis loop is obtained. Detailed analysis of Eq. (2) can be found in Ref. [9]. One major advantage of the differential operator for modeling the hysteresis loop is that the rate dependence on the input is introduced into the model through the term $\mu\dot{v}$ and the hysteresis operator is simply a first order ordinary differential equation. The cost is that the equation is strongly nonlinear with bifurcation embedded in.

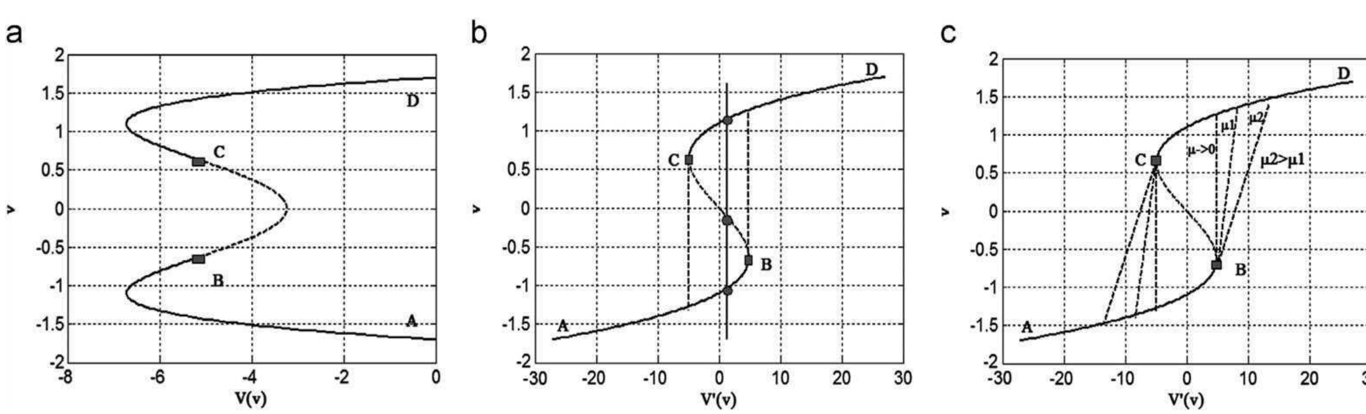


**Fig. 1.** (a) Nonconvex potential energy function. (b) Corresponding constitutive curve. (c) Influence of the parameter $\mu$ on the hysteresis loop.

## 3. Preisach-type model of hysteresis loop

For the modeling of hysteresis loops in various engineering applications, the above dynamic hysteresis operator could be modified accordingly without losing generality. From the analysis of Eq. (2) it can be observed that the profile of the hysteresis loops is actually controlled by the non-convex potential energy function. For different hysteretic behaviors, different potential functions $V(v)$ need to be constructed [7,8]. By following the same strategy as for the classical Preisach model, the overall output of the dynamic Preisach model is also formulated as a weighted sum as follows:

$$Z(t) = \iint \omega(\alpha, \beta) v_{\alpha,\beta}(t) d\alpha\, d\beta,$$
$$\mu \dot{v}_{\alpha,\beta} + V'\left(v_{\alpha,\beta}\right) = u, \quad (3)$$

where $Z(t)$ is the overall output of the dynamic Preisach model, $\omega(\alpha, \beta)$ is the density function, and $v_{\alpha,\beta}$ is the output of the dynamic hysteresis operator associated with the threshold pair $(\alpha, \beta)$. It is obvious that the overall output $Z(t)$ must be rate dependent since all $v_{\alpha,\beta}$ are dependent on the input $u(t)$ and its time derivative. It is worth to note that, the non-convex potential function was constructed in Ref. [9] as a high order polynomial for the sake of clarification and simplicity, but this may not be the best choice for the approximation to measured hysteresis loops. In real applications, there is no such a restriction on the energy function. Therefore, the profile of the elementary hysteresis loop, as sketched in Fig. 1, could be replaced by any continuous curve as required. A convenient choice is a piecewise polynomial determined by experimental data.

To illustrate the capability of the above constructed model, the experimental data in Ref. [6] is used here as an example. The major loops of a ZDKH electrical steel, which was strongly oriented, were measured. The parameter values used for the experiment are as follows, thickness $d=0.23$ mm, and width $\omega=4.0$ mm. Data were measured using a Single Sheet Tester at frequencies: 1, 50, 500, 1000 and 2000 Hz. The experimental results at 50, 500 and 1000 Hz are taken here for comparison purpose.

