1 Proof of Mutual Exclusion

Lemma 1: $q2 \implies in \neq out$

- Initially $q2$ is false, lemma is true
- Only statement that progresses to $q2$ is $q1$ which requires $in \neq out$
- $in \neq out$ cannot become false between $q1$ and $q2$
  - Only other statement which can change in or out is $p4$
  - Since **lemma 2**, p cannot make $in \neq out$

Lemma 2: $p3..4 \implies out \neq (in + 1)\ mod\ N$

- Initially holds, as $p3..4$ is false
- Only statement that progresses to $p3..4$ is $p2$ which requires $out \neq (in + 1)$
- $out! = (in + 1) modN$ cannot become false between $p2..p4$
- Thus cannot increment in such that $in = out$
  - Only statement which can change in or out is $q3$ ($out = out + 1$)
  - Thus can increment to $out + 1$, so $(out + 1) \neq (in + 1)\ mod\ N$

Theorem 1: $\sim (p3 \wedge q2)$

- Assume $p3\hat{q}2$
- $q2 \implies in! = out \longrightarrow \sim q2 \vee in \neq out$
- $p3 \implies out \neq (in + 1)\ mod\ N$
- $(\neq q2 \vee in \neq out) \wedge (\sim p3 \vee out \neq (in + 1)\ mod\ N$
- If we assume $p3 \wedge q2$, then $\sim q2 = false$ and $\sim p3 = false$
- $(in \neq out) \wedge (out \neq (in + 1)\ mod\ N)$
- $in \neq (in + 1)\ mod\ N$
- Therefore theorem holds

2 Proof of Freedom from Starvation

Theorem 2: $\Box(p1 \implies \Diamond p3) \wedge \Box(q1 \implies \Diamond q2)$

$\Box(p1 \implies \Diamond p3)$

- From $p1$, progresses to $p2$
- To progress to $p3$, $out \neq (in + 1)\ mod\ N$ must be true
- Intially $in = out = 0$, so $0 \neq (0 + 1)\ mod\ N$ is true, thus progresses to $p3$

$\Box(q1 \implies \Diamond q2)$

- To progress to $q2$, $in \neq out$ must be true
- When an item is added, $in \neq out$ will be true and will progress to $q2$