Take-home assignment

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1 Graphs

We have a simple connected graph G with vertices V. Given an arbitrary $v \in V$ we would like to find all directed subgraphs g of G where: (1) v is in g, (2) no vertex is a source or a sink, (3) an edge in G appears at most once in g, i.e., g is an oriented graph, and (4) the number of edges in g is less than k and greater than 0.

Demonstrate your algorithm with $k = 10$ on a random graph with 10 vertices and 20 edges. If you can't enumerate all valid subgraphs, find as many as you can. Explain if your solution is practical for much larger graphs and how it could be improved.

It may be fruitful to think in terms of directed cycles and their unions.

2 Optimization

Given functions $f_i : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ with $f_i(0) = 0$ and $\forall a < b, f_i(a) \leq f_i(b)$, we want to numerically solve the optimization problem:

$$u^* = \underset{\substack{0 \leq u_i \leq 1 \\ \sum_i u_i = 1}}{\arg\max} \sum_i f_i(u_i)$$

Note that f_i are not necessarily differentiable or even continuous. Your solution can be approximate and must be more efficient than a simple grid search. How would you get the best performance if there were thousands of functions? Can you make use of the fact that the functions are non-decreasing?

Demonstrate your algorithm on the following problem and plot the objective function over the two dimensional simplex with the optimal value overlaid:

$$f_1(x) = 2x^{\frac{1}{3}} \tag{1}$$

$$f_2(x) = 5x^2 \tag{2}$$

$$f_3(x) = \begin{cases} 0 & x \leq 0.5 \\ x + 1 & x > 0.5 \end{cases} \tag{3}$$