For the purpose of illustration, the constitutive curve $V'(v)$ needs to be constructed first. Here $V'(v)$ function is constructed as a piecewise cubic Hermite polynomial, which passes through the points $(-1.02, -1000)$, $(-1, \alpha)$, $(1, \beta)$ and $(1.02, 1000)$ with natural boundary conditions. All these control points for the piecewise polynomial are chosen directly from the experimental data empirically. Here $\alpha$ and $\beta$, $\alpha \geq \beta$, provide thresholds at which the operator switches as in the classical Preisach model. To reduce the computational cost, $\alpha$ and $\beta$ are not distributed in the interval $(-930, 930)$ uniformly. With a given time constant $\mu$, different Preisach operators with different thresholds will have different switching behaviors. For the estimation of the density function and the time constant $\mu$, the least square approximation strategy is chosen to formulate the estimation problem as a nonlinear optimization problem as follows:

$$\min_{\omega(\alpha,\beta),\mu} G = \sum_{i=1}^{M} \left(\tilde{B}_i - B_i\right)^2, \quad (4)$$

where $M$ is the number of experimental data samples, $\tilde{B}_i$ are experimental values of $B$ at the $i$th time instant, and $B_i$ are the simulated values at the $i$th time instant.

By using the given experimental data, the estimated density function is shown in Fig. 2a and the time constant is estimated to be 0.01. The comparison between the experimental loop and the numerical simulation is presented in Fig. 2b by plotting the corresponding hysteresis loops in the same figure. It can clearly be seen that, by using the same density function and the same time constant, the simulated hysteresis loops with three input frequencies fit their experimental counterparts very well. It indicates that the rate-dependent property of the system is modeled successfully. The density function mainly occupies the area near the diagonal line of the $\alpha$–$\beta$ plane, because only the major loops of the experimental data are taken for the estimation purpose here.

## 4. One-dimensional density function

One of the disadvantages of the classical Preisach model in engineering applications is that the estimation of the density function is computationally demanding. By replacing the classical Preisach operator with the dynamical one defined by an ordinary differential equation, it is easy to see that the density function estimation will be a bit more computationally expensive.

So far in the discussion, all the dynamic Preisach operators have been regarded as independent and have nothing to do with each other. However, in smart materials and structures, the hysteresis loops are always associated with certain kind of phase transitions, and the mechanism of the phase transition is the same for the same materials. Therefore it is physically justifiable to assume that there are similarities among these Preisach operators. By following the same approach proposed in Ref. [10], one can treat the different Preisach operators in the model as operators with the same phase transition dynamics, but located in different grains which have different principal axes in the material. Thus the similarity among the dynamic Preisach operators could be drawn

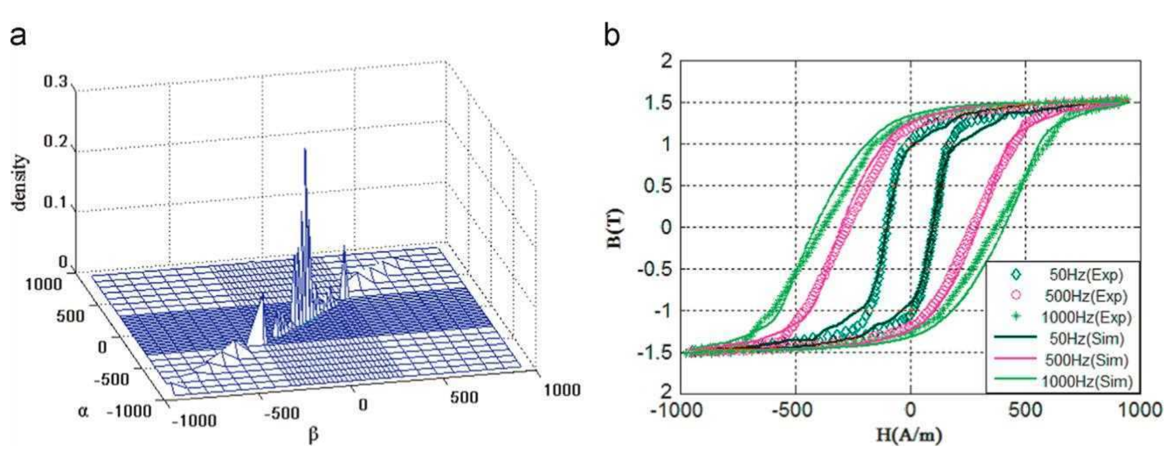


**Fig. 2.** Simulation and validation of the Preisach-type model with two-dimensional density function. (a) The estimated two-dimensional density function. (b) Comparison between the simulated results and the experimental counterparts.

