Information Visualization

W05: Rendering Pipeline

Graduation School of System Informatics

Department of Computational Science

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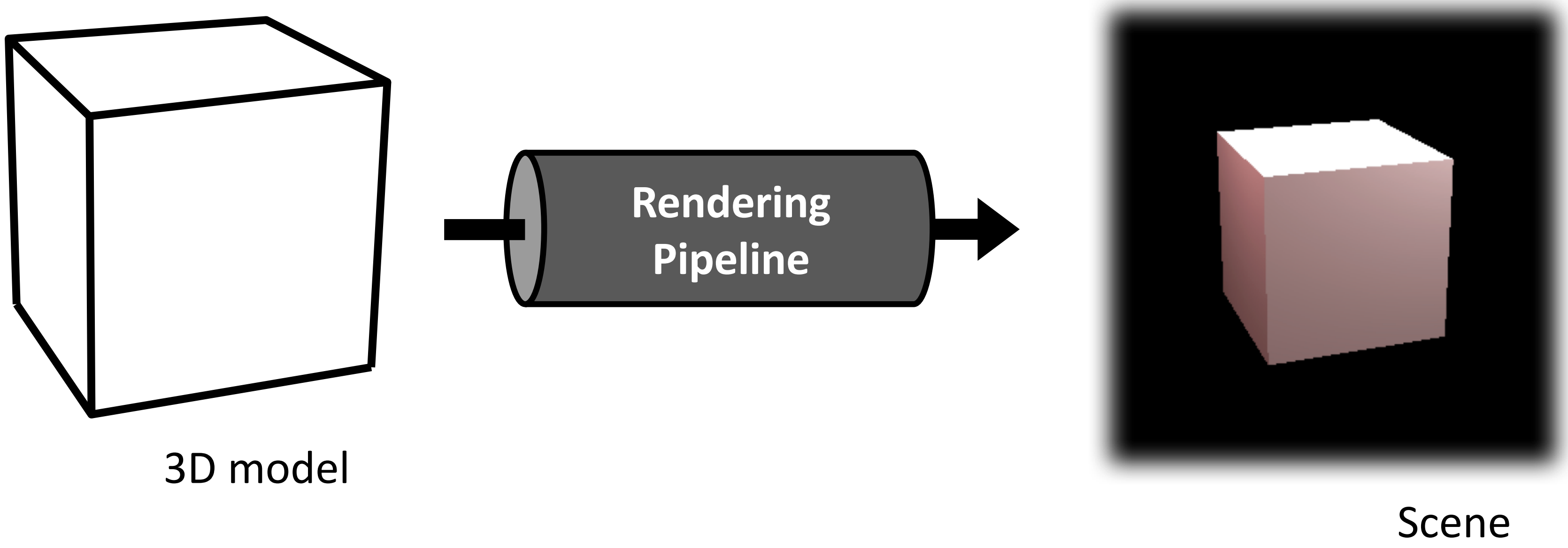
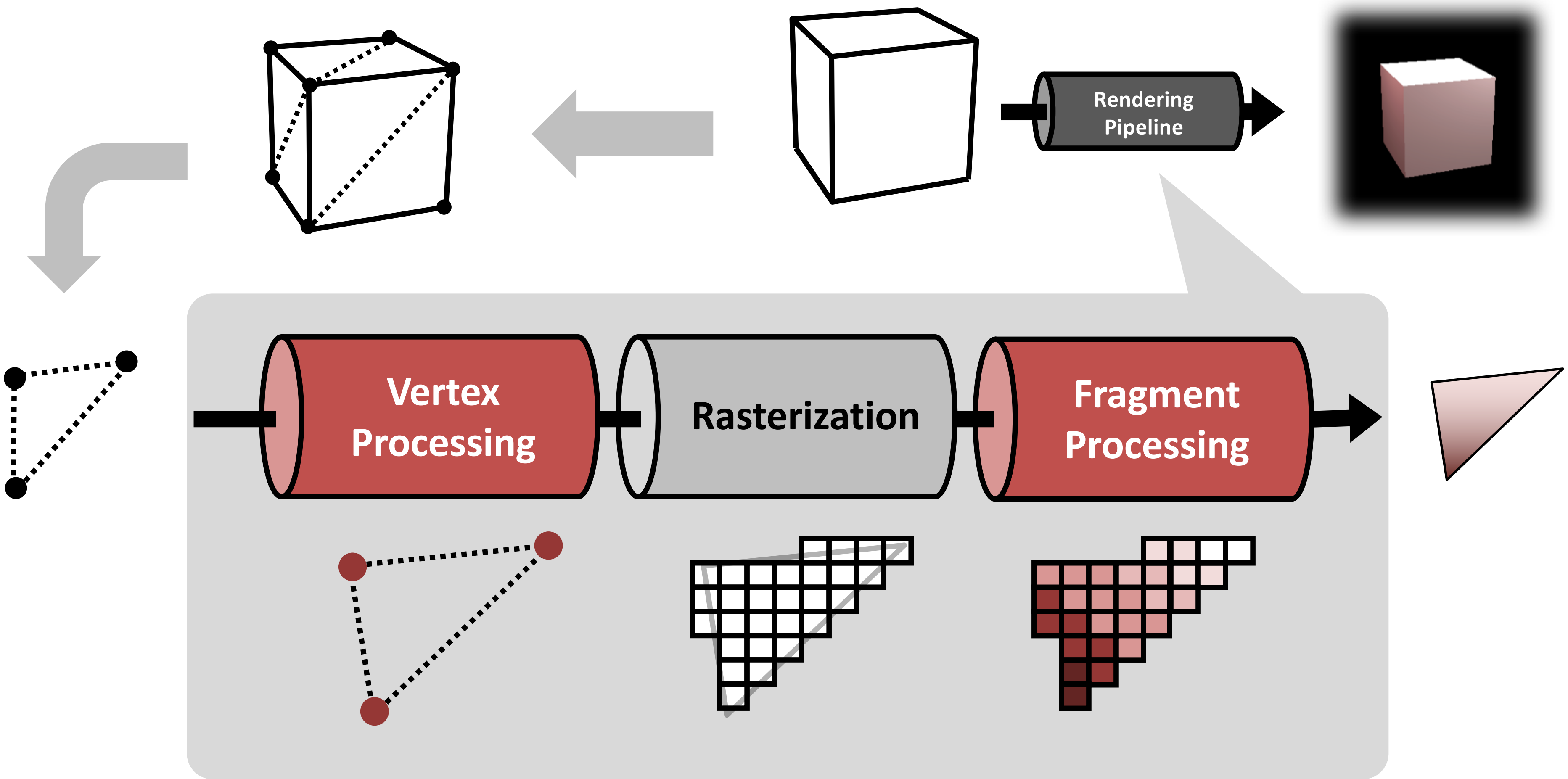
Rendering Pipeline

- Rendering pipeline (Graphics pipeline)
  - Sequence of steps when rendering objects



Rendering Pipeline

- Processing steps on rendering pipeline



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- Rendering pipeline
- Geometric Transformations
- Coordinate System and Transformations

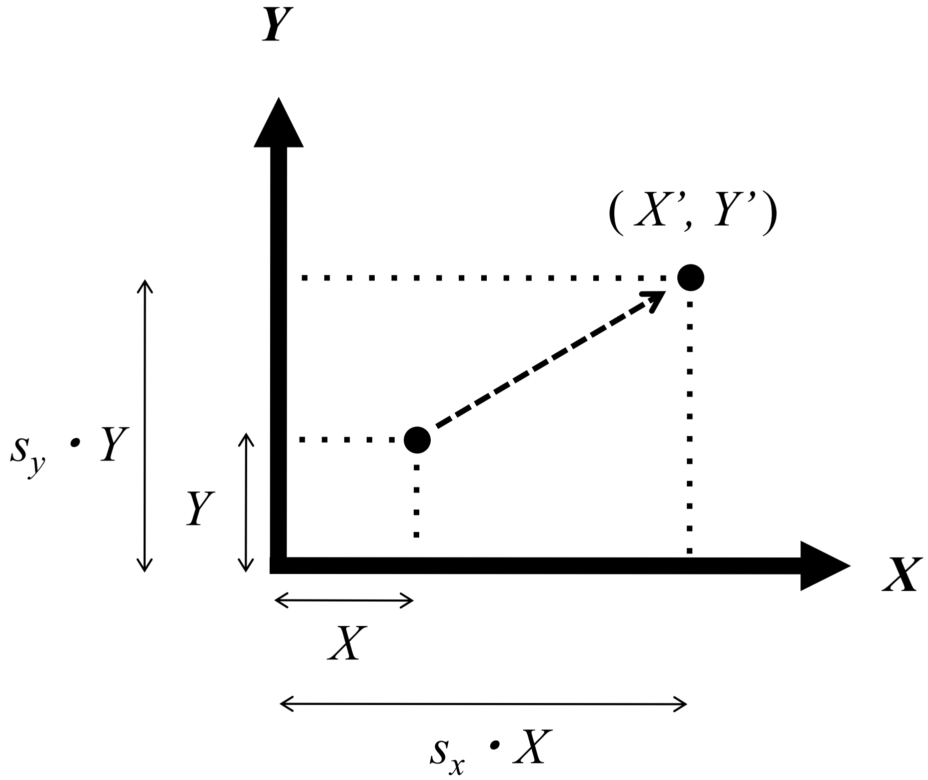
Geometric Transformations

- Geometric objects consist of a set of points and a point $\mathbf{P}$ is represented in Cartesian coordinates.
  - $\mathbf{P} = (X, Y)$
- Size, position, and orientation of objects can be changed by matrix operations.
  - Scaling
  - Translation
  - Rotation

Scaling

- Scaling an object from the point of origin by the factors sx and sy in x- and y-direction, respectively.

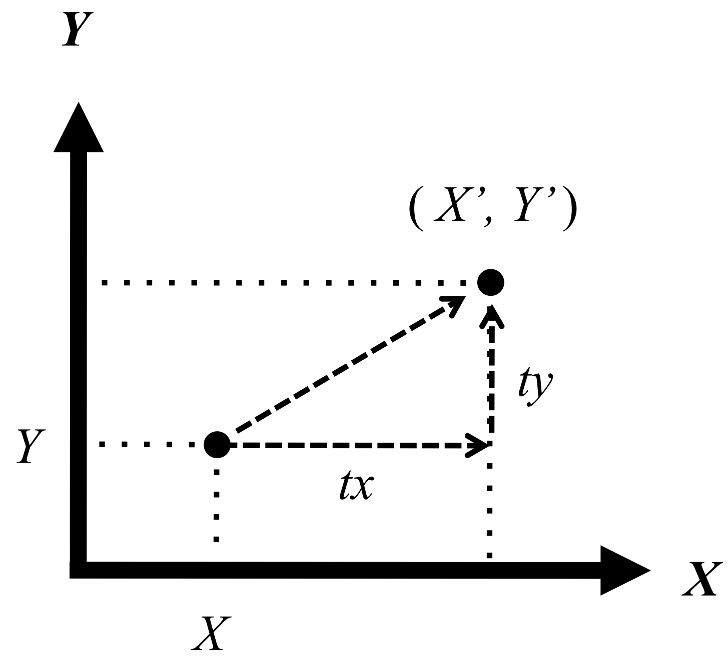
$$
\begin{aligned}
X' &= sx \cdot X \\
Y' &= sy \cdot Y
\end{aligned}
$$



Translation

- Translation of a point (X, Y) by the factors tx and ty in x- and y-direction.

$$X' = X + tx$$
$$Y' = Y + ty$$



Rotation

- Rotating a point (X, Y) around the point of origin by an angle θ.

