

# 1 Proof

**Lemma 1:**  $q2 \implies in \neq out$

- Initially  $q2$  is false, lemma is true
- Only statement that progresses to  $q2$  is  $q1$  which requires  $in \neq out$
- $in \neq out$  cannot become false between  $q1$  and  $q2$ 
  - Only other statement which can change  $in$  or  $out$  is  $p4$
  - Since **lemma 2**,  $p$  cannot make  $in \neq out$

**Lemma 2:**  $p3..4 \implies out \neq (in + 1) \bmod N$

- Initially holds, as  $p3..4$  is false
- Only statement that progresses to  $p3..4$  is  $p2$  which requires  $out \neq (in + 1)$
- $out! = (in + 1) \bmod N$  cannot become false between  $p2..p4$
- Thus cannot increment  $in$  such that  $in = out$ 
  - Only statement which can change  $in$  or  $out$  is  $q3$  ( $out = out + 1$ )
  - Thus can increment to  $out + 1$ , so  $(out + 1) \neq (in + 1) \bmod N$