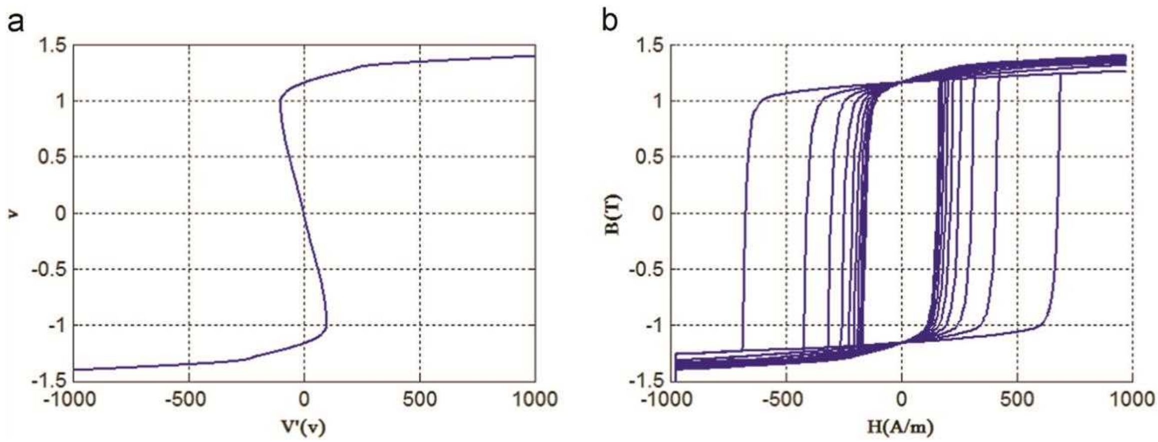


**Fig. 3.** (a) A typical $V'(v)$ function. (b) Simulated Preisach operators with the specific $V'(v)$ function and different $\theta$ values.

out.

Without losing generality, the dynamic Preisach operator in an arbitrarily chosen grain could be formulated as follows:

$$\mu \dot{v}_\theta + V'(v_\theta) = u \cos\theta, \tag{5a}$$

where $\theta$ is the angle between the principal axes of the chosen grain and that of the reference Preisach operator. Then, the overall output can be represented as follows:

$$v(t) = \int_0^{\pi/2} v_\theta w(\theta) d\theta, \tag{5b}$$

where $w(\theta)$ is the corresponding density function value for $v_\theta$. By comparing with the classical Preisach model, it is very clear that proposed model actually has the same structure. The main difference, however, is that the density function is now one-dimensional.

At the same time, one can obtain the dynamic responses of all Preisach operator variants from that of the reference Preisach operator simply by changing the landscapes of $V'(v)$, and by simulating its dynamic response. This relationship is shown clearly by Eq. (5), and is presented in Fig. 3. In other words, all elementary Preisach operators could be regarded as an enlargement (or shrinkage) of the reference hysteresis loop horizontally. For clarity, a typical $V'(v)$ is shown in Fig. 3a. Then with different $\theta$ values, one can get different hysteresis loops (as shown in Fig. 3b).

## 5. Numerical simulation and validation

To illustrate the capability of the proposed model, the same experimental datasets from Ribbenfjard [6] are used here for numerical simulation and validation. In the first numerical experiment, the density function is assumed to be a constant, which means that the distribution of all dynamic Preisach operators is uniform. The profile of the constitutive curve $V'(v)$ and the time constant $\mu$ are estimated with the constant density function. The optimized approximation of $V'(v)$ is depicted in Fig. 4a and $\mu$ is estimated to be 0.0077. By using these estimated $V'(v)$ and $\mu$, the dynamic hysteretic response of the system could be modeled by using Eq. (5). The simulated rate-dependent hysteresis loops are plotted together with their experimental counterparts together in Fig. 4b. The plots show clearly that, even without a carefully chosen density function, the proposed model can model the rate-dependent tendency quite well. But, it is also clearly seen that the discrepancy between the numerical and experimental loops is still easily noticeable, particular in those areas where phase transitions are initiated and being finished. This discrepancy could be easily explained by the fact that the density function has been simply assumed as a constant.

In the second numerical experiment, the density function is also estimated by employing the least square approximation strategy, like those performed in Eq. (4). The estimated density function is plotted in Fig. 5a. It is worth to note that the constitutive curve $V'(v)$, $\mu$ and the density function are not estimated simultaneously here. Instead, the constitutive curve $V'(v)$ and $\mu$,