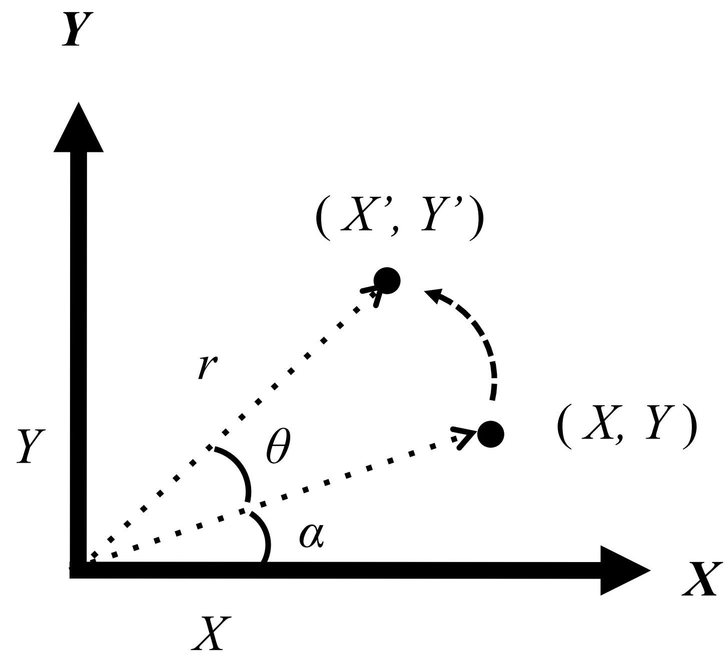
$$\begin{cases} X = r\cos\alpha \\ Y = r\sin\alpha \end{cases}$$

$$\begin{cases} X' = r\cos(\alpha+\theta) \\ Y' = r\sin(\alpha+\theta) \end{cases}$$

$$\Downarrow$$

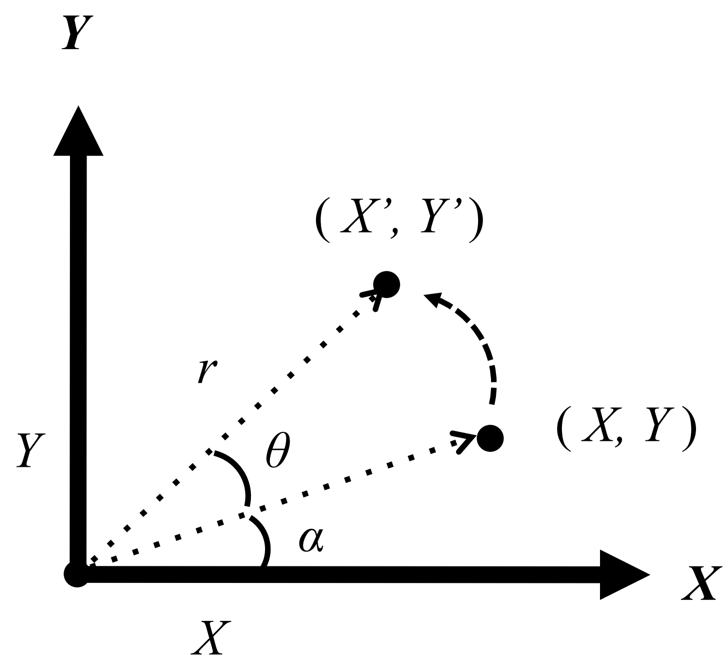
$$X' = \cos\theta \cdot X - \sin\theta \cdot Y$$

$$Y' = \sin\theta \cdot X + \cos\theta \cdot Y$$



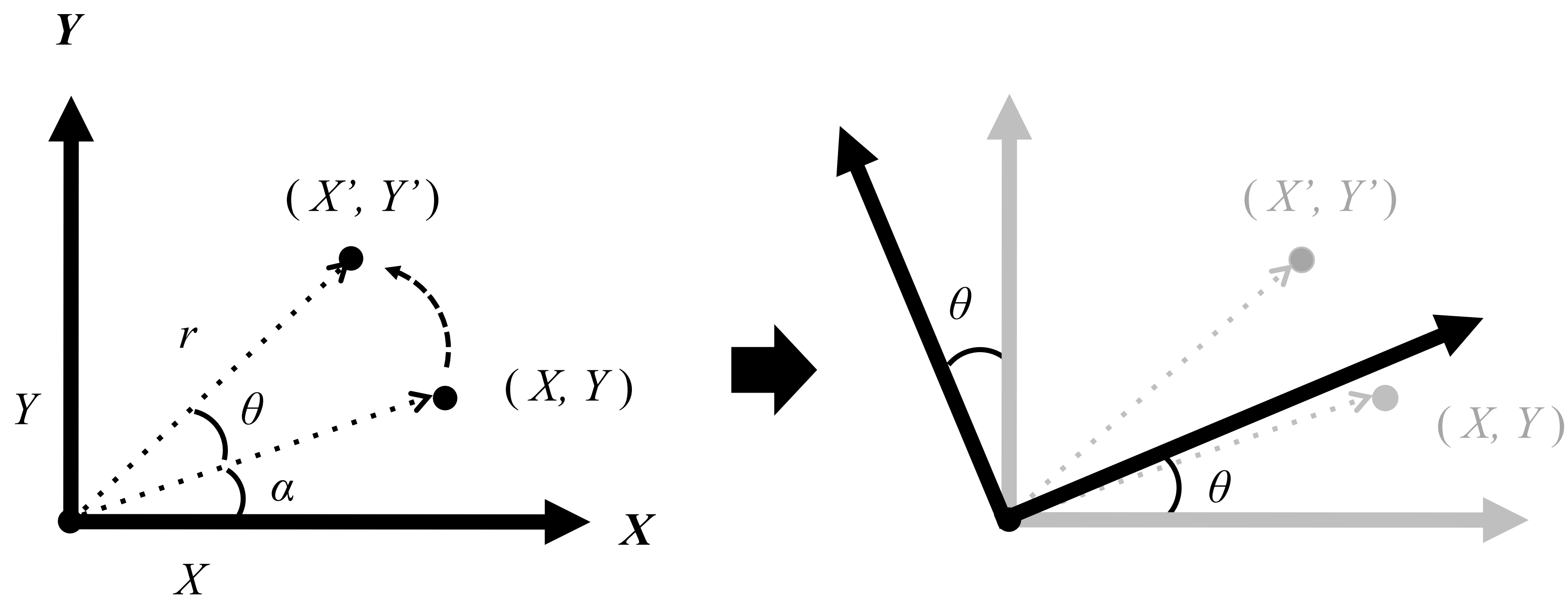
Rotation

- Rotation of X and Y axis.



Rotation

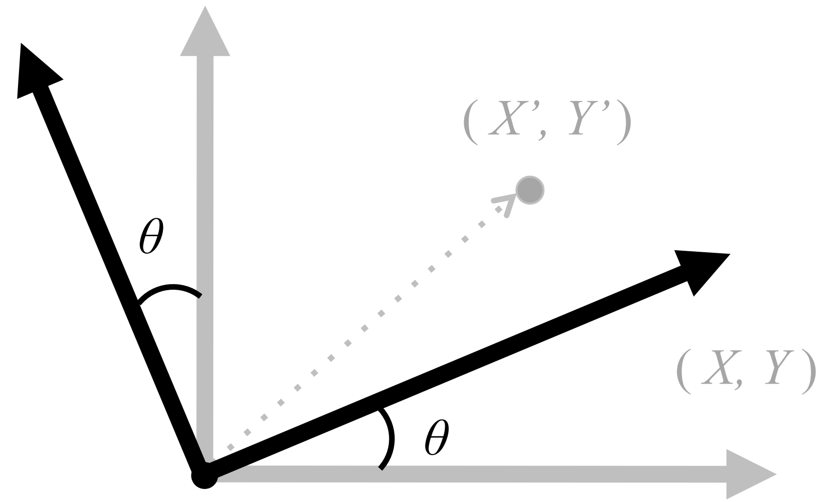
- Rotation of X and Y axis.



Rotation

- Rotation matrix
  - Column vectors represent unit direction vectors of the rotated axes.

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$



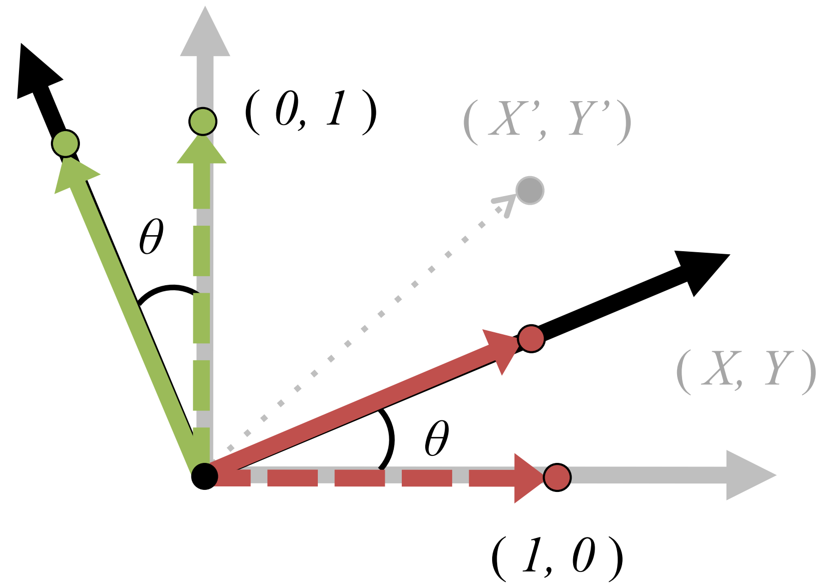
Rotation

- Rotation matrix
  - Column vectors represent unit direction vectors of the rotated axes.

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Unit direction vector of the rotated x-axis

Unit direction vector of the rotated y-axis



Matrix Representations

- Scaling

$$\begin{aligned} X' &= sx \cdot X \\ Y' &= sy \cdot Y \end{aligned}$$

$$\begin{bmatrix} X' \\ Y' \end{bmatrix} = \begin{bmatrix} sx & 0 \\ 0 & sy \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix}$$

- Translation

$$\begin{aligned} X' &= X + tx \\ Y' &= Y + ty \end{aligned}$$

?

- Rotation

$$\begin{aligned} X' &= \cos\theta \cdot X - \sin\theta \cdot Y \\ Y' &= \sin\theta \cdot X + \cos\theta \cdot Y \end{aligned}$$

$$\begin{bmatrix} X' \\ Y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix}$$

Matrix Representations

- Scaling

$$
\begin{aligned}
X' &= sx \cdot X \\
Y' &= sy \cdot Y
\end{aligned}
\qquad
\begin{bmatrix} X' \\ Y' \end{bmatrix} = \begin{bmatrix} sx & 0 \\ 0 & sy \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix}
$$

- Translation

$$
\begin{aligned}
X' &= X + tx \\
Y' &= Y + ty
\end{aligned}
\qquad
\begin{bmatrix} X' \\ Y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}
$$

- Rotation

$$
\begin{aligned}
X' &= \cos\theta \cdot X - \sin\theta \cdot Y \\
Y' &= \sin\theta \cdot X + \cos\theta \cdot Y
\end{aligned}
\qquad
\begin{bmatrix} X' \\ Y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix}
$$

Matrix Multiplication

- Transformations can be described as a matrix multiplication.
  - Homogeneous representation
  - Combined matrix $\boldsymbol{M}$ with scaling, translation and rotation matrices

$$\begin{bmatrix} X' \\ Y' \\ 1 \end{bmatrix} = M \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

Homogeneous Representations

- Scaling

$$\begin{bmatrix} X' \\ Y' \\ 1 \end{bmatrix} = \begin{bmatrix} sx & 0 & 0 \\ 0 & sy & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

- Translation

$$\begin{bmatrix} X' \\ Y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

- Rotation

$$\begin{bmatrix} X' \\ Y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

Ordering Transformations

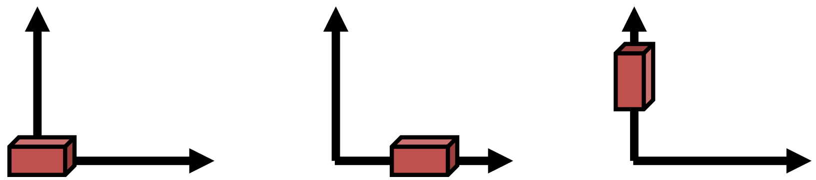
- Where is P'=(X',Y') transformed with a scaling matrix S, a translation matrix T and a rotation matrix R from P = (X,Y)?

$$\begin{bmatrix} X' \\ Y' \\ 1 \end{bmatrix} = STR \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

Ordering Transformations

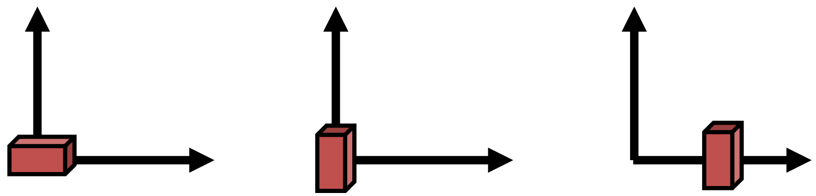
- Matrix multiplication is not cumulative.
  - Translation (***T***) and then rotation (***R***)

$$P' = RT\,P$$



  - Rotation (***R***) and then translation (***T***)

$$P' = TR\,P$$



Scaling

- Scaling a point (X, Y, Z) with (sx, sy, sz)

$$\begin{bmatrix} X' \\ Y' \\ Z' \\ 1 \end{bmatrix} = \begin{bmatrix} sx & 0 & 0 & 0 \\ 0 & sy & 0 & 0 \\ 0 & 0 & sz & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

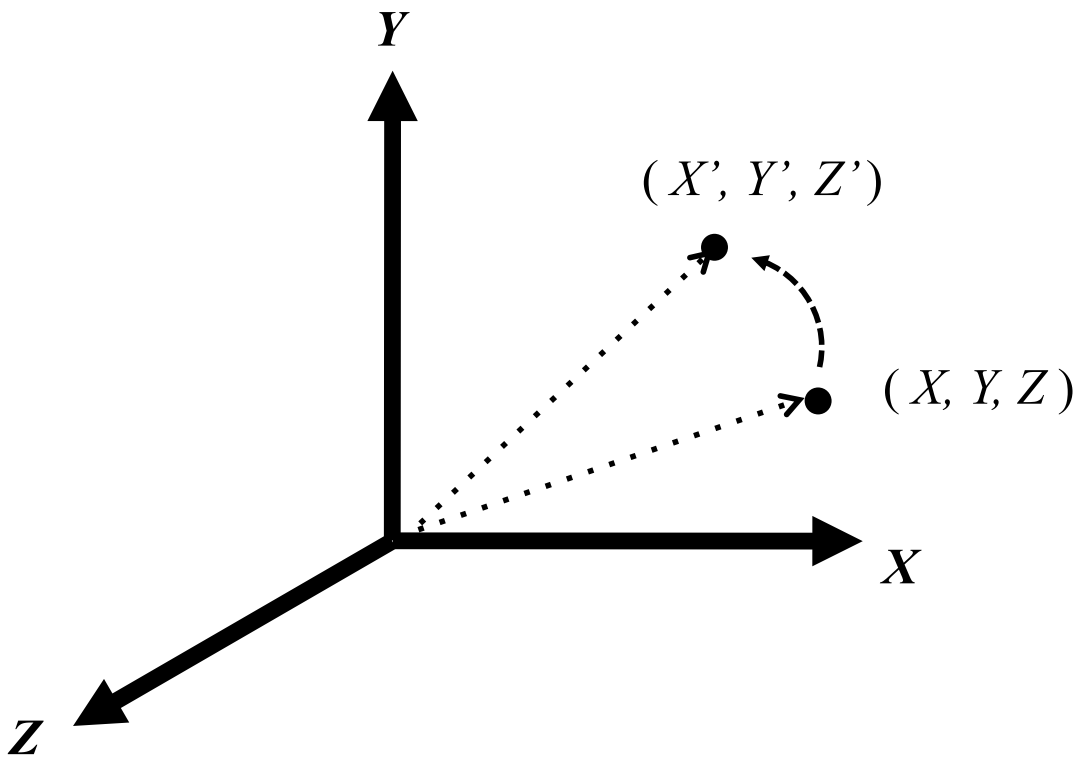
Translation

- Translation a point (X, Y, Z) with (tx, ty, tz)

$$\begin{bmatrix} X' \\ Y' \\ Z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & tx \\ 0 & 1 & 0 & ty \\ 0 & 0 & 1 & tz \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

