Ryan Skinner

Homework 1

8. Show how these polynomials can be efficiently evaluated:

b. $p(x) = 3(x - 1)^5 + 7(x - 1)^9$

$z = (x-1)$

$p = z^5(3+z^4(7))$

d. $p(x) = x^{127} - 5x^{37} + 10x^{17} - 3x^7$

$x^7(-3 + x^{10}(10 + x^{20}(-5 + x^{90})))$

11. Write segments of pseudocode to evaluate the following expressions efficiently:

a. $p(x) = \sum_{k=0}^{n-1} kx^k$

```
int sum = n-1;
for(int k = n-2; k > 0; k++){
        sum = sum * x + k;
}
```

d. $p(t) = \sum_{i=1}^{n} \prod_{j=1}^{i-1} (t - x_j)$

```
int sum = 0;
for(int i = 1; i < n; i++){
        int prod = 1;
        for(int j = 1; j < i; j++){
                prod *= (t – x[j]);
        }
        sum += prod;
}
```

4. Why do the following functions not possess Taylor series expansions at x = 0?

c. $f(x) = \arcsin(x - 1)$

$f'(0) = 1/0$

d. $f(x) = \cot x$

$f(0) = 1/0$

f. $f(x) = x^{\pi}$

$f''''(0) = 1/0$

6. Determine the first two nonzero terms of the series expansion about zero for the following:
c. $(\cos x)^2(\sin x)$

1 + -7

13. Use the Alternating Series Theorem to determine the number of terms in Series (5) needed for computing ln1.1 with error less than $1/2\times10^{-8}$

8 terms

26. In the Taylor series for the function $3x^2 - 7 + \cos x$ (expanded in powers of x), what is the coefficient of x^2?

6 – cos x

29. Find the value of ξ that serves in Taylor's Theorem when f (x) = sinx, with x = π/4, c = 0, and n = 4.

$$\sum_{k=0}^{4}\frac{f^k(0)}{k!}\left(\frac{\pi}{4}\right)^k+\frac{f^{n+1}(\xi)}{(n+1)!}\left(\frac{\pi}{4}\right)^{n+1}\quad \text{where}\quad f(0)=\sin(0);\quad \xi = 0.170864067757$$

36. Using the Taylor series expansion in terms of h, determine the first three terms in the series for $e^{\sin(x+h)}$. Evaluate $e^{\sin 90.01}$ accurately to ten decimal places as *Ce* for constant C.

$$f(x+h)=e^{\sin(x)}+e^{\sin(x)}\cos(x)h+\frac{e^{\sin(x)}\cos^2(x)-e^{\sin(x)}\sin(x)h^2}{2}\qquad f(90.01)=2.43384254499$$

41. Establish the Taylor series in terms of h for the following:
c. $\ln[(x - h^2)/(x + h^2)]$

$$\sum_{k=0}^{n}\frac{f^{(k)}(x)}{k!}(-2h^{2k})+\frac{f^{(n+1)}(\xi)}{(n+1)!}(-2h^{2(n+1)})\quad \text{where}\quad f(x)=\ln(x)$$

45. Determine the first three terms in the Taylor series to represent sinh(x + h). Evaluate sinh(0.0001) to 20 decimal places (rounded) using this series.

$$f(a+h)=\sinh(a)+\cosh(a)h+\frac{\sinh(a)}{2}h^2\qquad f(0.0001)=0.00010000000016666667$$