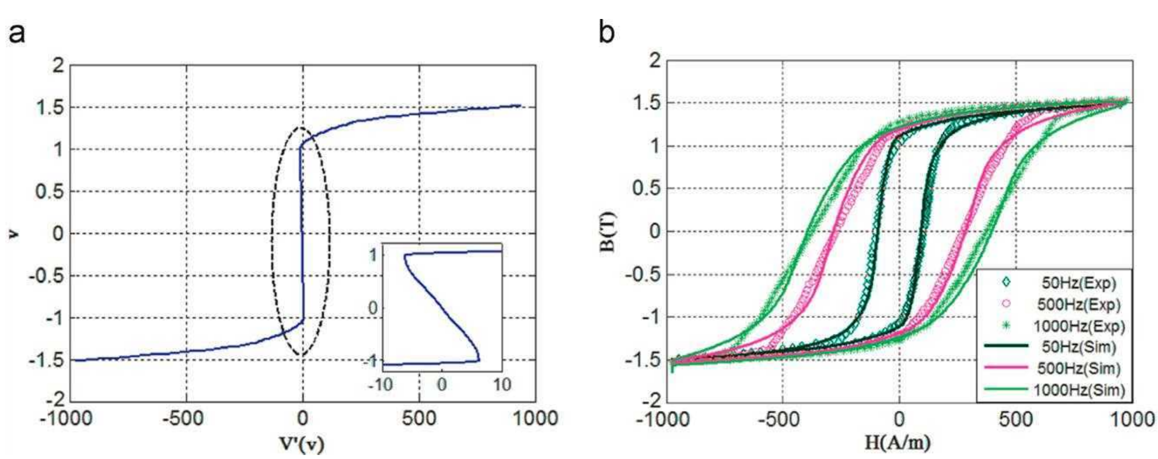


**Fig. 4.** Simulation and validation of the differential Preisach-type model with a constant distribution density. (a) The estimated $V'(v)$ function. (b) Comparison between the simulated results and the experimental counterparts.

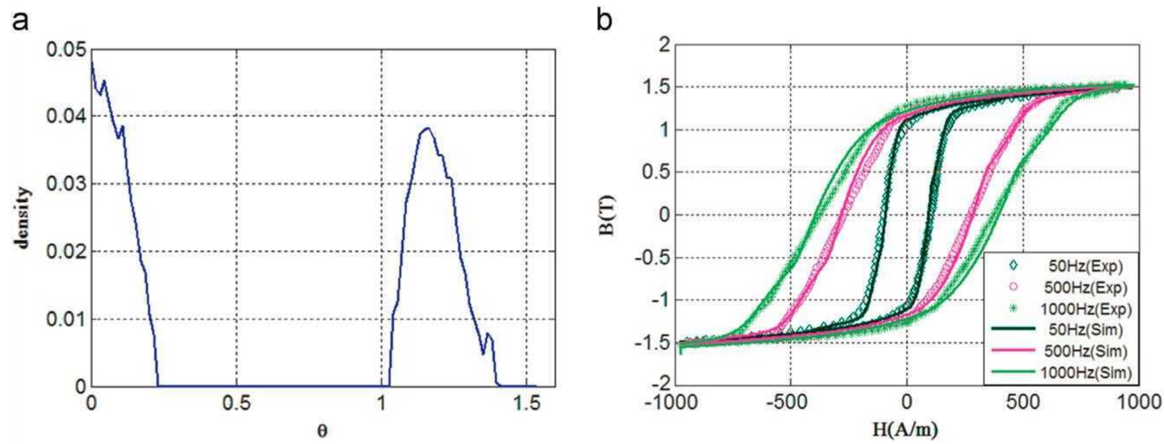


**Fig. 5.** Simulation and validation of differential Preisach-type models with one-dimensional density function. (a) The estimated one-dimensional density function. (b) Comparison between the simulated results and their experimental counterparts.

estimated in the first numerical experiment, are used here for the estimation of the density function. Similarly, the numerical and experimental hysteresis loops with three different loading frequencies are plotted together in Fig. 5b. It is shown by comparison that, when the density function is estimated by using experimental data, the model results could fit the experimental ones perfectly well, with the discrepancy hardly noticeable for all considered frequencies.

If one considers the fact that the constitutive curve $V'(v)$, time constant $\mu$, and the density function are not estimated simultaneously, but separately, it is natural to expect that the agreement between the model results and experimental measurements will be even better. The reason for making a separate estimation here for the density function is two-fold. On one side, the model results already fit their experimental counterparts perfectly well, and it will not be easily noticed even when the fit is improved. On the other side, the optimization problem is much easier when the estimation is performed separately. But, compared to the classical Preisach model, the simultaneous estimation of the density function, the constitutive equation, and the time constant will not be more difficult than the estimation of the two-dimensional density function, since here the density function is one-dimensional.

To further reduce the computation cost, one can do the simulation for only one Preisach model, whilst all other Preisach operators could be obtained by a specific affine transformation, as indicated by Eq. (5a).

## 6. Conclusion

In this paper, we have proposed a novel Preisach-type model for rate-dependent hysteresis based on dynamic elementary Preisach operators. The elementary operator is formulated as a first order ordinary differential equation. It is inherently dynamic and easy to implement. By introducing a one-dimensional density function for the operators, the overall output of the model can be formulated as a weighted sum of the dynamic responses of the differential Preisach operators. Comparisons between the model results and experimental measurements demonstrate that the proposed model quantifies dynamic hysteresis phenomena very well with all input frequencies considered.

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