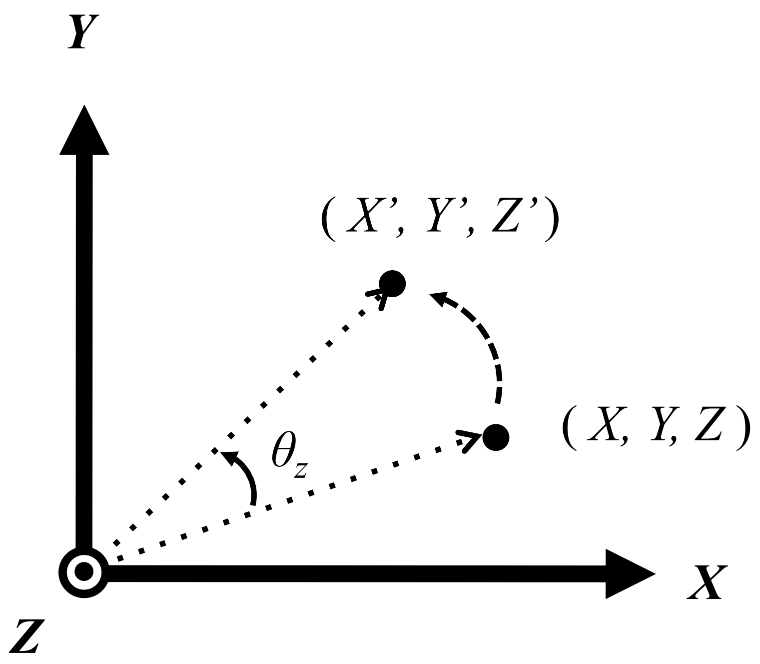
Rotation

- Rotation in 3D space



Rotation around z-axis

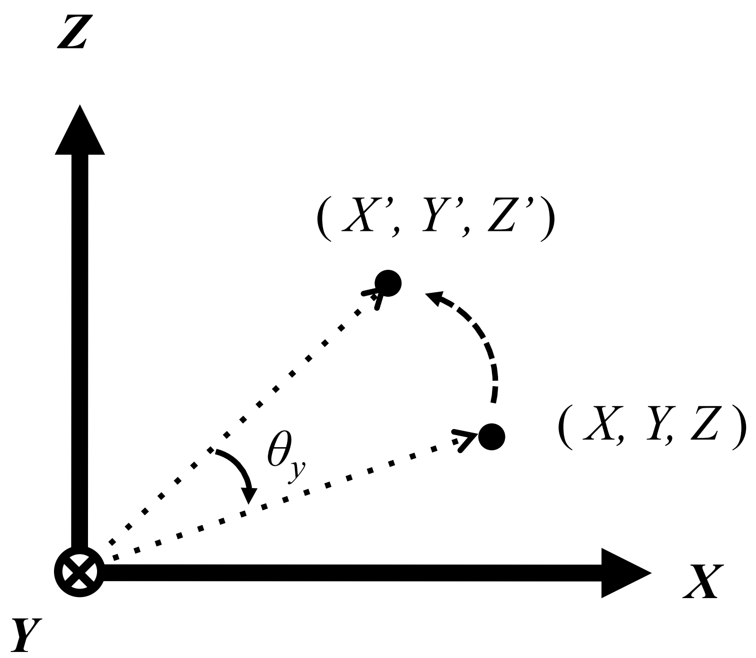
- Rotation around the z-axis by an angle θ_z



$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} \cos\theta_z & -\sin\theta_z & 0 \\ \sin\theta_z & \cos\theta_z & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Rotation around y-axis

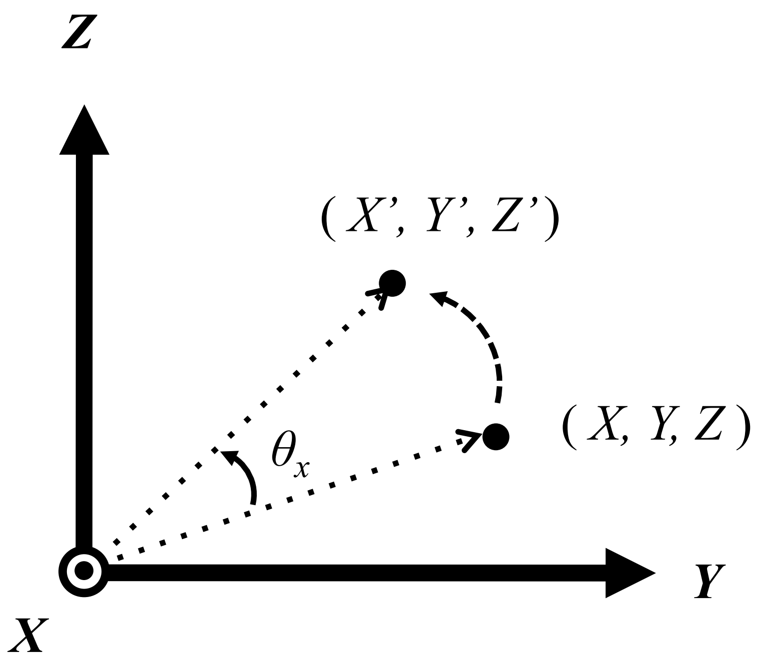
- Rotation around the y-axis by an angle θ_y



$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} \cos\theta_y & 0 & \sin\theta_y \\ 0 & 1 & 0 \\ -\sin\theta_y & 0 & \cos\theta_y \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Rotation around x-axis

- Rotation around the x-axis by an angle θ_x



$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_x & -\sin\theta_x \\ 0 & \sin\theta_x & \cos\theta_x \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

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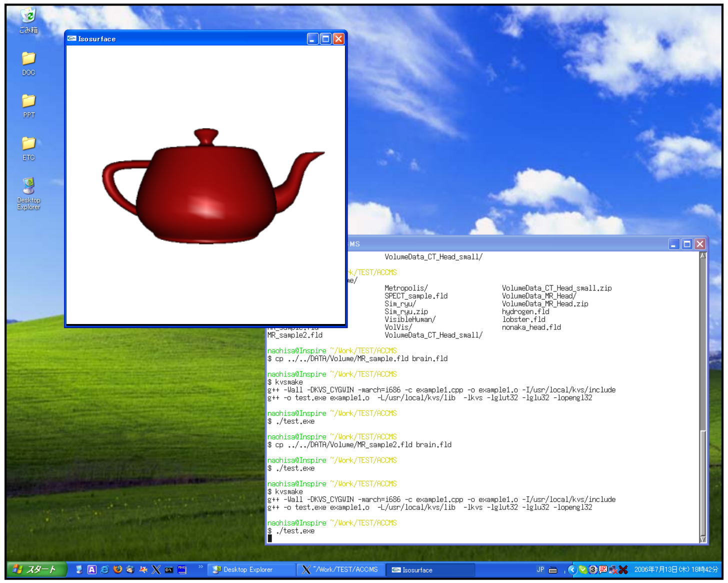
Coordinate Transformations

- 3D rendering
  - Converting 3D objects into 2D images

Teapot (3D)



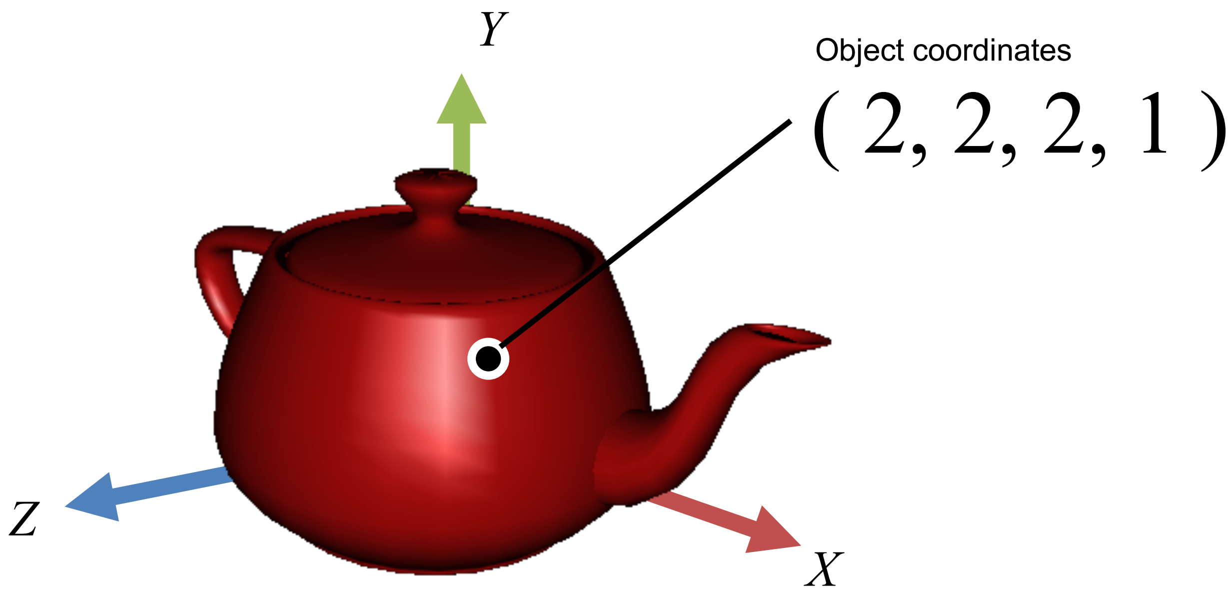
Desktop (2D)

Coordinate Systems

- In 3D computer graphics, objects are projected onto a image plane through several coordinate systems.
- Coordinate Systems
  1. Object Coordinate System
  2. World Coordinate System
  3. Camera Coordinate System
  4. Clip Coordinate System
  5. Normalized Device Coordinate System
  6. Window Coordinate System

Object Coordinate System

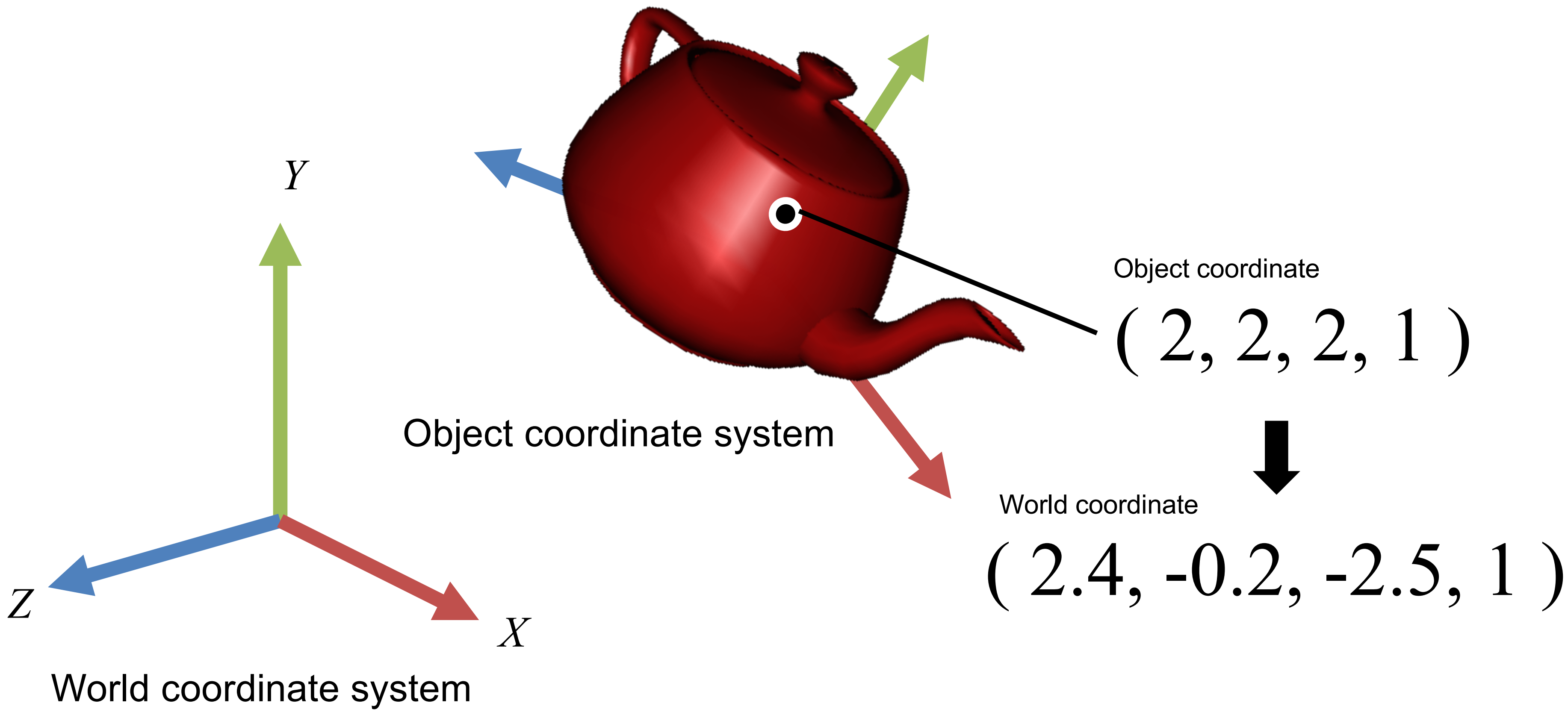
- 3D objects are often defined in an original local coordinate system.



Object coordinate system

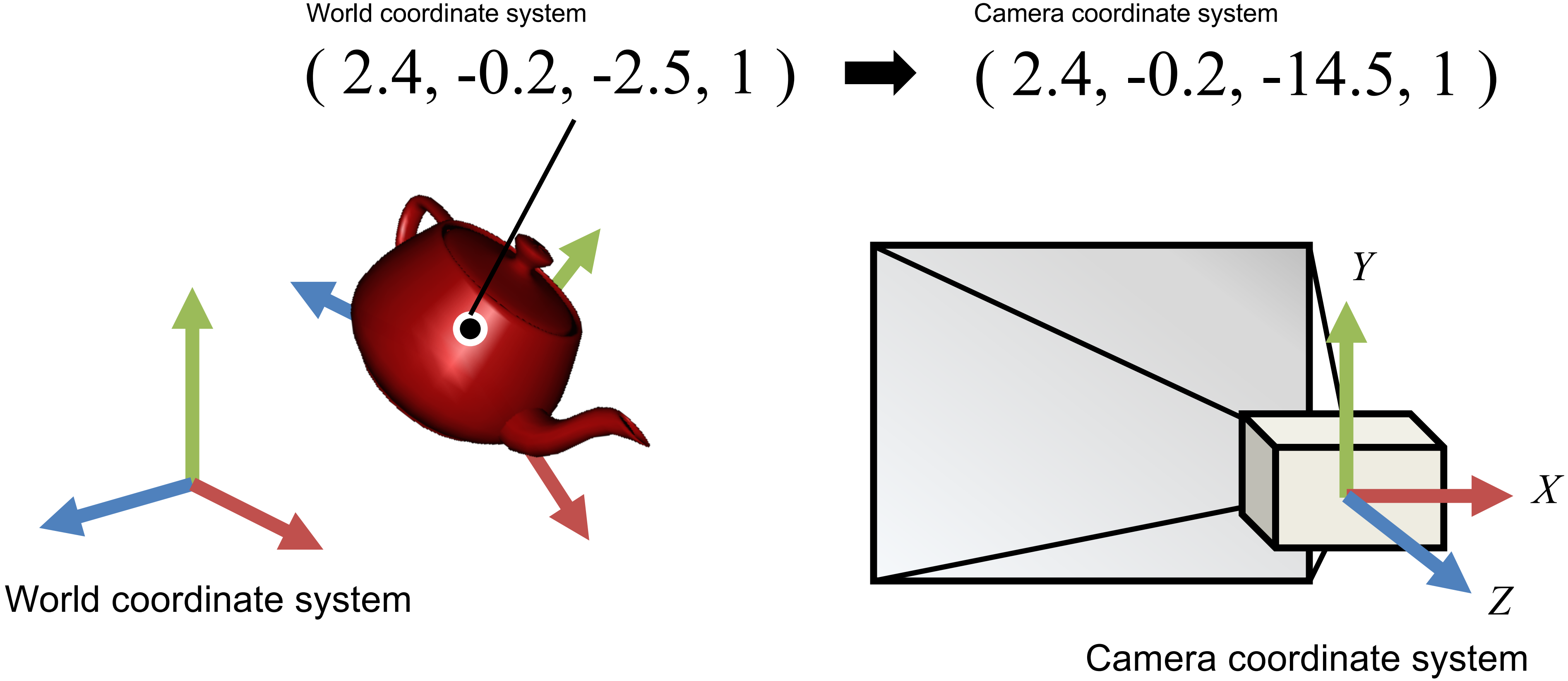
World Coordinate System

- A basic reference coordinate system for all objects.



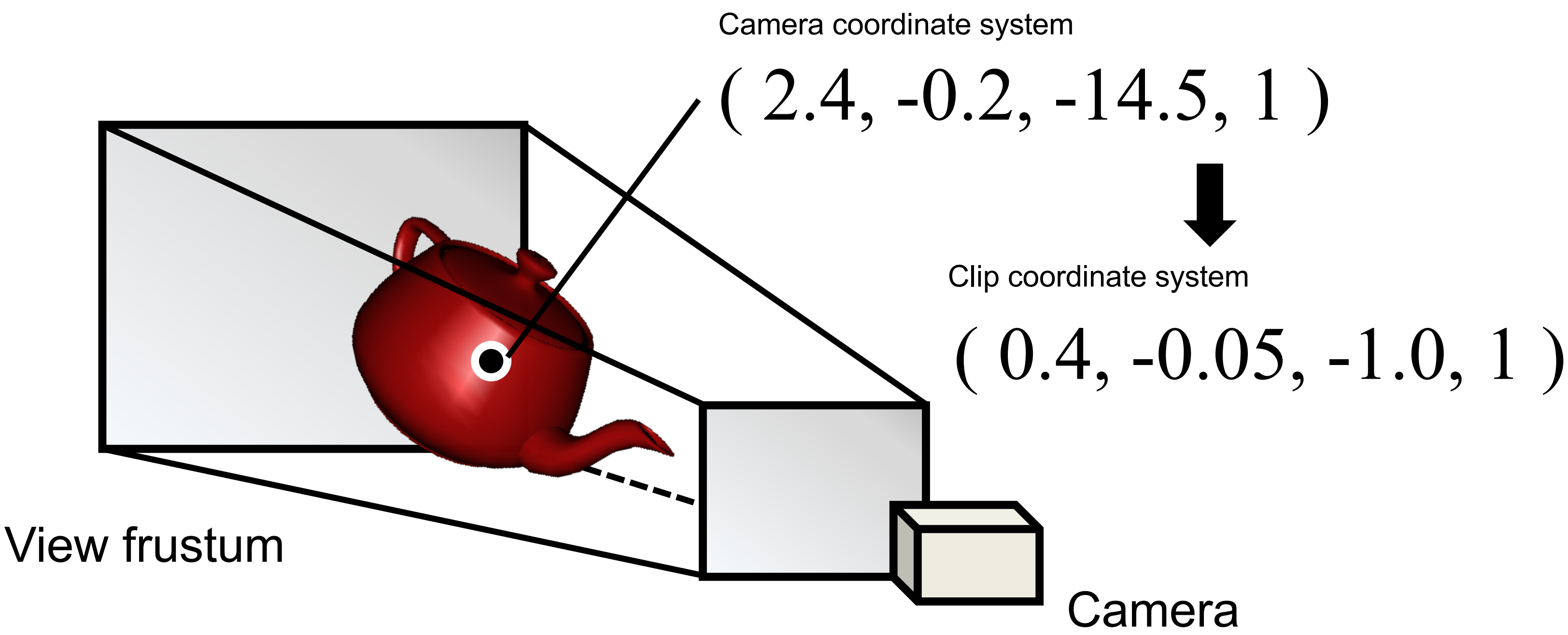
Camera Coordinate System

- A coordinate system on the basis of a camera for rendering the objects.



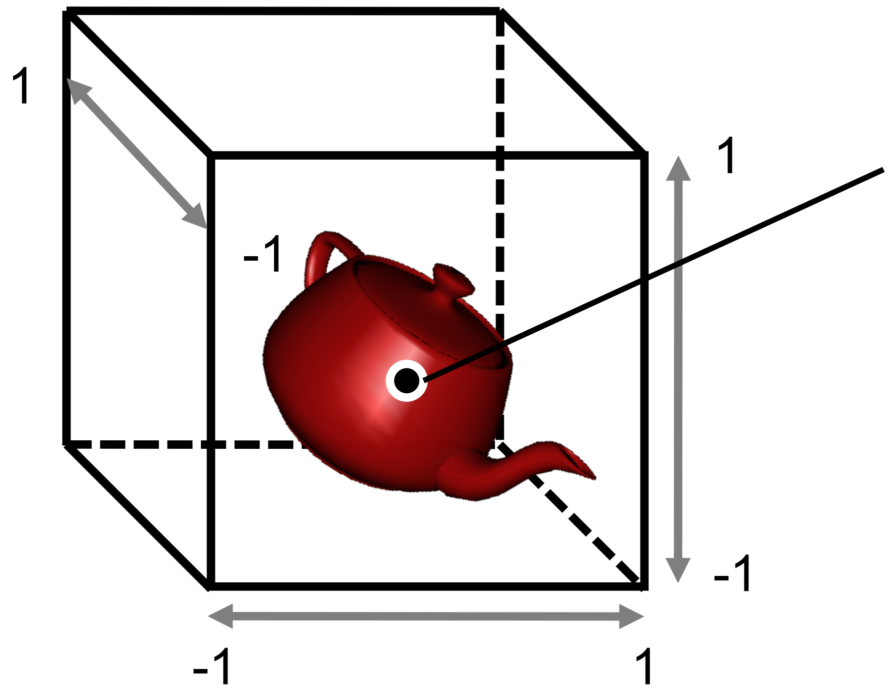
Clip Coordinate System

- A coordinate system to clip points in the camera coordinate system based on the view frustum.
  - Range: $-w \leqq x \leqq w$, $-w \leqq y \leqq w$, $-w \leqq z \leqq w$



Normalized Device Coordinate System

- A coordinate system yielded by dividing the clip coordinates by w.
  - Range: $-1 \leqq x \leqq 1$, $-1 \leqq y \leqq 1$, $-1 \leqq z \leqq 1$



Clip coordinate system

$(0.4, -0.05, -1.0, 1)$

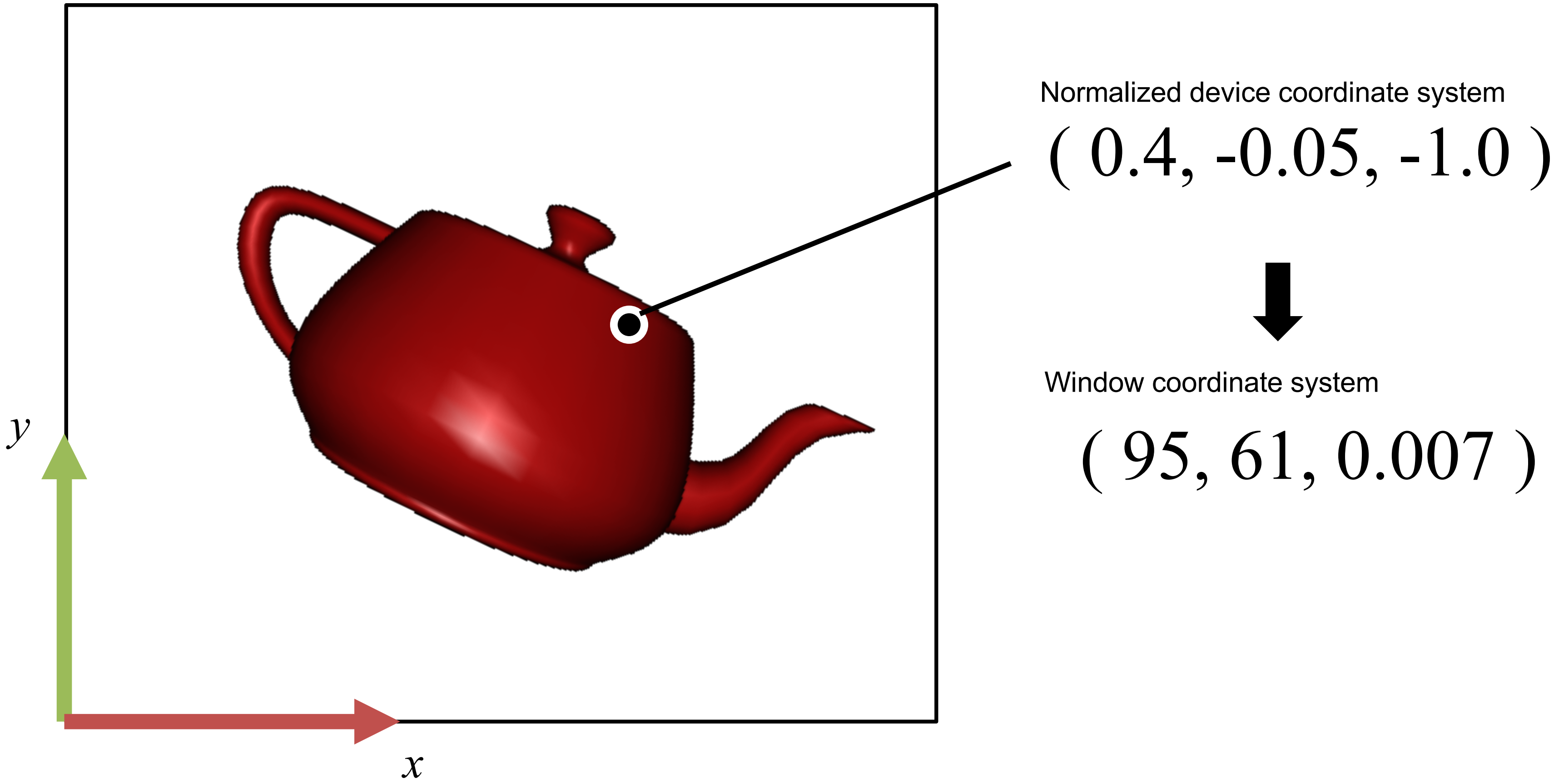
⬇

Normalized device coordinate system

$(0.4, -0.05, -1.0)$

Window Coordinate System

- A 2D coordinate system defined on a projection plane.



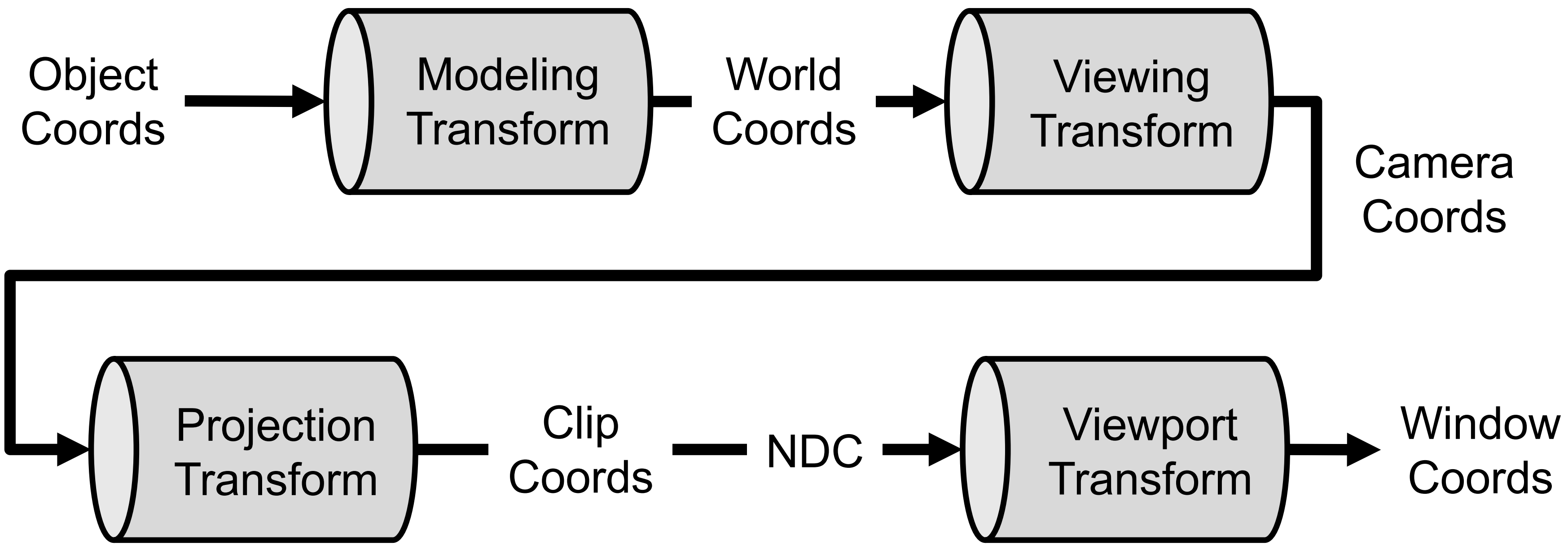
Coordinate Transformations

- 3D Objects are rendered through the following coordinate transformations

1. Modeling transformation
    - Object coordinates to world coordinates
2. Viewing transformation
    - World coordinates to camera coordinates
3. Projection transformation
    - Camera coordinates to clip coordinates (NDC)
4. Viewport transformation
    - NDC to window coordinates

Coordinate Transformations

- Transformation pipeline



Modeling Transformation

- Object coordinates to world coordinates
  - Scaling matrix $\boldsymbol{S}$
  - Translation matrix $\boldsymbol{T}$
  - Rotation matrix $\boldsymbol{R}$

$\left.\begin{matrix} \\ \\ \\ \end{matrix}\right\}$ Modeling matrix (Combined matrix) $\boldsymbol{M}_{\text{model}}$

$$\begin{bmatrix} x_{\text{world}} \\ y_{\text{world}} \\ z_{\text{world}} \\ w_{\text{world}} \end{bmatrix} = M_{\text{model}} \begin{bmatrix} x_{\text{obj}} \\ y_{\text{obj}} \\ z_{\text{obj}} \\ w_{\text{obj}} \end{bmatrix}$$

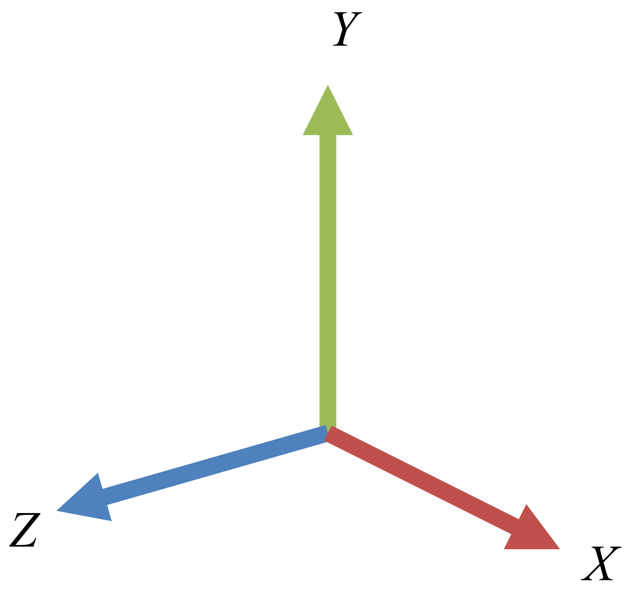
Viewing Transformation

- World coordinates to camera coordinates
  - Camera position and orientation in world coords

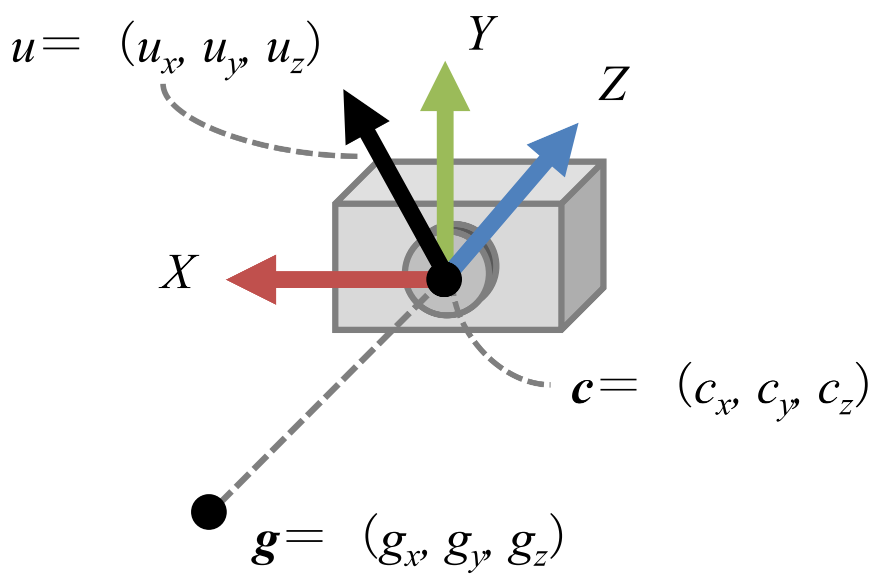
$$\begin{bmatrix} x_{\text{cam}} \\ y_{\text{cam}} \\ z_{\text{cam}} \\ w_{\text{cam}} \end{bmatrix} = M_{\text{view}} \begin{bmatrix} x_{\text{world}} \\ y_{\text{world}} \\ z_{\text{world}} \\ w_{\text{world}} \end{bmatrix}$$

Viewing Transformation

- Camera position $\boldsymbol{c} = (c_x, c_y, c_z)$
- Up vector $\boldsymbol{u} = (u_x, u_y, u_z)$
- Look-at point $\boldsymbol{g} = (g_x, g_y, g_z)$



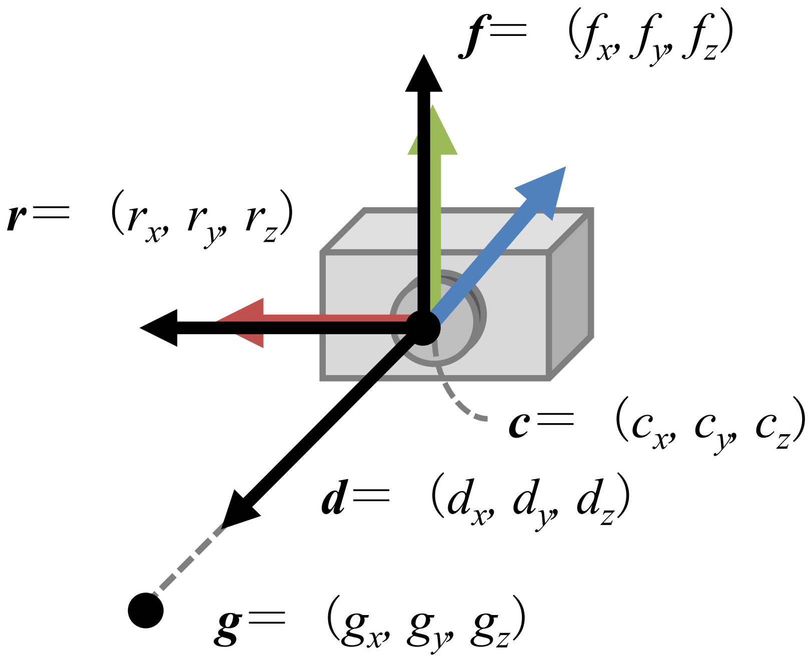
World coordinate system



Viewing Transformation

- Viewing matrix $\boldsymbol{M}_{\text{view}} = \boldsymbol{C}_{\text{dir}}\,\boldsymbol{C}_{\text{pos}}$
  - $\boldsymbol{C}_{\text{dir}}$: Camera direction matrix

$$C_{\text{dir}} = \begin{bmatrix} r_x & r_y & r_z & 0 \\ f_x & f_y & f_z & 0 \\ -d_x & -d_y & -d_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$$d = \frac{g-c}{|g-c|} \quad f = \frac{r \times d}{|r \times d|} \quad r = \frac{d \times u}{|d \times u|}$$

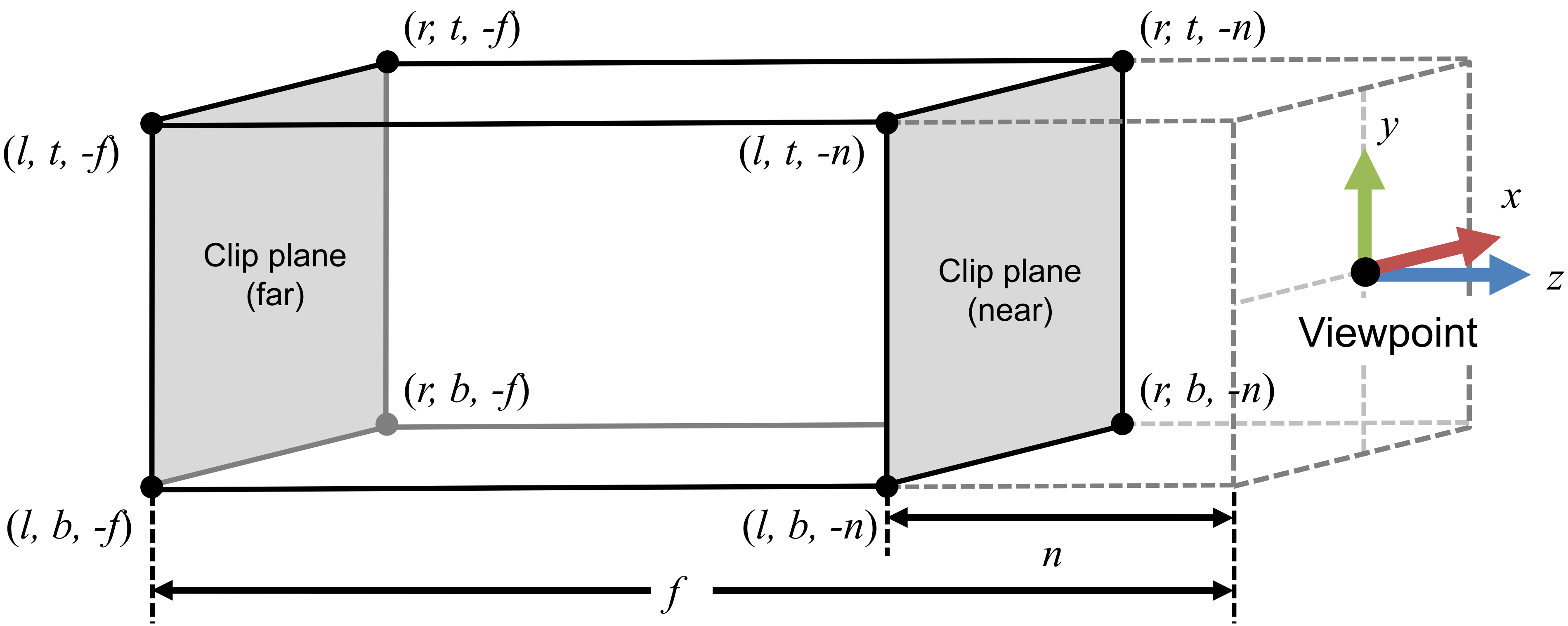
Projection Transformation

- Camera coordinates to clip coordinates (normalized device coordinates)
  - Orthogonal Projection
  - Perspective Projection

$$\begin{bmatrix} x_{\text{clip}} \\ y_{\text{clip}} \\ z_{\text{clip}} \\ w_{\text{clip}} \end{bmatrix} = M_{\text{proj}} \begin{bmatrix} x_{\text{cam}} \\ y_{\text{cam}} \\ z_{\text{cam}} \\ w_{\text{cam}} \end{bmatrix} \qquad \begin{bmatrix} x_{\text{NDC}} \\ y_{\text{NDC}} \\ z_{\text{NDC}} \end{bmatrix} = \begin{bmatrix} x_{\text{clip}} / w_{\text{clip}} \\ y_{\text{clip}} / w_{\text{clip}} \\ z_{\text{clip}} / w_{\text{clip}} \end{bmatrix}$$

Orthogonal Projection

- All projection lines are orthogonal to the projection plane.



Orthogonal Projection

- Orthogonal projection matrix $\boldsymbol{P}_{\text{orth}}$

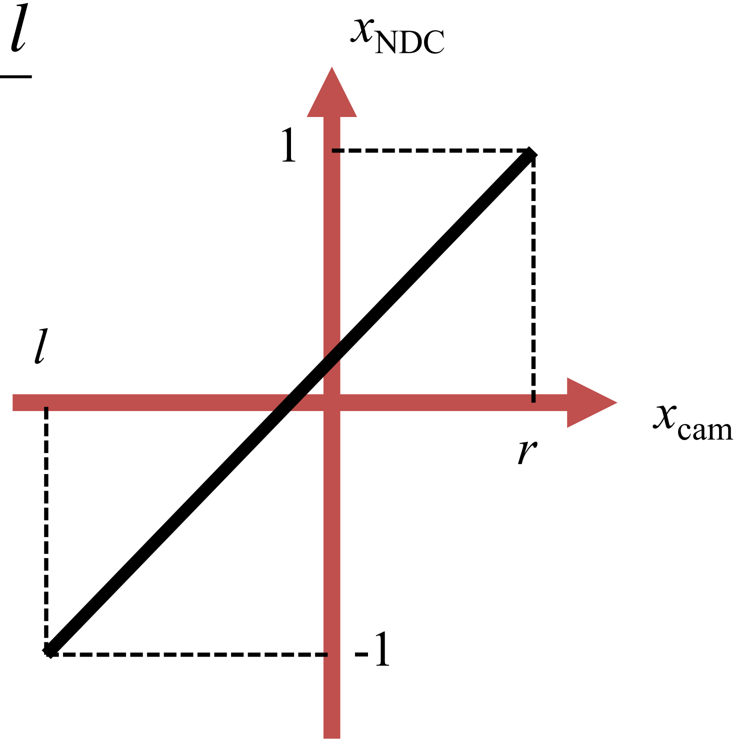
$$M_{\text{proj}} = P_{\text{orth}} = \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & -\frac{2}{f-n} & -\frac{f+n}{f-n} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Orthogonal Projection Matrix

- Mapping from xcam to xNDC
  - $[l, r]$ to $[-1, 1]$

$$x_{\text{NDC}} = \frac{1-(-1)}{r-l} x_{\text{cam}} + \frac{r \cdot (-1) - 1 \cdot l}{r-l}$$

$$= \frac{2}{r-l} x_{\text{cam}} - \frac{r+l}{r-l}$$

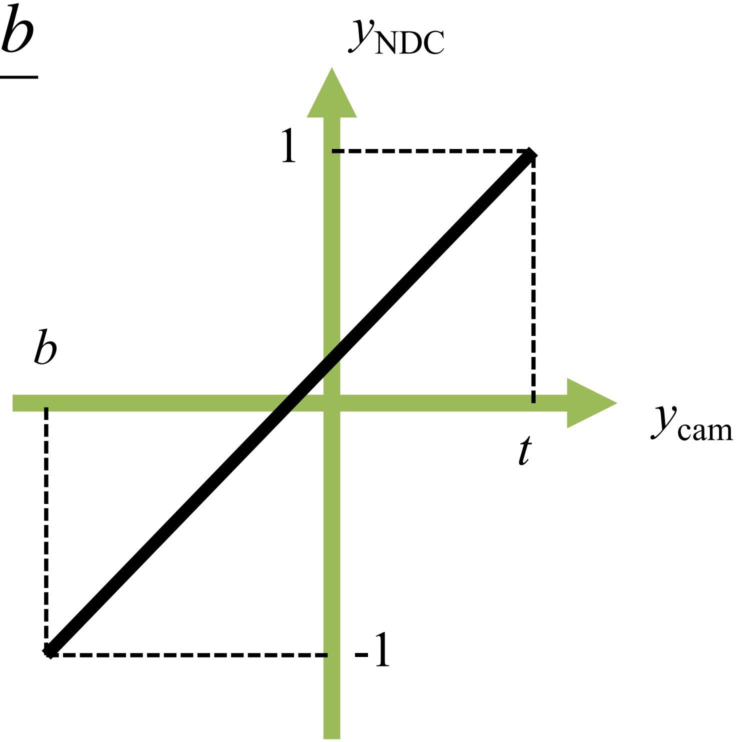


Orthogonal Projection Matrix

- Mapping from ycam to yNDC
  - $[b, t]$ to $[-1, 1]$

$$y_{\text{NDC}} = \frac{1-(-1)}{t-b} y_{\text{cam}} + \frac{t \cdot (-1) - 1 \cdot b}{t-b}$$

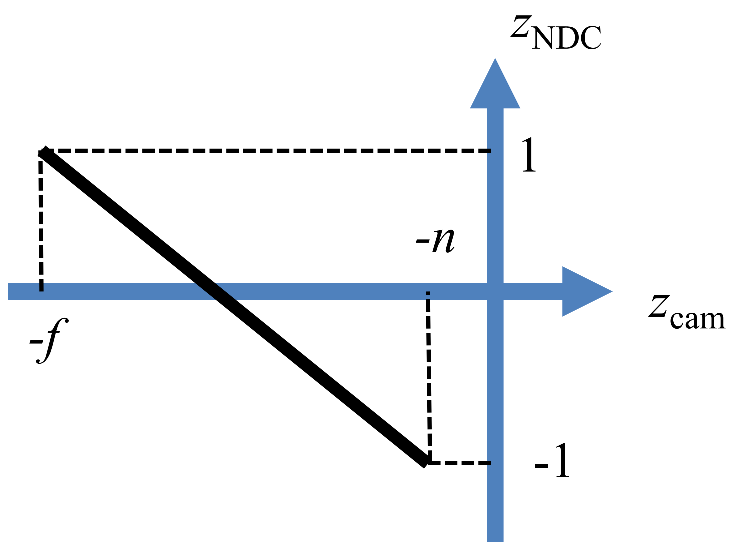
$$= \frac{2}{t-b} y_{\text{cam}} - \frac{t+b}{t-b}$$



Orthogonal Projection Matrix

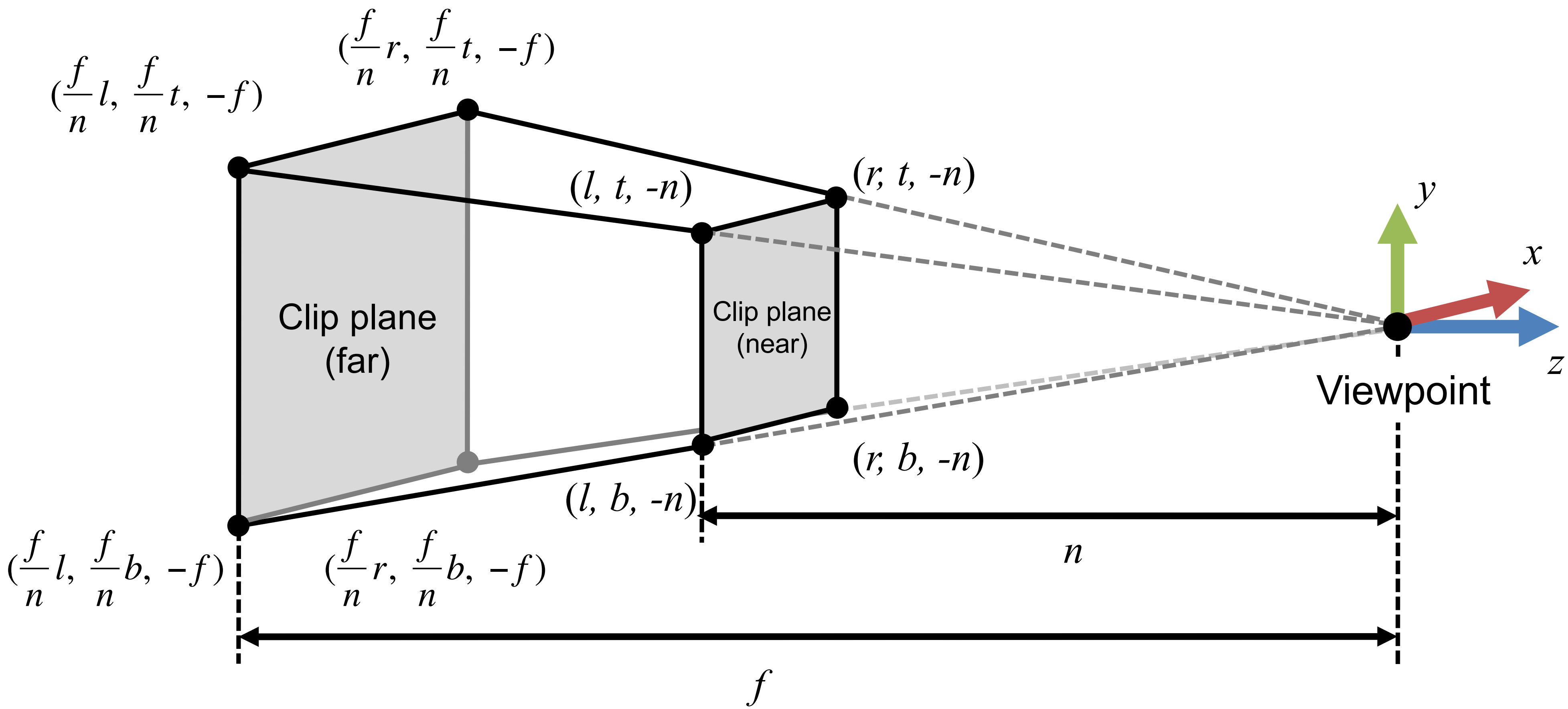
- Mapping from zcam to zNDC
  - $[-n, -f]$ to $[-1, 1]$

$$
\begin{aligned}
z_{\mathrm{NDC}} &= \frac{1-(-1)}{-f-(-n)} z_{\mathrm{cam}} + \frac{(-f)\cdot(-1) - 1\cdot(-n)}{-f-(-n)} \\
&= -\frac{2}{f-n} z_{\mathrm{cam}} - \frac{f+n}{f-n}
\end{aligned}
$$



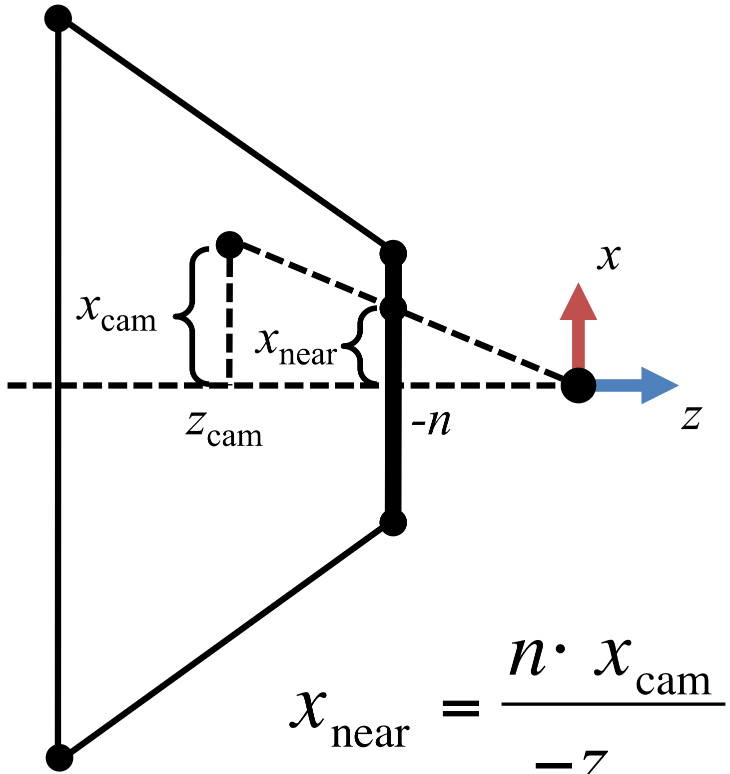
Perspective Projection

- Perspective projection has a center of projection (viewpoint)

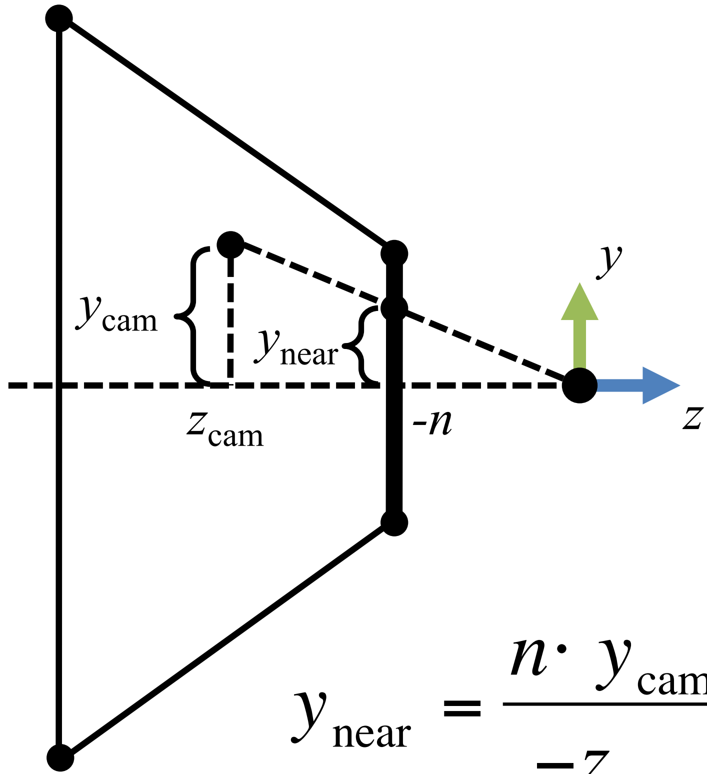


Perspective Projection Matrix

- Projected point $(x_{\text{near}}, y_{\text{near}})$ onto the near clipping plane



$$x_{\text{near}} = \frac{n \cdot x_{\text{cam}}}{-z_{\text{cam}}}$$



$$y_{\text{near}} = \frac{n \cdot y_{\text{cam}}}{-z_{\text{cam}}}$$

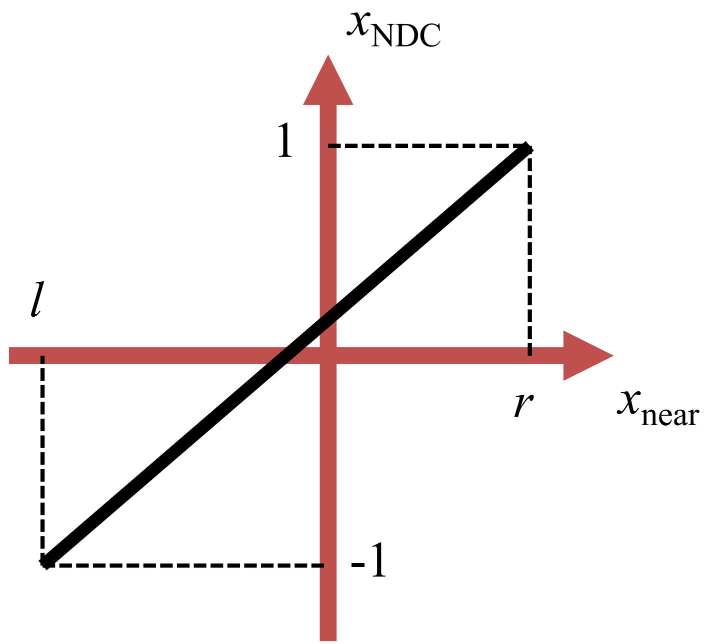
Perspective Projection

- Perspective projection matrix $\boldsymbol{P}_{\text{pers}}$

$$M_{\text{proj}} = P_{\text{pers}} = \begin{bmatrix} \frac{2n}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ 0 & \frac{2n}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & -\frac{f+n}{f-n} & -\frac{2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

Perspective Projection Matrix

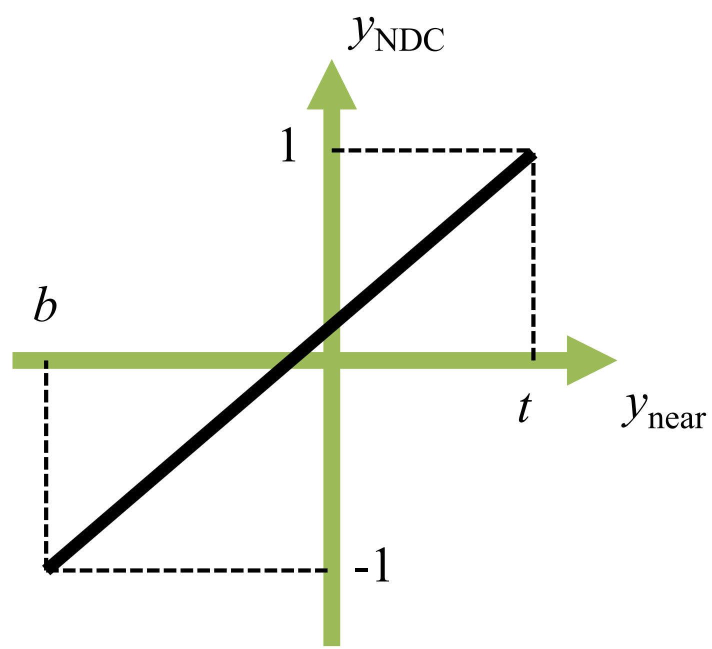
- Mapping from xnear to xNDC
  - $[l, r]$ to $[-1,1]$



$$
\begin{aligned}
x_{\text{NDC}} &= \frac{2}{r-l}x_{\text{near}} - \frac{r+l}{r-l} \\
&= \frac{2}{r-l}\cdot\frac{n\cdot x_{\text{cam}}}{-z_{\text{cam}}} - \frac{r+l}{r-l} \\
&= \left(\frac{2n}{r-l}x_{\text{cam}} + \frac{r+l}{r-l}z_{\text{cam}}\right)\Big/ -z_{\text{cam}}
\end{aligned}
$$

Perspective Projection Matrix

- Mapping from ynear to yNDC
  - $[b, t]$ to $[-1, 1]$



$$
\begin{aligned}
y_{\text{NDC}} &= \frac{2}{t-b} y_{\text{near}} - \frac{t+b}{t-b} \\
&= \frac{2}{t-b} \cdot \frac{n \cdot y_{\text{cam}}}{-z_{\text{cam}}} - \frac{t+b}{t-b} \\
&= \left(\frac{2n}{t-b} x_{\text{cam}} + \frac{t+b}{t-b} z_{\text{cam}} \right) \Big/ -z_{\text{cam}}
\end{aligned}
$$

Perspective Projection Matrix

- Mapping from zcam to zNDC
  - $[-n, -f]$ to $[-1, 1]$
  - The projection does not depend on xcam and ycam.
  - Solve for A and B in the following matrix.

$$\begin{bmatrix} x_{clip} \\ y_{clip} \\ z_{clip} \\ w_{clip} \end{bmatrix} = \begin{bmatrix} \frac{2n}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ 0 & \frac{2n}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & A & B \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_{cam} \\ y_{cam} \\ z_{cam} \\ w_{cam} \end{bmatrix} \Rightarrow$$

$$\begin{aligned} z_{clip} &= Az_{cam} + Bw_{cam} \\ w_{clip} &= -z_{cam} \\ \therefore z_{NDC} &= \frac{z_{clip}}{w_{clip}} = \frac{Az_{cam} + Bw_{cam}}{-z_{cam}} \end{aligned}$$

Perspective Projection Matrix

- $w_{\text{cam}}=1$
- $(z_{\text{cam}}, z_{\text{NDC}})=(-n, -1), (-f, 1)$

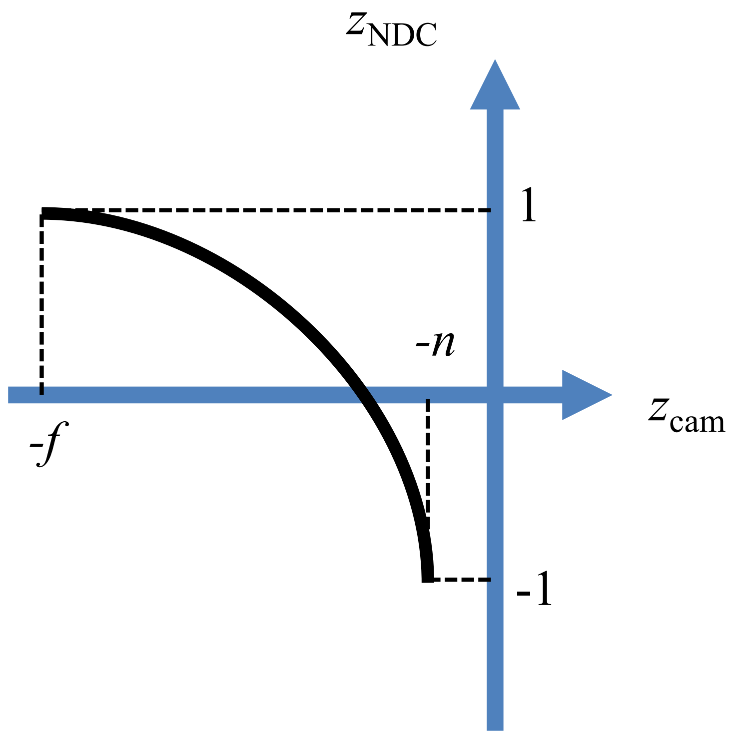
$$\begin{cases} \dfrac{-An+B}{n} & = & -1 \\ \dfrac{-Af+B}{f} & = & 1 \end{cases} \Rightarrow \begin{cases} A & = & -\dfrac{f+n}{f-n} \\ B & = & -\dfrac{2fn}{f-n} \end{cases}$$

$$\therefore z_{NDC} = \frac{Az_{cam} + Bw_{cam}}{-z_{cam}} = \frac{-\dfrac{f+n}{f-n} z_{cam} - \dfrac{2fn}{f-n}}{-z_{cam}}$$

z_{NDC} and z_{cam}

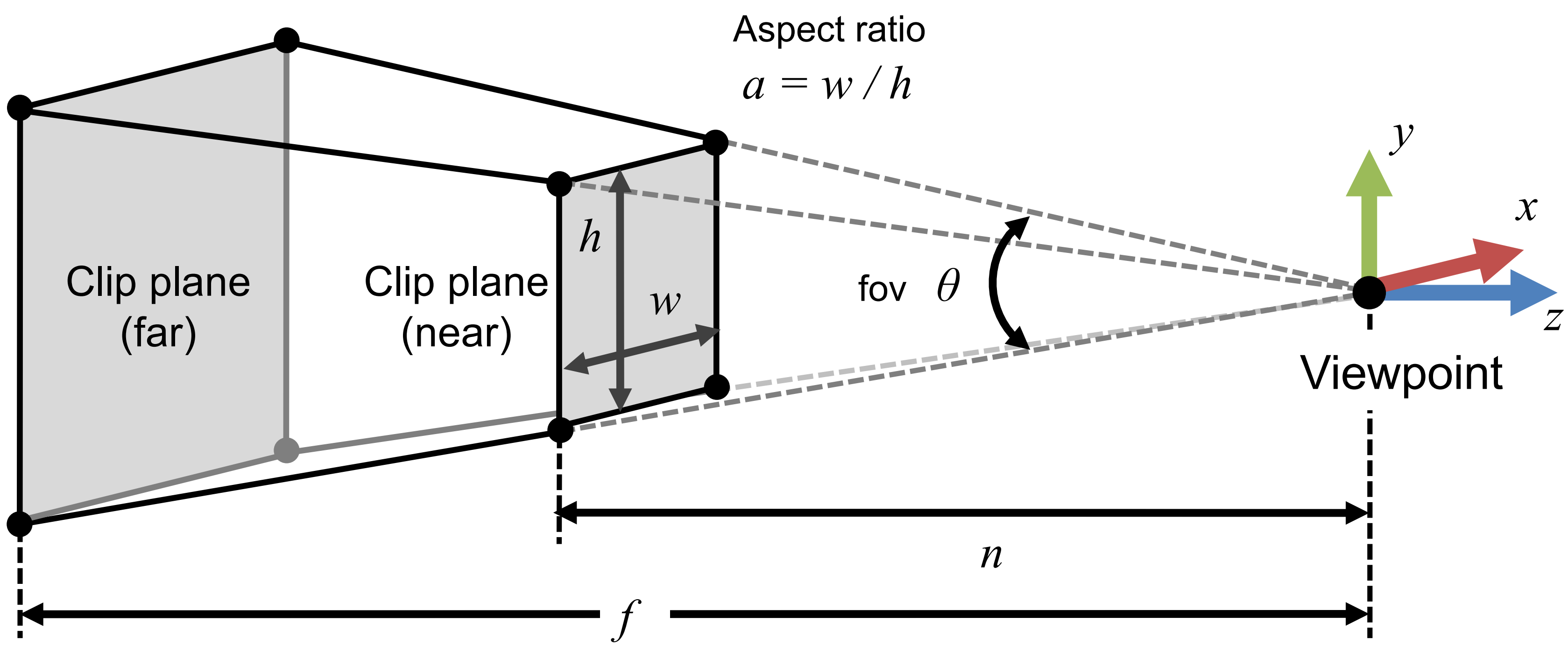
- Nonlinear mapping of z_{cam} to z_{NDC}

$$
\begin{aligned}
z_{\mathrm{NDC}} &= \frac{Az_{\mathrm{cam}} + Bw_{\mathrm{cam}}}{-z_{\mathrm{cam}}} \\
&= \frac{-\dfrac{f+n}{f-n} z_{\mathrm{cam}} - \dfrac{2fn}{f-n}}{-z_{\mathrm{cam}}} \\
&= \frac{2fn}{f-n}\frac{1}{z_{\mathrm{cam}}} + \frac{f+n}{f-n}
\end{aligned}
$$



Perspective Projection with Field-of-View

- The parameters in the perspective projection matrix can be reduced by representing it with an angle θ called as field-of-view (fov).



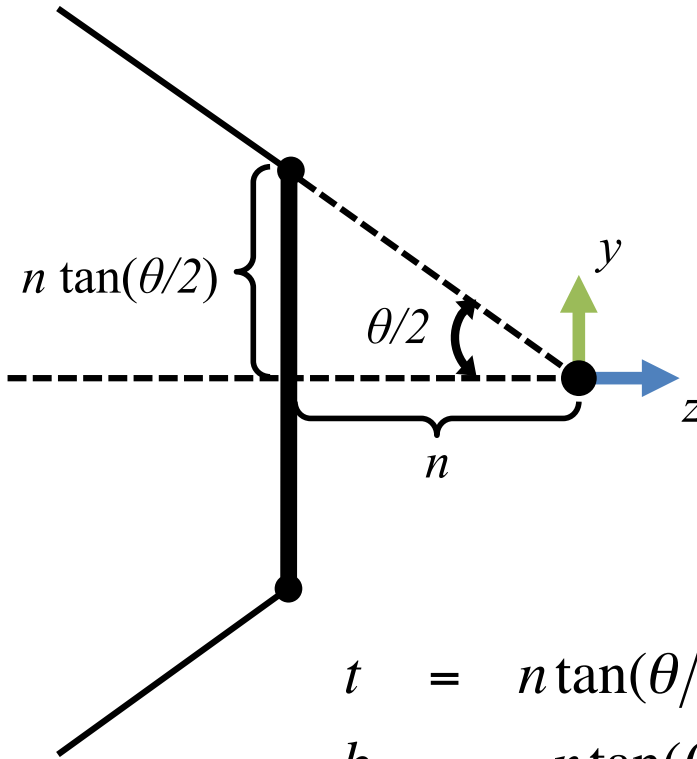
Perspective Projection with Filed-of-View

- Perspective projection matrix $\boldsymbol{P}_{\text{pers}}$

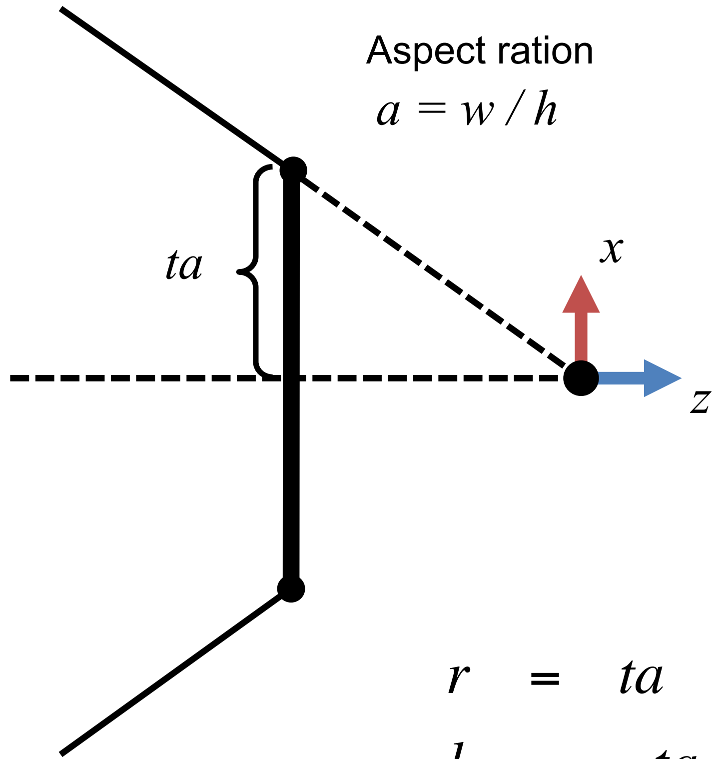
$$M_{\text{proj}} = P_{\text{pers}} = \begin{bmatrix} \dfrac{1}{a\tan(\theta/2)} & 0 & 0 & 0 \\ 0 & \dfrac{1}{\tan(\theta/2)} & 0 & 0 \\ 0 & 0 & -\dfrac{f+n}{f-n} & -\dfrac{2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

Perspective Projection Matrix with Field-of-View

- The near clipping plane represented with (r, l, t, b) is calculated as follows:



$$
\begin{aligned}
t &= n\tan(\theta/2) \\
b &= -n\tan(\theta/2)
\end{aligned}
$$



$$
\begin{aligned}
r &= ta \\
l &= -ta
\end{aligned}
$$

Viewport Transformation

- Normalized device coordinates (NDC) to window coordinates

$$\begin{bmatrix} x_{\text{win}} \\ y_{\text{win}} \\ z_{\text{win}} \\ 1 \end{bmatrix} = M_{\text{viewport}} \begin{bmatrix} x_{\text{NDC}} \\ y_{\text{NDC}} \\ z_{\text{NDC}} \\ 1 \end{bmatrix}$$

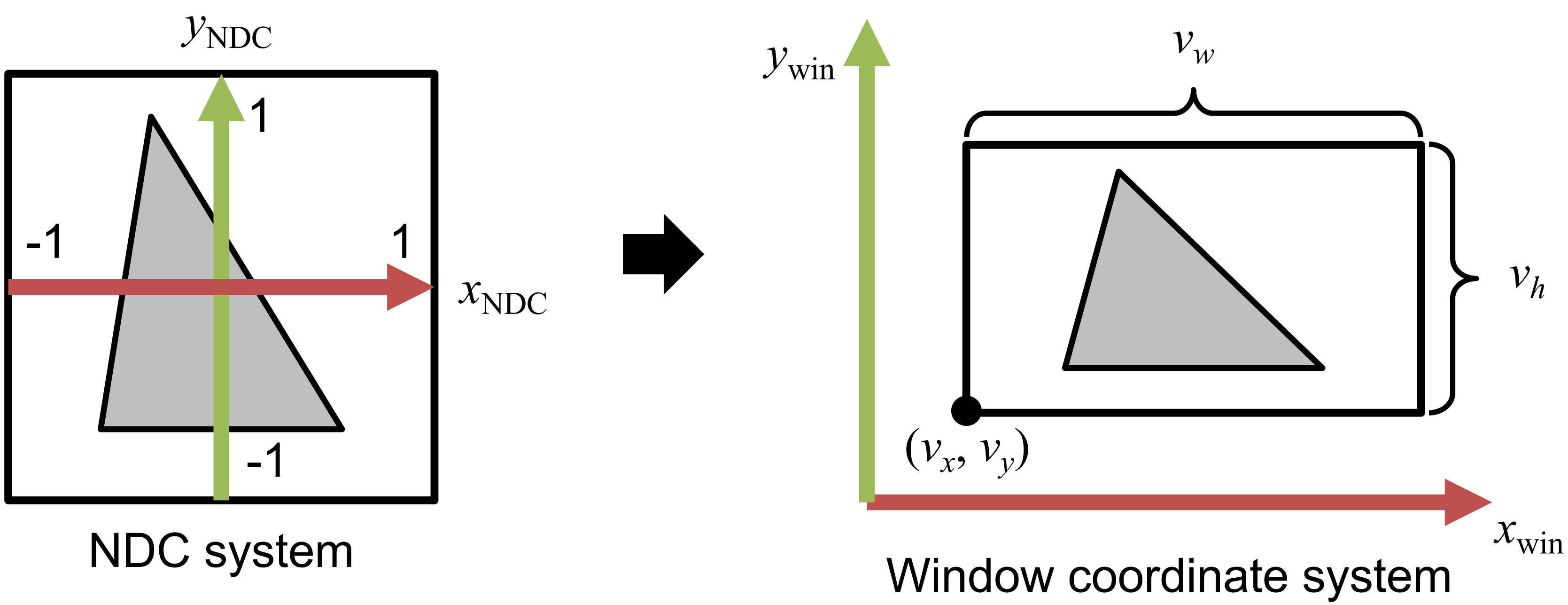
Viewport Transformation

- Viewport matrix M_{viewport}

$$M_{\text{viewport}} = \begin{bmatrix} \frac{v_w}{2} & 0 & 0 & v_x + \frac{v_w}{2} \\ 0 & \frac{v_h}{2} & 0 & v_y + \frac{v_h}{2} \\ 0 & 0 & \frac{d_f - d_n}{2} & \frac{d_f + d_n}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Viewport Transformation

- Viewport (v_x, v_y, v_w, v_h)
- Range of depth value $[d_n, d_f]$

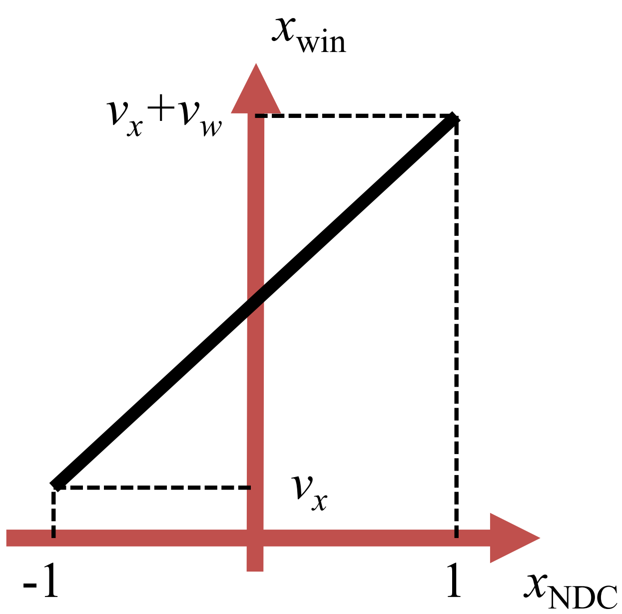


Viewport Matrix

- Mapping from xNDC to xwin
  - [-1, 1] to [v_x, v_x+v_w]

$$x_{\text{win}} = \frac{(v_x + v_w) - v_x}{1-(-1)} x_{\text{NDC}} + \frac{1\cdot v_x - (-1)\cdot (v_x + v_w)}{1-(-1)}$$

$$= \frac{v_w}{2} x_{\text{NDC}} + \left(v_x + \frac{v_w}{2} \right)$$

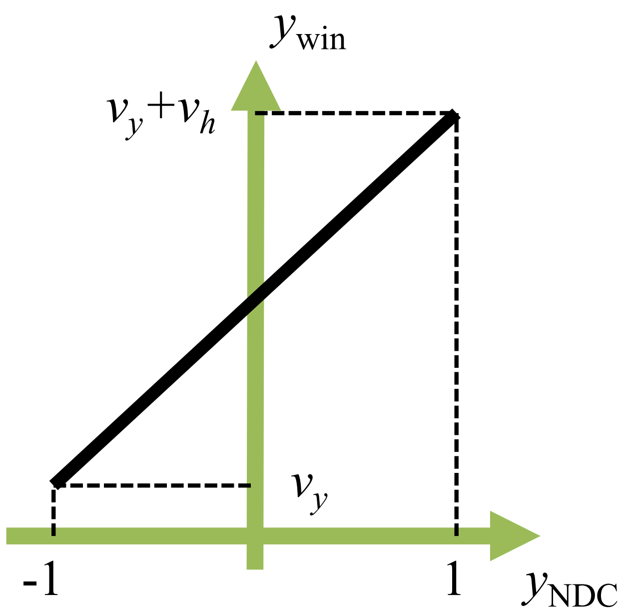


Viewport Matrix

- Mapping from yNDC to ywin
  - [-1, 1] to $[v_y, v_y+v_h]$

$$y_{\text{win}} = \frac{(v_y + v_h) - v_y}{1-(-1)} y_{\text{NDC}} + \frac{1 \cdot v_y - (-1) \cdot (v_y + v_h)}{1-(-1)}$$

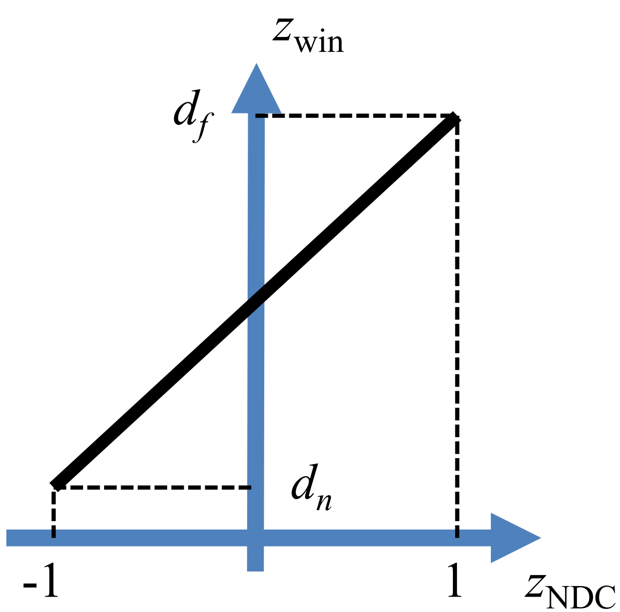
$$= \frac{v_h}{2} y_{\text{NDC}} + \left(v_y + \frac{v_h}{2}\right)$$



Viewport Matrix

- Mapping from zNDC to zwin
  - $[-1, 1]$ to $[d_n, d_f]$

$$
\begin{aligned}
z_{\text{win}} &= \frac{d_f - d_n}{1-(-1)} z_{\text{NDC}} + \frac{1\cdot d_n - (-1)\cdot d_f}{1-(-1)} \\
&= \frac{d_f - d_n}{2} z_{\text{NDC}} + \frac{d_f + d_n}{2}
\end{aligned}
$$



Polling

- Take the poll
  - Student ID Number
  - Name